NICPB KBFI

CONNECTING SCIENCES

Beyond (and back to) Palatini quadratic gravity and inflation

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based on
2212.11869 and 2307.02963
with
C. Dioguardi (Taltech & NICPB) & E. Tomberg (NICPB)

Euroopa Liit
Euroopa Sotsiaalfond
Eesti tuleviku heaks

• Metric vs Palatini •

The properties of spacetime are essentially described by:

- the affine connection: $\Gamma^{\lambda}_{\alpha\beta} \rightarrow$ parallel transport
- the metric tensor: $g_{\mu\nu} \rightarrow$ distance

The connection coefficients and metric tensor are fundamentally independent quantities. They exhibit no *a priori* known relationship. If they are to have any relationship, it must derive from

- additional constraints (metric formalism $\nabla_{\alpha} g_{\mu\nu} = 0$)
- EoM (Palatini formalism)

If Einsteinian gravity ($\sim R$), EoM $\Rightarrow \nabla_{\alpha} g_{\mu\nu} = 0$ (i.e Palatini $\equiv$ metric)

With non-minimal theories, metric and Palatini formalism generate different physical theories. (Koivisto & Kurki-Suonio: arXiv:0509422)

• Palatini $F(R) + \mathcal{L}(\phi)$ •

- We start with the following action in the Palatini formulation

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{1}{2} F(R(\Gamma)) + \mathcal{L}(\phi) \right]$$
$$\mathcal{L}(\phi) = -\frac{1}{2} K(\phi) g_J^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

- we rewrite the $F(R)$ term using the auxiliary field ζ, obtaining

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{1}{2} \left(F(\zeta) + F'(\zeta) \left(R(\Gamma) - \zeta \right) \right) + \mathcal{L}(\phi) \right]$$

- we move to the Einstein frame: $g^E_{\mu\nu} = F' g^J_{\mu\nu}$ N.B. now $\boxed{\Gamma^E = \Gamma^J}$

$$S_E = \int d^4x \sqrt{-g_E} \left[\frac{R}{2} - \frac{1}{2} g_E^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi, \zeta) \right]$$
$$U(\chi, \zeta) = \frac{V(\phi(\chi))}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)}$$
$$\frac{\partial \chi}{\partial \phi} = \sqrt{\frac{K(\phi)}{F'(\zeta)}} \quad \text{(canonically normalized scalar)}$$

- no $-\frac{3}{2}\left(\frac{\partial F'}{F'}\right)^2$ like in metric gravity! ζ still auxiliary! still single field setup!

• Computational method •

- The full EoM for ζ is

$$G(\zeta) = \frac{1}{2} K(\phi)\, \partial^\mu \phi \partial_\mu \phi\, F'(\zeta) + V(\phi)$$

with

$$G(\zeta) = \frac{1}{4} \left[2F(\zeta) - \zeta F'(\zeta) \right]$$

- The standard procedure would be now to solve the EoM and determine $\zeta(\phi, \partial^\mu \phi \partial_\mu \phi)$ and insert it back into the action.

- However not always solvable for any $F(R)$
- ζ as a computational variable ← A.R. et al., JHEP 06 (2022) 106
  - SR approximation: $G(\zeta) = V(\phi)$
  - $V \to G \Rightarrow \boxed{U(\zeta) = \frac{\zeta}{4F'(\zeta)}}$

Inflationary observables

- SR parameters

$$\epsilon(\zeta) = \frac{1}{2}\left(\frac{\partial U/\partial\chi}{U}\right)^2 = \frac{1}{2}g^2\left(\frac{U'}{U}\right)^2 \qquad \boxed{g = \frac{\partial\zeta}{\partial\chi}}$$

$$\eta(\zeta) = \frac{\partial^2 U/\partial\chi^2}{U} = \frac{gg'U' + g^2U''}{U}$$

- observables

$$N_e = \int_{\chi_f}^{\chi_N} \frac{U}{\partial U/\partial\chi}\mathrm{d}\chi = \int_{\zeta_f}^{\zeta_N} \frac{U}{g^2\,U'}\mathrm{d}\zeta$$

$$r(\zeta) = 16\epsilon(\zeta) = 8g^2\left(\frac{U'}{U}\right)^2$$

$$n_s(\zeta) = 1 + 2\eta(\zeta) - 6\epsilon(\zeta) = 1 + \frac{2g}{U^2}\left(g'U'U + gU''U - 3gU'^2\right)$$

$$A_s(\zeta) = \frac{U}{24\pi^2\epsilon(\zeta)} = \frac{U^3}{12\pi^2g^2U'^2}$$

- more details in A.R. et al., JHEP 06 (2022) 106

• Examples •

We study the following $F_{>2}(R)$ (i.e. $F(R)$ diverging faster than R^2 at big R):

A. $F(R) = R + \alpha R^3$

B. $F(R) = \frac{1}{\alpha} e^{\alpha R}$

C. $F(R) = R + \alpha R^2 \ln(\alpha R)$

combined with the following potentials (with $K(\phi) = 1$):

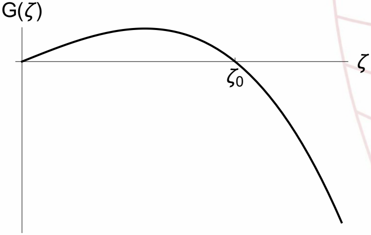
i. $V_k(\phi) = -\lambda_k \phi^k \qquad \lambda_k = \frac{\lambda^k}{k!}$

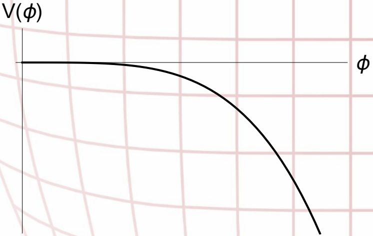
ii. $V_e(\phi) = -e^{\lambda\phi}$

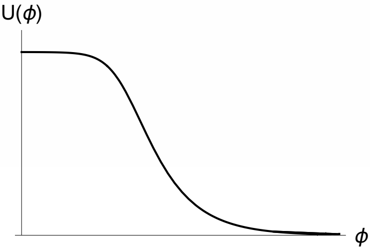
N.B. $\alpha = 0$ not allowed for A, B, C scenarios!!!

$U(\phi)$

$$G(\zeta) = \left[2F(\zeta) - \zeta F'(\zeta)\right]/4 \quad = V(\phi)$$

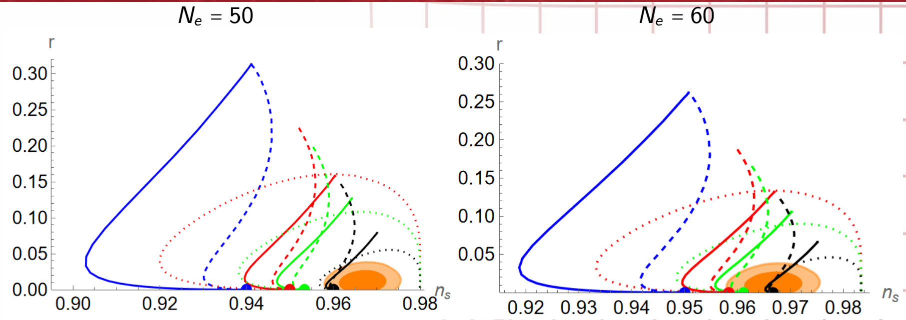






- $G(\zeta_0) = 0 \Rightarrow \zeta_0$ vacuum sol.
- $U(\phi) = \frac{\zeta(\phi)}{4F'_{>2}(\zeta(\phi))} > 0$
- small $\phi \Rightarrow \zeta \approx \zeta_0 \Rightarrow U$ has a max
- U decreases for ζ increasing
- a horizontal asymptote at $U = 0$
- quintessential tail → not today

• Inflationary predictions •



■ $V_4 = -\lambda_4 \phi^4$

■ $V_6 = -\lambda_6 \phi^6$

■ $V_8 = -\lambda_8 \phi^8$

■ $V_e = -e^{\lambda \phi}$

— $F(R) = R + \alpha R^3$

- - $F(R) = \frac{1}{\alpha} e^{\alpha R}$

· · $F(R) = R + \alpha R^2 \ln(\alpha R)$

■ BICEP & Planck, 2110.00483

N.B. Universal strong coupling ($\alpha \gg 1$) limit predictions!!!

• Back to R^2 gravity •

- $G(\zeta_0) = 0$, $\zeta_0 = \zeta(0) \Leftrightarrow$ no matter i.e. $\phi = \partial_\mu \phi = 0 \Rightarrow$ GR+CC: $U(\zeta_0) = \zeta_0/(4F'(\zeta_0))$
- $\alpha \to \infty$, $\zeta \to \zeta_0$
- expand $F(\zeta)$ in Taylor series around ζ_0 up to the quadratic order:

$$\begin{aligned} F_2(\zeta) &= F(\zeta_0) + F'(\zeta_0)(\zeta - \zeta_0) + \frac{1}{2}F''(\zeta_0)(\zeta - \zeta_0)^2 \\ &= 2\Lambda - \omega\zeta + \alpha\zeta^2 , \end{aligned}$$

where

$$\begin{aligned} \Lambda &= -\zeta_0 G'(\zeta_0) > 0 \\ \omega &= -4G'(\zeta_0) > 0 , \\ 2\alpha &= F''(\zeta_0) > 0 , \end{aligned}$$

- a negative sign for the EH term
- unfamiliar but a priori allowed as long as $F_2'(R) > 0$ and $F_2''(R) \neq 0$
- unfortunately $F_2' > 0$ only under SR
- good only as an effective description

• Back to R^2 gravity •

$$F_2(\zeta) = 2\Lambda - \omega\zeta + \alpha\zeta^2$$

- $G = V \Rightarrow \zeta(\phi) = \frac{-4V(\phi)+4\Lambda}{\omega}$
- $\zeta \to \zeta_0$ can be interpred as $V(\phi) \to 0$
- In the Einstein frame:

$$\left(\frac{\partial\chi}{\partial\phi}\right)^2 = \frac{\omega}{8\alpha\left(\Lambda + \lambda_k\phi^k\right) - \omega^2} \qquad \lambda_k = \frac{\lambda^k}{k!} \qquad V(\phi) = -\lambda_k\phi^k$$

$$U(\phi) = \frac{\lambda_k\phi^k + \Lambda}{8\alpha(\lambda_k\phi^k + \Lambda) - \omega^2}$$

- Leading order terms for $V(\phi) \to 0 \Rightarrow$ $\boxed{k - \text{hilltop } U}$:

$$U(\chi) \simeq U_0\left(1 - \bar{\lambda}\chi^k\right) \quad \text{with} \begin{cases} U_0 = \frac{1}{8\alpha}\frac{\Lambda}{\Lambda - \frac{\omega^2}{8\alpha}} \\ \bar{\lambda} = \frac{\lambda_k\,\omega}{\Lambda}\left(\frac{8\alpha\Lambda}{\omega} - \omega\right)^{\frac{k}{2}-1} \end{cases} \Rightarrow \boxed{\begin{array}{l} r \ll 1 \\ n_s = 1 - \frac{k-1}{k-2}\frac{2}{N_e} \\ k \geq 3\ \&\ \bar{\lambda}, \alpha \gg 1 \end{array}}$$

- universal strong coupling limit for any $F_{>2}(R)$

After some manipulations, the full Einstein frame EoMs read:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{V'(\phi)}{F'(\zeta)} = \frac{\dot{\phi}\dot{\zeta}F''(\zeta)}{F'(\zeta)}$$

$$3H^2 = \frac{1}{2}\frac{\dot{\phi}^2}{F'(\zeta)} + U(\phi,\zeta)$$

$$-\frac{1}{2}\dot{\phi}^2 F'(\zeta) + 2V(\phi) - 2G(\zeta) = 0$$

These can be used to also derive

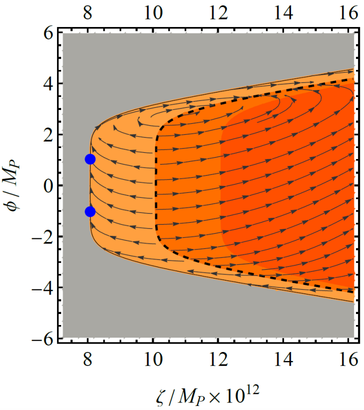
$$\dot{H} = -\frac{1}{2}\frac{\dot{\phi}^2}{F'(\zeta)}$$

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{12V(\phi) - 6F(\zeta) + 3\zeta F'(\zeta)}{6V(\phi) - 3F(\zeta) + 2\zeta F'(\zeta)}$$

$$\dot{\zeta} = \frac{3H\dot{\phi}^2 F'(\zeta) + 3V'(\phi)\dot{\phi}}{2G'(\zeta) + \frac{3}{2}\dot{\phi}^2 F''(\zeta)}$$

N.B. Even though ζ is only auxiliary, it has an implicit time dependence via ϕ

• Beyond SR. $F_{>2}(R)$ •



- $F(R) = \frac{1}{\alpha}\exp(\alpha R)$, $V(\phi) = V_8(\phi)$, $\lambda = 0.047$, $\alpha \simeq 2.48 \times 10^{11}$

  $\sim$ for any $F_{>2}(R)$, V

- $H > 0$, $\dot\phi > 0$ branch
- ■ no ζ's EoM
- ■ light $\to \epsilon_H < 1$
- ■ dark $\to \epsilon_H > 1$
- • $n_s \simeq 0.961$, $r \simeq 8.83 \times 10^{-6}$, $N_e = 60$

- - $\dot\zeta \to \infty$: trajectories diverging/converging from/to there. no crossing
  - in the patch where SR happens, it never finishes
  - in the other patch, fast roll $\to$ too low N_e
  - $F_{>2}(R)$ can only work as an effective theory for SR. New Physics $\Rightarrow$ a) remove the pole from ζ's EoM b) graceful exit before the pole

• Conclusions •

- We studied single field inflation embedded in Palatini $F_{>2}(R)$
- We found that

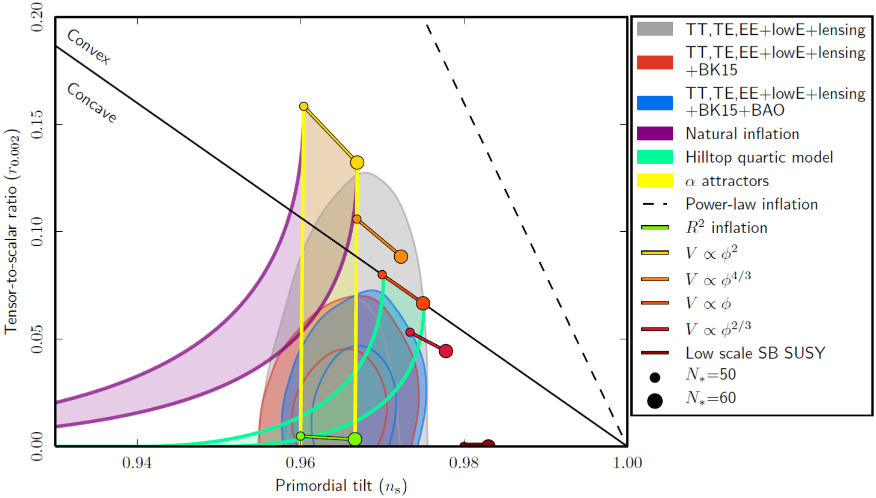
$$F_{>2}(R) \;\Rightarrow V \leq 0$$
$\rightarrow U > 0$
$\rightarrow$ universal strong coupling limit $F_2(R)$
$\rightarrow$ k-hilltop inflation
$\rightarrow$ only valid under SR

- Possible solution: $F(R - X) \rightarrow$ ~~C. Dioguardi~~ still me ☺

Grazie! - Thank you! - Aitäh!

BACKUP SLIDES

• Planck 2018 data •



- FLAT potentials are strongly FAVORED!!!

• Non-minimal setups •

setups:

generic: $S \sim$ gravity + inflaton kin. term - potential

minimal: $S_E = \int d^4x\sqrt{-g_E}\left[\frac{R}{2} - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi)\right]$

non-minimal: $S_J \sim \int d^4x\sqrt{-g_J}\,[F(R,\phi,\dots) - K(\phi)g_J^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)]$

$\uparrow$

Palatini $F(R)$ $\leftarrow$ our choice

Why? Because:

- Palatini $\phi^2 R$ $\rightarrow$ done $\rightarrow$ Bauer & Demir, 1012.2900 (many others after)
- Palatini R^2 $\rightarrow$ done $\rightarrow$ Enckell et al., 1810.05536 (many others after)
- Palatini $F(R)$ $\rightarrow$ not yet done $\rightarrow$ we do it! $\rightarrow$ A.R. et al., JHEP 06(2022)106, **2212.11869 and 2305.xxxxx**

Palatini $F(R)$

- we start with Palatini $F(R)$ action alone

$$S_J = \int \mathrm{d}^4x \sqrt{-g_J} \frac{1}{2} F(R_J(\Gamma))$$

- we rewrite the $F(R)$ term using the auxiliary field ζ

$$S_J = \int \mathrm{d}^4x \sqrt{-g_J} \left[\frac{1}{2} F'(\zeta) R_J(\Gamma) - V_J(\zeta) \right]$$

$$V_J(\zeta) = \frac{-F(\zeta)}{2} + \frac{\zeta F'(\zeta)}{2} \qquad F' = \frac{\partial F}{\partial \zeta}$$

- we move to the Einstein frame:

$$\left. \begin{array}{l} g^E_{\mu\nu} = F' g^J_{\mu\nu} \\ \Gamma^E = \Gamma^J \end{array} \right\} \Rightarrow \left\{ \begin{array}{ll} g^J_{\mu\nu} = g^E_{\mu\nu}/F' & \Rightarrow \quad \sqrt{-g^J} = \sqrt{-g^E}/(F')^2 \\ R^J_{\mu\nu} = R^E_{\mu\nu} & \Rightarrow \quad R_J = F' R_E \end{array} \right.$$

$$S_E = \int \mathrm{d}^4x \sqrt{-g_E} \left[\frac{R_E}{2} - V_E(\zeta) \right]$$

$$V_E(\zeta) = -\frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)}$$

- N.B. ζ stays auxiliary!

  Not like in metric gravity (cf. Starobinsky model): no $-\frac{3}{2}\left(\frac{\partial F'}{F'}\right)^2$

• Palatini $F(R)$ •

- ζ's EoM: $V_E'(\zeta) = 0 \rightarrow$ a bit of albegra:

$$G(\zeta) = \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right] = 0$$

$$\zeta = \zeta_0 \quad \rightarrow \quad F(\zeta_0) = \frac{1}{2}\zeta_0 F'(\zeta_0)$$

with $\boxed{F', F'' \neq 0}$

- inserting $\zeta = \zeta_0$ in V_E we obtain

$$V_E(\zeta_0) = -\frac{\zeta_0 F'(\zeta_0)}{2F'(\zeta_0)^2} + \frac{\zeta_0}{4F'(\zeta_0)} = \frac{1}{4}\frac{\zeta_0}{F'(\zeta_0)}$$

i.e. a CC

- therefore pure Palatini $F(R)$ is equivalent to GR + CC
- completely different from metric $F(R)$ gravity!

- first of all $V \to G$ in U:

$$
\begin{aligned}
U(\chi,\zeta) &= \frac{V(\phi(\chi))}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} \\
&= \frac{G(\zeta)}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} \\
&= \cancel{\frac{F(\zeta)}{2F'(\zeta)^2}} - \frac{\zeta}{4F'(\zeta)} - \cancel{\frac{F(\zeta)}{2F'(\zeta)^2}} + \frac{\zeta}{2F'(\zeta)} \\
&= \boxed{\frac{1}{4}\frac{\zeta}{F'(\zeta)} = U(\zeta)}
\end{aligned}
$$

- valid for any $F(R)$ and $V(\phi)$
- what changes is the actual *solution* for ζ
- also valid for the pure $F(R)$ case ($\mathcal{L}(\phi) = 0$)
- $F(R) = R + \alpha R^2$ is the only $F(R)$ $\Rightarrow$ universally flat ($\neq 0$) U for $\alpha \gg 1$ (unless for $\alpha \gg 1$ $\zeta \to \zeta_0 \neq 0$)
- $\zeta > 0 \Rightarrow U > 0$ regardless of the sign of V

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• Calculus •

- We want to compute inflationary observables
- First of all we need to compute the first derivative of the potential:

$$\frac{\partial}{\partial\chi}U(\zeta) = \boxed{\frac{\partial\zeta}{\partial\chi}\frac{\partial}{\partial\zeta}U(\zeta)} \quad \leftarrow \text{we need this!}$$

$$= \frac{\partial\phi}{\partial\chi}\frac{\partial}{\partial\phi}U(\zeta) = \sqrt{\frac{F'(\zeta)}{k(\phi)}}\frac{\partial}{\partial\phi}U(\zeta)$$

- $G(\zeta) = V(\phi) \;\Rightarrow\; \phi = V^{-1}(G)$, the inverse function of $V(\phi)$

$$= \sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}}\frac{\partial\zeta}{\partial\phi}\frac{\partial U}{\partial\zeta} =$$

$$= \sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}}\frac{1}{\frac{\partial\phi}{\partial\zeta}}\frac{\partial U}{\partial\zeta} = \sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}}\frac{1}{\frac{\partial V^{-1}}{\partial\zeta}}\frac{\partial U}{\partial\zeta} =$$

$$= \boxed{\sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}}\frac{1}{\frac{\partial G}{\partial\zeta}\frac{\partial V^{-1}}{\partial G}}\frac{\partial U}{\partial\zeta} = \frac{\partial}{\partial\chi}U(\zeta)}$$

• Calculus •

- Summarizing:

$$\frac{\partial \zeta}{\partial \chi} = \sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}} \frac{1}{\frac{\partial G}{\partial \zeta}\frac{\partial V^{-1}}{\partial G}} \equiv g(\zeta)\,, \qquad \frac{\partial}{\partial \chi} f(\zeta) = g(\zeta)\frac{\partial f(\zeta)}{\partial \zeta}$$

N.B. The derivative can be explicitly computed as long as V is invertible.

- This allows us to easily express higher derivatives:

$$\frac{\partial^2}{\partial \chi^2} U(\zeta) = g(\zeta)\frac{\partial}{\partial \zeta}\left(g(\zeta)\frac{\partial U}{\partial \zeta}\right) = gg'U' + g^2 U''\,, \quad \ldots$$

where primes denote derivatives w.r.t. ζ.

- we have a method for computing SR parameters and inflationary observables!

• Requirements •

- Usual condition: $F'(\zeta) > 0$
  - GR(+CC) as low energy limit: $F(R \sim 0) \approx F(0) + R\,F'(0) + \dots$
  - correct Weyl transformation $\Omega = F'$
- New requirements coming from the method:
  - consistent definition of $g(\zeta)$
    a. $V(\phi)$ invertible (at least in a subdomain)
    b. $G(\zeta)$ bijective
  - existence of a real solution for $G(\zeta) = V(\phi)$

• $G = V$ problem •

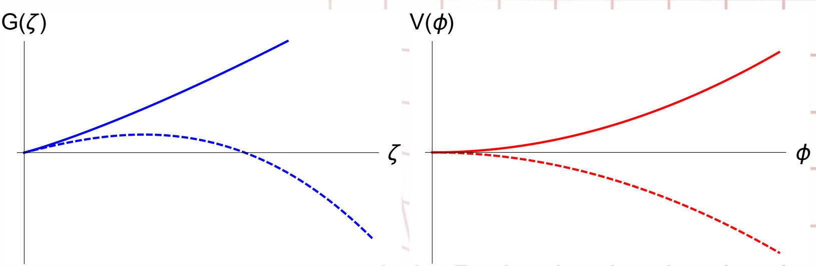


Figure: $G(\zeta)$ for $F(R) = R + R^n$, $n < 2$ (full), $n > 2$ (dashed), $V(\phi) = \pm\phi^2$

- $$\boxed{G(\zeta) = \left[2F(\zeta) - \zeta F'(\zeta)\right]/4 \quad = V(\phi)}$$
- For $\zeta \to +\infty$, if $F(\zeta) \approx \zeta^n \Rightarrow G(\zeta) \approx (2-n)\zeta^n \Rightarrow$ with $n \lessgtr 2 \Rightarrow G \to \pm\infty$
- configurations:

  $n < 2 \;\Rightarrow G(+\infty) \to +\infty \Rightarrow V \geq 0 \quad \Rightarrow F_{\leq 2}(R) \to$ A.R. et al., 2112.12149

  $n > 2 \;\Rightarrow G(+\infty) \to -\infty \Rightarrow V \leq 0 \quad \Rightarrow F_{>2}(R) \to$ **A.R. et al. 2212.11869**
- otherwise *ad hoc* tuned V

• Beyond SR •

After some manipulations, the full Einstein frame EoMs read:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{V'(\phi)}{F'(\zeta)K(\phi)} = \frac{\dot{\phi}\dot{\zeta}F''(\zeta)}{F'(\zeta)} - \frac{1}{2}\frac{K'(\phi)}{K(\phi)}\dot{\phi}^2$$

$$3H^2 = \frac{1}{2}\frac{\dot{\phi}^2}{F'(\zeta)}K(\phi) + U(\phi,\zeta)$$

$$-\frac{1}{2}\dot{\phi}^2 F'(\zeta)K(\phi) + 2V(\phi) - 2G(\zeta) = 0$$

These can be used to also derive

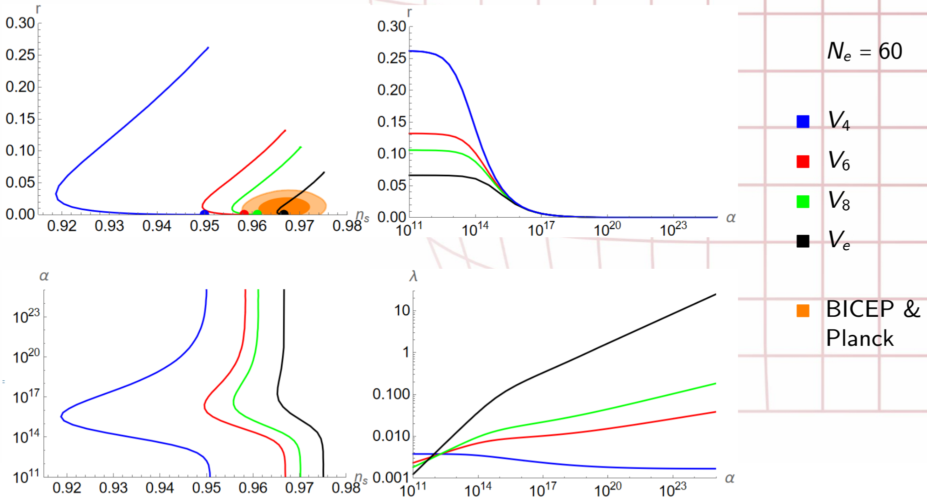
$$\dot{H} = -\frac{1}{2}\frac{\dot{\phi}^2}{F'(\zeta)}K(\phi)$$

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{12V(\phi) - 6F(\zeta) + 3\zeta F'(\zeta)}{6V(\phi) - 3F(\zeta) + 2\zeta F'(\zeta)}$$

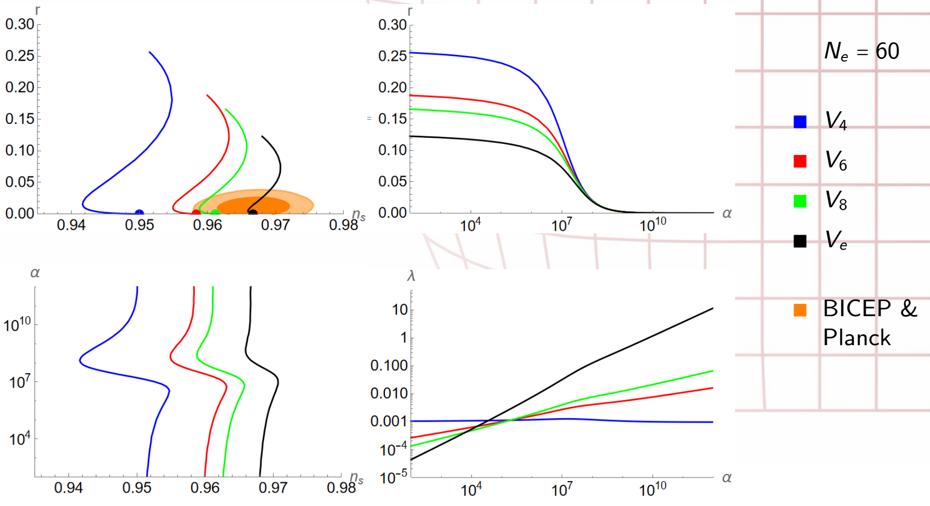
$$\dot{\zeta} = \frac{3H\dot{\phi}^2 F'(\zeta)K(\phi) + 3V'(\phi)\dot{\phi}}{2G'(\zeta) + \frac{3}{2}\dot{\phi}^2 F''(\zeta)K(\phi)}$$

N.B. Even though ζ is only auxiliary, it has an implicit time dependence via ϕ

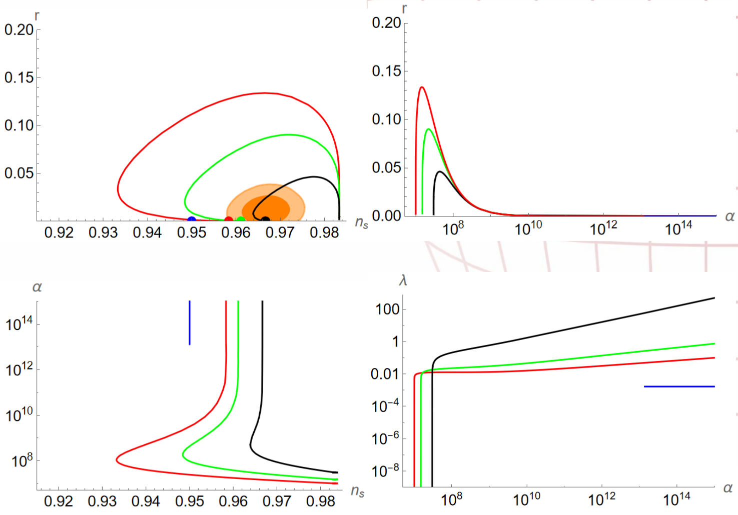
Example 2. $F(R) = R + \alpha R^3$



• Example 2. $F(R) = e^{\alpha R}/\alpha$ •



• Example 2. $F(R) = R + \alpha R^2 \ln(\alpha R)$ •



$N_e = 60$

- V_4
- V_6
- V_8
- V_e
- BICEP & Planck

• Beyond SR. $F_{>2}(R)$ •