Constraints on Lorentz Invariance Violation using the extraordinary flare of Mrk 421 in 2014

Jelena Strišković*, Giacomo D'Amico, Tomislav Terzić and Daniel Kerszberg on behalf of the MAGIC collaboration

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Department of Physics

Naples, 14/09/2023

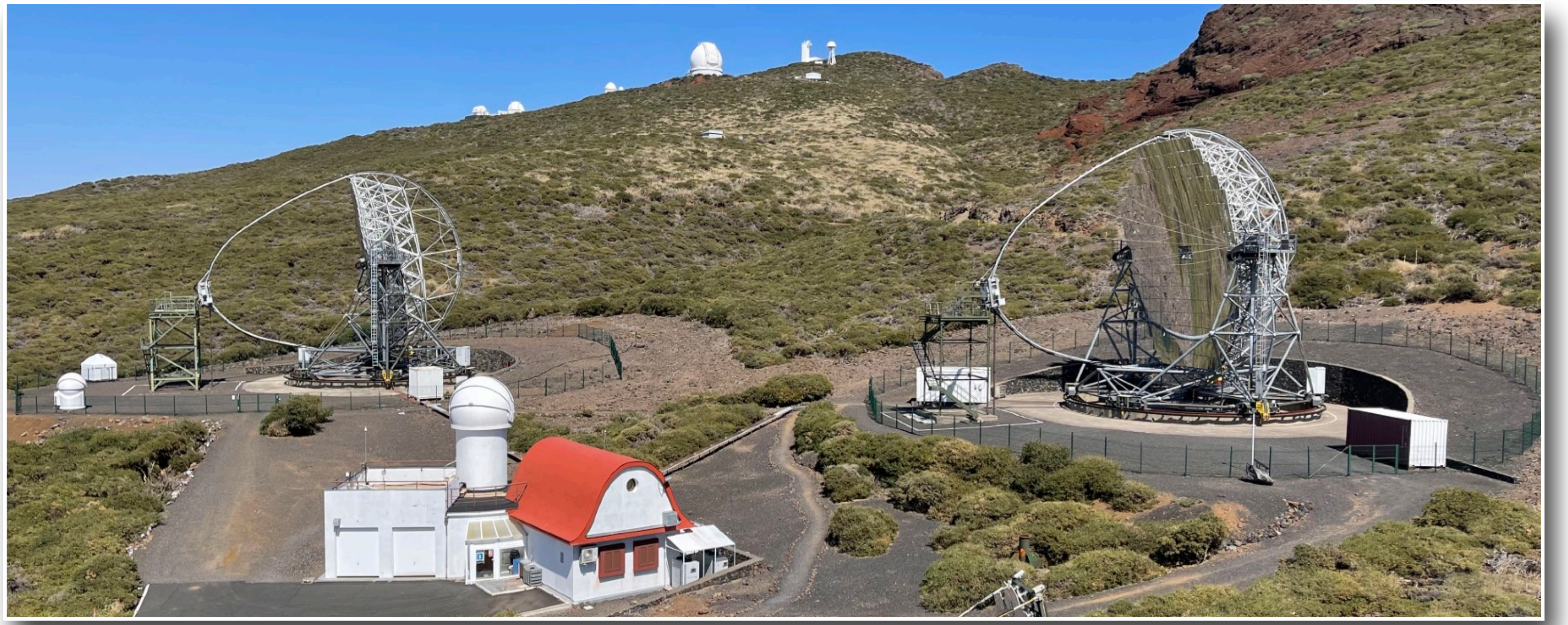
Today I will talk about:

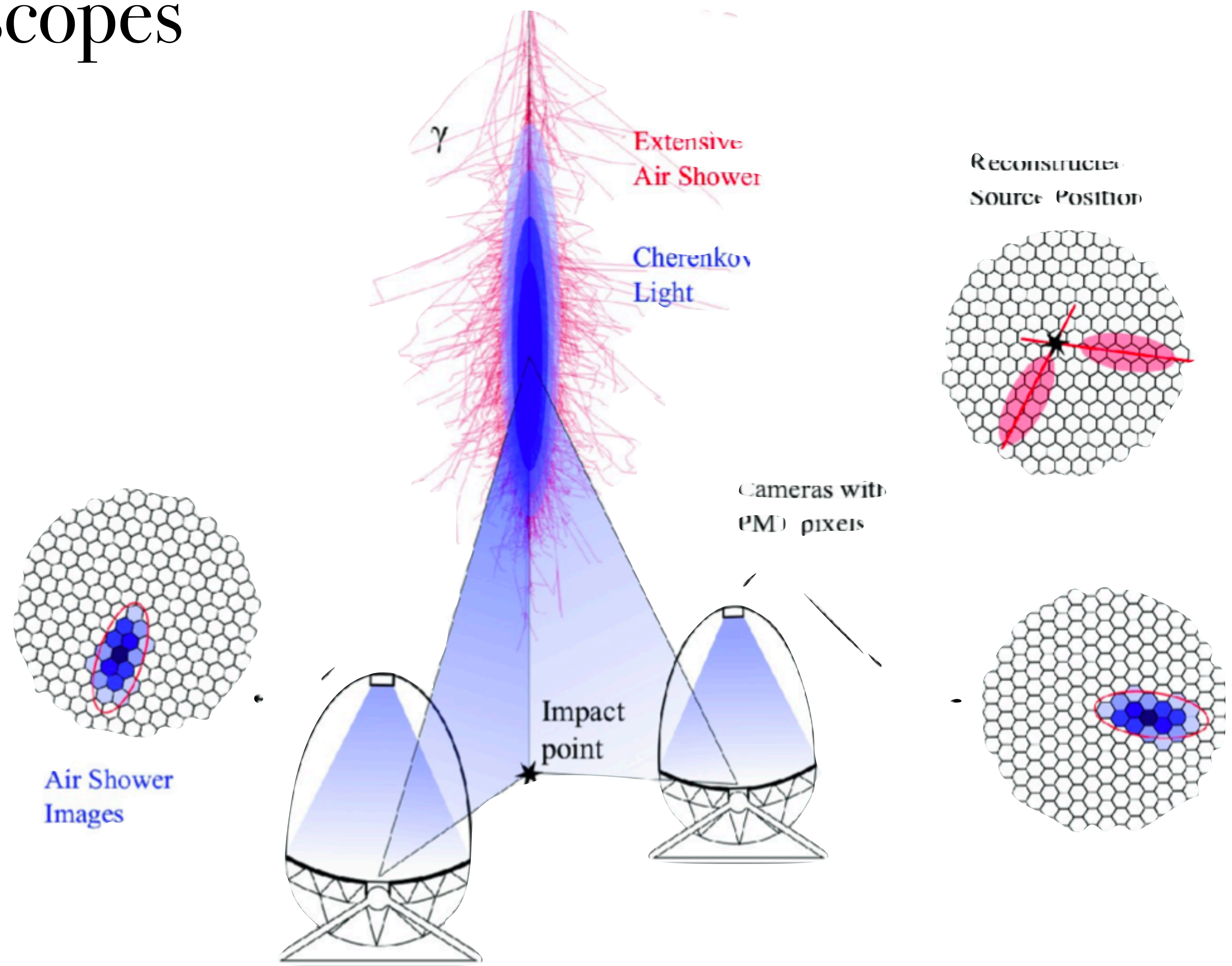
- MAGIC telescopes
- Lorentz invariance violation (LIV)
  - Time of Flight (ToF) studies
  - Suitable sources for LIV ToF studies
- Mrk 421 and used VHE gamma-ray flare
- Binned likelihood
- Results


Credit: Urs Leutenegger

MAGIC telescopes

* Major Atmospheric Gamma Imaging Cherenkov telescopes
* Energy range: between 20 GeV and 100 TeV
* Energy resolution: ~15-25%
* Field of view: ~ 3.5°





* 2200 m a.s.l. at La Palma

Motivation

* **Quantum gravity (QG)**: fundamental **incompatibility** issues between **general relativity** and **quantum field theory**

Motivation

* **Quantum gravity (QG)**: fundamental **incompatibility** issues between **general relativity** and **quantum field theory**

String theory	Space-time foam	Loop quantum gravity	Brane-world backgrounds	…

allow for

Lorentz invariance violation (LIV)

Lorentz invariance violation

- Modified photon dispersion relation:

$$E^2 = p^2c^2 \times \left[1 + \sum_{n=1}^{\infty} S_n \left(\frac{E}{E_{\mathrm{QG},n}}\right)^n\right]$$

n = 1 linear contribution

n = 2 quadratic contribution

Lorentz invariance violation

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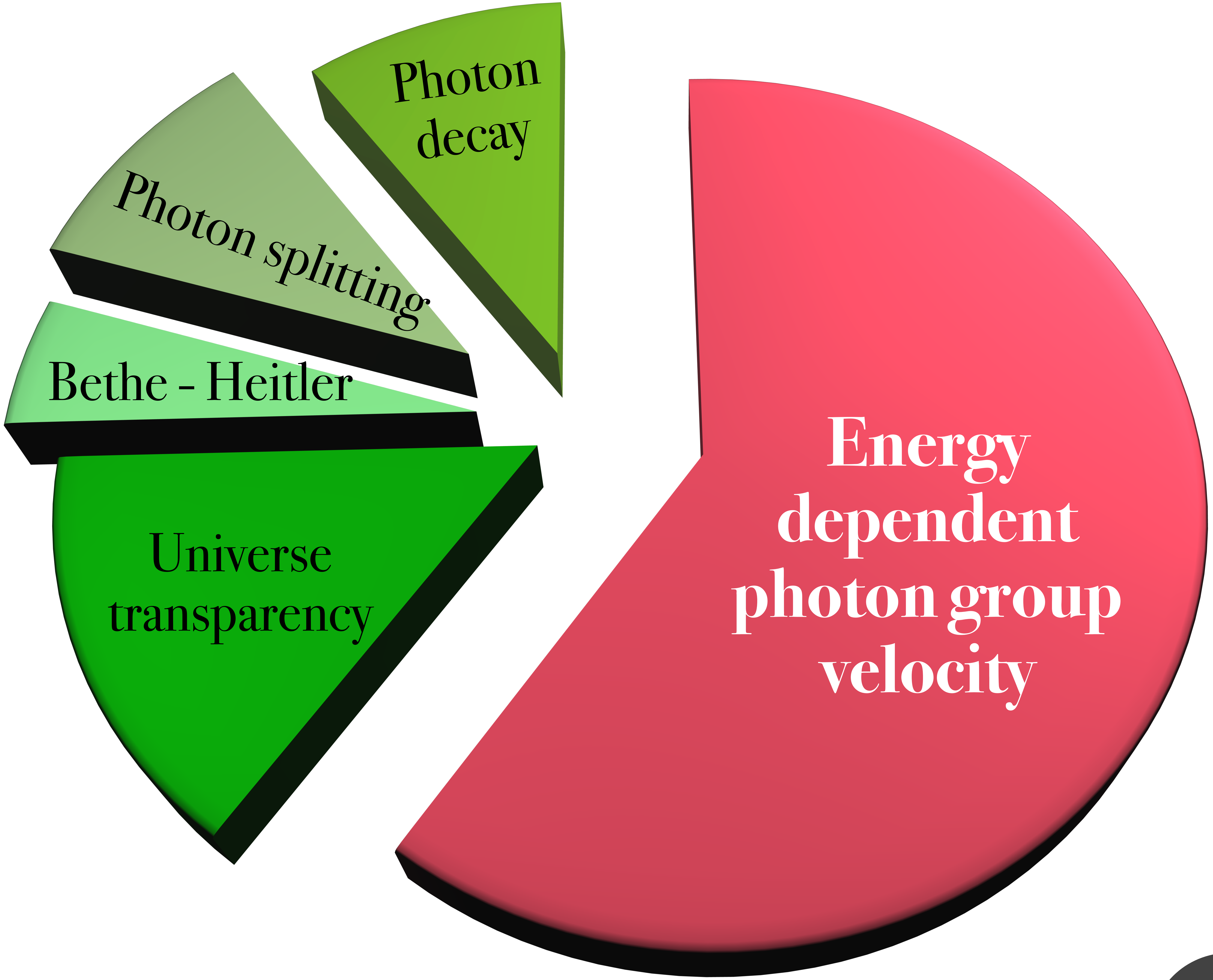
n = 1 linear contribution

n = 2 quadratic contribution

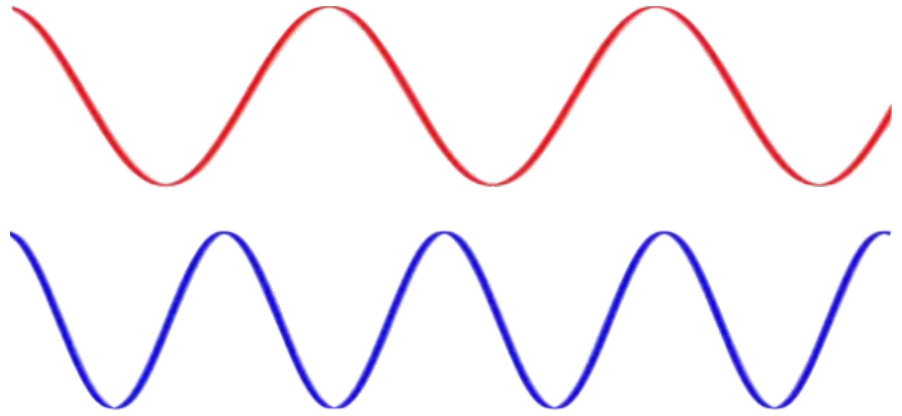
Expected $E_{QG,n}$: Planck energy (~10^{28} eV)

very tiny effects

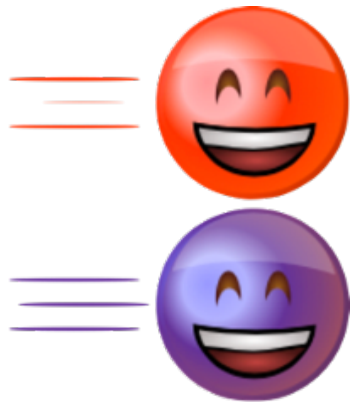
Consequences of modifying photon dispersion relation



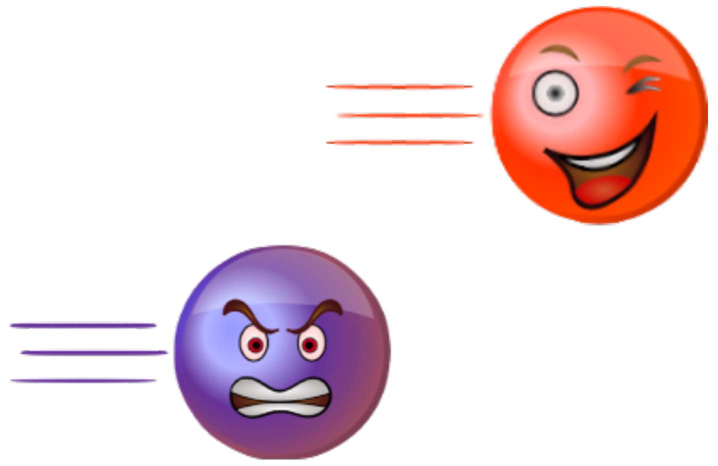
Time of Flight



- Energy dependent photon group velocity:



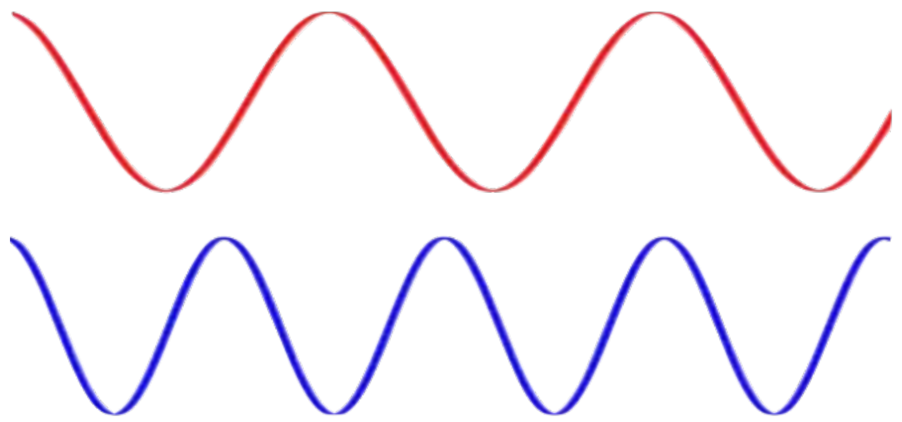
$$v_\gamma = \frac{\partial E}{\partial p} \simeq c \left[1 + \sum_{n=1}^{\infty} S_n \frac{n+1}{2} \left(\frac{E}{E_{\mathrm{QG},n}} \right)^n \right]$$



$S_n = +1$ superluminal behaviour
$S_n = -1$ subluminal behaviour

-

Time of Flight



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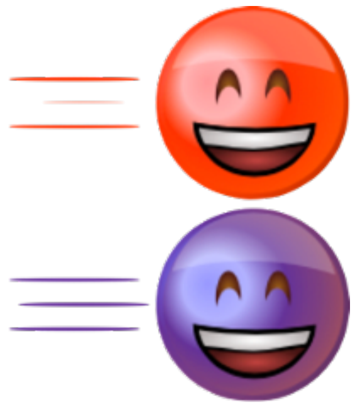

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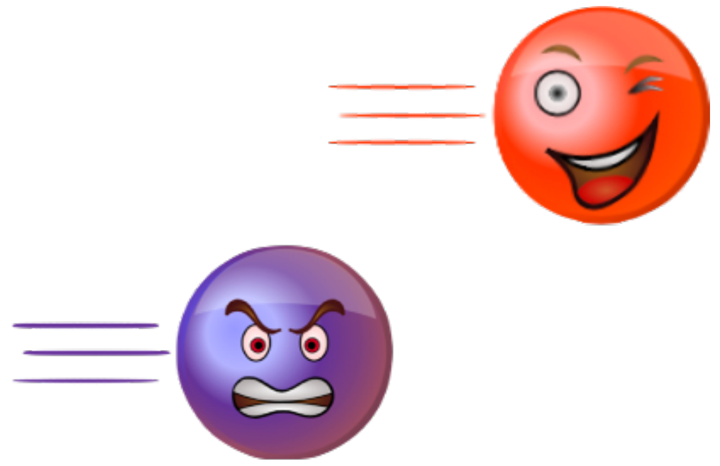

$S_n = +1$ superluminal behaviour
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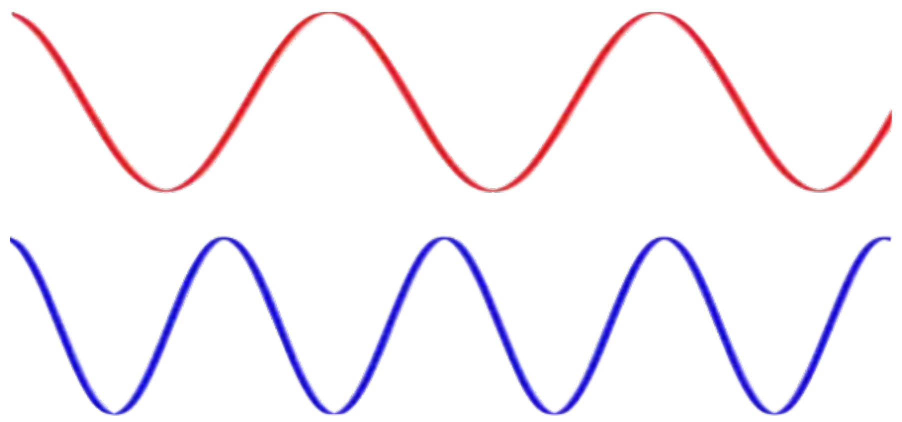
- Energy dependent time delay:

Distance contribution

$$-S_n \frac{n+1}{2}\frac{\kappa_n(z_s)}{E^n_{\mathrm{QG},n}}$$

$$\Delta t_n \cong -S_n \frac{n+1}{2}\frac{E^n}{E^n_{\mathrm{QG},n}} \cdot \kappa_n(z_s) \equiv \eta_n \times E^n$$

Time of Flight



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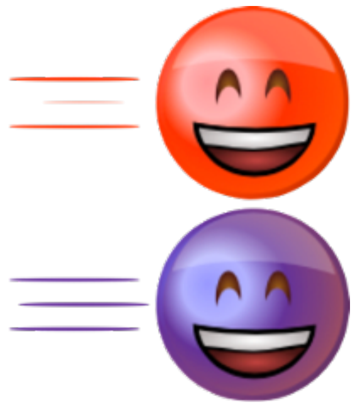

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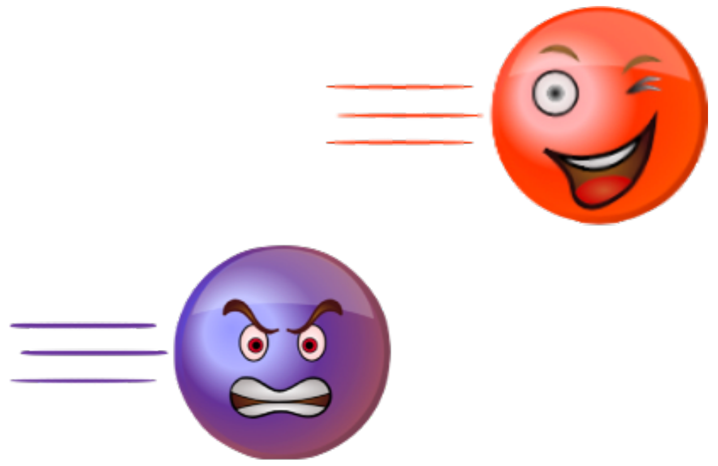

$S_n = +1$ superluminal behaviour
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Distance contribution

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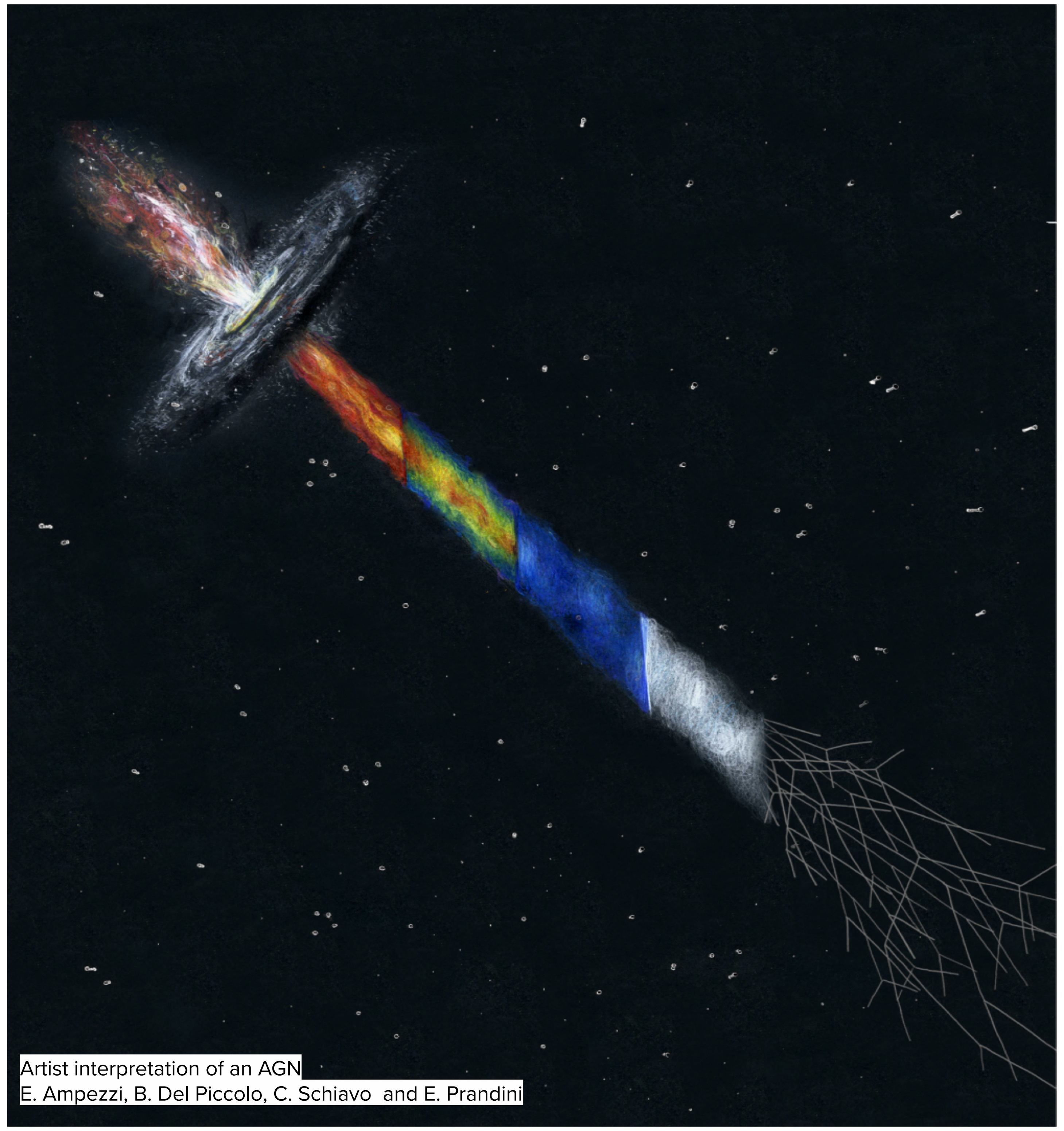
$E_{\mathrm{QG},1}$	$\propto$	E_{max}	t_{var}^{-1}	$z_\mathrm{s}^{\sim 1}$
$E_{\mathrm{QG},2}$	$\propto$	E_{max}	$t_{\mathrm{var}}^{-1/2}$	$z_\mathrm{s}^{\sim 2/3}$

Terzić et al. (2021)
arXiv: 2109.09072

VHE gamma-ray sources suitable for LIV studies

Active galactic nuclei



$E_{\rm max} > 10$ TeV
$t_{\rm var} \gtrsim 1$ min
$z_{\rm s} \lesssim 1$

Mrk 501, Mrk 421
PKS 2155-304
PG 1553+113

VHE gamma-ray sources suitable for LIV studies

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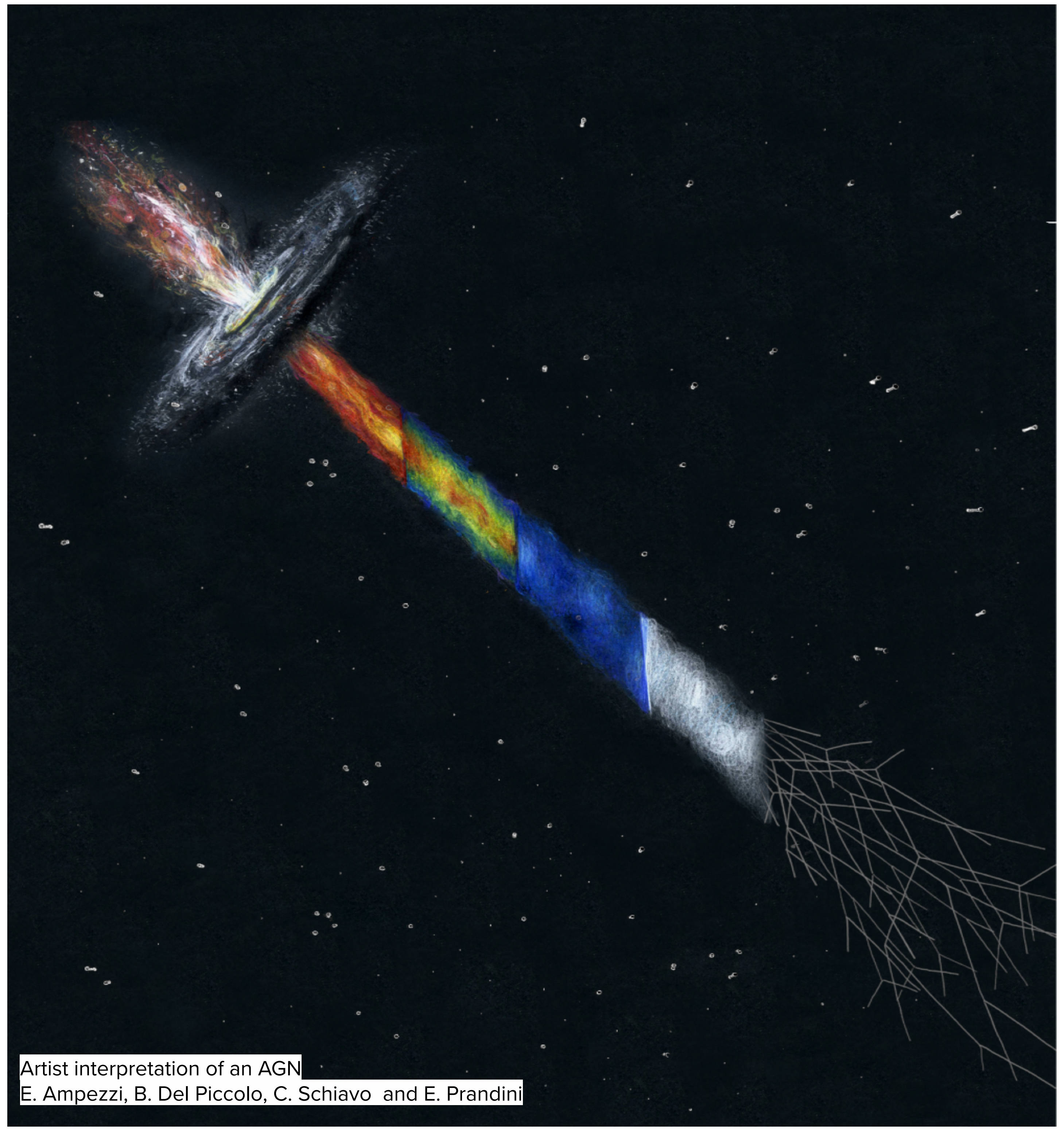

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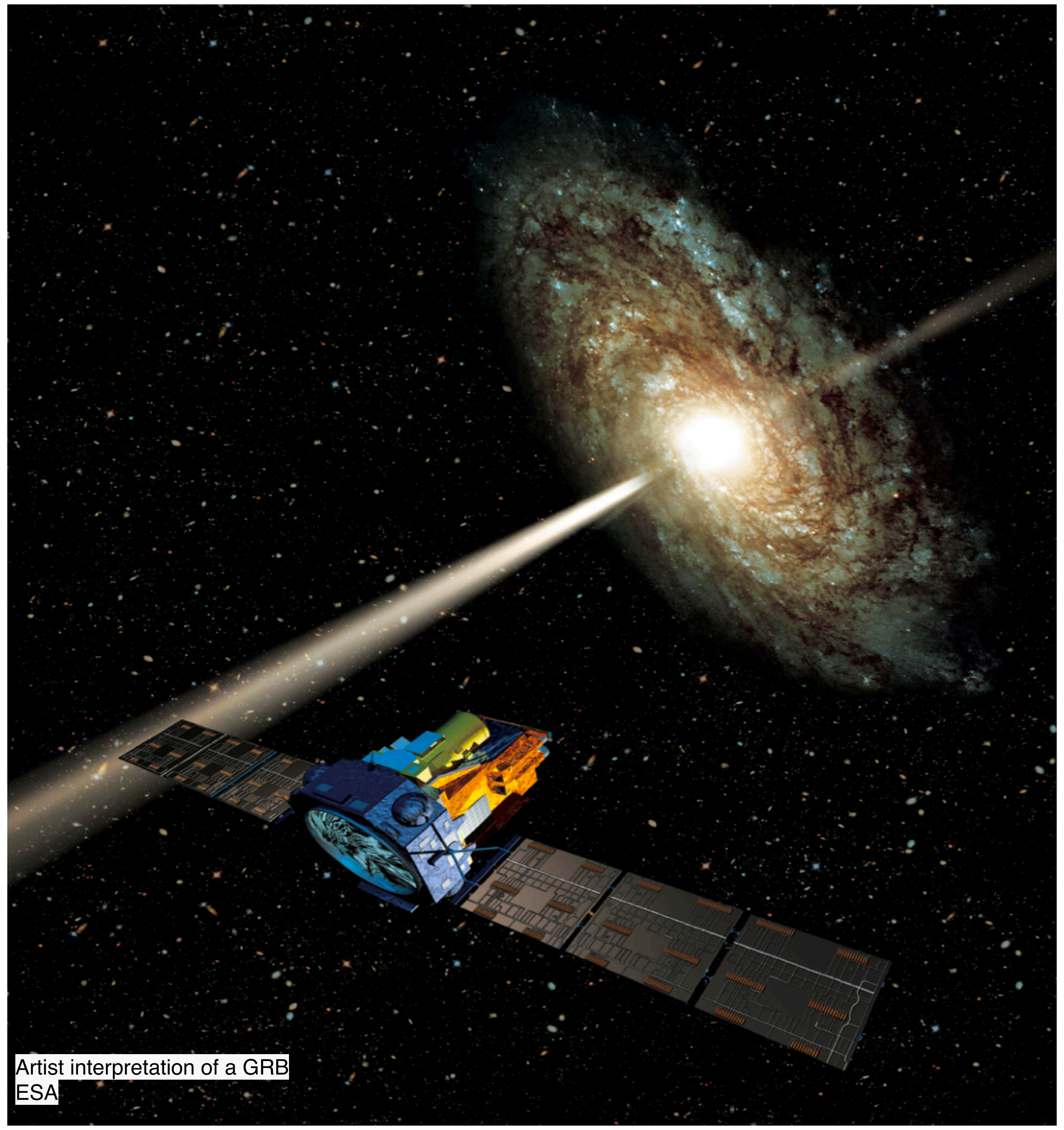
Gamma-ray bursts

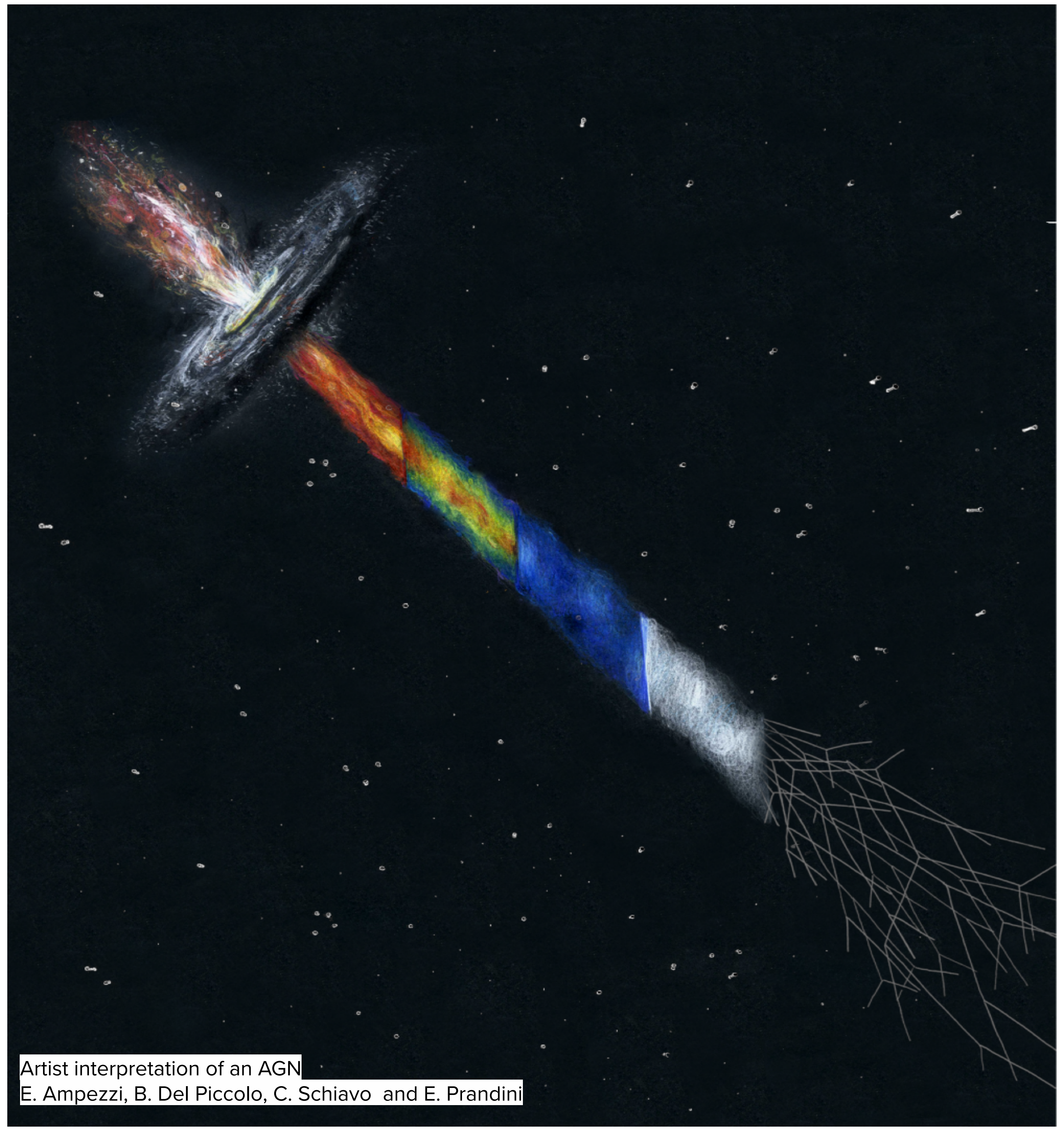


$E_{\text{max}} \lesssim 0.1\ \text{TeV}$ $\quad E_{\text{max}} \sim 1\ \text{TeV}$
$t_{\text{var}} \sim 1\text{msec} - 1\ \text{min}$
$z_s \sim 0.01 - 8.2$

GRB 190114C

VHE gamma-ray sources suitable for LIV studies

Active galactic nuclei



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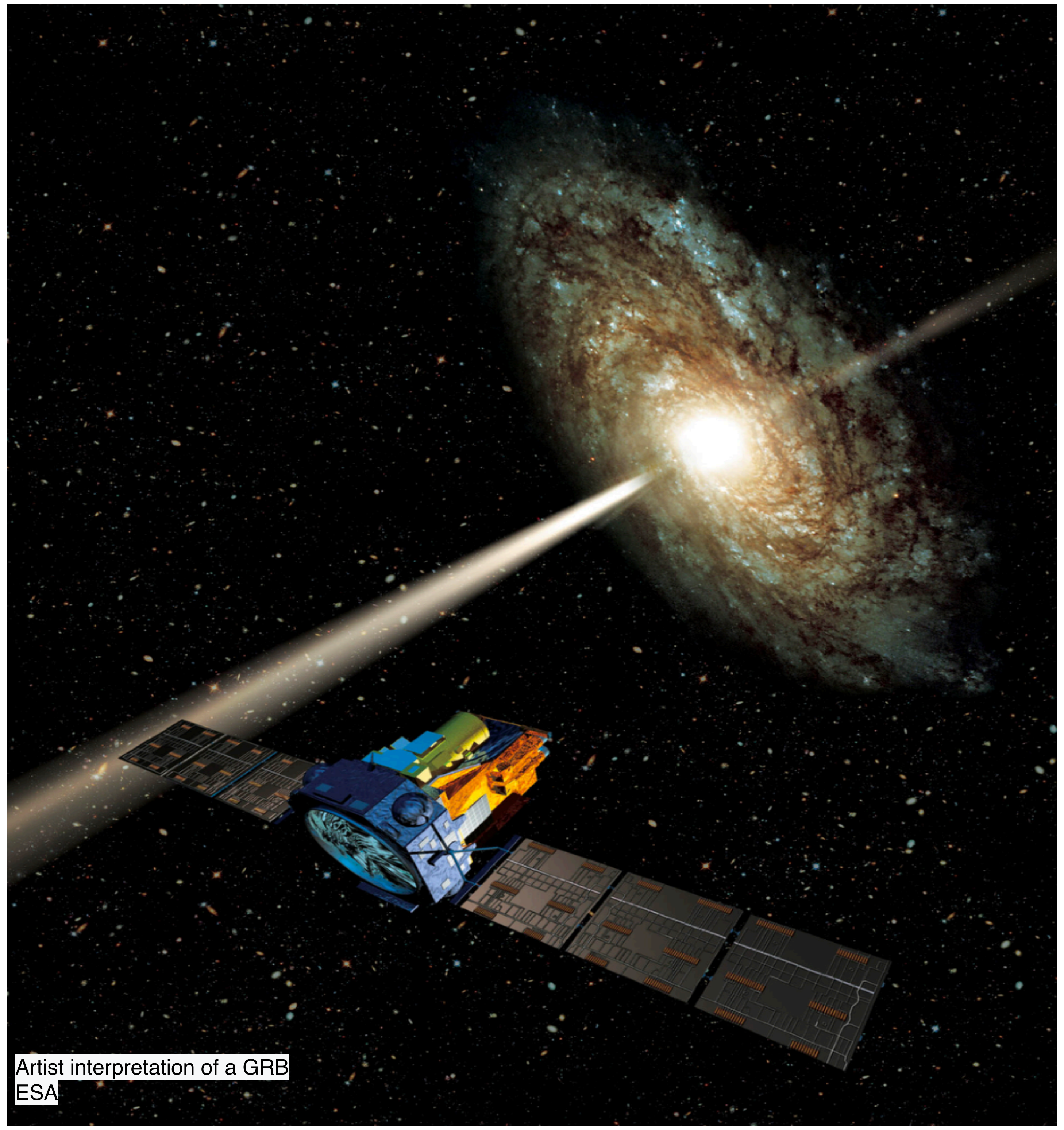
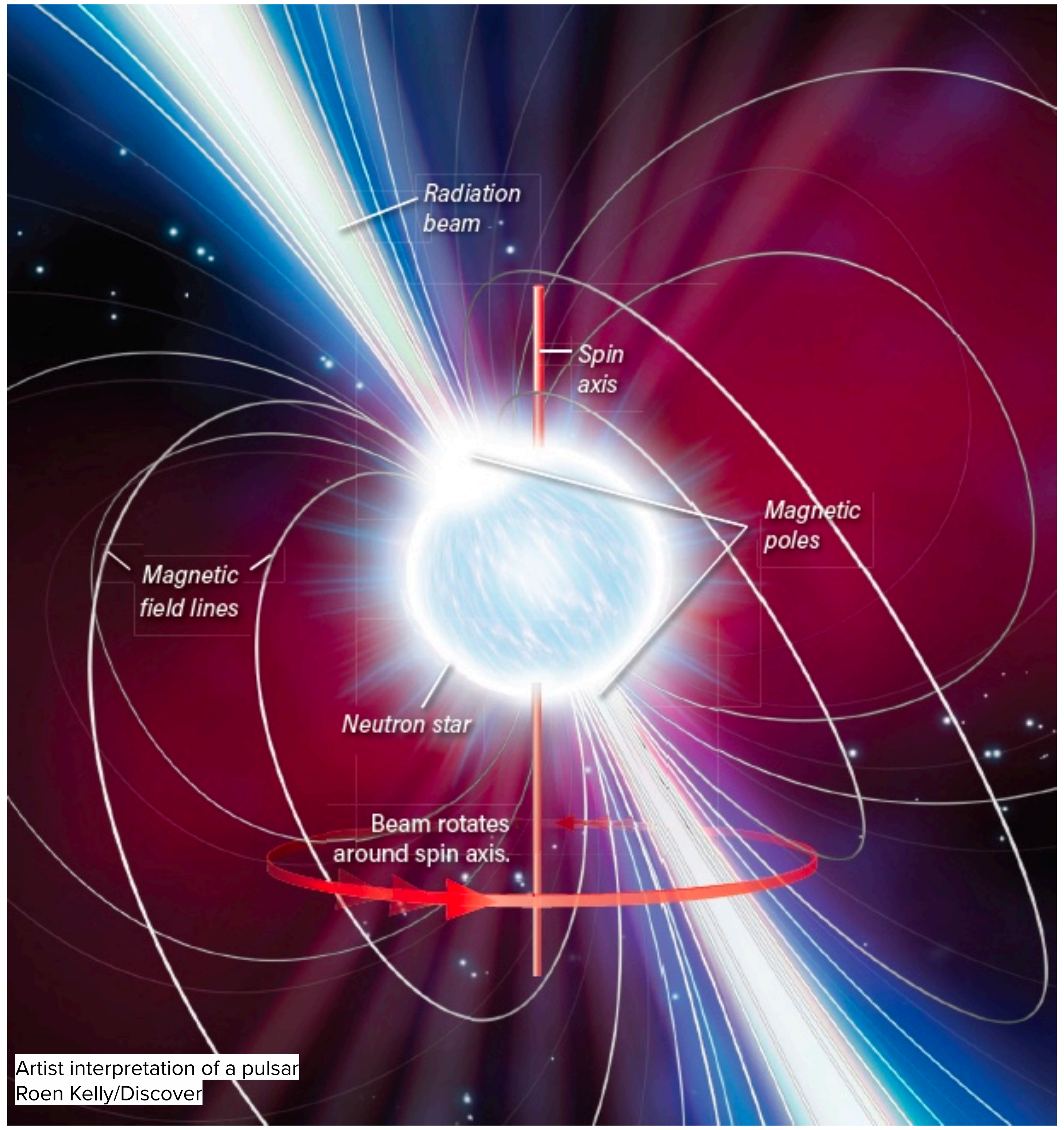
Gamma-ray bursts



$E_{\mathrm{max}} \lesssim 0.1$ TeV $E_{\mathrm{max}} \sim 1$ TeV
$t_{\mathrm{var}} \sim 1$msec – 1 min
$z_{\mathrm{s}} \sim 0.01 - 8.2$

GRB 190114C

Pulsars

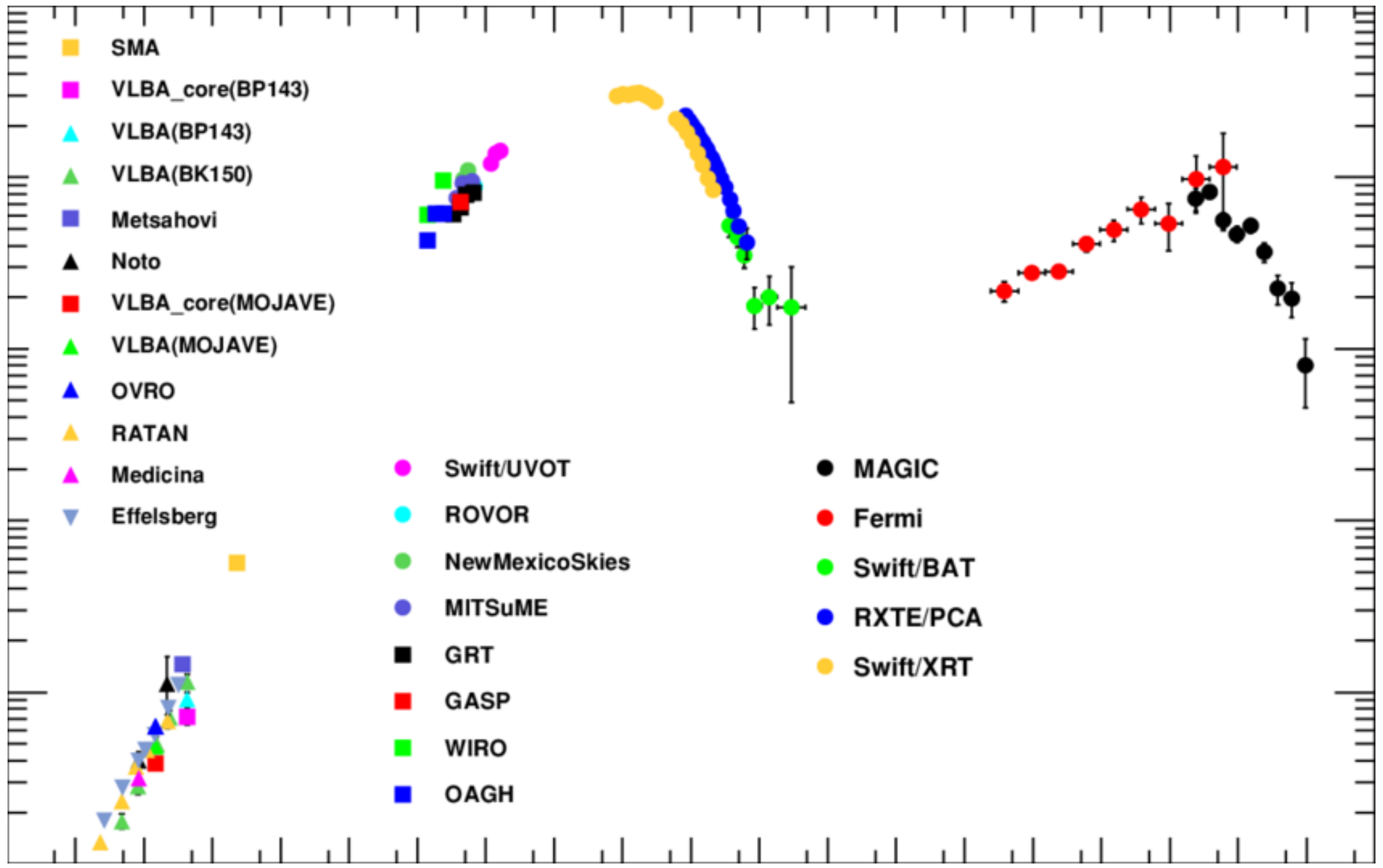


$E_{\mathrm{max}} \sim 1$ TeV
$t_{\mathrm{var}} \sim 1$ msec
~ 1 kpc

Crab
Vela

Source used in our study: Mrk 421

- Active galactic nucleus at z = 0.031
- First extragalactic object detected in VHE gamma ray band
- Monitoring observation on the night of April 25th to 26th 2014
  - Good atmospheric conditions
- After data quality cuts ~ 9000 events > 100 GeV in the signal region → ~7% expected background contamination



A. A. Abdo et al. (2011)
arXiv: 1106.1348

Spectrum and Light Curve

- The resulting spectrum:

$$\phi(E) = A\left(\frac{E}{E_0}\right)^{-\gamma-\beta\log\left(\frac{E}{E_0}\right)}$$

$$\boxed{\begin{aligned}\gamma &= 2.11 \pm 0.03 \\ \beta &= 0.119 \pm 0.014\end{aligned}}$$

Spectrum and Light Curve

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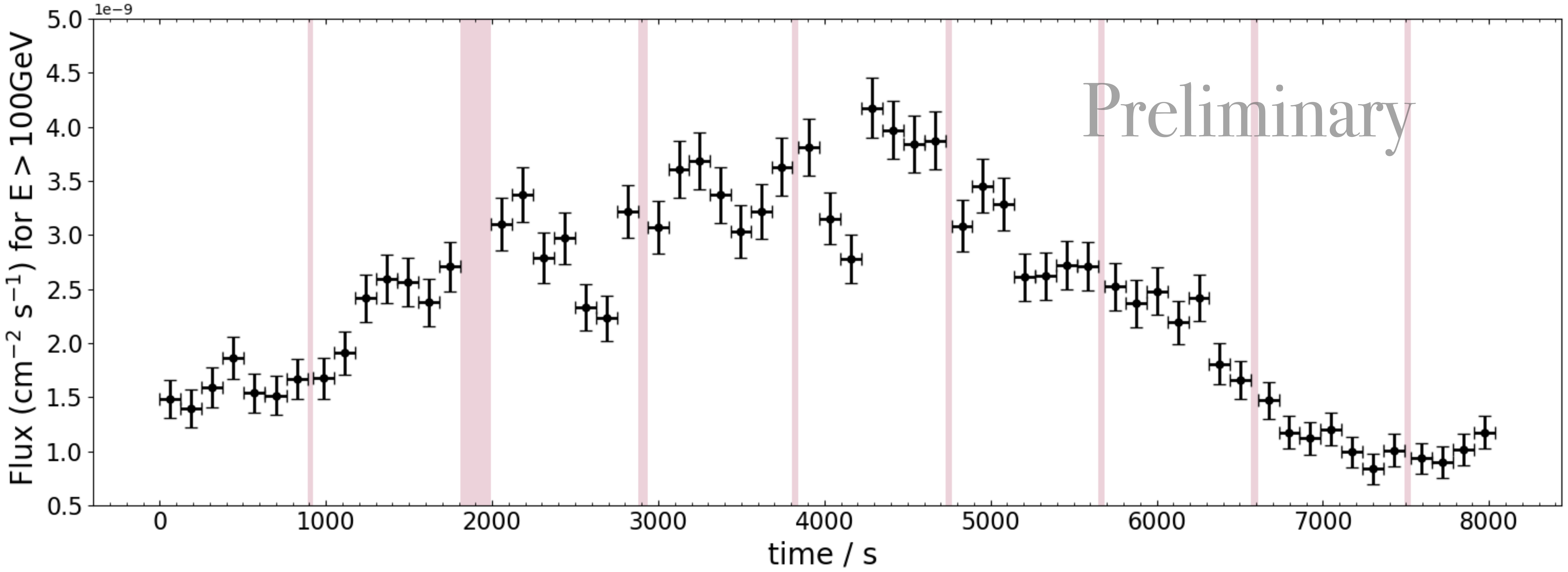
- Light curve:



~ 8 C.U.

LIV analysis: unbinned likelihood method

* Standard, unbinned likelihood:

$$\mathscr{L}\left(\eta_n\right) = \prod_{i=1}^{N} P\left(E_i, t_i \mid \eta_n\right)$$

* Obtaining intrinsic properties poses a considerable challenge

* The selection of the interpolation algorithm is somewhat arbitrary and challenging to justify

* After obtaining the LC template, it becomes difficult to incorporate the Poissonian uncertainties

LIV analysis: binned likelihood

- Novel, binned likelihood:

$$\mathscr{L} = \prod_{i=1}^{N_1} \prod_{j=1}^{N_2} \mathscr{P}\left(s_{i,j}, b_{i,j} \mid N_{\mathrm{ON},i,j}, N_{\mathrm{OFF},i,j}\right)$$

Expected signal counts

Expected background counts

The observed counts in ON region

The observed counts in OFF region

- Poissonian term:

$$\mathscr{P}(s,b) = \frac{(s+\alpha b)^{N_{\mathrm{ON}}}}{N_{\mathrm{ON}}!} e^{-(s+\alpha b)} \frac{b^{N_{\mathrm{OFF}}}}{N_{\mathrm{OFF}}!} e^{-b}$$

LIV analysis: binned likelihood

- The expected signal counts $s_{i,j}$ for the i-th time bin and j-th energy bin:

$$s_{i,j} = \int_{\Delta E_j} dE \sum_{k=1}^{N_1} \int dE' \, \mathrm{EBL}\left(E'\right) \phi_k\left(E', \mu\right) \mathrm{IRF}_i\left(E, E'\right) \Delta t_{i,k}\left(\eta_n \cdot E'\right)$$

j-th energy bin

Extragalactic background light absorption

Instrument response function

Time-width migration for the i-th bin relative to the k-th one

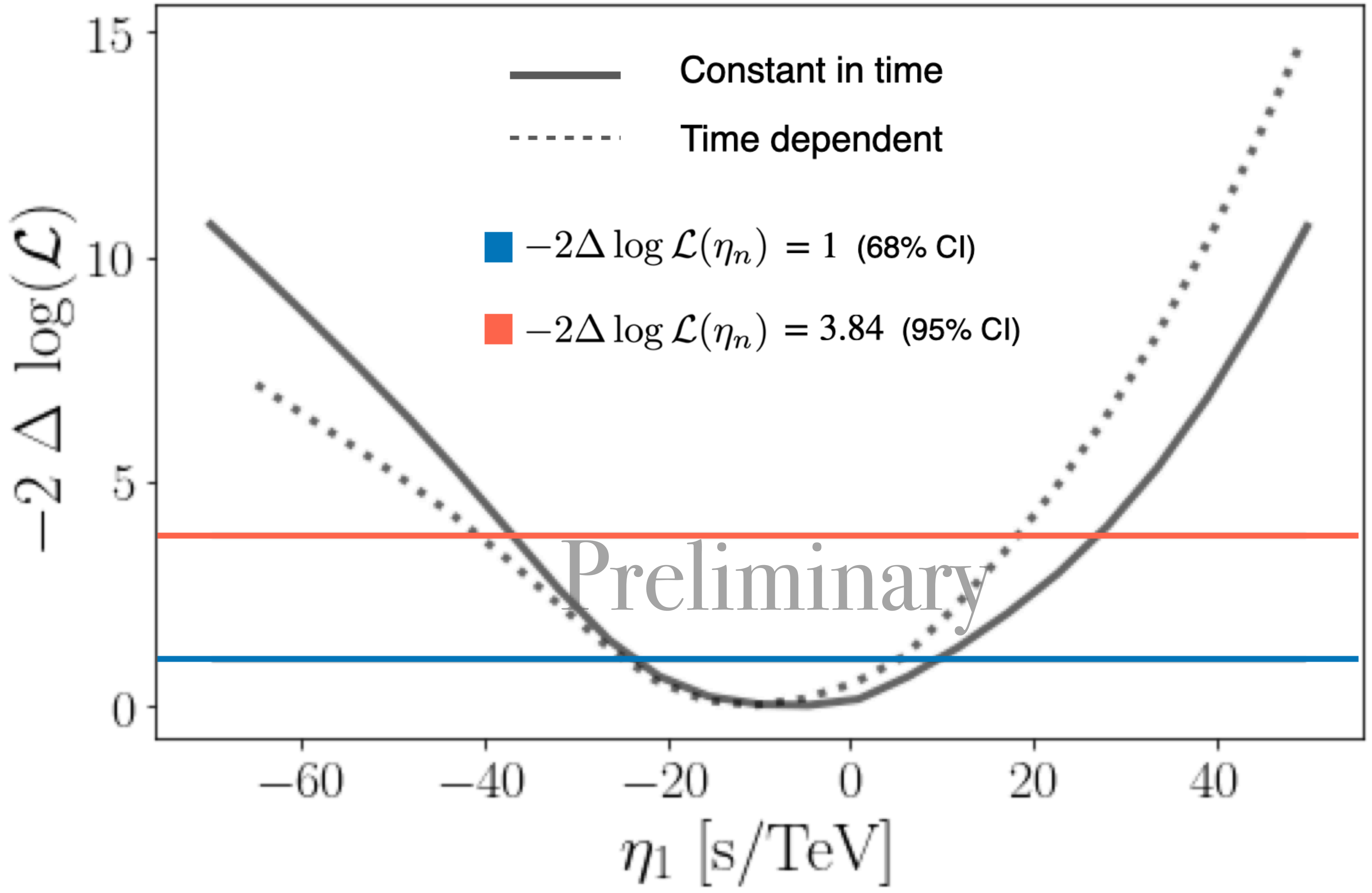
Intrinsic flux per energy and time in the k-th time bin

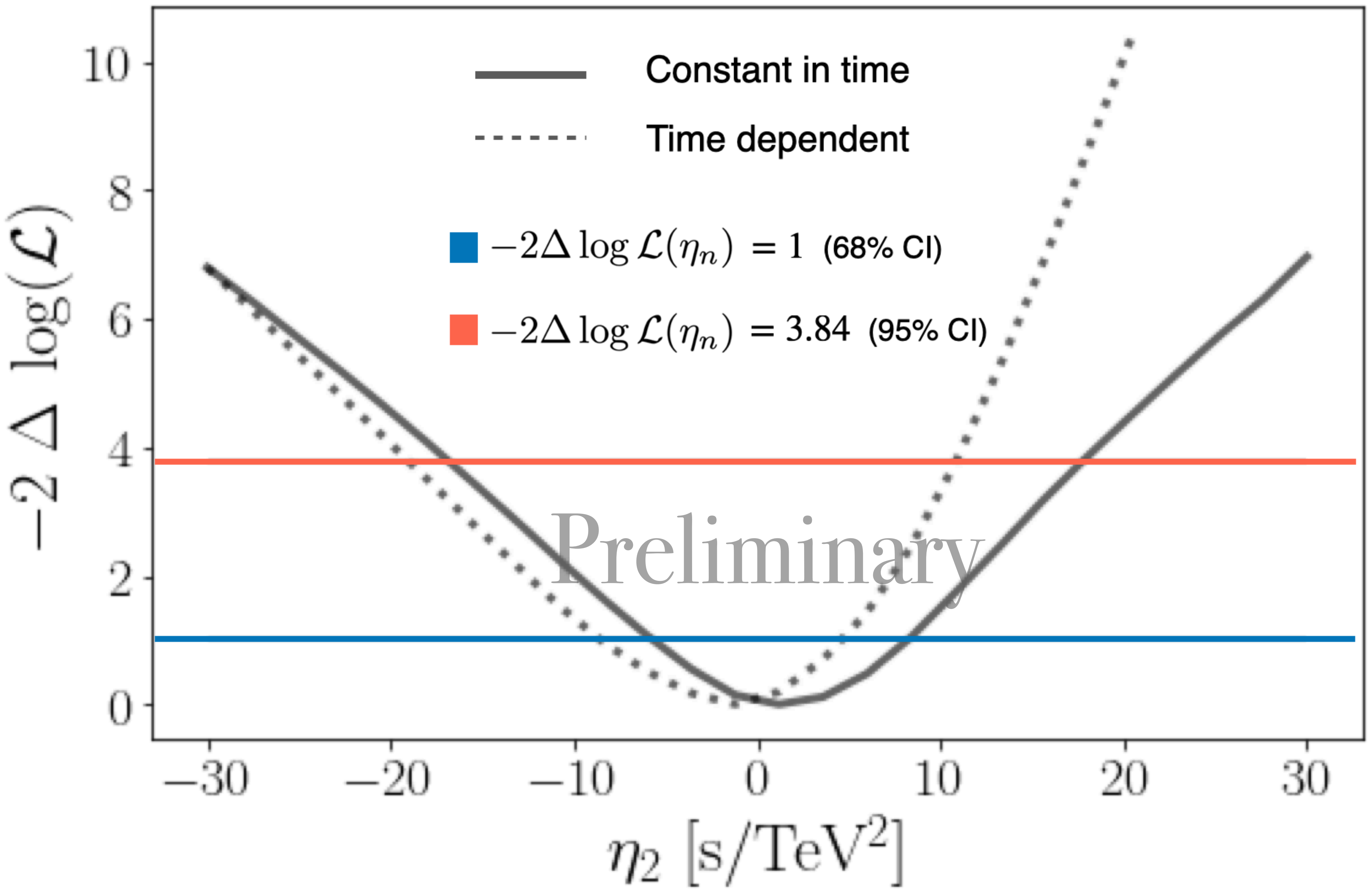
For us: log-parabola function in energy with parameters γ and β constant in time

$\gamma = 2.11 \pm 0.03$

$\beta = 0.119 \pm 0.014$

Time dependence of SED parameters





- SED parameters constant in time → relaxing this assumption → variations in the derived lower limits on the QG energy scale by 8-45% in the linear scenario and 9-22% in the quadratic case.

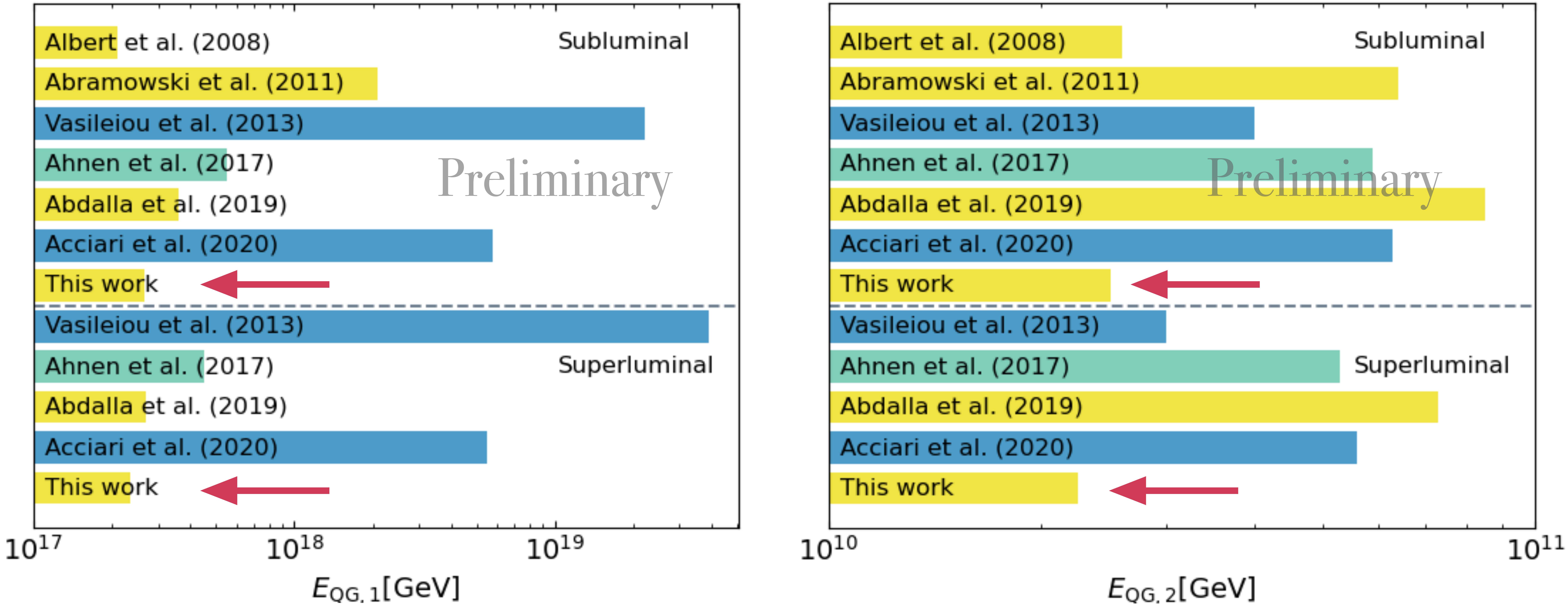
Results

* After taking into account number of bins (70 in time & 10 in energy), energy scale, background normalisation, cosmological model, time dependence of SED parameters:

Obtained limits		
Case	**No systematics**	**Including systematics**
Linear scenario: $E_{\mathrm{QG},1}/\mathrm{GeV}$		
superluminal	$3.55 \cdot 10^{17}$	$2.66 \cdot 10^{17}$
subluminal	$4.82 \cdot 10^{17}$	$2.36 \cdot 10^{17}$
Quadratic scenario: $E_{\mathrm{QG},2}/\mathrm{GeV}$		
superluminal	$3.58 \cdot 10^{10}$	$2.51 \cdot 10^{10}$
subluminal	$3.52 \cdot 10^{10}$	$2.25 \cdot 10^{10}$

Lower bounds on the LIV energy scale obtained with ToF method

* Lower limits on the QG energy scale as reported in prior studies



Conclusions

- The null hypothesis of no spectral lag is compatible with the observation

- Our limits are more robust since they do not depend on defining a LC template → fully integrating Poissonian statistics

Thank you for your attention :)

Interested in proposing observations with MAGIC ?

Next MAGIC observing call (Cycle-19) will come very soon. It will be posted here:

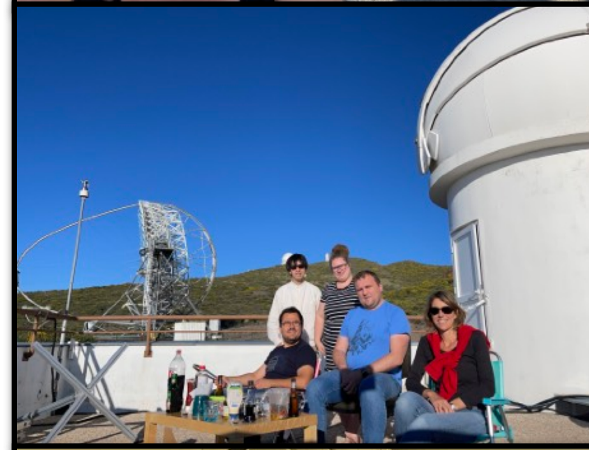
https://magic.mpp.mpg.de/public/magicop/

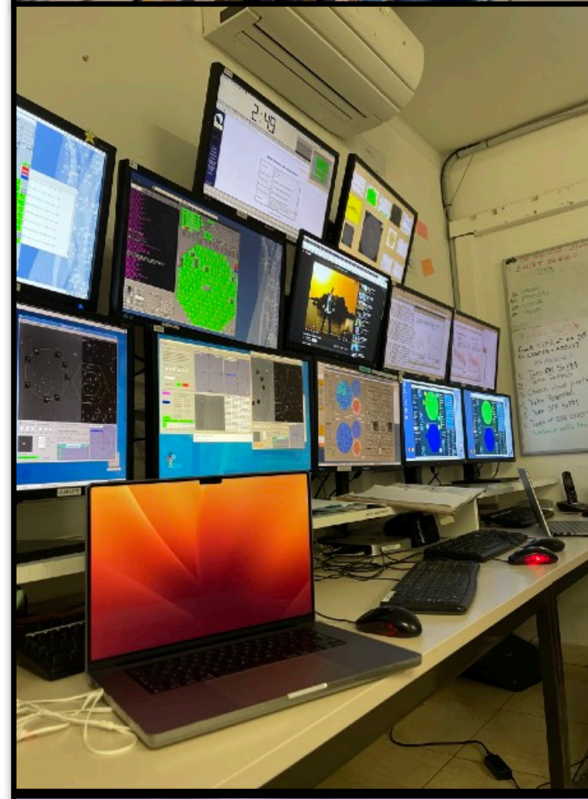
(Deadline for submitting proposals in the end of October or beginning of November)

PoS (LIV with Mrk 421)

Get in contact

jelena.striskovic@gmail.com



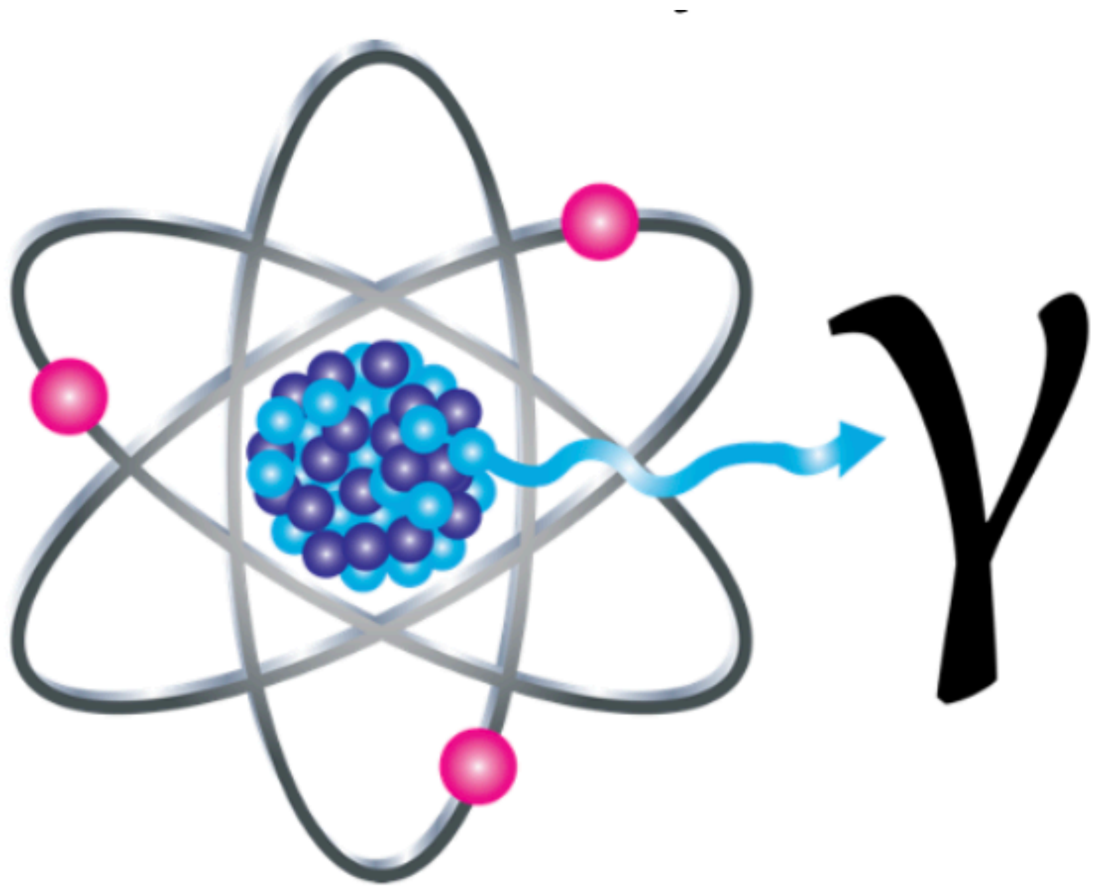


Backup slides

Expected energy scale of QG

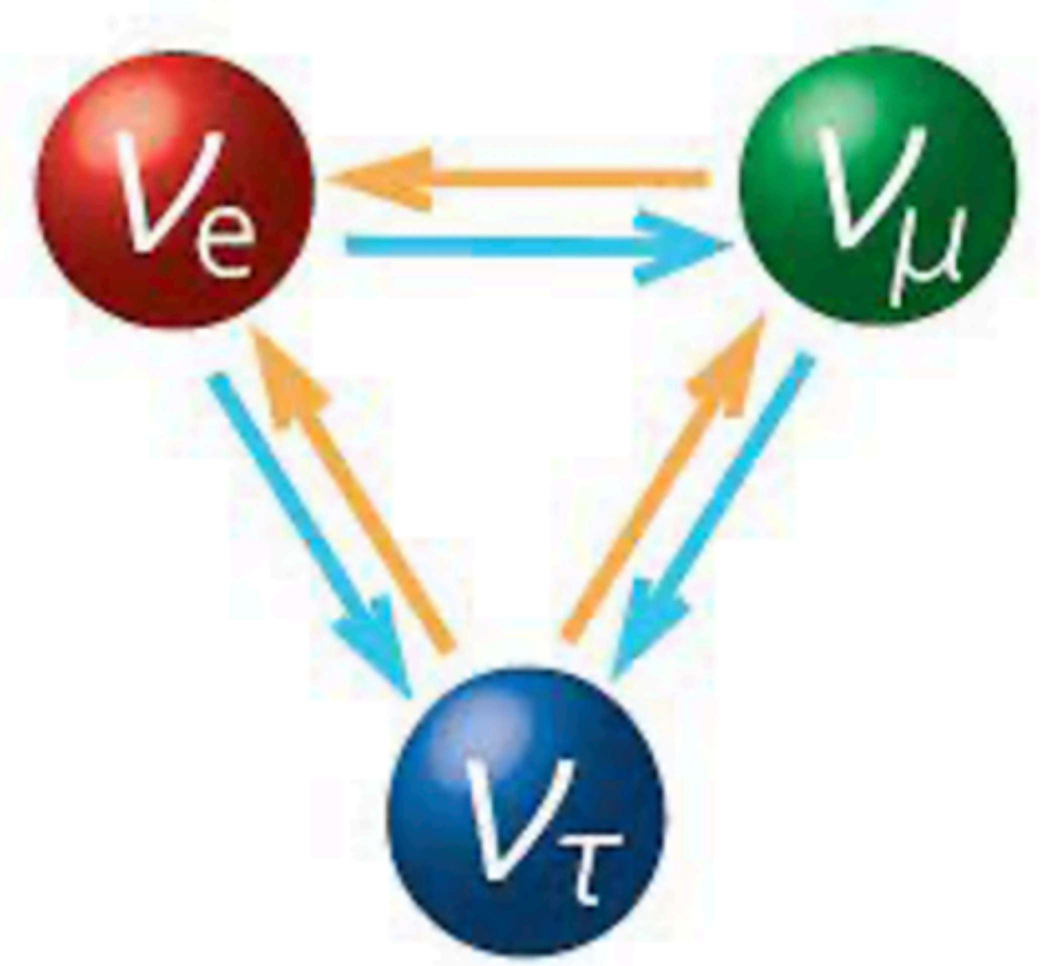
- **Planck energy** (~ 10^{28} eV)
- Vastly above the highest energies accessible in human-built accelerators

Gamma rays



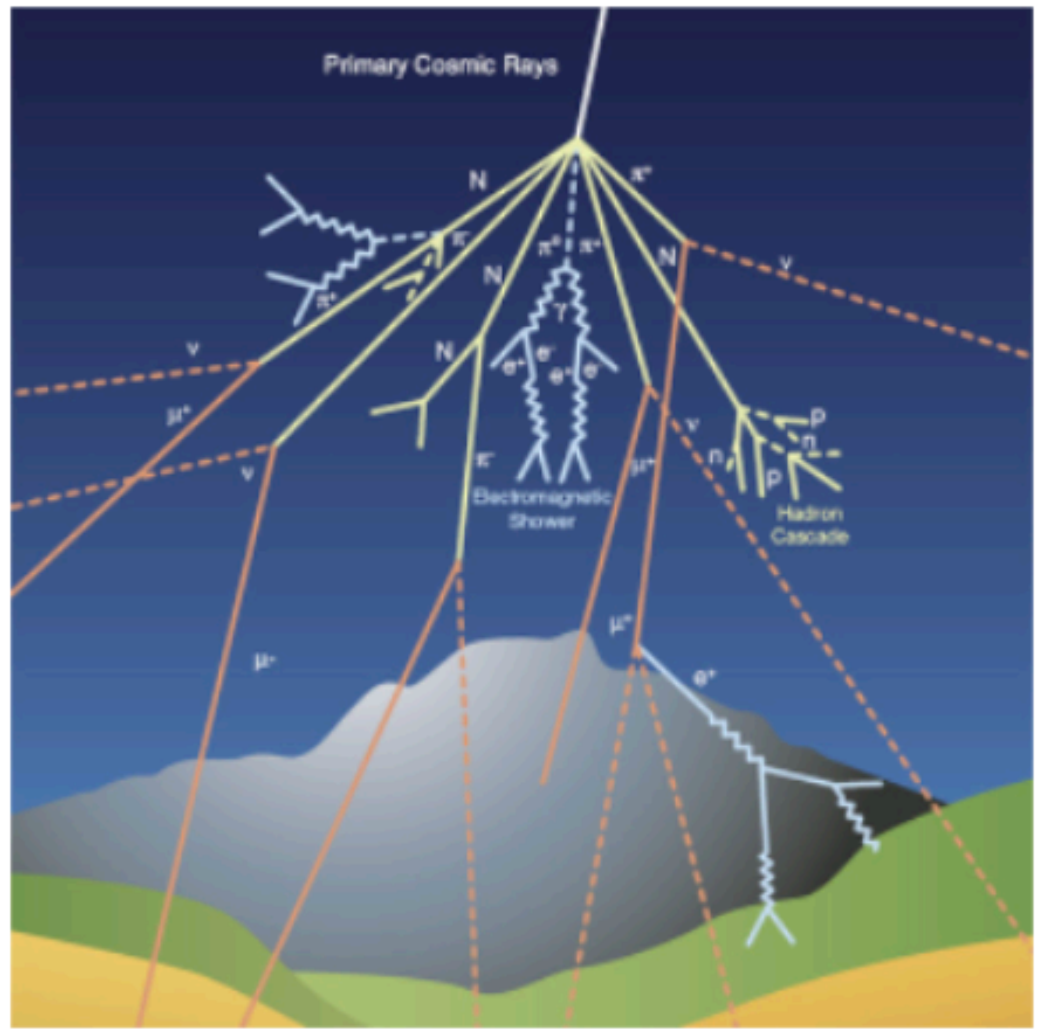
$\sim 1.4 \times 10^{15}$ eV

Neutrinos



$\sim 6.3 \times 10^{15}$ eV

Cosmic rays



$\sim 3.2 \times 10^{20}$ eV

- Huge cosmological distances → **cumulative effect**

The time-width migration

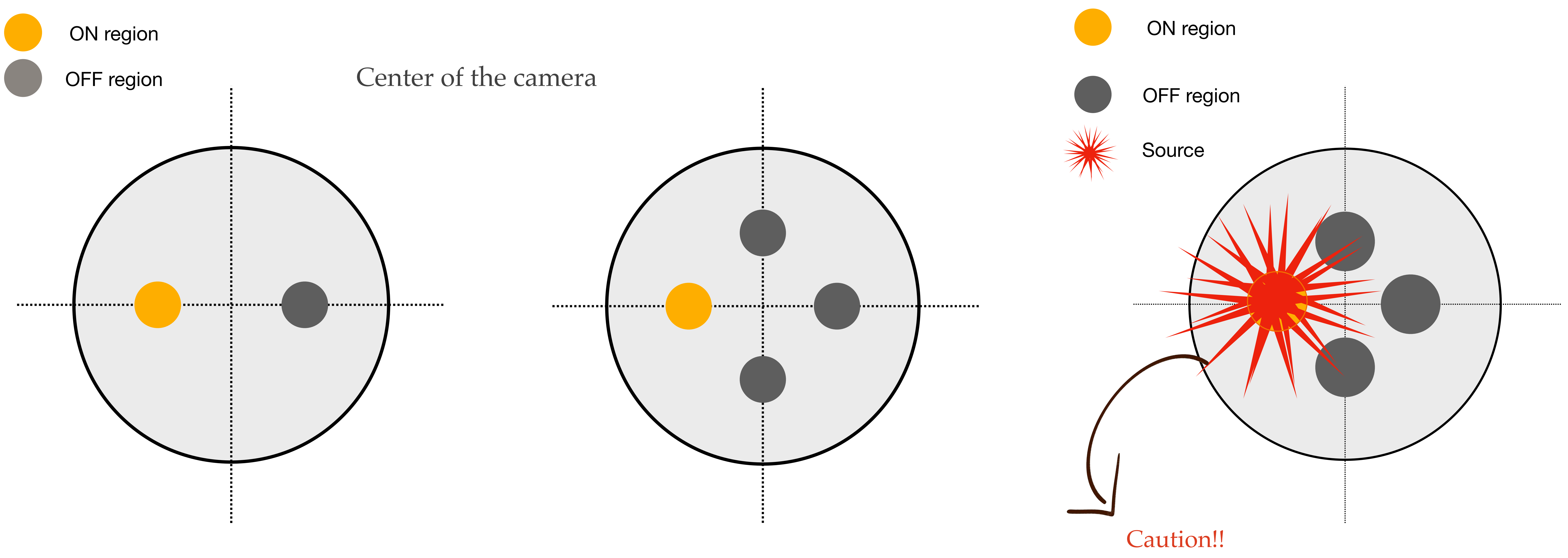
- Matrix that considers the intrinsic flux contribution from the k-th bin to the i-th one, factoring in the LIV-induced delays
- Trivial example in case with: $\eta_1 = 0$ s/TeV:

$$\Delta t_{i,k}(0\,\mathrm{s}) = \delta_{\mathrm{i,k}} \Delta \mathrm{t_i} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \cdot \mathrm{s}$$

- Trivial example in case with: E'=1 TeV and $\eta_1 = 1$ s/TeV:

$$\Delta t_{i,k}(1\,\mathrm{s}) = \begin{bmatrix} 9 & 0 \\ 1 & 9 \end{bmatrix} \cdot \mathrm{s}$$

ON/OFF regions



$$p_i^{(s)} = \frac{N_{\mathrm{ON}} - \alpha N_{\mathrm{OFF}}}{N_{\mathrm{ON}}}$$

$$p_i^{(b)} = \frac{\alpha N_{\mathrm{OFF}}}{N_{\mathrm{ON}}}$$

ON/OFF normalisation

Systematic uncertainties

Study of systematic uncertainties				
Systematic effect	**Size($E_{QG,1}$)**		**Size($E_{QG,2}$)**	
	superl.	subl.	superl.	subl.
Number of bins in time	$< 14\%$	$< 14\%$	$< 16\%$	$<16\%$
Number of bins in energy	$<11\%$	$< 11\%$	$< 17\%$	$<17\%$
Energy scale	$\sim 15\%$	$\sim 15\%$	$\sim 15\%$	$\sim 15\%$
Background normalization	$<0.1\%$	$<0.1\%$	$<0.1\%$	$<0.1\%$
Cosmological model	$<4\%$	$<4\%$	$<5\%$	$<5\%$
Time-evolution of SED parameters	$\sim 8\%$	$\sim 45\%$	$\sim 9\%$	$\sim 22\%$
Total	$\lesssim 25\ \%$	$\lesssim 51\ \%$	$\lesssim 30\ \%$	$\lesssim 36\ \%$

Unbinned maximum likelihood analysis

- Likelihood function for observed number of events in the signal region:

$$\mathscr{L}(\eta_n) = \prod_{i=1}^{N_{\mathrm{ON}}} \left(p_i^{(\mathrm{s})} \frac{f^{(\mathrm{s})}(E_i, t_i)}{\int_{E_{\min}}^{E_{\max}} dE \int_{t_{\min}}^{t_{\max}} f^{(\mathrm{s})}(E,t)dt} + p_i^{(\mathrm{b})} \frac{f^{(\mathrm{b})}(E_i, t_i)}{\int_{E_{\min}}^{E_{\max}} dE \int_{t_{\min}}^{t_{\max}} f^{(\mathrm{b})}(E,t)dt} \right)$$

$$p_i^{(\mathrm{s})} = \frac{N_{\mathrm{ON}} - \alpha N_{\mathrm{OFF}}}{N_{\mathrm{ON}}}$$

ON/OFF normalisation

$$p_i^{(\mathrm{b})} = \frac{\alpha N_{\mathrm{OFF}}}{N_{\mathrm{ON}}}$$

- Probability distribution function:

$$f^{(\mathrm{s})}(E,t) = \int_0^{\infty} F\left(t + \eta_n E_{\mathrm{true}}^n\right) \Phi_{\mathrm{obs}}(E_{\mathrm{true}}) G\left(E, E_{\mathrm{true}}\right) A_{\mathrm{eff}}\left(E_{\mathrm{true}}, t\right) dE_{\mathrm{true}}$$

F(t) - observed light curve (LC); LC template

energy resolution and bias

collection area

observed spectral distribution of gamma rays