Introduction to Deep-Inelastic Scattering

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Introduction /1

Measurements of Deep-Inelastic scattering have been seminal in our understanding of the proton (hadronic) substructure. They have provided us with an insight on:

- the constituents (quarks and gluons) from which the proton is built;
- the theory (QCD) that describes the interactions between them.

The advent of the electron-proton collider HERA (1992-2007) has allowed to extend these measurements to a new kinematic region and test pQCD at an unprecedented level and to greatly improve our understanding of proton's structure.

The forthcoming experiment(s) at the Electron-Ion Collider (EIC) will continue this fantastic journey, giving us the possibility to address the yet remaining (and most challenging) questions on QCD, proton's structure and spin.

Introduction /2

Lecture's plan:

- Lecture 1:
  We will give a general overview of the most important experimental and phenomenological results on Structure Functions, parton distribution functions (PDFs) and DGLAP analyses. The focus will be mostly on inclusive unpolarised Deep-Inelastic Scattering.
- Lecture 2:
  From an historical perspective we will discuss the impact of past, present DIS experiments (as well as non DIS experiments) on the collinear PDFs
- Lecture 3 (Hands-on session)
  Hands-on session on how to perform a "real" DGLAP analysis

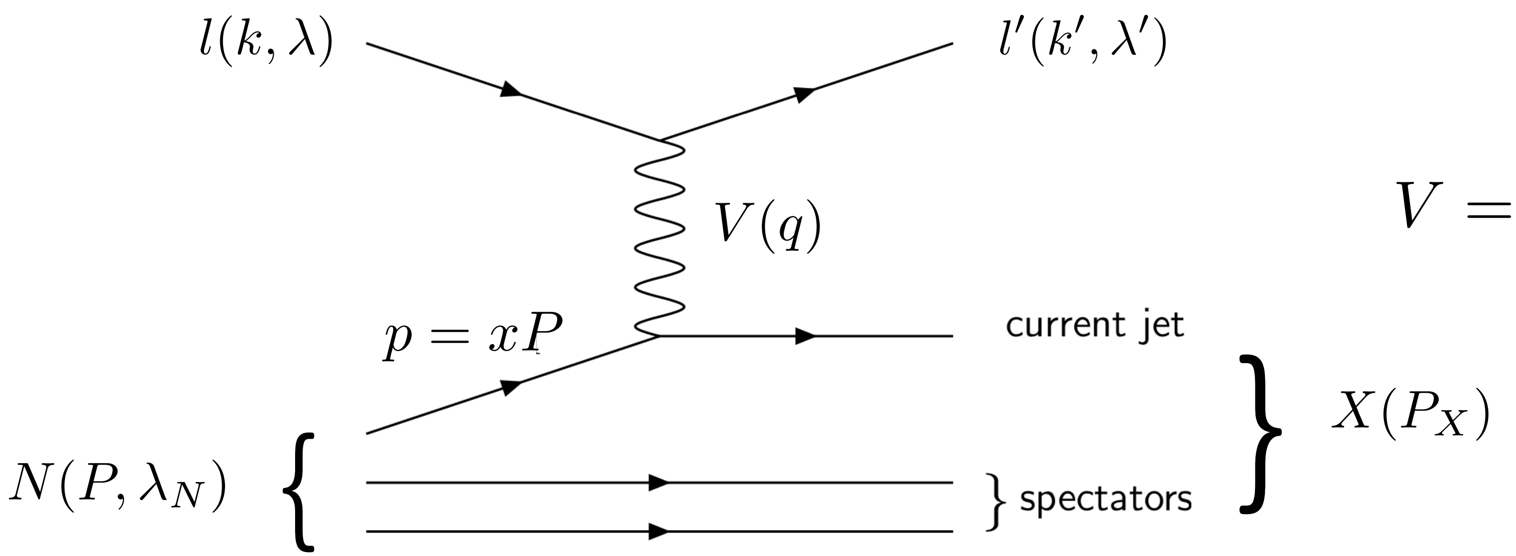
Overall main goal of these lectures is to stimulate your interest towards this fascinating research field.

Outline (Lecture 1)

- The DIS process, the kinematic variables and their reconstruction
- The Differential Cross section and the Structure Functions
- Overview of experimental results on the Structure Functions (including EW)
- The QCD-improved partonic picture of the nucleon
- The proton's parton distribution functions and the DGLAP analyses

The Deep-Inelastic process

$$l(k) + N(P) \longrightarrow l'(k') + X(P_X)$$



$$V = \gamma, Z \text{ and } W^{\pm}$$

Typical initial and final state particles:

$$l = e^{\pm},\ \mu^{\pm}, \nu_\mu(\bar{\nu_\mu}), \ldots \qquad l' = e^{\pm},\ \nu_e(\bar{\nu_e}),\ \mu^{\pm}, \nu_\mu(\bar{\nu_\mu}), \ldots$$
$$N = p, nuclei\ (D, Fe, \ldots)$$

DIS kinematic variables /1

The following, Lorentz-invariant quantities, are used to express Cross-sections and Structure Functions:

Centre-of-mass energy $\sqrt{s}$: $$s = (k + P)^2$$

Virtuality of the exchanged gauge boson (V) : $$Q^2 = -q^2 = -(k - k')^2$$

Bjorken variables x and y : $$x = \frac{Q^2}{2P \cdot q} \quad \text{and} \quad y = \frac{P \cdot q}{P \cdot k}$$

Invariant mass W of the final state system X: $$W^2 = P_X^2 = (q + P)^2 = Q^2 \frac{1 - x}{x} + m_N^2$$

Energy (ν) trasported by V in the N rest frame: $$\nu = \frac{P \cdot q}{M_N}$$

DIS kinematic variables/2

$$\nu = \frac{q \cdot P}{m_N} = E_e - E'_e \text{ is the lepton's energy loss in the nucleon rest frame}$$

$$x = \frac{Q^2}{2m_N\nu} \text{ at LO is the fraction of the nucleon's momentum carried by the struck quark}$$

$$y = \frac{q \cdot P}{k \cdot P} = \frac{\nu}{E_e} \text{ is the fraction of the lepton's energy lost in the nucleon rest frame}$$

Additional important kinematical relationships:

$$Q^2 = (s - m_N^2)xy \simeq sxy \ , \quad x = \frac{Q^2}{2M_N\nu} = \frac{Q^2}{W^2 + Q^2 - M_N^2} \ , \ y = \frac{2M_N\nu}{(s - M_N^2)} = \frac{W^2 + Q^2 - M_N^2}{(s - M_N^2)}$$

DIS kinematic: Bjorken limit

The Bjorken limit is formally defined as:

$$\nu, Q^2 \longrightarrow +\infty \quad \text{with} \quad x \quad \text{fixed}$$

The inclusive lepton-nucleon process is denoted as deep(ly) inelastic if:

$$W > 2 \text{ GeV (outside of the resonance region) and } \; Q^2 \gg \Lambda_{QCD}$$

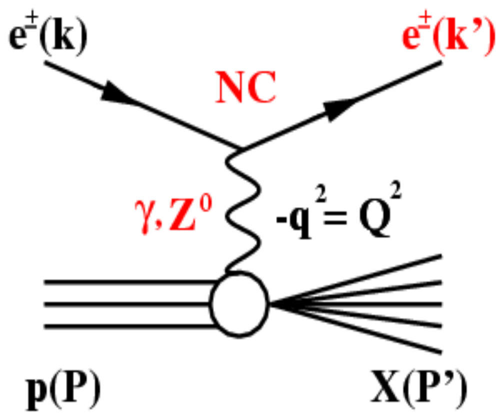
Resolution power of the gauge boson V (probe):

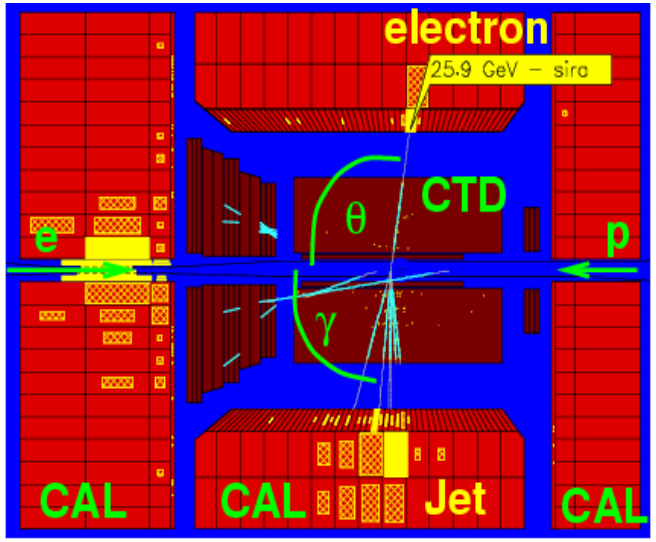
$$d = \frac{\hbar c}{\sqrt{Q^2}} \simeq \frac{0.197}{\sqrt{Q^2}} \text{ GeV fm} \quad , \quad d \sim \mathcal{O}(10^{-18}\text{m}) \;\; \text{for} \;\; Q^2 \sim 10^5 \text{GeV}^2$$

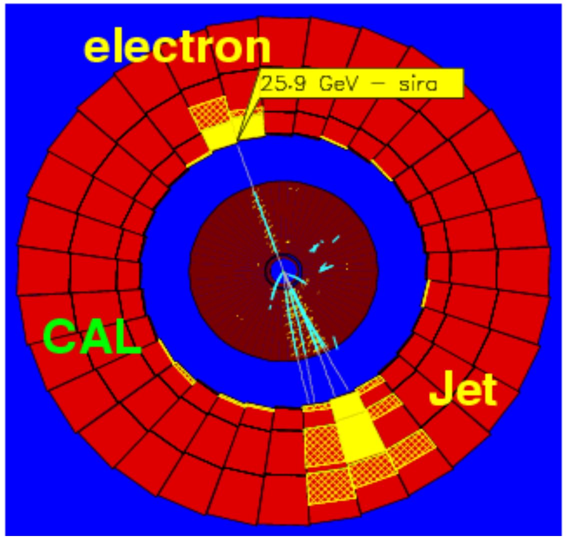
DIS Neutral and charges current processes

Neutral current (NC) : $V = \gamma, Z$ Charged current (CC) : $V = W^{\pm}$

Taking HERA (ZEUS and H1 Experiments) as an example







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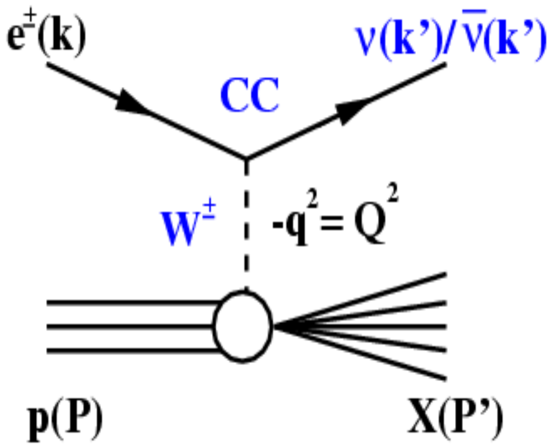
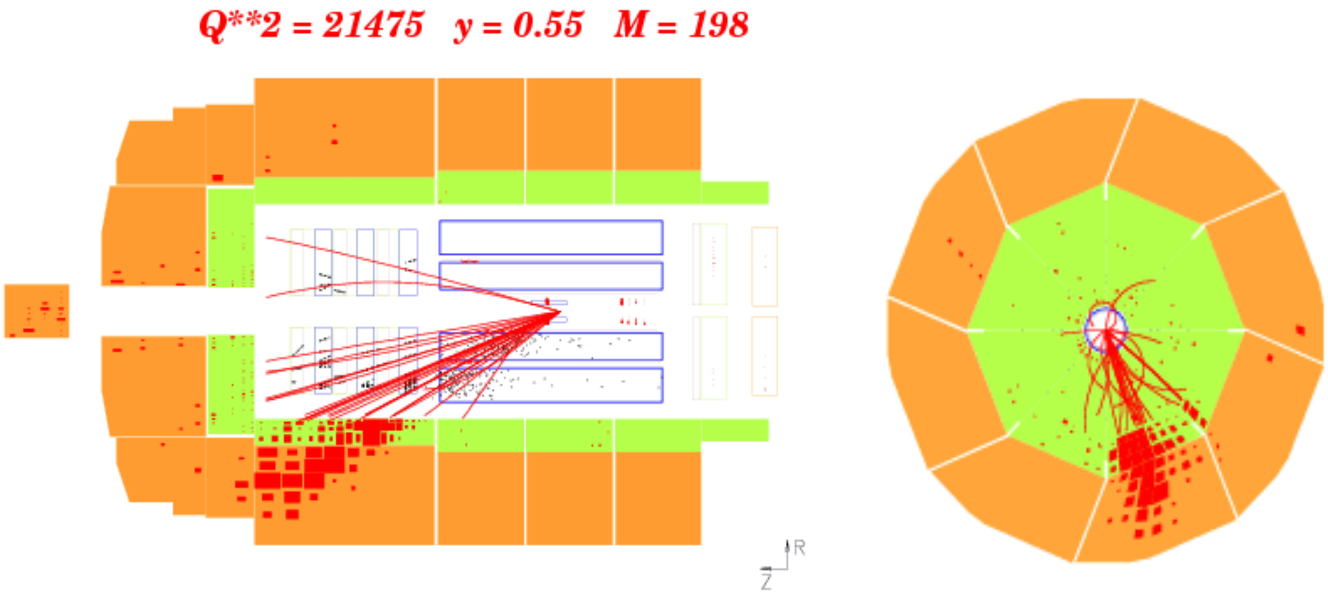
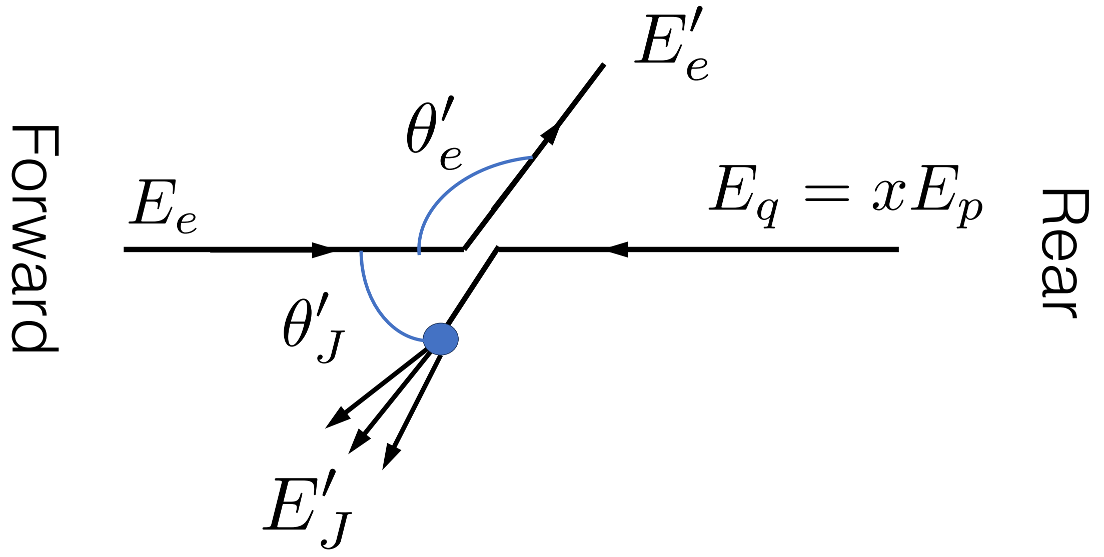




Reconstruction methods of kinematic variables

The quality of the Differential Cross Sections and Structure Functions measurements fully rely on the best possible reconstruction of the variables Q^2 and x.



$$xE_p + E_e = E'_e + E'_J$$
$$xE_p - E_e = E'_e \cos\theta'_e + E'_J \cos\theta'_J$$
$$E'_e \sin\theta'_e = E'_J \sin\theta'_J$$

Any pair out the set $(E'_e, E'_J, \theta'_e, \theta'_J)$ uniquely identify a point in the (x, Q^2) plane.

Please note that the angles θ'_e and θ'_J are referred to the incoming nucleon/proton direction.

Reconstruction methods of kinematic variables

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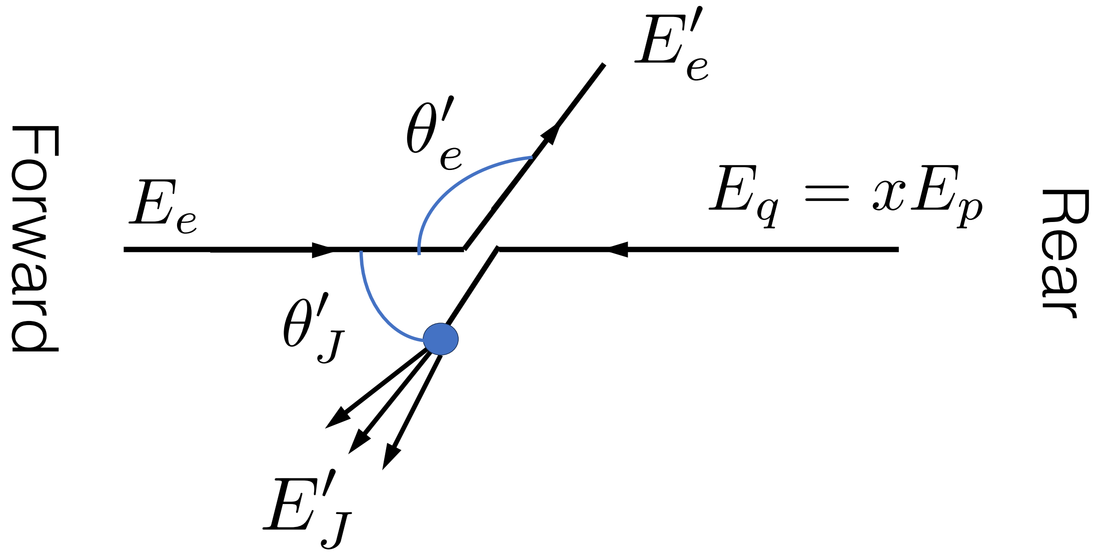
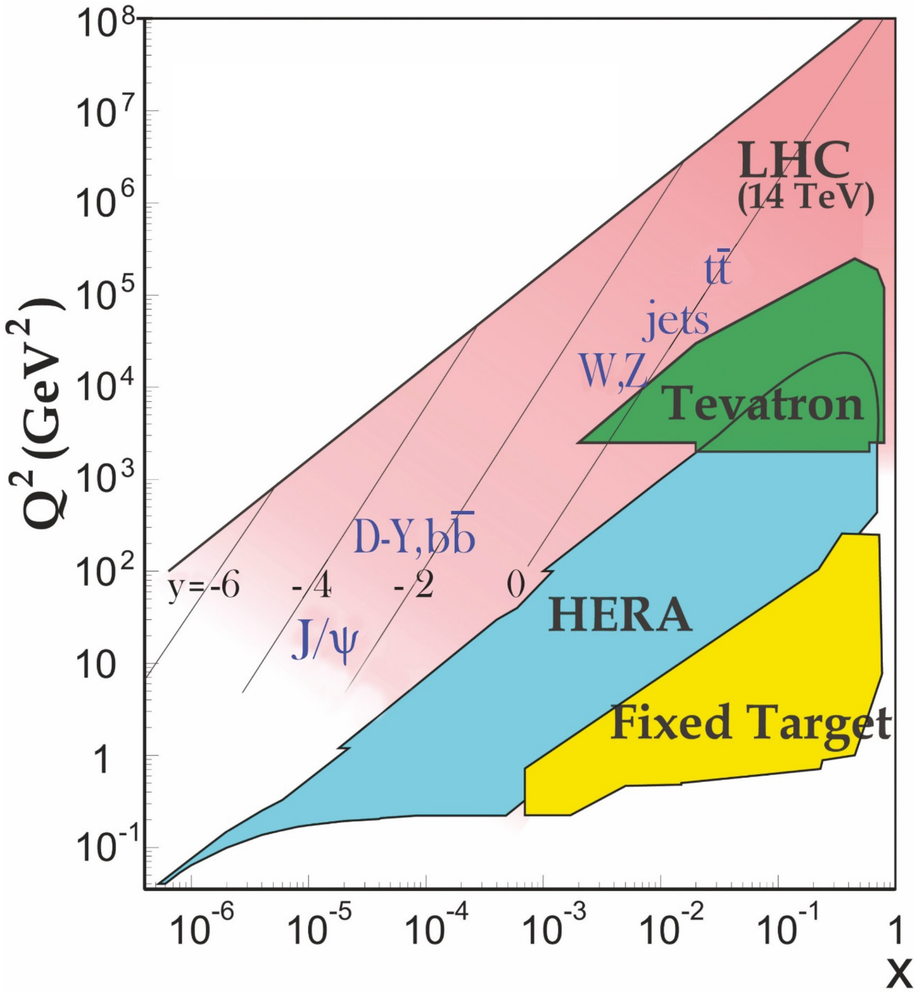


6 Reconstruction methods:

$$[M1]: \quad Q^2 = Q^2[E'_e, \theta'_e] \quad , \quad x = x[E'_e, \theta'_e]$$
$$[M2]: \quad Q^2 = Q^2[E'_e, \theta'_J] \quad , \quad x = x[E'_e, \theta'_J]$$
$$[M3]: \quad Q^2 = Q^2[\theta'_e, \theta'_J] \quad , \quad x = x[\theta'_e, \theta'_J]$$
$$[M4]: \quad Q^2 = Q^2[E'_e, E'_J] \quad , \quad x = x[E'_e, E'_J]$$
$$[M5]: \quad Q^2 = Q^2[E'_J, \theta'_e] \quad , \quad x = x[E'_J, \theta'_e]$$
$$[M6]: \quad Q^2 = Q^2[E'_J, \theta'_J] \quad , \quad x = x[E'_J, \theta'_J]$$

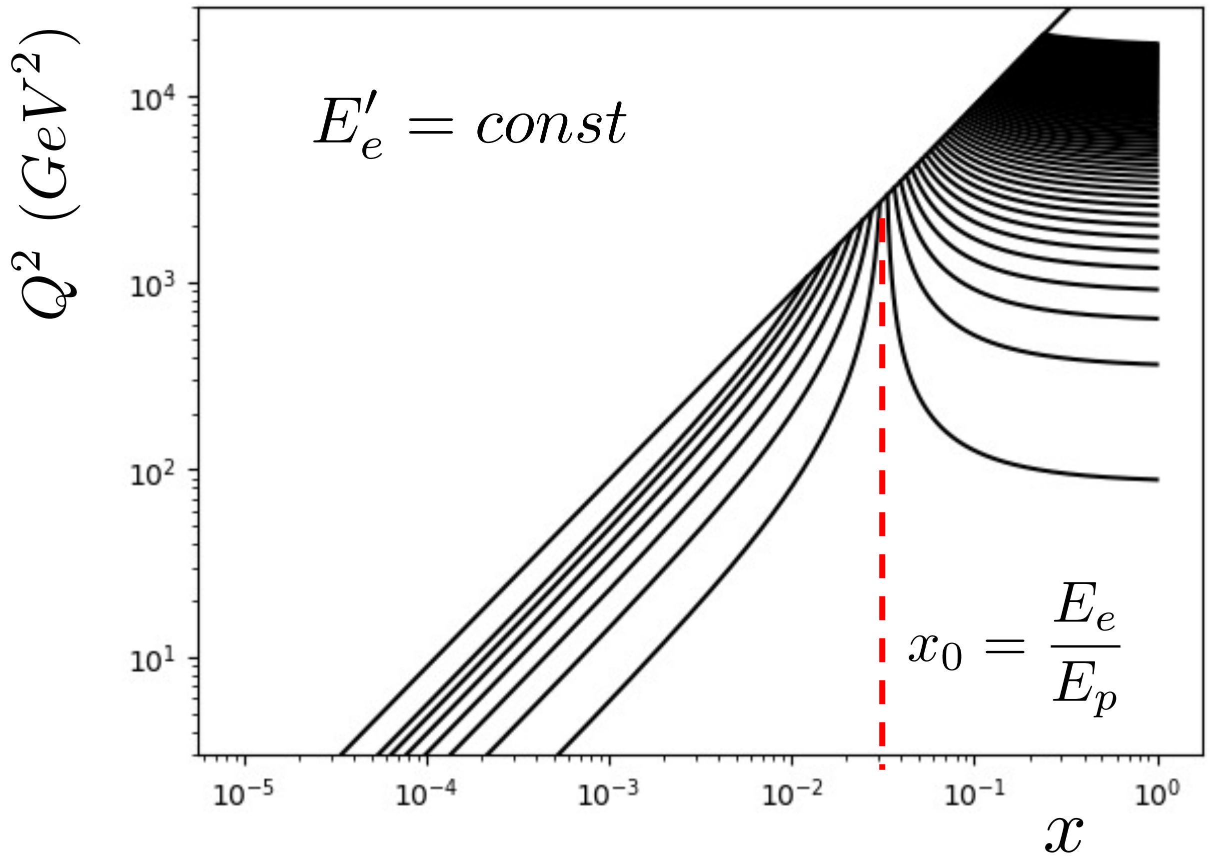
Any pair out of the set $(E'_e, E'_J, \theta'_e, \theta'_J)$ uniquely identify a point in the (x, Q^2) plane

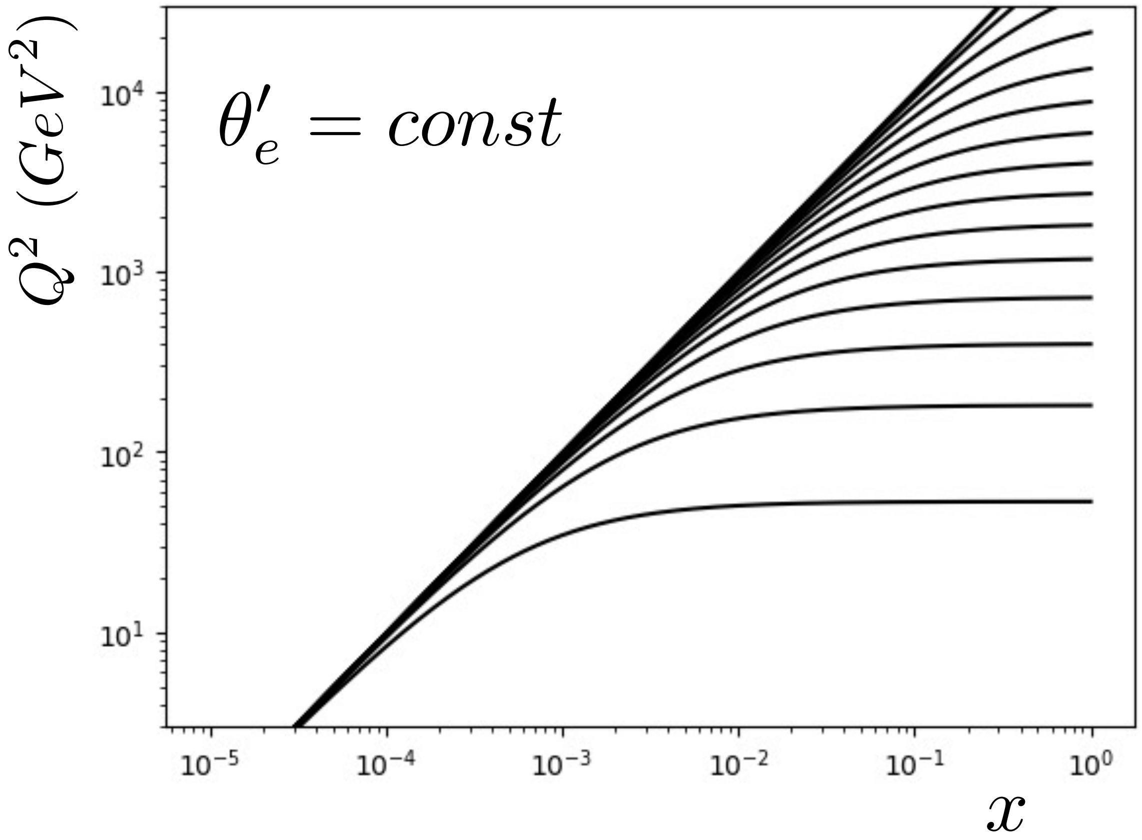
DIS kinematics – (x,Q²) plane



DIS kinematics – (x,Q^2) plane and isolines

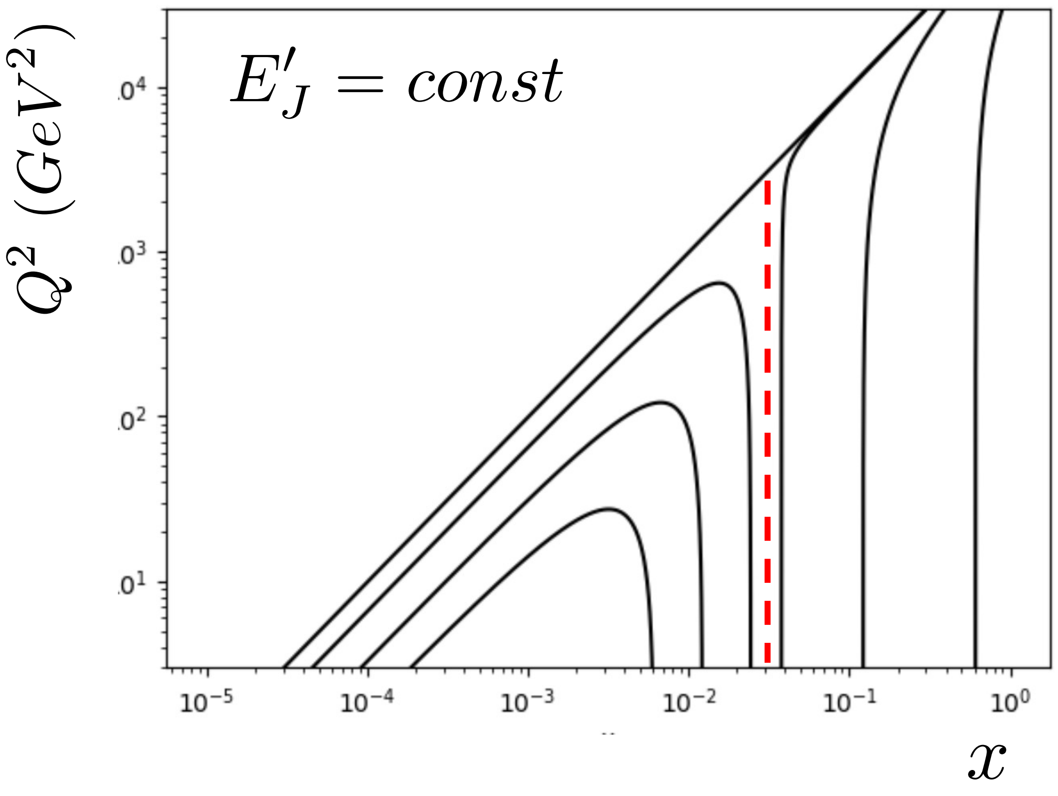
The x-Q^2 plane, isolines

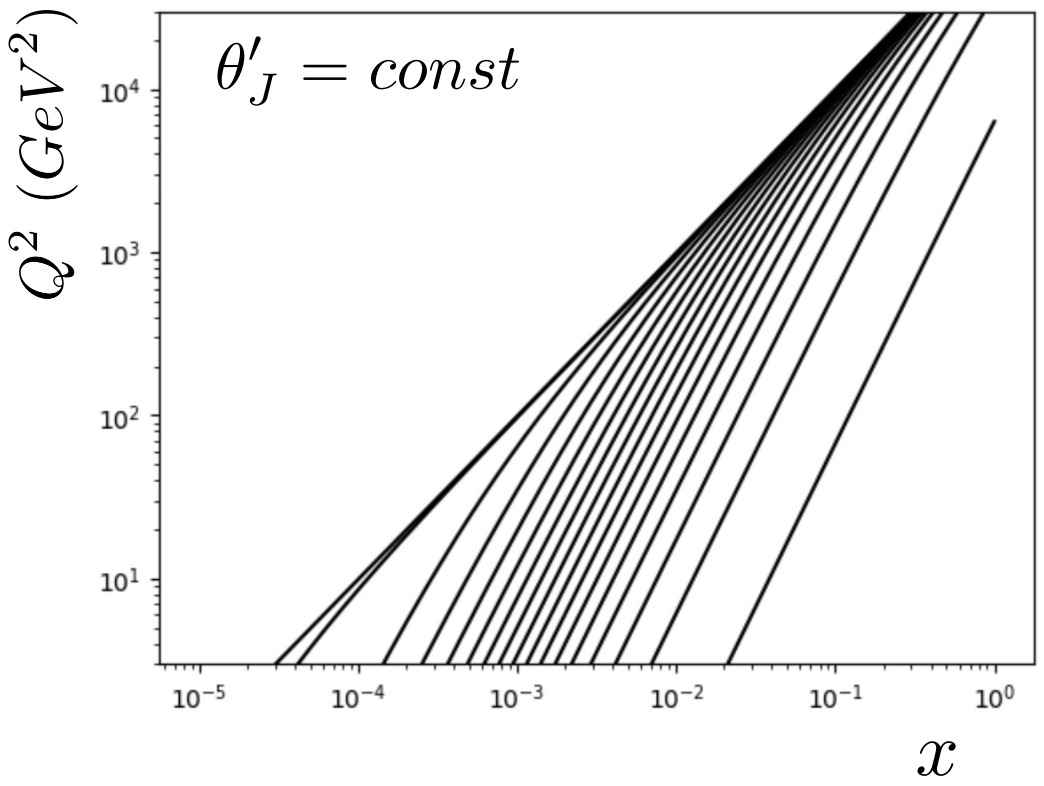




DIS kinematics – (x,Q^2) plane and isolines

The x-Q^2 plane, isolines





Reconstruction Methods: Electron Method

Take first the Electron method (M1) as an example:

$$Q^2_{el} = 2E_e E'_e(1 - cos\theta'_e) \qquad y_{el} = \frac{Q^2_{el}}{s x_{el}}$$

$$x_{el} = x_0 \frac{E'_e(1+\cos\theta'_e)}{2E_e - E'_e(1-\cos\theta'_e)} \qquad x_0 = \frac{E_e}{E_p}$$

By simple uncertainties propagation one gets:

$$\frac{\delta Q^2_{el}}{Q^2_{el}} = \frac{\delta E'_e}{E'_e} \oplus \tan(\theta'_e)\delta\theta'_e$$

$$\frac{\delta x_{el}}{x_{el}} = \frac{1}{y_{el}}\frac{\delta E'_e}{E'_e} \oplus \left[\tan\left(\frac{\theta'_e}{2}\right) + \left(\frac{1}{y_{el}} - 1\right)\cot\left(\frac{\theta'_e}{2}\right)\right]\delta\theta'_e$$

$$\frac{\delta y_{el}}{y_{el}} = \left(1 - \frac{1}{y_{el}}\right)\frac{\delta E'_e}{E'_e} \oplus \left(\frac{1}{y_{el}} - 1\right)\cot\left(\frac{\theta'_e}{2}\right)\delta\theta'_e$$

Reconstruction Methods: JB Method

Take Jacquet-Blondel (M6) (charged current):

$$Q^2_{JB} = \frac{\sum p_\perp^2}{1 - y_{JB}} \simeq \frac{E_J^2 \sin^2\theta_J}{1 - y_{JB}} \qquad x_{JB} = \frac{Q^2_{JB}}{s y_{JB}}$$

$$y_{JB} = \frac{\sum(E - p_z)}{2E_e} \simeq \frac{E_J}{E_e}\sin^2\left(\frac{\theta_J}{2}\right)$$

By simple uncertainties propagation one gets:

$$\frac{\delta Q^2_{JB}}{Q^2_{JB}} = \frac{2 - y_{JB}}{1 - y_{JB}}\frac{\delta E_J}{E_J} \oplus \left[2\cot\theta_J + \frac{y_{JB}}{1 - y_{JB}}\cot\left(\frac{\theta_J}{2}\right)\right]\delta\theta_J$$

$$\frac{\delta x_{JB}}{x_{JB}} = \frac{1}{1 - y_{JB}}\frac{\delta E_J}{E_J} \oplus \left[2\cot\theta_J + \frac{2y_{JB} - 1}{1 - y_{JB}}\cot\left(\frac{\theta_J}{2}\right)\right]\delta\theta_J$$

$$\frac{\delta y_{JB}}{y_{JB}} = \frac{\delta E_J}{E_J} \oplus \cot\left(\frac{\theta_J}{2}\right)\delta\theta_J$$

Reconstruction Methods: DA Method

Double-Angle method (M3):

$$Q^2_{DA} = 4E_e^2 \cdot \frac{\sin\theta_J(1+\cos\theta'_e)}{\sin\theta_J + \sin\theta'_e - \sin(\theta'_e + \theta_J)}$$

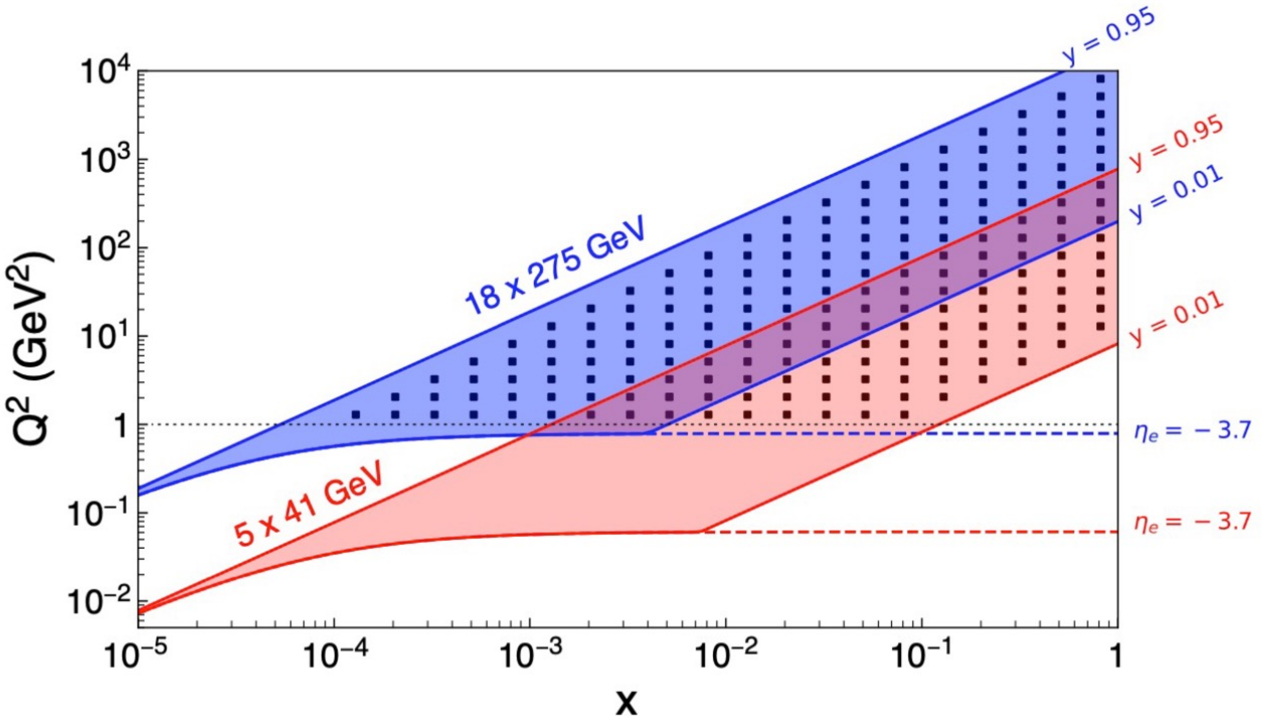
$$y_{DA} = \frac{\sin\theta'_e(1-\cos\theta_J)}{\sin\theta_J + \sin\theta'_e - \sin(\theta'_e + \theta_J)}$$

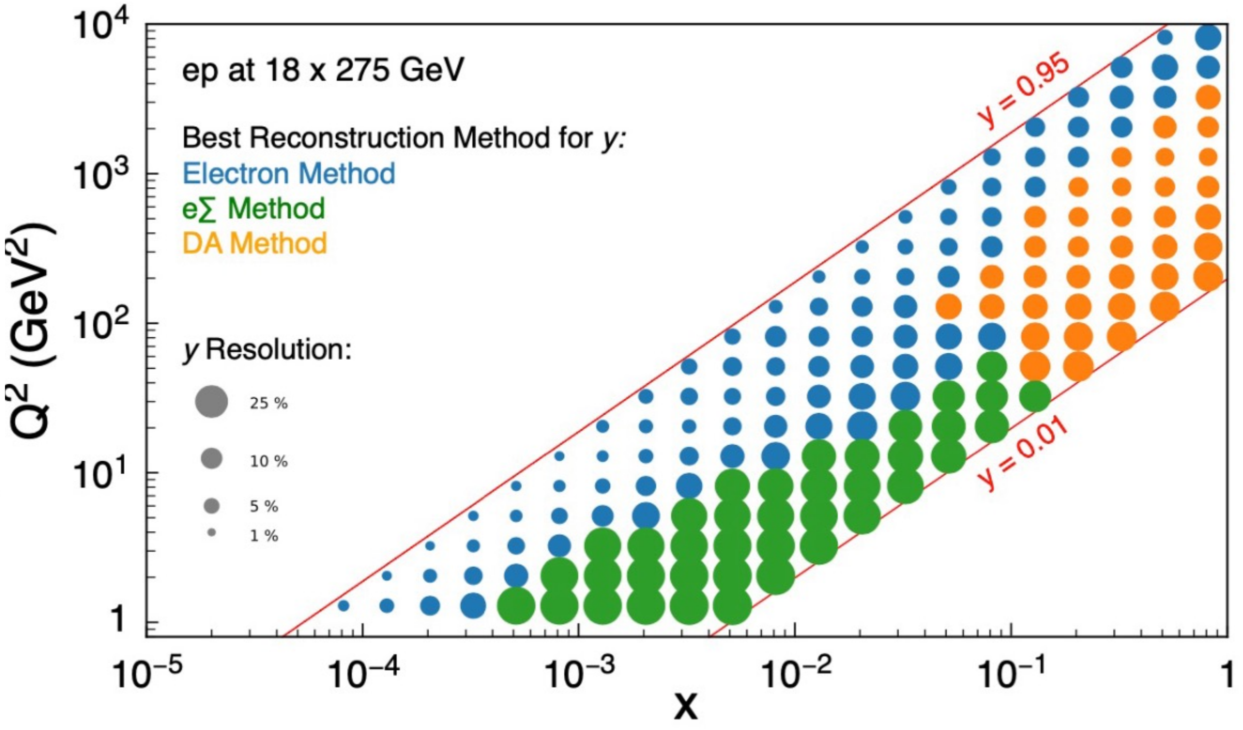
$$x_{DA} = \frac{Q^2_{DA}}{s y_{DA}}$$

Other methods exploited at HERA: “PT method”, “Sigma method”, “e-Sigma method”, …

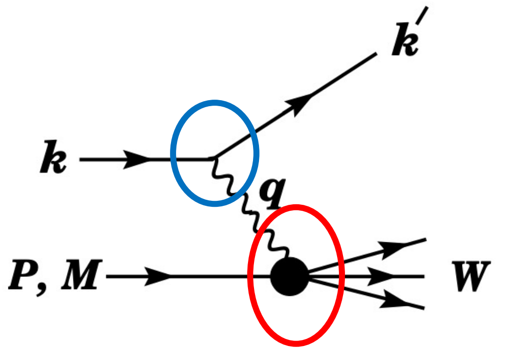
All methods should be studied with detailed MC simulations (to quantify resolutions and migrations)

EIC: x-Q^2 plane





Double differential cross sections



$$\frac{d^2\sigma}{dxdy} = \frac{2\pi y\alpha^2}{Q^4} \sum_j \eta_j L_j^{\mu\nu} W_{\mu\nu}^j$$

$j = \gamma$ neutral current (EM)

$j = \gamma, \gamma Z, Z$ neutral current (EW)

$j = W^{\pm}$ charged current (EW)

Leptonic tensors:

$$L_{\mu\nu}^{\gamma} = 2\left(k_\mu k'_\nu + k'_\mu k_\nu - (k\cdot k' - m_\ell^2)g_{\mu\nu} - i\lambda\varepsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta\right),$$

$$L_{\mu\nu}^{\gamma Z} = (g_V^e + e\lambda g_A^e)\, L_{\mu\nu}^{\gamma}, \quad L_{\mu\nu}^{Z} = (g_V^e + e\lambda g_A^e)^2\, L_{\mu\nu}^{\gamma},$$

$$L_{\mu\nu}^{W} = (1 + e\lambda)^2\, L_{\mu\nu}^{\gamma},$$

with: $g_V^e = -\frac{1}{2} + 2\sin^2\theta_W, \quad g_A^e = -\frac{1}{2} \quad e = \pm 1$

Double differential cross sections

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi y\alpha^2}{Q^4} \sum_j \eta_j \, L_j^{\mu\nu} \, W_{\mu\nu}^j$$

$j = \gamma$ neutral current (EM)

$j = \gamma, \gamma Z, Z$ neutral current (EW)

$j = W^{\pm}$ charged current (EW)

Propagator/coupling ratio factors:

$$\eta_\gamma = 1 \; ; \; \eta_{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\right)\left(\frac{Q^2}{Q^2+M_Z^2}\right);$$

$$\eta_Z = \eta_{\gamma Z}^2 \; ; \; \eta_W = \frac{1}{2}\left(\frac{G_F M_W^2}{4\pi\alpha}\frac{Q^2}{Q^2+M_W^2}\right)^2.$$

Double differential cross sections

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi y\alpha^2}{Q^4} \sum_j \eta_j \, L_j^{\mu\nu} \, W_{\mu\nu}^j$$

$j = \gamma$ neutral current (EM)

$j = \gamma, \gamma Z, Z$ neutral current (EW)

$j = W^{\pm}$ charged current (EW)

Hadronic tensor:

$$W_{\mu\nu}^j = \frac{1}{4\pi} \int d^4z \; e^{iq\cdot z} < P, S| \left[J_\mu^{\dagger i_1}(z), J_\nu^{i_2}(0) \right] |P, S > \quad \text{with} \;\; i_1, i_2 = \gamma, Z, W$$

J_α is the hadronic contribution to the electromagnetic, or weak current

S denots the nucleon spin 4-vector, with $S^2 = -M_N^2$ and $S \cdot P = 0$

Double differential cross sections

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi y\alpha^2}{Q^4} \sum_j \eta_j \, L_j^{\mu\nu} \, W_{\mu\nu}^j$$

$j = \gamma \quad \text{neutral current (EM)}$

$j = \gamma, \gamma Z, Z \quad \text{neutral current (EW)}$

$j = W^{\pm} \text{ charged current (EW)}$

Diff. cross section (alternative forms ad higher-order EW contributions):

$$\frac{d^2\sigma}{dx\,dy} = x\,(s - M^2)\,\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\,M\nu}{E'}\,\frac{d^2\sigma}{d\Omega_{\mathrm{Nrest}}\,dE'}$$

$$\frac{d^2\sigma}{dxdQ^2} = \frac{d^2\sigma_0}{dxdQ^2} + \frac{\alpha}{2\pi}\frac{d^2\sigma_1}{dxdQ^2} + \mathcal{O}(\alpha^2) = \frac{d^2\sigma_0}{dxdQ^2}(1 + \delta_{EW})$$

Hadronic tensor and Structure Functions

$$
\begin{aligned}
W^j_{\mu\nu} = &\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F^j_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F^j_2(x,Q^2) \\
&- i\epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha P^\beta}{2P\cdot q} F^j_3(x,Q^2) \\
&+ i\epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha}{P\cdot q}\left[S^\beta g^j_1(x,Q^2) + \left(S^\beta - \frac{S\cdot q}{P\cdot q}P^\beta\right) g^j_2(x,Q^2)\right] \\
&+ \frac{1}{P\cdot q}\left[\frac{1}{2}\left(\hat{P}_\mu \hat{S}_\nu + \hat{S}_\mu \hat{P}_\nu\right) - \frac{S\cdot q}{P\cdot q}\hat{P}_\mu \hat{P}_\nu\right] g^j_3(x,Q^2) \\
&+ \frac{S\cdot q}{P\cdot q}\left[\frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} g^j_4(x,Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) g^j_5(x,Q^2)\right]
\end{aligned}
$$

where $\hat{P}_\mu = P_\mu - \frac{P\cdot q}{q^2}$, $\hat{S}_\mu = S_\mu - \frac{S\cdot q}{q^2}$ and $j = \gamma, \gamma Z, Z, W$

Hadronic tensor and Structure Functions

$$
\begin{aligned}
W^j_{\mu\nu} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F^j_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F^j_2(x,Q^2) \\
&- i\epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha P^\beta}{2P\cdot q} F^j_3(x,Q^2) \\
&+ i\epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha}{P\cdot q}\left[S^\beta g^j_1(x,Q^2) + \left(S^\beta - \frac{S\cdot q}{P\cdot q}P^\beta\right) g^j_2(x,Q^2)\right] \\
&+ \frac{1}{P\cdot q}\left[\frac{1}{2}\left(\hat{P}_\mu \hat{S}_\nu + \hat{S}_\mu \hat{P}_\nu\right) - \frac{S\cdot q}{P\cdot q}\hat{P}_\mu \hat{P}_\nu\right] g^j_3(x,Q^2) \\
&+ \frac{S\cdot q}{P\cdot q}\left[\frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} g^j_4(x,Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) g^j_5(x,Q^2)\right]
\end{aligned}
$$

Unpolarised SFs

Polarised SFs

where $\hat{P}_\mu = P_\mu - \frac{P\cdot q}{q^2}$, $\hat{S}_\mu = S_\mu - \frac{S\cdot q}{q^2}$ and $j = \gamma, \gamma Z, Z, W$

Hadronic tensor and Structure Functions

$$W^j_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F^j_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F^j_2(x,Q^2)$$
$$-i\epsilon_{\mu\nu\alpha\beta}\frac{q^\alpha P^\beta}{2P\cdot q} F^j_3(x,Q^2)$$
$$+i\epsilon_{\mu\nu\alpha\beta}\frac{q^\alpha}{P\cdot q}\left[S^\beta g^j_1(x,Q^2) + \left(S^\beta - \frac{S\cdot q}{P\cdot q}P^\beta\right) g^j_2(x,Q^2)\right]$$
$$+\frac{1}{P\cdot q}\left[\frac{1}{2}\left(\hat{P}_\mu \hat{S}_\nu + \hat{S}_\mu \hat{P}_\nu\right) - \frac{S\cdot q}{P\cdot q}\hat{P}_\mu \hat{P}_\nu\right] g^j_3(x,Q^2)$$
$$+\frac{S\cdot q}{P\cdot q}\left[\frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} g^j_4(x,Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) g^j_5(x,Q^2)\right]$$

where $\hat{P}_\mu = P_\mu - \frac{P\cdot q}{q^2}$, $\hat{S}_\mu = S_\mu - \frac{S\cdot q}{q^2}$ and $j = \gamma, \gamma Z, Z, W$

Parity conserving SFs

Parity violating SFs

Double differential cross sections and Structure Functions

The unpolarised double diff. cross sections for e+- p NC and CC DIS are

$$\frac{d^2\sigma^k}{dxdy}(e^{\pm}p) = \eta^k \frac{2\pi\alpha^2}{xyQ^2}\left[Y_+\mathcal{F}_2^k - y^2\mathcal{F}_L^k \mp Y_- x\mathcal{F}_3^k\right]$$

$$\text{with } k = NC, CC \text{ and } \eta^{NC} = 1\ ,\ \eta^{CC} = 2\eta^W \text{ and } Y_{\pm} = 1 \pm (1-y)^2$$

In this expression we have introduced the Longitudinall Structure Function:

$$\mathcal{F}_L^k = \mathcal{F}_2^k - 2x\mathcal{F}_1^k$$

$\mathcal{F}_L$ contributes at very high-y and $\mathcal{F}_3$ at very high-Q^2

Double differential cross sections and Structure Functions

The unpolarised double diff. cross sections for e+- p NC and CC DIS are

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$$\text{with } k = NC, CC \text{ and } \eta^{NC} = 1 \ , \ \eta^{CC} = 2\eta^W \text{ and } Y_\pm = 1 \pm (1-y)^2$$

For the structure functions we have:

$$\mathcal{F}_a^{CC} = F_a^W \text{ with } a = 1,2,3$$

$$\mathcal{F}_a^{NC} = F_a^\gamma - g_V^e \eta_{\gamma Z} F_a^{\gamma Z} + ({g_V^e}^2 + {g_A^e}^2)\eta_Z F_a^Z \quad \text{with } a = 2, L$$

$$\mathcal{F}_3^{NC} = -g_A^e \eta_{\gamma Z} F_3^{\gamma Z} + 2 g_V^e g_A^e \eta_Z F_3^Z$$

Double differential cross sections and Structure Functions

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For the structure function we have:

$$\mathcal{F}_a^{CC} = F_a^W \text{ with } a = 1, 2, 3$$

$$\mathcal{F}_a^{NC} = F_a^\gamma - g_V^e \eta_{\gamma Z} F_a^{\gamma Z} + ({g_V^e}^2 + {g_A^e}^2)\eta_Z F_a^Z \text{ with } a = 2, L$$

$$\mathcal{F}_3^{NC} = -g_A^e \eta_{\gamma Z} F_3^{\gamma Z} + 2 g_V^e g_A^e \eta_Z F_3^Z$$

Double differential cross sections and Structure Functions

The unpolarised double diff. cross sections for e± p NC and CC DIS are

$$\frac{d^2\sigma^k}{dxdy}(e^{\pm}p) = \eta^k \frac{2\pi\alpha^2}{xyQ^2}\left[Y_+\mathcal{F}_2^k - y^2\mathcal{F}_L^k \mp Y_- x\mathcal{F}_3^k\right]$$

with $k = NC, CC$ and $\eta^{NC} = 1$, $\eta^{CC} = 2\eta^W$ and $Y_\pm = 1 \pm (1-y)^2$

Results are also presented in terms of <u>reduced cross sections</u>:

$$\mathcal{F}_a^{NC} = \mathcal{F}_a$$

$$\mathcal{F}_a^{CC} = \mathcal{F}_a^{W^\pm} = \mathcal{W}_a^\pm$$

$$\sigma_{r,NC}^{\pm} = \frac{d^2\sigma_{NC}(e^\pm p)}{dxdQ^2} \cdot \frac{Q^4 x}{2\pi\alpha^2 Y_+} = \mathcal{F}_2 - \frac{y^2}{Y_+}\mathcal{F}_L \mp \frac{Y_-}{Y_+}x\mathcal{F}_3$$

$$\sigma_{r,CC}^{\pm} = \frac{d^2\sigma_{CC}(e^\pm p)}{dxdQ^2} \cdot \frac{2\pi x}{G_F^2}\left[\frac{M_W^2 + Q^2}{M_W^2}\right]^2 = \frac{1}{2}\left(Y_+\mathcal{W}_2^\pm - y^2\mathcal{W}_L^\pm \mp Y_- x\mathcal{W}_3^\pm\right)$$

DIS Experiments: Typical analysis (HERA)

Very precise reconstruction of event's characteristics and kinematic variables:

$$Q^2_{\mathrm{PT}} = 4E_e^2 \frac{\sin\gamma_{\mathrm{PT}}(1+\cos\theta_e)}{\sin\gamma_{\mathrm{PT}}+\sin\theta_e-\sin(\gamma_{\mathrm{PT}}+\theta_e)};$$

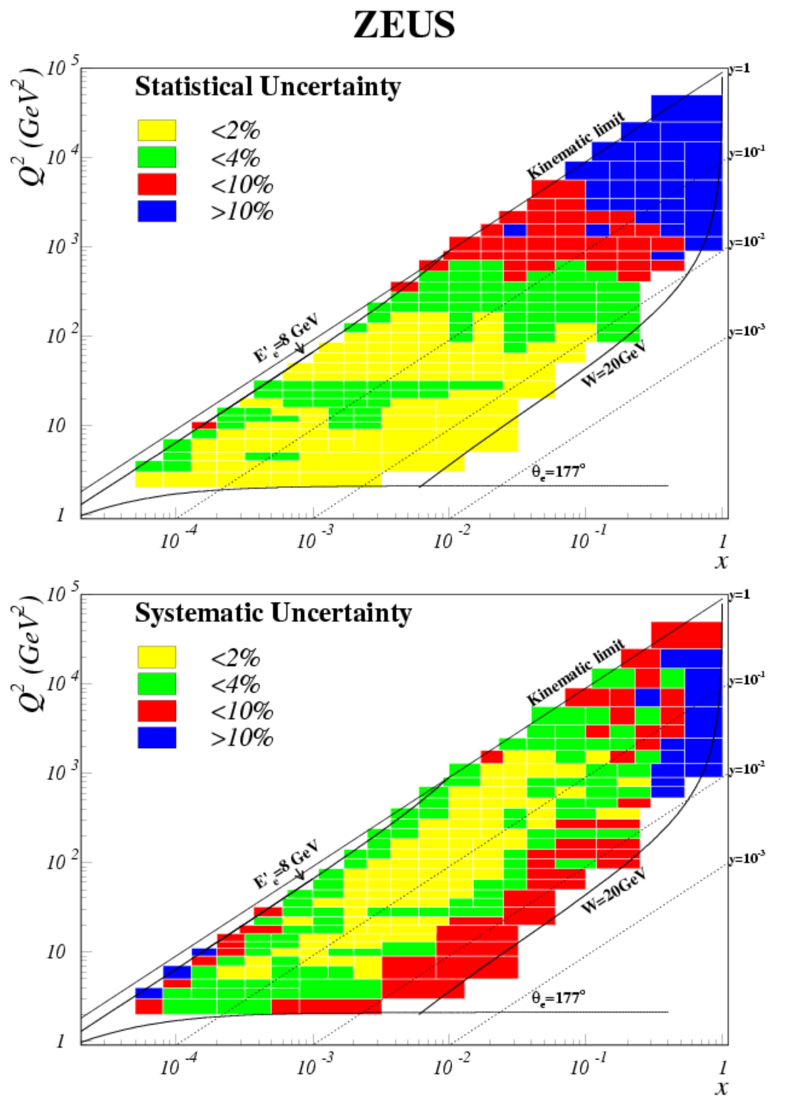
$$x_{\mathrm{PT}} = \frac{E_e}{E_p}\frac{\sin\gamma_{\mathrm{PT}}+\sin\theta_e+\sin(\gamma_{\mathrm{PT}}+\theta_e)}{\sin\gamma_{\mathrm{PT}}+\sin\theta_e-\sin(\gamma_{\mathrm{PT}}+\theta_e)};$$

Count nr. of events in appropriately defined (x,Q^2) bins

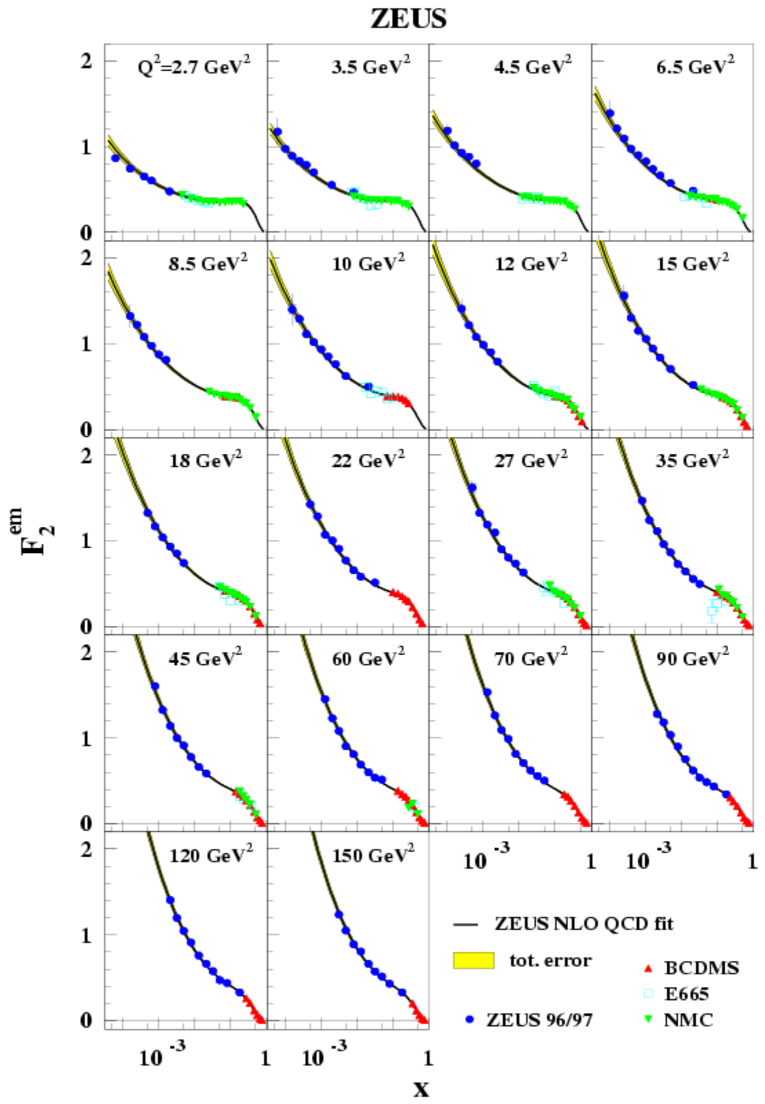
Extract reduced cross section and correct radiative EW corrections

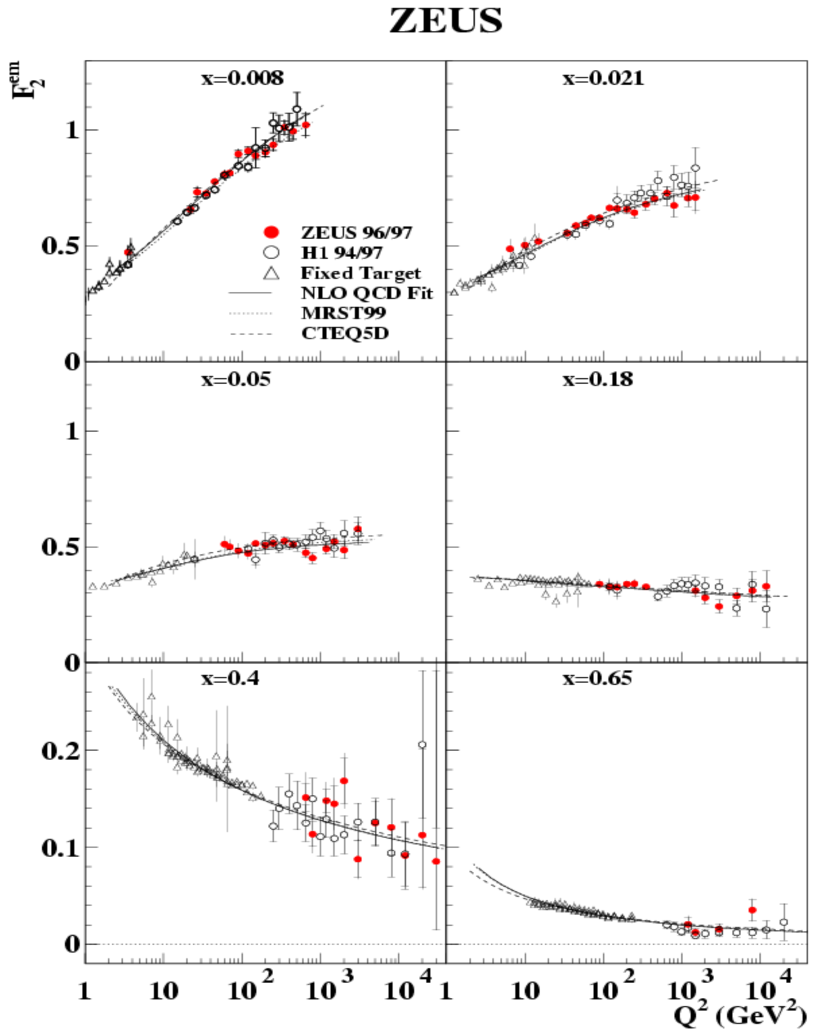
$$\sigma_{r,NC} = \frac{1}{1+\delta_{EW}}\left(\frac{N_{data}-N_{bkg}}{N_{MC}}\right)\sigma^{NLO}_{r,NC}$$

Extract F_2



DIS Experiments: Typical analysis (HERA)





$$F_2^{em} = F_2^{\gamma}$$

DIS Experiments: HERA Combined Results

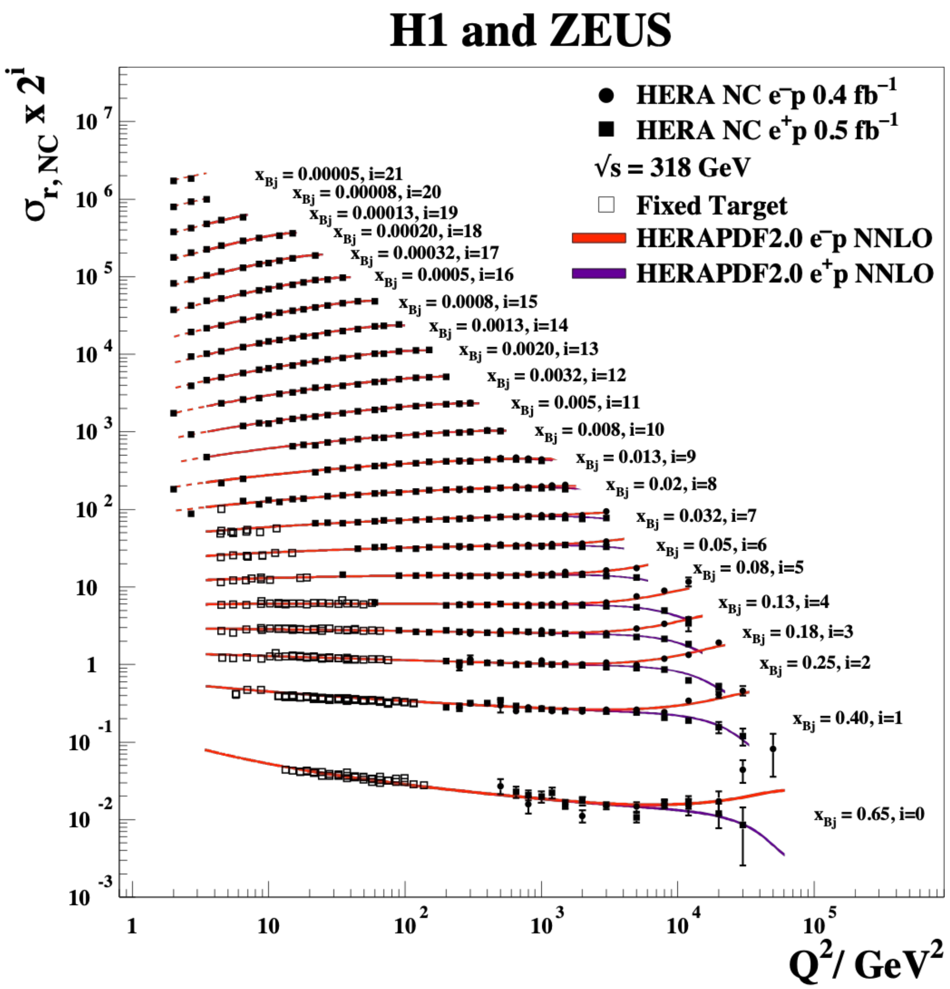
Data Set		x_{Bj} Grid from	to	Q^2[GeV^2] Grid from	to	$\mathcal{L}$ pb^{-1}	e^+/e^-	$\sqrt{s}$ GeV	x_{Bj}, Q^2 from equations	Ref.
I E_p = 820 GeV and E_p = 920 GeV data sets										
H1 svx-mb [2]	95-00	0.000005	0.02	0.2	12	2.1	e^+p	301, 319	13,17,18	[3]
H1 low Q^2 [2]	96-00	0.0002	0.1	12	150	22	e^+p	301, 319	13,17,18	[4]
H1 NC	94-97	0.0032	0.65	150	30000	35.6	e^+p	301	19	[5]
H1 CC	94-97	0.013	0.40	300	15000	35.6	e^+p	301	14	[5]
H1 NC	98-99	0.0032	0.65	150	30000	16.4	e^-p	319	19	[6]
H1 CC	98-99	0.013	0.40	300	15000	16.4	e^-p	319	14	[6]
H1 NC HY	98-99	0.0013	0.01	100	800	16.4	e^-p	319	13	[7]
H1 NC	99-00	0.0013	0.65	100	30000	65.2	e^+p	319	19	[7]
H1 CC	99-00	0.013	0.40	300	15000	65.2	e^+p	319	14	[7]
ZEUS BPC	95	0.000002	0.00006	0.11	0.65	1.65	e^+p	300	13	[11]
ZEUS BPT	97	0.0000006	0.001	0.045	0.65	3.9	e^+p	300	13, 19	[12]
ZEUS SVX	95	0.000012	0.0019	0.6	17	0.2	e^+p	300	13	[13]
ZEUS NC [2] high/low Q^2	96-97	0.00006	0.65	2.7	30000	30.0	e^+p	300	21	[14]
ZEUS CC	94-97	0.015	0.42	280	17000	47.7	e^+p	300	14	[15]
ZEUS NC	98-99	0.005	0.65	200	30000	15.9	e^-p	318	20	[16]
ZEUS CC	98-99	0.015	0.42	280	30000	16.4	e^-p	318	14	[17]
ZEUS NC	99-00	0.005	0.65	200	30000	63.2	e^+p	318	20	[18]
ZEUS CC	99-00	0.008	0.42	280	17000	60.9	e^+p	318	14	[19]
HERA II E_p = 920 GeV data sets										
H1 NC $^{1.5p}$	03-07	0.0008	0.65	60	30000	182	e^+p	319	13, 19	[8][1]
H1 CC $^{1.5p}$	03-07	0.008	0.40	300	15000	182	e^+p	319	14	[8][1]
H1 NC $^{1.5p}$	03-07	0.0008	0.65	60	50000	151.7	e^-p	319	13, 19	[8][1]
H1 CC $^{1.5p}$	03-07	0.008	0.40	300	30000	151.7	e^-p	319	14	[8][1]
H1 NC med Q^2 $^{*y.5}$	03-07	0.0000986	0.005	8.5	90	97.6	e^+p	319	13	[10]
H1 NC low Q^2 $^{*y.5}$	03-07	0.000029	0.00032	2.5	12	5.9	e^+p	319	13	[10]
ZEUS NC	06-07	0.005	0.65	200	30000	135.5	e^+p	318	13,14,20	[22]
ZEUS CC $^{1.5p}$	06-07	0.0078	0.42	280	30000	132	e^+p	318	14	[23]
ZEUS NC $^{1.5}$	05-06	0.005	0.65	200	30000	169.9	e^-p	318	20	[20]
ZEUS CC $^{1.5}$	04-06	0.015	0.65	280	30000	175	e^-p	318	14	[21]
ZEUS NC nominal ^{*y}	06-07	0.000092	0.008343	7	110	44.5	e^+p	318	13	[24]
ZEUS NC satellite ^{*y}	06-07	0.000071	0.008343	5	110	44.5	e^+p	318	13	[24]
HERA II E_p = 575 GeV data sets										
H1 NC high Q^2	07	0.00065	0.65	35	800	5.4	e^+p	252	13, 19	[9]
H1 NC low Q^2	07	0.0000279	0.0148	1.5	90	5.9	e^+p	252	13	[10]
ZEUS NC nominal	07	0.000147	0.013349	7	110	7.1	e^+p	251	13	[24]
ZEUS NC satellite	07	0.000125	0.013349	5	110	7.1	e^+p	251	13	[24]
HERA II E_p = 460 GeV data sets										
H1 NC high Q^2	07	0.00081	0.65	35	800	11.8	e^+p	225	13, 19	[9]
H1 NC low Q^2	07	0.0000348	0.0148	1.5	90	12.2	e^+p	225	13	[10]
ZEUS NC nominal	07	0.000184	0.016686	7	110	13.9	e^+p	225	13	[24]
ZEUS NC satellite	07	0.000143	0.016686	5	110	13.9	e^+p	225	13	[24]

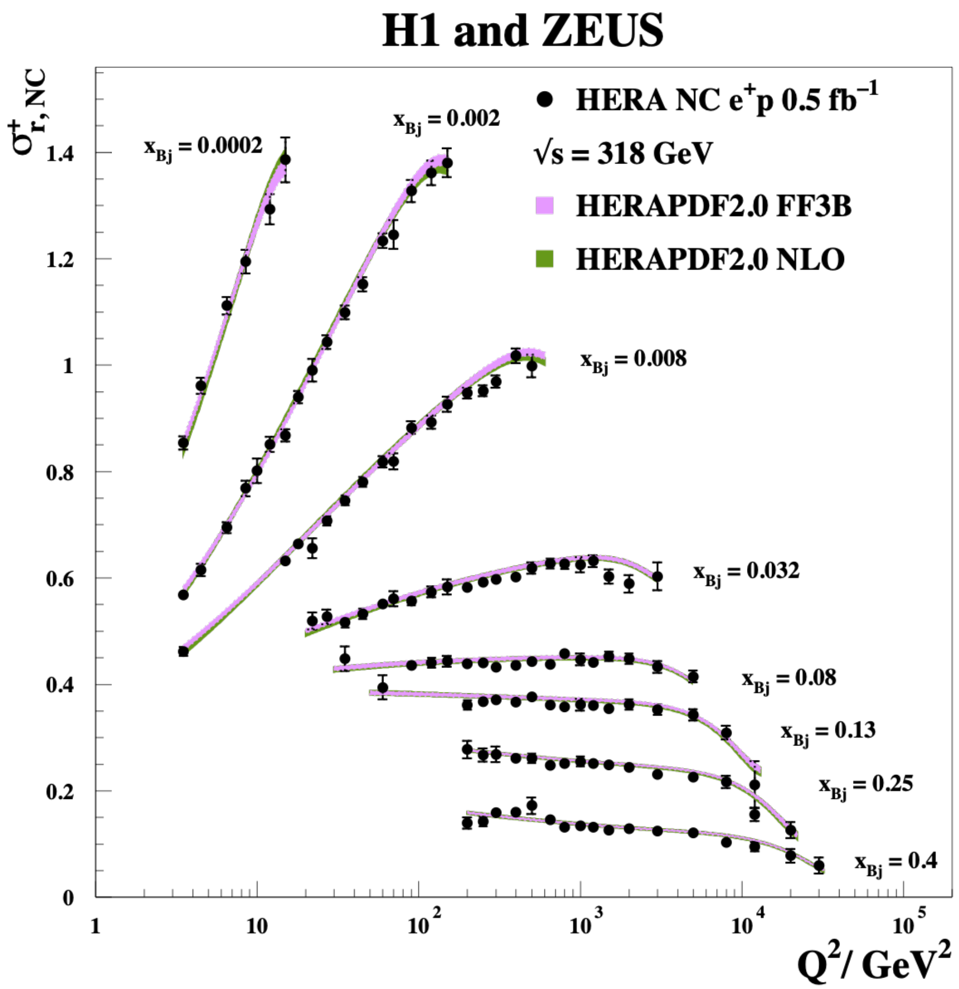
H1 & ZEUS have now published all their inclusive measurements (1992-2007)
- HERA-I
- HERA-II measurements at high-Q^2
- HERA-II measurements at reduced $\sqrt{s}$

$0.6 \times 10^{-6} < x < 0.65$, $0.045 < Q^2 < 50000$

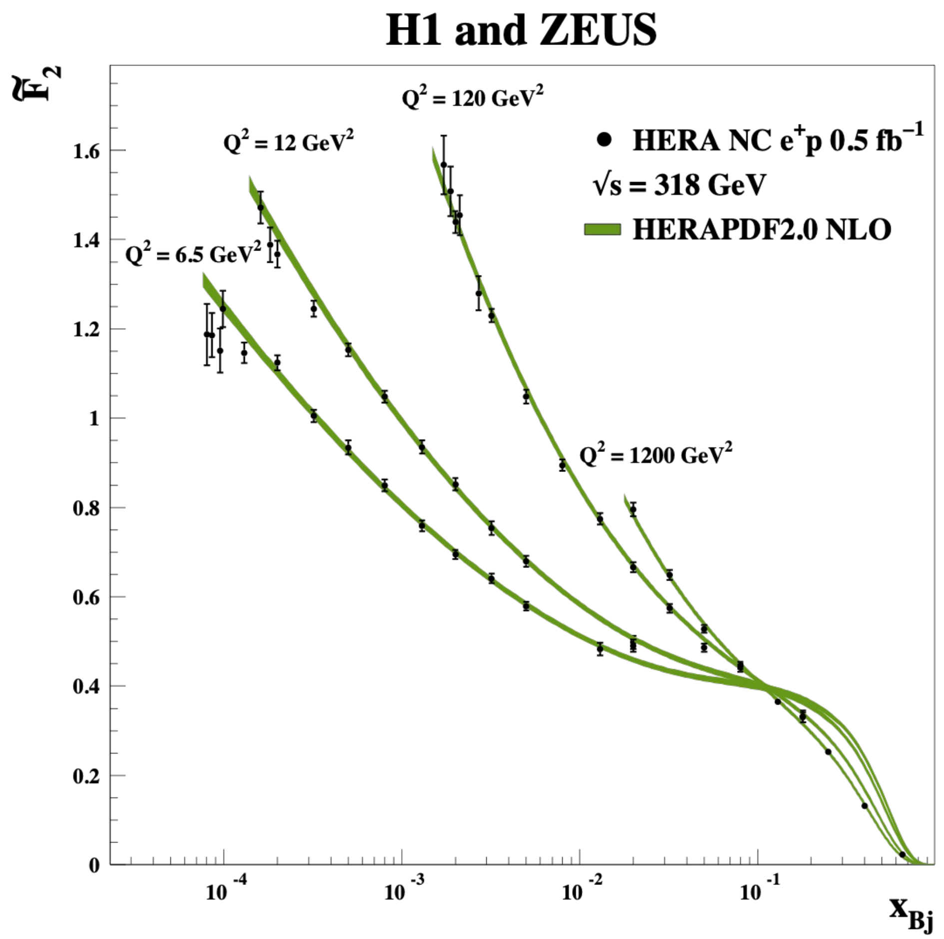
41 data sets are combined:
- NC & CC cross sections
- e^+p and e^-p scattering
- 4 different $\sqrt{s}$ (318, 301, 252 and 225 GeV)

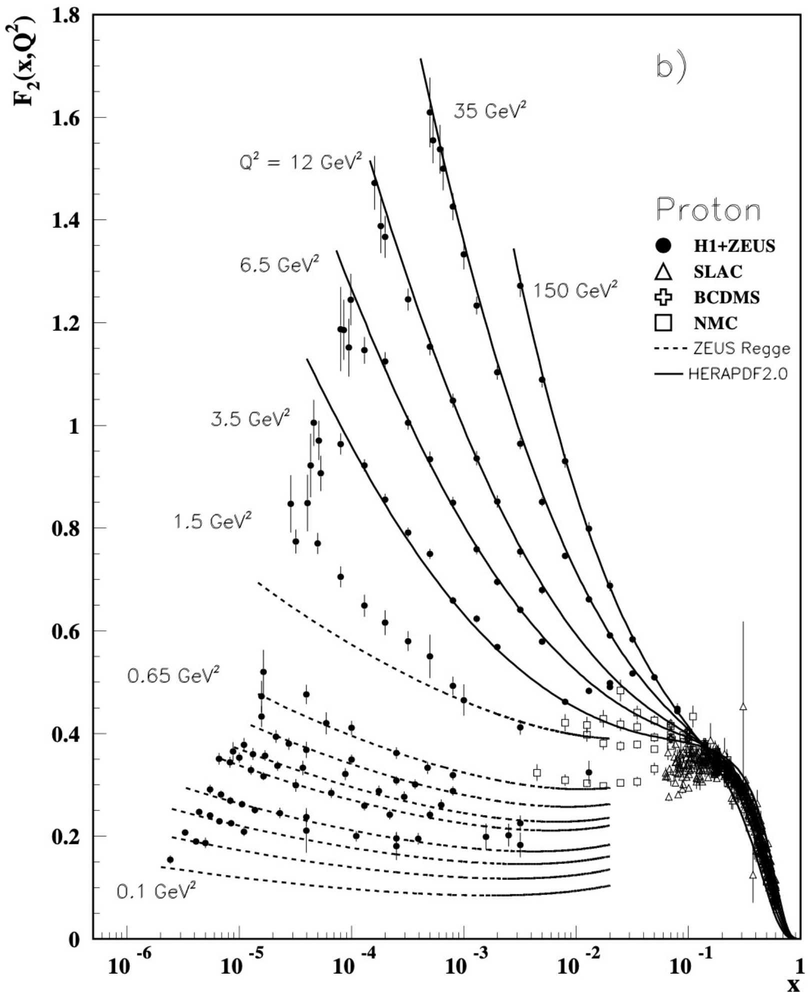
DIS Experiments: HERA Combined results



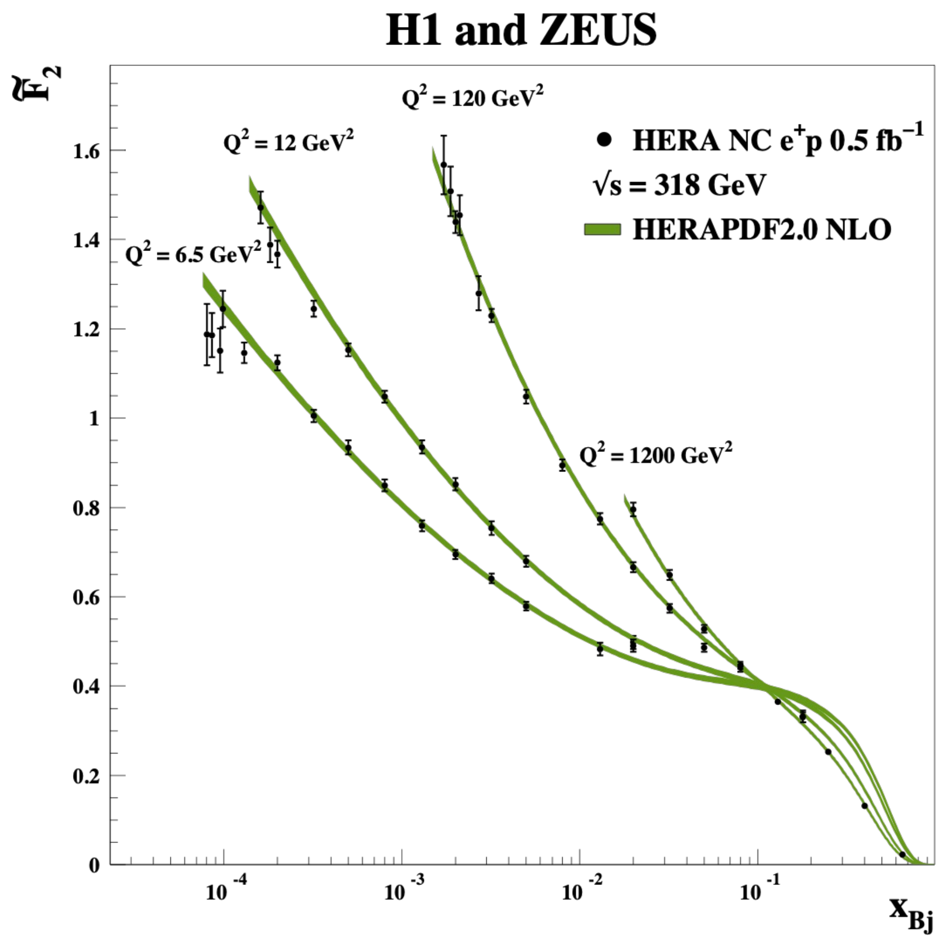


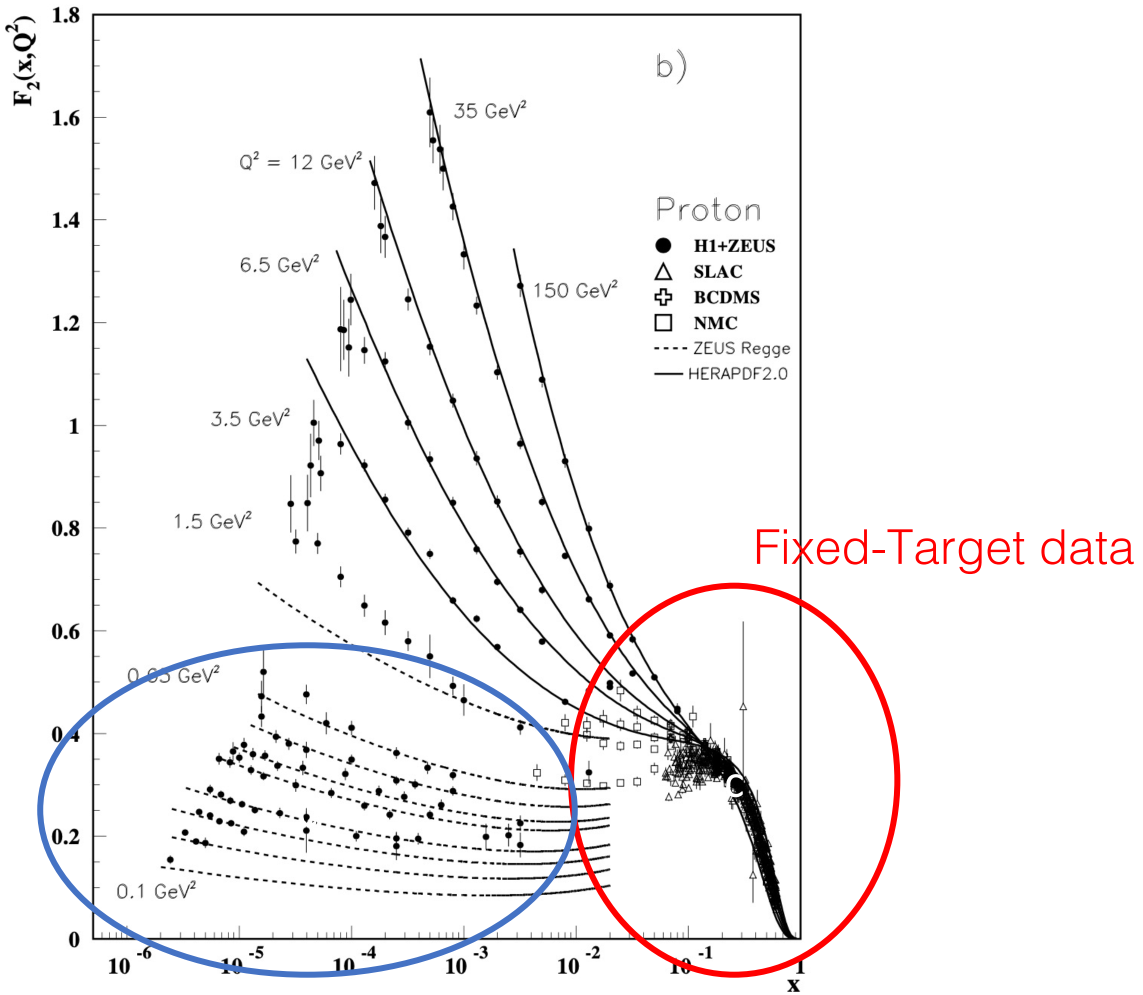
DIS Experiments: Fivex target and HERA





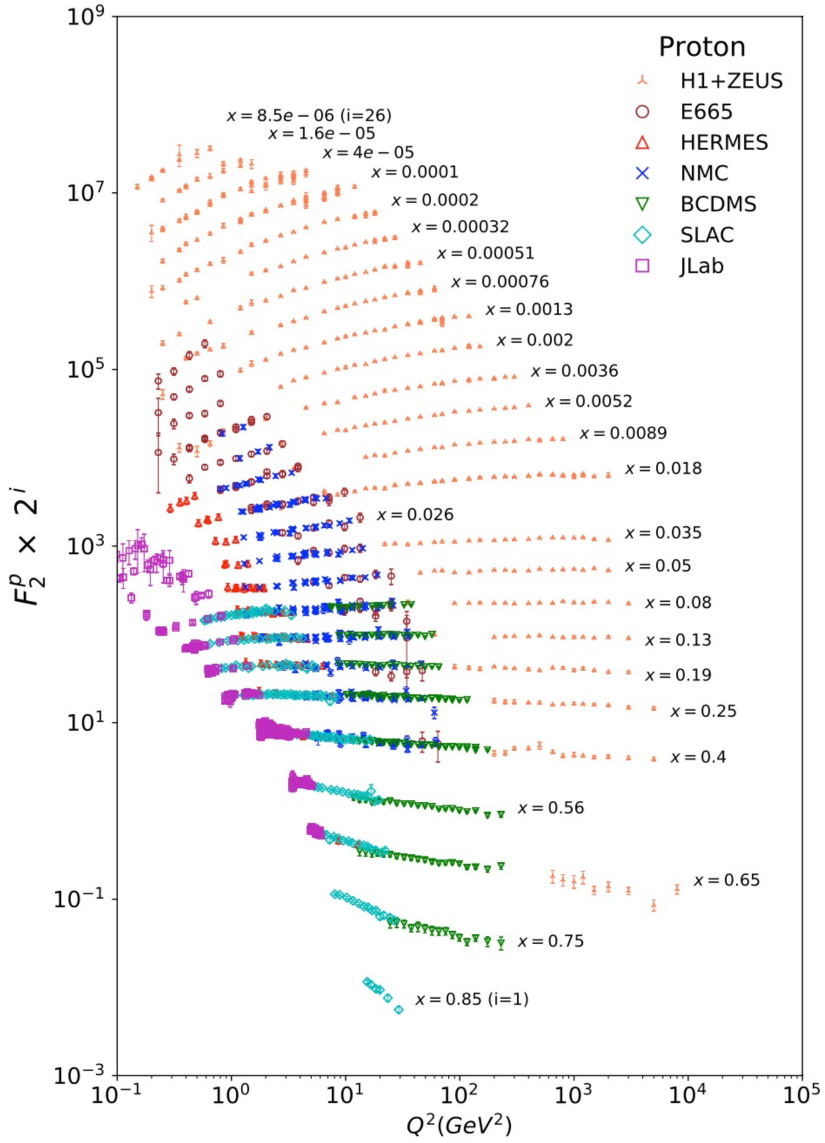
DIS Experiments: Fivex target and HERA





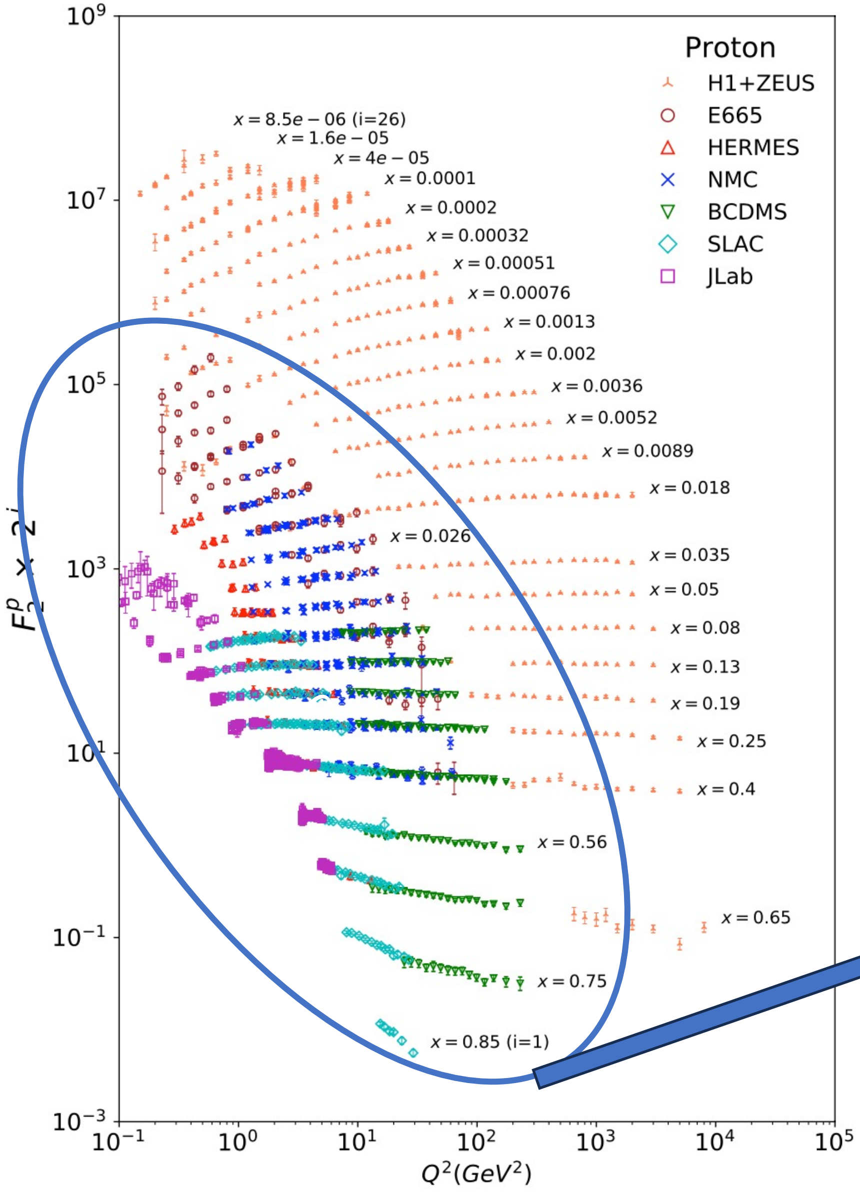
Very low-x and Q2 HERA data

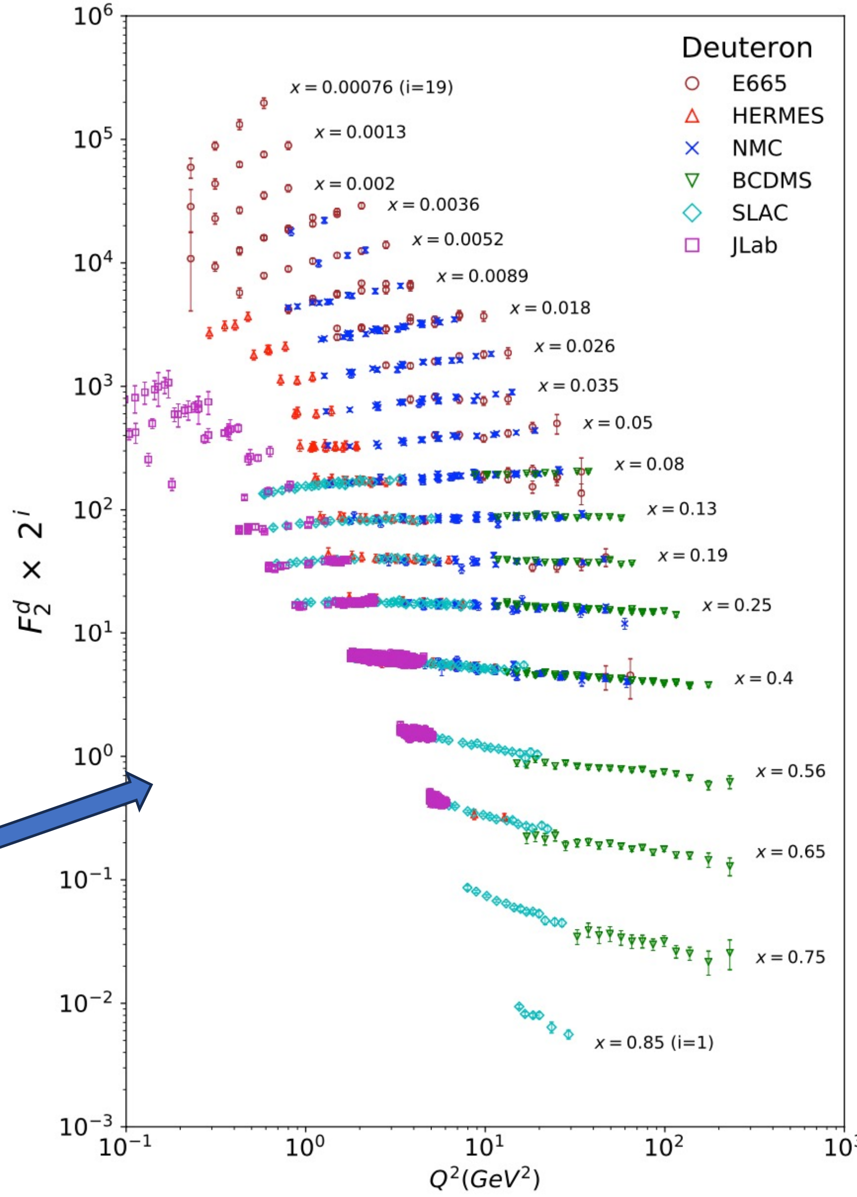
DIS Experiments: Fixed Target and HERA



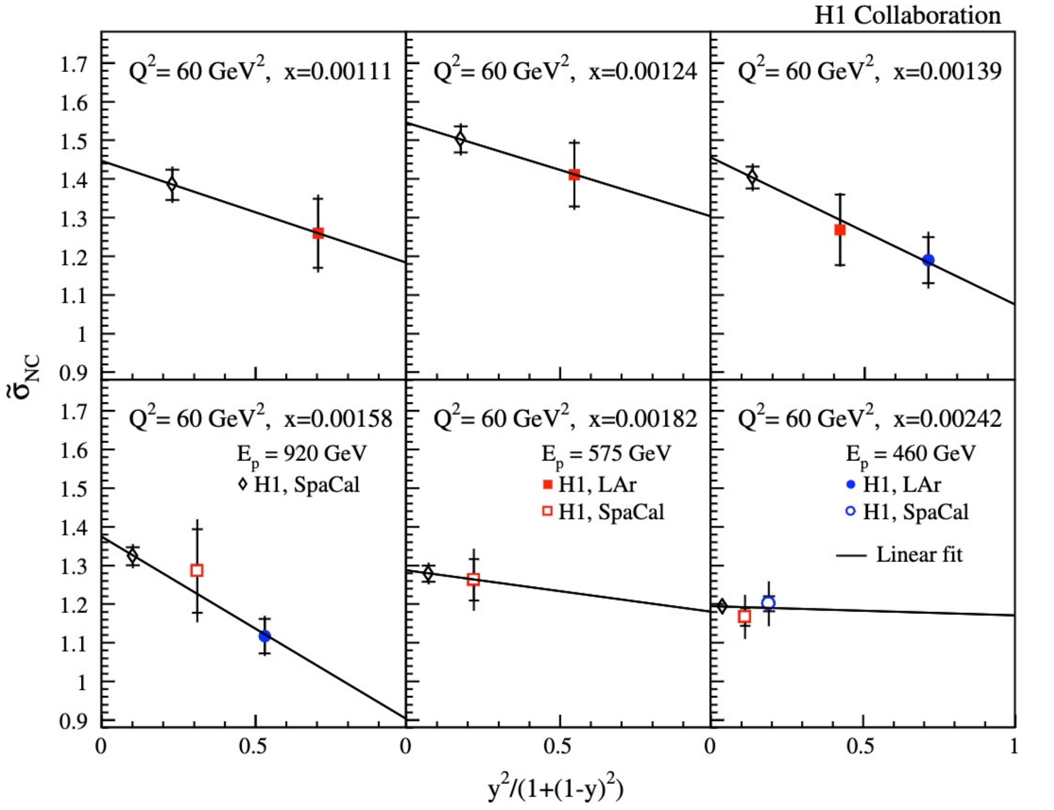
More on Fixed Target exps in Lecture 2

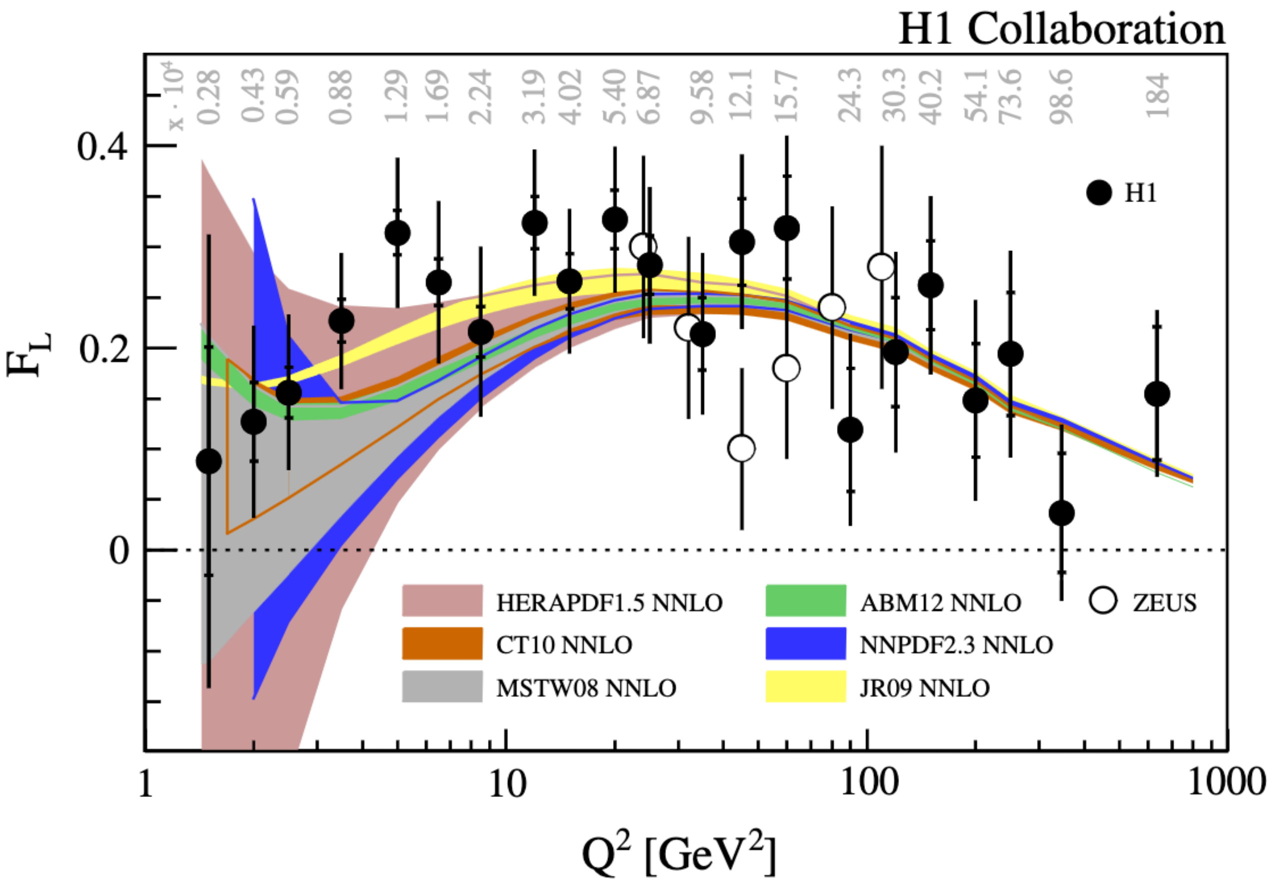
DIS Experiments: Fixed Target and HERA





DIS Experiments: Long. SF F_L (H1)





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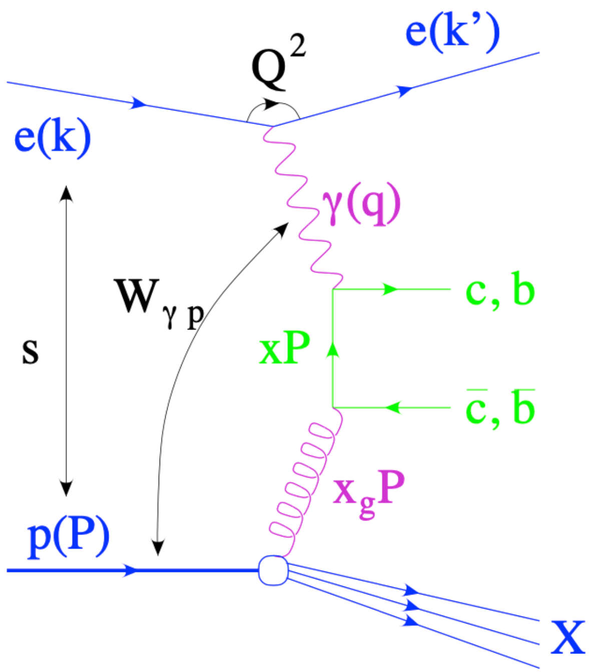
DIS Experiments: Heavy quarks (c and b)

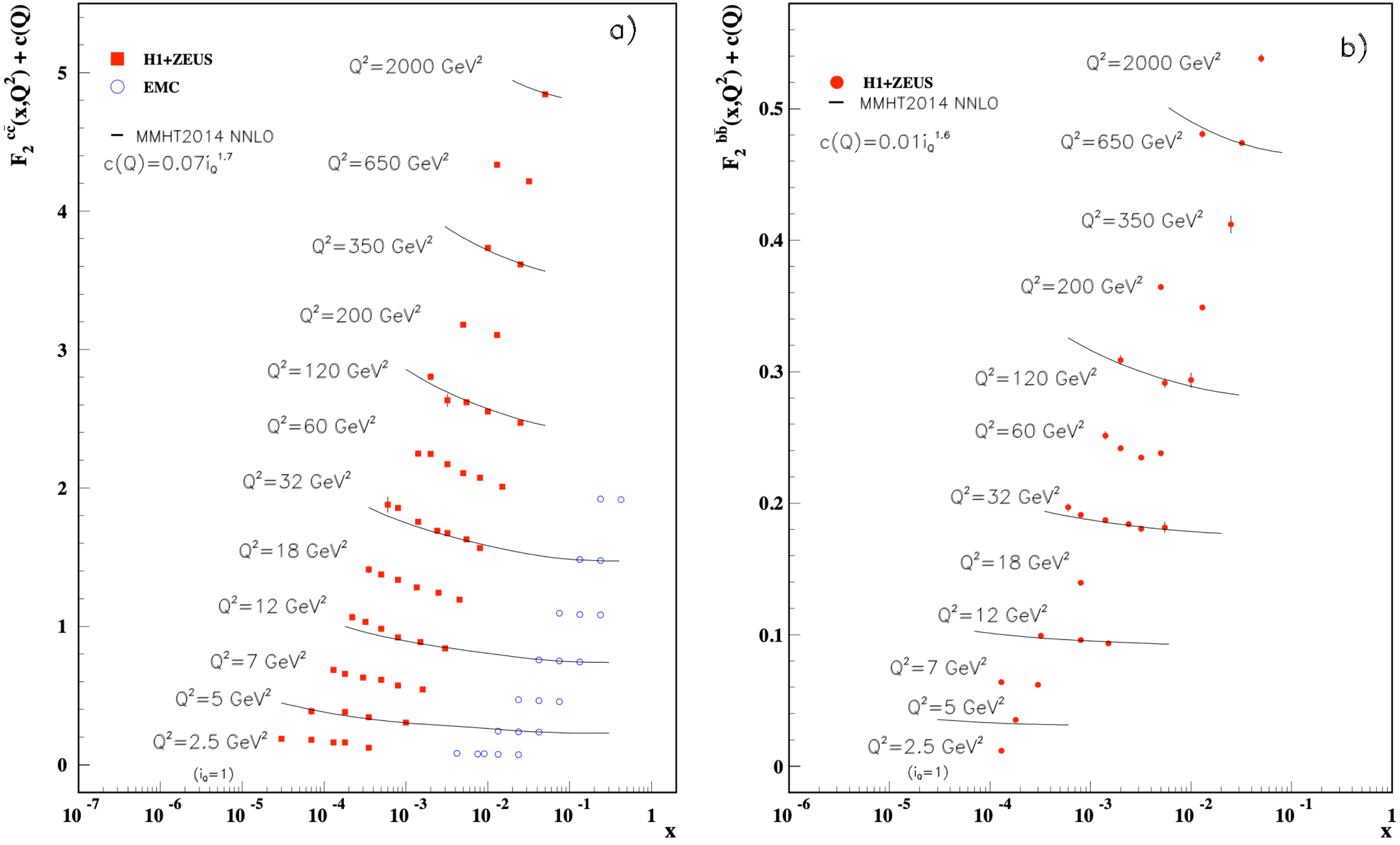


$$\frac{d\sigma^{Q\bar{Q}}(e^{\pm}p)}{dx\,dQ^2} = \frac{2\pi\alpha^2}{x\,Q^4}\left(\left(1+(1-y)^2\right)F_2^{Q\bar{Q}} - y^2\,F_L^{Q\bar{Q}} \mp x\left(1-(1-y)^2\right)F_3^{Q\bar{Q}}\right)$$

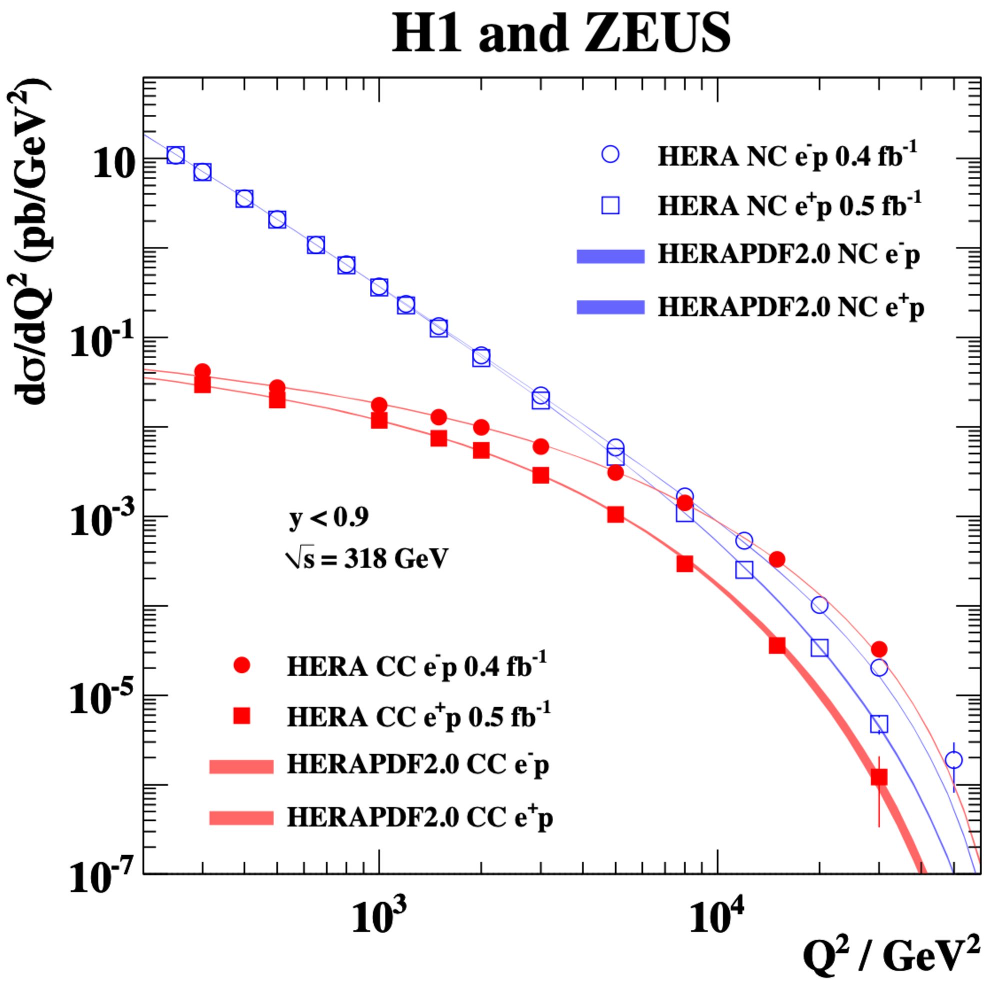
$$\sigma_{red}^{Q\bar{Q}}(x,Q^2) = \frac{d\sigma^{Q\bar{Q}}(e^{\pm}p)}{dx\,dQ^2}\cdot\frac{x\,Q^4}{2\pi\alpha^2 Y_+} = F_2^{Q\bar{Q}} - \frac{y^2}{Y_+}\,F_L^{Q\bar{Q}},$$

Important for testing FFNS and VFNS….

DIS Experiments: Heavy quarks (c and b)

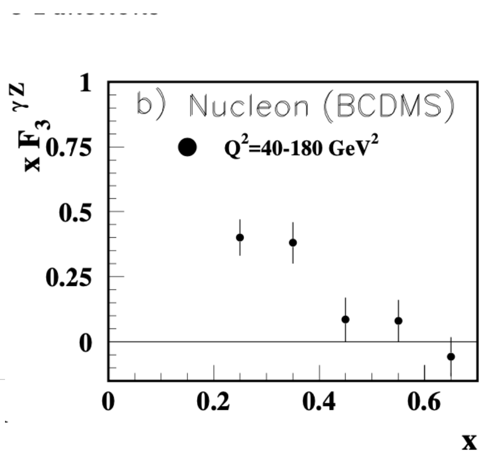


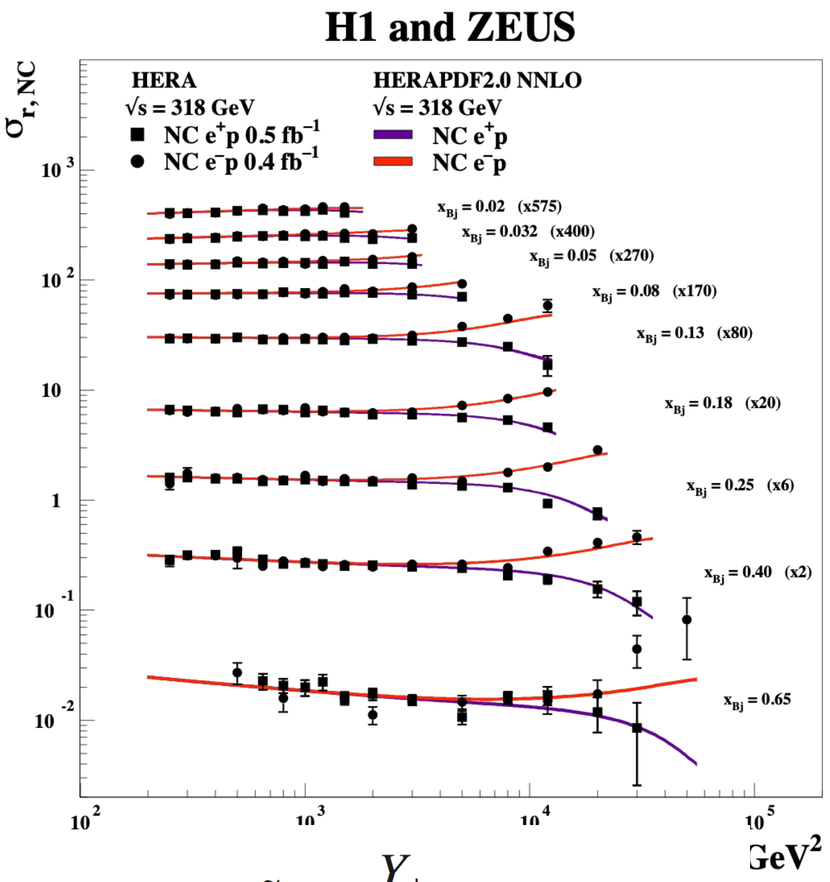
DIS Experiments: EW unification



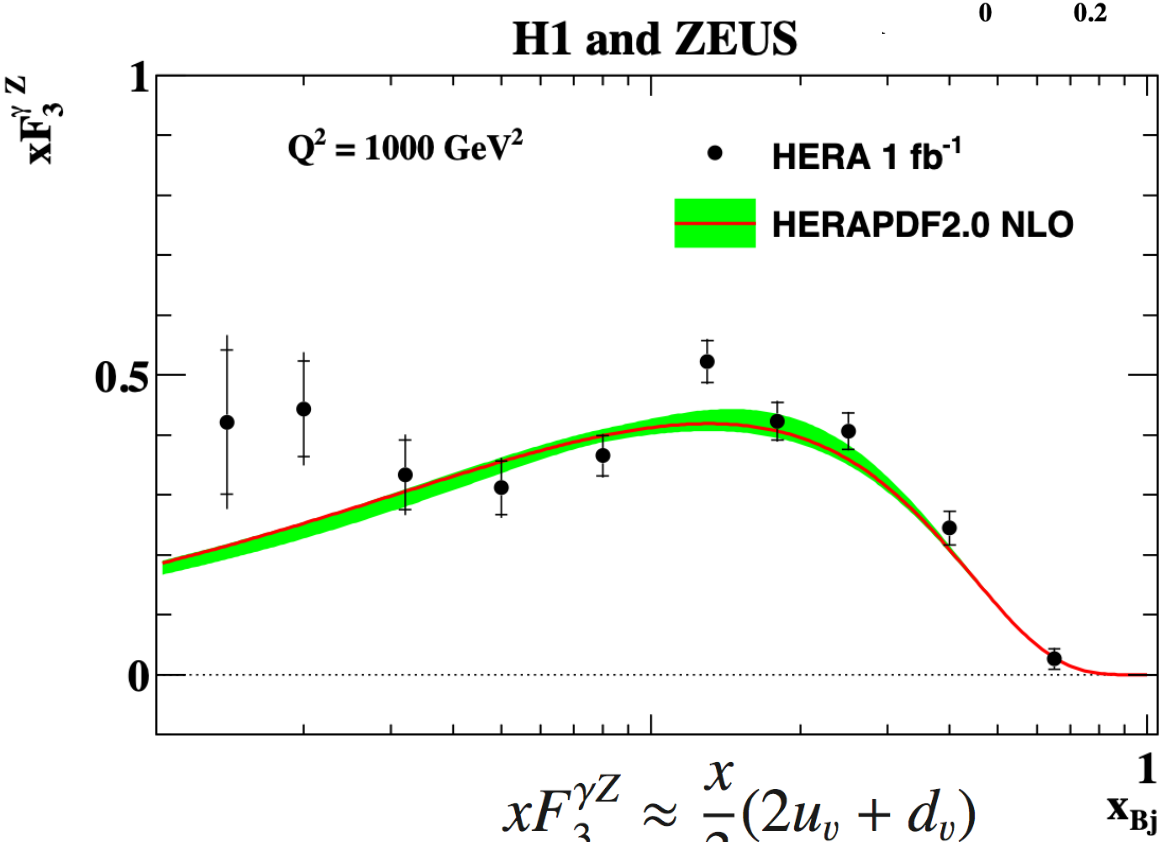
A beautiful textbook plot that shows the unification of the NC and CC interactions at the EW scale

DIS Experiments: EW effects





$$x\tilde{F}_3 = \frac{Y_+}{2Y_-}(\sigma^-_{r,\mathrm{NC}} - \sigma^+_{r,\mathrm{NC}})$$



$$xF_3^{\gamma Z} \approx \frac{x}{3}(2u_v + d_v)$$

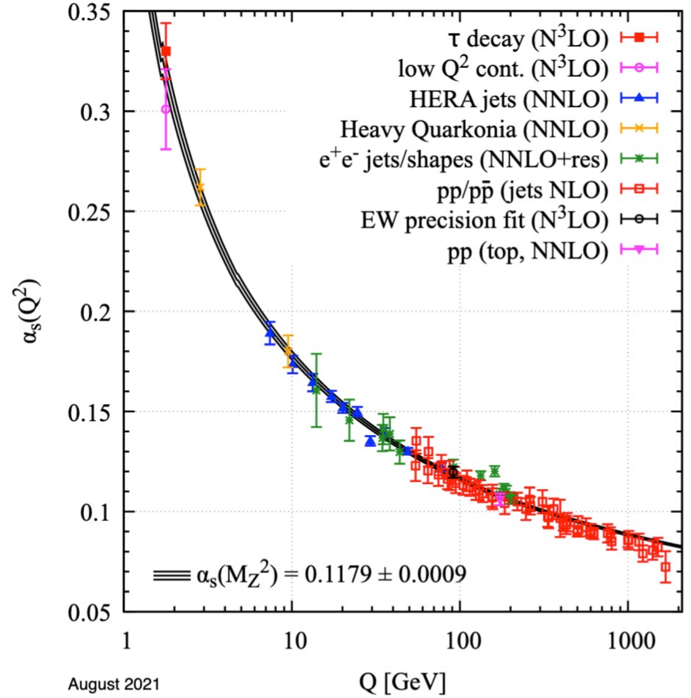
QCD

$$\mathcal{L} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavours}} \bar{q}_a (i\not{D} - m)_{ab} q_b + \mathcal{L}_{\text{gauge-fixing}}$$

$$F^A_{\alpha\beta} = \partial_\alpha \mathcal{A}^A_\beta - \partial_\beta \mathcal{A}^A_\alpha - g f^{ABC} \mathcal{A}^B_\alpha \mathcal{A}^C_\beta$$

$$(D_\alpha)_{ab} = \partial_\alpha \delta_{ab} + ig \left(t^C \mathcal{A}^C_\alpha \right)_{ab}$$

$$\frac{\partial \alpha_S(Q)}{\partial \tau} = \beta(\alpha_S(Q)) \quad \tau = \ln\left(\frac{Q^2}{\mu^2}\right)$$



QCD Collinear Factorization:

$$d\sigma(ep \to eX)(x, Q^2) = \sum_q [d\hat{\sigma}_{eq}(Q^2, \mu) \otimes f_q(Q^2, \mu)](x)$$

$$\sigma \otimes f = \int_x^1 \frac{dy}{y} \sigma(y) f\left(\frac{x}{y}\right)$$

QCD Factorization and Structure functions

$$\mathcal{F}_a^k(x,Q^2) = \sum_i C_{a,i}^k(a_s, \frac{Q^2}{\mu_F^2}, m_c, m_b, \frac{x}{z}) \otimes f_i(a_s, z, \mu_F^2)$$

with $a = 2, L, 3$ and $k = NC, CC$ and $i = q, \bar{q}, g$

Coefficient functions

$$C_{a,i}^k = C_{a,i}^{k\ (0)} + \frac{\alpha_s}{4\pi} C_{a,i}^{k\ (1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{a,i}^{k\ (2)} + \dots \text{ with } a = 2, 3, L \text{ and } k = NC, CC$$

QCD Factorization and Structure functions

$$\mathcal{F}_a^k(x, Q^2) = \sum_i C_{a,i}^k(a_s, \frac{Q^2}{\mu_F^2}, m_c, m_b, \frac{x}{z}) \otimes f_i(a_s, z, \mu_F^2)$$

$$\text{with } a = 2, L, 3 \text{ and } k = NC, CC \text{ and } i = q, \bar{q}, g$$

Parton distribution functions, DGLAP evolution equations and Splitting functions

$$\frac{dq_i(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q_i(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{qg}\left(\frac{x}{y}\right) \right]$$

$$\frac{dg(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[\sum_i q_i(y, Q^2) P_{gq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{gg}\left(\frac{x}{y}\right) \right]$$

QCD Factorization and Structure functions

$$\mathcal{F}_a^k(x,Q^2) = \sum_i C_{a,i}^k(a_s, \frac{Q^2}{\mu_F^2}, m_c, m_b, \frac{x}{z}) \otimes f_i(a_s, z, \mu_F^2)$$

$$\text{with } a = 2, L, 3 \text{ and } k = NC, CC \text{ and } i = q, \bar{q}, g$$

Parton distribution functions, DGLAP evolution equations and Splitting functions

$$P_{\rm qq}^{(0)}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$P_{\rm qg}^{(0)}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

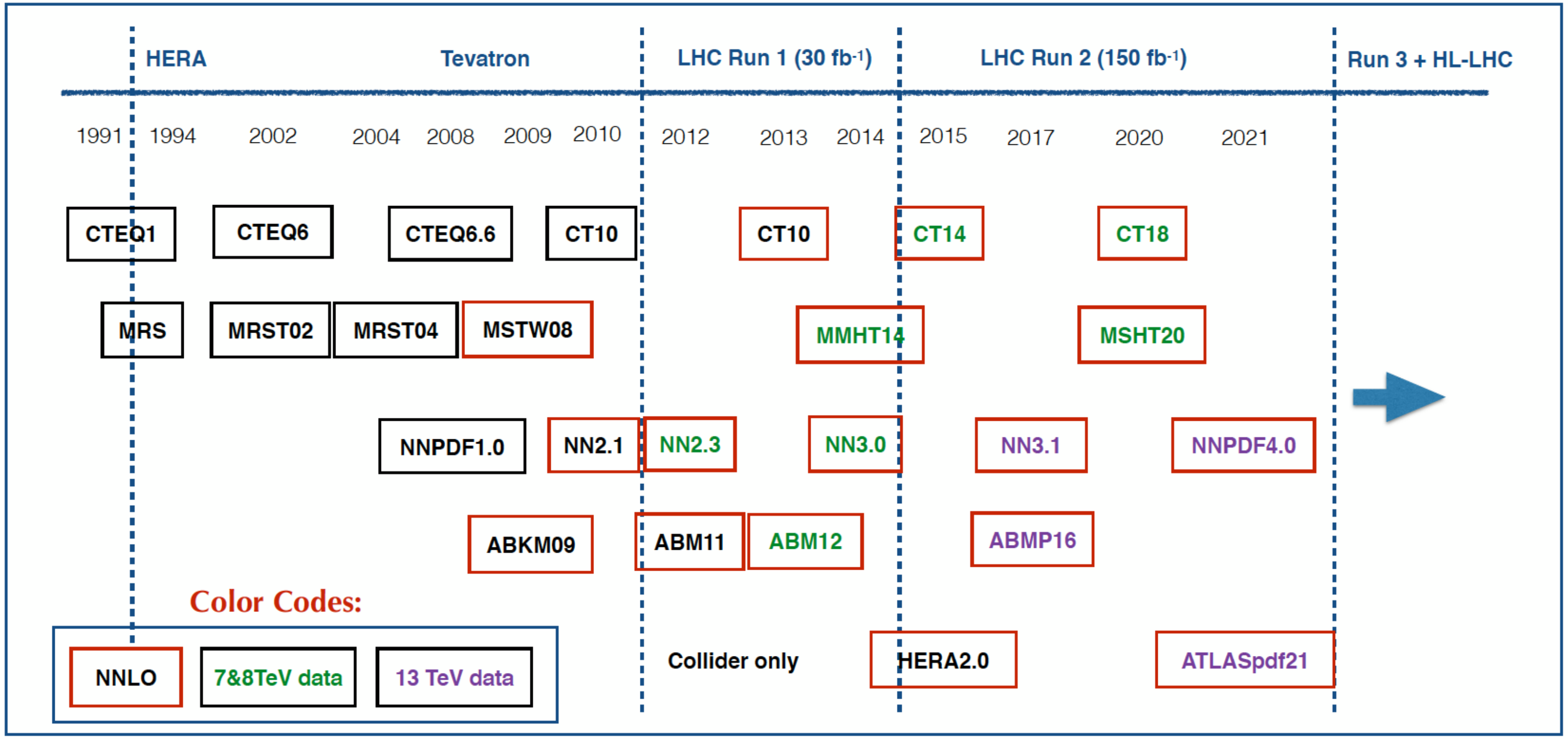
$$P_{\rm gq}^{(0)}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$

$$P_{\rm gg}^{(0)}(z) = 6\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-z)\right]$$

$$\int_0^1 dx f(x)[F(x)]_+ = \int_0^1 dx\, (f(x) - f(1))F(x)$$

Groups/Collaborations active on PDFs

Many groups active since the mid 80’s in the DGLAP analyses. The “modern era” of PDF fitting started at the beginning of the 90s (CTEQ1, MRS).



QCD DGLAP analysis

Parameterisation at the starting scale:

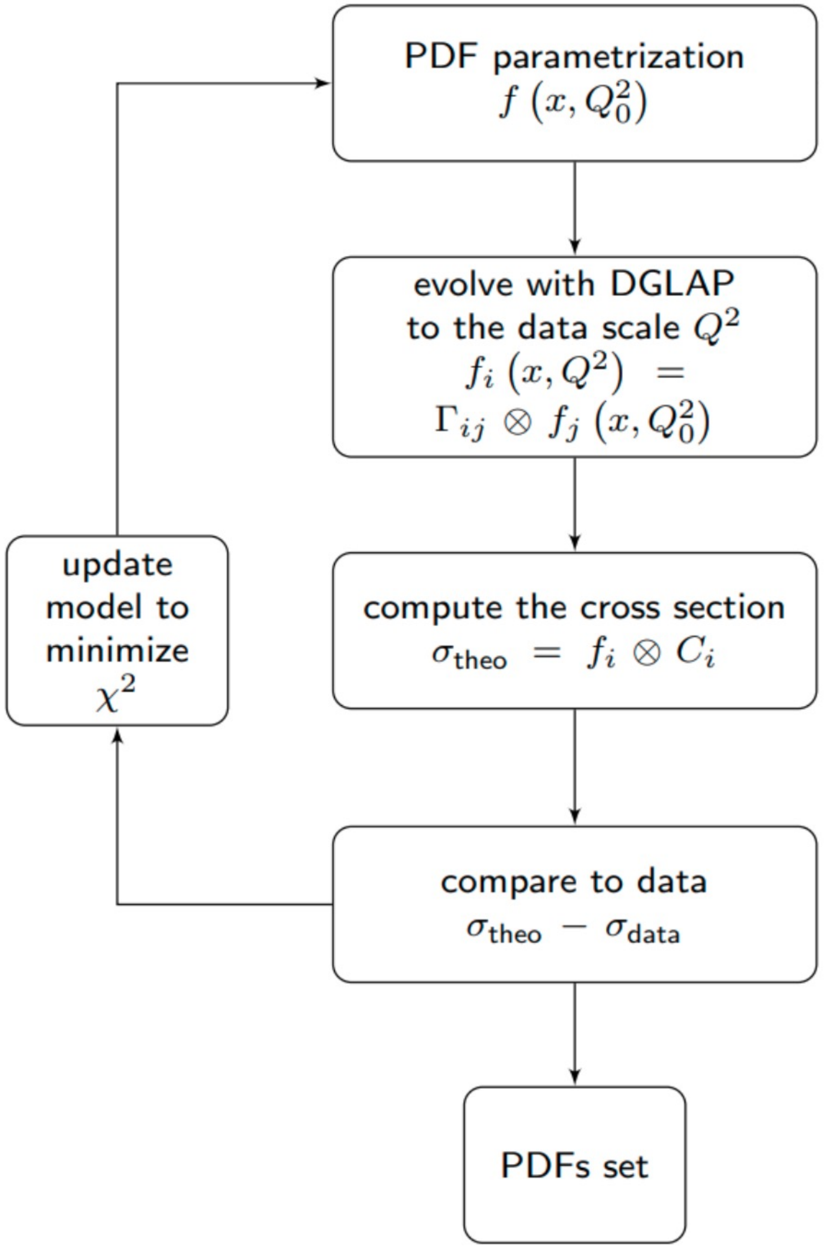
$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

Parforme numerical evolution of PDFs and compute theoretical observables:

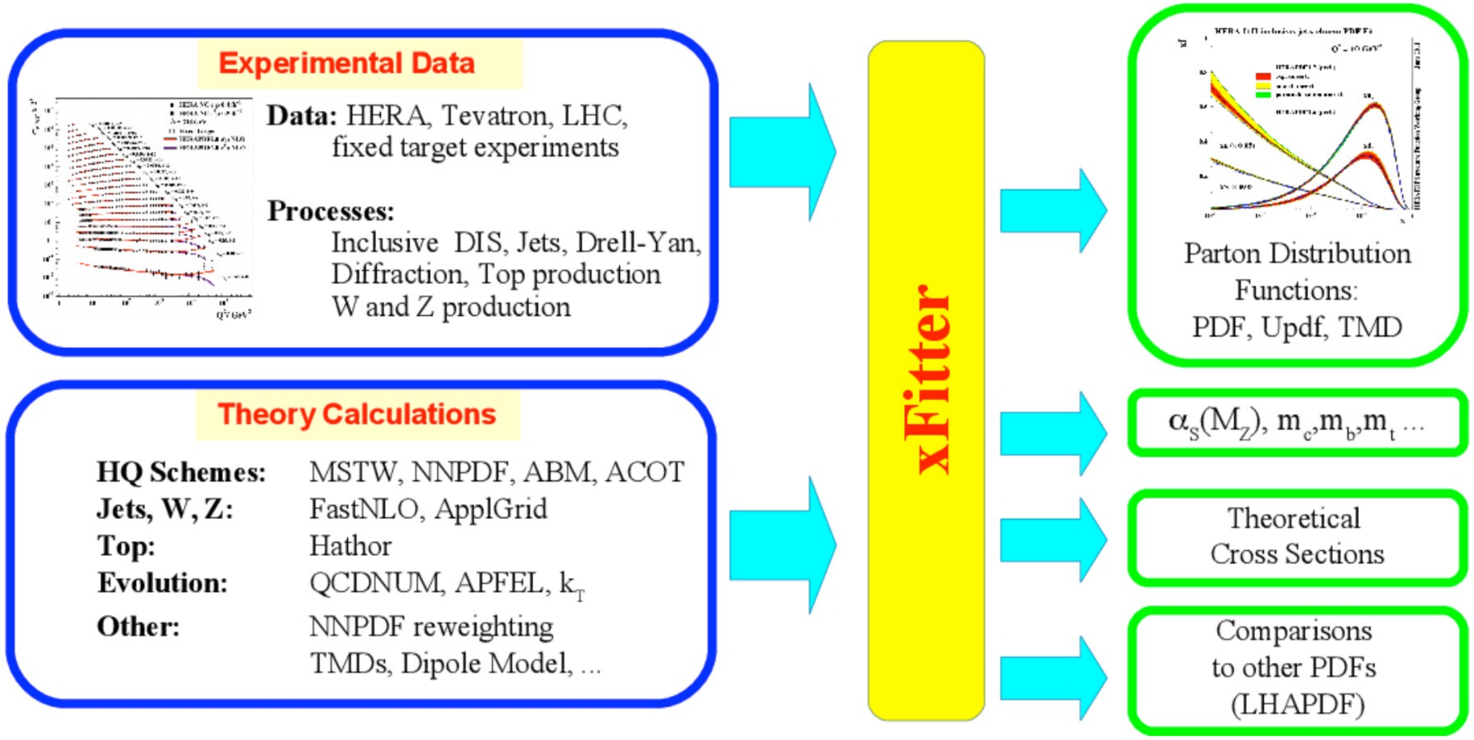
$$\sigma_{theo} = C_i \otimes f_i = C_i \otimes \Gamma_{i,j} \otimes f_i(Q_0^2)$$

Determine the PDFs parameters (and their uncertainties) at the starting scale via a chi2 minimization procedure:

$$\chi^2_{\text{exp}}(\boldsymbol{m}, \boldsymbol{s}) = \sum_i \frac{\left[m^i - \sum_j \gamma^i_j m^i s_j - \mu^i\right]^2}{\delta^2_{i,\text{stat}} \mu^i m^i + \delta^2_{i,\text{uncor}} (m^i)^2} + \sum_j s_j^2$$



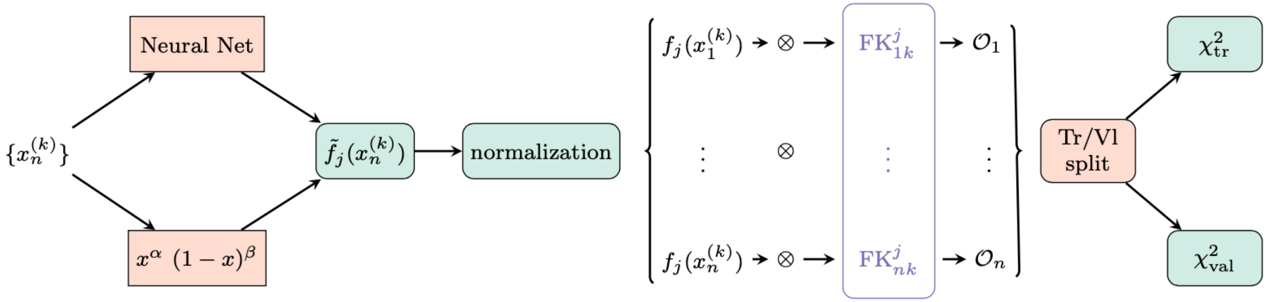
xFitter

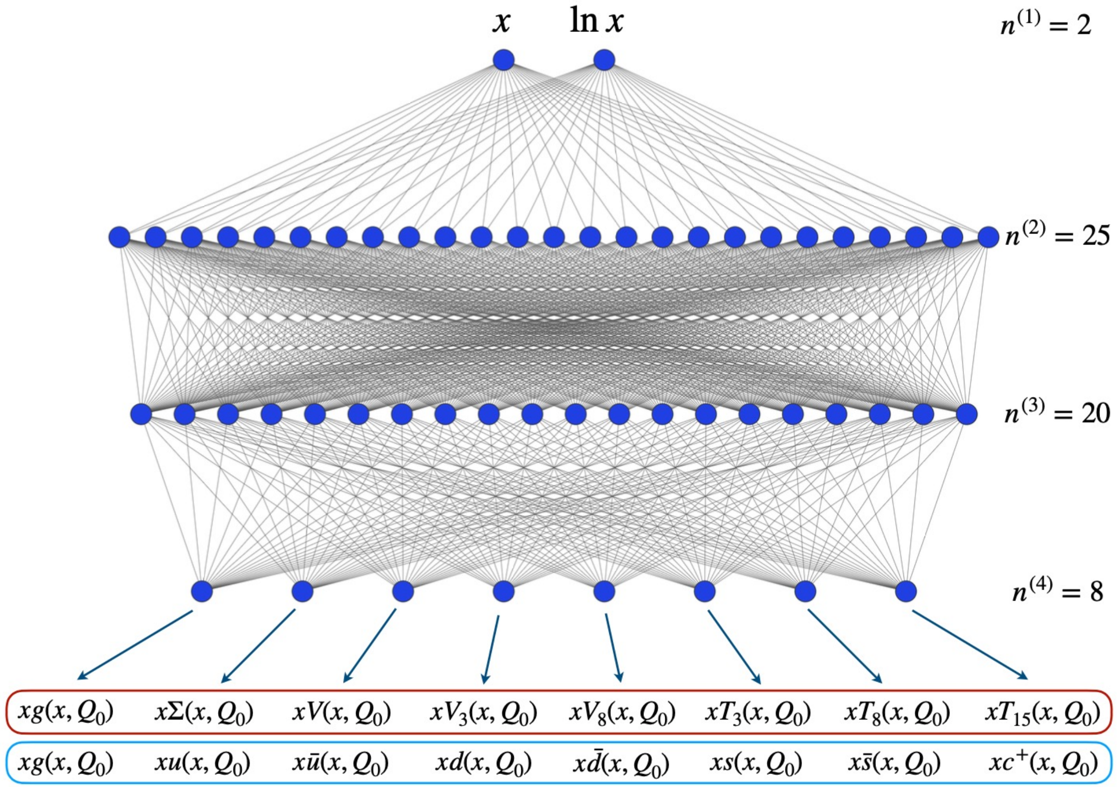


More on xFitter in Lecture 3 ("hands-on" session)

NNPDF4.0

$$xf_k(x, Q_0; \boldsymbol{\theta}) = A_k\, x^{1-\alpha_k}(1-x)^{\beta_k}\mathrm{NN}_k(x; \boldsymbol{\theta})\,, \quad k = 1, \ldots, 8\,,$$





More on NNPDF in Lecture 2 and Lecture 3 (“hands-on” session)

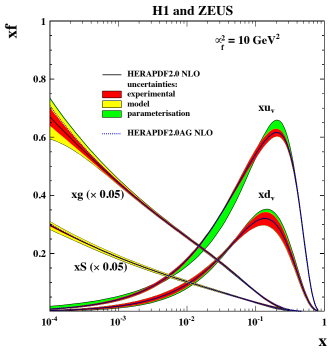
HERAPDF2.0: Settings DGLAP analysis

- HERA combined data set

- Final analytical form (14 params) obtained via parameter scan:

$$
\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g}, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1 + E_{u_v} x^2\right), \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\left(1 + D_{\bar{U}} x\right), \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}
$$

- Heavy Flavours: Roberts-Thorne VFNS (RTOPT) and two FF Schemes
- Fits performed at LO, NLO and NNLO and for Q_{2min}=3.5 and 10 GeV^2
- Detailed study of PDFs uncertainties: experimental, model and parameterisation



HERAPDF2.0: Uncertainties

Three different uncertainty components are considered:

Experimental uncertainties

Consistent data sets ➔ use Hessian Method with $\Delta\chi^2 = 1$
(Cross checked with a MC method based on pseudo data sets (replicas))

Model uncertainties

The following variations were considered:

Variation	Standard Value	Lower Limit	Upper Limit
$Q^2_{\rm min}$ [GeV^2]	3.5	2.5	5.0
$Q^2_{\rm min}$ [GeV^2] HiQ2	10.0	7.5	12.5
M_c(NLO) [GeV]	1.47	1.41	1.53
M_c (NNLO) [GeV]	1.43	1.37	1.49
M_b [GeV]	4.5	4.25	4.75
f_s	0.4	0.3	0.5

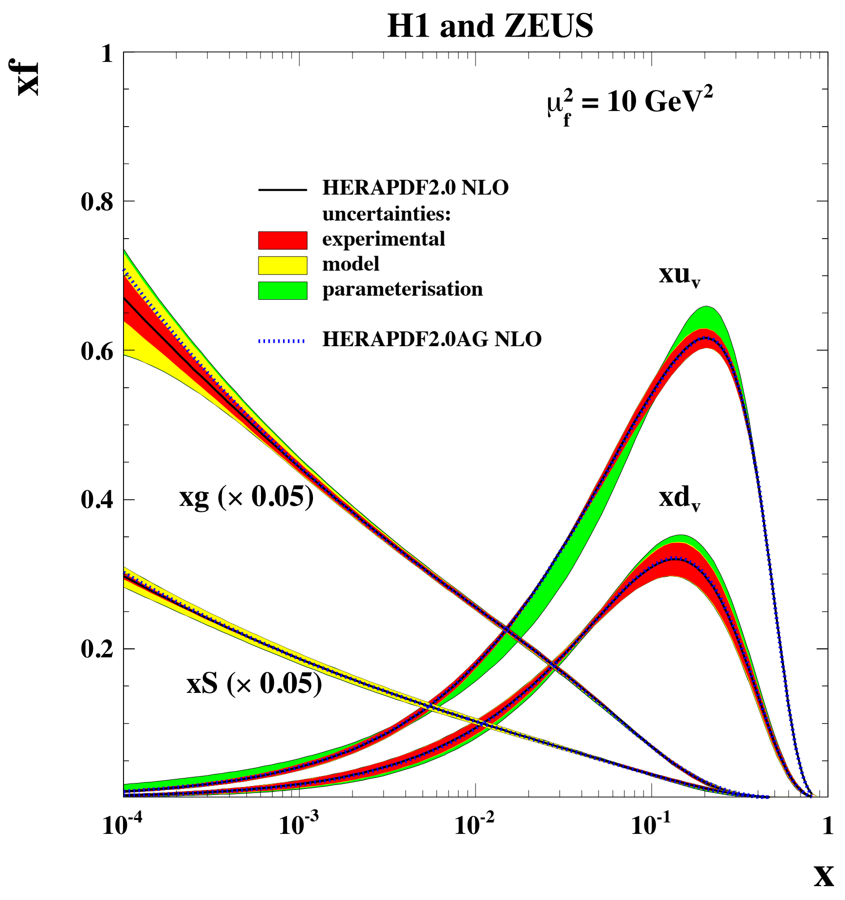
Parameterisation uncertainties

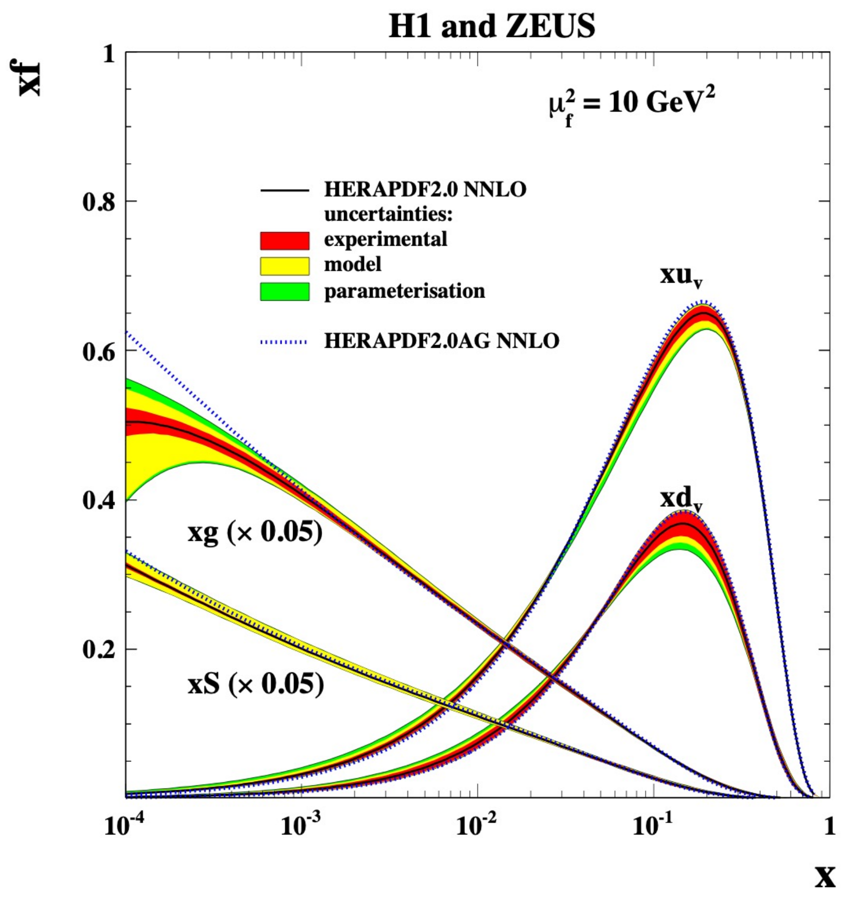
- Addition of the parameters D and E in the parameterisation formula:

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

- Starting scale Q_0^2

HERAPDF2.0: NLO and NNLO PDFs





To be continued tomorrow morning… (Lecture 2)

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