Axion and FIMP Dark Matter in a U(1) extension of the Standard Model

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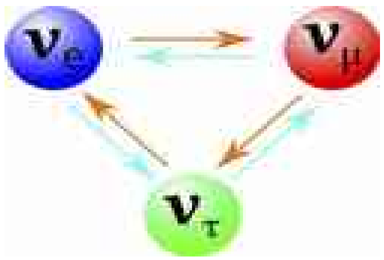
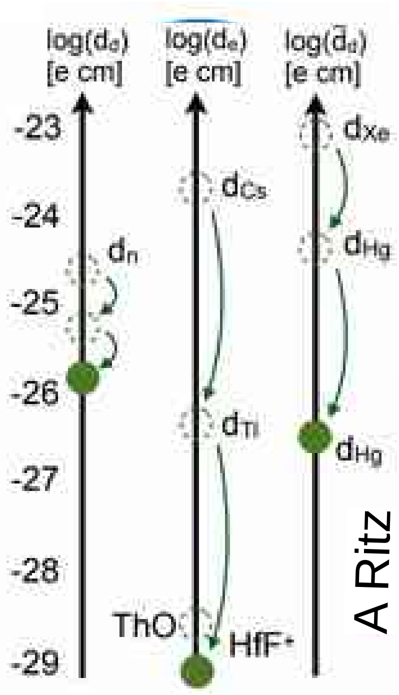
Problems in the SM

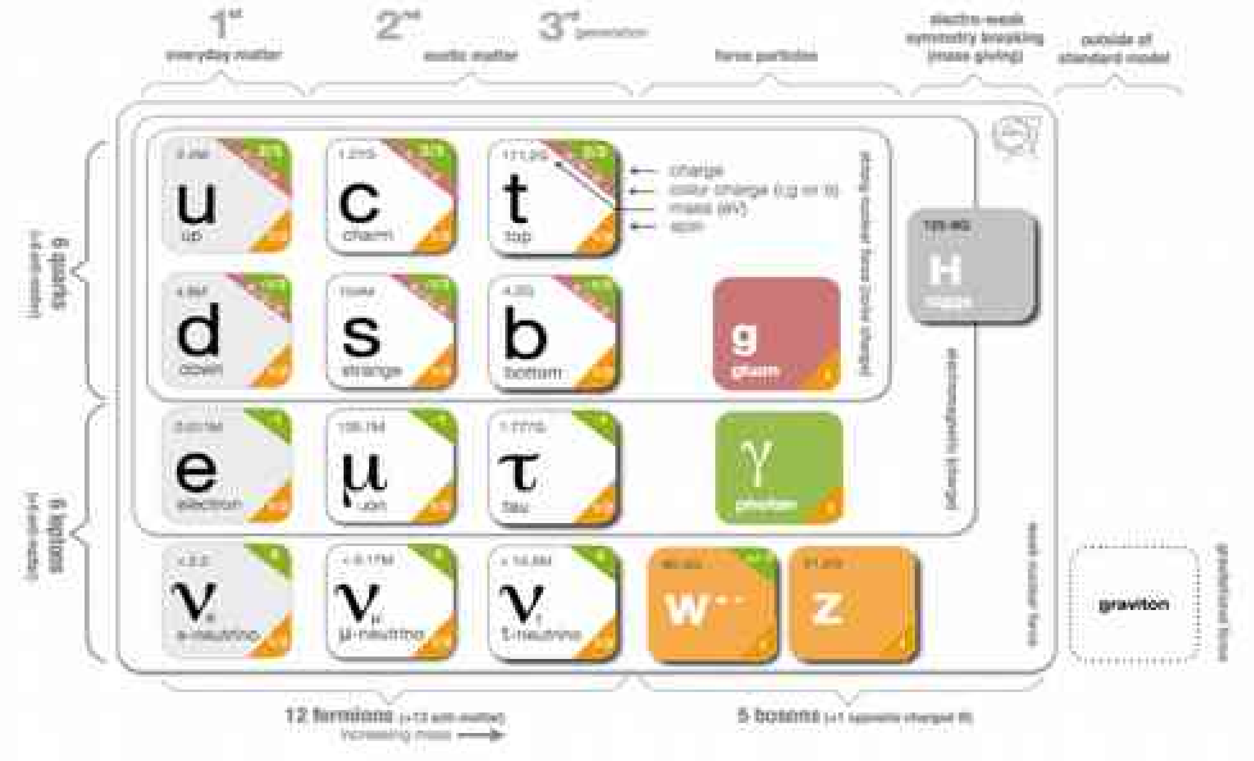


- SM fails to explain neutrino mass and mixings.
- SM doesn't have a DM candidate.
- SM can not explain the observed baryon asymmetry.
- The origin of smallness of the θ-parameter.



A Ritz

SM Particle Spectrum



- Neutrino has all the properties mentioned before
- It can not account the whole amount of DM
- Relativistic in nature so no structure formation
- SM is incomplete, need new BSM physics to address DM

Strong CP Problem

- The measurement of nEDM d_n will imply P, CP violation and could be related to the early matter-antimatter asymmetry.

nEDM

$$H = -d_n E.\hat{S}$$

$$L = -d_n \frac{i}{2} \bar{n} \sigma_{\mu\nu} \gamma_5 n F^{\mu\nu}$$

- nEDM puts bound on $\mathrm{d_n}$ $i.e.$ $|\mathrm{d_n}| < 1.5 \times 10^{-12} \mathrm{e\,GeV^{-1}}$

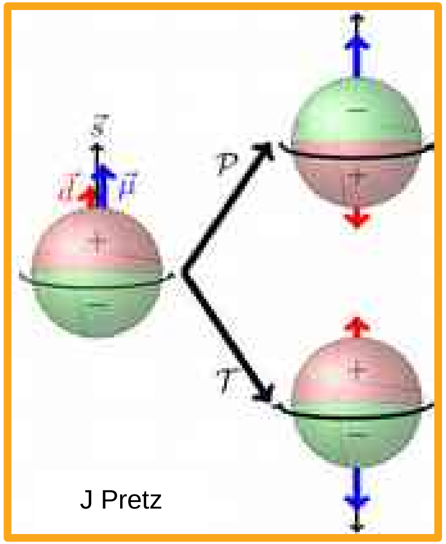
Abel et al, PRL 20

- $\mathrm{L}_\theta = \theta \frac{\mathrm{g_s^2}}{32\pi^2} \mathrm{G\tilde{G}}$ contribution to nEDM which comes out as $\mathrm{d_n} \sim 1.2 \times 10^{-2} \theta \mathrm{e\,GeV^{-1}}$

Pospelov, Ritz '99

- Comparing theoretical and the experimental values of nEDM, we obtain $\theta < 10^{-10}$

- The problem arises why the θ parameter is so small



Gauge group and Particle content

$U(1)_{PQ}$

Accidental Symmetry

Complete gauge group → $G_{SM} \times U(1)_X \times \mathbb{Z}_2$

Gauge Group	Baryon Fields			Lepton Fields						Scalar Fields
	Q_L^i	u_R^i	d_R^i	L_L^e	L_L^μ	L_L^τ	e_R	μ_R	τ_R	ϕ_h
$SU(2)_L$	2	1	1	2	2	2	1	1	1	2
$U(1)_Y$	1/6	2/3	$-1/3$	$-1/2$	$-1/2$	$-1/2$	-1	-1	-1	1/2
$U(1)_X$	m	m	m	n_e	n	n	n_e	n	n	0
$U(1)_{PQ}$	0	0	0	$-2a$	0	0	$-2a$	0	0	0

Gauge Group	Fermions							Scalars	
	N_1	N_2	N_3	ψ_L	ψ_R	χ_L	χ_R	ϕ_1	ϕ_2
$SU(3)_c, SU(2)_L$	(1, 1)	(1, 1)	(1, 1)	(3, 1)	(3, 1)	(3, 1)	(3, 1)	1	1
$U(1)_X$	n_e	n	n	α_L	α_R	β_L	β_R	$\alpha_L - \alpha_R$	$\beta_L - \beta_R$
$U(1)_{PQ}$	$-2a$	0	0	$-a$	a	a	$-a$	$-2a$	$2a$
$\mathbb{Z}_2$	-1	1	1	1	1	-1	-1	1	1
No. of flavors	1	1	1	N_ψ	N_ψ	N_χ	N_χ	1	1

- KSVZ type axion model has been considered
- $\mathbb{Z}_2$-symmetry forbids mixing among the exotic quarks and also stabilise the FIMP DM
- We have two DM namely axion and right handed neutrino which is odd under $\mathbb{Z}_2$
- $U(1)_{\rm PQ}$ symmetry is accidental and extracted from $U(1)_X$ gauge symmetry

Gauge anomaly will put bound on the additional abelian gauge group charges

Choice of α_L and β_L

$U(1)_\chi^3$ and $[Gravity]^2 \times U(1)_\chi$ → $(n_\chi^2 - 1)y^2 + 3(n_\chi - z)y + 3(1 - z^2) = 0$

where $z = \dfrac{\beta_L}{\alpha_R}$, $n_\chi = \dfrac{N_\chi}{N_\psi}$ and $y = \dfrac{\beta_R - \beta_L}{\alpha_R} = \dfrac{\Delta\beta}{\alpha_R}$

Roots and condition for real eigenvalues →

$$y_\pm = \frac{-3(n_\chi - z) \pm \sqrt{9(n_\chi - z)^2 - 12(n_\chi^2 - 1)(1 - z^2)}}{2(n_\chi^2 - 1)}$$

A physical solution has to be real

$$(4n_\chi^2 - 1)z^2 - 6n_\chi z - n_\chi^2 + 4 \geq 0\,.$$

$$z_\pm = \frac{3n_\chi \pm 2|n_\chi^2 - 1|}{(4n_\chi^2 - 1)}$$

Choice of α_L and β_L

Depending on the sign of $4n_\chi^2 - 1$, different ranges for z are allowed, i.e.

- when $n_\chi > 1/2$, we must require $z \leq z_-$ or $z \geq z_+$; as for large n_χ, $z_\pm \to \pm 1/2$ this reduces to the requirement $|z| > 1/2$ in the limit $n_\chi \gg 1$,
- when $n_\chi < \frac{1}{2}$, we must require $z_- \leq z \leq z_+$ and this reduces to $|z| > 2$ for $n_\chi \ll 1$.

Considered $n_\chi > 1$ because $n_\chi < 1$ → $\alpha_i \leftrightarrow \beta_i, N_\psi \leftrightarrow N_\chi, n_\chi \leftrightarrow 1/n_\chi$.

Sets of Allowed Charge assignments →

n_χ	z	y	α_L	β_L	β_R	$\alpha_L - \alpha_R$	$\beta_L - \beta_R$	n_e	n	m
10	1	$-\frac{3}{11}$	$-\frac{19}{11}\alpha_R$	α_R	$\frac{8}{11}\alpha_R$	$-\frac{30}{11}\alpha_R$	$\frac{3}{11}\alpha_R$	$-\frac{3}{2}\alpha_R$	$\frac{27}{22}\alpha_R$	$-\frac{7}{66}\alpha_R$
10	-1	$-\frac{1}{3}$	$-\frac{7}{3}\alpha_R$	$-\alpha_R$	$-\frac{4}{3}\alpha_R$	$-\frac{10}{3}\alpha_R$	$\frac{1}{3}\alpha_R$	$-\frac{11}{6}\alpha_R$	$\frac{3}{2}\alpha_R$	$-\frac{7}{54}\alpha_R$
11	1	$-\frac{1}{4}$	$-\frac{7}{4}\alpha_R$	α_R	$\frac{3}{4}\alpha_R$	$-\frac{11}{4}\alpha_R$	$\frac{1}{4}\alpha_R$	$-\frac{3}{2}\alpha_R$	$\frac{5}{4}\alpha_R$	$-\frac{1}{9}\alpha_R$
11	-1	$-\frac{3}{10}$	$-\frac{23}{10}\alpha_R$	$-\alpha_R$	$-\frac{13}{10}\alpha_R$	$-\frac{33}{10}\alpha_R$	$\frac{3}{10}\alpha_R$	$-\frac{9}{5}\alpha_R$	$\frac{3}{2}\alpha_R$	$-\frac{2}{15}\alpha_R$

$$n_e = -\frac{1+n_\chi}{2}(\beta_L - \beta_R), \; n = \frac{n_\chi - 1}{2}(\beta_L - \beta_R) \text{ and } m = -\frac{n_e + 2n}{9}$$

Gravitational Effect in axion Potential

Kamionkowski et al PLB 92,
Fukuda et al JHEP 18,
Ibe et al JHEP 18

$$g = |g| e^{i\delta}$$

PQ breaking higher dimensional operator at the Planck scale

$$\mathcal{V}_{PL}(\Phi_1, \Phi_2) = \frac{g}{N_\psi! N_\chi!} \frac{\Phi_1^{N_\psi} \Phi_2^{N_\chi}}{M_{PL}^{N_\psi + N_\chi - 4}} + h.c.$$

$$r_g = \frac{(M_A^g)^2}{(M_A)^2}$$

Total axion potential

$$V(\bar{\theta}_a) = F_a^2 M_a^2 \left[(1 - \cos \bar{\theta}_a) + r_g \left(1 - \cos(p\, \bar{\theta}_a + \delta)\right) \right]$$

$$(M_A^g)^2 = \frac{|g|}{N_\psi! N_\chi!} \frac{\langle \Phi_1 \rangle^{N_\psi} \langle \Phi_2 \rangle^{N_\chi}}{(\sqrt{2})^{N_\psi + N_\chi} M_{PL}^{N_\psi + N_\chi - 4} F_A^2}$$

quantum-gravitational induced axion mass

Extra potential term shift the minima from $\theta = 0$ by an amount

$$\Delta\theta = \frac{r_g |p \sin\delta|}{\left[1 + p^4 r_g^2 + 2p^2 r_g \cos\delta\right]^{1/2}}$$

$r_g \ll 1$ and $|p \sin\delta| \sim 1$

$$\Delta\theta \sim r_g = \frac{|g|}{N_\psi! N_\chi! \, (\sqrt{2})^{N_\psi + N_\chi}} \frac{v_1^{N_\psi} v_2^{N_\chi}}{M_{PL}^{N_\psi + N_\chi - 4} (f_\pi m_\pi)^2} \frac{(m_u + m_d)^4}{m_u^2 m_d^2}$$

Analytical Estimate of $\Delta\theta$

$v_1 = v_2$ and $|g| \sim 1$ $\Rightarrow$ $$\Delta\theta \sim \frac{1}{n_\chi!}\left[\frac{1+n_\chi^2}{2}\right]^{\frac{1+n_\chi}{2}}\left[\frac{F_a^2}{f_\pi m_\pi}\right]^2\left[\frac{F_a}{M_{PL}}\right]^{n_\chi-3}\frac{(m_u+m_d)^4}{m_u^2 m_d^2}$$

Stirling's formula for $n_\chi \gg 1$

$n_\chi(1+\ln\frac{F_a}{M_{PL}}) < 0$

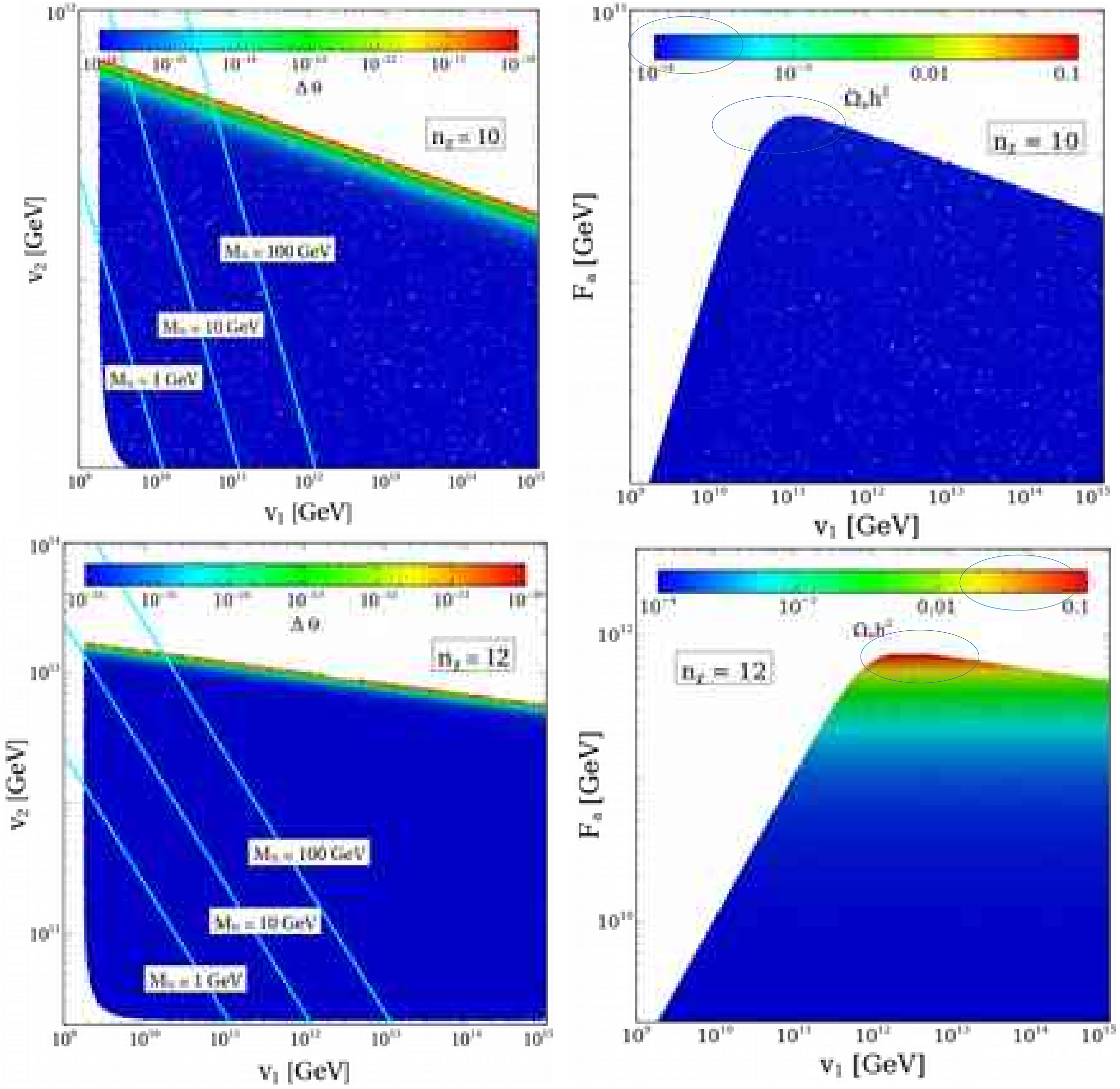
$$\Delta\theta \sim \frac{e^{n_\chi}}{(\sqrt{2})^{1+n_\chi}}\left[1+\frac{1}{n_\chi^2}\right]^{\frac{1+n_\chi}{2}}\sqrt{\frac{n_\chi}{2\pi}}\left[\frac{F_a^2}{f_\pi m_\pi}\right]^2\left[\frac{F_a}{M_{PL}}\right]^{n_\chi-3}\frac{(m_u+m_d)^4}{m_u^2 m_d^2}$$

Ruled out by nEDM

$n_\chi \leq 8$ $\Rightarrow$ $\Delta\theta \geq 10^{-10}$

$n_\chi = 9$ $\Rightarrow$ $\Delta\theta \sim 29.41 \times 10^{-10}\left[\frac{F_a}{10^{10}\mathrm{GeV}}\right]^{10}$

Running of SU(3) coupling above the EQ mass scale $\Rightarrow$ $\beta_3(\alpha_3) = -\frac{\alpha_3^2}{2\pi}\left[7-\frac{2(N_\psi+N_\chi)}{3}\right]$ $\Rightarrow$ $N_\psi + N_\chi \leq 10$



Axion relic density using misalignment:

$$\Omega_a h^2 \simeq 0.18\,\theta_i^2 \left(\frac{F_a}{10^{12}\,\mathrm{GeV}}\right)^{1.19}$$

- $n_\chi = 10$ can not give us total amount for dark matter although not true for higher n_χ values.
- $n_\chi = 12$ Can give right relic density for part of the parameter space.
- From the asymptotic freedom we can not take high value of n_χ.
- We need a second component to fill the gap to the total DM relic density.

$\boxed{N_\psi + N_\chi \leq 10}$

Axion coupling with $SU(2)_L$ and $U(1)_Y$ Gauge Bosons

- Non – trivial contribution from $SU(2)_L \times SU(2)_L \times U(1)_{PQ}$ anomaly :

$$\mathcal{L}_{WWA} = \frac{g_2^2}{64\pi^2}\frac{A}{F_a}\tilde{W}^i_{\mu\nu}W_i^{\mu\nu} = \frac{g_2^2}{64\pi^2}\frac{A}{F_a}\tilde{W}^i_{\mu\nu}W_i^{\mu\nu}.$$

- Non – trivial contribution from $U(1)_Y \times U(1)_Y \times U(1)_{PQ}$ anomaly :

$$\mathcal{L}_{AYY} = (2Y_{L_e}^2 - Y_e^2)\frac{g_1^2}{32\pi^2}\frac{A}{F_a}\tilde{F}^Y_{\mu\nu}F^{\mu\nu\,Y} = -\frac{g_1^2}{64\pi^2}\frac{A}{F_a}\tilde{F}^Y_{\mu\nu}F^{\mu\nu\,Y},$$

- Combining W3 and BY we get exact cancellation of axion coupling with photons

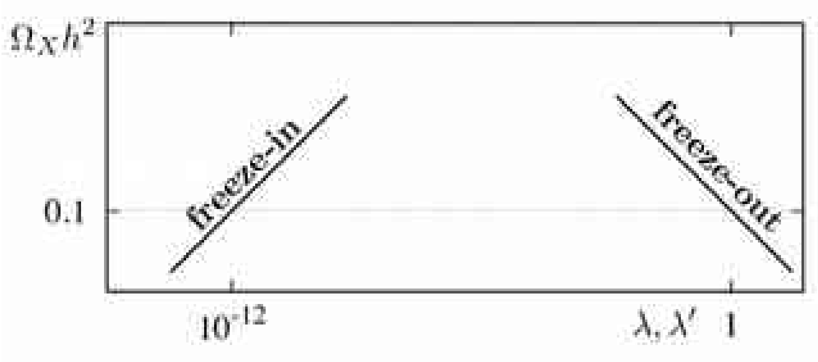
- Nevertheless a non-vanishing coupling arises from the pion-axion mixing giving Luzio et al ‘20

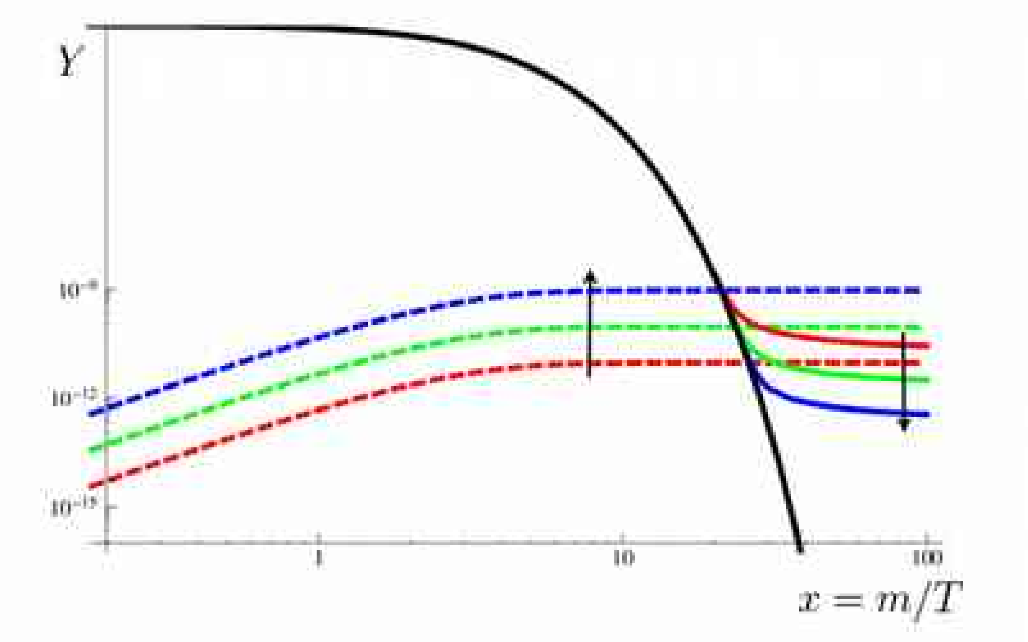
$$\mathcal{L}_{A\gamma\gamma} = -\frac{e^2}{12\pi^2}\left(\frac{4m_d + m_u}{m_d + m_u}\right)\frac{A}{F_a}\tilde{F}_{\mu\nu}F^{\mu\nu}.$$

- MADMAX can explore F_A in between $(1.4 - 14) \times 10^{10}$ GeV which corresponds to $(40 - 400)\mu$eV axion mass → babyIAXO will explore even higher mass range i.e. from meV to eV range

FIMP DM

Hall et al' 09





- WIMP DM is easy to detect but no signal puts bound on its parameter space
- FIMP DM is difficult to probe in different experiments due to its feeble interaction
- This work has both Axion and FIMP type DM depending on choice one can dominate over other

- In the present model vevs are very heavy from the axion study
- To have TeV scale extra gauge boson and DM, their associated interactions become very suppressed

N_1 as FIMP

- In the present work, we can consider one of the RHN as DM which is odd under $\mathbb{Z}_2$

- Lagrangian for the DM candidate N_1 : $\mathcal{L}_{N_1} = \frac{i}{2}\bar{N}_1\gamma^\mu\left(\partial_\mu - i\, g_X^{eff} Z_X\right)N_1 + \lambda \bar{N}_1^c N_1 \frac{\phi_1^\dagger \phi_2}{M_{Pl}} + h.c.$

- Boltzmann equation for the DM evolution:

$$\frac{dY_{Z_X}}{dz} = \frac{2\, M_{\rm Pl}\, z\, \sqrt{g_\star(z)}}{1.66\, M_{h_1}^2\, g_s(z)}\left[\sum_{i=1,2}\langle\Gamma_{h_i\to Z_X Z_X}\rangle_{TH}\left(Y_{h_i} - Y_{Z_X}^2\right) - \langle\Gamma_{Z_X\to N_1 N_1}\rangle_{\rm NTH}\, Y_{Z_X}\right]$$

$$\frac{dY_{N_1}}{dz} = \frac{2\, M_{\rm Pl}\, z\, \sqrt{g_\star(z)}}{1.66\, M_{h_1}^2\, g_s(z)}\left[\sum_{i=1,2}\langle\Gamma_{h_i\to N_1 N_1}\rangle Y_{h_i} + \langle\Gamma_{Z_X\to N_1 N_1}\rangle_{\rm NTH}\, Y_{Z_X}\right].$$

- Non-thermal average of gauge boson decay:

$$\langle\Gamma_{Z_X\to N_1 N_1}\rangle_{\rm NTH} = M_{Z_X}\Gamma_{Z_X\to N_1 N_1}\frac{\int \frac{f_{Z_X}(p)}{\sqrt{p^2+M_{Z_X}^2}}d^3p}{\int f_{Z_X}(p)d^3p}.$$

- DM relic density:

$$\Omega_{N_1}h^2 = 2.755\times 10^8\left(\frac{M_{N_1}}{\rm GeV}\right)Y_{N_1}(T_{\rm Now}).$$

N_1 as FIMP

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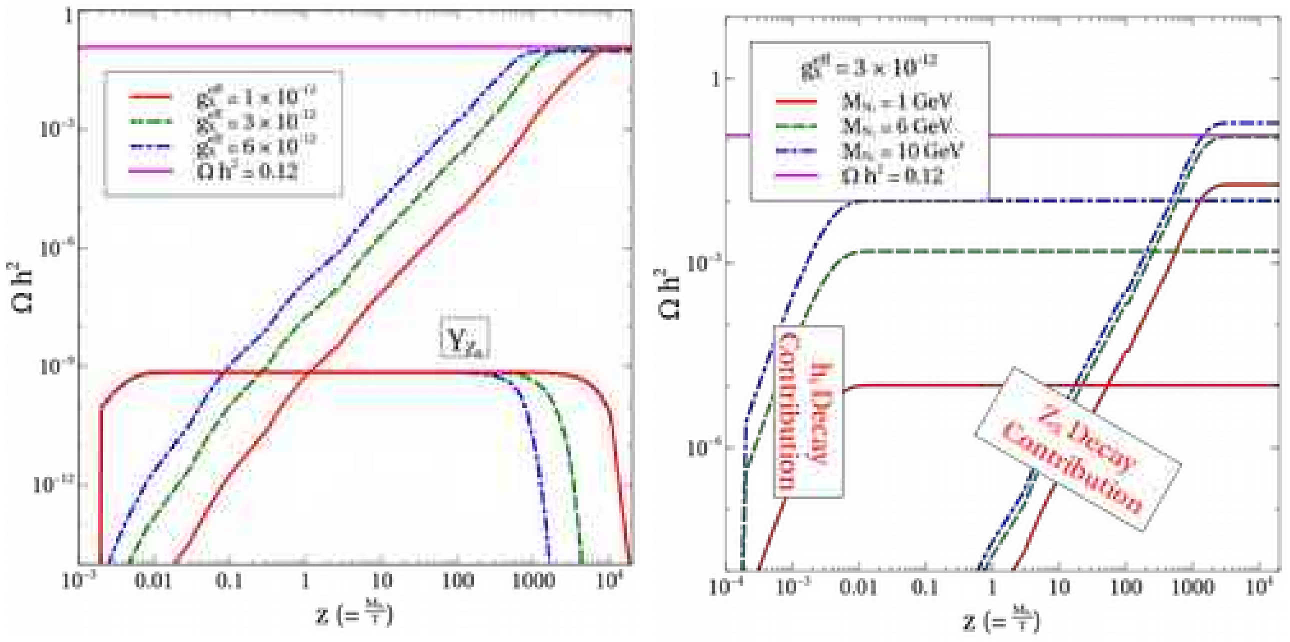


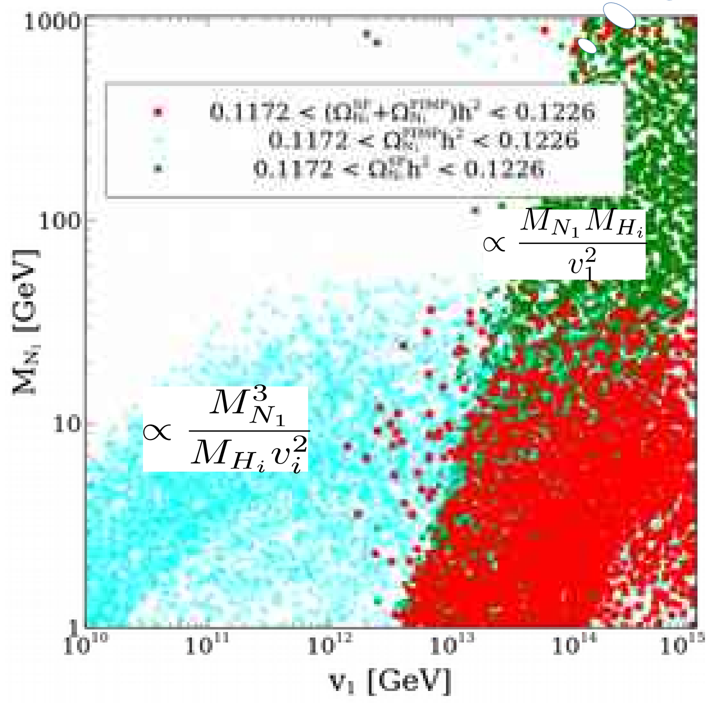
- FIMP is produced from the decay of h_i and Z_X

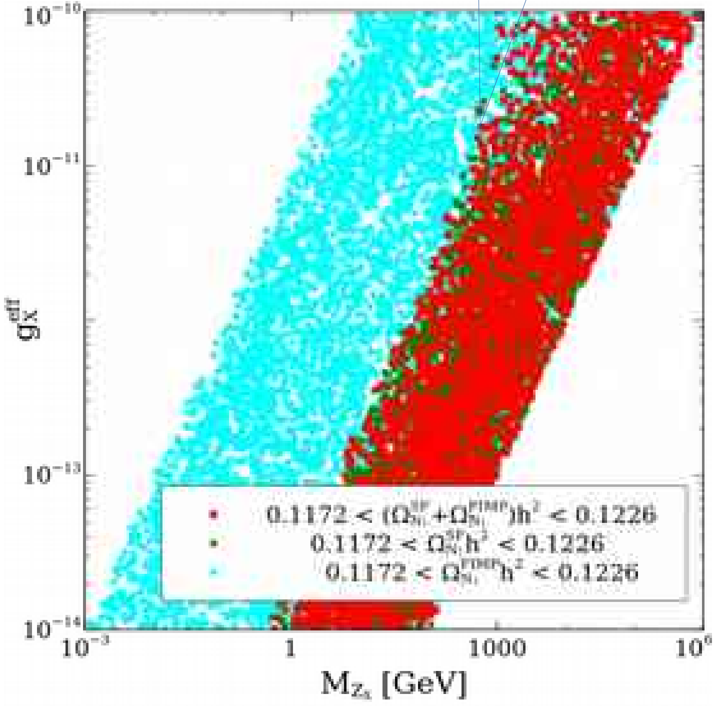
- Z_X never reaches thermal equilibrium so we have determined its distribution function.

Scatter Plots

Points due to PS Suppression

$M_{Zx} \propto g_X^2 \sqrt{n_\chi^2 v_1^2 + v_2^2}$





- Cyan region is from the higgses decay.
- Green region is from additional gauge boson decay.
- Red region is the total contribution.

➡ Higher values of FIMP DM mass are ruled out due to over production of DM

Neutrino Oscillation Parameters Range

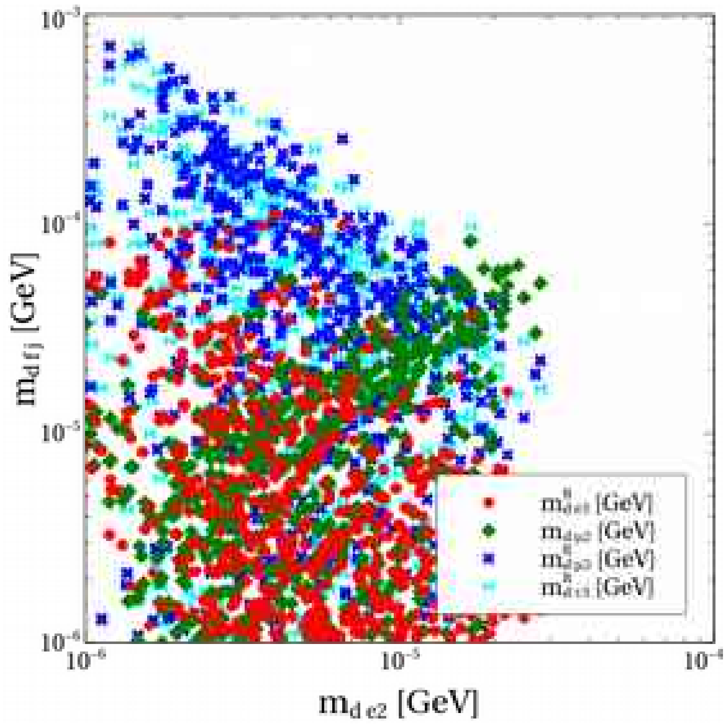
θ_{13}Narrow range

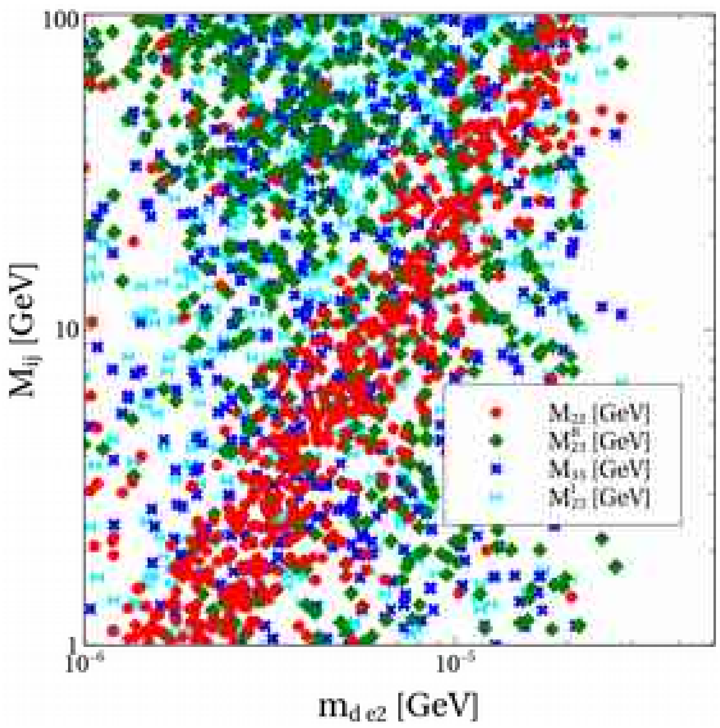
NuFIT 5.2 (2022)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
	$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	$0.02029 \to 0.02391$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \to 8.89$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$
	$\delta_{CP}/^\circ$	197^{+42}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$
		Normal Ordering (best fit)		**Inverted Ordering ($\Delta\chi^2 = 6.4$)**	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
	$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
	$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
	$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
	$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
	$\delta_{CP}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

Associated model parameters are tightly constrained by the neutrino oscillation data

Neutrino mass





Model parameters have the correlations as shown in the two plots

- Present model can generate oscillation parameters in the correct range by varying the model parameters
- The lightest **eigenvalue** among the active neutrinos **is zero** since the mixing involves only two RHN

Conclusion

- Present model can accommodate neutrino mass with the allowed range of the neutrino oscillation parameters.

- It also explain the smallness of the θ-parameter and solves the strong CP problem naturally.
- With asymptotic freedom, we could have a not so small contribution to θ which corresponds to small F_a and may be measured in near future experiments.

- Unless we choose very high value of n_χ (≥ 12)which might ruin the asymptotic freedom of QCD coupling, axion can not accommodate whole amount of DM relic density.

- ADMX, MADMAX, babyIAXO can explore the present model for axion mass range from $\mu\mathrm{eV}$ and above, even if axion is not the total DM density.

- One of the right handed neutrino can be a FIMP DM and fill the deficit in the total DM relic density.

- RH FIMP DM is produced from the decay of thermal Higgses and non-thermal gauge boson.

Thank you for your attention

Back up Slides

Lagrangian

Full Lagrangian $\rightarrow$ $$\mathcal{L} = \underbrace{\mathcal{L}_{SM}}_{\text{SM Lagrangian}} + \overbrace{\mathcal{L}_N^{Yuk}}^{\text{Yukawa for RHN}} + \underbrace{\mathcal{L}_{BSM}^{Yuk}}_{\text{Yukawa for EQ}}$$

Lagrangian associated with the right handed neutrinos:

$$\mathcal{L}_N^{Yuk} = y_{\mu 2}\bar{L}_\mu \phi_h N_2 + y_{\mu 3}\bar{L}_\mu \phi_h N_3 + y_{\tau 2}\bar{L}_\tau \phi_h N_2 + y_{\tau 3}\bar{L}_\tau \phi_h N_3 + y_{e2}\bar{L}_e \phi_h N_2 \frac{\phi_1}{M_{PL}}$$ $\rightarrow$ Dirac mass terms

$$+ y_{e3}\bar{L}_e \phi_h N_3 \frac{\phi_1}{M_{PL}} + y_{22} N_2 N_2 \frac{\phi_1 \phi_2}{M_{PL}} + y_{23} N_2 N_3 \frac{\phi_1 \phi_2}{M_{PL}} + y_{33} N_3 N_3 \frac{\phi_1 \phi_2}{M_{PL}} + h.c.,$$ $\rightarrow$ RHN mass terms

Terms associated with the exotic quarks: $$\mathcal{L}_{BSM}^{Yuk} = \sum_{i,j=1}^{N_\psi 0} \lambda_{ij} \bar{\psi}_L^i \psi_R^j \phi_1 + \sum_{i,j=1}^{N_\chi} y_{ij} \bar{X}_L^i X_R^j \phi_2 + h.c.,$$

Redefining the fields: $$\psi_L \rightarrow e^{i\frac{a_1}{2v_1}}, \; \psi_R \rightarrow e^{-i\frac{a_1}{2v_1}}, \; \chi_L \rightarrow e^{i\frac{a_2}{2v_2}}, \; \chi_R \rightarrow e^{-i\frac{a_2}{2v_2}}$$

Axion gluon coupling: $$\mathcal{L}_{AGG} = \left(\frac{N_\psi a_1}{v_1} + \frac{N_\chi a_2}{v_2}\right)\frac{g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$
$$= N_\psi \frac{A}{F_a}\frac{g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu},$$

$$A = \frac{v_2 a_1 + n_\chi v_1 a_2}{\sqrt{n_\chi^2 v_1^2 + v_2^2}} \qquad F_a = \frac{v_1 v_2}{\sqrt{n_\chi^2 v_1^2 + v_2^2}}$$

Axion Density

- Misalignment mechanism gives us the axion density:

$$\Omega_a h^2 \simeq 0.18\,\theta_i^2 \left(\frac{F_a}{10^{12}\,\mathrm{GeV}}\right)^{1.19}.$$

Preskill et al PLB 1983
Turner et al '85

- PQ symmetry breaking happens before or during inflation then the axion field as massless field contains quantum fluctuations:

$$\delta a = \frac{H_{\rm inf}}{2\pi},$$

Kawasaki et al PLB 2018

- These fluctuations of the axionic field contribute to the axion energy density:

$$\Omega_a h^2 \simeq 0.18 \left[\theta_i^2 + \left(\frac{H_{\rm inf}}{2\pi F_a}\right)^2\right] \left(\frac{F_a}{10^{12}\,\mathrm{GeV}}\right)^{1.19},$$

- axion fluctuations generate an isocurvature perturbation (SDM) on top of the curvature perturbation given by the inflaton:

$$S_{\rm DM} = \frac{\Omega_a h^2}{\Omega_{\rm DM} h^2} \frac{\delta\rho_a}{\rho_a},$$

- Hubble parameter H_inf and F_a must satisfy the following relation to be consistent with the CMB data

$$H_{\rm inf} < 2.4 \times 10^7\,\mathrm{GeV} \left(\frac{F_a}{10^{12}\,\mathrm{GeV}}\right)^{0.405}.$$

Analytical estimate of FIMP DM

$\mathrm{h_i \to N_1 N_1}$ decays contribute to the FIMP DM :

$$\Omega^{FIMP}_{N_1} h^2 \sim \frac{2.038 \times 10^{27}}{g_s \sqrt{g_\rho}} \sum_i \frac{M^3_{N_1}}{16\pi M_{h_i} F_a^2 (n_\chi^2 + 1)}$$

$\mathrm{h_i \to Z_X Z_X \to N_1 N_1}$ decays contribute to the FIMP DM :

$$(\Omega^{SF}_{N_1} h^2) \sim \frac{2.038 \times 10^{27}}{g_s \sqrt{g_\rho}} 2BR_{Z_X \to N_1 N_1} \sum_i \frac{M_{N_1} q_i^2 M_{h_i}}{32\pi q_2^2 (n_\chi^2 + 1)^2 F_a^2}$$

$\mathrm{Z_X \to N_1 N_1}$ branching analytically can be approximated as

$$2BR_{Z_X \to N_1 N_1} = \frac{2}{24} \frac{(n_\chi + 1)^2}{n_\chi^2 - 8n_\chi + 28/3} \to \frac{1}{12}, \quad \text{for } n_\chi \gg 1$$

→ In the DM scatter plots, we have considered contribution both axion and FIMP DM

Neutrino Mass

- Neutrino mass matrix in the basis $(\nu_L^c \quad N_R) \rightarrow \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix}$

- Dirac mass matrix takes the form: $$m_d = m_{d\,fi} = \begin{pmatrix} \frac{y_{e2} v v_1}{2M_{pl}} & \frac{(y^R_{e3}+iy^I_{e3}) v v_1}{2M_{pl}} \\ \frac{y_{\mu 2} v}{\sqrt{2}} & \frac{(y^R_{\mu 3}+y^I_{\mu 3}) v}{\sqrt{2}} \\ \frac{y_{\tau 2} v}{\sqrt{2}} & \frac{(y^R_{\tau 3}+y^I_{\tau 3}) v}{\sqrt{2}} \end{pmatrix},$$

- Right handed neutrino mass matrix: $$m_R = \begin{pmatrix} M_{22} & M^R_{23} + iM^I_{23} \\ M^R_{23} + iM^I_{23} & M_{33} \end{pmatrix},$$

- Neutrino mass is generated by Type-I Seesaw mechanism $$m_\nu = -m_D^T M_R^{-1} M_D$$

Parameters range: $\rightarrow$ $$10^{-6}\,\mathrm{GeV} \leq m_{d\,fi} \leq 10^{-3}\,\mathrm{GeV}\,,$$ $$1\,\mathrm{GeV} \leq m_R \leq 100\,\mathrm{GeV}\,.$$