Determination of heavy mesons fragmentation functions in the phenomenological approach

By

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November 23, 2022

Outline

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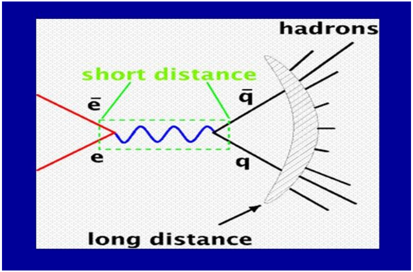
- Introduction.
- Study of fragmentation function.
- FFs in phenomenological approach.
- QCD framework of our analysis.
- Fit results.
- Summery and conclusion.

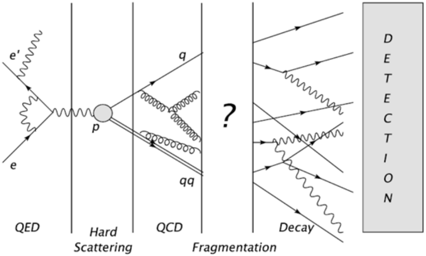
INTRODUCTION



In perturbative QCD the hadron production cross section is factorized into two parts.

- Perturbative
  - The first part is the hard scattering subprocess.
  - We need to calculate $e^-e^+ \to q\bar{q}$ cross section.
  - This part is calculable with perturbation theory.
- Non-Perturbative
  - This part contains FFs.
  - We need to extract non-perturbative FFs describing $q/\bar{q} \to H$
  - This part is extracted by global analysis.

Fragmentation functions (FFs)



FFs

The FFs are the probability density in which outgoing parton fragments into a hadron with a certain fraction of the parton's momentum.

Study of fragmentation functions

FFs

FFs in hadronization processes

Since FFs are universal, they play important roles in all hadronization processes.

- $e^- + e^+$ (SIA)
- $P + \bar{P}$
- $P + P$
- SIDIS

Single Inclusive electron-positron Annihilation(SIA)

Heavy hadron production in electron-positron annihilation

$$e^- e^+ \to (z,\gamma) \to q\bar{q} \to H + X, \tag{1}$$

where X stands for the residual final state including unobservable jet produced together with the hadron H, which is observed.

- This process has in general less contributions by background processes compared to hadron collisions.
- We don't have to deal with the uncertainty introduced by parton density functions.
- The most experimental data sets belong to SIA

Structure of the cross section

Differential cross section

The cross section of process (1) is expressed as

$$\sigma_{(SIA)} = \hat{\sigma} \otimes D_i^H$$

The general fom of differential cross section is

$$\frac{d\sigma(x_H, s)}{dx_H} = \sum_{i=q,g,x_H} \int_H^1 \left(\frac{dx_i}{x_i}\right) \frac{d\hat{\sigma}}{dx_i}(x_i, \mu_r, \mu_f) D_i^H\left(\frac{x_H}{x_i}, \mu_f\right). \tag{2}$$

$x_i = 2E_i/\sqrt{s}$ and $x_H = 2E_H/\sqrt{s}$, $z = \frac{x_H}{x_i}$ and $\mu_r = \mu_s = \sqrt{s}$

Total cross section

Most of experimental data are presented in the form of $1/\sigma_{tot} \times \frac{d\sigma}{dx_H}$ to be able to compare our theoretical results with experimental data we need to normalize the last equation to the total cross section

$$\sigma_{tot} = \frac{4\pi\alpha^2(Q)}{(Q^2)} \left(\sum_i^{n_f} \tilde{e}_i^2(Q) \right) (1 + \alpha_s K_{QCD}^{(1)} + \alpha_s^2 K_{QCD}^{(2)} + \ldots), \qquad (3)$$

where α and α_s are the electromagnetic and strong-coupling constants, respectively, $\tilde{e}_i$ is the effective electroweak charge of quark i, and the coefficient $K_{\mathrm{QCD}}^{(n)}$ contains the N^{n}LO correction. $K_{\mathrm{QCD}}^{(1)} = 3C_F/4\pi$ where $C_F = 4.3$ and $K_{\mathrm{QCD}}^{(2)} \approx 1.411$.

sub-processes cross section

Wilson coefficients

The cross sections of the relevant partonic sub-processes for quarks are given by[1]

$$\frac{d\sigma_{q_i}}{dx_q}(x_q,\mu) = N_c\sigma_0(V_{qi}^2+A_{qi}^2)\Big\{\delta(1-x_q) + \frac{\alpha_s(\mu)}{2\pi}\left[P^{(0,T)}_{q\to q}(x_q) ln\frac{s}{\mu^2}+C_q(x_q)\right]\Big\}, \quad (4)$$

and for glouns we have[2]

$$\frac{d\sigma_g}{dx_g}(x_g,\mu) = 2N_c\sigma_0\sum_{i=1}^{n_f}(V_{qi}^2+A_{qi}^2)\frac{\alpha_s(\mu)}{2\pi}\left[P^{(0,T)}_{q\to g}(x_g) ln\frac{s}{\mu^2}+C_g(x_g)\right], \quad (5)$$

Here, $N_c = 3$ is the number of color and $\sigma_0 = \frac{4\pi\alpha^2}{3}$.

[1] S. Moch and A. Vogt, Phys. Lett. B 659, 290 (2008).
[2] A. Mitov, S. Moch and A. Vogt, Phys. Lett. B 638, 61 (2006).

Splliting functions

$P^{(0,T)}_{a\to b}$ are the LO splitting functions

$$\begin{aligned}
P^{(0,T)}_{q\leftrightarrow q}(x_q) =& C_F\left[\frac{3}{2}\delta(1-x_q)+\frac{1+x_q^2}{(1-x_q)_+}\right], \\
P^{(0,T)}_{q\leftrightarrow g}(x_g) =& C_F\frac{1+(1-x_g)^2}{x_g},
\end{aligned} \tag{6}$$

where $C_F = 3/4$ and the coefficient functions read

$$\begin{aligned}
C_q(x_q) =& C_F\left\{\left(-\frac{9}{2}+\frac{2}{3}\pi^2\right)\delta(1-x_q)-\frac{3}{2}\left(\frac{1}{1-x_q}\right)_+ +2\left[\frac{ln(1-x_q)}{1-x_q}\right]_+\right. \\
&\left. +\frac{5}{2}-\frac{3}{2}x_q+4\frac{lnx_q}{1-x_q}-(1+x_q)[2lnx_q+ln(1-x_q)]\right\}, \\
C_g(x_g) =& C_F\frac{1+(1-x_g)^2}{x_g}[2lnx_g+ln(1-x_g)].
\end{aligned} \tag{7}$$

Fragmentation Functions in phenomenological approach

Determination of FFs

Steps

1. Considering a parametrization form with unknown parameter.
2. Finding relevant experimental data.
3. Doing a fit to experimental data for determination of unknown parameter.
   - To get the best value of parameter we need to introduce χ^2 function and minimized it to obtain the optimum values

$$\chi^2 = \Sigma_i \frac{(\Delta_i^{data} - \Delta_i^{theo})^2}{(\sigma_i^{data})^2} \tag{8}$$

heavy mesons FFs

In this work we want to determine:

- B-mesons fragmentation function.
- D-mesons fragmentation functions.

QCD framework our analysis

Extraction of the FFs

PRAMETRIZATION FORM

Parameterization of the B-meson FFs

Simple power form

For B-meson we have simple power form

$$D_b^B(z,\mu_0^2) = N_b z^{\alpha_b}(1-z)^{\beta_b} \tag{9}$$

Parameterization of the D^0 and D^+ mesons FFs

- $Z \to c/\bar{c}$ followed by $c/\bar{c} \to H_c$ fragmentation.
- $Z \to b/\bar{b}$ followed by $b/\bar{b} \to H_b$ fragmentation then $H_b \to H_c + X$.

Bowler Parametric form

We adopt the optimal functional form suggested by Bowler for the parametrization of c and b quark FFs

$$D_{c,b}^{(D^0,D^+)}(z,\mu_0^2) = N_i z^{-(\alpha_i^2+1)}(1-z)^{\beta_i} e^{-\alpha_i^2/z}. \tag{10}$$

Parameterization of the D_s^+-meson FFs

Peterson and simple power form

We use the optimal functional form for the parameterization of charm suggested by Peterson as

$$D_c^{D_s^+}(z,\mu_0^2) = N_c \frac{z(1-z)^2}{[(1-z)^2+\varepsilon z]^2}, \tag{11}$$

and for $b \to D_s^+$ transition, we use the following simple power form

$$D_b^{D_s^+}(z,\mu_0^2) = N_b z^{\alpha}(1-z)^{\beta} \tag{12}$$

EXPERIMENTAL INPUT

B-mesons

Experimental data for B-mesons

LEP(CERN)	ALEPH	OPAL	DELPHI
SLC(SLAC)		SLD	

D-mesons

Experimental data for D^0, D^+ and D_s^+

LEP(CERN)	OPAL
CESR	CLEO

D-mesons

OPAL

OPAL collaboration released their results in the form of $\frac{1}{N_{had}}\frac{dN}{dz}$. We need to divide it by the branching fractions of the decays

$$\begin{aligned} Br(D^0 \to K^-\pi^+) =& (3.84 \pm 0.13)\% \\ Br(D^+ \to K^-\pi^+\pi^-) =& (9.1 \pm 0.6)\% \\ Br(D_s^+ \to \varphi\pi^+) =& (3.5 \pm 0.4)\% \end{aligned} \tag{13}$$

D-mesons

Mass corrections

To incorporate the hadron mass effects, we used a specific choice of scaling variables by working in the light-cone coordinates.

$$\frac{d\sigma}{dx_H}(x_H,s) = \frac{1}{1-\frac{m_H^2}{s\eta^2(x_H)}}\frac{d\sigma}{d\eta}(\eta(x_H),s), \tag{14}$$

where $\eta = x_H/2 \times (1-4m_H^2/(sx_H^2))$ and

$$\frac{d\sigma}{d\eta}(\eta,s) = \sum_i \int_\eta^1 \frac{dy}{y}\frac{d\hat{\sigma}_i}{dy} D_i^H(\frac{\eta}{y},\mu_F). \tag{15}$$

UNCERTAINTIES

Hessian approach

$$[\Delta D_i^H(z)]^2 = \Delta\chi^2 \sum_{j,k}^{n} \frac{\partial D_i^H(z,a_j)}{\partial a_j} H_{jk}^{-1} \frac{\partial D_i^H(z,a_j)}{\partial a_k}, \tag{16}$$

where a_j are the free parameters and H^{-1} is the covariance matrix.

Conditions

1. Initial scales
   - for $b \to D^B(z,\mu_0^2) : \mu_0^2 = 20.25 GeV^2$
   - for $(c,b) \to D^{D_c}(z,\mu_0^2) : \mu_0^2 = 18.5 GeV^2$
2. Scheme: ZM-VFN
3. $\alpha_s(M_Z) = 0.118$
4. D-mesons masses
   - $m(D^0) = 1.864 GeV$
   - $m(D^+) = 1.869 GeV$
   - $m(D_s^+) = 1.968 GeV$

Fit Results

- We use APFEL package and MINUIT program for determination of FFs.

Fit results for B-mesons

M. Salajegheh, S.M.M. Nejad, H. Khanpour, B. A. Kniehl, and M. Soleymaninia, Phys. Rev. D 99, 114001 (2019).

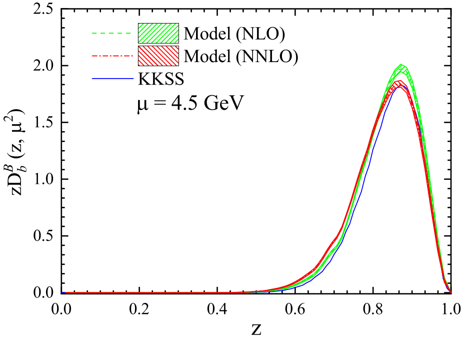
B-meson

Collaboration	data properties	$\sqrt{s}$ GeV	data points	χ^2(NLO)	χ^2(NNLO)
ALEPH	Inclusive	91.2	18	14.376	12.269
DELPHI	Inclusive	91.2	8	7.535	15.377
OPAL	Inclusive	91.2	15	35.594	20.002
SLD	Inclusive	91.2	18	25.675	14.195
TOTAL:			59	83.180	61.844
(χ^2/d.o.f)				1.48	1.104

flavor b	N_i	α_i	β_i
NLO	2575.014	15.424	2.394
NNLO	1805.896	14.168	2.341

B-meson

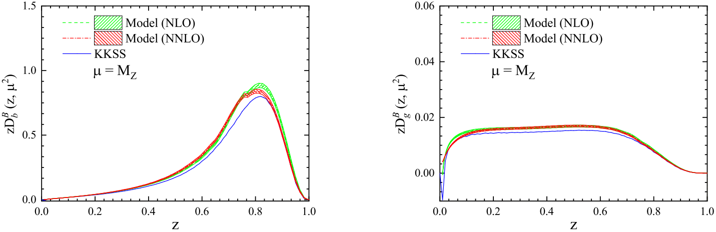
FFs of b quark to B-Meson at $Q_0 = 4.5$ GeV obtained from QCD analysis at NLO and NNLO accuracies along with the error uncertainties.The KKSS results are shown for comparison.[3]



[3] B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, Phys. Rev. D 77, 014011 (2008).

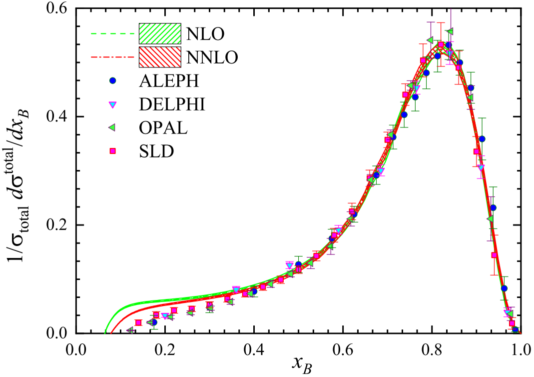
B-meson

Line shapes of $zD_b^B(z, M_Z)$ (left panel) and $zD_g^B(z, M_Z)$ (right panel) at NLO and NNLO and their experimental uncertainty bands.



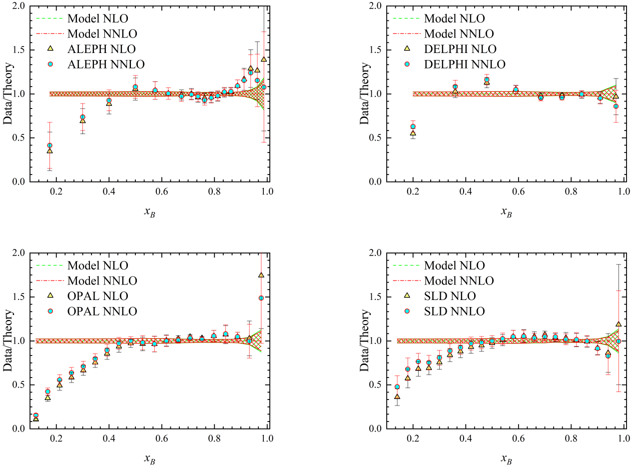
B-meson

Comparison between the dataset for *B*-meson production from the ALEPH, DELPHI, OPAL and SLD experiments and the corresponding NLO and NNLO theoretical predictions.



B-meson

For better visibility, we present the information contained in below as data over theory plots one for each experiment.



Fit results for D^0-mesons

M. Salajegheh, S. M. M. Nejad, M. Soleymaninia, H. Khanpour, and S. A. Tehrani, Eur.Phys.J. C79 (2019) no.12, 999.

D^0-meson

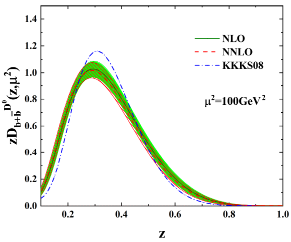
The individual χ^2 values for inclusive and b-tagged cross sections obtained at NLO and NNLO for D^0

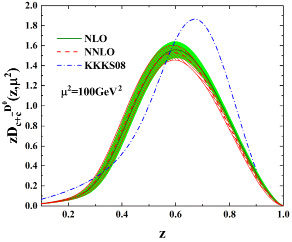
Collaboration	data properties	$\sqrt{s}$ GeV	data points	χ^2 (NLO)	χ^2 (NNLO)
OPAL	Inclusive	91.2	13	8.36	7.08
	b-tagged	91.2	13	14.62	14
TOTAL:			26	22.98	21.08
(χ^2/ d.o.f)				1.149	1.05

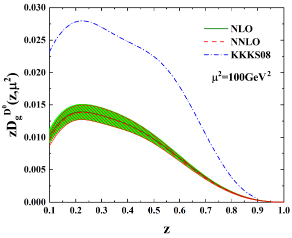
D^0-meson

The optimal values for the input parameters of the D^0-FF at the initial scale $\mu_0^2 = 18.5$ GeV^2

Parameter	Best values	
	NLO	NNLO
N_c	284.513	261.214
α_c	1.341	1.402
β_c	1.981	1.953
N_b	13.127	12.701
α_b	3.944	4.014
β_b	0.904	0.891



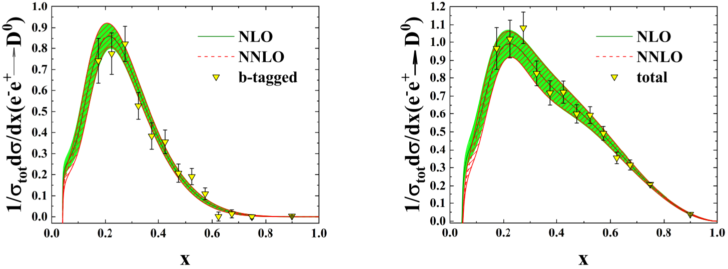




T. Kneesch, B. A. Kniehl, G. Kramer and I. Schienbein, Nucl. Phys. B 799, 34 (2008) (KKKS08).

D^0-meson

Our NLO and NNLO theoretical predictions are compared with the normalized inclusive total (right) and b-tagged (left) data sets for D^0 meson production from OPAL experiment.



Fit results for D^+-mesons

M. Salajegheh, S. M. M. Nejad, M. Soleymaninia, H. Khanpour, and S. A. Tehrani, Eur.Phys.J. C79 (2019) no.12, 999.

D^+-meson

The individual χ^2 values for inclusive and b-tagged cross sections obtained at NLO and NNLO for D^+

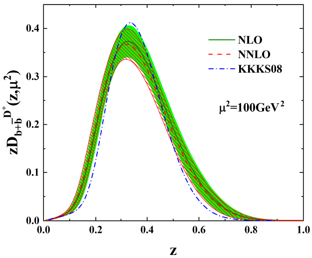
Collaboration	data properties	$\sqrt{s}$ GeV	data points	χ^2 (NLO)	χ^2 (NNLO)
OPAL	Inclusive	91.2	13	7.24	6.2
	b-tagged	91.2	13	8.51	8.32
TOTAL:			26	15.75	14.52
(χ^2/ d.o.f)				0.75	0.69

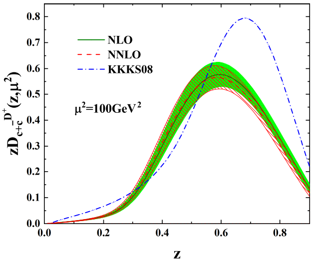
D^+-meson

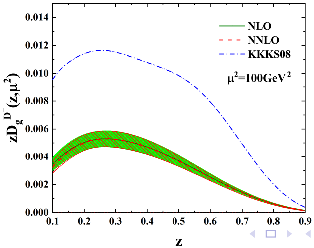
The optimal values for the input parameters of the D^+-FF at the initial scale $\mu_0^2 = 18.5\ \mathrm{GeV}^2$

Parameter	Best values	
	NLO	NNLO
N_c	49.817	44.357
α_c	1.20	1.253
β_c	1.841	1.806
N_b	11.664	10.653
α_b	4.308	4.343
β_b	1.095	1.073

D^+-meson

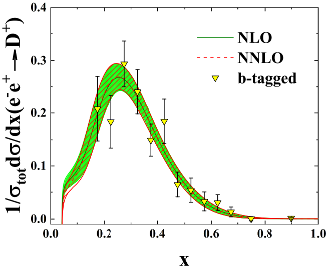

NLO
NNLO
KKKS08
μ^2=100GeV^2
$zD_{b+\bar{b}}^{D^+}(z,\mu^2)$
z

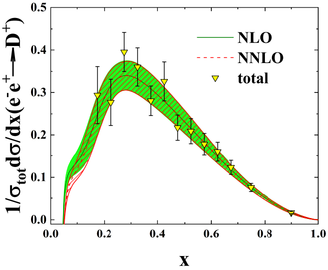

NLO
NNLO
KKKS08
μ^2=100GeV^2
$zD_{c+\bar{c}}^{D^+}(z,\mu^2)$
z


NLO
NNLO
KKKS08
μ^2=100GeV^2
$zD_{g}^{D^+}(z,\mu^2)$
z

D^+-meson

Our NLO and NNLO theoretical predictions are compared with the normalized inclusive total (right) and b-tagged (left) data sets for D^+ meson production from OPAL experiment.





Fit results for D_s^+-mesons

M. Salajegheh, S.M.M. Nejad and M. Delpasand, Phys.Rev. D100 (2019) no.11, 114031.

D_s^+-meson

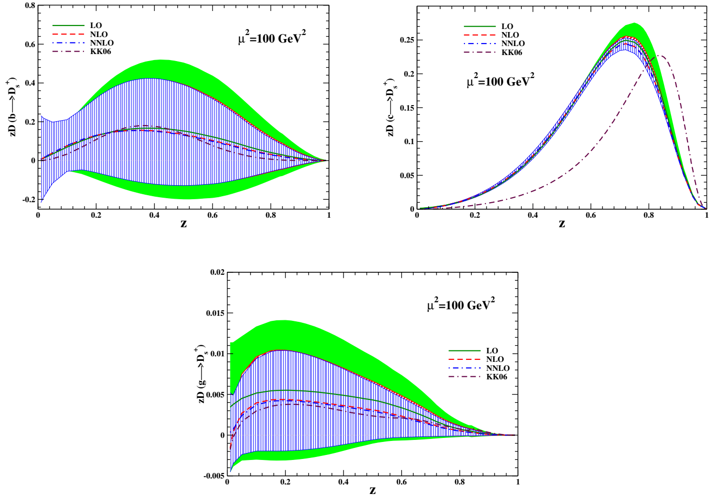
The individual χ^2 values for inclusive and b-tagged cross sections obtained at LO, NLO and NNLO for D_s^+

data properties	$\sqrt{s}$ GeV	data points	χ^2 (LO)	χ^2 (NLO)	χ^2 (NNLO)
Inclusive	91.2	4	0.037	0.025	0.022
b-tagged	91.2	4	0.123	0.103	0.098
TOTAL:		8	0.160	0.128	0.121
(χ^2/ d.o.f)			0.082	0.064	0.060

D_s^+-meson

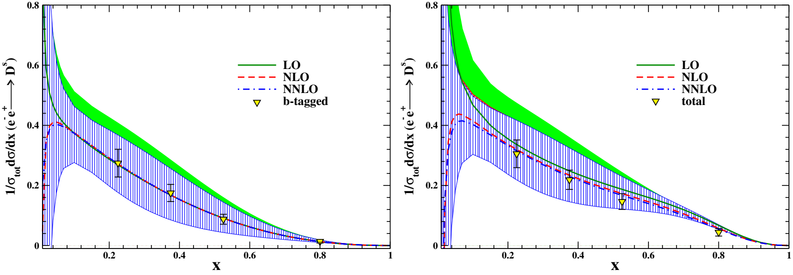
The optimal values for the input parameters of the D_s^+-FF at the initial scale $\mu_0^2 = 18.5\ \mathrm{GeV}^2$

Parameter	Best values		
	LO	NLO	NNLO
N_c	0.176	0.176	0.176
ε	0.108	0.141	0.155
N_b	1.555	1.359	1.380
α	0.318	0.206	0.203
β	1.859	1.923	1.983



B. A. Kniehl and G. Kramer, Phys. Rev. D 74, 037502 (2006) (KK06).

D_s^+-meson


LO
NLO
NNLO
b-tagged
total
$1/\sigma_{tot} d\sigma/dx\ (e^+e^- \longrightarrow D^s)$
x
0
0.2
0.4
0.6
0.8
1

SUMMERY

AND

CONCLUSION

Conclusion

- We had a brief study on fragmentation functions and their formalism.
- The framework of our analysis has been studied.
- We introduced the parametrization form for both of B and D mesons FFs and The relevant experimental data sets.
- We extracted B and D mesons FFs up to NNLO.
- Obtained FFs were compared with the other theoretical groups.
- Our results have been compared with experimental data and we saw that they are in good agreement.


THANK
YOU

BACKUP

Total fragmentation function

The SIA differential cross-section involving a hadron H in the final state can be expressed as

$$\frac{d\sigma}{dz}(e^- e^+ \to H + X)\frac{1}{\sigma_{tot}} = F^H(z,s) \tag{17}$$

$F^H(z,s)$ is the fragmentation (structure) function, defined in analogy with the structure function F_2 in DIS. In the literature $F^H(z,s)$ is often called total fragmentation function. From Eq. (2), the relation between $F^H(z,s)$ and $D_i^H(z,\mu)$ is as below

$$F^H(z,s) = \sum_i \int_{x_H}^1 \frac{dy}{y} D_i^H(\frac{x_H}{x_i},\mu_f)\frac{1}{\sigma_{\rm tot}}\frac{d\sigma_i}{dx_i}(x_i,\mu_r,\mu_f), \tag{18}$$

In the above equation μ_r and μ_f are the renormalization and factorization scales respectively.

$$\mu_r = \mu_f = \sqrt{s}$$

D-meson

CLEO

CLEO collaboration published their results in the form of $\frac{d\sigma}{dx_p}$. These data sets are located much closer to the thresholds $\sqrt{s} = 2m_c$ and $\sqrt{s} = 2m_b$ of the transitions $c \to H_c$ and $b \to H_b$, than those from OPAL. Hence, including the CLEO data sets in the analysis might be a reason for tension. To check this point, we converted the `CLEO` data to the desired form $d\sigma/dx_D$ as follow

$$\frac{d\sigma}{dx_D} = \frac{d\sigma}{dx_p}\left(1 + \frac{4}{x_p^2}\frac{m_H^2}{s}\right)^{\frac{1}{2}}. \tag{19}$$

Heavy meson production in top quark decay

Top quark decay

Meson production

Top quark decay process

$$t \to b + W^+(g) \to W^+ + H + X, \tag{20}$$

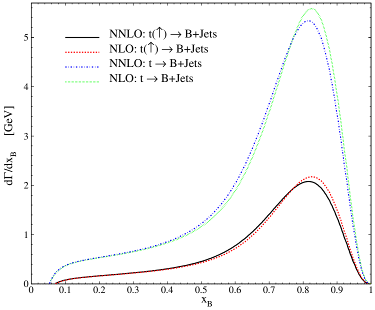
where, $H = B, D$ mesons.
We apply the extracted heavy meson FFs to make our phenomenological predictions for the energy distribution of heavy mesons produced[4]

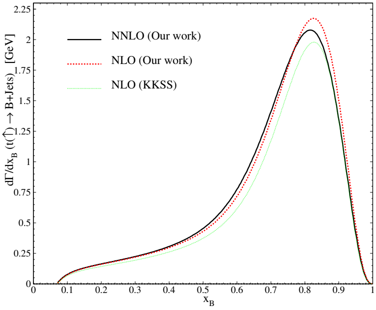
$$\frac{d\Gamma}{dx_H} = \sum_{i=b,g} \int_{x_i^{min}}^{x_i^{max}} \frac{dx_i}{x_i} \frac{d\Gamma}{dx_i}(\mu_r, \mu_f) D_i^H\left(\frac{x_H}{x_i}, \mu_f\right), \tag{21}$$

where $x_H = 2E_H/(m_t(1-\omega))$, $\omega = m_w^2/m_t^2$ and $\mu_r = \mu_f = m_t$.

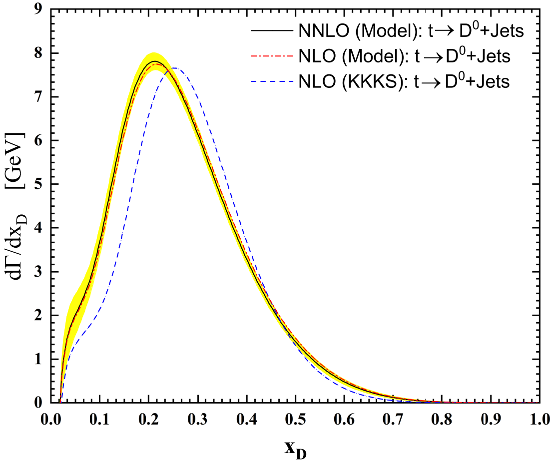
[4] M. Moosavi Nejad, Eur.Phys.J. C72 (2012) 2224

B-mason energy spectrum

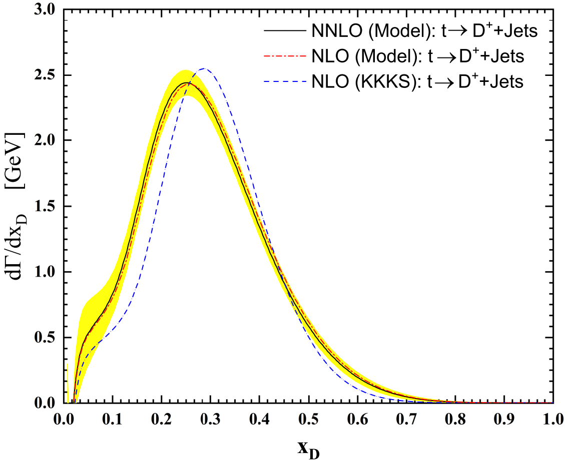




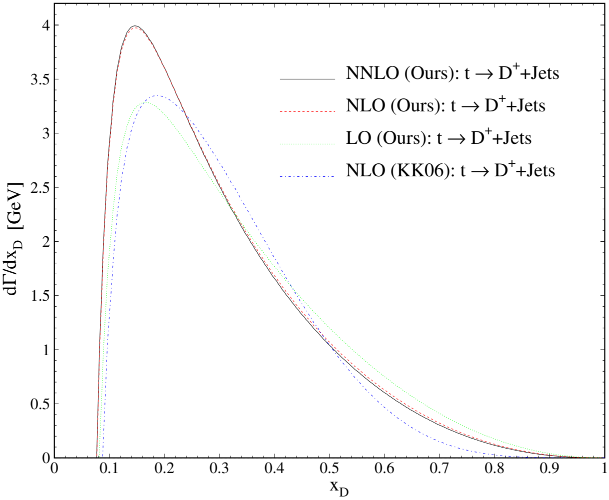
D^0-mason energy spectrum



D^+-mason energy spectrum



D_s^+-mason energy spectrum



$$\sigma_{\text{tot}} = \frac{4\pi\alpha^2(Q)}{Q^2}\left(\sum_i^{n_f}\tilde{e}_i^2(Q)\right)\left(1+\alpha_s K_{\text{QCD}}^{(1)}+\alpha_s^2 K_{\text{QCD}}^{(2)}+\cdots\right), \tag{22}$$

where α and α_s are the tiny structure and the strong coupling constants, respectively, and the coefficients $K_{QCD}^{(i)}$ including $K_{\text{QCD}}^{(1)} = 3C_F/4\pi$ indicate the perturbative QCD corrections to the lowest order result and are currently known up to $\mathscr{O}(\alpha_s^3)$.