

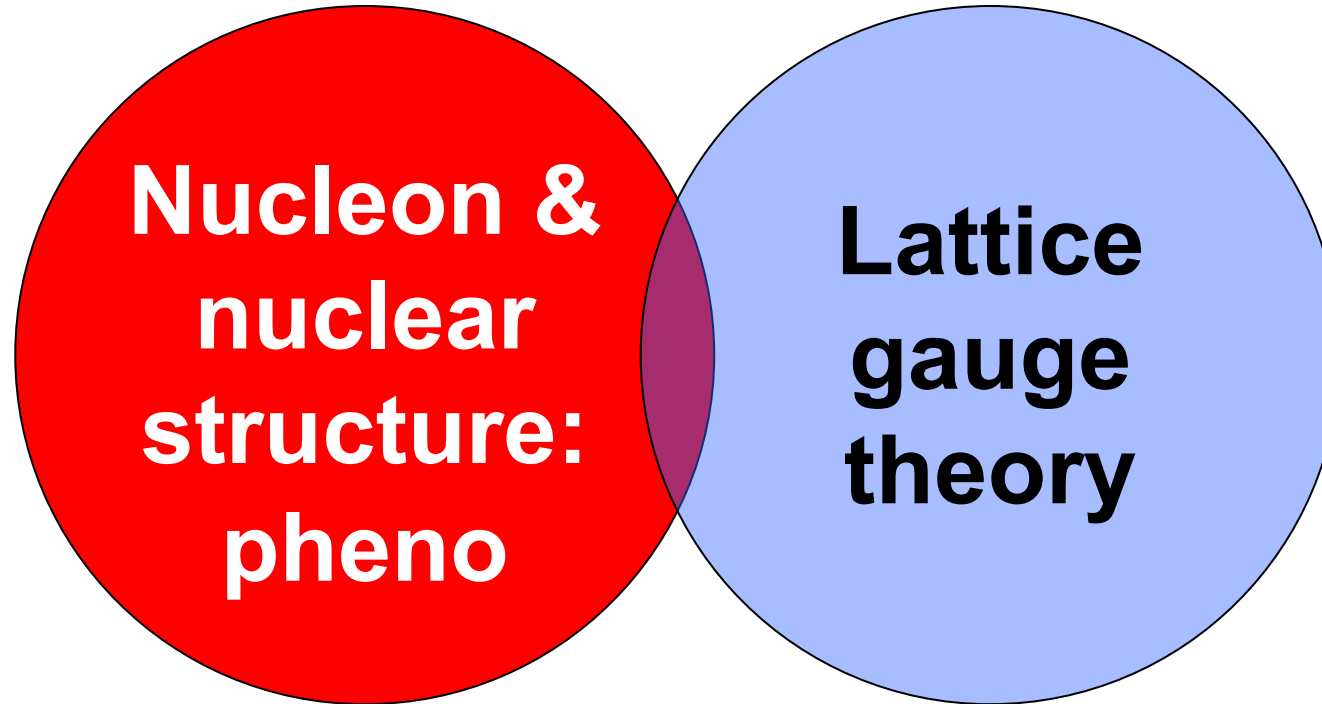
# Summary of workshop 2: *QCD analysis of nucleon structure*





# Topics here (and in EIC era!)

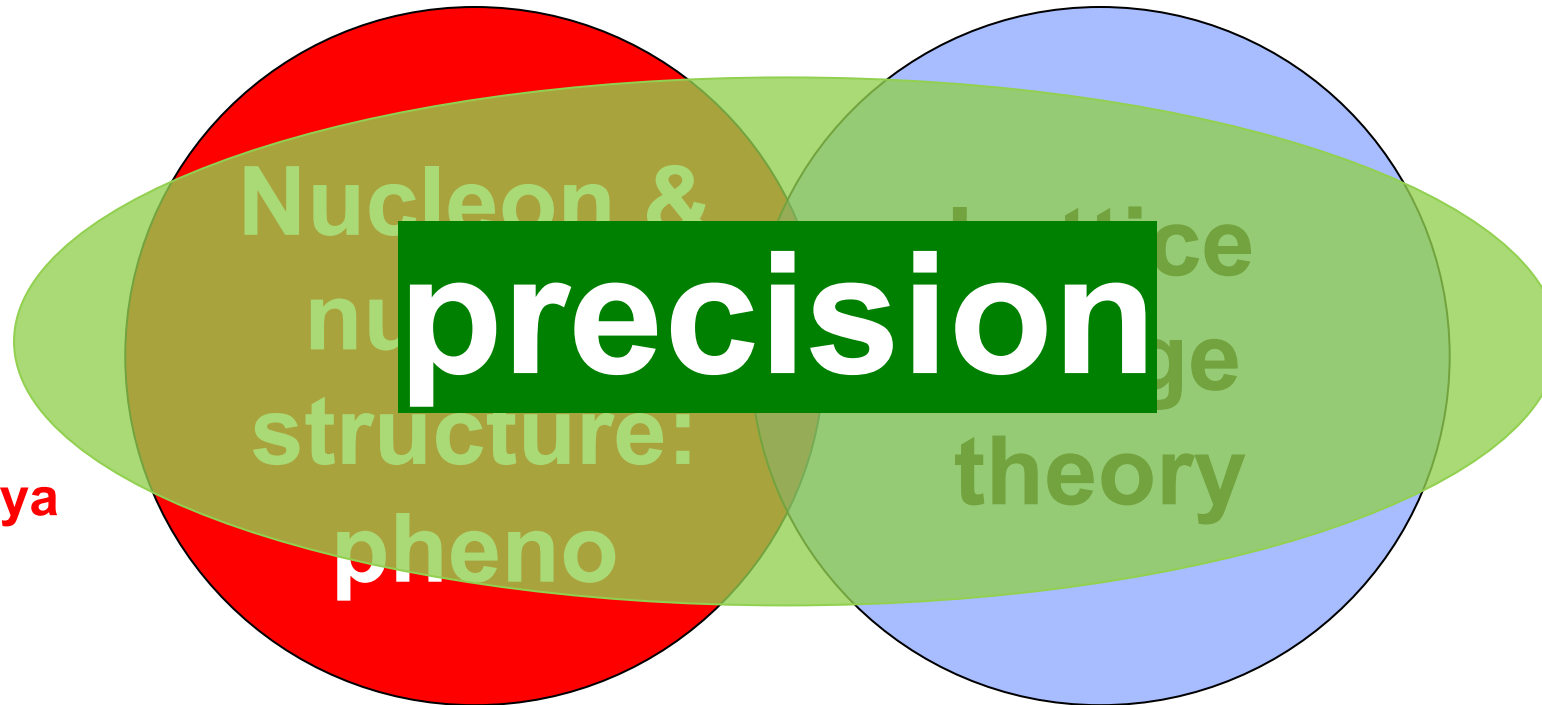
**Moos  
Schlegel  
Gamberg  
Zurita  
Borsa  
Magni  
Bertone  
Braun  
Bhattacharya  
Pedron  
Tomalak  
Hobart**



**Bacchio  
Koutsou  
Pefkou  
Zhang  
Mukherjee  
Constantinou  
Pittler  
Li**

# Topics here (and in EIC era!)

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# Disclaimers:

- “personal highlights”: will be selective  
-- apologies for talks not discussed in detail
- will make no attempt to do justice to wider literature (not a review of this field...)
- excellent introductions already by plenary talks



$$\mathcal{W}(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$\int d^2 b_\perp$$

$$\int d^2 k_\perp$$

$$f(x, \vec{k}_\perp)$$

$$F(x, \vec{b}_\perp)$$

**TMD**

**“impact par. PDF”**

$$\int d^2 k_\perp$$

$$\int d^2 b_\perp$$

$$\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$$

$$q(x)$$

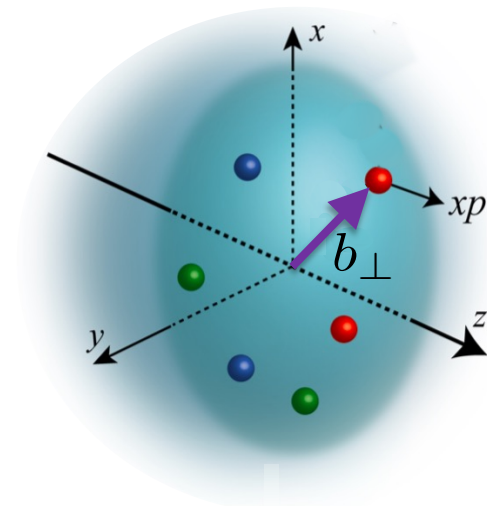
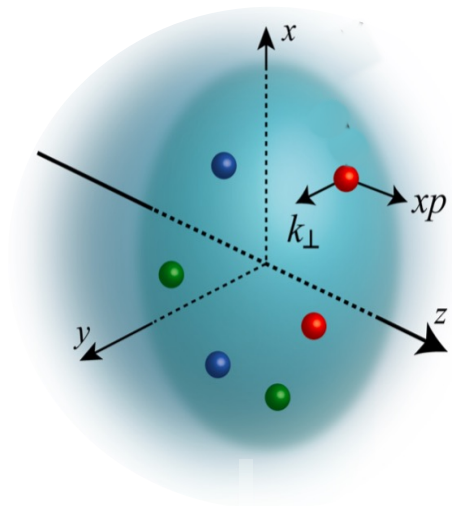
**PDF**

**GPD**

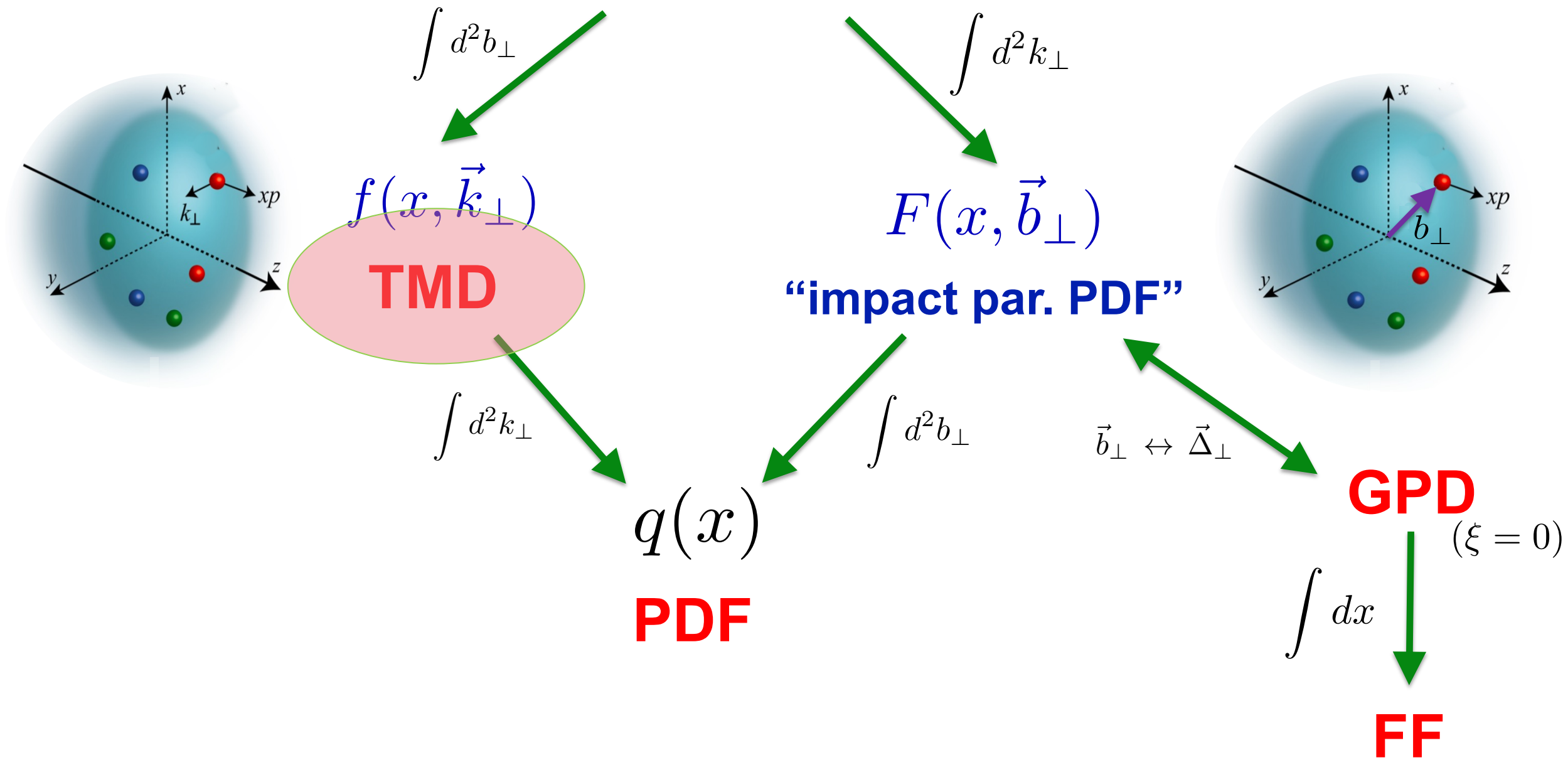
( $\xi = 0$ )

$$\int dx$$

**FF**



$$\mathcal{W}(x, \vec{b}_\perp, \vec{k}_\perp)$$





|       |   |   |   |
|-------|---|---|---|
| N \ q | U | L | T |
| U     |   |   |   |
| L     |   |   |   |
| T     |   |   |   |

**TMDPDF**

**CS**

$$W_{f_1 f_1}^f(Q, q_T; x_1, x_2) \sim \int_0^\infty db b J_0(bq_T) f_{1, f \leftarrow h}(x_1, b) f_{1, \bar{f} \leftarrow h}(x_2, b) \left( \frac{Q^2}{\zeta_\mu(b)} \right)^{-2\mathcal{D}(b, \mu)}$$

$$\mu^2 \frac{d}{d\mu^2} f_{1, q \leftarrow h}(x, b; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} f_{1, q \leftarrow h}(x, b; \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} f_{1, q \leftarrow h}(x, b; \mu, \zeta) = -\mathcal{D}(b, \mu) f_{1, q \leftarrow h}(x, b; \mu, \zeta)$$

$$-\zeta \frac{d\gamma_F(\mu, \zeta)}{d\zeta} = \mu \frac{d\mathcal{D}(b, \mu)}{d\mu} = \Gamma_{\text{cusp}}(\mu)$$

# FLAVOUR DEPENDENCE

P. Zurita

# OF TMDs

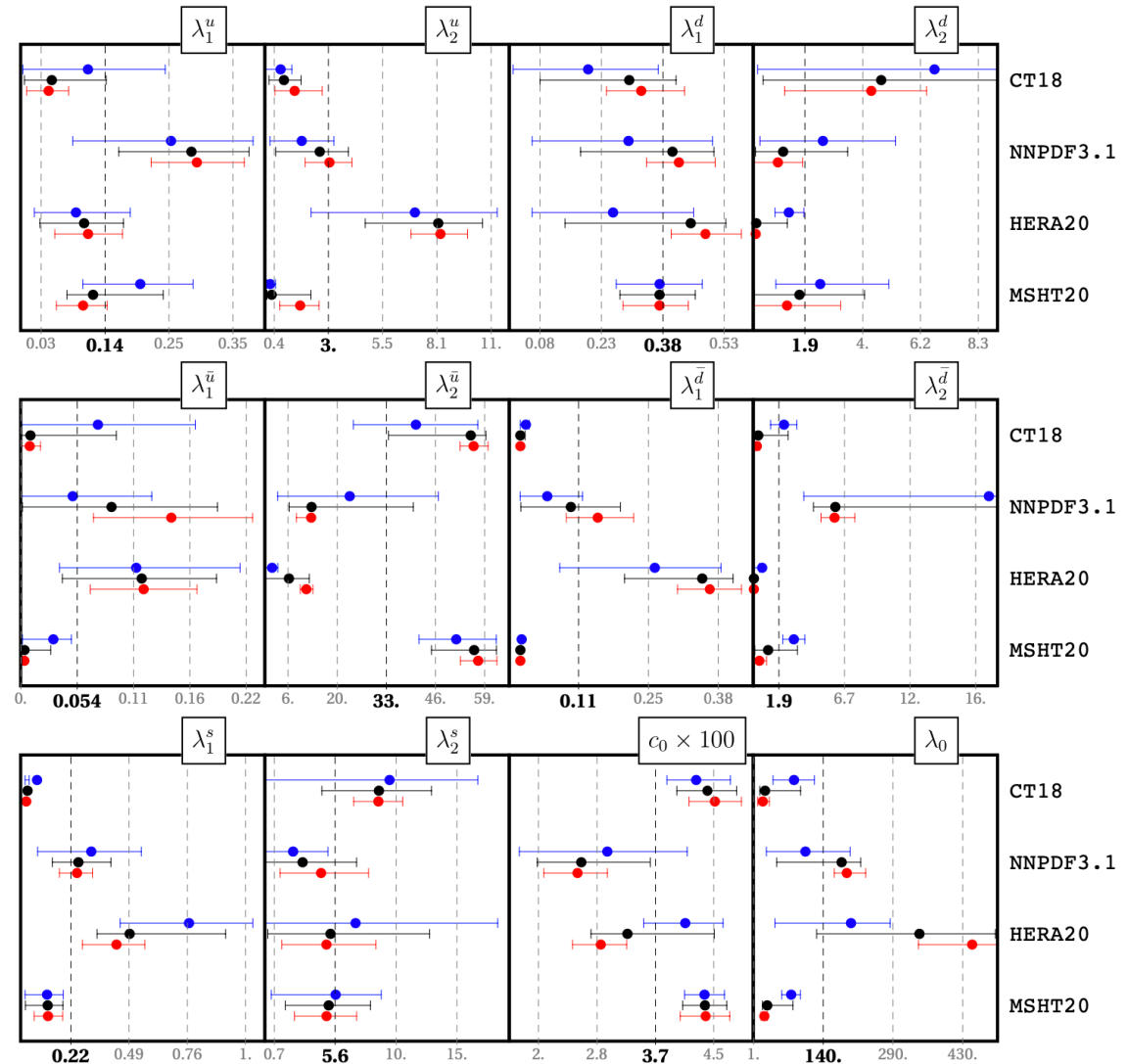
P. Zurita

$$f_{1,f}(x, b) = \int_x^1 \frac{dy}{y} \sum_{f'} C_{f \rightarrow f'}(y, \mathbf{L}, a_s) q_{f'}\left(\frac{x}{y}\right) f_{NP}^f(x, b)$$

$$f_{NP}^f(x, b) = \exp\left(-\frac{\lambda_1^f(1-x) + \lambda_2^f x}{\sqrt{1 + \lambda_0 x^2} \mathbf{b}^2}\right)$$

$$f = u, \bar{u}, d, \bar{d}, sea$$

**Black:** final result.

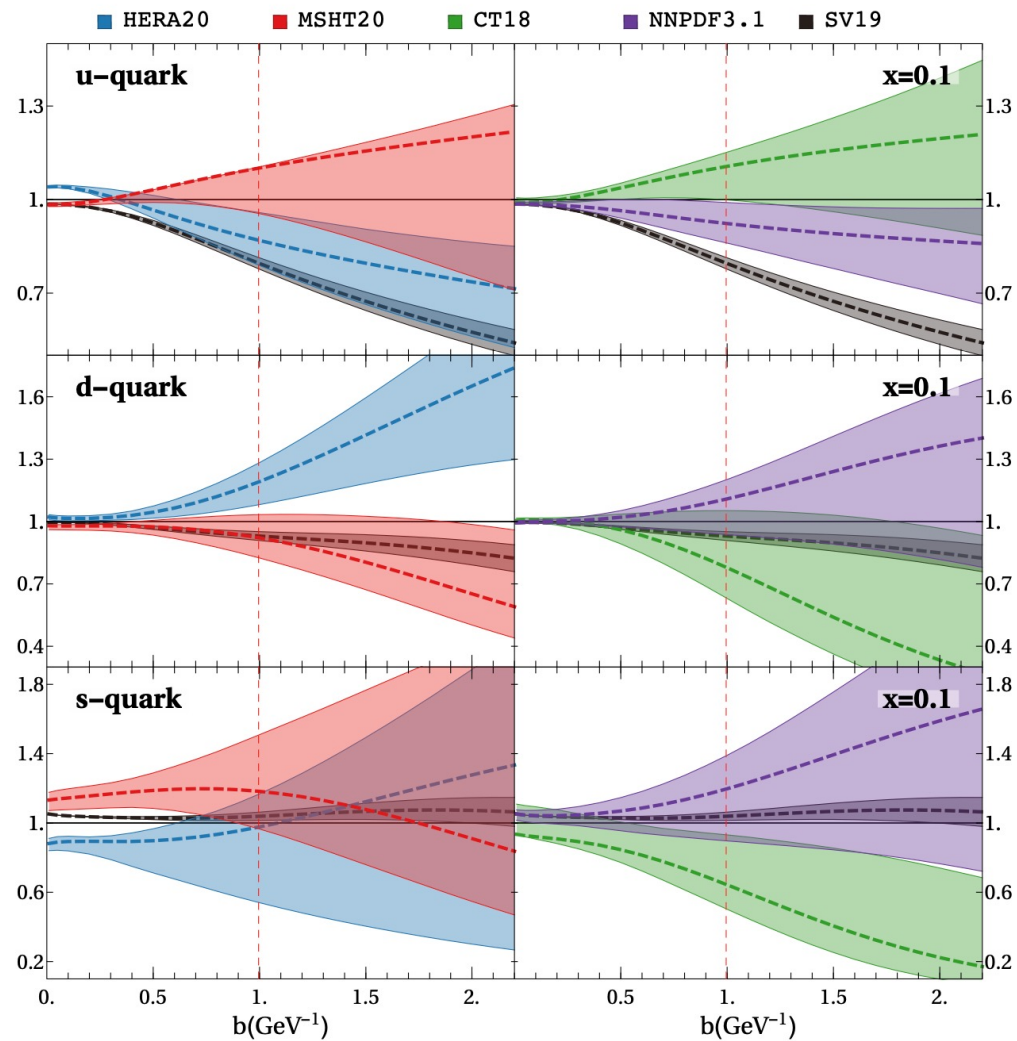






We obtain more realistic uncertainty bands for the TMDPDFs:

P. Zurita

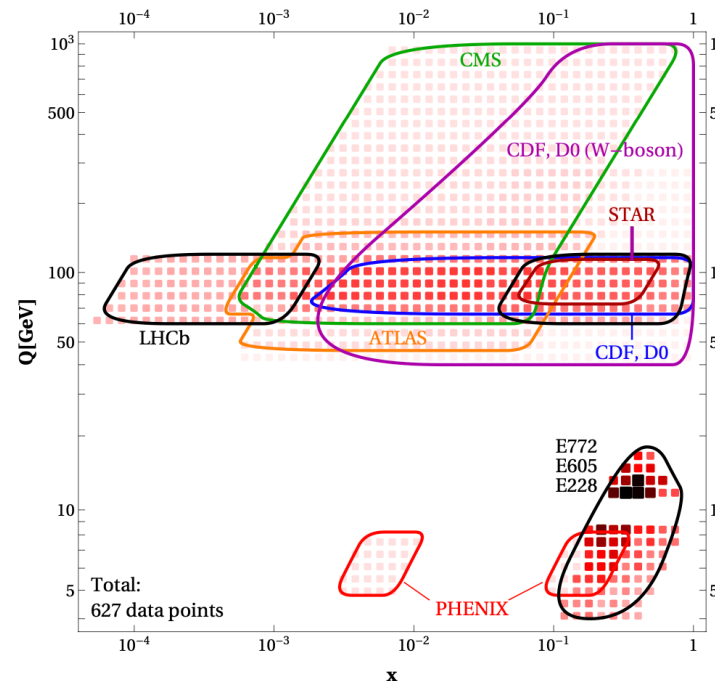


# Now full analysis at N<sup>4</sup>LL:

$$f_{1,f}(x, b) = \int_x^1 \frac{dy}{y} \sum_{f'} C_{f \rightarrow f'}(y, \mathbf{L}, a_s) q_{f'} \left( \frac{x}{y} \right) f_{\text{NP}}^f(x, b)$$

$$f_{\text{NP}}^f(x, b) = \frac{1}{\cosh \left( \left( \lambda_1^f (1-x) + \lambda_2^f x \right) b \right)}$$

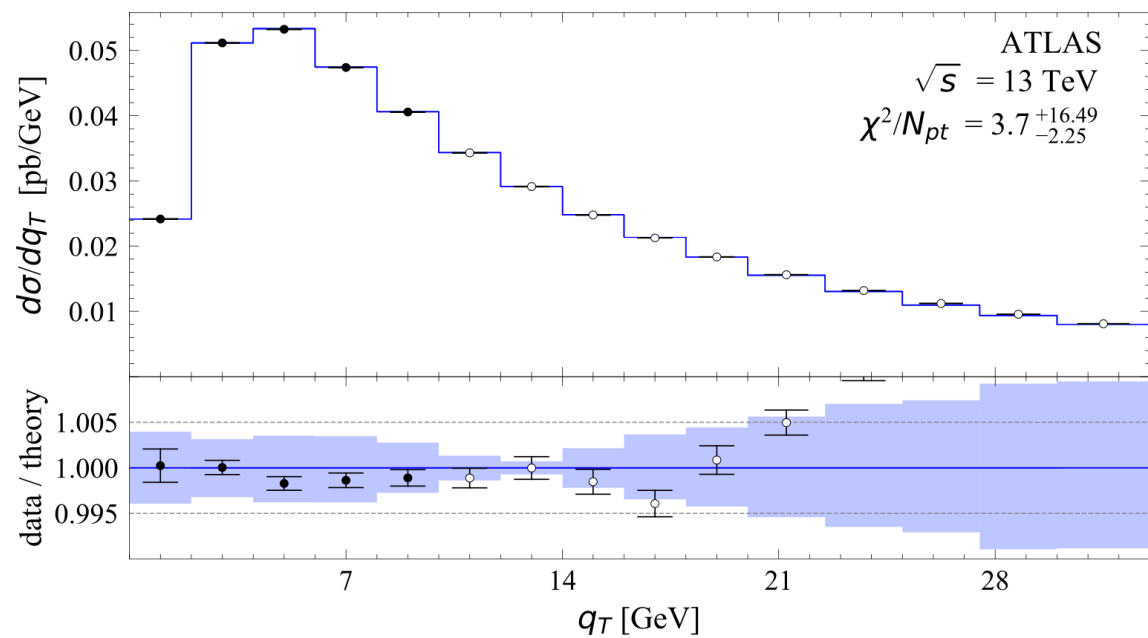
- ▶  $f \in \{u, \bar{u}, d, \bar{d}, sea\}$   
→ 2 × 5 independent parameters!
- ▶ flavour dependent ansatz!  
(=NEW feature!)



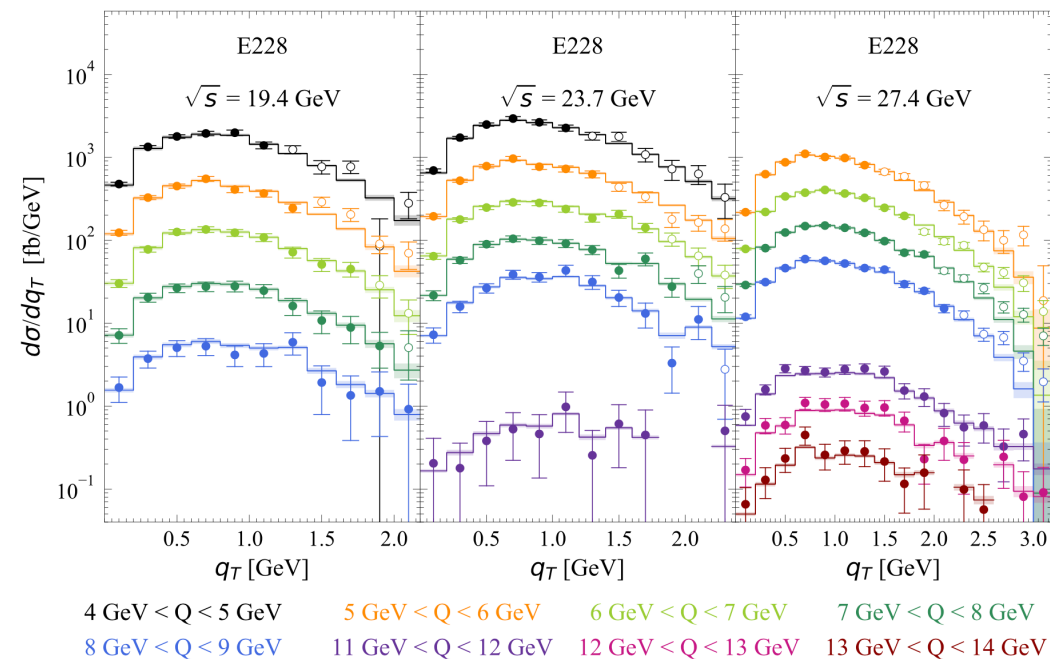
Features:

- ▶ large range of resolution scale:  
4 GeV → 1 TeV
- ▶ including DY W production
- ▶  $\frac{q_T}{Q} < 0.25$  (TMD region!)
- ▶ 627 datapoints included in fit  
vs. 457 (SV19),  
vs. 484 (MAP22)





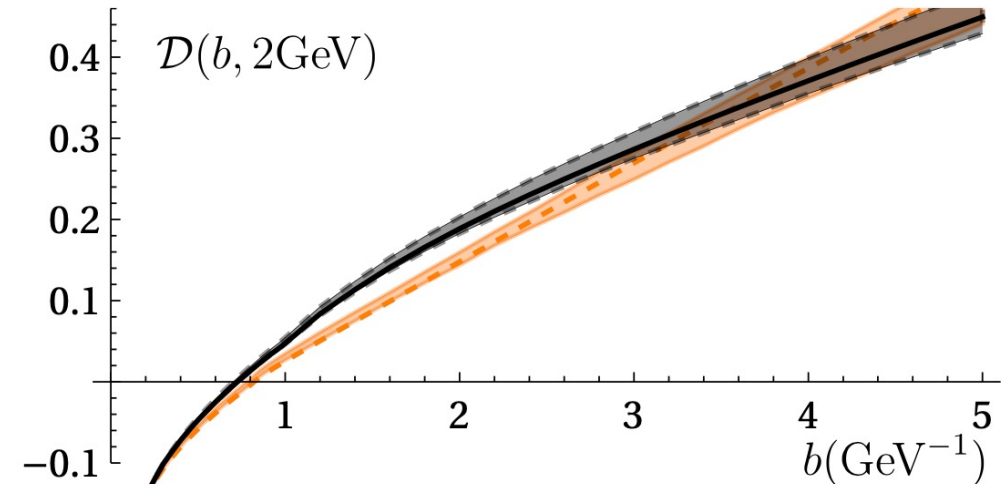
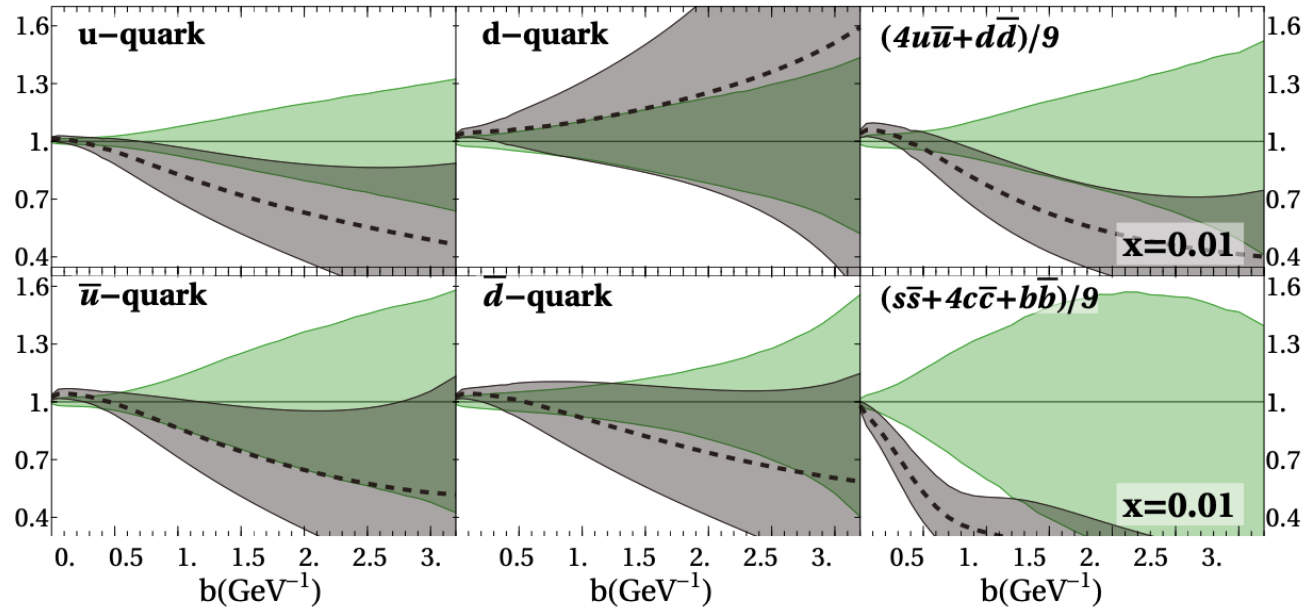
...and also at very low  $Q$  for fixed target experiments.



We can describe this VERY precise data!

$$\text{overall } \chi^2/N_{pt} = 0.96^{+0.09}_{-0.01}$$

Still, rather large dependence on coll. PDF set and its uncertainty:



Wider ramifications of TMD flavor dependence?  $W$  mass?

Extractions using MSHT20 and NNPDF3.1

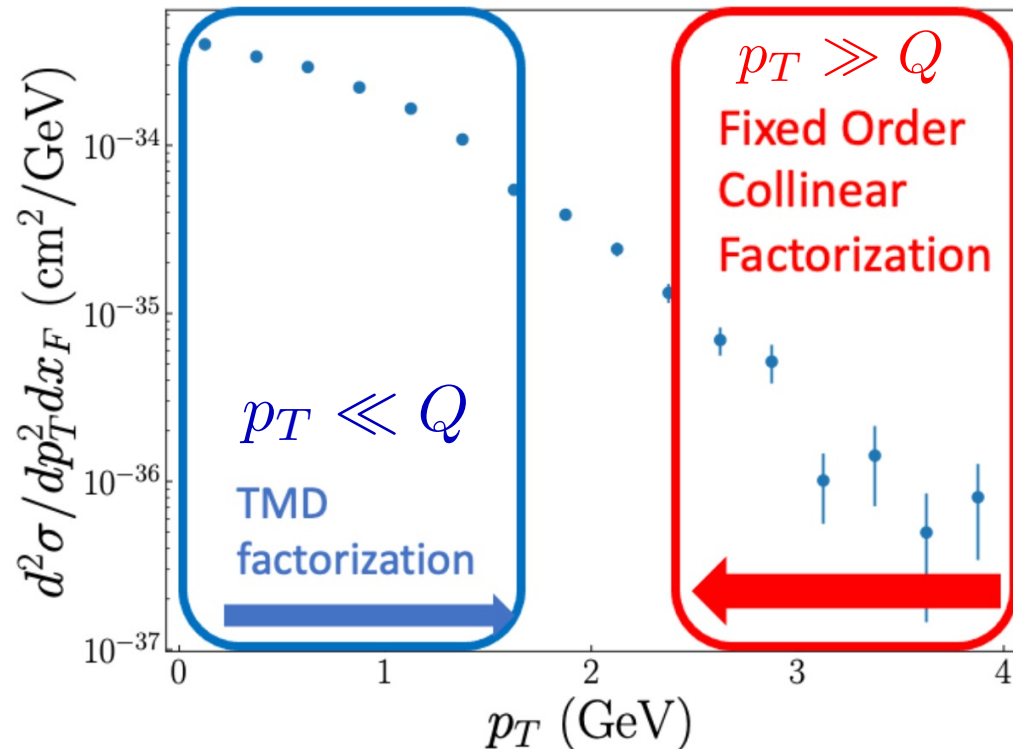


# TMDs also exist beyond leading power.

L. Gamberg

## Can one establish factorization?

- relevant as “background” to LP TMD analysis (often not small)
- interesting physics in itself (e.g., quark-gluon correlations)



### Various sources for power suppressed terms identified and disc

Tree level Studies, Mulders, Tangerman (1996), Bacchetta et al.

- This includes corrections associated to kinematic prefactors referred to as **kinematic power corrections**.
- Another involve subleading terms in quark-quark correlators referred to as **intrinsic power corrections**— e.g. *Cahn function*
- Another from hadronic matrix elements of (interaction deper referred to **dynamic power corrections** multi-parton *qgq* corr

(linked by e.o.m.)

# Cahn intrinsic $k_T$

Volume 78B, number 2,3

PHYSICS LETTERS

25 September 1978

## AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION<sup>☆</sup>

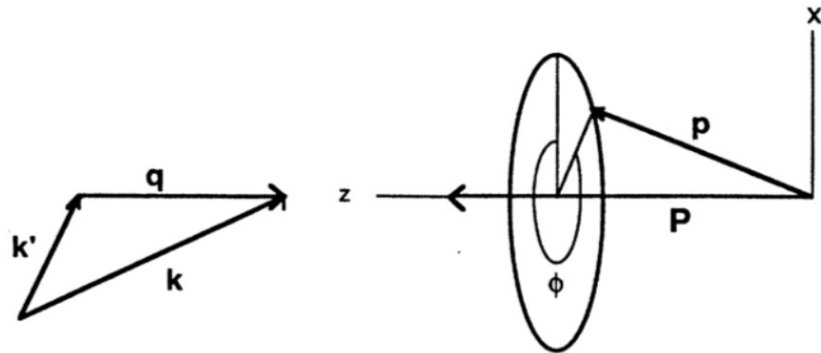
Robert N. CAHN

*Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA*

Received 5 June 1978

parton model argument allowing  
for transverse momentum  
in Mandelstam variables...

Semi-inclusive leptonproduction,  $\ell + p \rightarrow \ell' + h + X$ , is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in  $ep$ ,  $\nu p$  and  $\bar{\nu} p$  scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. **The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.**



NLP

$$\langle \cos \phi \rangle_{ep} = - \left( \frac{2p_{\perp}}{Q} \right) \frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

## TMD factorization at NLO and NLP

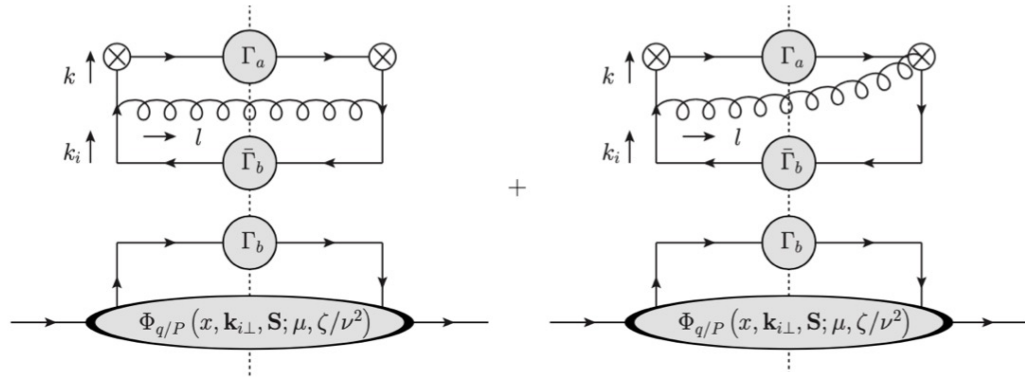
- We perform one loop calculation
- & attempt to establish renormalization group consistency: Regions hard, soft, collinear

$$\begin{aligned}
 F_{\text{DIS}}^3(x, z, \mathbf{P}_{h\perp}) &= H_{\text{DIS}}^{\text{LP}}(Q; \mu) C^{\text{DIS}} \left[ \frac{q_{\perp}}{Q} f_1 D_1 \mathcal{S}^{\text{LP}} \right] \\
 &\quad - H_{\text{DIS}}^{\text{int}}(Q; \mu) C^{\text{DIS}} \left[ \left( x \frac{\mathbf{k}_{\perp} \cdot \hat{x}}{Q} f_{\perp} D_1 - \frac{\mathbf{p}_{\perp} \cdot \hat{x}}{zQ} f_1 D^{\perp} \right) \mathcal{S}^{\text{int}} \right] \\
 &\quad - \int \frac{dx_g}{x_g} H_{\text{DIS}}^{\text{dyn}}(x_g, Q; \mu) C^{\text{DIS}} \left[ x \frac{\mathbf{k}_{\perp} \cdot \hat{x}}{Q} \tilde{f}_{\perp} D_1 \mathcal{S}^{\text{dyn}} \right] \\
 &\quad + \int \frac{dz_g}{z_g} H_{\text{DIS}}^{\text{dyn}}(z_g, Q; \mu) C^{\text{DIS}} \left[ \frac{\mathbf{p}_{\perp} \cdot \hat{x}}{zQ} f_1 \tilde{D}^{\perp} \mathcal{S}^{\text{dyn}} \right].
 \end{aligned}$$

# NLO Ingredients collinear factor

L. Gamberg

Diagrams associated with the evolution of the collinear region



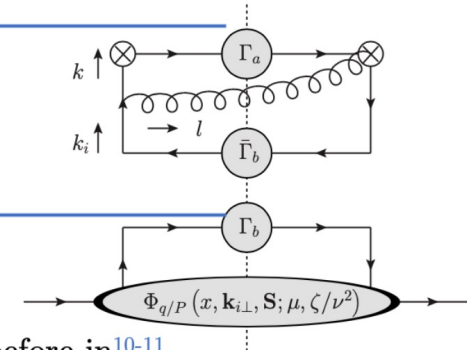
Renormalize TMDs: soft & UV subtraction

$$\hat{\Phi}^{[\Gamma^a]}(x, \mathbf{b}, \mathbf{S}; \mu, \zeta/\nu^2) = Z_{\Gamma^a \Gamma^b}(b, \mu, \zeta/\nu^2) \Phi^{[\Gamma^b]0}(x, \mathbf{b}, \mathbf{S}; xP^+)$$

Evolution equations naturally enter as matrices due to mixing

$$\frac{\partial}{\partial \ln \mu} \begin{bmatrix} \Phi^{[\#]} \\ \Phi^{[\#\gamma^\delta]} \\ \Phi^{[i\sigma^{i+}\gamma^\delta]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^\delta]} \\ \Phi^{[\gamma^i]} \\ \Phi^{[\gamma^i\gamma^\delta]} \\ \Phi^{[i\sigma^{ij}\gamma^\delta]} \\ \Phi^{[i\sigma^{+-}\gamma^\delta]} \end{bmatrix} = \Gamma^\mu \begin{bmatrix} \Phi^{[\#]} \\ \Phi^{[\#\gamma^\delta]} \\ \Phi^{[i\sigma^{i+}\gamma^\delta]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^\delta]} \\ \Phi^{[\gamma^i]} \\ \Phi^{[\gamma^i\gamma^\delta]} \\ \Phi^{[i\sigma^{im}\gamma^\delta]} \\ \Phi^{[i\sigma^{+-}\gamma^\delta]} \end{bmatrix}$$

$$\frac{\partial}{\partial \ln \nu} \begin{bmatrix} \Phi^{[\#]} \\ \Phi^{[\#\gamma^\delta]} \\ \Phi^{[i\sigma^{i+}\gamma^\delta]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^\delta]} \\ \Phi^{[\gamma^i]} \\ \Phi^{[\gamma^i\gamma^\delta]} \\ \Phi^{[i\sigma^{ij}\gamma^\delta]} \\ \Phi^{[i\sigma^{+-}\gamma^\delta]} \end{bmatrix} = \Gamma^\nu \begin{bmatrix} \Phi^{[\#]} \\ \Phi^{[\#\gamma^\delta]} \\ \Phi^{[i\sigma^{i+}\gamma^\delta]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^\delta]} \\ \Phi^{[\gamma^i]} \\ \Phi^{[\gamma^i\gamma^\delta]} \\ \Phi^{[i\sigma^{im}\gamma^\delta]} \\ \Phi^{[i\sigma^{+-}\gamma^\delta]} \end{bmatrix}$$



We find operator mixing in the Collins-Soper equation. Seen before in<sup>10-11</sup>

$$\Gamma^\mu = \begin{bmatrix} \Gamma_2^\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Gamma_2^\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_2^\mu \delta_i^i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Gamma_3^\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_3^\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_3^\mu \delta_i^i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^\mu \delta_i^i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^\mu \delta_i^i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \Gamma_3^\mu (\delta_i^i \delta_m^j - \delta_l^j \delta_m^i) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^\mu \end{bmatrix}$$

$$\Gamma^\nu = \frac{\alpha_s C_F}{2\pi} \begin{bmatrix} 2L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2L \delta_i^i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2ib_l}{xP^+} \frac{\partial L}{\partial b^2} & 0 & L & 0 & 0 & 0 & 0 & 0 \\ \frac{2ib_l}{xP^+} \frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L \delta_i^i & 0 & 0 & 0 & 0 \\ 0 & \frac{2ib_l}{xP^+} \frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L \delta_i^i & 0 & 0 & 0 \\ 0 & 0 & \frac{i}{xP^+} \frac{\partial L}{\partial b^2} (b^j \delta_l^i - b^i \delta_l^j) & 0 & 0 & 0 & 0 & L (\delta_i^i \delta_m^j - \delta_l^j \delta_m^i) & 0 & 0 \\ 0 & 0 & \frac{2ib_l}{xP^+} \frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & 0 & 0 & L \end{bmatrix}$$

LP to LP

LP to NLP

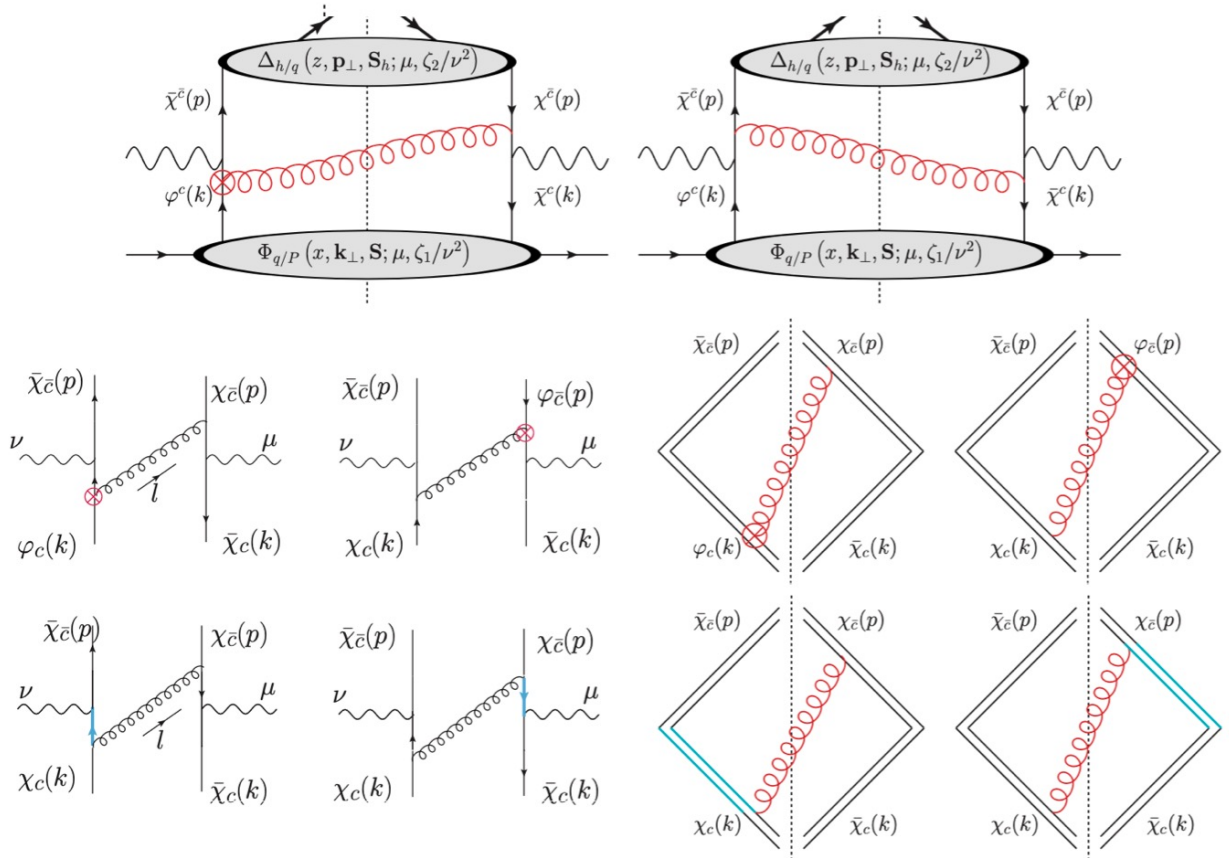
NLP to NLP



# NLO Ingredients soft factor

## The soft region

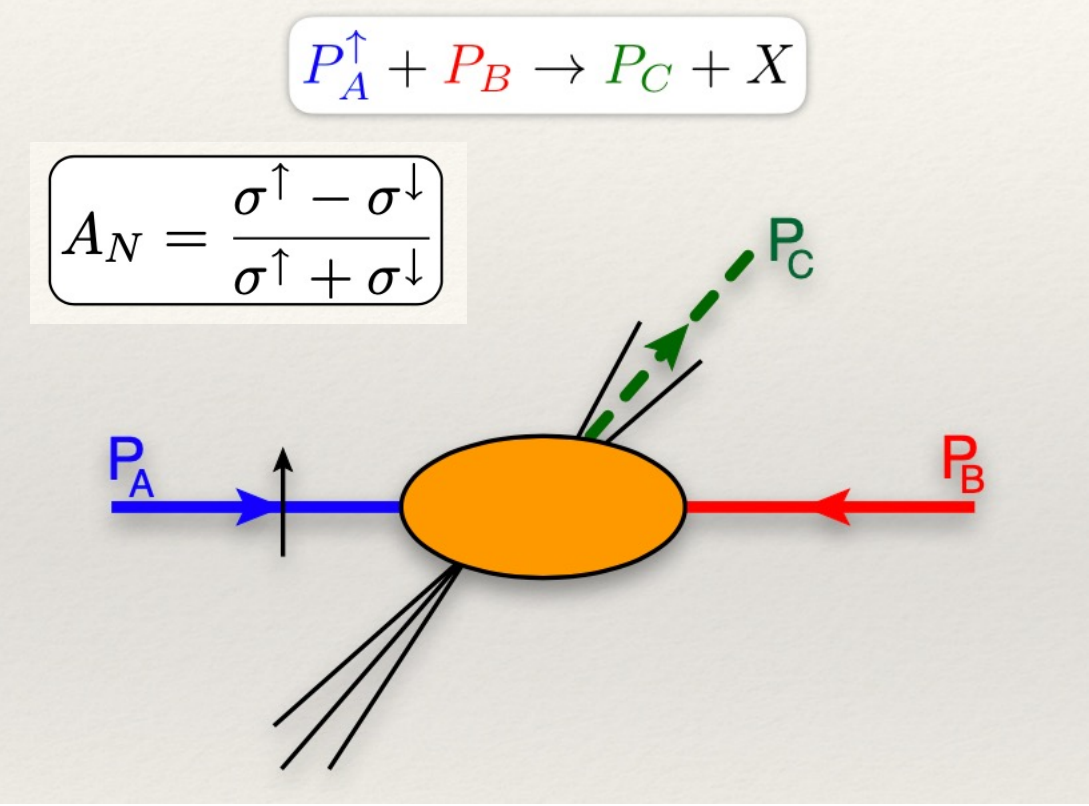
The soft function is generated through the emissions of soft gluons in the partonic cross section



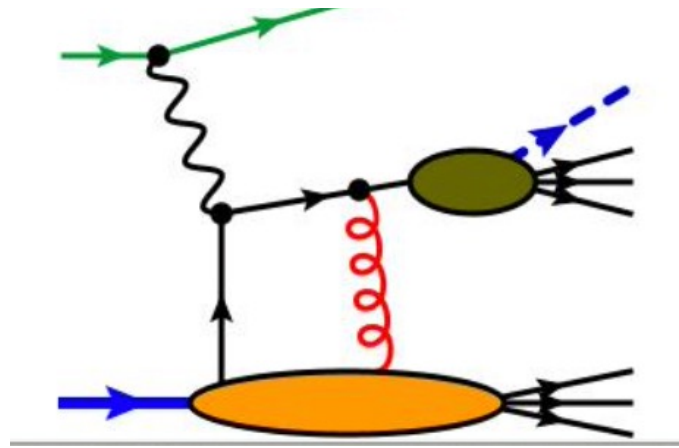
Progress Report  
Stay tuned ...

*Contributions to the soft factor after applying the eikonal approximation and including the effect from the transverse momentum contributions from the quark propagators.*

- long-standing struggle to understand single-spin asymmetries



$e N^\uparrow \rightarrow \pi X$



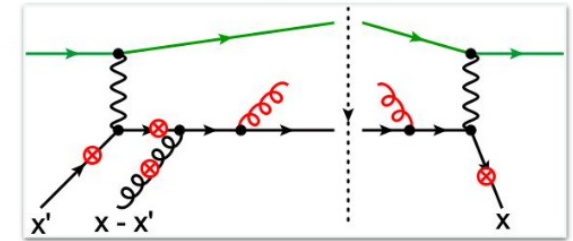
$A_N \propto \sum_q e_q^2 \int dz (1 - x \frac{d}{dx}) F_{FT}^q(x, x) D_1^{\pi/q}(z)$

Four partonic channels at NLO:

- $qg \rightarrow g$ : gluon fragmentation ✓

The easiest channel:

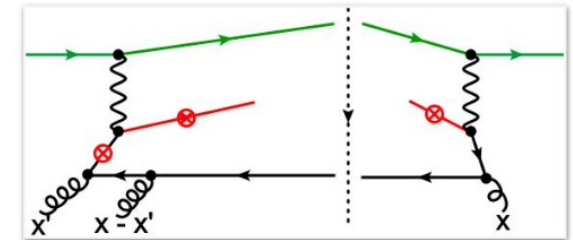
- 12 real diagrams, no virtual contributions
- no  $\overline{MS}$  - subtraction of collinear divergences from SGP, but from FF



- $gg \rightarrow q$ : triple gluon correlations ⓧ

The next-to-easiest (?) channel:

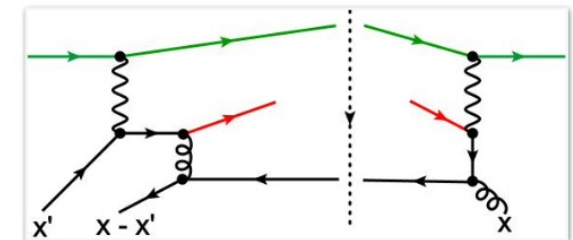
- 12 real diagrams, no virtual contributions
- $\overline{MS}$  - subtraction of collinear divergences from SGP (?)



- $qq \rightarrow q$ : quark-antiquark correlations ✓

The second most subtle channel: (suppressed in  $1/N_c$ )

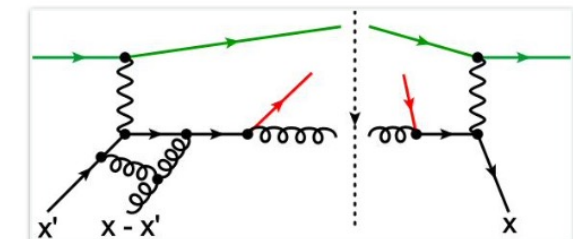
- 12 real diagrams & virtual contributions ( $x' < 0$ )
- $\overline{MS}$  - subtraction of collinear divergences from SGP needed



- $qg \rightarrow q$ : quark-antiquark correlations (partially) ✓

The most difficult channel:

- 12 real diagrams & virtual contributions ( $x' > 0$ )
- $\overline{MS}$  - subtraction of collinear divergences from SGP & FF needed





LO

$$E_\pi \frac{d\sigma}{d\mathbf{P}_\pi} = \sigma_0(S) \sum_q e_q^2 \int_{v_0}^{v_1} dv \frac{1+v^2}{(1-v)^4} [(F_{FT}^q - x_0 (F_{FT}^q)') (x_0, x_0) D_1^q(z)]$$

## Integral contribution

$$+\sigma_0(S) \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \int_0^1 d\zeta [C_F \hat{\sigma}_1^{\text{int}}(v, w, \zeta) + \frac{1}{2} N_c \hat{\sigma}_2^{\text{int}}(v, w, \zeta)] \frac{F_{FT}^q(x, x \zeta) - \zeta(2 - \zeta) F_{FT}^q(x, x) + \zeta(1 - \zeta) \frac{1}{2} x \frac{dF_{FT}^q}{dx}(x, x) - (1 - \zeta)^2 F_{FT}^q(x, 0)}{\zeta(1 - \zeta)^2} D_1^q(z)$$

$$+\sigma_0(S) \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \int_0^1 d\zeta [C_F \hat{\sigma}_{5,1}^{\text{int}}(v, w, \zeta) + \frac{1}{2} N_c \hat{\sigma}_{5,2}^{\text{int}}(v, w, \zeta)] \frac{G_{FT}^q(x, x \zeta) + \zeta(1 - \zeta) x \frac{\partial G_{FT}^q}{\partial x'}(x, x) - (1 - \zeta)^2 G_{FT}^q(x, 0)}{\zeta(1 - \zeta)^2} D_1^q(z)$$

$$+\sigma_0(S) \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \left[ C_F \hat{\sigma}_1^{\text{SGP}}(v, w, \frac{su}{t\mu^2}) + \frac{1}{2} N_c \hat{\sigma}_2^{\text{SGP}}(v, w, \frac{su}{tm_e^2}) \right] F_{FT}^q(x, x) D_1^g(z)$$

$$+\sigma_0(S) \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} [C_F \hat{\sigma}_{5,1}^{\text{SGP}}(v, w) + \frac{1}{2} N_c \hat{\sigma}_{5,2}^{\text{SGP}}(v, w)] x \frac{\partial G_{FT}^q}{\partial x'}(x, x) D_1^g(z)$$

$$+\sigma_0(S) \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \left[ C_F \hat{\sigma}_1^{\text{SFP}}(v, w, \frac{su}{tm_e^2}) + \frac{1}{2} N_c \hat{\sigma}_2^{\text{SFP}}(v, w, \frac{su}{tm_e^2}) \right] F_{FT}^q(x, 0) D_1^g(z)$$

$$+\sigma_0(S) \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \left[ C_F \hat{\sigma}_{5,1}^{\text{SFP}}(v, w, \frac{su}{tm_e^2}) + \frac{1}{2} N_c \hat{\sigma}_{5,2}^{\text{SFP}}(v, w, \frac{su}{tm_e^2}) \right] G_{FT}^q(x, 0) D_1^g(z)$$

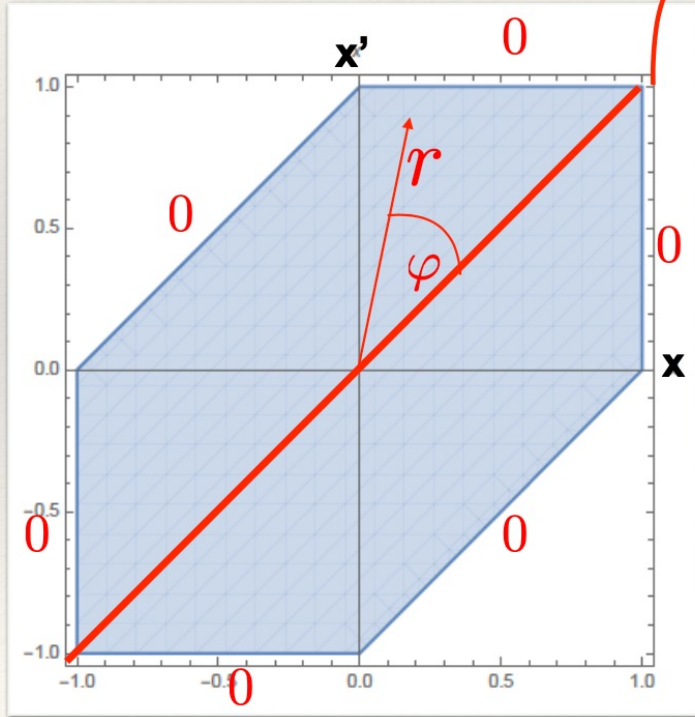
## Soft-Gluon Pole

$$\ln\left(\frac{su}{t\mu^2}\right) \quad \ln\left(\frac{su}{tm_e^2}\right) \quad \frac{1}{(1-w)_+}$$

## Soft-Fermion Pole

$$\ln\left(\frac{su}{tm_e^2}\right)$$

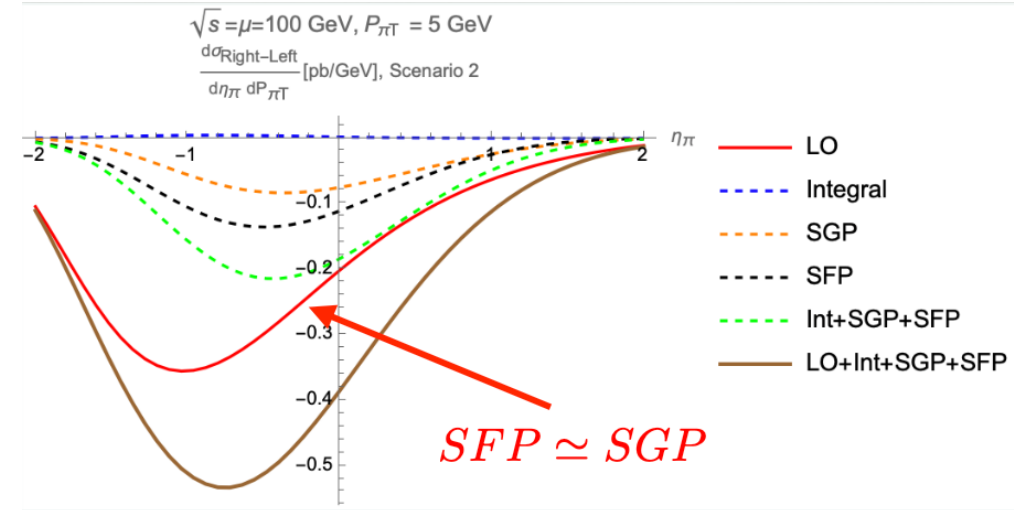
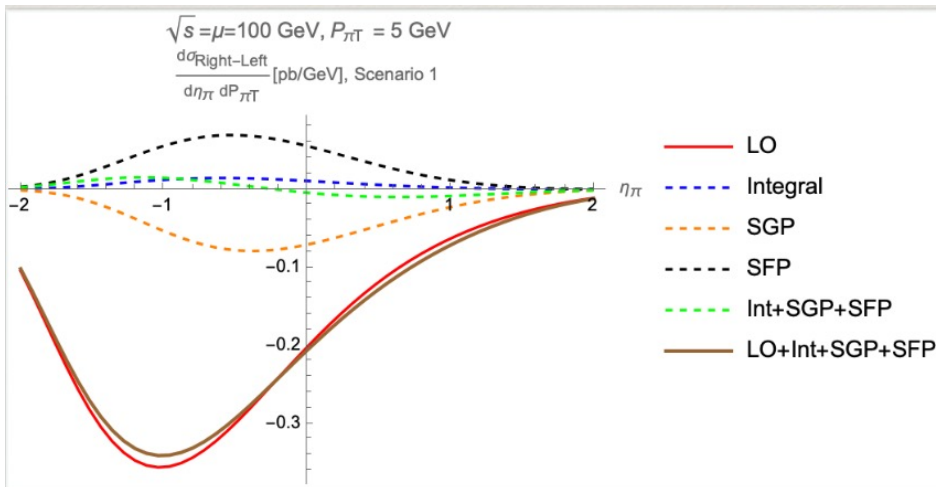
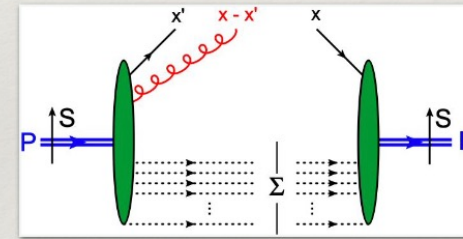




“Soft gluon pole” (SGP)

$$\pi F_{FT}^q(x, x) = f_{1T}^{\perp(1),q}(x)$$

$$G_{FT}^q(x, x) = 0$$



$$\mathcal{W}(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$\int d^2 b_\perp$$

$$\int d^2 k_\perp$$

$$f(x, \vec{k}_\perp)$$

$$F(x, \vec{b}_\perp)$$

**TMD**

**“impact par. PDF”**

$$\int d^2 k_\perp$$

$$\int d^2 b_\perp$$

$$\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$$

$$q(x)$$

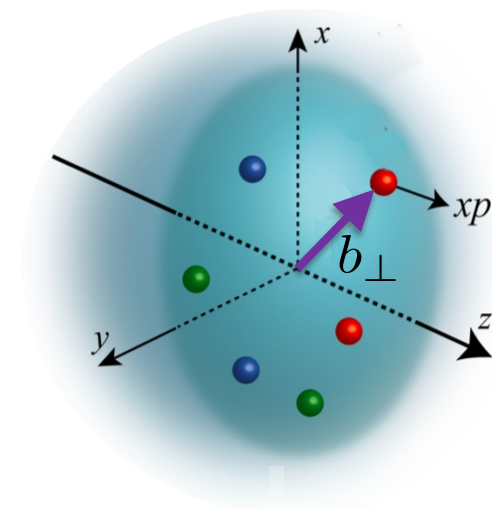
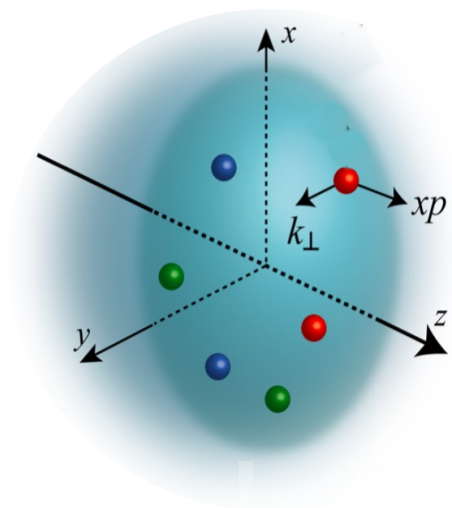
**PDF**

**GPD**

$(\xi = 0)$

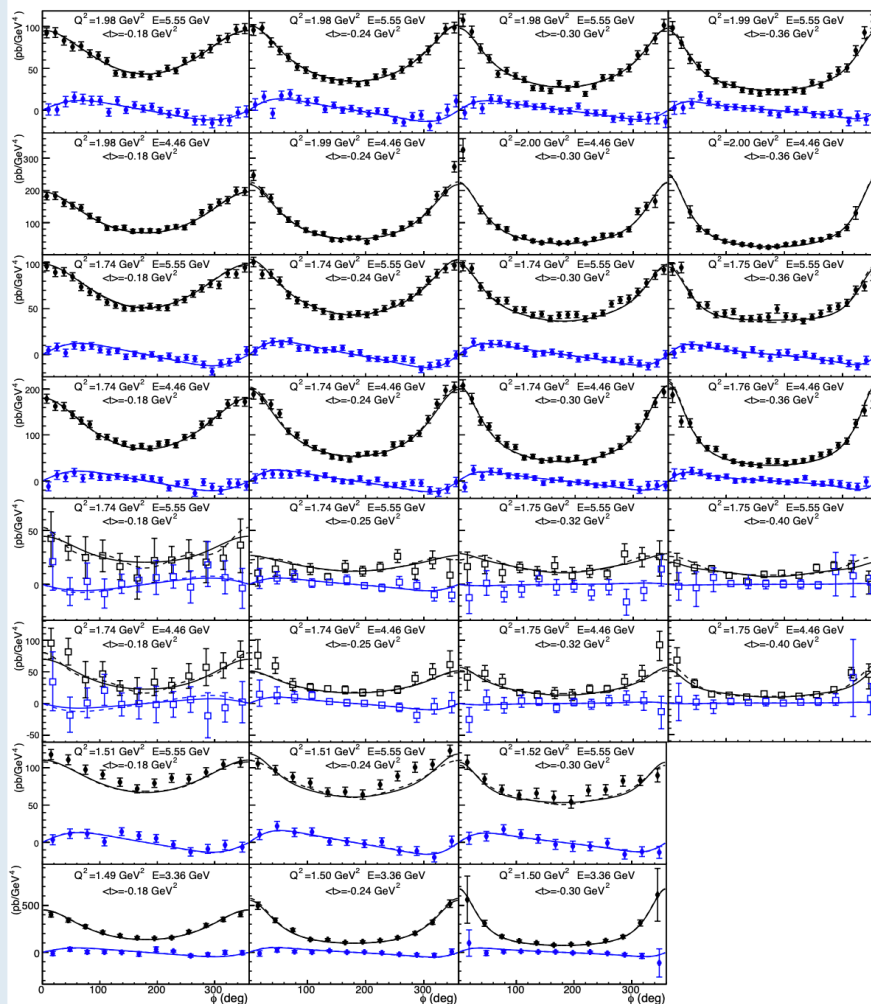
$$\int dx$$

**FF**



# DVCS at the Precision Frontier

V. Braun



- High statistical accuracy
- Several beam energies
- Neutron/deuteron
- Coherent DVCS from  $^4\text{He}$
- Transverse polarization

@ leading tw.

$$\mathcal{A}_{\mu\nu}^{\text{DVCS}} = -g_{\mu\nu}^{\perp} V + \epsilon_{\mu\nu}^{\perp} A + \dots$$

$$V(\xi, Q^2) = \sum_q e_q^2 \int_{-1}^1 \frac{dx}{\xi} C_V(x/\xi, Q^2/\mu^2) F_q(x, \xi, t, \mu).$$

$$F_q(x, \xi) = \frac{1}{2P_+} \left[ H_q(x, \xi, t) \bar{u}(p') \gamma_+ u(p) + E_q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\nu} \Delta_\nu}{2m_N} u(p) \right].$$

$$C(x/\xi, Q^2/\mu^2) = C^{(0)}(x/\xi) + a_s C^{(1)}(x/\xi, Q^2/\mu^2) + a_s^2 C^{(2)}(x/\xi, Q^2/\mu^2) + \dots$$

- Flavor-nonsinglet calculated using two different techniques

 $C_V^{(2)}$  :
V.Braun, A.Manashov, S.Moch, J.Schönleber, JHEP **09**, 117 (2020)

J. Schönleber, unpublished

 $C_A^{(2)}$  :

V.Braun, Manashov, Moch, Schönleber, 2106.01437

J.Gao, T.Huber, Y.Ji and Y.M.Wang, 2106.01390

- Flavor-singlet CFs:

 $C_V^{(2)}$  :
V.Braun, Y. Ji, J. Schönleber, PRL **129** 172001 (2022)
 $C_A^{(2)}$  :

Y. Ji, J. Schönleber, e-Print: 2310.05724



## Example (flavor-nonsinglet)

$$C_V^{(2)}(x) = C_F^2 C_P^{(2)}(x) + \frac{C_F}{N_c} C_{NP}^{(2)}(x) + \beta_0 C_F C_\beta^{(2)}(x)$$

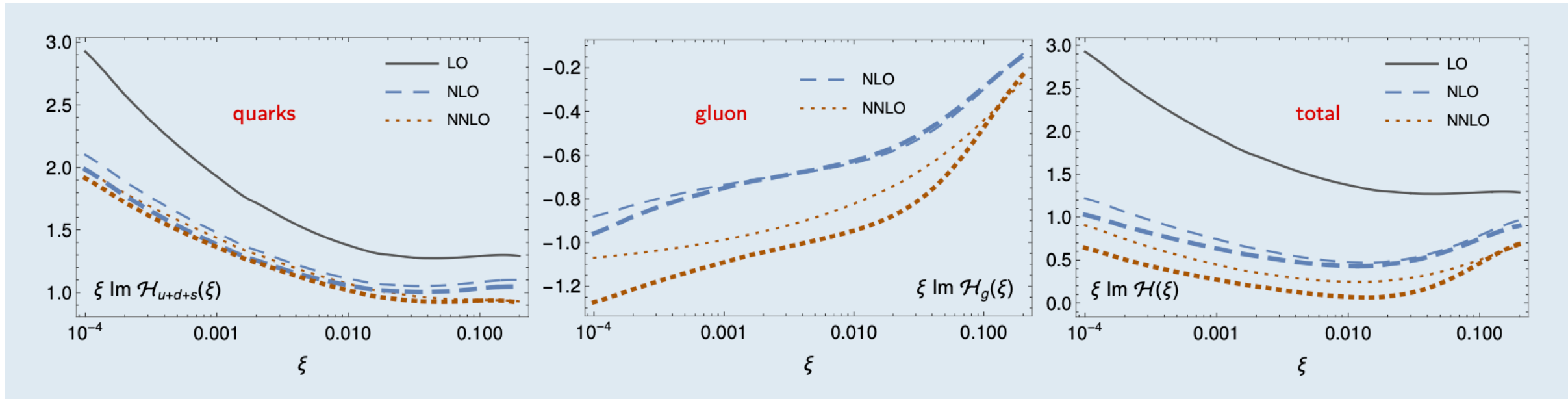
$$\begin{aligned} C_{NP}^{(2)} = & 6(1 - 2\omega) \left\{ H_{20} - H_3 + H_{110} - H_{12} + \zeta_2 (H_0 + H_1) - 3\zeta_3 \right\} \\ & + 12 \left( H_{10} - H_2 - H_0 - H_1 + \zeta_2 \right) + \frac{3}{\bar{\omega}} H_0 + \frac{3}{\omega} H_1 \\ & + \left\{ \frac{1}{\omega} \left( 12\zeta_3 - \frac{3}{2}\zeta_2^2 - \frac{5}{2}\zeta_2 - \frac{73}{24} \right) - \frac{3}{\omega} H_{200} - \left( \frac{2}{\omega} - \frac{1}{\bar{\omega}} \right) H_{30} + \left( \frac{4}{\omega} - \frac{1}{\bar{\omega}} \right) H_4 \right. \\ & - \left( \frac{2}{\omega} - \frac{1}{\bar{\omega}} \right) H_{210} + \left( \frac{3}{\omega} - \frac{2}{\bar{\omega}} \right) H_{22} - \left( \frac{2}{\omega} - \frac{1}{\bar{\omega}} \right) H_{31} - \frac{5}{\bar{\omega}} H_3 + \frac{5}{\bar{\omega}} H_{20} \\ & + \left( \frac{1}{\bar{\omega}} \left( \zeta_2 - \frac{9}{2} \right) + \frac{1}{\omega} \left( \frac{4}{3} - 2\zeta_2 \right) \right) H_{00} - \left( \frac{2}{\omega} (\zeta_2 - 1) - \frac{1}{\bar{\omega}} \left( \zeta_2 + \frac{7}{6} \right) \right) H_2 \\ & \left. + \left( \frac{1}{\bar{\omega}} \left( \frac{19}{6} + 5\zeta_2 - 3\zeta_3 \right) + \frac{1}{\omega} \left( 7\zeta_3 - \frac{16}{9} \right) \right) H_0 - (\omega \leftrightarrow \bar{\omega}) \right\} \end{aligned}$$

where  $\omega = (1 - x)/2$ ,  $\bar{\omega} = (1 + x)/2$ , and  $H_{\vec{m}} \equiv H_{\vec{m}}(\omega)$  are harmonic polylogarithms



# CFF $\text{Im}(H)$ :

$$t = -0.1 \text{ GeV}^2$$



- even relevant for Jlab kinematics
- based on GK model  $\rightarrow$  need to “retune”

### ① Towards NNLO accuracy

- Two-loop coefficient functions for DVCS
  - sizeable corrections, completed for light quarks
- Three-loop evolution equations for GPDS
  - flavor nonsinglet in position space, singlet for the first few moments
  - pressing issue: numerical implementation, also in NLO

### ② Kinematic power corrections $(\sqrt{-t}/Q)^k$ , $(m/Q)^k$

- Twist-four accuracy,  $(\sqrt{-t}/Q)^2$ ,  $(m/Q)^2$ 
  - complete results available, numerical code (B.Pirnay)
  - large effects for parts of phase space and in collider kinematics
  - Coherent DVCS from nuclei: Target mass corrections do not spoil factorization
- Higher powers (work in progress)
  - all-order results on OPE level
  - cancellation of IR divergences checked up to twist-6
  - scalar target completed, nucleon in progress

# Evolution equations

V. Bertone

🍏 Defining the **anti-quark** distributions as:

$$F_{\bar{q}/H}^{[U,T]}(x, \xi, \Delta^2; \mu) = -F_{q/H}^{[U,T]}(-x, \xi, \Delta^2; \mu)$$

$$F_{\bar{q}/H}^{[L]}(x, \xi, \Delta^2; \mu) = +F_{q/H}^{[L]}(-x, \xi, \Delta^2; \mu)$$

🍏 one can construct **non-singlet** and **singlet** combinations:

$$F^{[\Gamma],-} = F_{q/H}^{[\Gamma]} - F_{\bar{q}/H}^{[\Gamma]} \quad F^{[\Gamma],+} = \begin{pmatrix} \sum_{q=1}^{n_f} F_{q/H}^{[\Gamma]} + F_{\bar{q}/H}^{[\Gamma]} \\ F_{g/H}^{[\Gamma]} \end{pmatrix}$$

🍏 The evolution equations **decouple** and can be written in a **DGLAP-like** fashion:

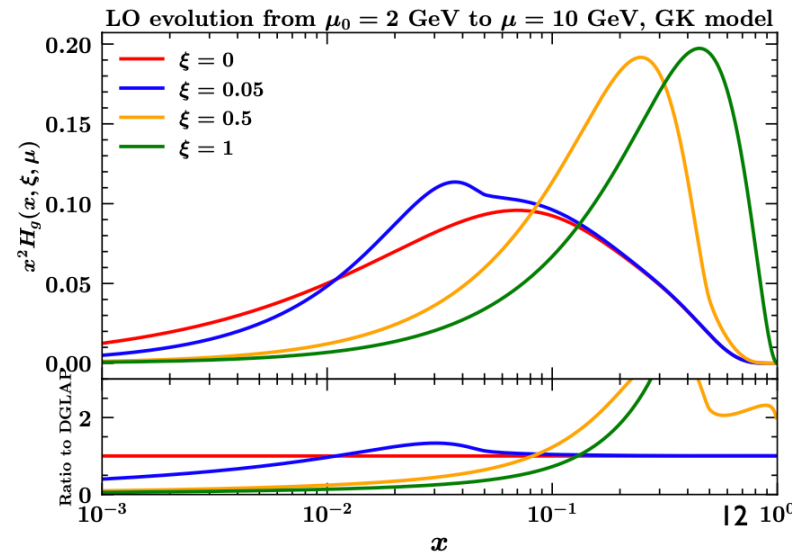
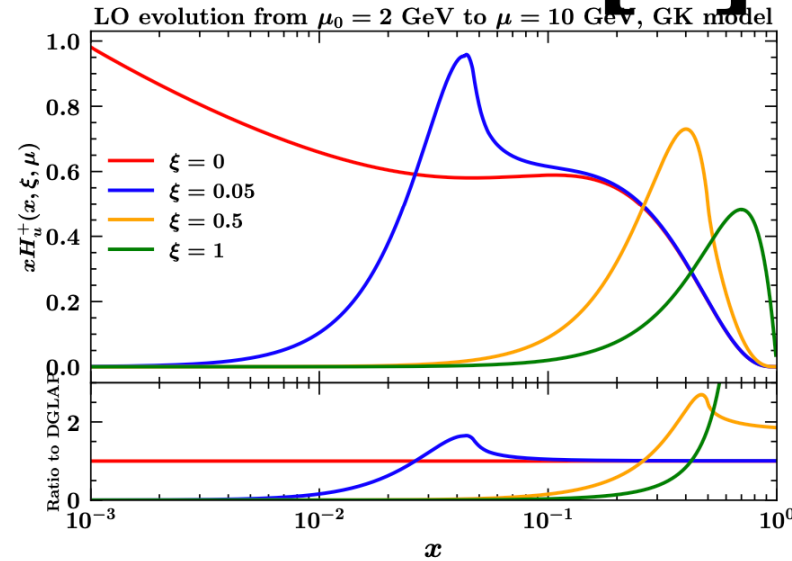
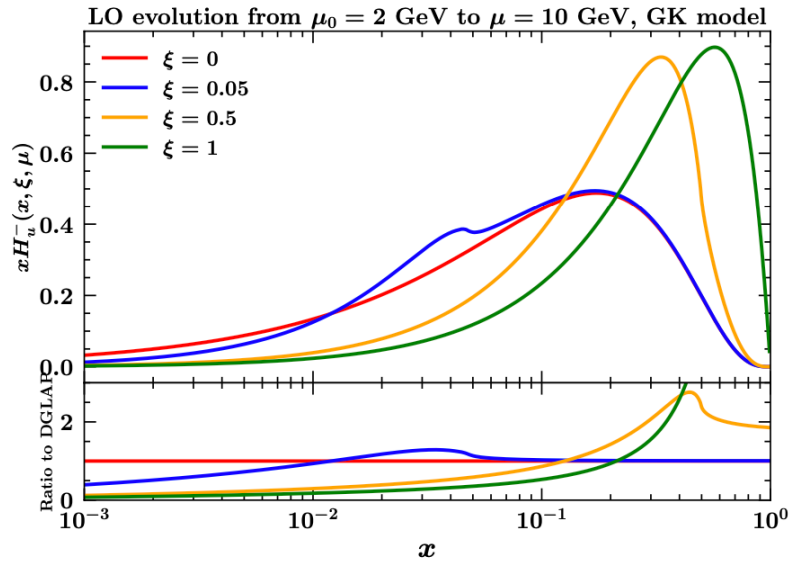
$$\frac{dF^{[\Gamma],\pm}(x, \xi, \mu)}{d \ln \mu^2} = \frac{\alpha_s(\mu)}{4\pi} \int_x^\infty \frac{dy}{y} \mathcal{P}^{[\Gamma]\pm,[0]} \left( y, \frac{\xi}{x} \right) F^{[\Gamma],\pm} \left( \frac{x}{y}, \xi, \mu \right)$$

$$\mathcal{P}^{[\Gamma]\pm,[0]} \left( y, \frac{\xi}{x} \right) = \theta(1-y) \mathcal{P}_1^{[\Gamma]\pm,[0]} \left( y, \frac{\xi}{x} \right) + \theta(\xi-x) \mathcal{P}_2^{[\Gamma]\pm,[0]} \left( y, \frac{\xi}{x} \right)$$

**DGLAP region**
**ERBL contribution**

🍏  $\mathcal{P}_{1,2}^{[\Gamma]\pm,[0]}$  are appropriate combinations of the functions  $p_{ij}^\Gamma$  presented before. 10

# Evolution and DGLAP limit [U]



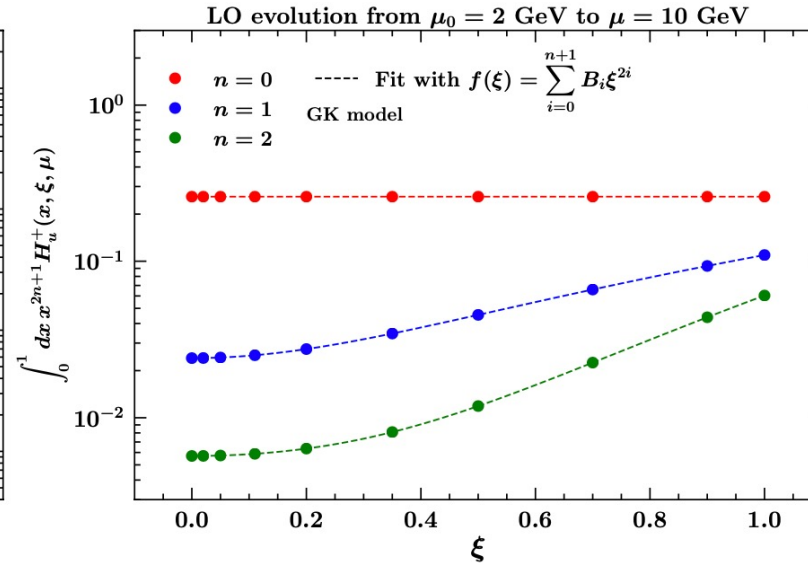
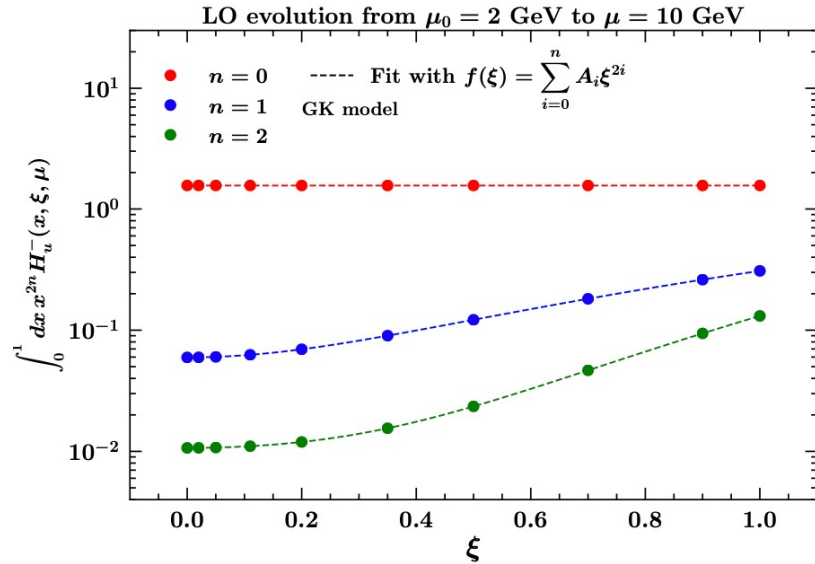
- 🍏 **DGLAP limit** reproduced within  $10^{-5}$  relative accuracy.
- 🍏 GPD evolution may significantly deviate from DGLAP evolution.
- 🍏 The evolution generates a cusp at  $x = \xi$  but the distribution remains **continuous** at this point.

so far, LO



# Polynomiality [U]

$$\int_0^1 dx x^{2n} F_q^{[\Gamma]-}(x, \xi, \mu) = \sum_{k=0}^n A_k^{[\Gamma]}(\mu) \xi^{2k}$$



🍎 **First moment** for both singlet and non-singlet is indeed **constant** in  $\xi$ :

🍎 this was expected and the expectation is very nicely fulfilled.

🍎 **Second and third moments** follow the expected law:

🍎 including odd-power terms in the fit gives coefficients very close to zero.

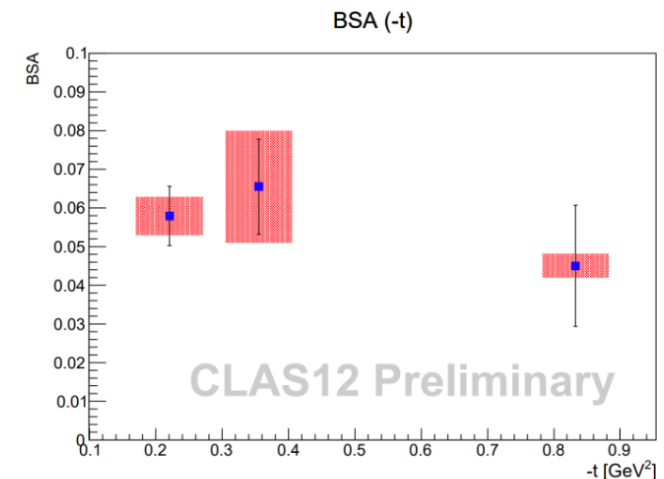
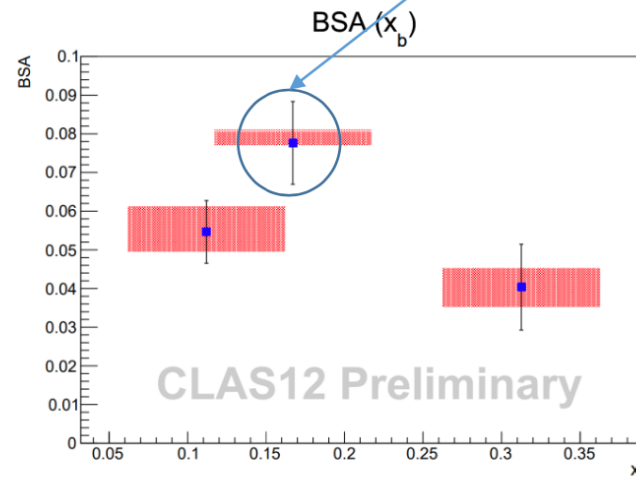
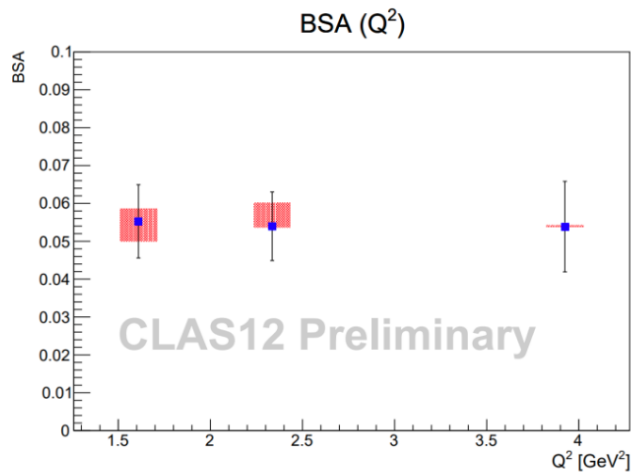
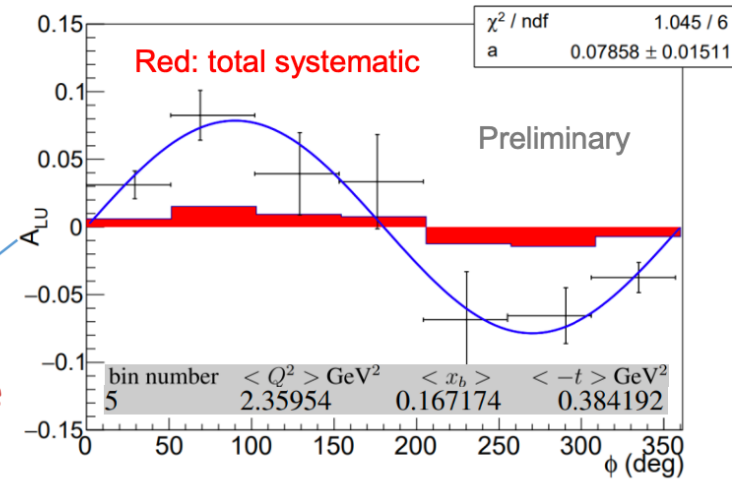
🍎  $B_{n+1}$  in the singlet is consistently found to be compatible with zero (no D-term)<sub>16</sub>



CLAS12: nDVCS with an unpolarized deuterium target

Under internal review

- Observation of positive BSA for nDVCS
- Systematic errors include:
  - Error due to beam polarization
  - Error due to selection cuts
  - Error due to residual proton contamination
  - Error due to merging of data sets with different energies
- Statistics is expected to double with remaining scheduled beam time and improvements with reconstruction software



- The **exclusivity** of the event is insured by:
  - Electron detection**: Cerenkov detector, drift chambers and electromagnetic calorimeter
  - Photon detection**: sampling calorimeter or a small PbWO4-calorimeter close to the beamline
  - Proton detection**: Silicon and Micromegas detector OR **Neutron detection**: Central Neutron Detector
- For Neutron Detection:
  - Machine Learning techniques are applied to improve the Identification and reduce charged particle contamination



## $\pi^0$ background subtraction

Under internal review

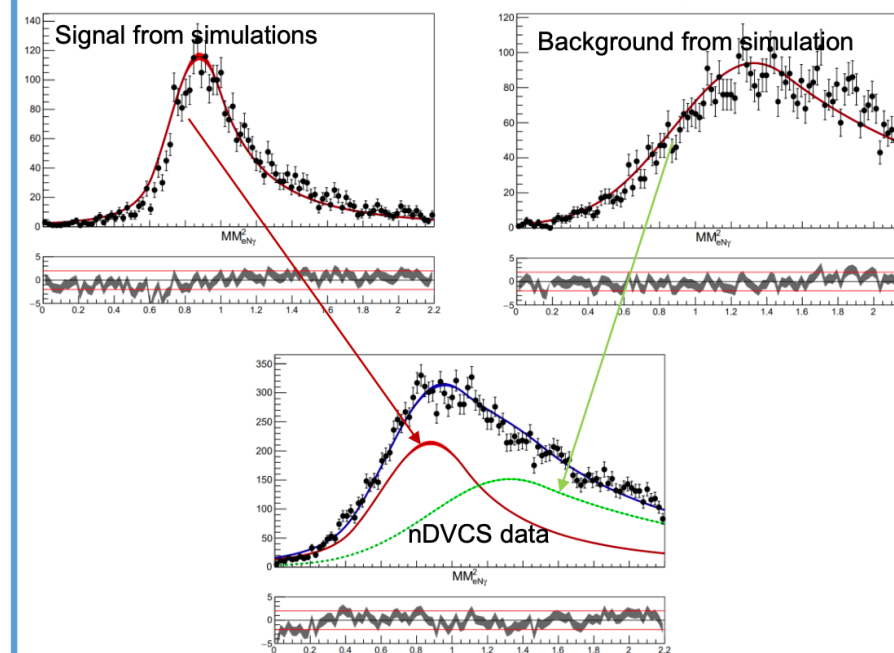
- Subtraction using simulations of the background channel
    - Monte Carlo simulations:
      - GPD-based event generator for DVCS/ $\pi^0$  on deuterium
      - DVCS amplitude calculated according to the BKM formalism
      - Fermi-motion distribution evaluated according to Paris potential
- Estimate the ratio of partially reconstructed  $eN\pi^0$  (1 photon) decay to fully reconstructed  $eN\pi^0$  decays in MC
  - This is done for each kinematic bin to minimize MC model dependence
  - Multiply this ratio by the number of reconstructed  $eN\pi^0$  in data to get the number of  $eN\pi^0$  (1 photon) in data
  - Subtract this number from DVCS reconstructed decays in data per each kinematical bin

$$\text{Simulations: } R = \frac{N(eN\pi_{1\gamma}^0)}{N(eN\pi^0)}$$

$$\text{Data: } N(eN\pi_{1\gamma}^0) = R * N(eN\pi^0)$$

$$N(DVCS) = N(DVCS_{recon}) - N(eN\pi_{1\gamma}^0)$$

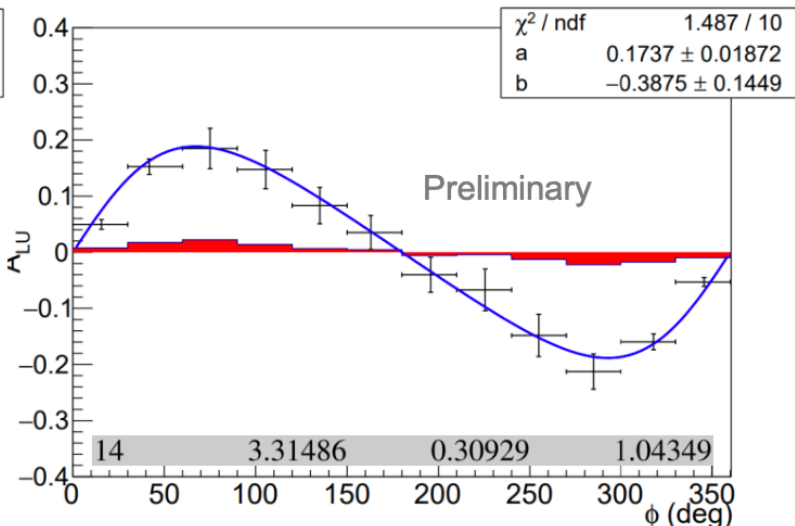
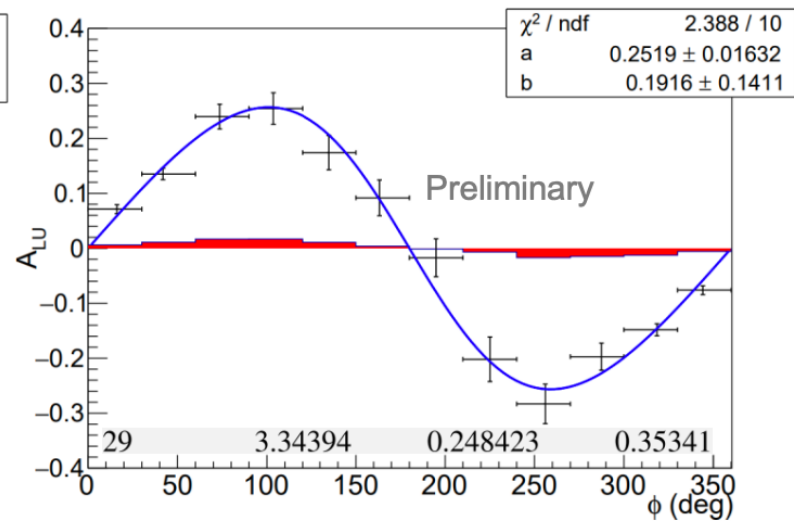
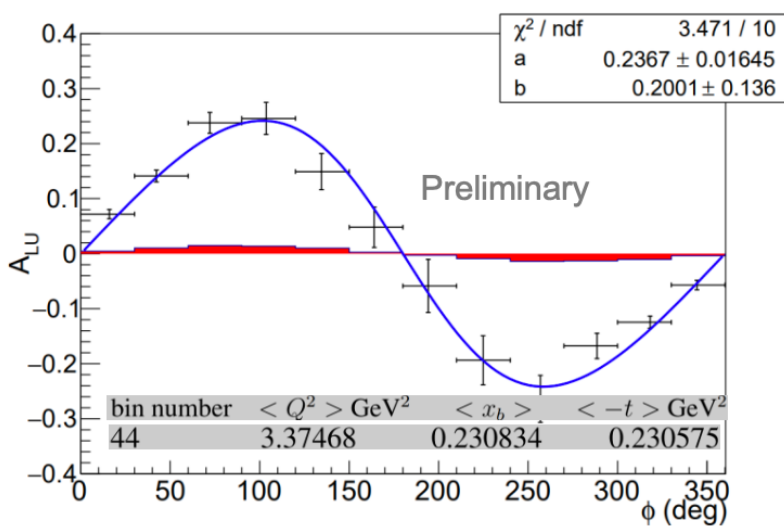
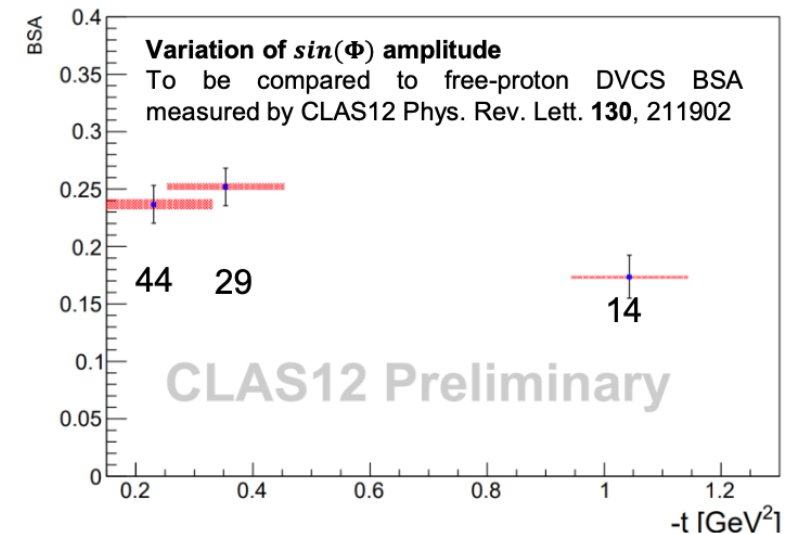
- $\pi^0$  background subtraction is also performed by statistical unfolding of contribution to the missing mass spectrum  
M. Pivk and F.R. Le Diberder, NIMA 555 1 2005



The difference between the estimations of background from both methods is considered as a systematic



- **Systematic errors include:**
  - Error due to beam polarization
  - Error due to selection cuts
  - Error due to merging of data sets with different energies
- **Statistics is expected to triple with remaining scheduled beam time and improvements with reconstruction software**



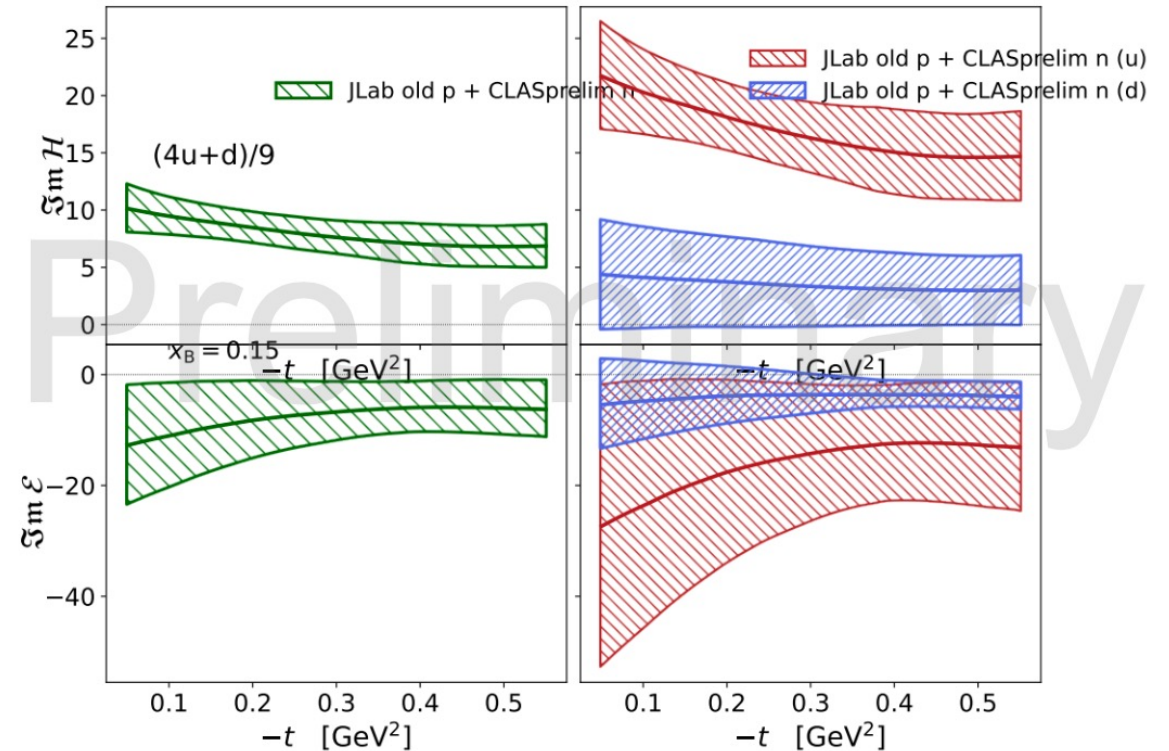




- Train new models with old JLab and new CLAS12 data
  - 40 NN trained on old data and another 40 NN trained on old JLAB pDVCS + new CLAS12 nDVCS data
  - separate models for u and d flavors of CFFs  $\text{Im } H$  and  $\text{Im } E$
  - flavor agnostic (flavor summed) models for CFFs  $\text{Re } H$  and  $\text{Im } \tilde{H}$

Flavor separation of  $\text{Im } H$  is better than before

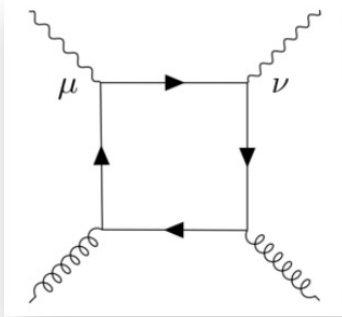
Less evident Flavor separation of  $\text{Im } E$







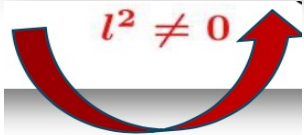
# Imprint of Anomalies in QCD Compton scattering



**Antisymmetric part of Compton amplitude**

**Collinear singularity regularized by  $l^2$**

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \times T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left( \sum_f e_f^2 \right) \bar{u}(P_2) \left[ \left( \tilde{\kappa}_{qg} \ln \frac{Q^2}{-l^2} + \delta \tilde{C}_g^{\text{off}} \right) \otimes \langle F^{+\mu} \tilde{F}_\mu^+ \rangle \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$



$l^2 \neq 0$

$$\tilde{\mathcal{F}}(x, l^2) = \frac{iP^+}{\bar{u}(P_2) \gamma_5 u(P_1)} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | F_a^{\mu\nu}(-z^-/2) \tilde{F}_{\mu\nu}^a(z^-/2) | P_1 \rangle$$

**Twist-4 GPD**

**But no suppression in  $1/Q^2$ !**

**The QCD factorization theorem:** Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \times T_{\mu\nu}^{\text{asym}} = \frac{1}{2} \sum_f e_f^2 \bar{u}(P_2) \left[ \gamma^\alpha \gamma_5 (\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{2M} (\tilde{E}_f^{\text{bare}}(x_B, \xi, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, \xi, l^2)) \right] u(P_1) + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2),$$

$$\boxed{\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2)} = \boxed{\tilde{E}_f^{\text{bare}}(x_B, l^2) + \tilde{E}_f^{\text{bare}}(-x_B, l^2)} + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

↑
↑  
 “Bare GPD” (tree level)                      Perturbative pole (one loop)

Postulate that the “renormalized” GPD integrates to  $g_P(l^2)$  :

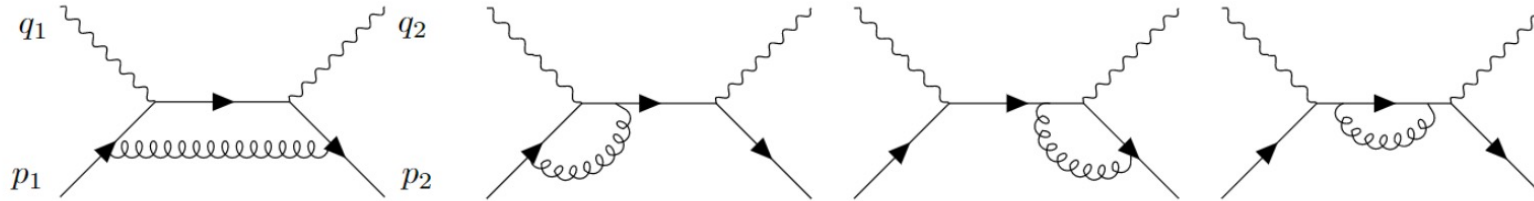
$$g_P(l^2) = \sum_f \int_{-1}^1 dx \tilde{E}_f(x, \xi, l^2) = \sum_f \int_0^1 \boxed{dx(\tilde{E}_f(x, \xi, l^2) + \tilde{E}_f(-x, \xi, l^2))}$$

obtain

$$\frac{g_P(l^2)}{2M} = -\frac{i}{\ell^2} \left( \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \Big|_{\ell^2=0} - \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} \right) \quad \text{finite at } \ell^2 = 0$$

similarly, symmetric part / trace anomaly

**Quark-channel diagrams in DVCS**



**Example: Antisymmetric case**

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta\tilde{C}_1^q(\hat{x}, \hat{\xi})$$

**Coefficient function**

$$\delta\tilde{C}_1^q(\hat{x}, \hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat{x})} + \frac{-1+2\hat{x}-4\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)} \ln \frac{\hat{x}-1}{\hat{x}} + \frac{(\hat{x}-\hat{\xi})(1+2\hat{x}^2+3\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}}$$

$$+ \frac{\hat{x}^2-2\hat{\xi}^2}{\hat{x}(1-\hat{\xi}^2)} \ln^2 \frac{\hat{x}-1}{\hat{x}} + \frac{\hat{x}^2-2\hat{\xi}^2}{\hat{x}(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} \ln \frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$

$$+ \frac{\hat{\xi}}{1-\hat{\xi}^2} \text{Li}_2 \frac{2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{1+\hat{x}^2}{(1-\hat{x})}$$

**Unexpected double IR pole**

**Unexpected single IR pole**

**Sudakov logs!**  $\ln\left(\frac{Q^2}{-l^2}\right), \ln^2\left(\frac{Q^2}{-l^2}\right)$

establishes factorization at one loop in regime  $\Lambda_{\text{QCD}}^2 \ll -t \ll Q^2$

★ **Twist-3:** poorly known, but very important:

- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g.  $g_2$ )
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

While twist-3  $f_i^{(1)}$  share some similarities with twist-2  $f_i^{(0)}$  in their extraction, there are several challenges both experimentally and theoretically

## Theoretical setup

★ **Correlation functions in coordinate space**

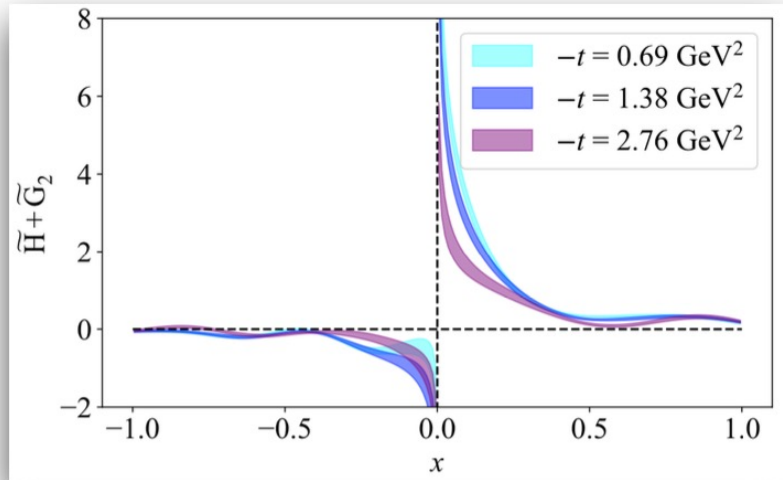
$$F^{[\Gamma]}(x, \Delta; P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi} e^{ik \cdot z} \langle p_f, \lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W}(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) | p_i, \lambda \rangle \Big|_{z^0=0, \vec{z}_\perp=\vec{0}_\perp}$$

★ **Parametrization of coordinate-space correlation functions**

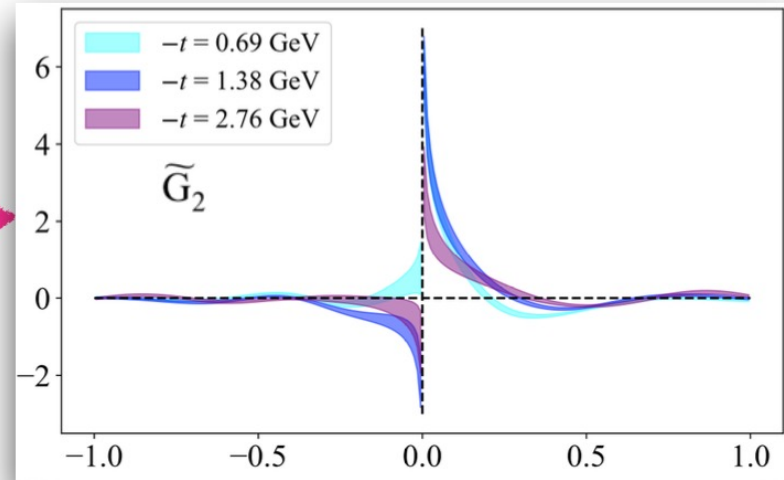
[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105]

[F. Aslan et al., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu \gamma_5]}(x, \Delta; P^3) = \frac{1}{2P^3} \bar{u}(p_f, \lambda') \left[ P^\mu \frac{\gamma^3 \gamma_5}{P^0} F_{\tilde{H}}(x, \xi, t; P^3) + P^\mu \frac{\Delta^3 \gamma_5}{2m P^0} F_{\tilde{E}}(x, \xi, t; P^3) \right. \\ + \Delta_\perp^\mu \frac{\gamma_5}{2m} F_{\tilde{E}+\tilde{G}_1}(x, \xi, t; P^3) + \gamma_\perp^\mu \gamma_5 F_{\tilde{H}+\tilde{G}_2}(x, \xi, t; P^3) \\ \left. + \Delta_\perp^\mu \frac{\gamma^3 \gamma_5}{P^3} F_{\tilde{G}_3}(x, \xi, t; P^3) + i \varepsilon_\perp^{\mu\nu} \Delta_\nu \frac{\gamma^3}{P^3} F_{\tilde{G}_4}(x, \xi, t; P^3) \right] u(p_i, \lambda)$$



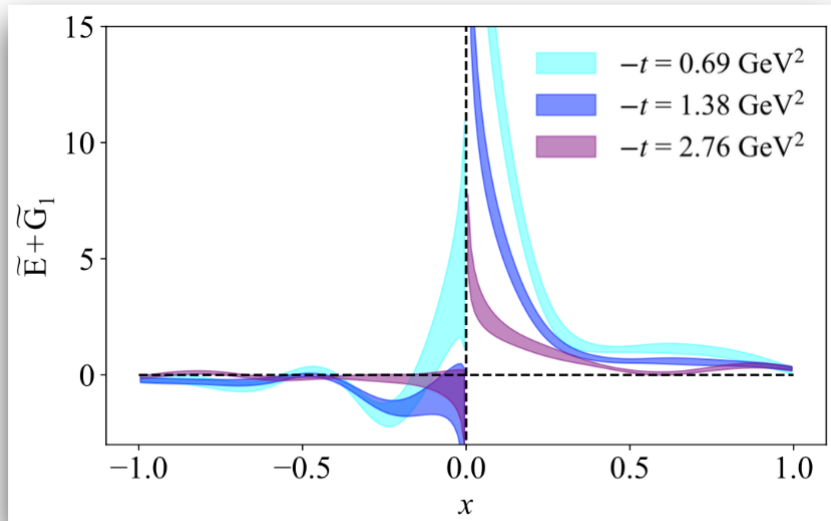
Isolating  $\tilde{G}_2$   
using  $\tilde{H}$



★ Direct access to  $\tilde{E}$ -GPD not possible for zero skewness

★ Glimpse into  $\tilde{E}$ -GPD through twist-3 :

$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\tilde{E}}(x, \xi, t; P^3)$$



★ LaMET formalism is applicable beyond leading twist. However, several improvements needed, e.g., mixing with quark-gluon-quark correlator

M. Constantinou



★ New proposal for Lorentz invariant decomposition has great advantages:

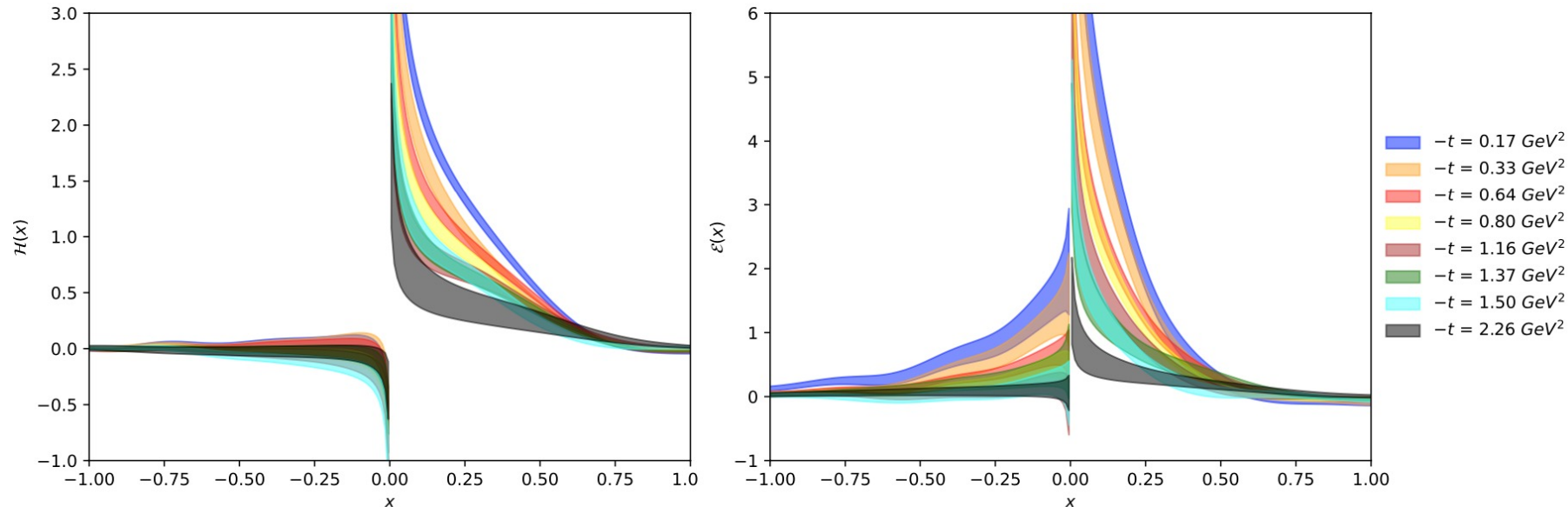
- significant reduction of computational cost
- access to a broad range of  $t$  and  $\xi$

M. Constantinou

$$F_{\lambda,\lambda'}^\mu = \bar{u}(p', \lambda') \left[ \frac{P^\mu}{M} A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M} A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M} A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M} A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M} A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M} A_8 \right] u(p, \lambda)$$

- symmetric frame:  $\vec{p}_f^s = \vec{P} + \frac{\vec{Q}}{2}, \quad \vec{p}_i^s = \vec{P} - \frac{\vec{Q}}{2} \quad -t^s = \vec{Q}^2 = 0.69 \text{ GeV}^2$

- asymmetric frame:  $\vec{p}_f^a = \vec{P}, \quad \vec{p}_i^a = \vec{P} - \vec{Q} \quad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65 \text{ GeV}^2$



$$\mathcal{W}(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$\int d^2 b_\perp$$

$$\int d^2 k_\perp$$

$$f(x, \vec{k}_\perp)$$

$$F(x, \vec{b}_\perp)$$

**TMD**

**“impact par. PDF”**

$$\int d^2 k_\perp$$

$$\int d^2 b_\perp$$

$$\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$$

$$q(x)$$

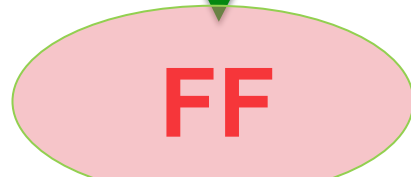
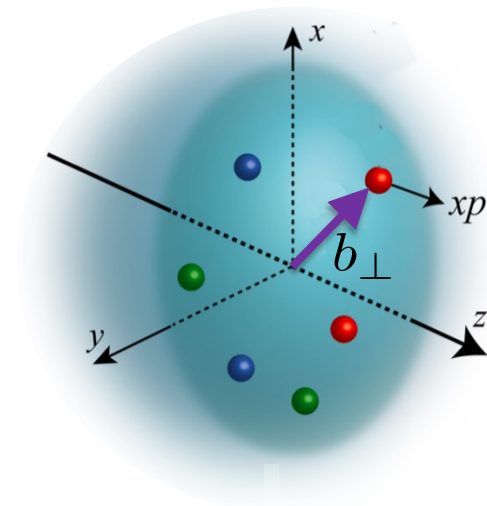
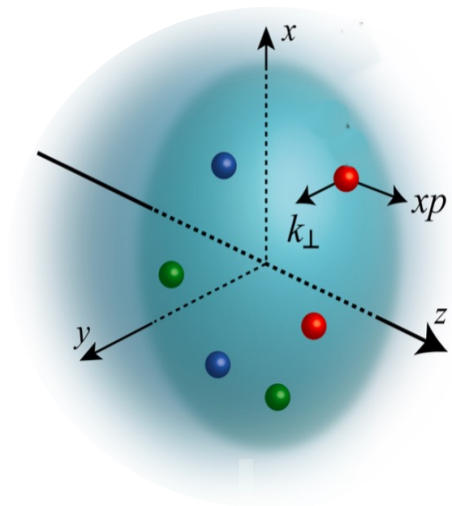
**PDF**

**GPD**

( $\xi = 0$ )

$$\int dx$$

**FF**



# Nucleon axial and pseudoscalar form factors from Lattice QCD simulations at the physical point



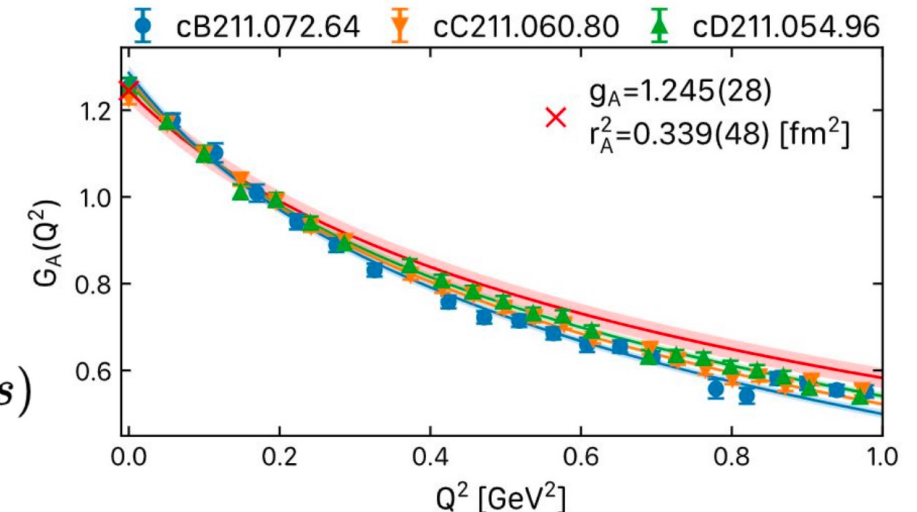
- Three physical point ensemble
- Thorough excited state analysis
- Combined fit of  $Q^2$ -dependence and continuum limit

**Dr. Simone Bacchio**

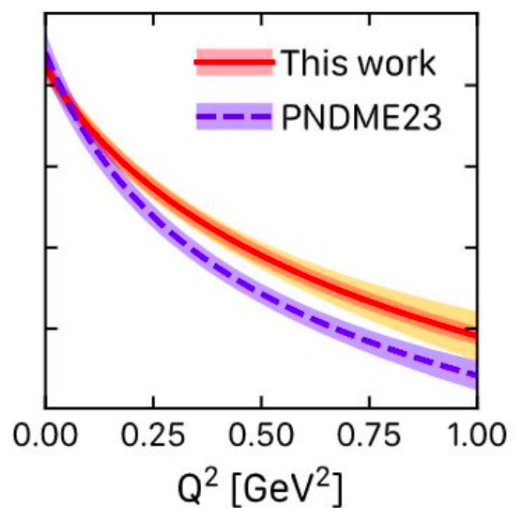
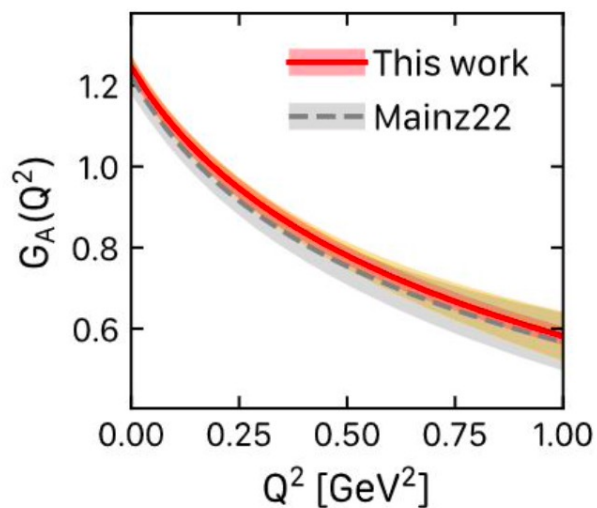
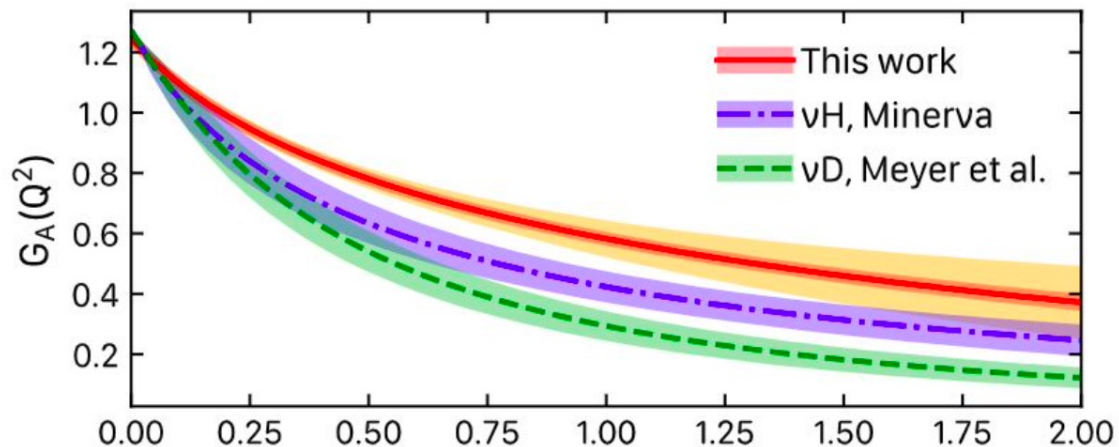
Computational Scientist  
 CaSToRC, The Cyprus Institute  
 National Competence Center in HPC

31/10/23 - EINN

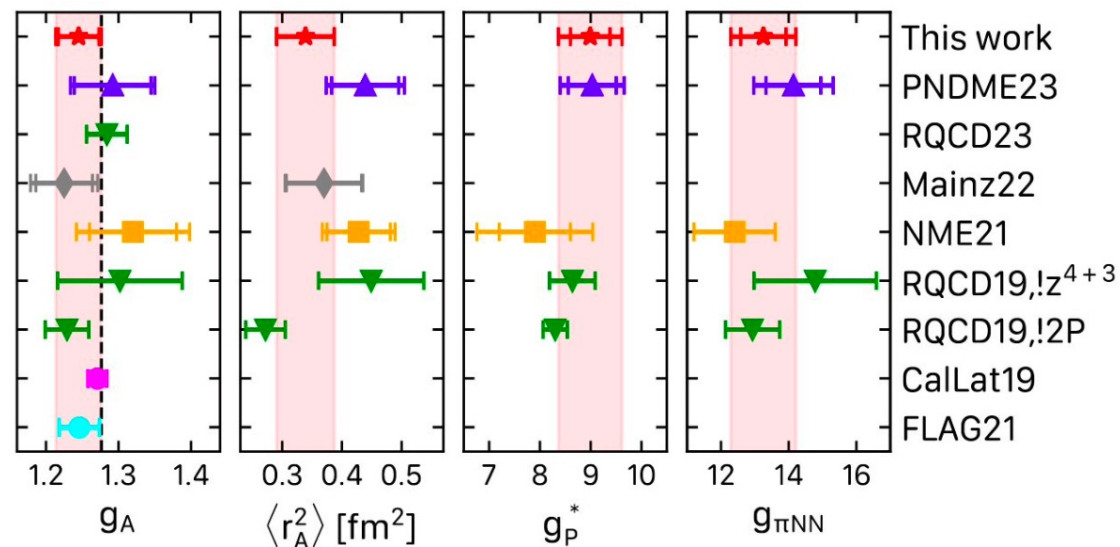
$$\langle N(p', s') | A_\mu | N(p, s) \rangle = \bar{u}_N(p', s') \left[ \gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N} G_P(Q^2) \right] \gamma_5 u_N(p, s)$$



# Conclusions

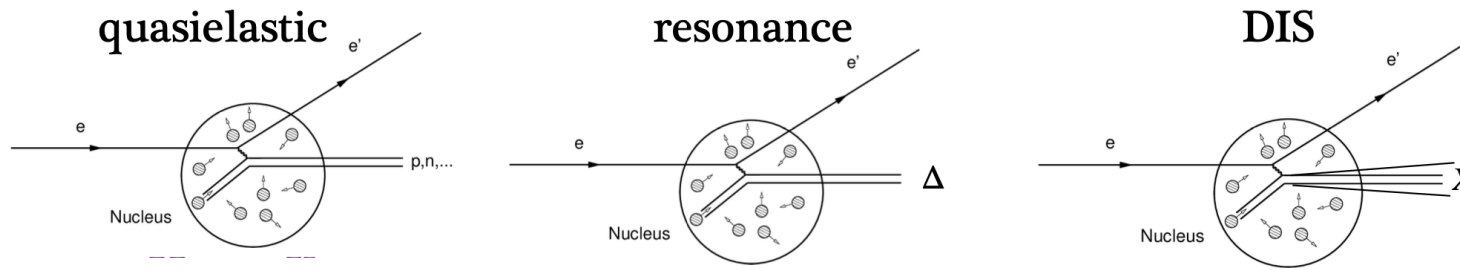


- Overall good agreement between recent lattice results and better agreement with the very recent results from Minerva

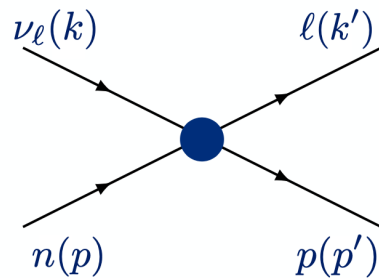


# Interaction mechanisms

O. Tomalak



## CCQE scattering on free nucleon



$$\nu = E_\nu/M - \tau - r^2$$

$$r = \frac{m_\ell}{2M} \quad \tau = \frac{Q^2}{4M^2}$$

unpolarized cross section

$$\frac{d\sigma}{dQ^2} \sim \frac{M^2}{E_\nu^2} \left( (\tau + r^2) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1 + \tau} C(Q^2) \right)$$

Llewellyn Smith (1972)

- structure-dependent functions

$$A = \tau (G_M^V)^2 - (G_E^V)^2 + (1 + \tau) F_A^2 - r^2 \left( (G_M^V)^2 + F_A^2 - 4\tau F_P^2 + 4F_A F_P \right)$$

$$B = \pm 4\tau F_A G_M^V$$

$$C = \tau (G_M^V)^2 + (G_E^V)^2 + (1 + \tau) F_A^2$$

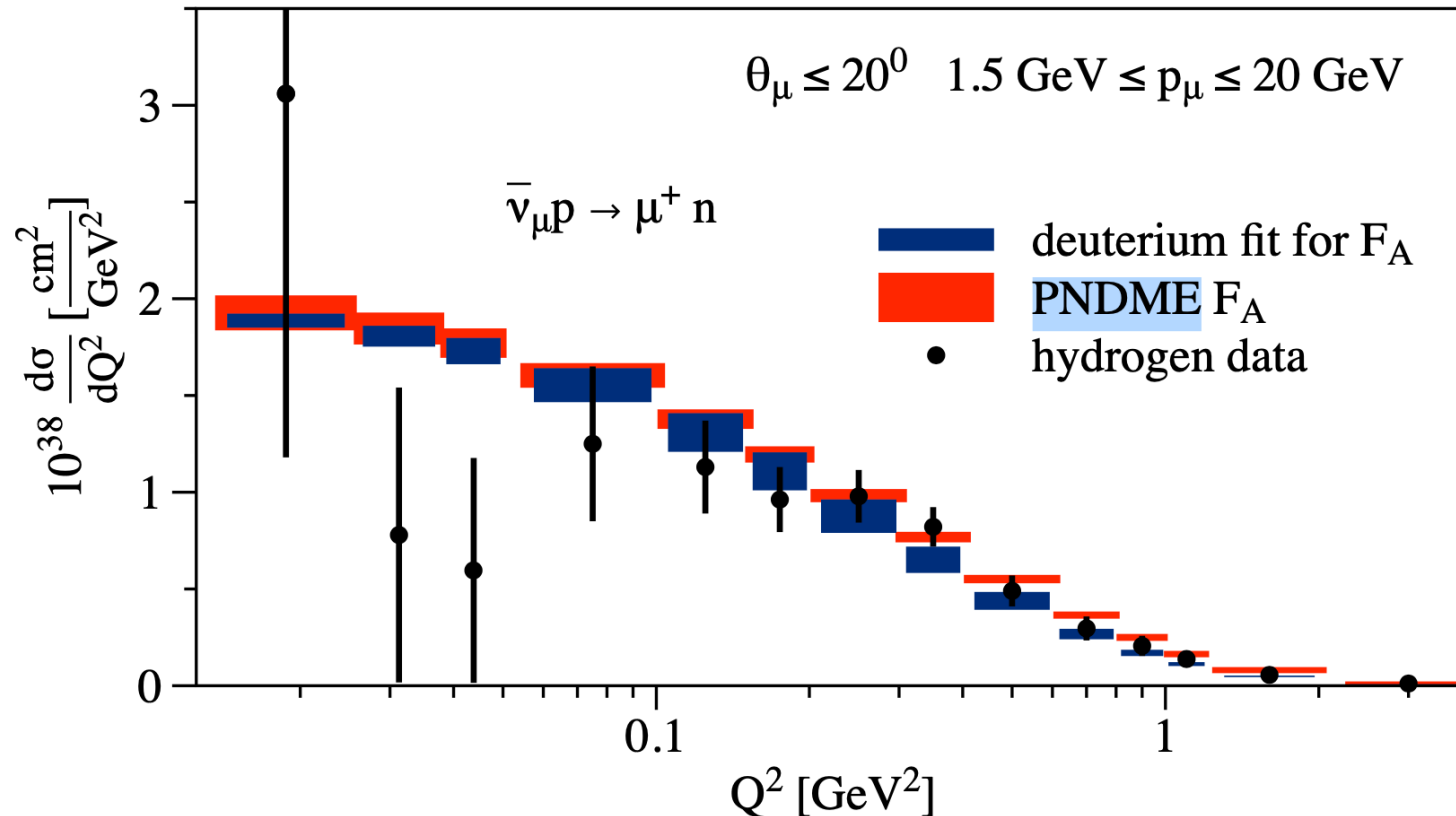
- pseudoscalar form factor contribution is suppressed by lepton mass
- cross section is sensitive to both vector and axial contributions



# Lattice QCD vs MINERvA

O. Tomalak

- PNDME 2023 axial-vector form factor as representative of lattice QCD



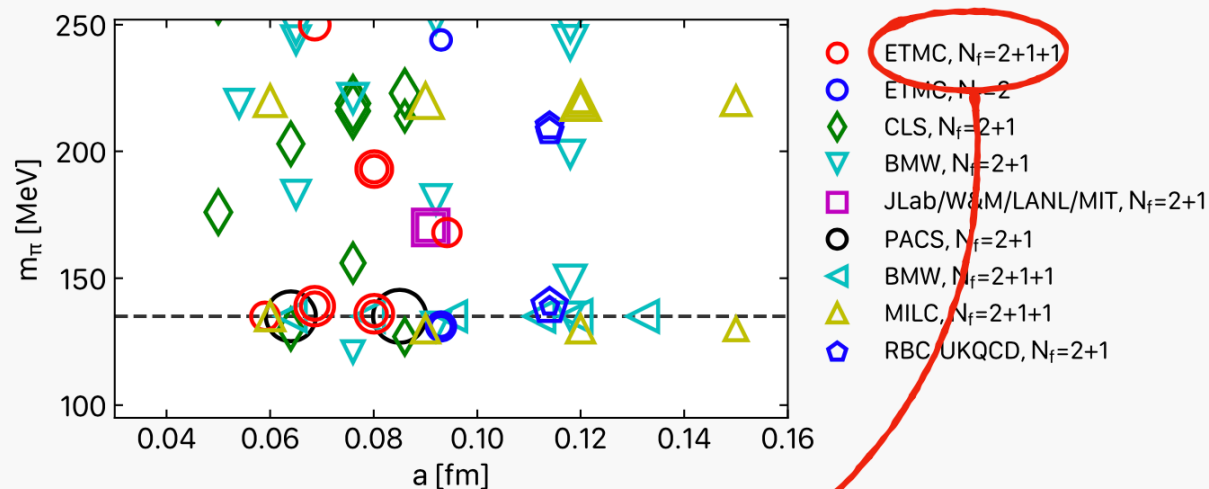
- $\sim 1\sigma$  agreement for each bin besides two at small  $Q^2$

- 2-3 $\sigma$  tension between lattice QCD and deuterium data
- MINERvA hydrogen data consistent with LQCD and deuterium

# Ensembles

G. Koutsou

Landscape of ensembles used for nucleon structure



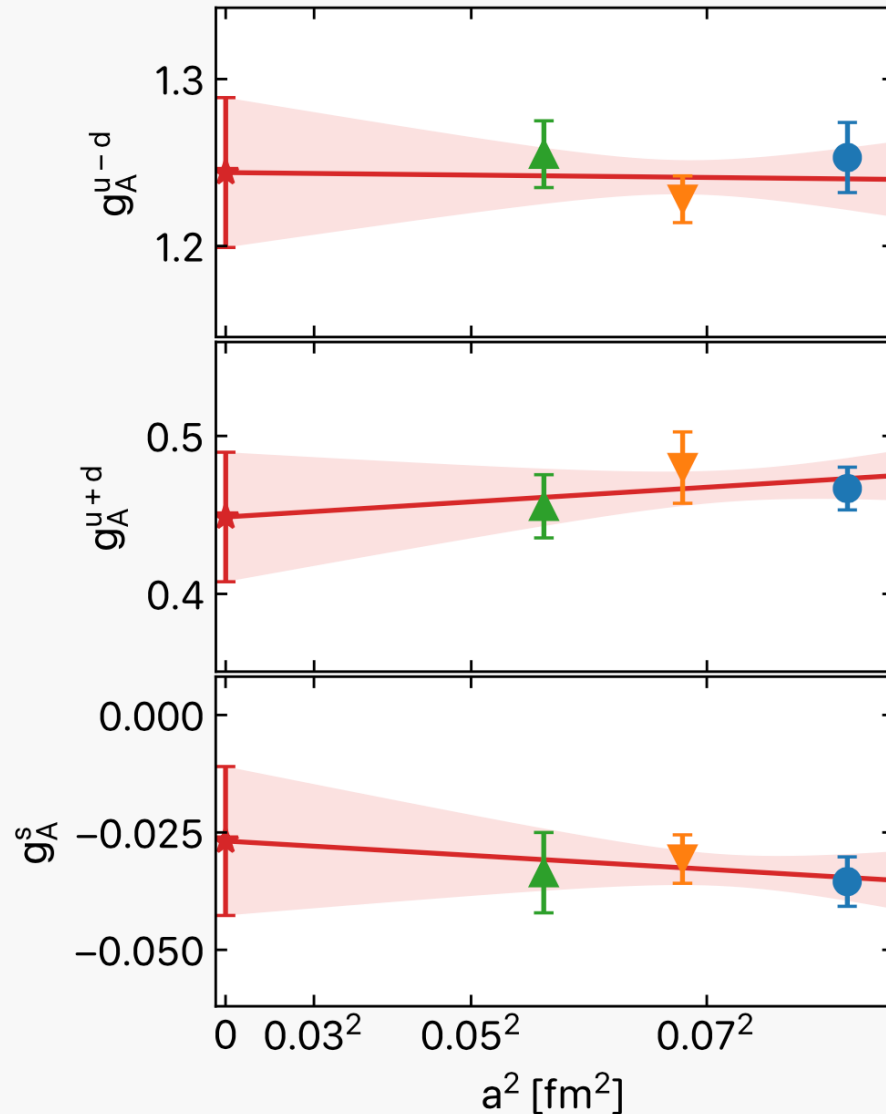
ETMC: three  $N_f=2+1+1$  ensembles at physical pion mass

| Ens. ID (abbrv.)    | Vol.            | $a$ [fm] |
|---------------------|-----------------|----------|
| cB211.072.64 (cB64) | $64 \times 128$ | 0.080    |
| cC211.060.80 (cC80) | $80 \times 160$ | 0.068    |
| cD211.054.96 (cD96) | $96 \times 192$ | 0.057    |

- Three lattice spacings at physical point
- Ongoing generation of finer ensembles and larger volumes
- **This talk:** 3 ensembles with:  
 $a = 0.057 - 0.068$  fm

# Nucleon axial charge

G. Koutsou



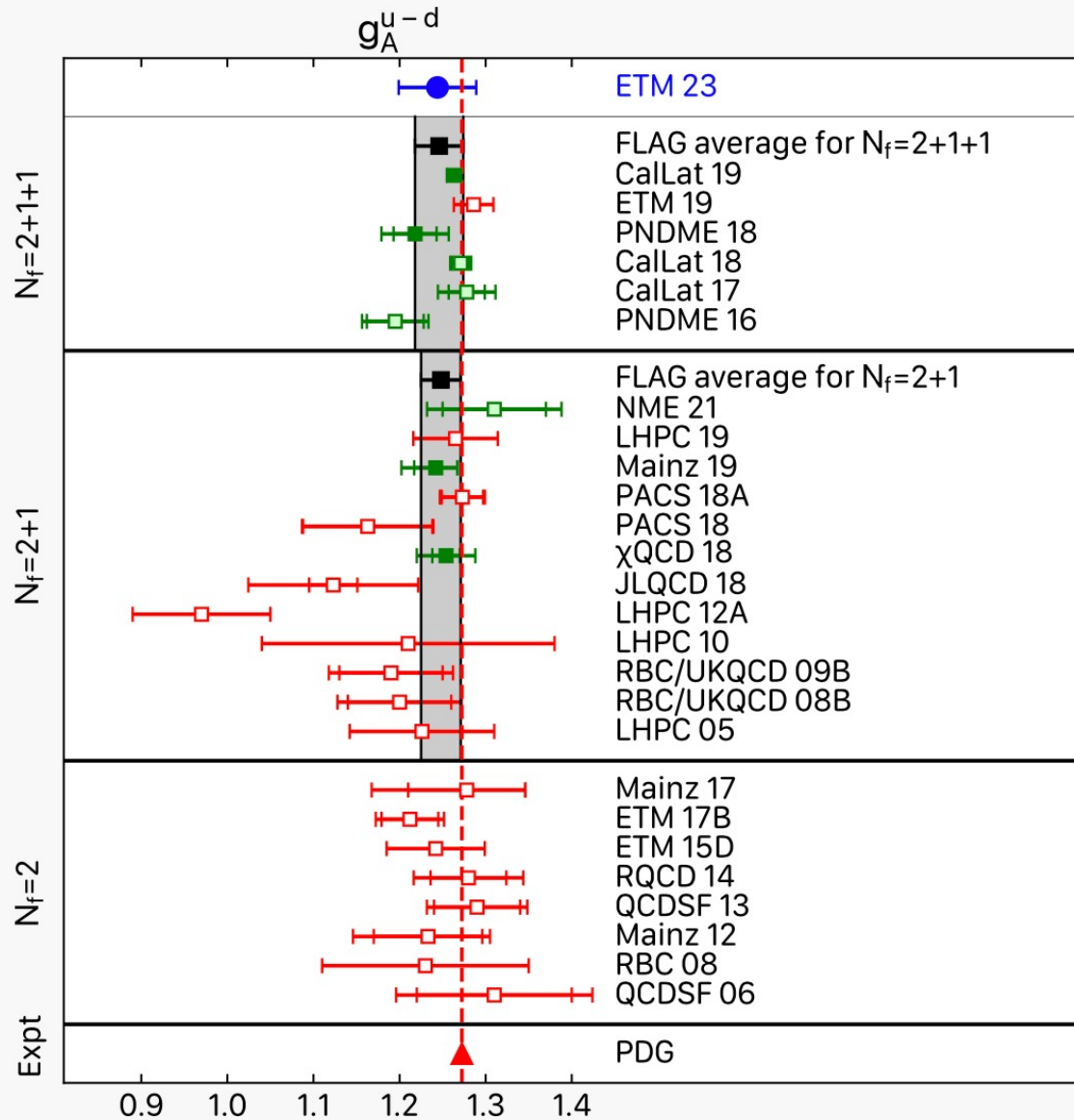
See arXiv:2309.05774 [hep-lat];

Includes isovector axial form-factors

- Errors for each ensemble include *statistical* and *systematic* due to excited state contamination
- Model averaged based on AIC (see e.g. arXiv:2208.14983)

Preliminary!

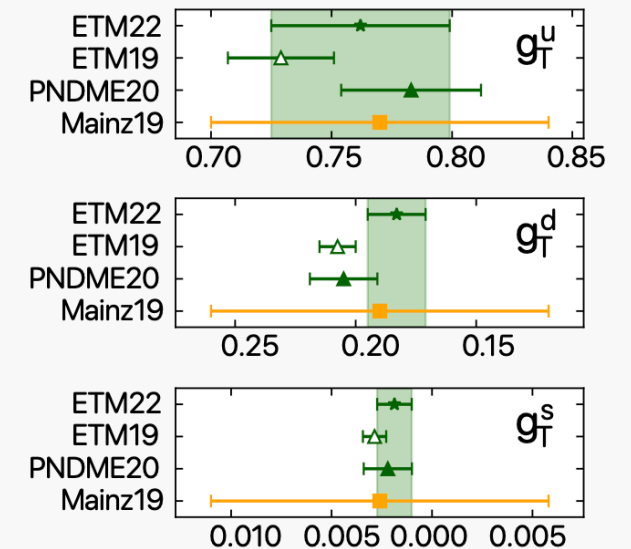
Complete analysis for flavour separated charges ongoing



Latest FLAG21 values

- ETM23 consistent with FLAG average
- Only result with three physical point ensembles
- Agreement for  $g_A$  means confidence for more challenging quantities
- E.g.

tensor charge



Preliminary

# Investigation of two-particle contributions to nucleon matrix elements

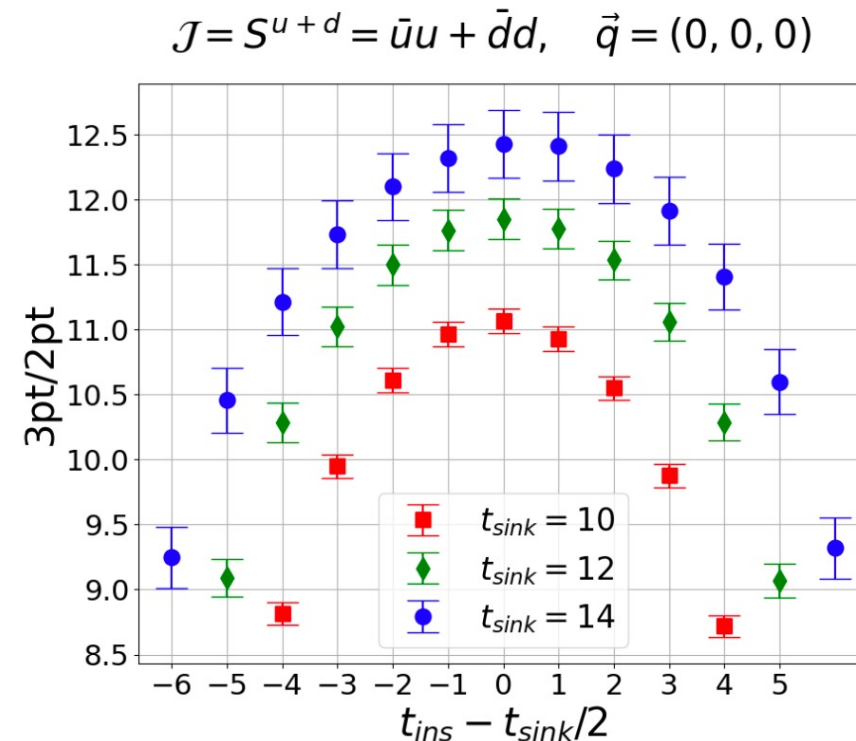
Yan Li

- Nucleon structure: nucleon matrix elements

$$\frac{\langle 0 | O_N(t_{\text{sink}}) J(t_{\text{ins}}) \bar{O}_N(0) | 0 \rangle}{\langle 0 | O_N(t_{\text{sink}}) \bar{O}_N(0) | 0 \rangle} \xrightarrow{\text{all } t \text{ well-separated}} \langle N | J | N \rangle$$

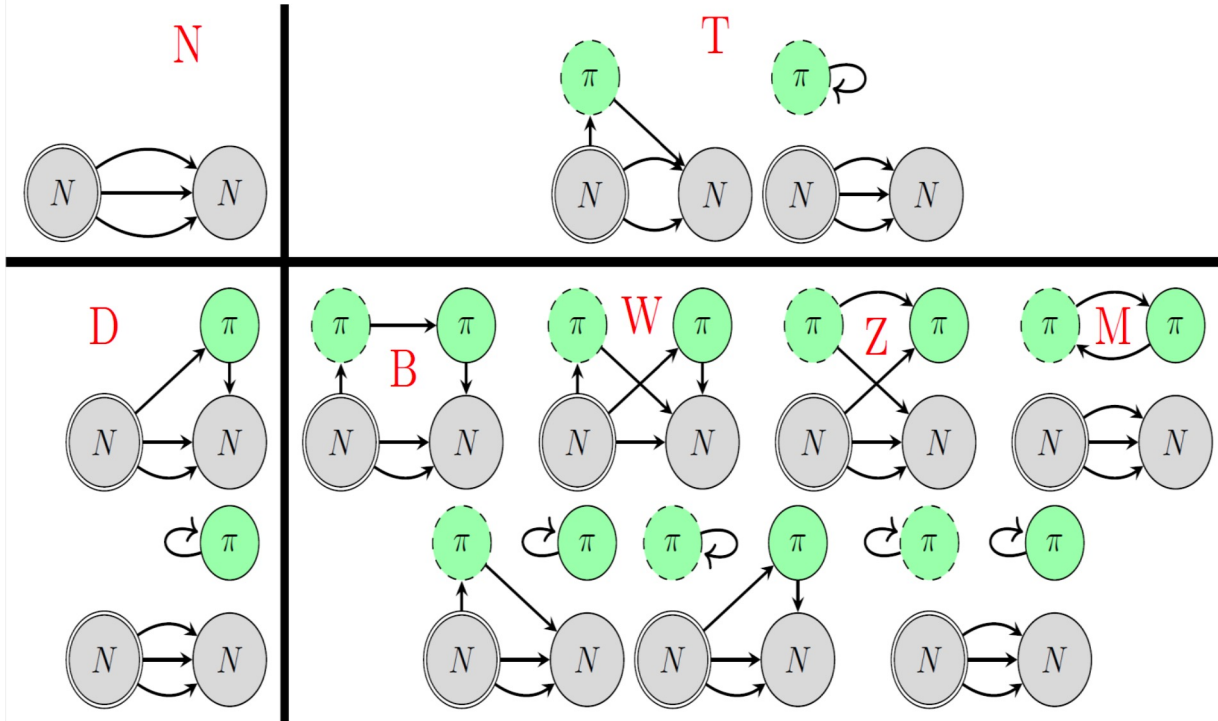
- Time-dependence indicates contamination from excited states

- Lowest excited state is a Nucleon-Pion state

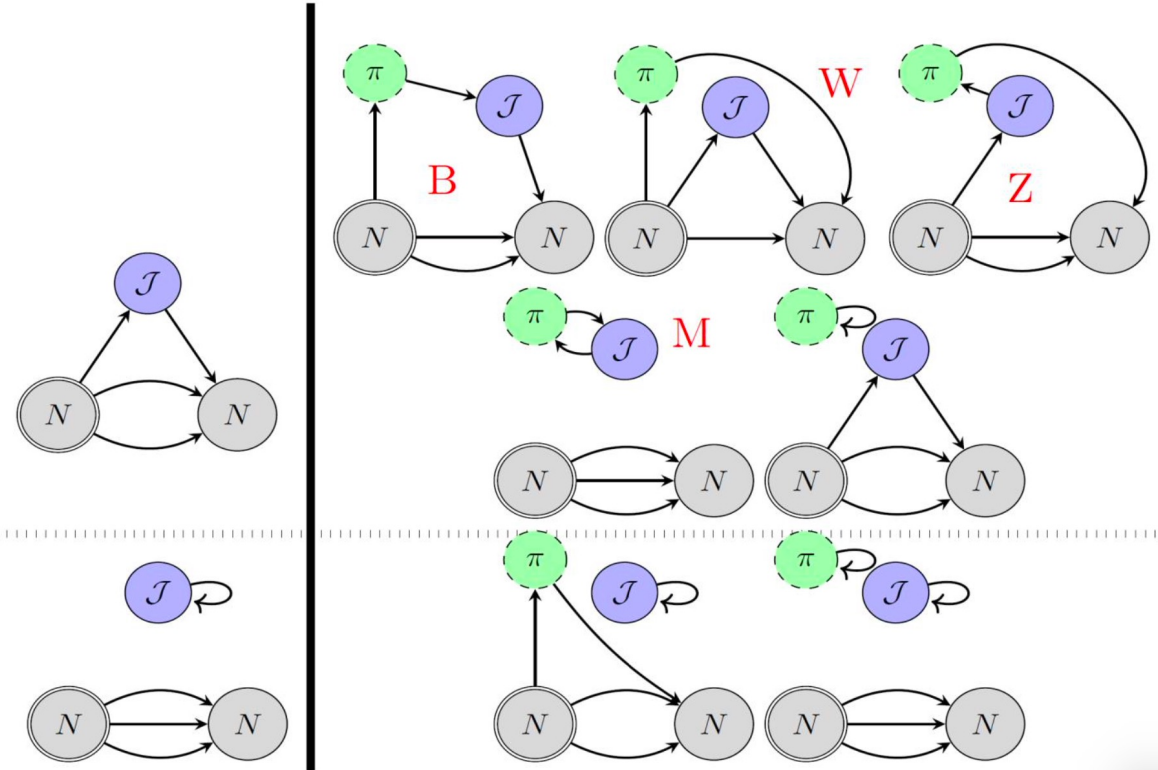




## Diagrams: 2pt functions



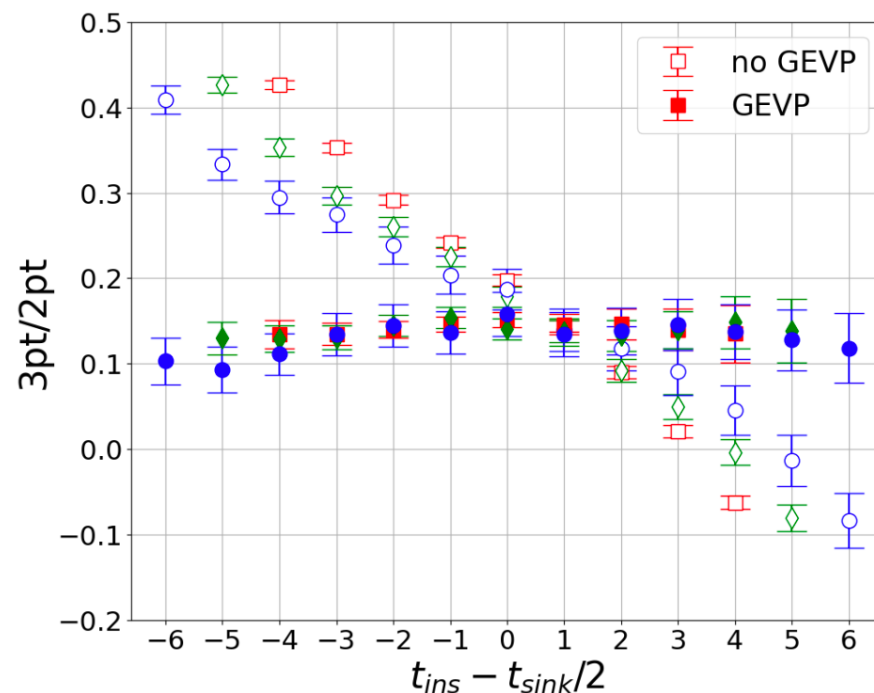
## Diagrams: 3pt functions



Generalized eigenvalue problem (GEVP)

GEVP improvement on 3pt functions:

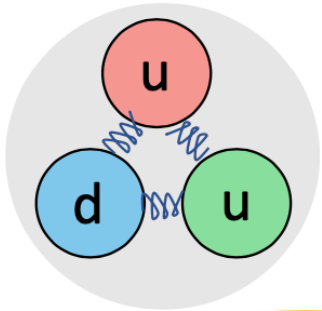
$$J = A_4^{u-d} = \bar{u}\gamma_5\gamma_4u - \bar{d}\gamma_5\gamma_4d; \quad \vec{q} = (0, 0, 1)$$



$$m_\pi = 346 \text{ MeV}$$

# Gravitational form factors - proton

D. Pefkou



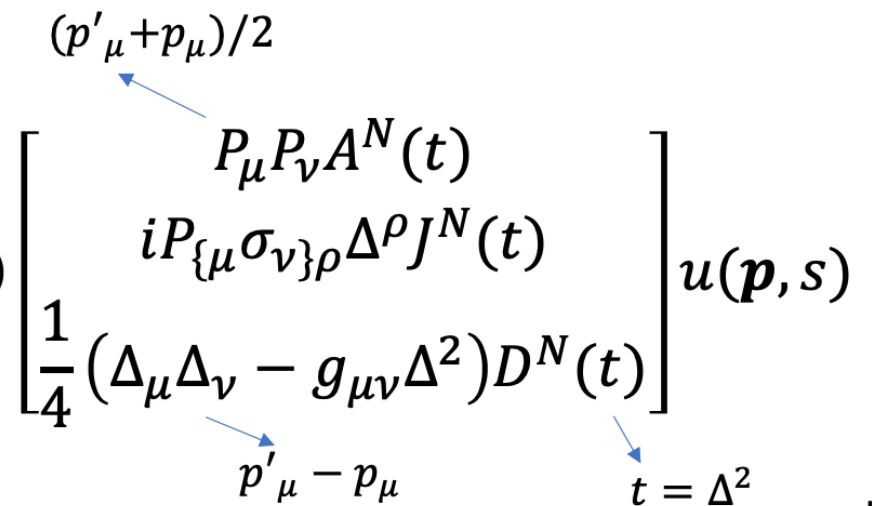
**EMT**

$$\langle N(p', s') | T_{\mu\nu} | N(p, s) \rangle = \frac{1}{m_N} \bar{u}(\mathbf{p}', s') \left[ \begin{array}{c} P_\mu P_\nu A^N(t) \\ iP_{\{\mu\sigma\nu\}\rho} \Delta^\rho J^N(t) \\ \frac{1}{4} (\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2) D^N(t) \end{array} \right] u(\mathbf{p}, s)$$

*t* –dependence:

much less is known

$$\begin{aligned} A^N(t) &= A_g^N(t) + A_q^N(t), & A^N(0) &= 1 \\ J^N(t) &= J_g^N(t) + J_q^N(t), & J^N(0) &= 1/2 \\ D^N(t) &= D_g^N(t) + D_q^N(t), & D^N(0) &= ? \end{aligned}$$



this talk

Lattice A+J+D for q + g : [Hackett DAP Shanahan 2310.08484](#)

Experiment (A+D):  
[Duran Meziani et al Nature 2023](#)

Lattice (A+J+D):  
[Shanahan Detmold PRL 2018,](#)  
[DAP Hackett Shanahan PRD 2022](#)

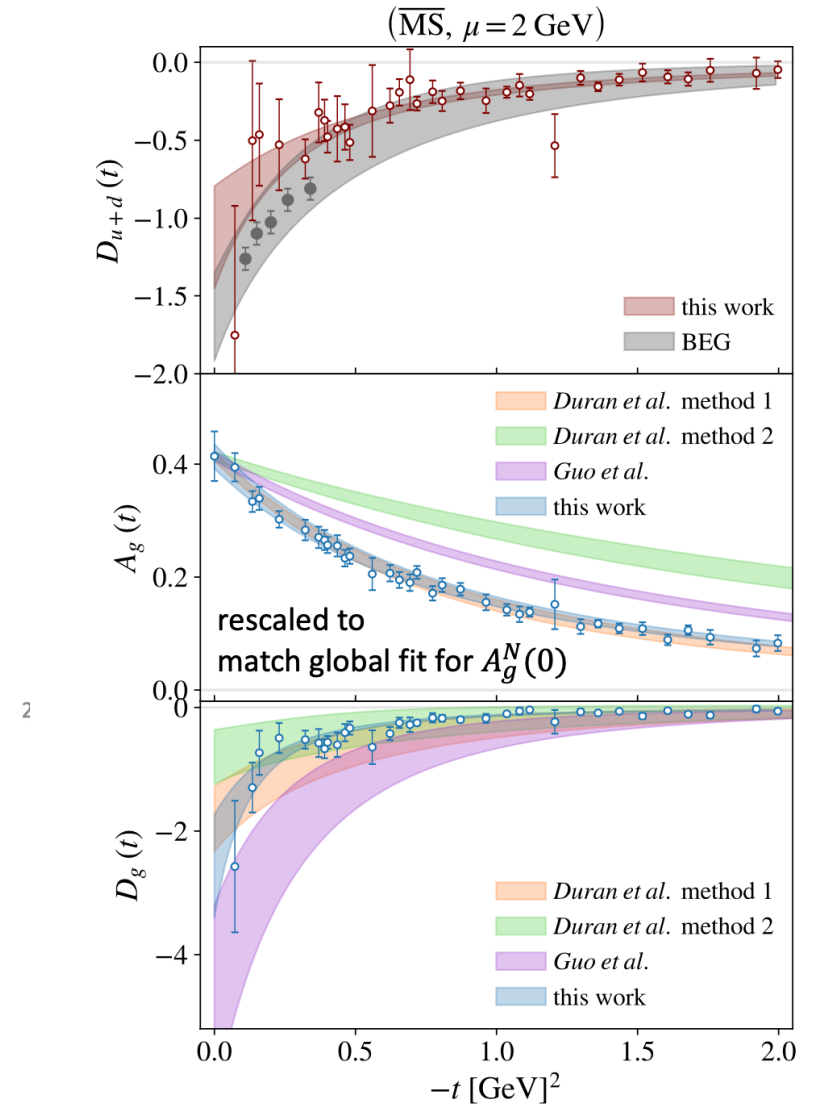
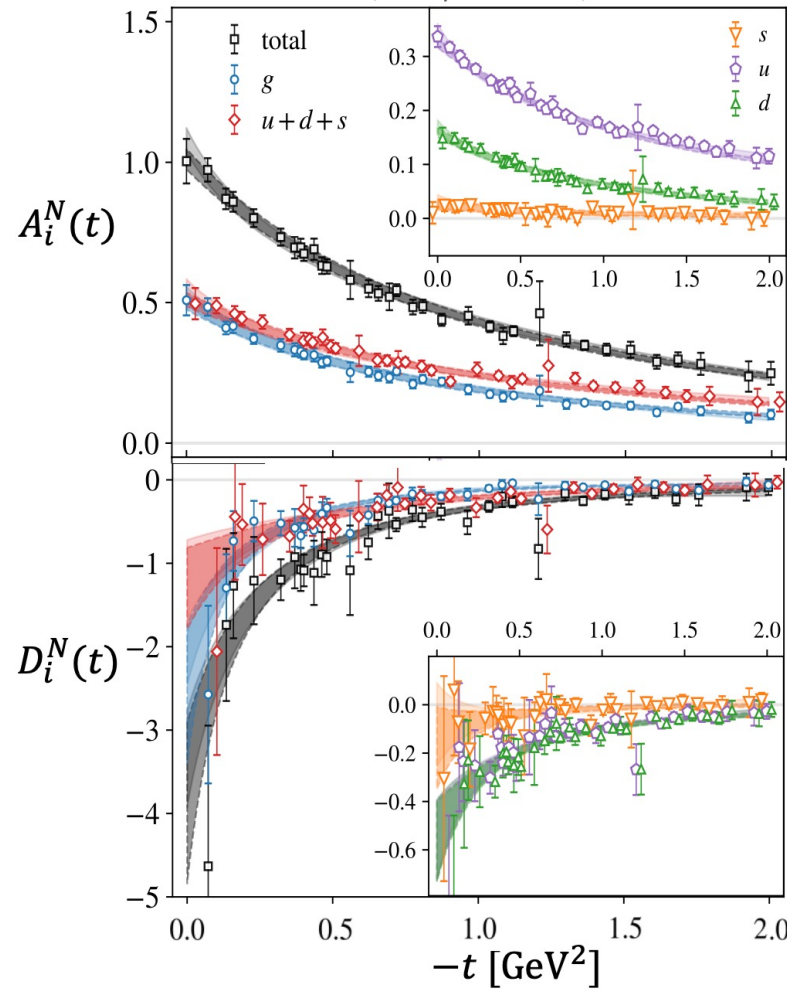
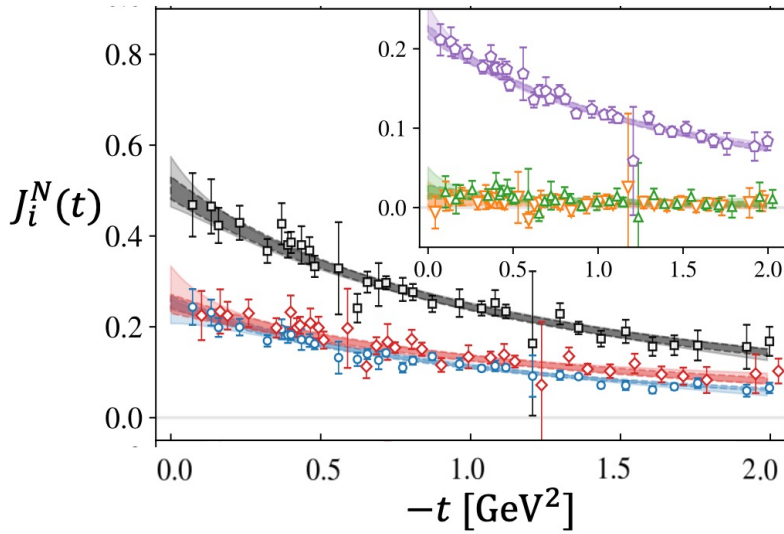
Lattice (A+J+D):  
[ETMC PRD 2020,](#)  
[LHPC PRD 2008](#)

Experiment (D):  
[Burkert Elouadrhiri Girod Nature 2018](#)  
Dispersive (D):  
[Pasquini et al 2014](#)

+ more from theory and models  
(see [2023 Colloquium Burkert et al](#) for review)

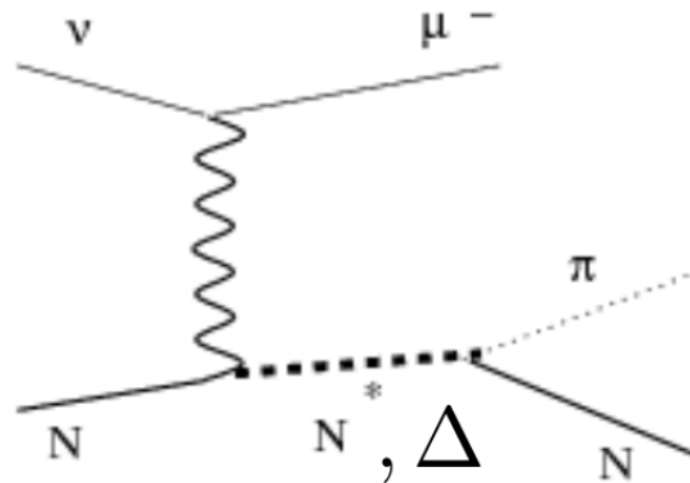
# Renormalized nucleon GFFs

D. Pefkou



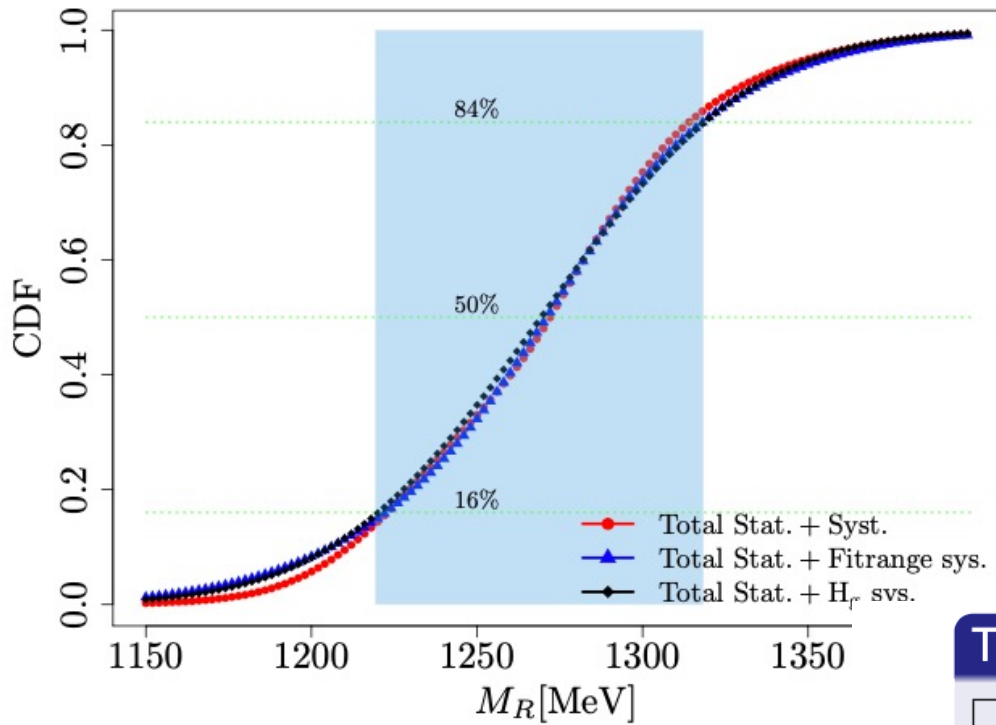
## Delta resonance

- $\Delta(1232)$
- Lightest baryonic resonance
- Dominant in the  $p$  wave  $N\pi$  scattering
- Simplest resonance: 3 quark and  $N\pi$  contribution
- Resonances are not eigenstates of the QCD Hamiltonian



- They decay via strong interactions
- Finite volume influences the two-hadron spectrum





This work (Details [arXiv:2307.12846])

| Ensemble       | $m_\pi$ [MeV] | $L$     | $m_\Delta$ [MeV] | $g_{\Delta-\pi N}$ |
|----------------|---------------|---------|------------------|--------------------|
| Twisted-Clover | 139 MeV       | 5.12 fm | 1267(46) MeV     |                    |
| Nf2+1 Clover   | 200 MeV       | 3.7 fm  | 1320(10) MeV     | 17.6(2.7)          |
| Nf2+1 Clover   | 250 MeV       | 2.8 fm  | 1380(7) MeV      | 13.6(5)            |
| Nf2+1 Clover   | 250 MeV       | 3.7 fm  | 1373(6) MeV      | 10.3(1.6)          |

| Collaboration            | $m_\pi$ [MeV] | Methodology                  | $m_\Delta$ [MeV]         | $g_{\Delta-\pi N}$      |
|--------------------------|---------------|------------------------------|--------------------------|-------------------------|
| Verduci(2014)            | 266           | Distillation, Lüscher        | 1396(19)                 | 19.9(8)                 |
| Alexandrou et.al. (2013) | 360           | LO pert., Michael & McNeile  | 1535(25)                 | 26.7(1.5)               |
| Alexandrou et.al. (2015) | 180           | LO pert., Michael & McNeile  | 1350(50)                 | 23.7(1.3)               |
| Andersen et.al. (2017)   | 280           | Stoch. distillation, Lüscher | 1344(20)                 | 37.1(9.2)               |
| Morningstar et.al.(2022) | 200           | Stoch. distillation, Lüscher | 1290(7)                  | 14.41(53) <sub>BW</sub> |
| Silvi et.al. (2021)      | 255           | Smearred sources, Lüscher    | 1380(7)(9) <sub>BW</sub> | 13.6(5) <sub>BW</sub>   |

$$\mathcal{W}(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$\int d^2 b_\perp$$

$$\int d^2 k_\perp$$

$$f(x, \vec{k}_\perp)$$

$$F(x, \vec{b}_\perp)$$

**TMD**

**“impact par. PDF”**

$$\int d^2 k_\perp$$

$$\int d^2 b_\perp$$

$$\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$$

$$q(x)$$

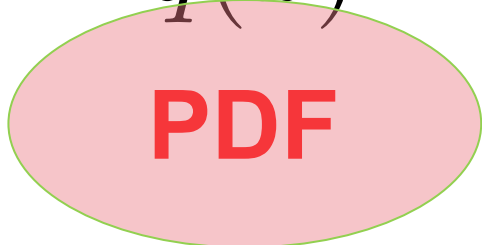
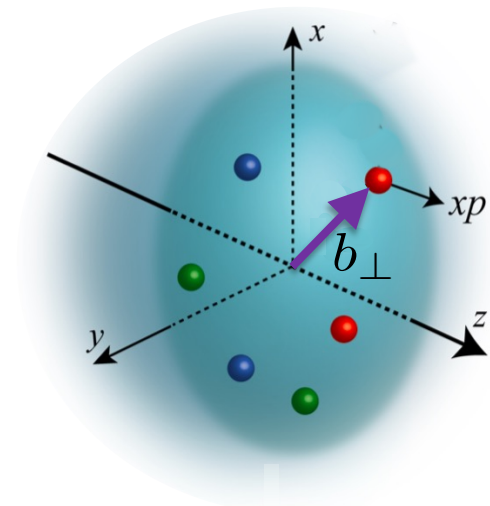
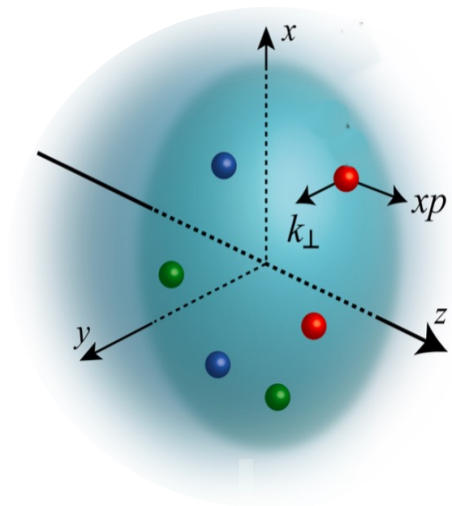
**PDF**

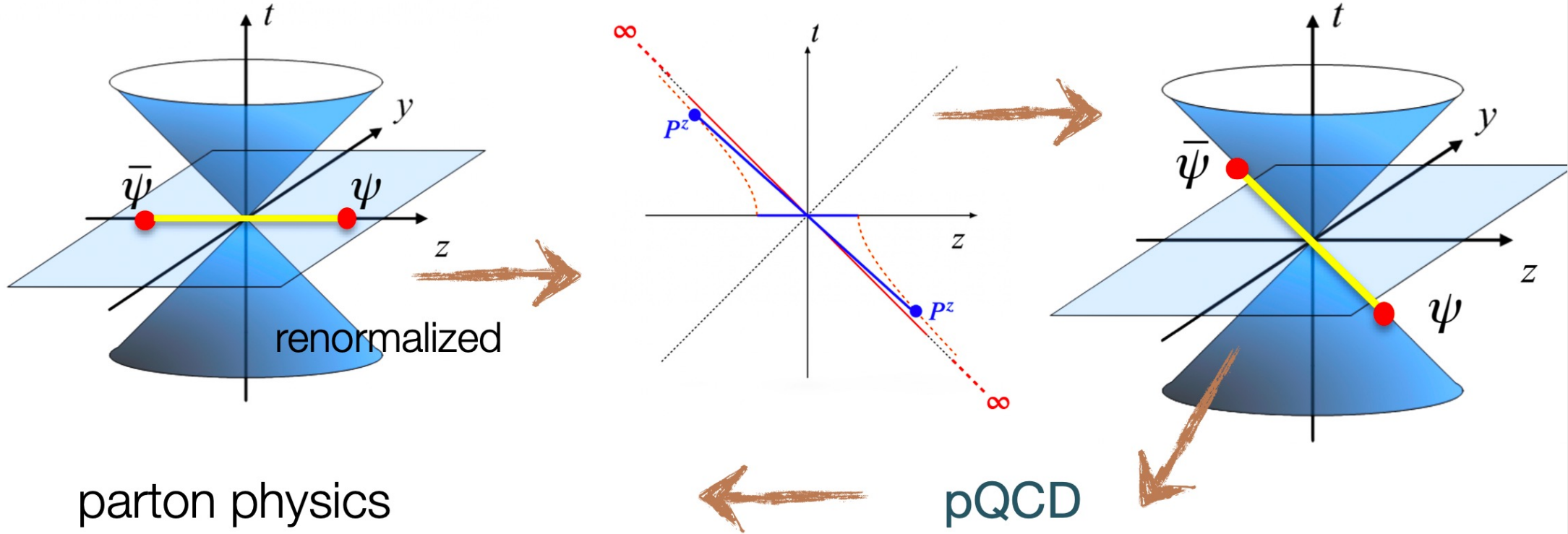
**GPD**

( $\xi = 0$ )

$$\int dx$$

**FF**





■ difference is UV physics, can be taken care of through pQCD matching

$$+\mathcal{O}\left[\frac{\Lambda_{\text{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\text{QCD}}}{(1-x)P_z}, \frac{M_H^2}{P_z^2}, \dots\right]$$

$$+\mathcal{O}\left[z^2 \Lambda_{\text{QCD}}^2, z^2 M_H^2, \dots\right]$$

$C(x, P_z, \mu) \otimes$  momentum space

$C(\alpha, z^2, \mu) \otimes$  position space

# pion valance PDF

NNLO momentum matching

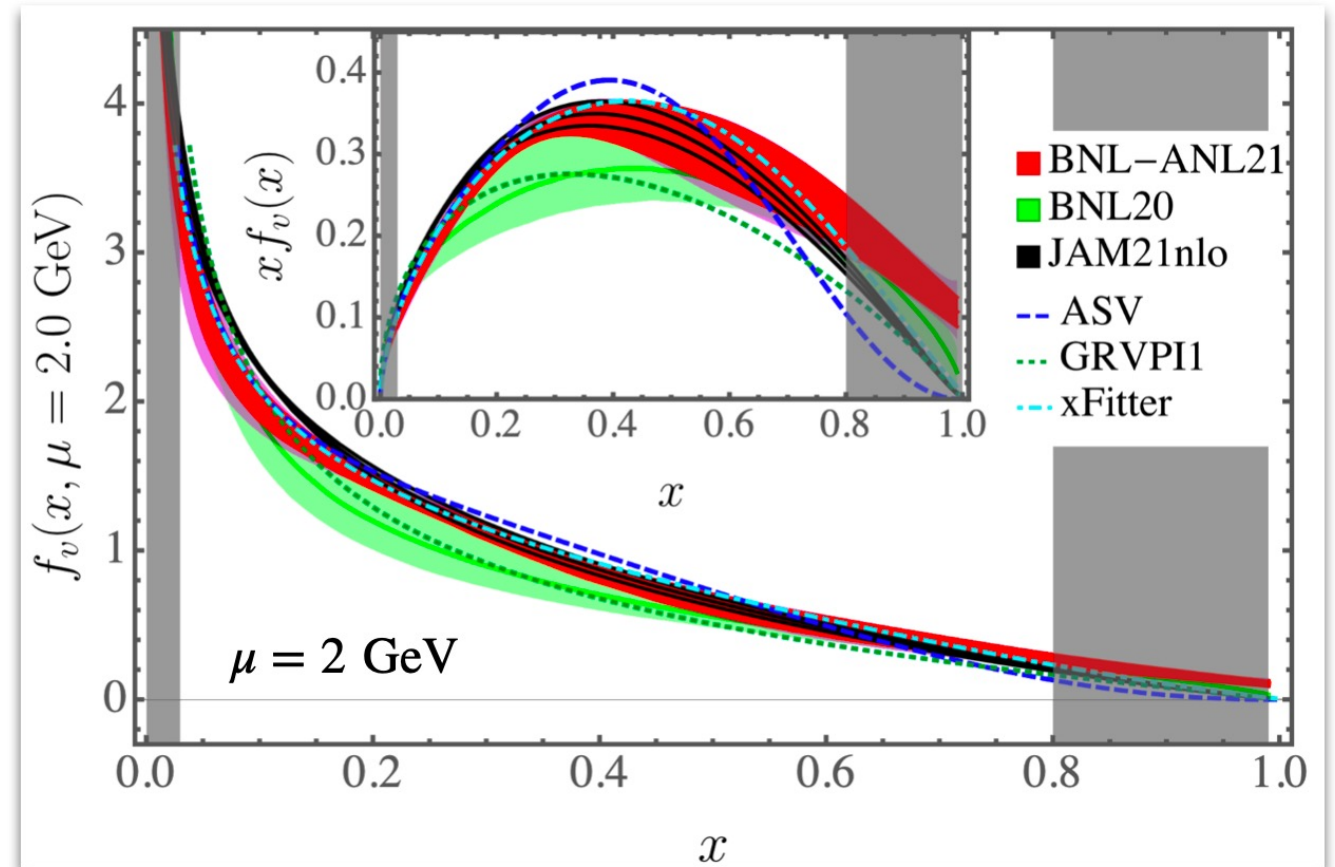
valance pion mass 300 MeV

lattice spacing 0.04 fm

pion momenta up to 2.4 GeV

first LQCD PDF at NNLO

Yong Zhao *et al.*, *Phys.Rev.Lett.* 128 (2022) 14, 142003



# proton unpolarized isovector PDF

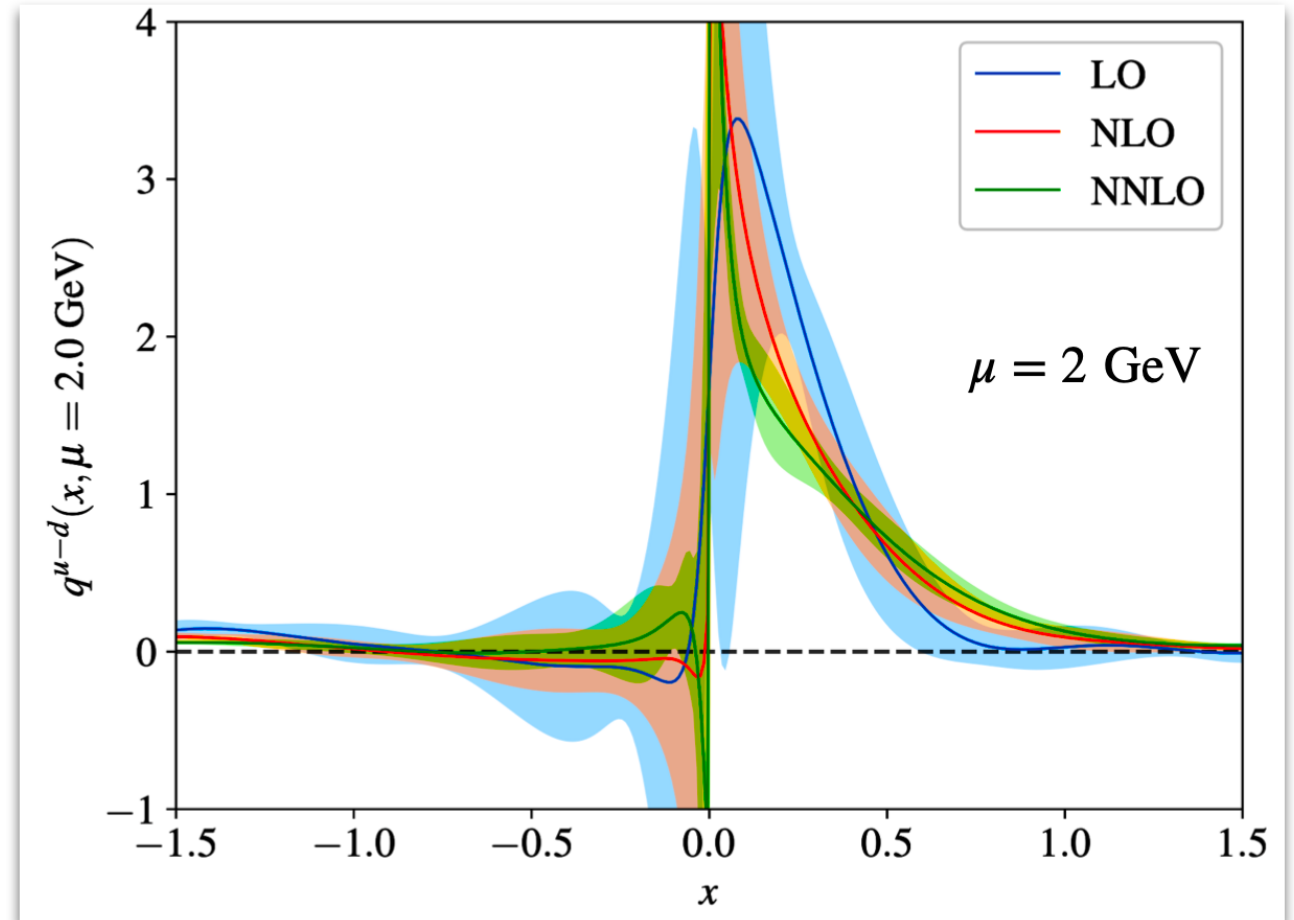
Andrew Hanlon *et al.*, *Phys.Rev.D* 107 (2023) 7, 074509

NNLO momentum matching

physical pion mass

lattice spacing 0.075 fm

proton momenta  $\leq 1.53$  GeV



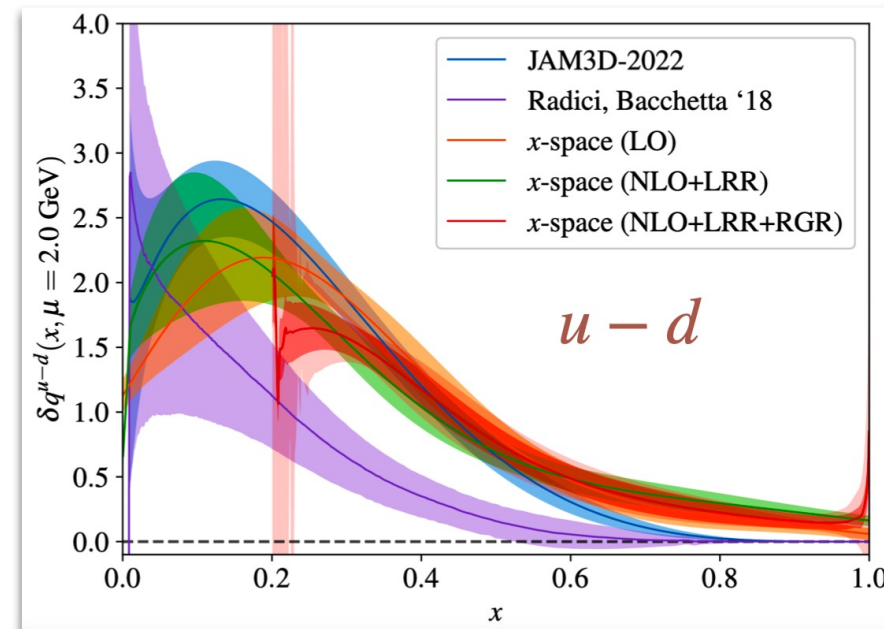
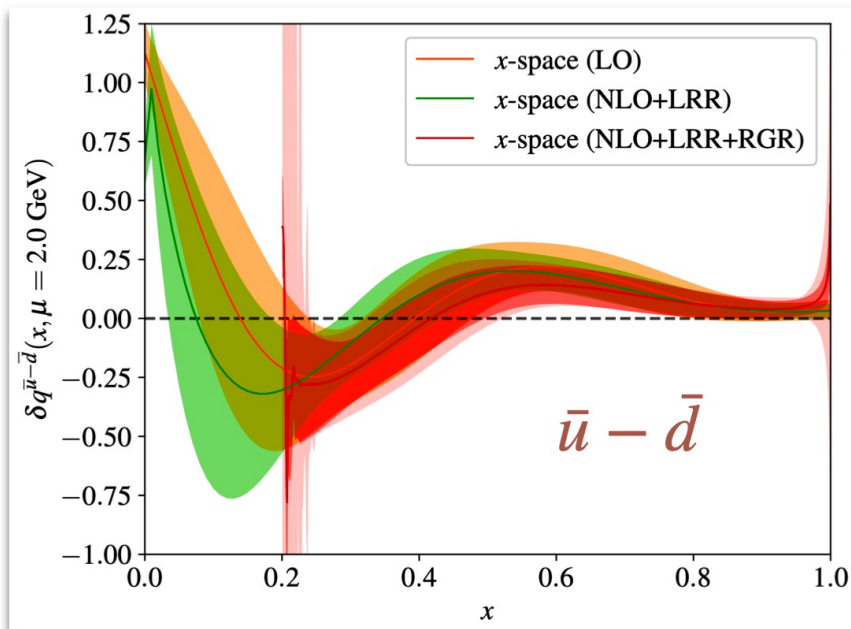


$$C(\mathcal{S}, \mu) \sim \alpha_s^0(\mu) + \alpha_s(\mu)f(\ln[\mathcal{S}\mu]) + \alpha_s^2(\mu)f(\ln[\mathcal{S}\mu]) + \dots$$

$$\mathcal{S} \sim 1/\mu \longrightarrow \ln[\mathcal{S}\mu] \text{ large} \quad \mathcal{S} = 2xP_z, z^2$$

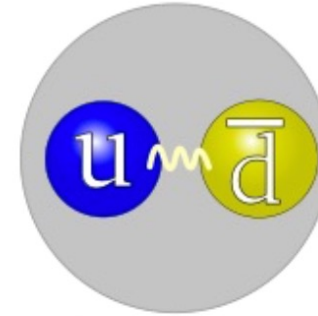
### proton transversity PDF

Andrew Hanlon *et al.*, arXiv:2310.19047



**NLO+LRR+NLL(RGR)** momentum matching

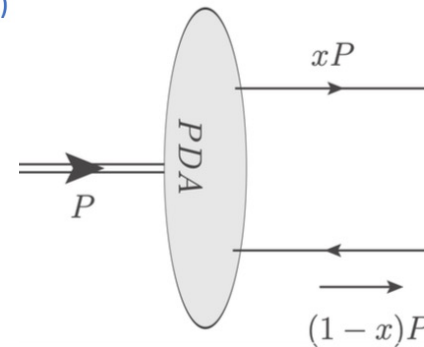
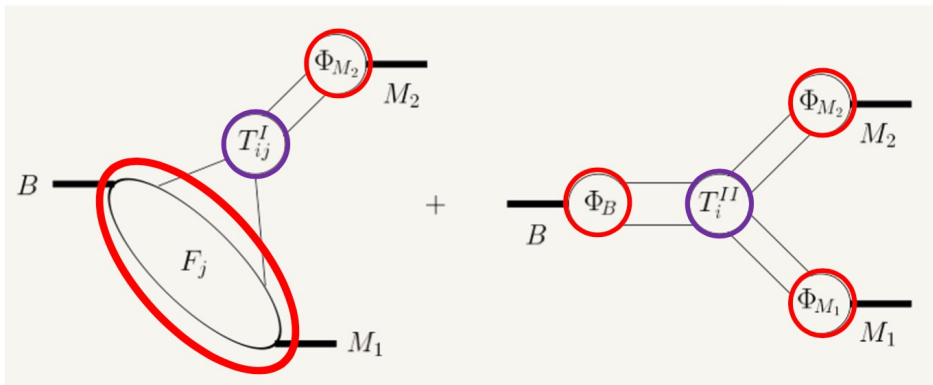
# Pion Distribution Amplitude (DA)



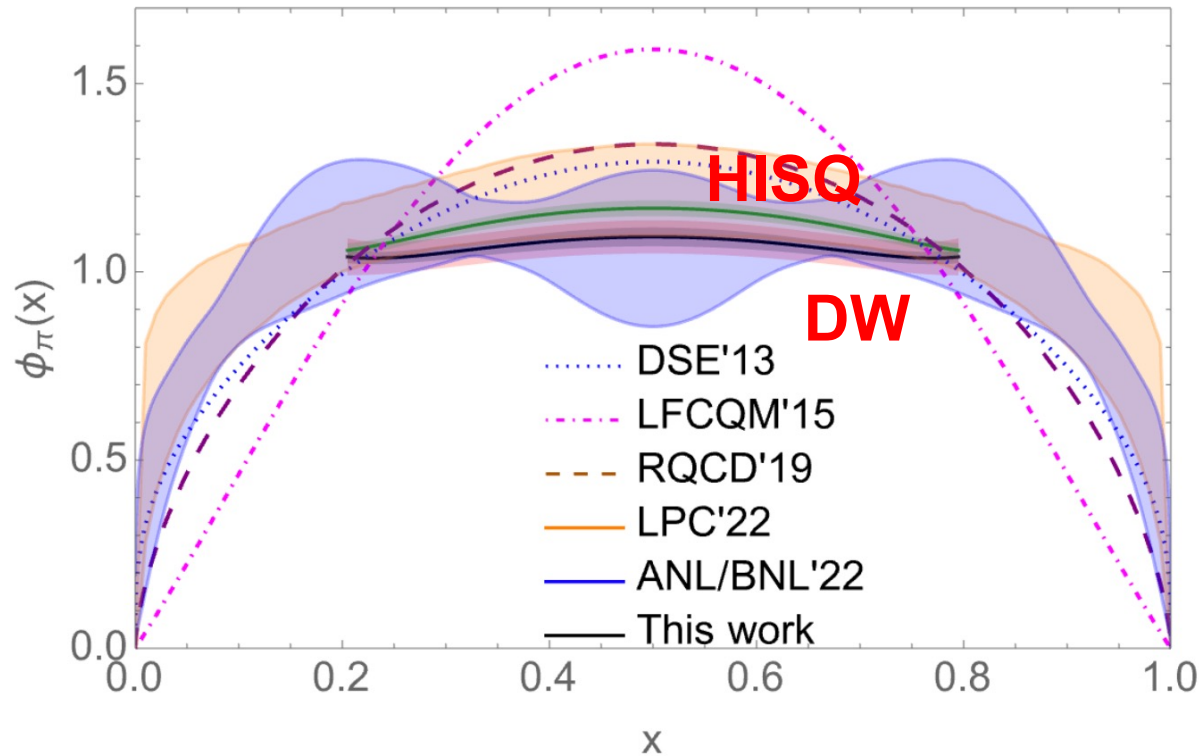
Pion lightfront DA  $\phi(x)$ : **probability amplitude** of pion in the bound state's minimal Fock component  $|q\bar{q}\rangle$

$$\phi(x, \mu) = \frac{1}{if_\pi} \int \frac{d\xi^-}{2\pi} e^{i(\frac{1}{2}-x)\xi^- p^+} \langle 0 | \bar{q} \left( \frac{\xi^-}{2} \right) \gamma^- \gamma_5 U \left( \frac{\xi^-}{2}, -\frac{\xi^-}{2} \right) q \left( -\frac{\xi^-}{2} \right) | \pi(p) \rangle$$

DA as important input to hard exclusive process at  $Q^2 \gg \Lambda_{\text{QCD}}^2$ : [Beneke, et al. NPB\(2001\)](#)



# Comparison with previous results



- One possible explanation is that the explicit chiral-symmetry breaking term in HISQ action has a similar effect as making the meson heavier (thus has a more narrow distribution)

- A continuum limit study is needed for a more conclusive comparison
- Threshold resummation is needed for more exact matching

# The Intrinsic Charm hypothesis

G. Magni

- ▶ We define the charm content of the proton for  $Q < m_c$  in the  $n_f = 3$  flavour scheme as **Intrinsic Charm (IC)**.
- ▶ To determine IC we will need to **separate the perturbative component** of the charm PDF. Results are based on:

Allowing for **Intrinsic Charm** means:

$$f_c^{(3)}(x) \neq 0 \rightarrow f_c^{(4)}(x, m_c^2) = A_{cc} \otimes f_c^{(3)}(x) + \sum_{i=g,q} A_{c,i} \otimes f_i^{(3)}(x, m_c^2)$$

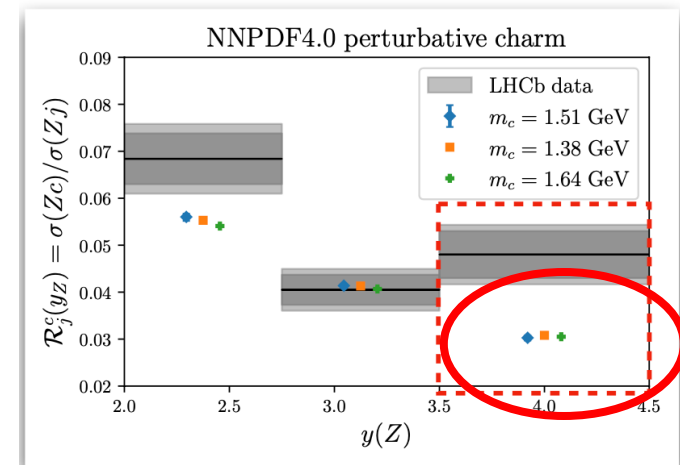
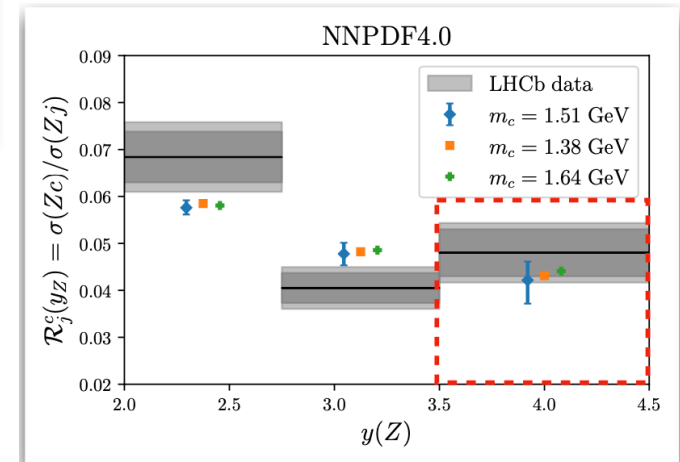
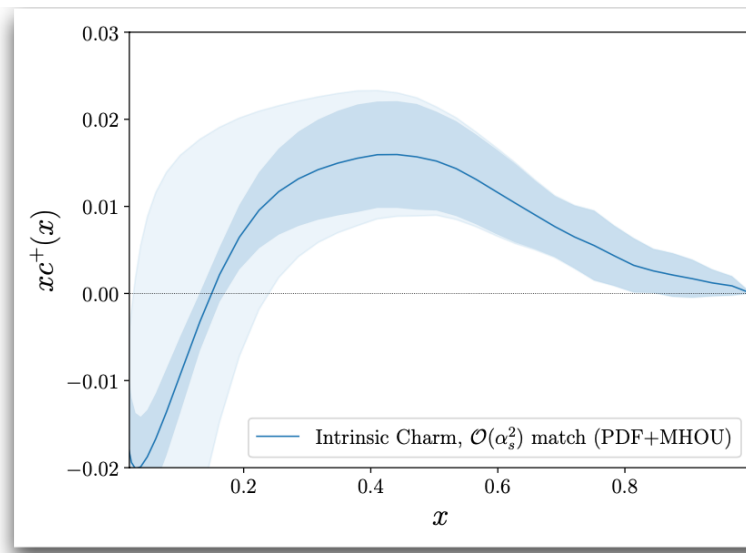
$f_c^{(4)}(x, Q)$  has to be treated as the other light flavour and **fitted to the data**.

# Probing Intrinsic Charm

G. Magni

## Z + c @ LHCb

- ▶ Starting from the fitting scale we **evolve** the NNPDF4.0 baseline to  $Q^2 = m_c^2$ .
- ▶ When passing the heavy quark threshold we need to **invert the matching conditions**  $A_{ij}$ .
- ▶ The **remaining part** of the charm PDF **is the intrinsic component**, which is scale independent for  $Q^2 < m_c^2$ .



The resulting **IC** in the  $n_f = 3$  scheme:

- ✓ Still contains **valence-like** peak.
- ✓ For  $x \leq 0.2$  the perturbative uncertainties are quite large.
- ✓ The carried **momentum fraction** is within **1%**.



## Current status of jets in polarized DIS

### Status on jet production in polarized DIS:

- Not much interest in fixed-target
- 1jet at NLO (N-jetiness)  
Boghezal, Petriello, Xing ('18)
- 2jets at NLO (dipoles)  
Photon - Borsa, de Florian, IP ('20)  
NC & CC - Borsa, de Florian, IP ('21)
- 1jet at NNLO (dipoles + P2B)  
Photon - Borsa, de Florian, IP ('20)  
NC & CC - Borsa, de Florian, IP ('23)

### Status on polarized inclusive DIS:

Structure function coefficients available at

- NNLO (photon -  $g_1$ )  
van Neerven, Zijlstra ('94)
- NNLO (NC & CC -  $g_1, g_4, g_5$ )  
Borsa, de Florian, IP ('22)
- N3LO (photon -  $g_1$ )\*
- Blumlein, Marquard, Schneider, Schönwald ('22)

# NNLO – Projection-to-Born method (P2B)

Obtain the **fully differential** cross section from

- The **inclusive** cross section at the desired order
- The exclusive cross section of the **observable + 1 jet at one lower order**

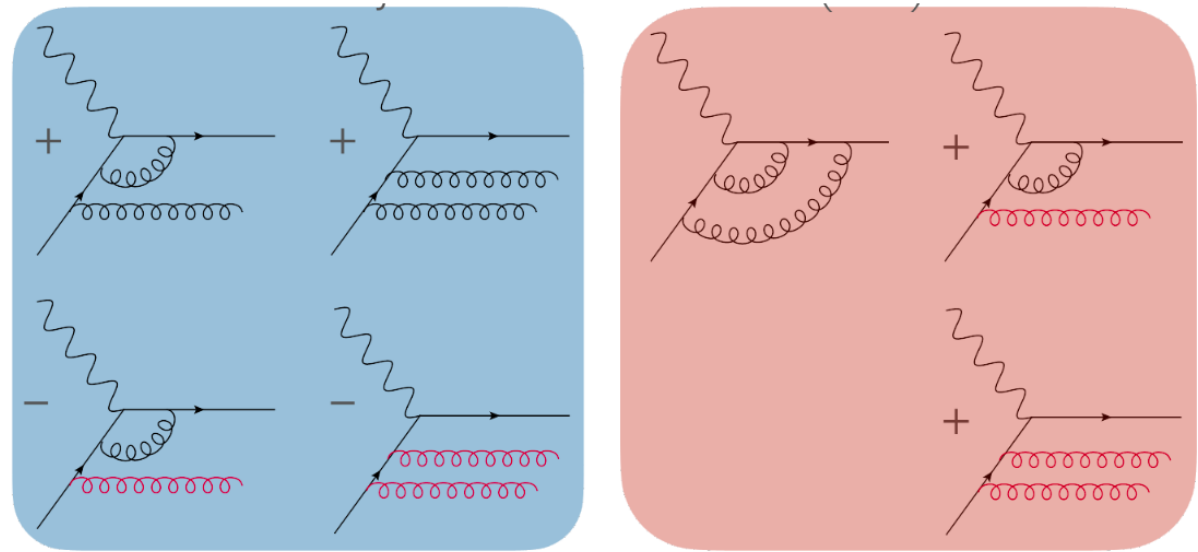
**In our case...**

Born kinematics mapping:

$$p_B = xP$$

$$p'_B = p_B + q$$

Not possible in the Breit-frame!!

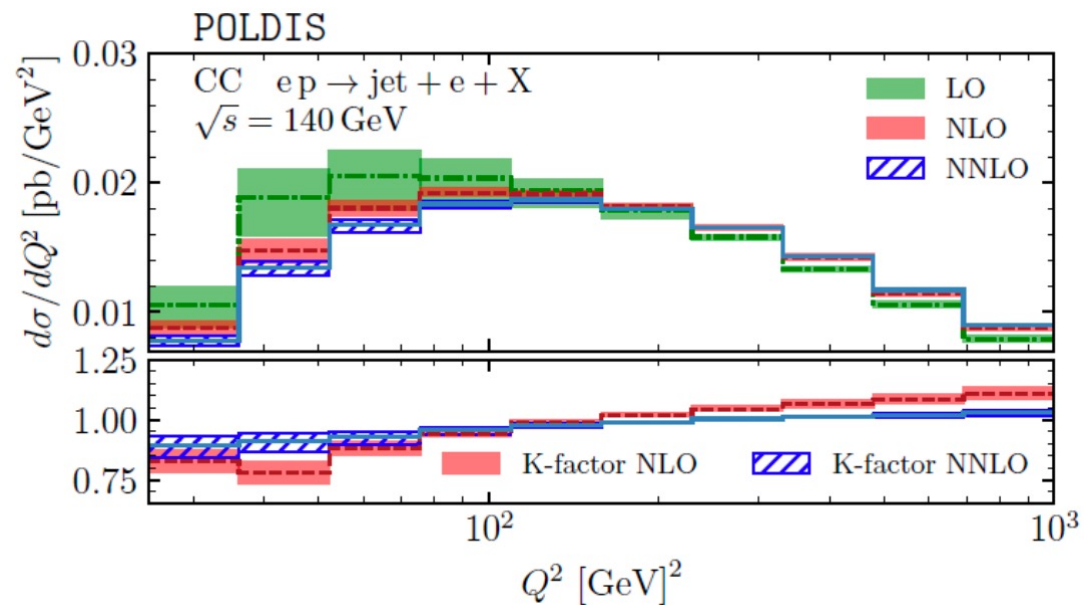
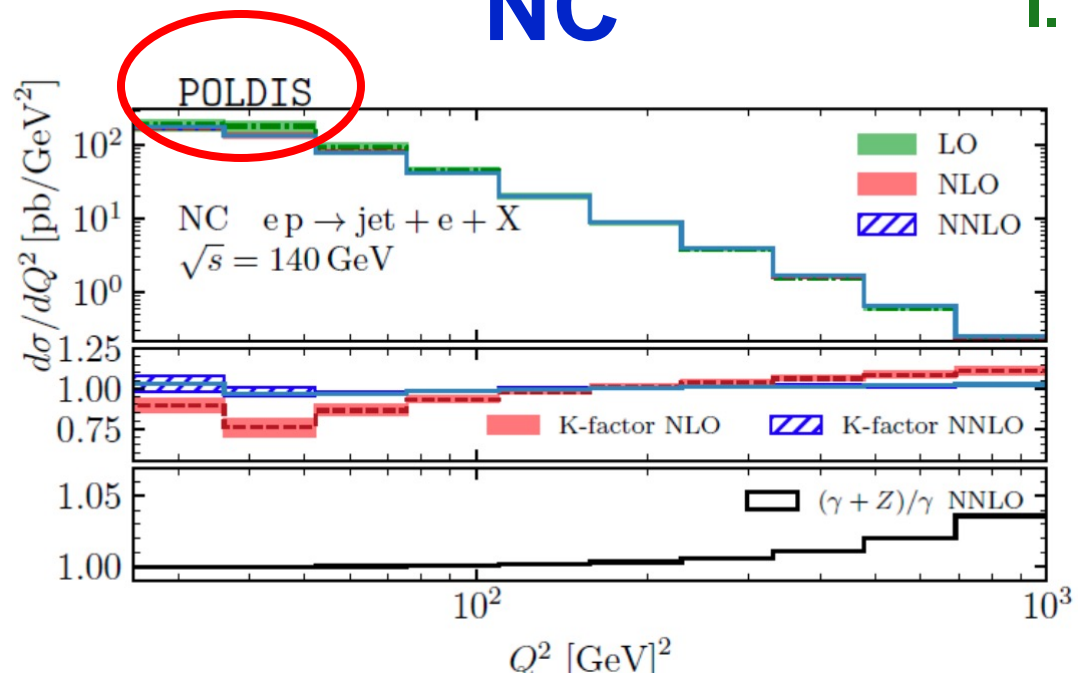


$$d\sigma_{1jet}^{NNLO} = \underbrace{d\sigma_{2jet}^{NLO}}_{\text{Dipoles}} - d\sigma_{2jet,P2B}^{NLO} + \underbrace{d\sigma_{1jet}^{NNLO, incl}}_{\text{Structure Functions}}$$

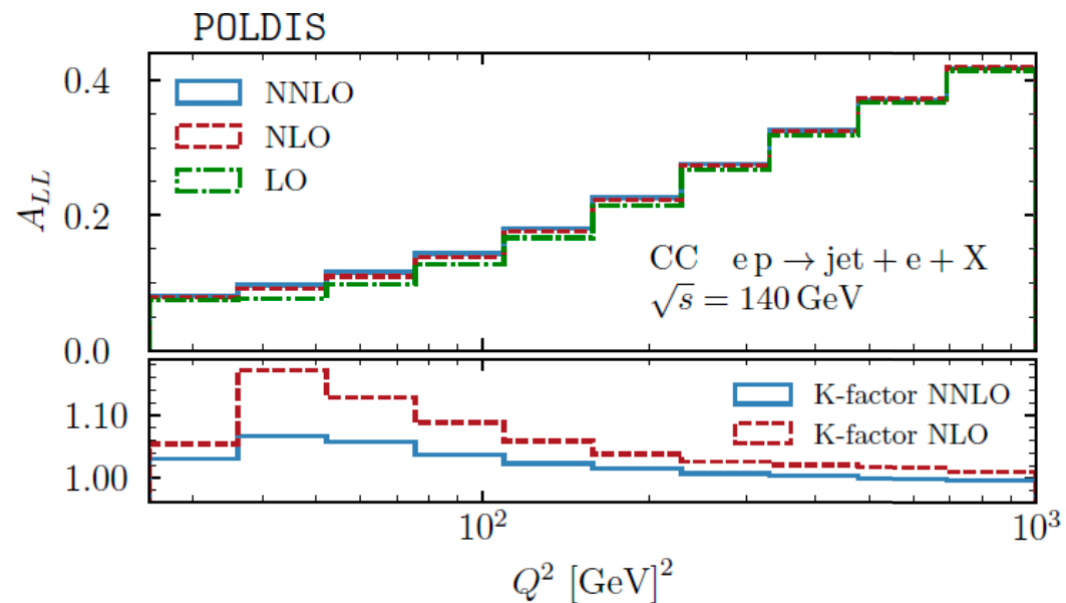
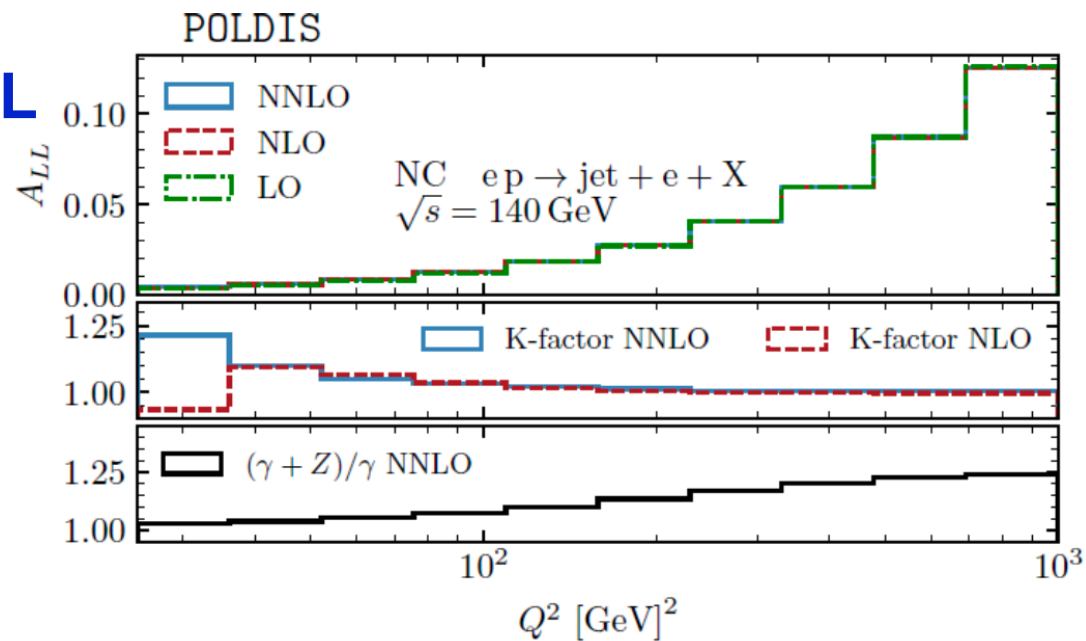
# NC

# I. Pedron

# CC



# ALL



# Nucleon helicity structure at NNLO

I. Borsa (WV)

- PDF evolution: 
$$\Delta\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \Delta P_{ij}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \Delta P_{ij}^{\text{NNLO}} + \dots$$

Moch, Vogt, Vermaseren  
Blümlein, Marquard, Schneider, Schönwald

- Partonic hard scattering: 
$$\Delta\hat{\sigma}_{ab} = \Delta\hat{\sigma}_{ab}^{\text{LO}} + \frac{\alpha_s}{\pi} \Delta\hat{\sigma}_{ab}^{\text{NLO}} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta\hat{\sigma}_{ab}^{\text{NNLO}} + \dots$$

Zijlstra, van Neerven  
Boughezal, Li, Petriello

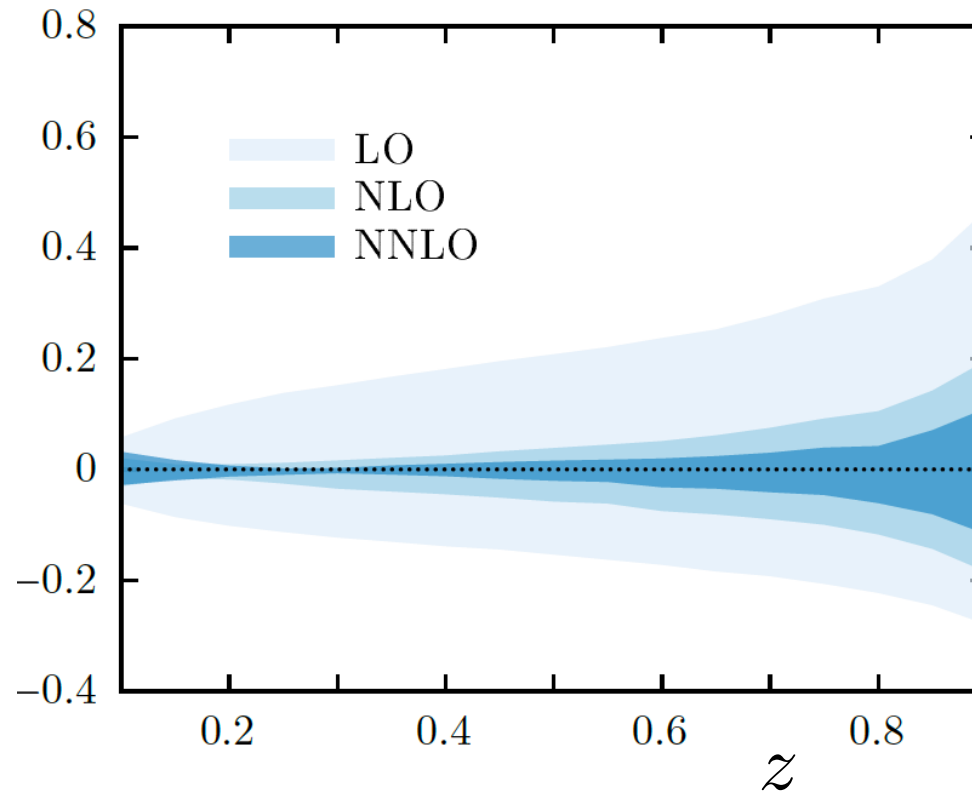
**+ soft-gluon  
techniques for  
SIDIS, pp**

# Why NNLO ?

- need per cent accuracy for EIC and JLab (cf. LHC experience)
- reduce theory uncertainty

$$\frac{\sigma(\mu) - \sigma(Q)}{\sigma(Q)}$$

$$Q/2 \leq \mu_{R,F} \leq 2Q$$



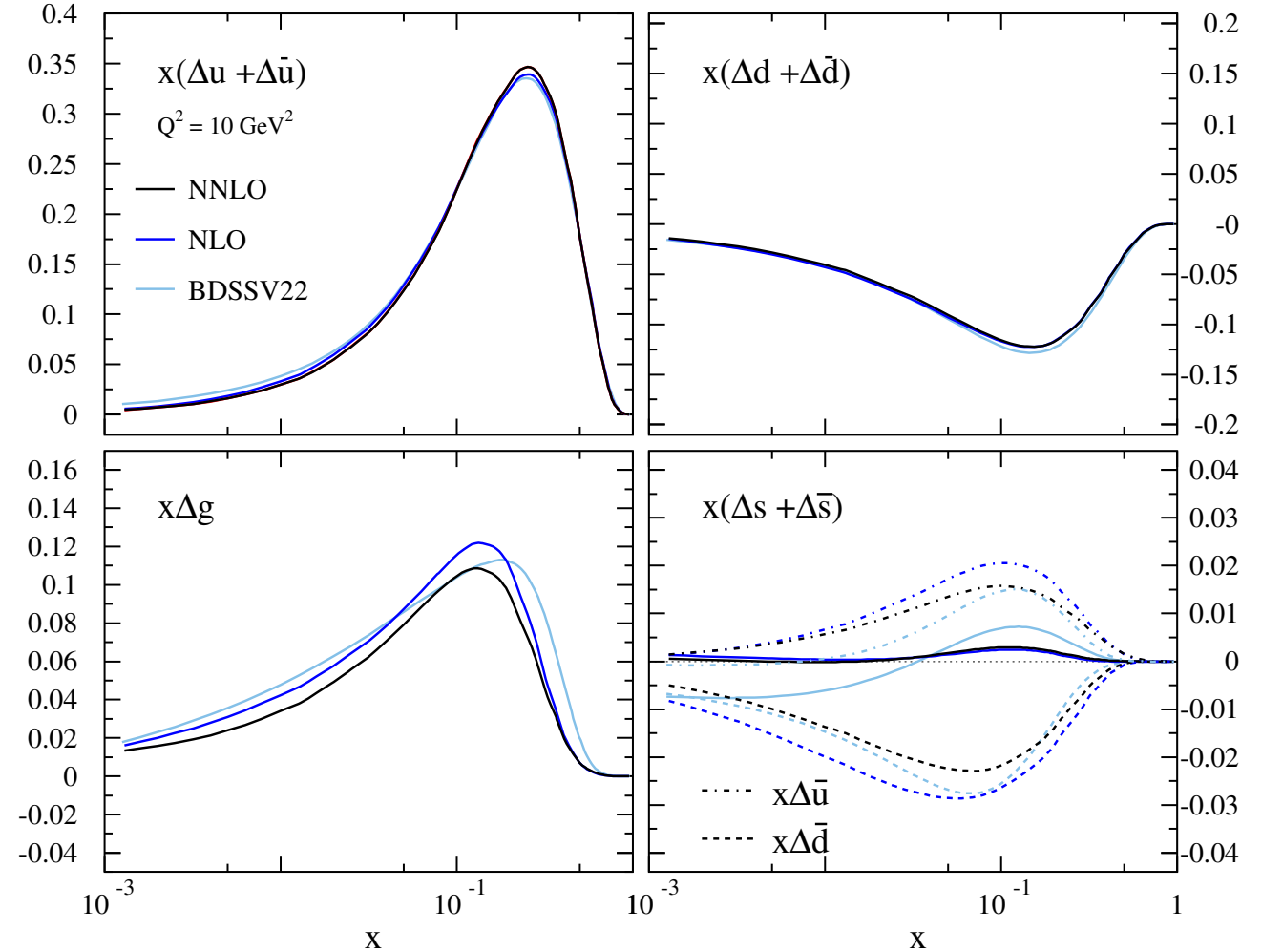
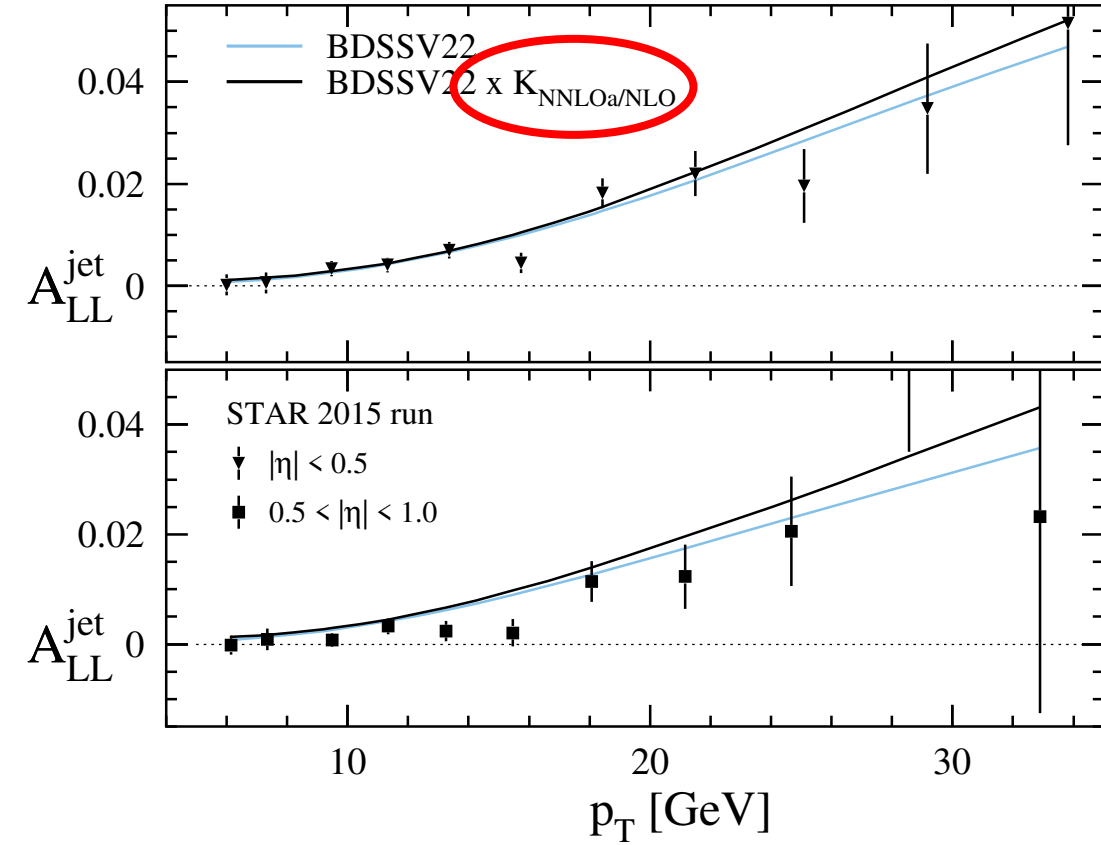
$$Q^2 > 5 \text{ GeV}^2$$

SIDIS@EIC

Abele, De Florian, WV

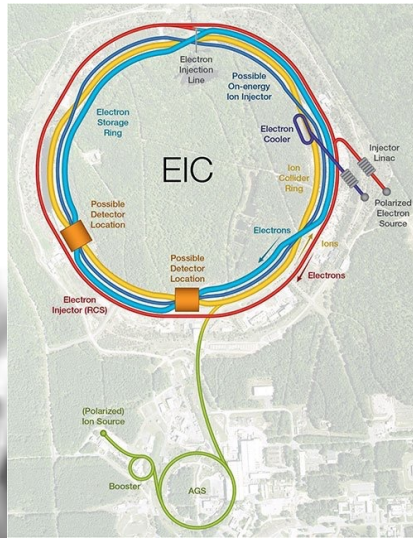
- progress in lattice computations





First ever NNLO global analysis of helicity PDFs !

**Moos  
Schlegel  
Gamberg  
Zurita  
Borsa  
Magni  
Bertone  
Braun  
Bhattacharya  
Pedron  
Tomalak  
Hobart**



**Bacchio  
Koutsou  
Pefkou  
Zhang  
Mukherjee  
Constantinou  
Pittler  
Li**

OBJECTS IN MIRROR ARE CLOSER  
THAN THEY APPEAR

**Thank you!**