Inflationary attractors in Palatini F(R,X)

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Slow-roll inflation in F(R) Palatini gravity

New method which allows to derive inflationary predictions in presence of higher order terms in the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(R) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

By means of a conformal transformation we obtain the Einstein frame action

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi,\zeta)\right]$$

with $\frac{\partial\chi}{\partial\phi} = \sqrt{\frac{1}{F'(\zeta)}}$; $U(\chi,\zeta) = \frac{V(\phi(\chi))}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)}$

Slow-roll inflation in F(R) Palatini gravity

EoM obtained by varying ζ gives

$$2F(\zeta) - \zeta F'(\zeta) - k(\phi)\partial^\mu\phi\partial_\mu\phi F'(\zeta) - 4V(\phi) = 0$$

In **slow-roll** limit the equation reduces to

$$G(\zeta) \equiv \frac{1}{4}[2F(\zeta) - \zeta F'(\zeta)] = V(\phi)$$

Cannot be solved for general $F(R)$

Trick: use ζ as computational variable using above equation as a constraint

Beyond Slow-Roll

However $F_{>2}(R)$ theories do not provide a graceful exit from inflation!

Using Friedmann equations one can show that

$$\dot{\zeta} = \frac{3H\dot{\phi}^2 F'(\zeta) + 3V'(\phi)\dot{\phi}}{2G'(\zeta) + \frac{3}{2}\dot{\phi}^2 F''(\zeta)}$$

The denominator has a pole whenever $F(\zeta)$ grows faster than ζ^2

It can be shown that $\epsilon_H = 1$ cannot be reached without hitting the pole hence we cannot have graceful exit

Beyond F(R): F(R,X)

This issue can be solved by extending the class of $F(R)$ theories to $F(R,X)$ where the X stands for the inflaton kinetic term $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(R,X) - V(\phi)\right]$$

If we consider $F(R,X) = F(R-X)$ we can rewrite the action in the Einstein frame as:

$$S = \int d^4x\sqrt{-g}\Big(\frac{R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\zeta,\phi)\Big)$$

Field ϕ is already canonically normalized!

Beyond F(R): F(R,X)

Slow-roll computations proceed in the same way

We just need to keep in mind that $\zeta = R - X$ and that field ϕ is already canonical

However in the strong coupling limit we get the same k-hilltop prediction of standard $F_{>2}(R)$: $r \sim 0, n_s = 1 - \frac{k-1}{k-2}\frac{2}{N_e}$

The only difference is that now we need to set α such that $\epsilon(\zeta) > 1$ at the local maximum

This sets α_{min} which is model dependent but always such that the strong coupling regime is realized

Beyond F(R): F(R,X)

New features for $F(R,X)$ that solve many of the issues of the $F(R)$ models:

1) $G(\zeta) = V(\phi)$ is an exact equation that holds outside of slow-roll as well

2) $\dot{\zeta} = \frac{V'(\phi)\dot{\phi}}{G'(\zeta)}$ is regular for any $\zeta > \zeta_0$

3) We have graceful exit from inflation and evolution is much more simple to study

Quadratic F(R,X) and inflationary attractors

Now we focus on the class of quadratic

$$F(R_X) = 2\Lambda + \omega R_X + \alpha R_X^2$$

with $\mathrm{R}_X = R - X$

with this choice the EoM for ζ becomes

$$\Lambda + \frac{\omega}{4}\zeta = V(\phi)$$

which upon substitution gives the Einstein frame potential

$$U(\phi) = \frac{\bar{V}(\phi)}{8\alpha\bar{V}(\phi) + \omega^2}$$

with $\bar{V}(\phi) = V(\phi) - \Lambda$

Quadratic $F(R, X)$ and inflationary attractors

Requiring

- positivity of $U(\phi)$
- consistency of the ζ's EoM
- $F'(\zeta) > 0$

only allows two configurations:

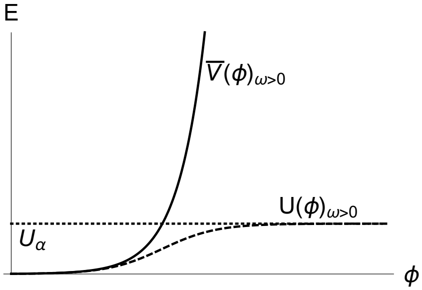
1) $\omega > 0, \Lambda \leq 0, V(\phi) \geq 0$
2) $\omega < 0, \Lambda > 0, V(\phi) \leq 0$

In both cases $\alpha > \frac{\omega^2}{8\Lambda}$ for $\Lambda \neq 0$

The vacuum solutions are given by $\zeta_0 = -\frac{\Lambda}{4\omega},\ U_\Lambda = \frac{\Lambda}{8\alpha\Lambda+\omega^2}$

In the strong field limit $V(\phi) \to \pm\infty$ the potential $U(\phi)$ reaches a plateau at $U_\alpha = \frac{1}{8\alpha}$

$\omega > 0$ configuration



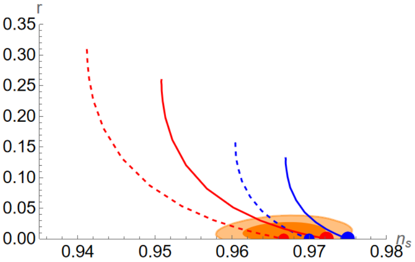
In this case we have $U_\alpha > U_\Lambda$

Inflation happens at large ϕ close to the U_α plateau

In this region the potential shape can be approximated by $U(\phi) \sim U_\alpha(1 - \frac{\omega^2 U_\alpha}{V(\phi)})$ which generalizes the polynomial α-attractors

Since for $\alpha\bar{V} >> \omega^2$ we generate asymptotically flat potentials $\rightarrow$ **canonical fractional attractors**

$\omega > 0$ configuration



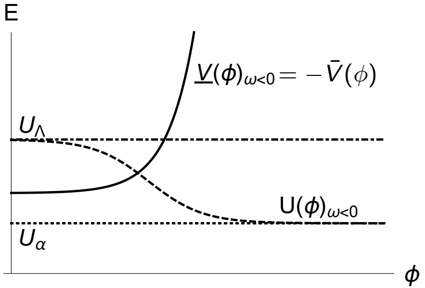
By choosing a monomial potential $V(\phi) = \frac{\lambda}{k!}\phi^k$ and taking the strong coupling limit we get the polynomial α-attractors prediction

$r \sim 0$

$n_s = 1 - \frac{k+1}{k+2}\frac{2}{N_e}$

The plot above shows the results for $V(\phi) = \frac{m^2}{2}\phi^2(blue)$ and $V(\phi) = \frac{\lambda}{4!}\phi^4(red)$

$\omega < 0$ configuration



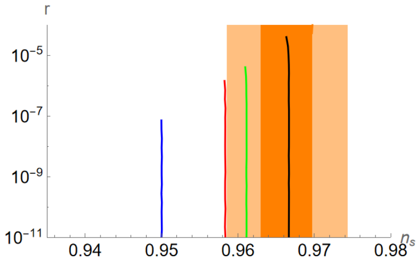
In this case we have $U_\Lambda > U_\alpha$

Inflation happens at small ϕ close to the U_Λ plateau

In this region the potential shape can be approximated by $U(\phi) \sim U_\Lambda(1 - \frac{U_\Lambda}{\Lambda U_\alpha}|V(\phi)|)$ which is a general version of hilltop potentials

Since for $\alpha \underline{V} >> \omega^2$ we generate hilltop-like potentials
$\rightarrow$ **tailed fractional attractors**

$\omega < 0$ configuration



By choosing a monomial potential $V(\phi) = -\frac{\lambda}{k!}\phi^k$ and taking the strong coupling limit we get the small r hilltop prediction

$r \sim 0$

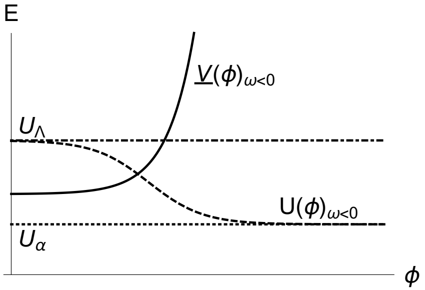
$n_s = 1 - \frac{k-1}{k-2}\frac{2}{N_e}$

The plot above shows the results for $k = 4(blue), 6(red), 8(green)$ and $V(\phi) = -e^{\lambda\phi}(black)$

Not every α is allowed in this case, $\alpha > \alpha_{min}$ model dependent, but always such that $r < 10^{-5}$

$\omega < 0$ configuration

In this case the U_α plateau can be chosen such that it matches the current value of the cosmological constant

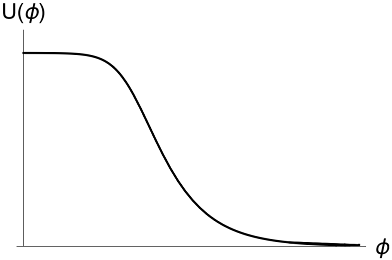


However α has to be set to very large values

Unfortunately this usually spoils the height of the potential at small ϕ, and inflation happens at too small scales

With an extreme tuning of Λ and α it is possible to keep U_Λ well separated from U_α $\rightarrow$ confirmation (not a solution!) of the problem

General behavior of $F_{>2}(R_X)$



The model $\omega < 0$ can be considered an approximation for all $F_{>2}(R_X)$ in the strong coupling regime

For large α:
$r \sim 0$, $n_s = 1 - \frac{k-1}{k-2}\frac{2}{N_e}$

In that regime the potential in the region where inflation happens can be approximated as a k-hilltop potential

$F_{>2}(R_X)$ also solve the issue of the horizontal asymptote: we don't need to fix α to enormous values, $U(\phi)$ asymptotically reaches 0

Recap

Palatini $F(R)$ theories provide flat potentials independently of the chosen model but they do not provide a graceful exit from inflation

Palatini $F(R_X)$ theories provide both flat potential and graceful exit

Quadratic theories produce classes of flat potentials that can be classified as attractors given their general predictions

Quadratic theories can also be used as approximations for more general $F(R_X)$ in the strong coupling regime

$F(R_X)$ exhibit viable inflation for a wide class of potentials $V(\phi)$ and $F(R_X)$ functions

Thank you for the attention!

Backup slides

Metric vs Palatini [2]

Metric formulation

Only DOF is the metric $g_{\mu\nu}$, connection Γ is algebrically related to the metric and is assumed to be the Levi-Civita one

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \mathcal{L}_m(g_{\mu\nu}, \phi, \partial\phi) \right]$$

Variation with respect to $g_{\mu\nu}$ yields EoM i.e. Einstein equations

Ricci scalar $R = g^{\alpha\beta} R_{\alpha\beta}$ depends on the metric through the

Levi-Civita connection $\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\lambda} (g_{\beta\lambda,\gamma} + g_{\lambda\gamma,\beta} - g_{\beta\gamma,\lambda})$

[2] T. Koivisto and H. Kurki-Suonio, Cosmological perturbations in the palatini formulation of modified gravity, Class. Quant. Grav. 23 (2006)

Metric vs Palatini

Palatini formulation

In Palatini we have both metric $g_{\mu\nu}$ and the connection Γ as DOF's

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R(\Gamma) + \mathcal{L}_m(g_{\mu\nu}, \Gamma, \phi, \partial\phi)\right]$$

Variation with respect to Γ yields EoM $\Gamma = \Gamma(g_{\mu\nu}, \phi)$

Variation with respect to $g_{\mu\nu}$ yields analogue of Einstein equations

Ricci tensor $R_{\alpha\beta}(\Gamma)$ is built from connection only

What if we have F(R) in the action?

Metric vs Palatini

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(R) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

F(R) term produces drastically different theories

In **metric formulation** there is one more scalar degree of freedom

Bi-field inflationary configuration

In **Palatini formulation** this scalar DOF is not present

Single field inflationary configuration

Different inflationary predictions

Slow-roll inflation in F(R) Palatini gravity

Then $U(\chi, \zeta) = \frac{G(\zeta)}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} = \frac{1}{4}\frac{\zeta}{F'(\zeta)} = U(\zeta)$

Notice that:

1) $U(\zeta) = \frac{\zeta}{4F'(\zeta)}$ implies $\zeta > 0$ since $F'(\zeta)$ must be positive to allow for correct sign of kinetic term (and Weyl transformation)

2) The Einstein frame potential $U(\zeta(\chi))$ is positive definite regardless of the choice of the Jordan frame $V(\phi)$

In order to compute the slow roll parameters we need the derivatives of $U(\zeta)$ with respect to χ

Slow-roll inflation in F(R) Palatini gravity

Slow-roll parameters:

$\epsilon(\zeta) = \frac{1}{2} g^2 (\frac{U'}{U})^2$

$\eta(\zeta) = \frac{g g' U' + g^2 U''}{U}$

$r(\zeta) = 8 g^2 \left(\frac{U'}{U}\right)^2$

$n_s(\zeta) = 1 + \frac{2g}{U^2} \left(g' U' U + g U'' U - 24 g U'^2\right)$

$A_s(\zeta) = \frac{U^3}{12 \pi^2 g^2 U'^2}$

$N_e = \int_{\zeta_f}^{\zeta_N} \frac{U}{g^2 \partial U / \partial \zeta} d\zeta$

Chain rule:

$\frac{\partial}{\partial \chi} f(\zeta) = \frac{\partial \phi}{\partial \chi} \frac{\partial \zeta}{\partial \phi} \frac{\partial}{\partial \zeta} f(\zeta) \equiv g(\zeta) f'(\zeta)$

with $g(\zeta) = \sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}} (\frac{\partial G}{\partial \zeta} \frac{\partial V^{-1}}{\partial G})^{-1}$