



Istituto Nazionale di Fisica Nucleare



HAS QCD
HADRONIC STRUCTURE AND
QUANTUM CHROMODYNAMICS



**UNIVERSITÀ
DI PAVIA**



Cooking show: How to extract a TMD from a global fit

Matteo Cerutti — EIC_Net Summer School — Corigliano Calabro - June 20th, 2023

Rules of the cooking show

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- A cooking show is NOT a lecture

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- I am going to USE formulas already discussed

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*DON'T
GIVE UP!!*

Rules of the cooking show

- A cooking show is NOT a lecture
- I am going to USE formulas already discussed
- I am going to discuss WITH YOU such formulas
- The aim of this cooking show is to discuss TOGETHER how to solve a “simple” exercise

*DON'T
GIVE UP!!*





Table of twist-2 TMDs

(A. Bacchetta lectures)

Quark Polarization

Nucleon Pol.

| | U | L | T |
|---|---|---|---|
| U | | | |
| L | | | |
| T | | | |

Table of twist-2 TMDs

(A. Bacchetta lectures)

Quark Polarization

Nucleon Pol.

| | U | L | T |
|---|-------|---|---|
| U | f_1 | | |
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| T | | | |

Table of twist-2 TMDs

(A. Bacchetta lectures)

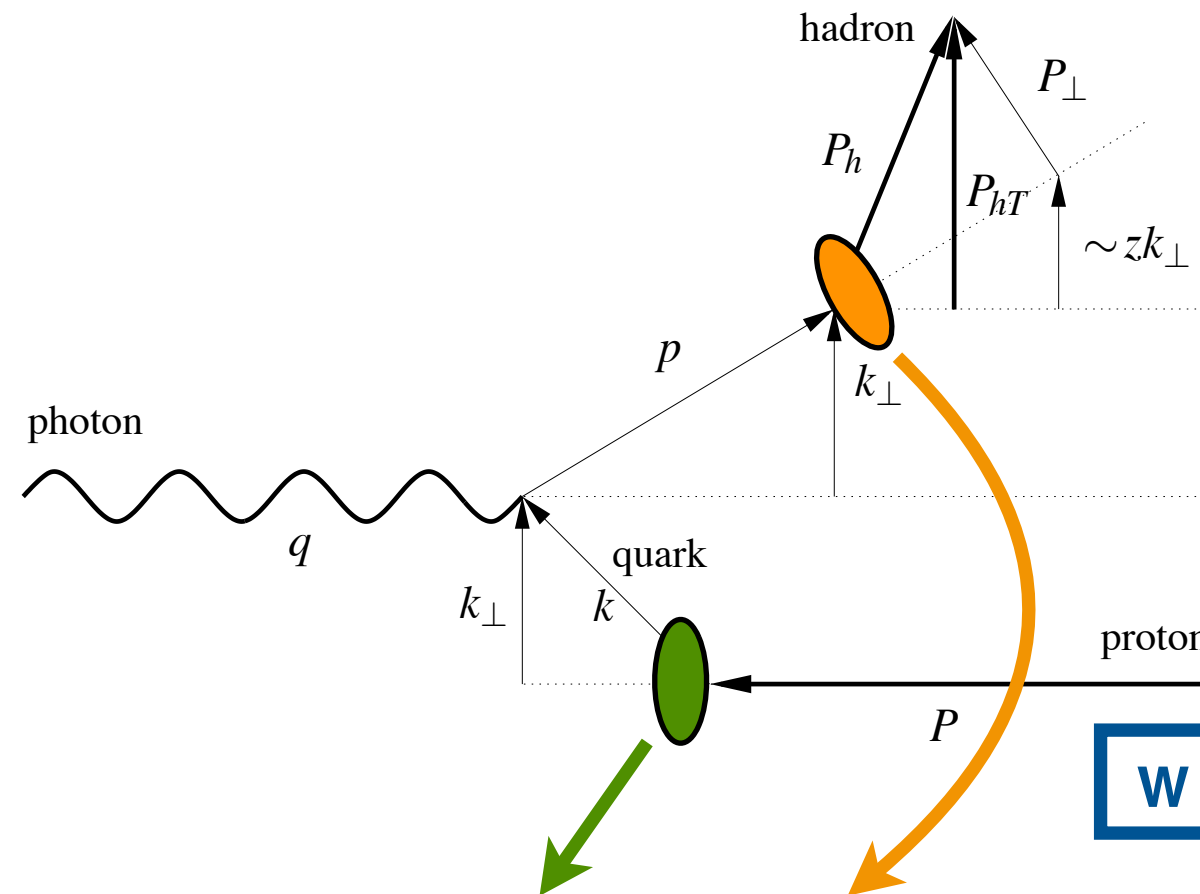
Quark Polarization

Nucleon Pol.

| | U | L | T |
|---|----------------|----------|--------------------------|
| U | f_1 | | h_1^\perp |
| L | | g_1 | h_{1L}^\perp |
| T | f_{1T}^\perp | g_{1T} | $h_1 \quad h_{1T}^\perp$ |

TMD Factorization - SIDIS

(A. Bacchetta lectures)



$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2)$$

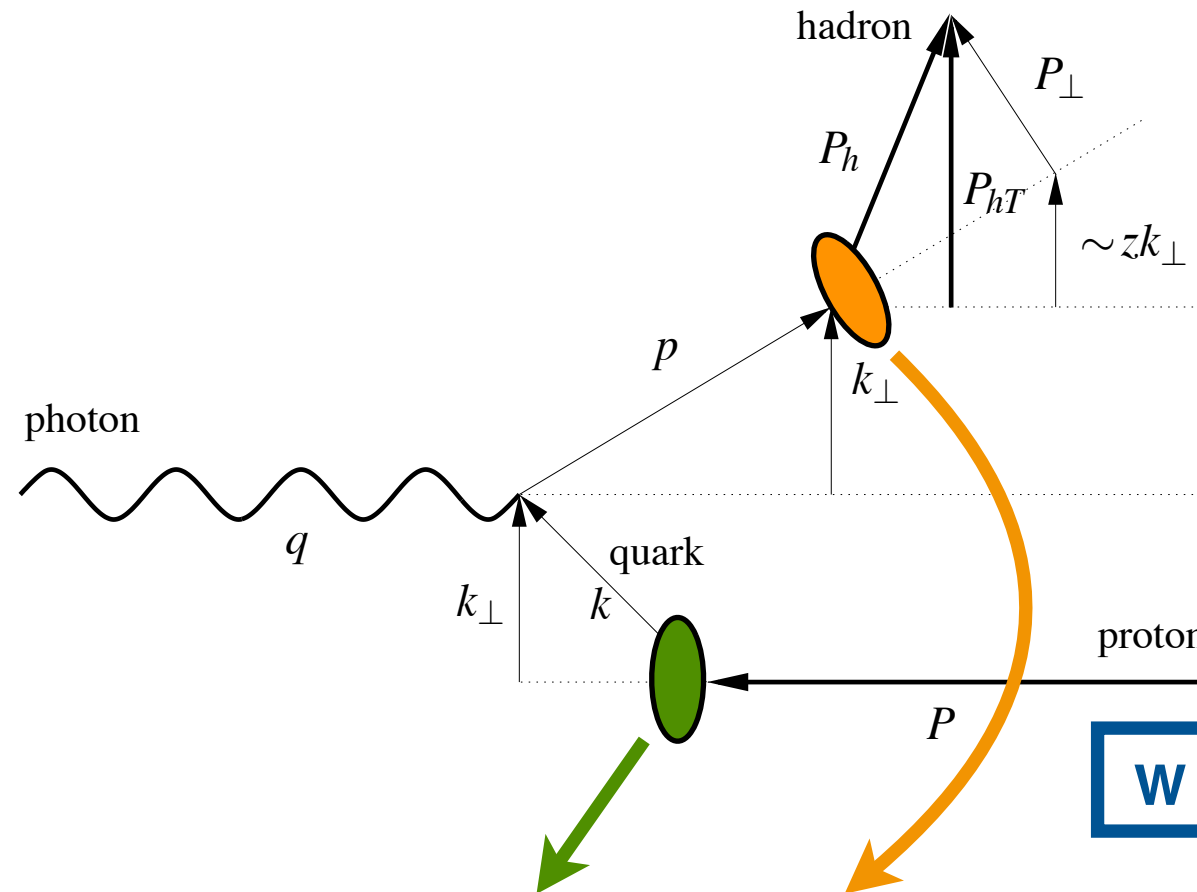
$$= x \sum_a \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

$$= x \sum_a \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_\perp^2; \mu^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

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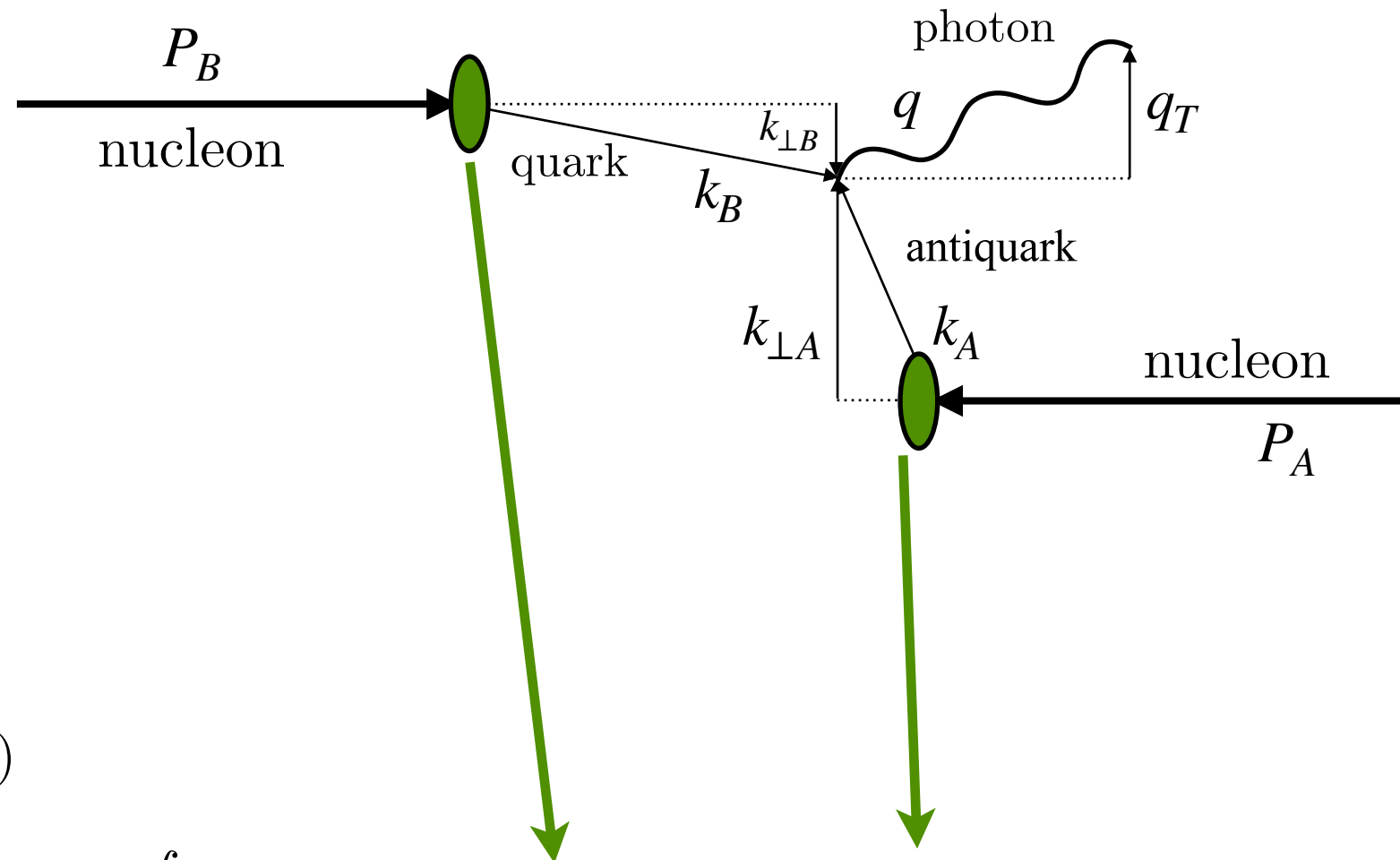
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Bacchetta, Diehl, et al., JHEP 02 (2007)

- The W term dominates in the region where $q_T \ll Q$
- Y term is usually excluded in TMD analyses

TMD Factorization - Drell–Yan

(A. Bacchetta lectures)



$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp A} d^2 \mathbf{k}_{\perp B} f_1^a(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{a}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

Structure of a TMD

(A. Bacchetta lectures)

Fourier Transform in b_T -space

$$\tilde{f}_1(x, b_T^2; Q^2) = \int \frac{d^2 k_T}{(2\pi)^2} e^{i b_T \cdot k_T} f_1(x, k_T^2; Q^2)$$

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How to model a TMD distribution?

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$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_{b_*}) e^{S_{\text{pert}}(b_*; \mu_{b_*}, \mu, \zeta)} e^{S_{\text{NP}}(b_T^2; \zeta)} \hat{f}_{NP}^q(x, b_T^2)$$

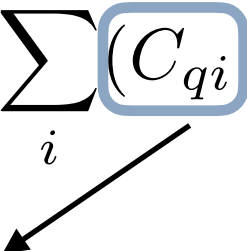
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Matching coefficients
(perturbatively
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Collinear PDF

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LHAPDF

<https://lhapdf.hepforge.org>

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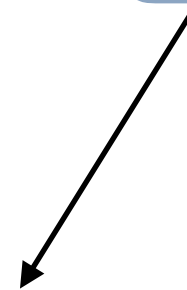
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Sudakov form factor
(Perturbatively calculable)

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Structure of a TMD - TMD evolution

(A. Bacchetta lectures)

Evolution in 2 dimensions

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(\mu)$$

$$\frac{\partial K(\mu)}{\partial \ln \mu} = -\gamma_K(\alpha_S(\mu))$$

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \mu} = \gamma(\mu, \zeta)$$

$$\frac{\partial \gamma(\mu, \zeta)}{\partial \ln \sqrt{\zeta}} = -\gamma_K(\alpha_S(\mu))$$

Collins and Soper, Nucl. Phys. B193 (1981)
J. Collins, "Foundation of Perturbative QCD"

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Sudakov form factor = solution of evolution equations

$$\hat{f}_1(x, b_T; \mu, \zeta) = e^{S(b_T; \mu, \zeta)} \hat{f}_1(x, b_T; \mu_0, \zeta_0)$$

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$$e^{S(b_T; \mu, \zeta)} = \exp \left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(\alpha_S(\mu')) - \gamma_K(\alpha_S(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\}$$

$$\gamma_F(\alpha_S(\mu)) = \gamma(\mu, \mu^2)$$

Structure of a TMD - TMD evolution

(A. Bacchetta lectures)

$$\mu_0 = \sqrt{\zeta_0} = \mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

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Ebert and Tackmann, JHEP 02 (2017)

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$$S_{\text{pert}}(\mu_b, Q) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(Q)}{4\pi} \right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n, 2n-k)} \quad L = \ln \left(\frac{Q^2}{\mu_b^2} \right)$$

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

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| Accuracy | H and C | K and γ_F | γ_K | α_S evol. |
|--------------------|-------------|--------------------|------------|-------------------|
| LL | 0 | - | 1 | - |
| NLL | 0 | 1 | 2 | NLO |
| NLL' | 1 | 1 | 2 | NLO |
| NNLL | 1 | 2 | 3 | NNLO |
| NNLL' | 2 | 2 | 3 | NNLO |
| N ³ LL | 2 | 3 | 4 | N ³ LO |
| N ³ LL' | 3 | 3 | 4 | N ³ LO |

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Model-dependent function

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Model-dependent function

FIT to data

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\mu_b = \frac{2e^{-\gamma E}}{b_T}$$

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T} \xrightarrow{b_T \gg 1} 0$$

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T} \xrightarrow{b_T \gg 1} 0 \quad \boxed{\alpha_S(\mu_b) \rightarrow +\infty}$$

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\infty \quad \xleftarrow{b_T \ll 1} \quad \mu_b = \frac{2e^{-\gamma_E}}{b_T} \quad \xrightarrow{b_T \gg 1} \quad 0 \quad \boxed{\alpha_S(\mu_b) \rightarrow +\infty}$$

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\boxed{\mu_b > Q} \quad \infty \quad \begin{array}{c} b_T \ll 1 \\ \longleftarrow \end{array} \quad \mu_b = \frac{2e^{-\gamma_E}}{b_T} \quad \begin{array}{c} b_T \gg 1 \\ \longrightarrow \end{array} \quad 0 \quad \boxed{\alpha_S(\mu_b) \rightarrow +\infty}$$

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b_* -prescription

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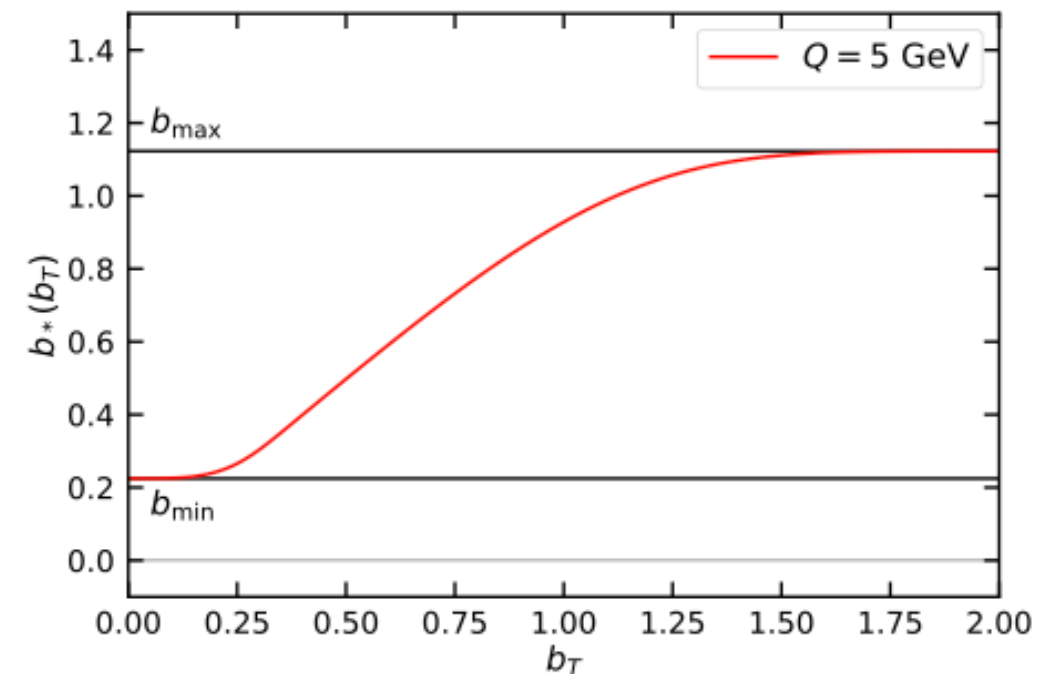
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$$b_*(b_T) = b_{\max} \left(\frac{1 - \exp\left(-\frac{b_T^4}{b_{\max}^4}\right)}{1 - \exp\left(-\frac{b_T^4}{b_{\min}^4}\right)} \right)^{\frac{1}{4}}$$

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J.C.Collins, D.E.Soper, and G.F.Sterman, Nucl.Phys. B250,199 (1985)

J.C.Collins, L.Gamberg, et al., Phys.Rev.D 94 (2016)

A.Bacchetta, M.G.Echevarria, P.J.G.Mulders, M.Radici, and A.Signori, JHEP 11, 076 (2015)

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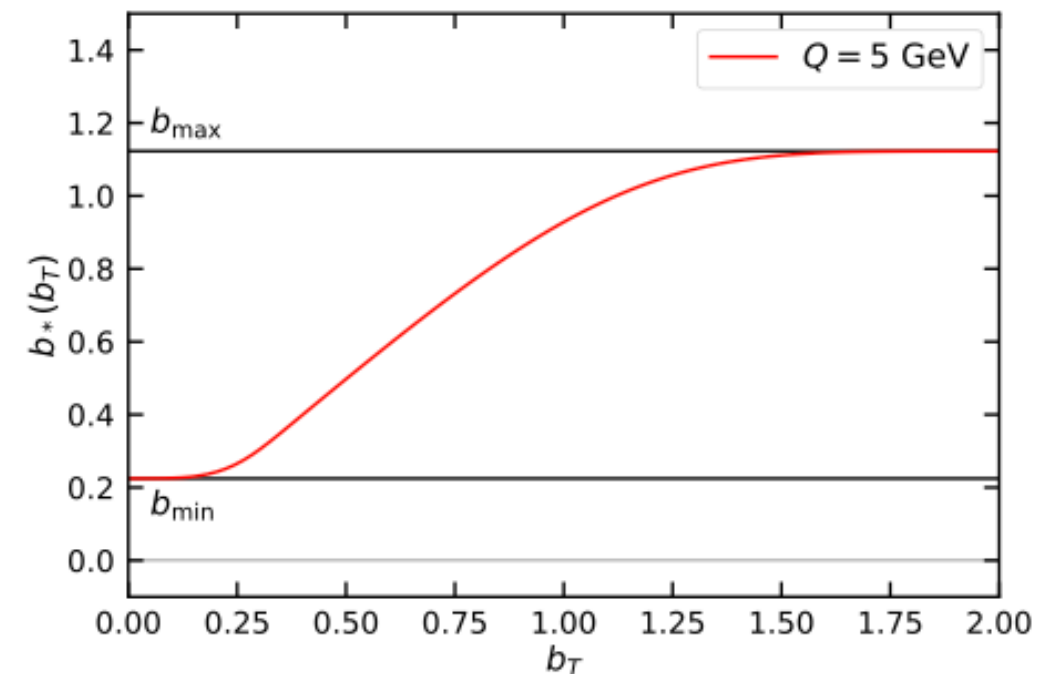
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A.Bacchetta, M.G.Echevarria, P.J.G.Mulders, M.Radici, and A.Signori, JHEP **11**, 076 (2015)

$$\hat{f}_1(x, b_T; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T); \mu, \zeta) \equiv f_{\text{NP}}(x, b_T; \zeta) \hat{f}_1(x, b_*(b_T); \mu, \zeta)$$

Structure of a TMD - NP content

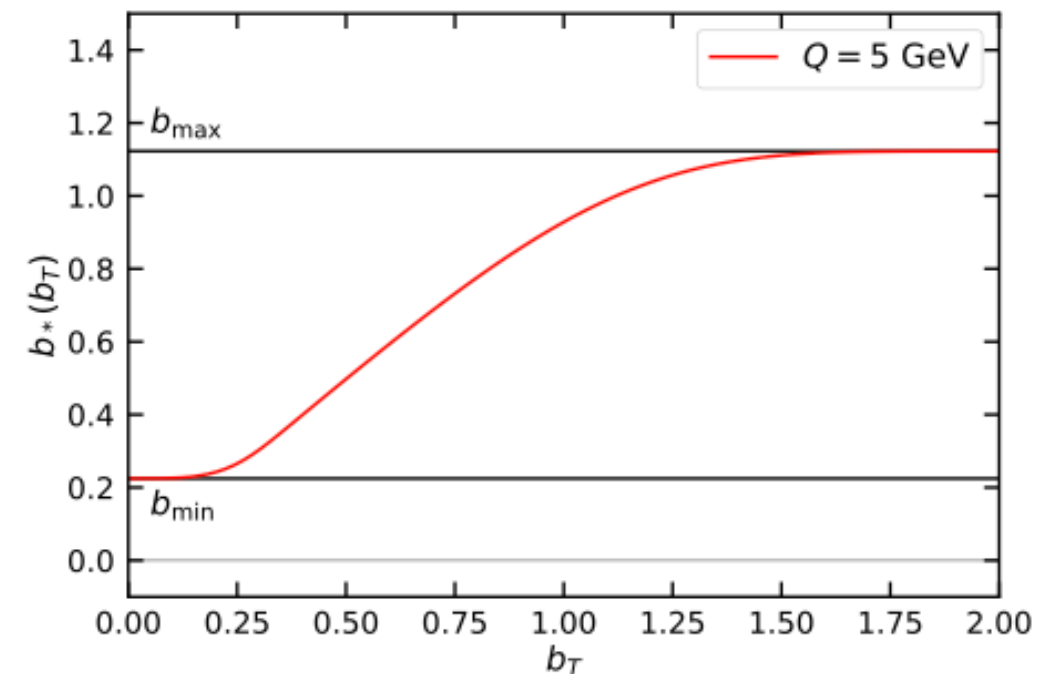
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GLOBAL FIT

Is it worth it??!



CSS formalism (NLL)

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$$\hat{f}_1(x, b_T^2; \mu, \zeta) = f_1(x, \mu_{b_*}) e^{S_{\text{pert}}(b_*; \mu_{b_*}, \mu, \zeta)} \\ \times e^{S_{\text{NP}}(b_T^2; \zeta)} \hat{f}_{NP}^q(x, b_T^2)$$

CSS formalism (NLL)

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CSS formalism (NLL)

(A. Bacchetta lectures)



CSS formalism (NLL)

(A. Bacchetta lectures)

SIDIS unpolarized structure function



CSS formalism (NLL)

(A. Bacchetta lectures)

SIDIS unpolarized structure function

$$F_{UU,T}(x, z, P_{hT}, Q) = x \sum_q e_q^2 \int db_T b_T J_0(b_T P_{hT}) \hat{f}_{1q/N}(x, b_T^2; Q, Q^2) \\ \times \hat{D}_{1h/q}(z, b_T^2; Q, Q^2)$$



CSS formalism (NLL)

(A. Bacchetta lectures)

SIDIS unpolarized structure function

$$F_{UU,T}(x, z, P_{hT}, Q) = x \sum_q e_q^2 \int db_T b_T J_0(b_T P_{hT}) \hat{f}_{1q/N}(x, b_T^2; Q, Q^2) \\ \times \hat{D}_{1h/q}(z, b_T^2; Q, Q^2)$$

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$$\hat{D}_1(z, b_T^2; \mu, \zeta) = D_1(z, \mu_{b_*}) e^{S_{\text{pert}}(b_*; \mu_{b_*}, \mu, \zeta)} \\ \times e^{S_{\text{NP}}(b_T^2; \zeta)} \hat{D}_{NP}(z, b_T^2)$$



Parton Model (LO)

(A. Bacchetta lectures)



Parton Model (LO)

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Parton Model (LO)

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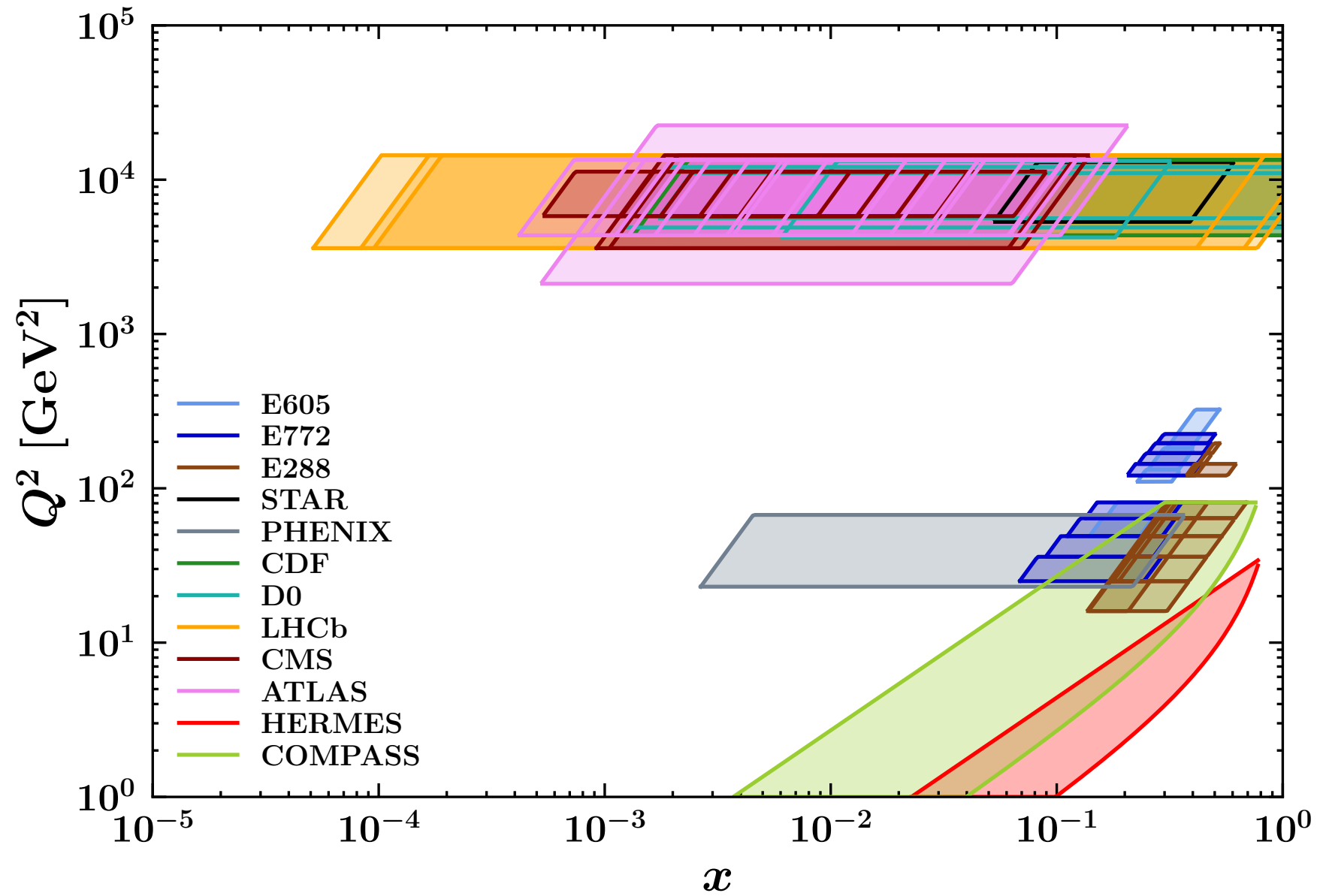
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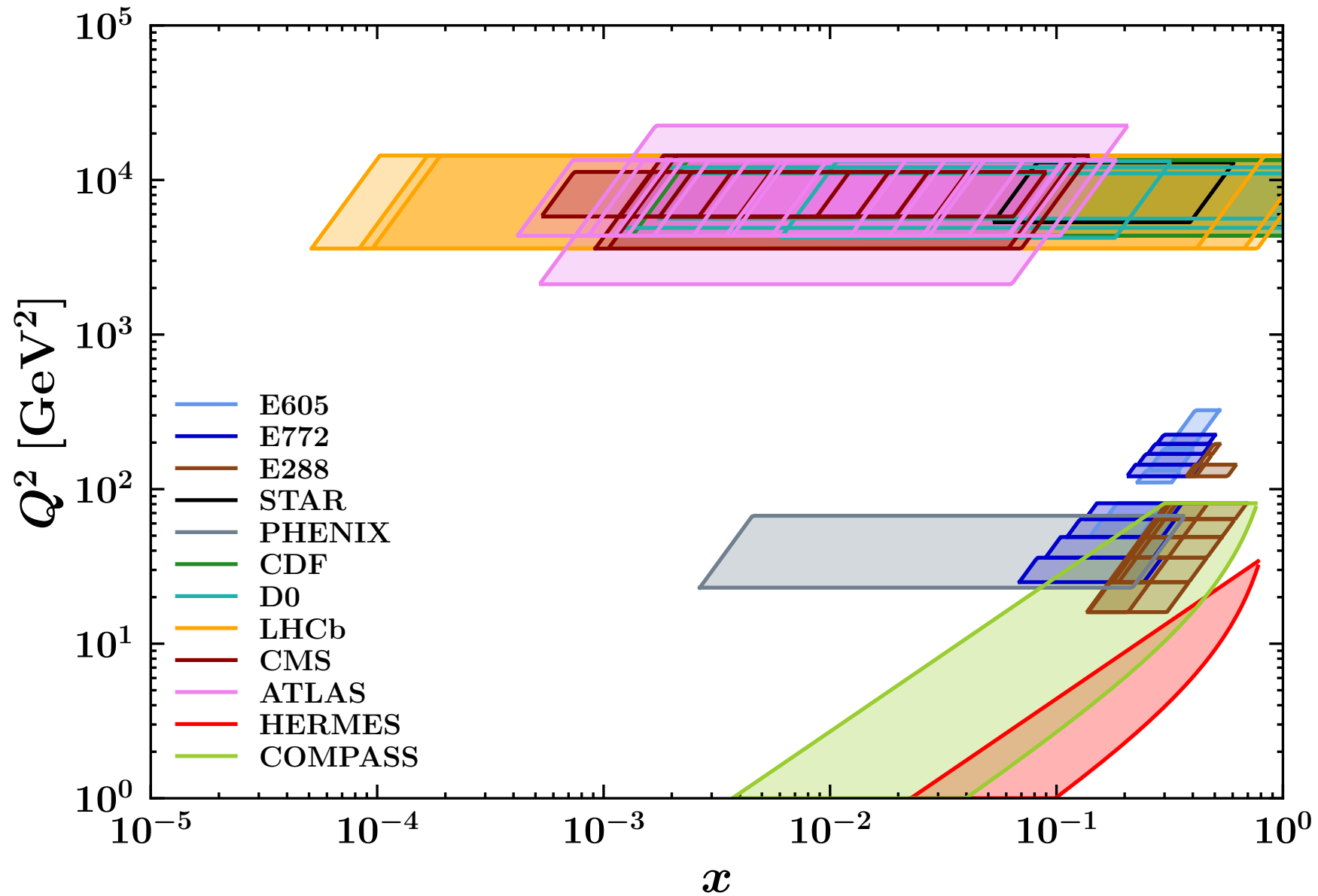


Phenomenology

Experimental data

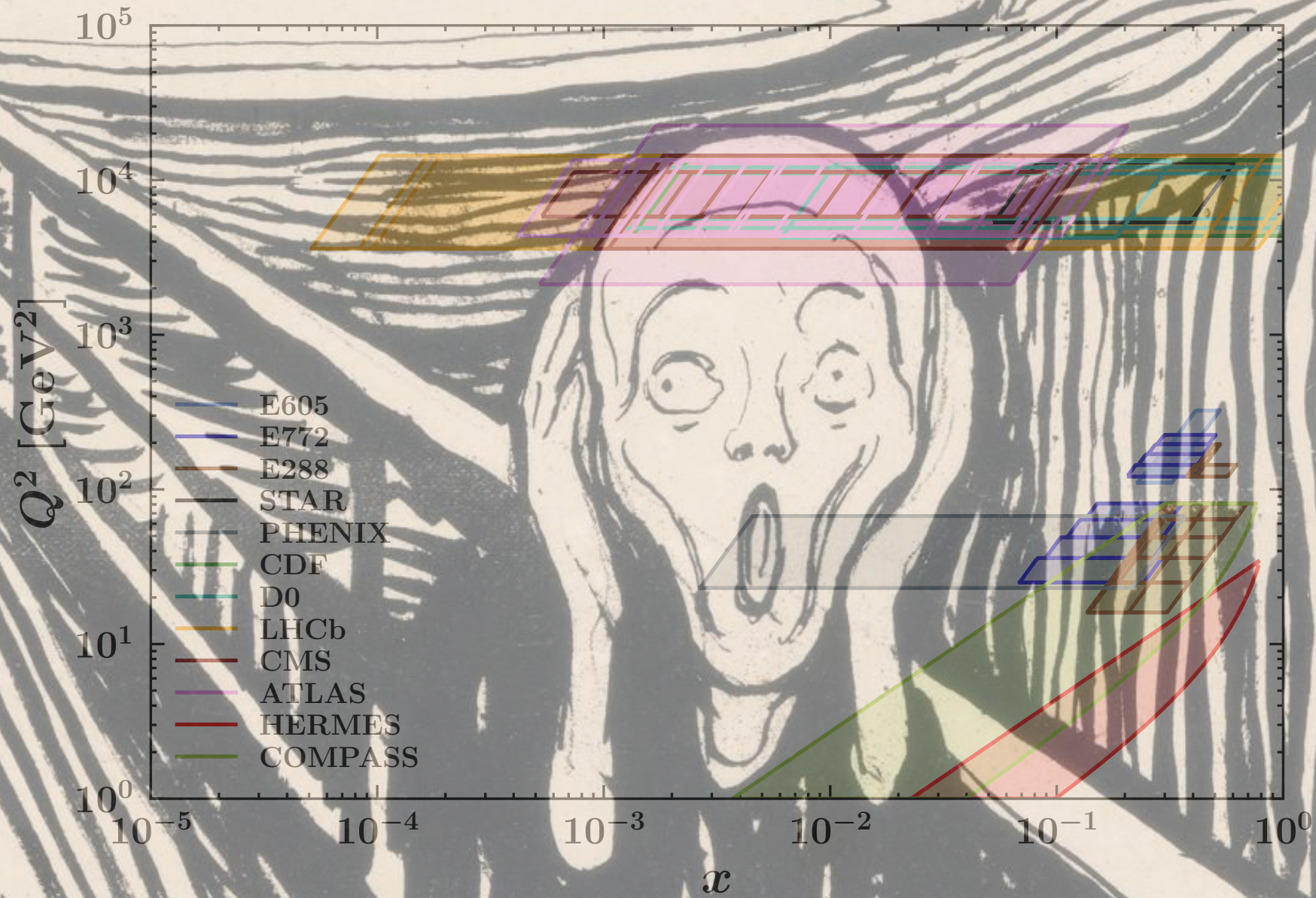


Experimental data



484 (DY) + 1547 (SIDIS)
2031 fitted experimental points

Experimental data



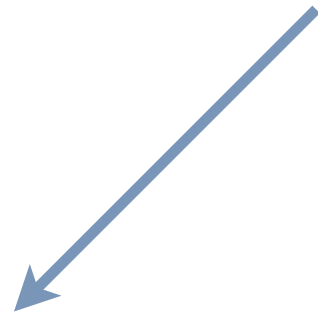
484 (DY) + 1547 (SIDIS)
2031 fitted experimental points

Nonperturbative parameterizations

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

Nonperturbative parameterizations

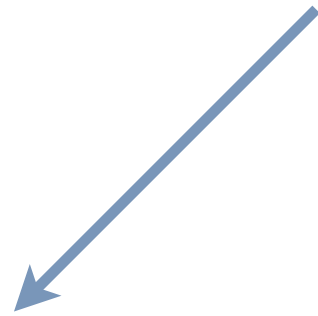
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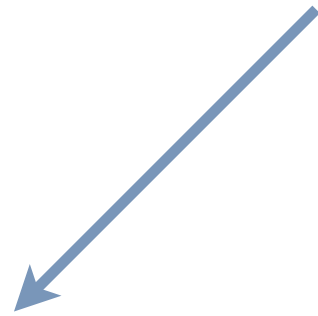


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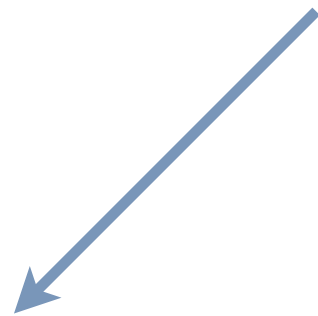
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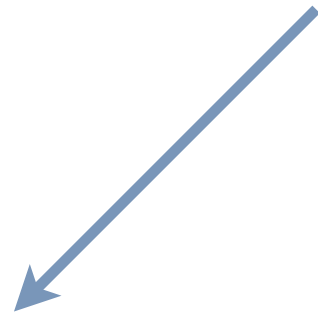


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Nonperturbative parameterizations

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**11 parameters for TMD PDF
+ 1 for NP evolution + 9 for TMD FF
= 21 free parameters**

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11 parameters for TMD PDF
+ 1 for NP evolution + 9 for TMD FF
= 21 free parameters



**KEEP
CALM
AND
CODE
ON**

An orange speech bubble with a white shadow, containing the word "HOW?" in a bold, dark grey, sans-serif font. The speech bubble has a pointed tail at the bottom left.

HOW?

Ask the MAP Collaboration

<https://github.com/MapCollaboration>

The screenshot shows the GitHub profile page for the organization 'MapCollaboration'. At the top, there is a navigation bar with the organization's name, a search bar, and icons for repository management. Below the navigation bar, the profile header includes a profile picture (a world map), the organization name 'M.A.P. Collaboration', a description 'Multi-dimensional Analyses of Partonic distributions', and location information 'Amsterdam, Edinburgh, Paris, Pavia'. A dropdown menu is open, showing options like 'New repository', 'New codespace', and 'New organization'. The main content area displays the 'README.md' file, which contains a greeting 'Hi there' and a detailed description of the MAP collaboration's focus on hadronic physics. Below the README, there are two pinned repositories: 'NangaParbat' and 'MontBlanc'. On the right side, there are sections for 'View as: Public', 'Discussions', and 'People'.

MapCollaboration

Overview Repositories 5 Projects Packages Teams People 12 Settings

M.A.P. Collaboration
Multi-dimensional Analyses of Partonic distributions
1 follower Amsterdam, Edinburgh, Paris, Pavia

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View as: Public
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[Get started with tasks](#) that most successful organizations complete.

Discussions
Set up discussions to engage with your community!
[Turn on discussions](#)

People

Invite someone

README .md

Hi there 🙌

The MAP (Multi-dimensional Analyses of Partonic distributions) collaboration is a pioneering initiative in the field of hadronic physics. It brings together a group of experts in QCD phenomenology to extract information about the distribution of partons inside hadrons across multidimensional momentum and configuration spaces. Key objects of study include Transverse Momentum Distributions (TMDs), Generalized Parton Distributions (GPDs), and Fragmentation Functions (FFs).

This GitHub repository is our central platform for sharing codes and disseminating research results.

NangaParbat Public
Nanga Parbat: a fitting framework for the determination of the non-perturbative component of TMD distributions
HTML 3 stars 4 forks

MontBlanc Public
A code for the determination of collinear distributions
Jupyter Notebook 1 star 3 forks

Repositories

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MapCollaboration

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View as: **Public**
You are viewing the README and pinned repositories as a public user.
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README .md

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Pinned Customize pins

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Repositories

NangaParbat fitting framework

<https://github.com/MapCollaboration/NangaParbat>



☰ README.md



Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

<https://github.com/MapCollaboration/NangaParbat>

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

NangaParbat fitting framework

git clone

Dependencies

In order to install the code a number of external but relatively standard libraries are required. Here is the list:

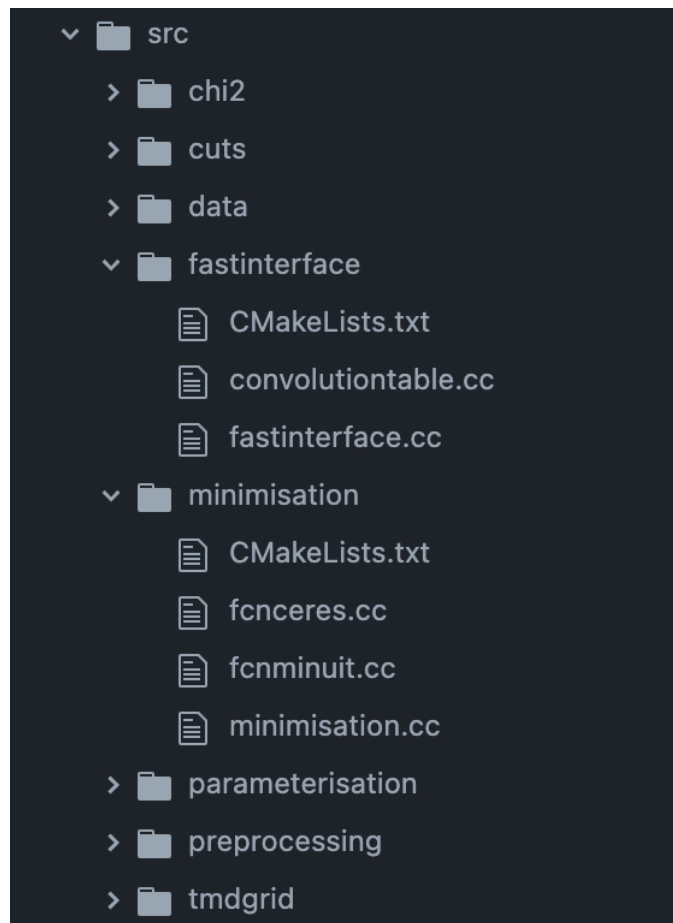
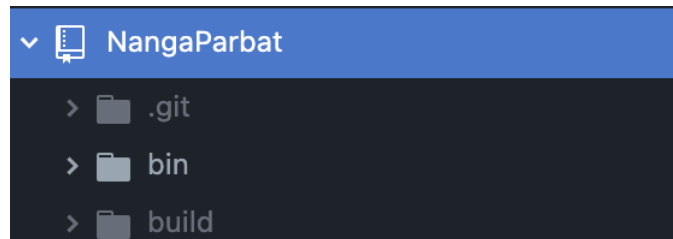
- `yaml-cpp` ,
- `eigen3` ,
- `ceres-solver` ,
- `GSL` ,
- `ROOT` with `Minuit2` ,
- `LHAPDF6` ,
- `APFEL++` .

Most of these libraries can be installed through standard package managers such as `Homebrew` on MacOS and `apt-get` on Linux.

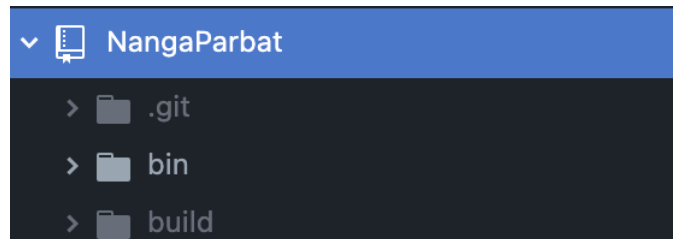
NangaParbat also has a **lite version**, that does not require all the dependencies listed above. The mandatory dependencies are `yaml-cpp` , `eigen3` , `LHAPDF6` , `APFEL++` .

During the installation, NangaParbat detects if one or more libraries among `GSL` , `ROOT` or `ceres-solver` are missing and installs and compiles only the part of the code that is possible to run.

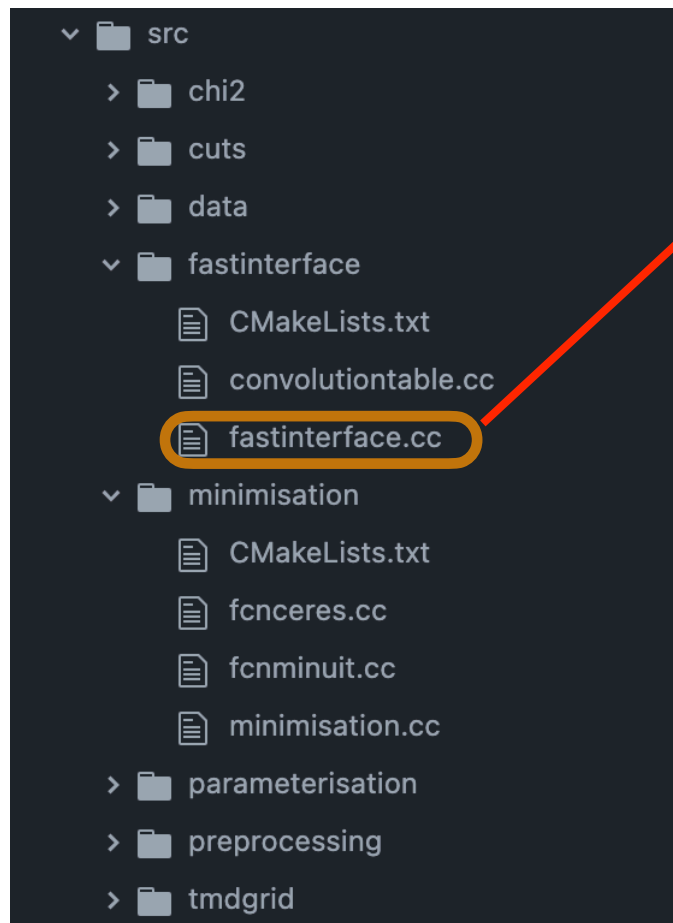
Implemented strategy to perform a fit



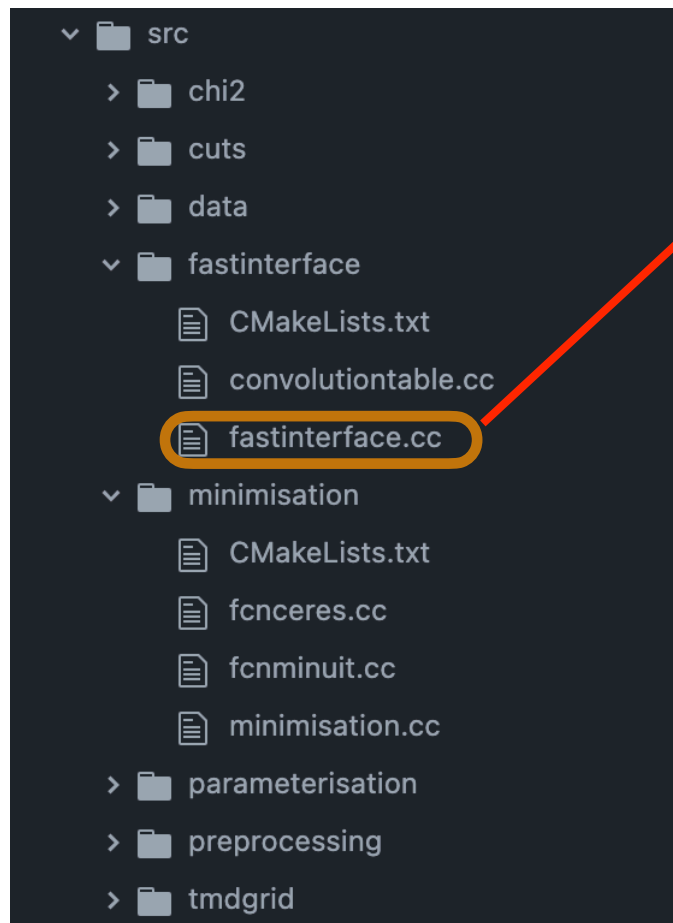
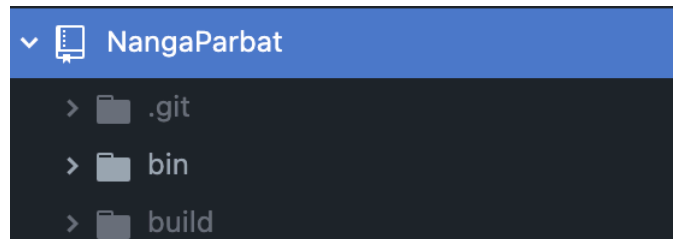
Implemented strategy to perform a fit



Generation of interpolation tables



Implemented strategy to perform a fit



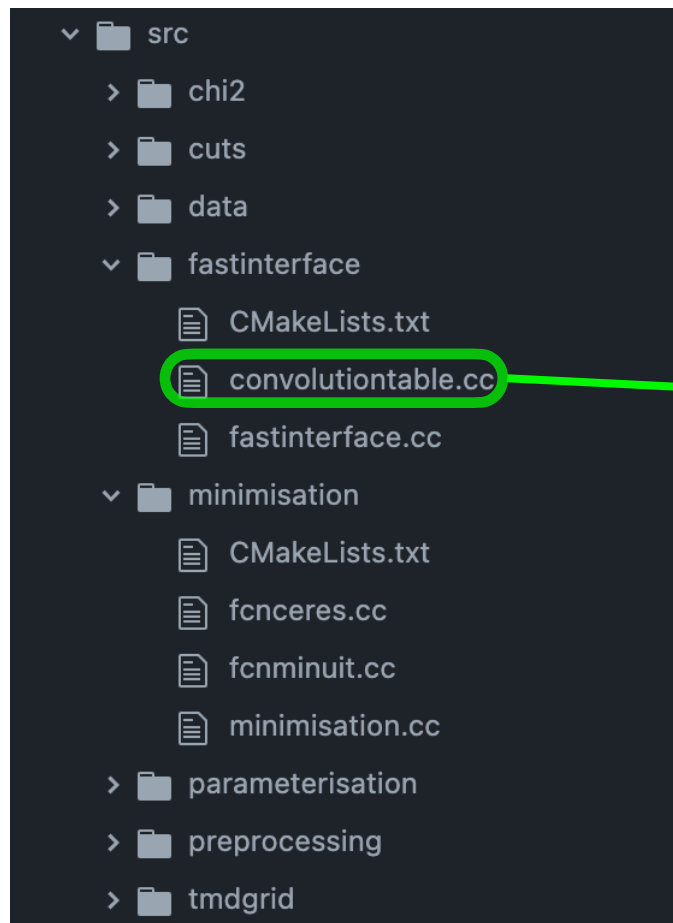
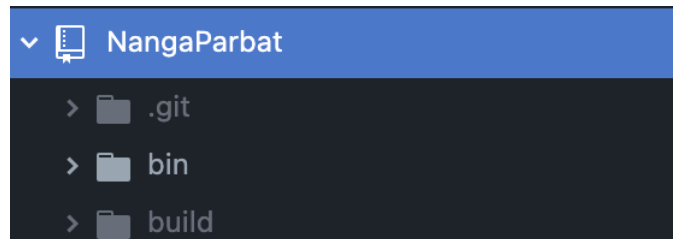
Generation of interpolation tables

Kinematical prefactors

Collinear distributions and evolution

TMD evolution

Implemented strategy to perform a fit



Generation of interpolation tables

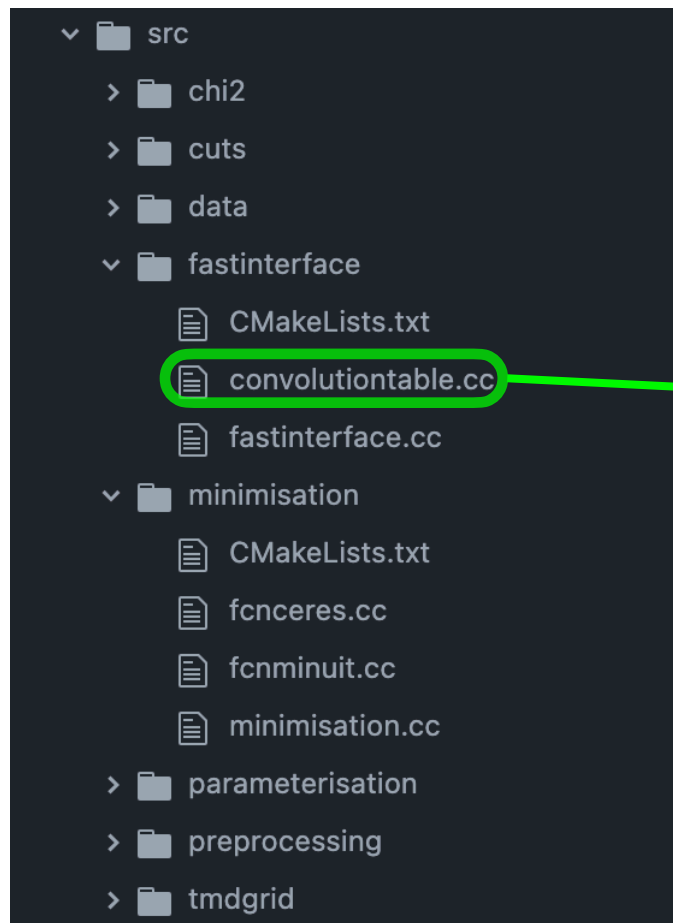
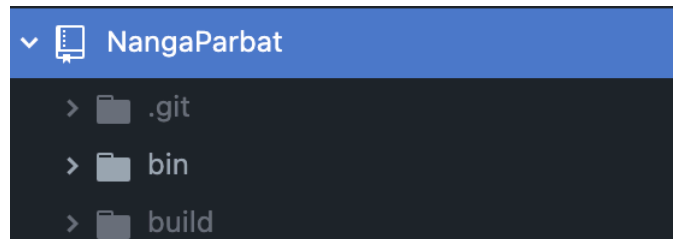
Kinematical prefactors

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Calculation of the prediction

Implemented strategy to perform a fit



Generation of interpolation tables

Kinematical prefactors

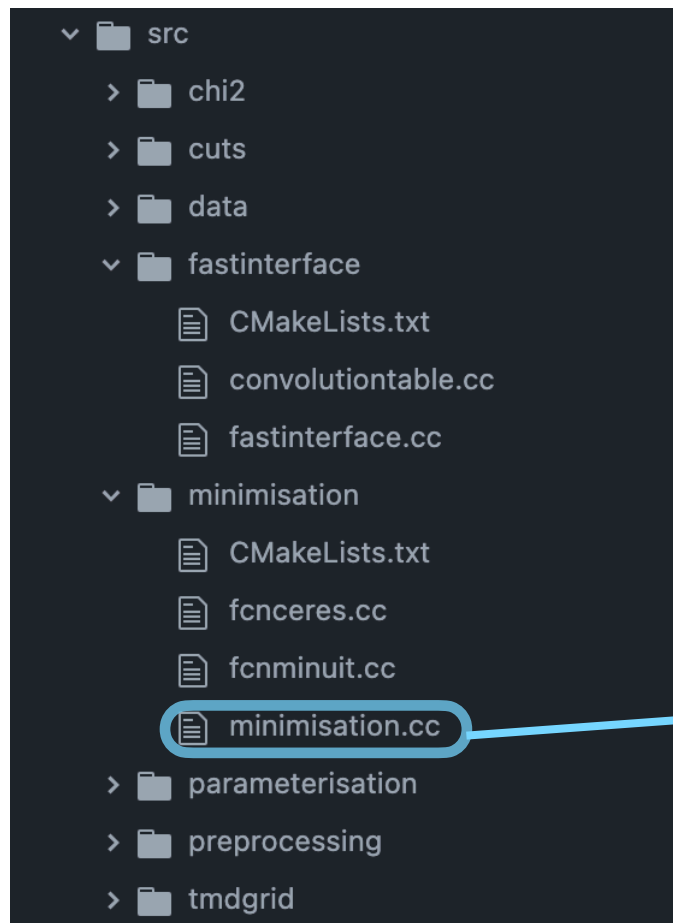
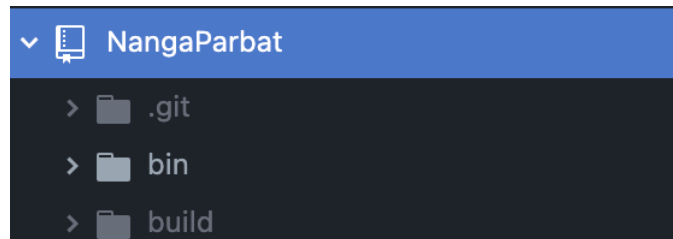
Collinear distributions and evolution

TMD evolution

Calculation of the prediction

NP model

Implemented strategy to perform a fit



Generation of interpolation tables

Kinematical prefactors

Collinear distributions and evolution

TMD evolution

Calculation of the prediction

NP model

Minimization of chi-squared function

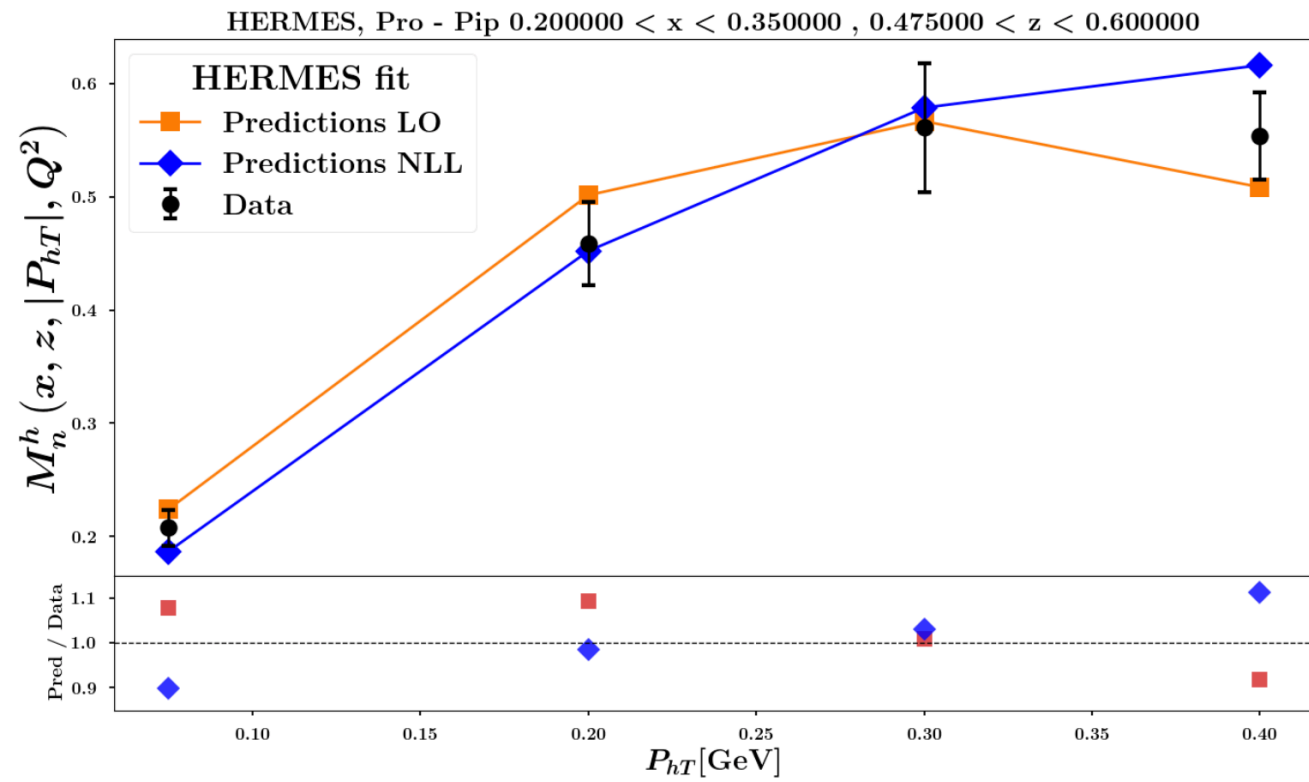
**LET'S
</CODE>**





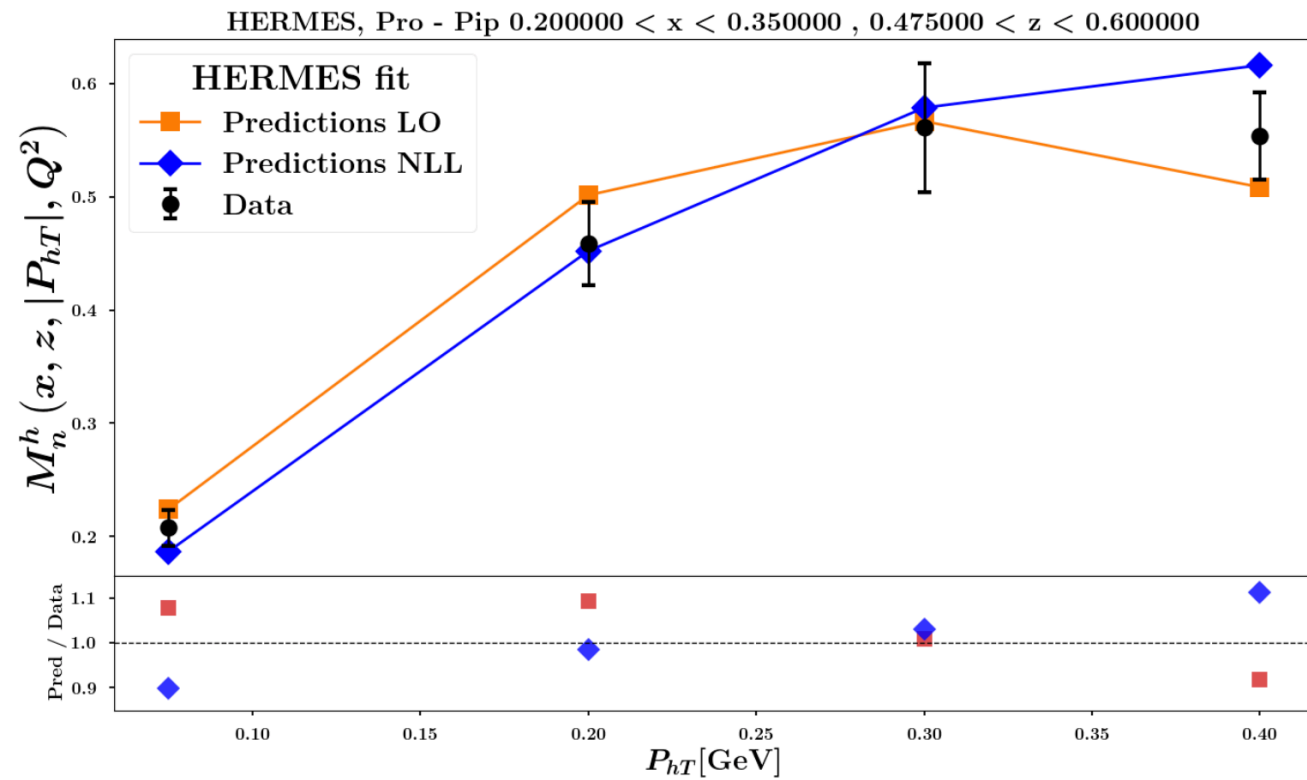
Results

It was a cooking show, wasn't it?



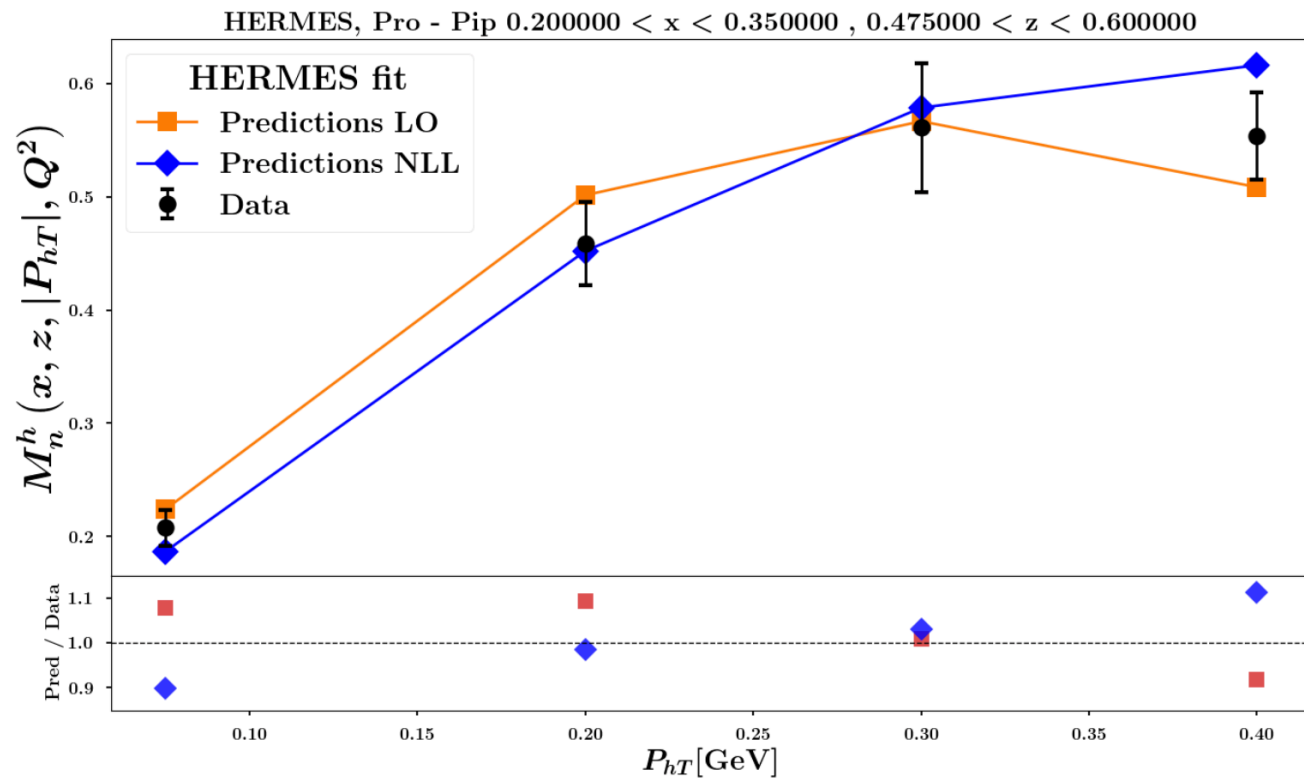
It was a cooking show, wasn't it?

At LO we can fit HERMES multiplicities



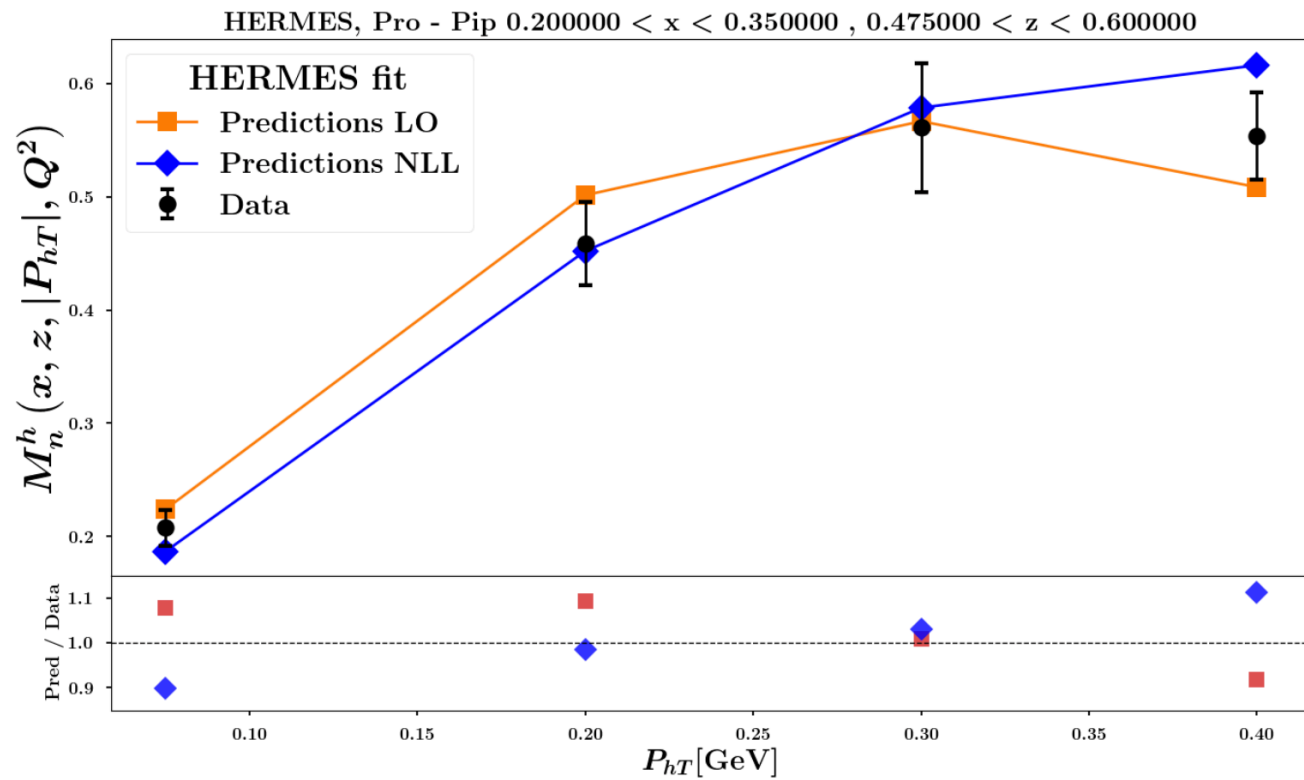
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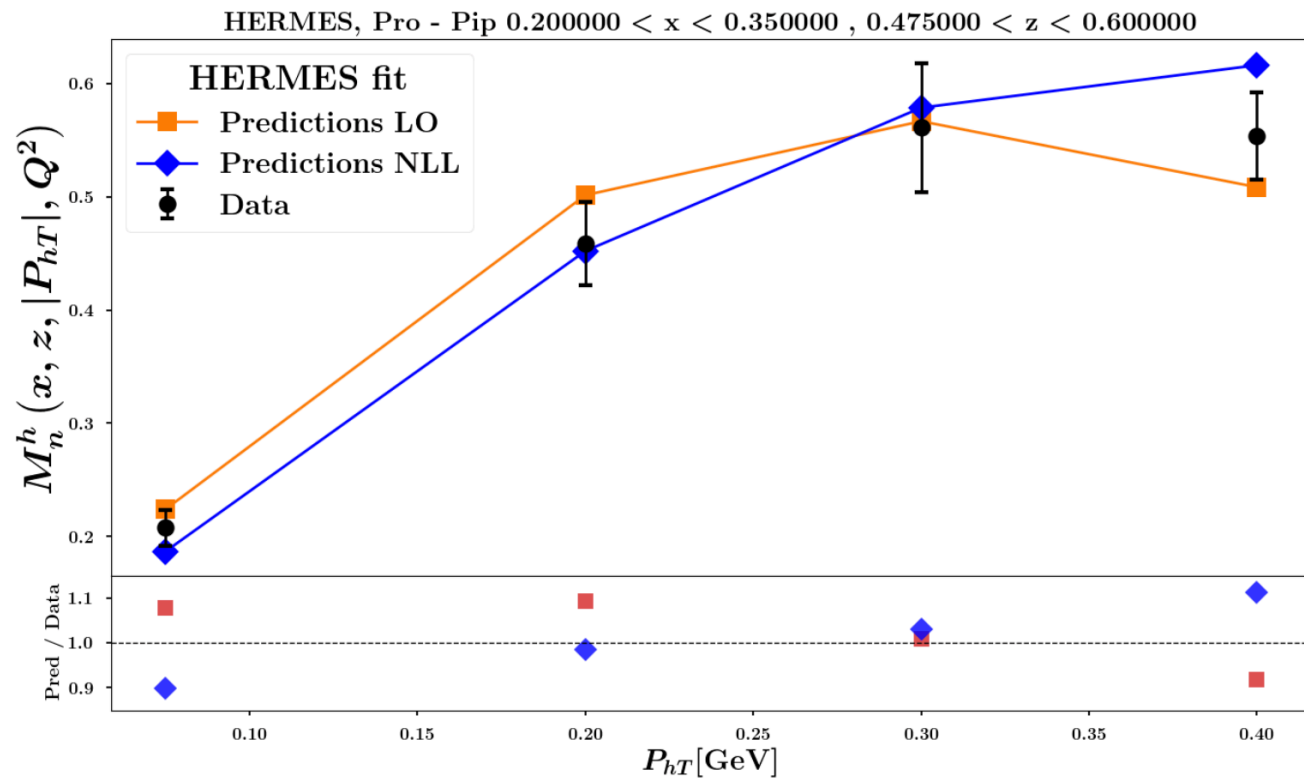
At LO we can fit HERMES multiplicities



At NLL we can fit HERMES multiplicities

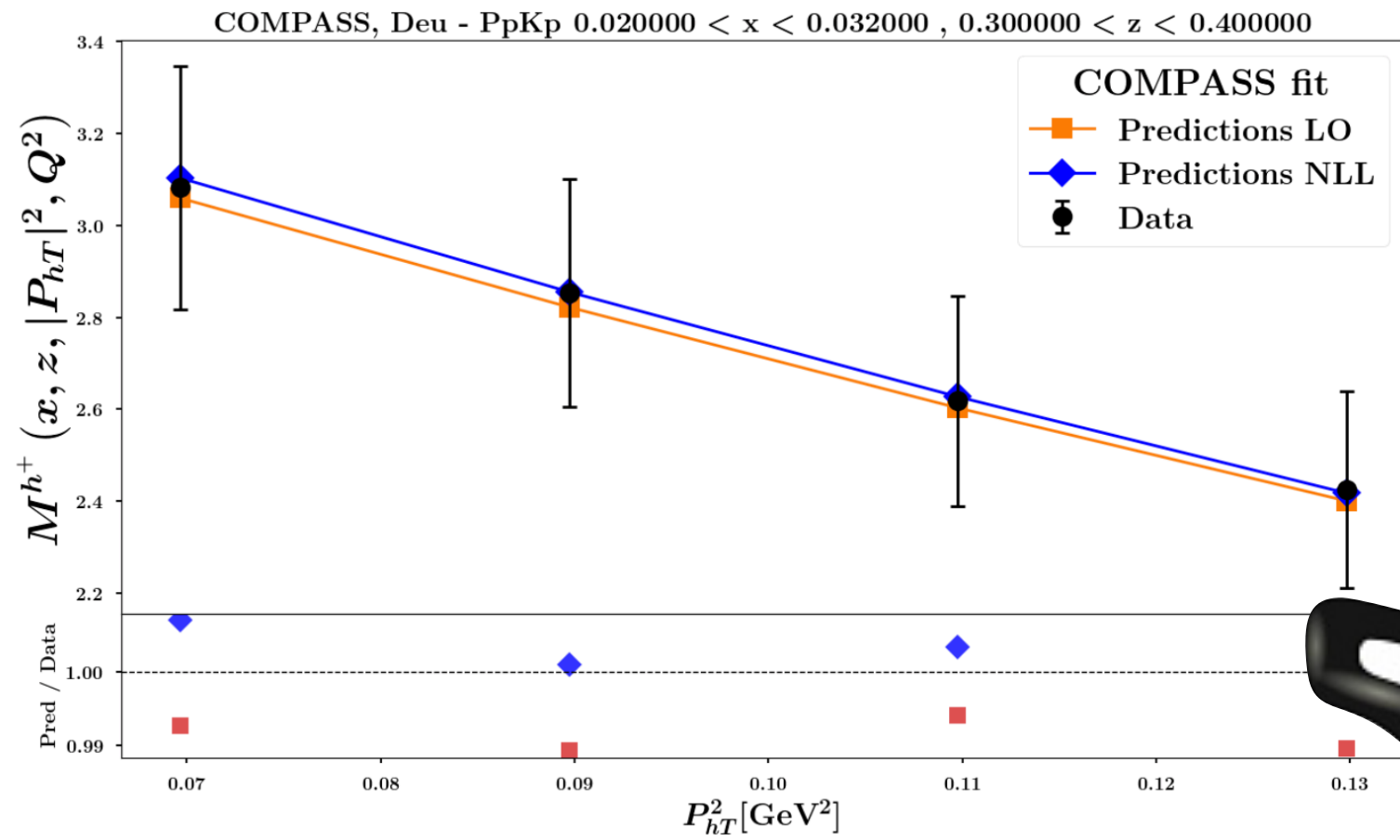
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At LO we can fit HERMES multiplicities



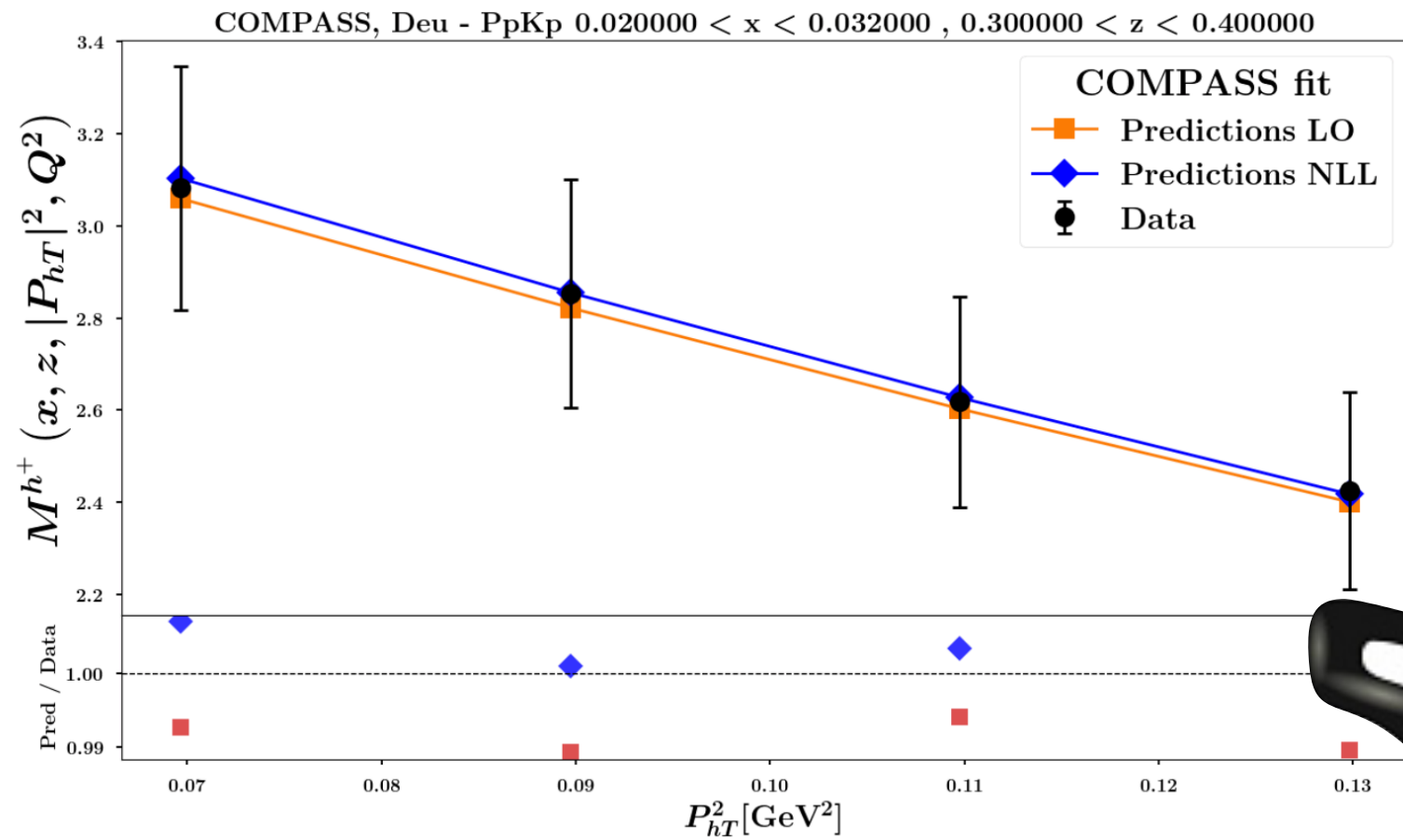
At NLL we can fit HERMES multiplicities

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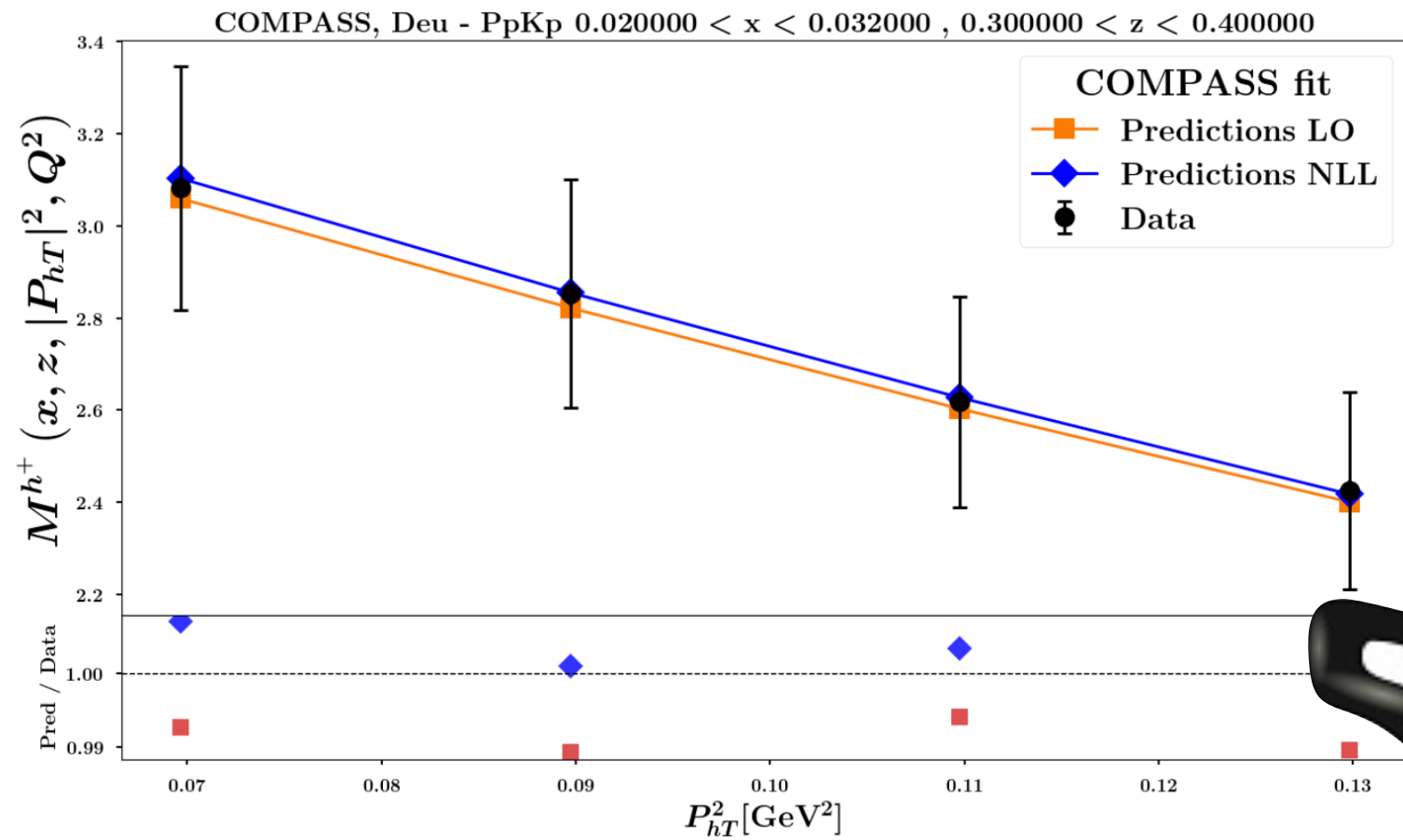
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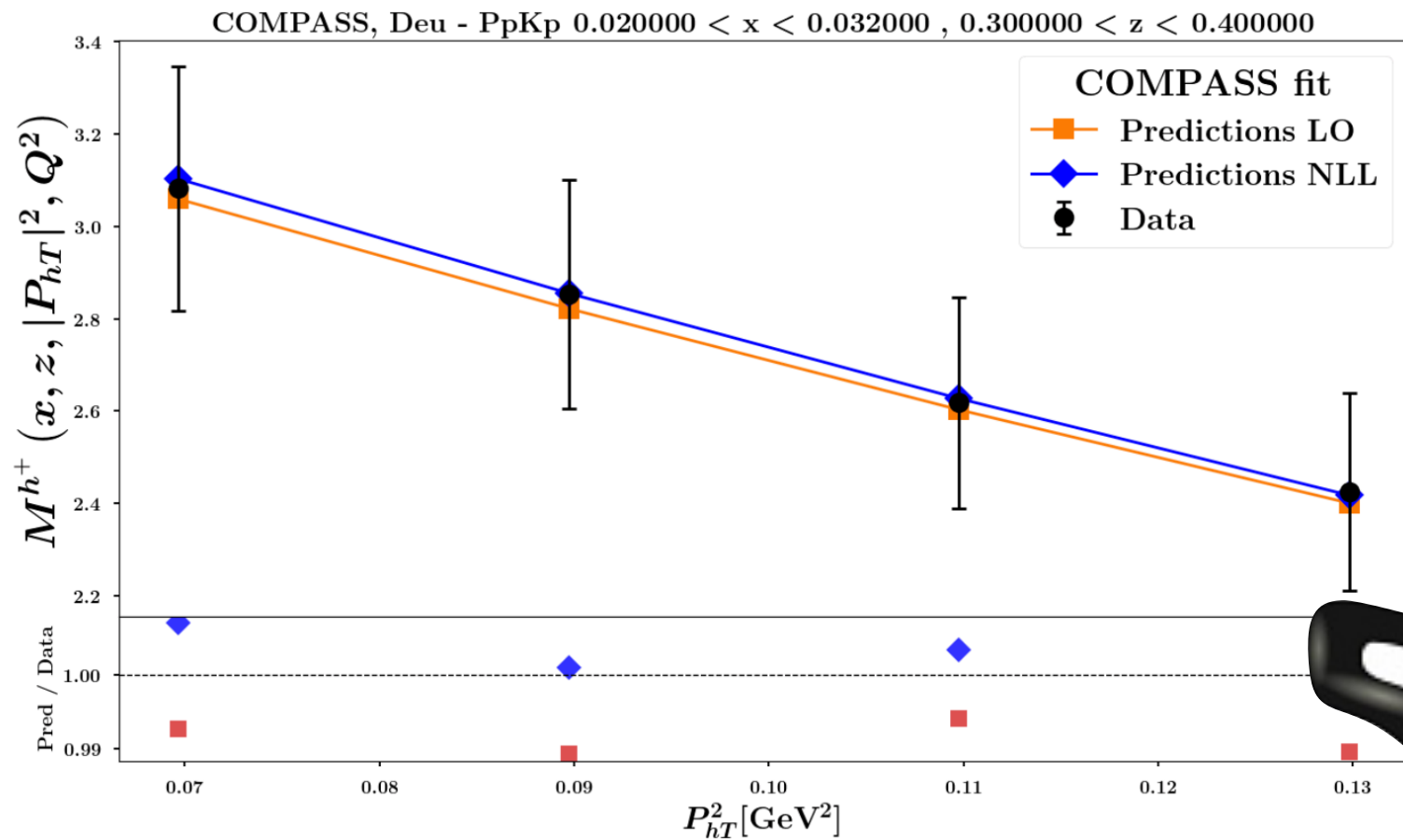
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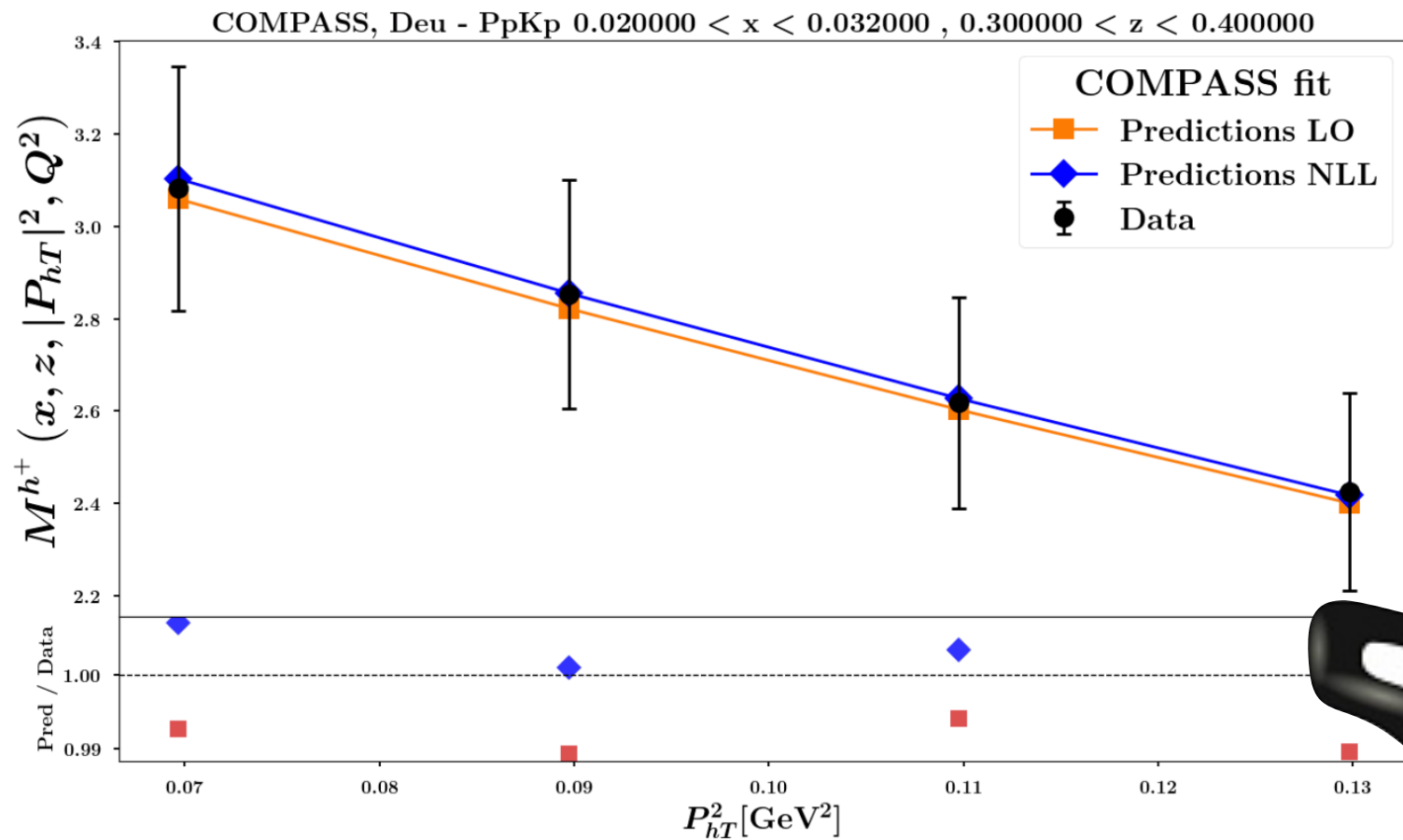
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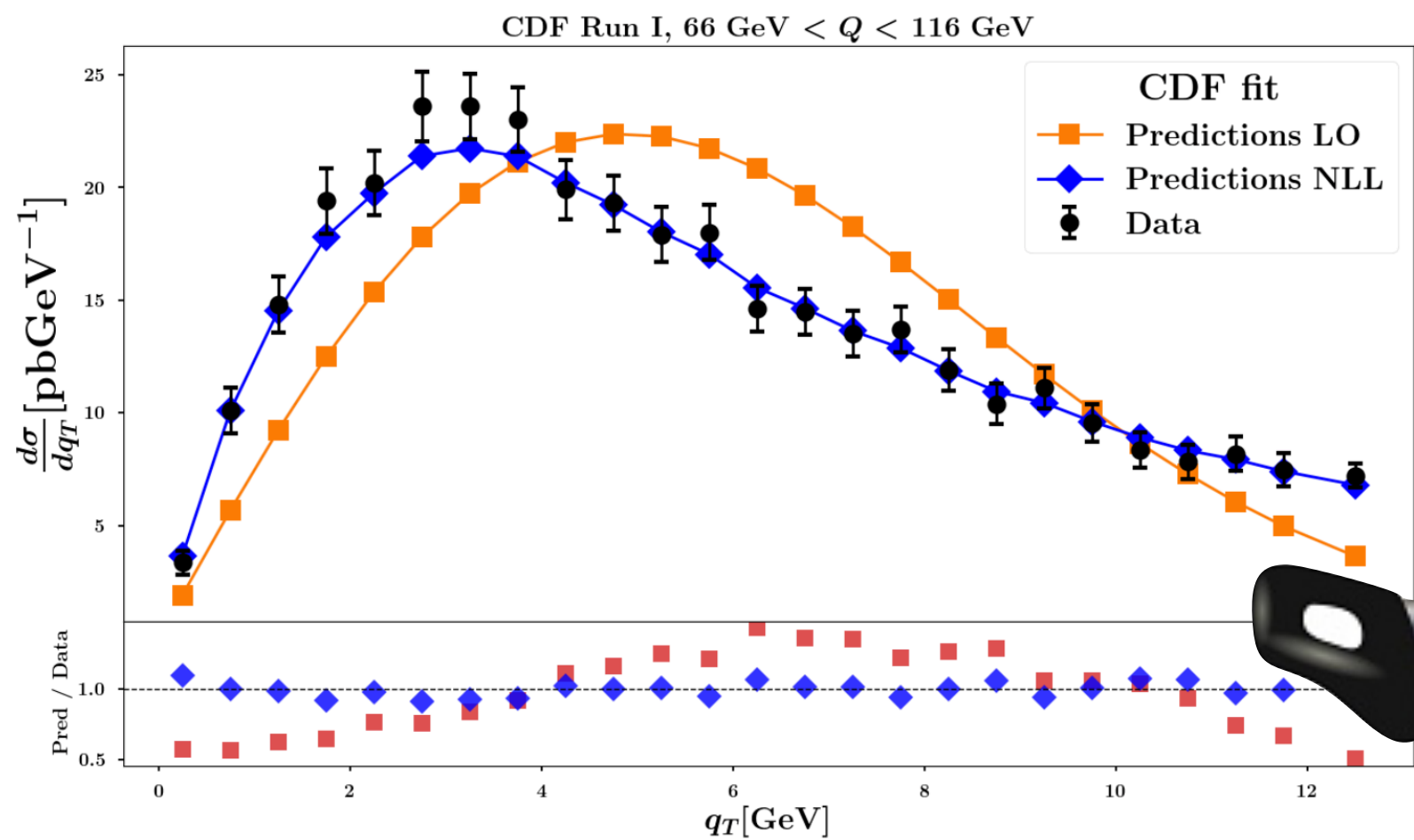
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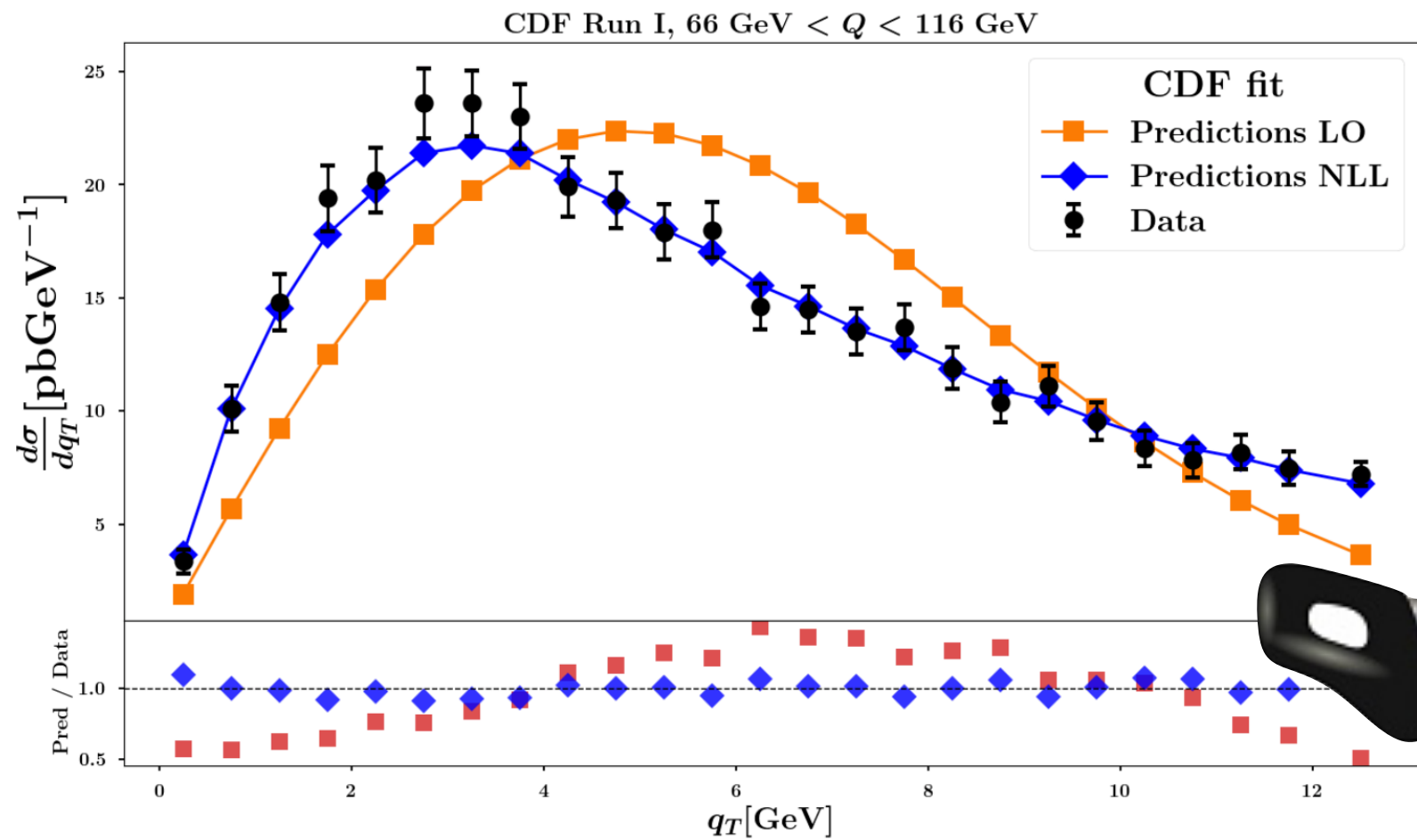
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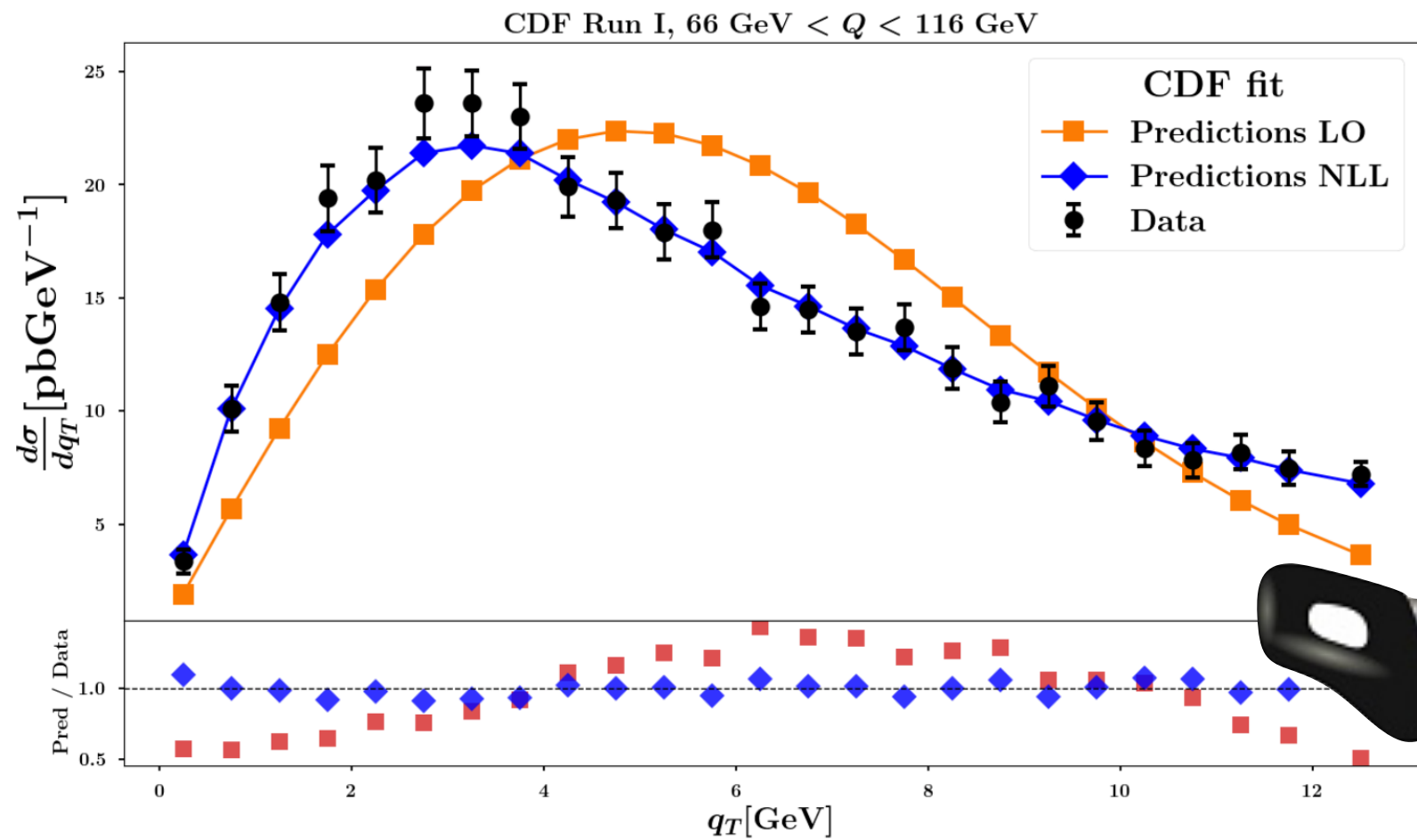
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At LO we canNOT fit CDF q_T -distribution



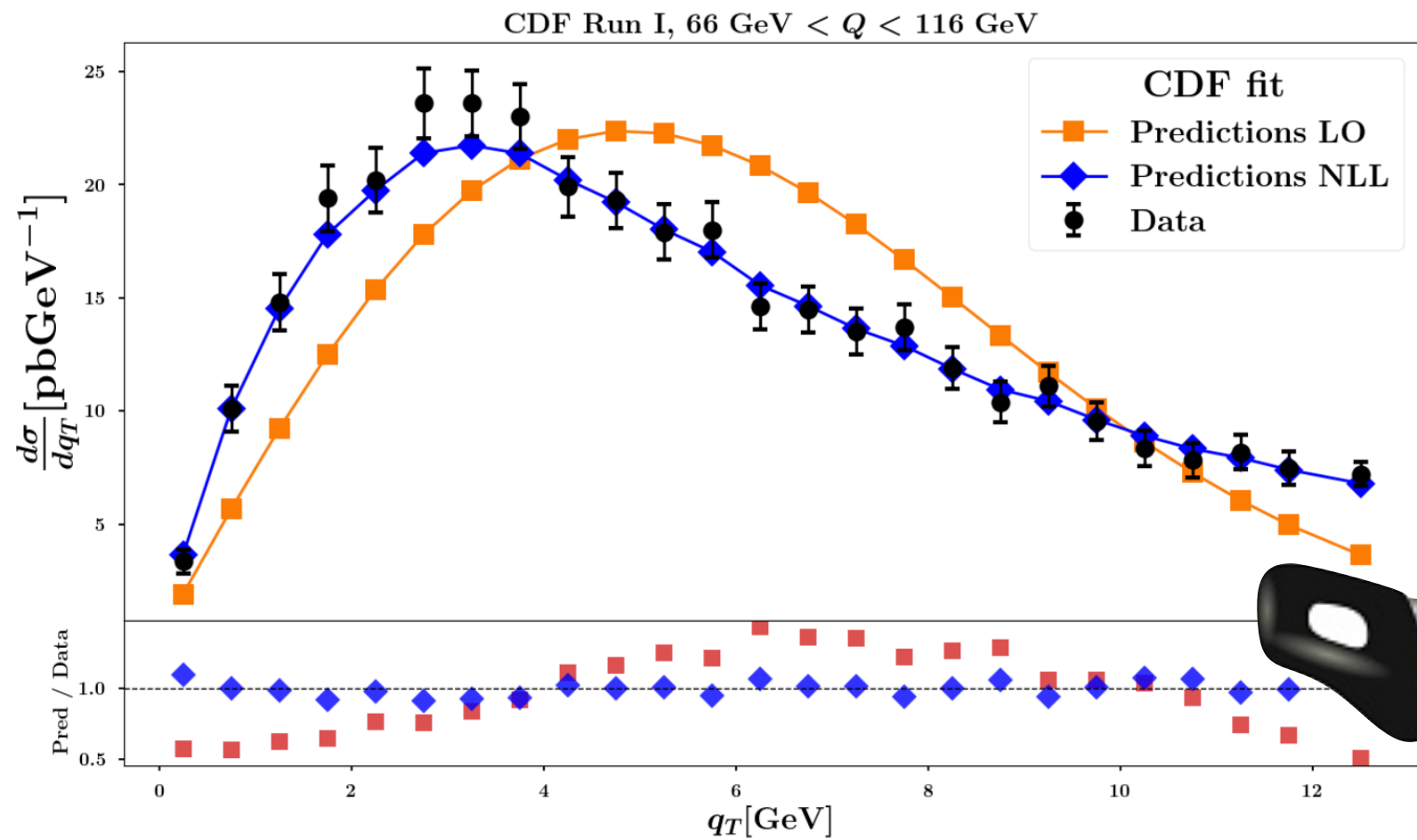
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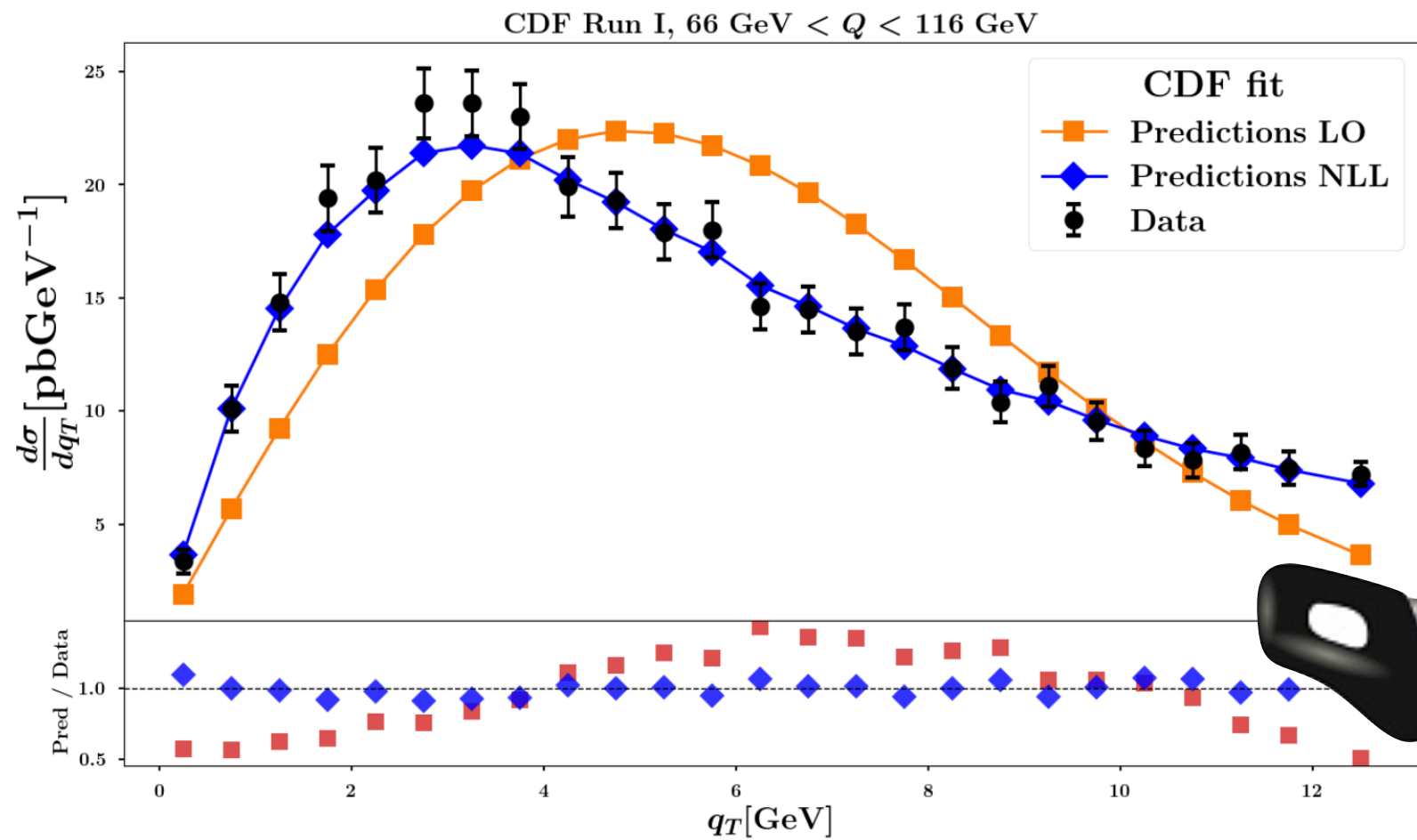


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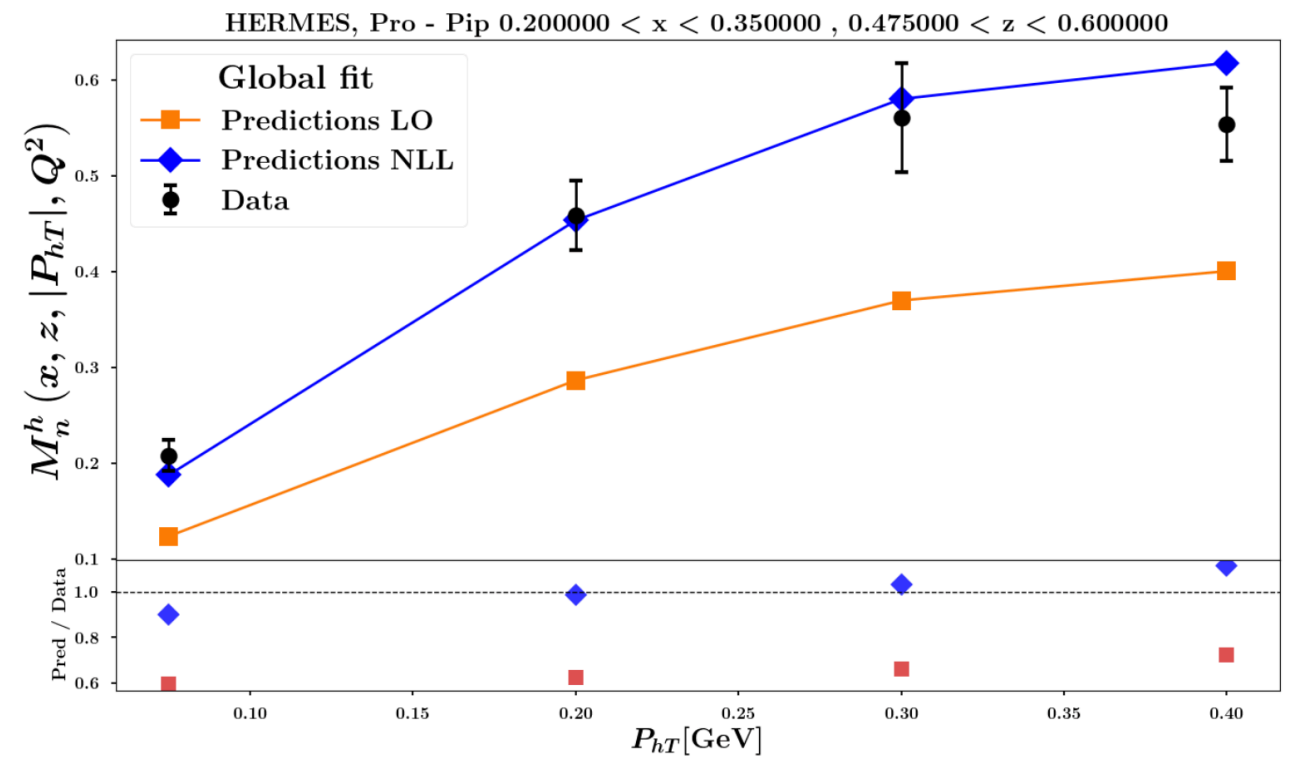
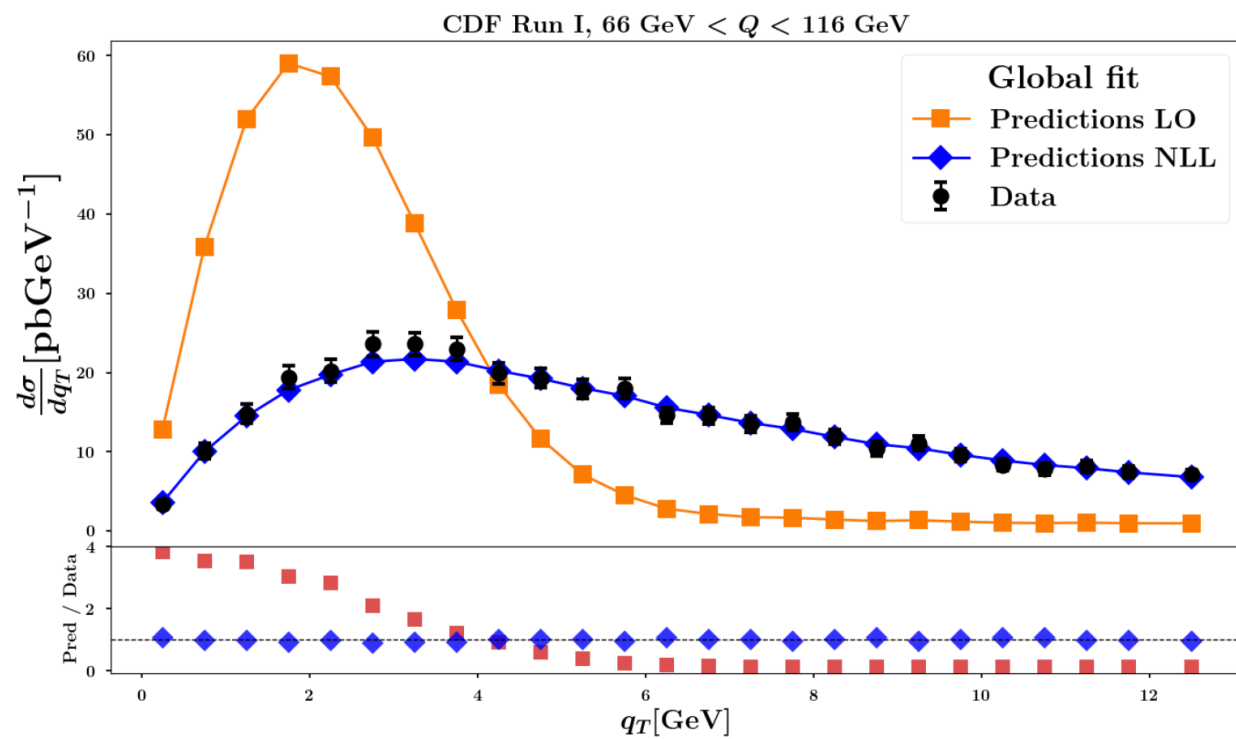
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At NLL we can fit CDF q_T -distribution

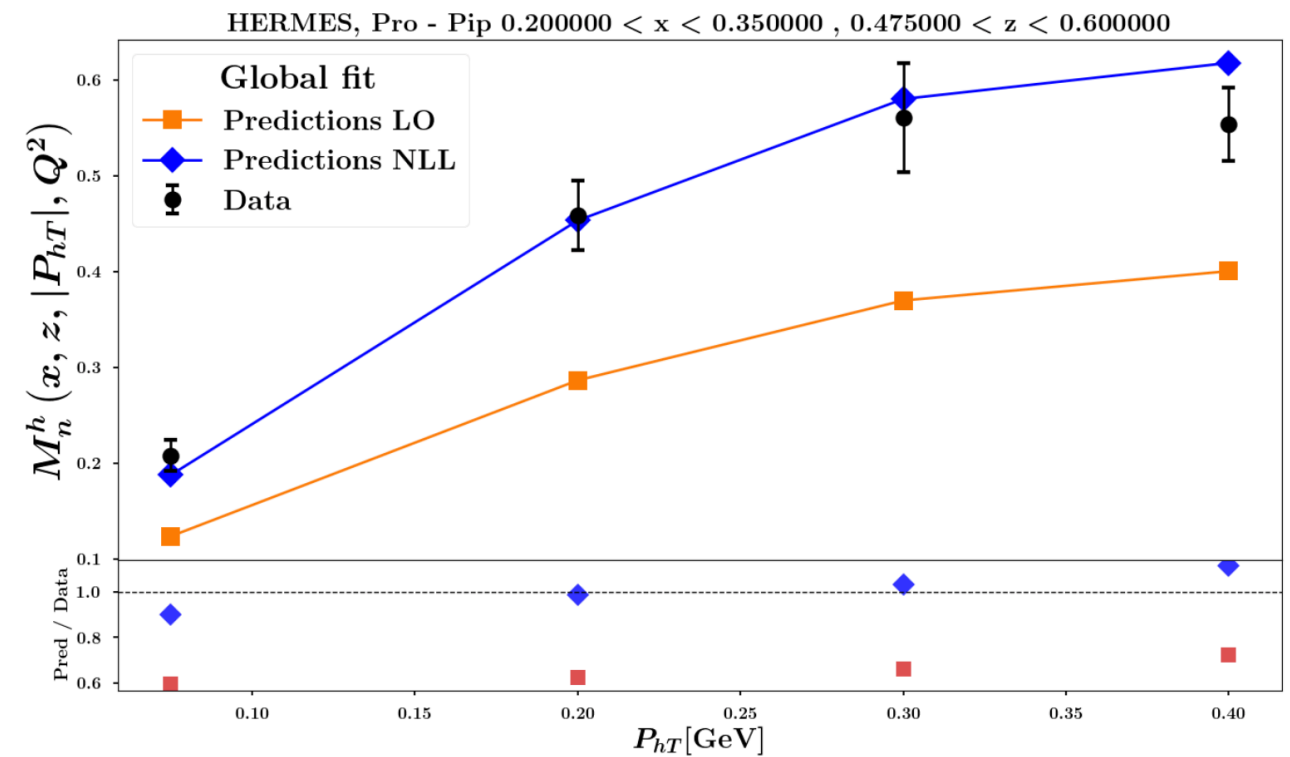
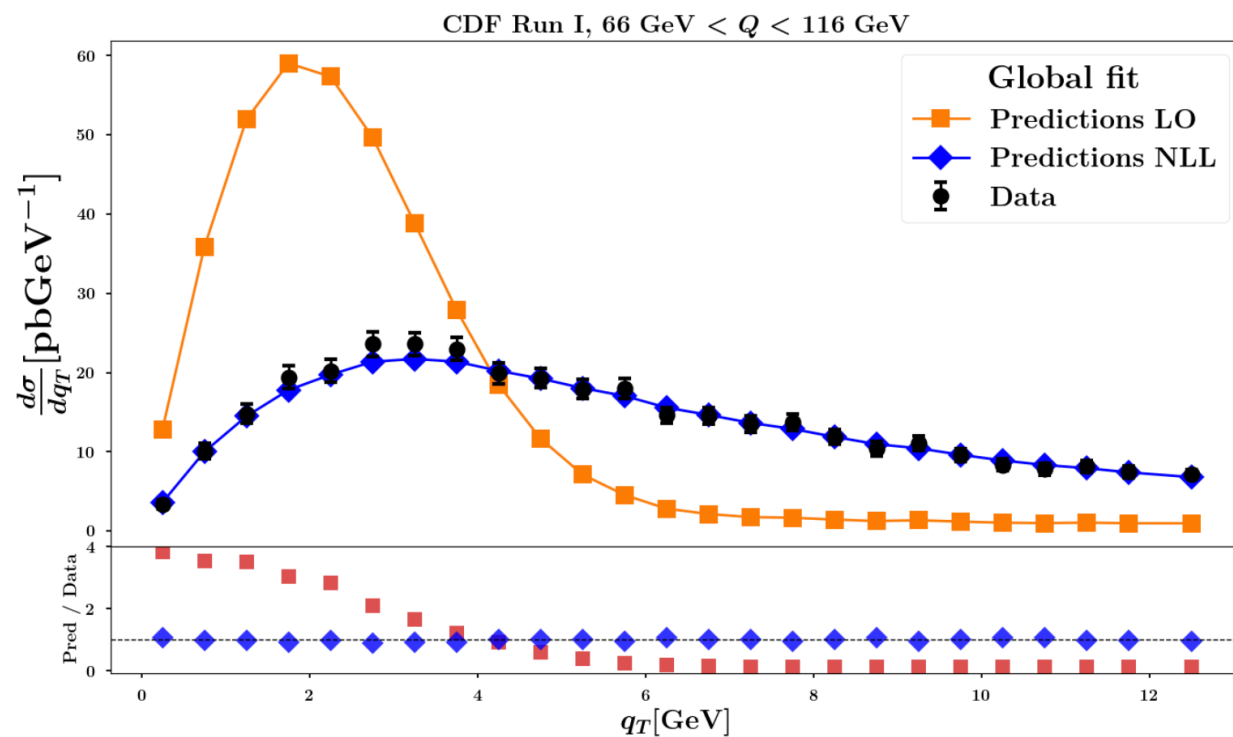


It was a cooking show, wasn't it?



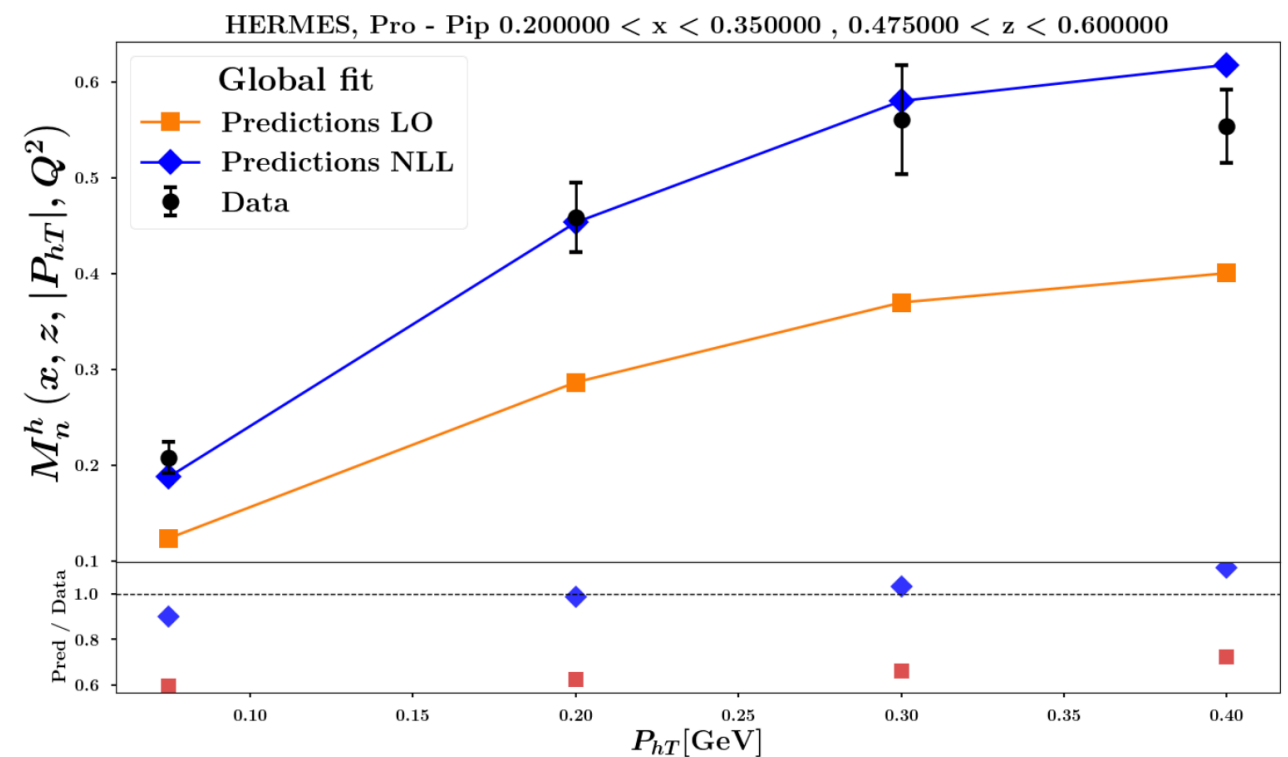
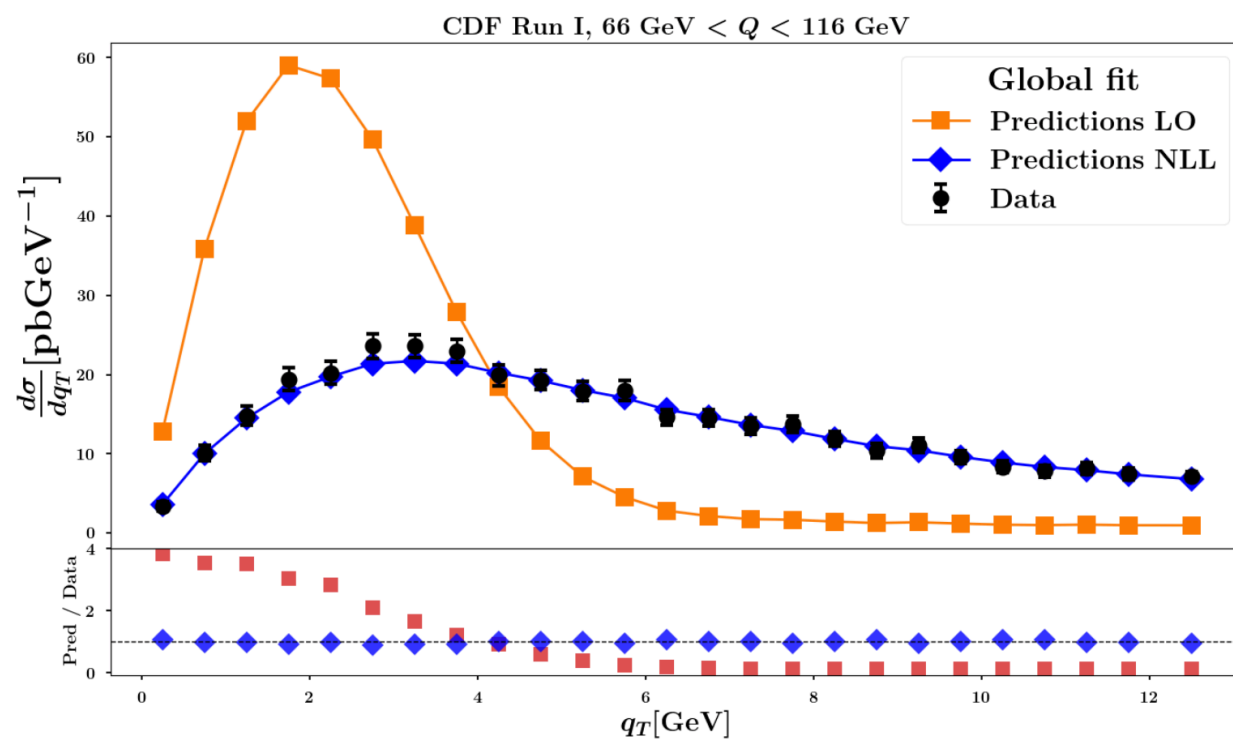
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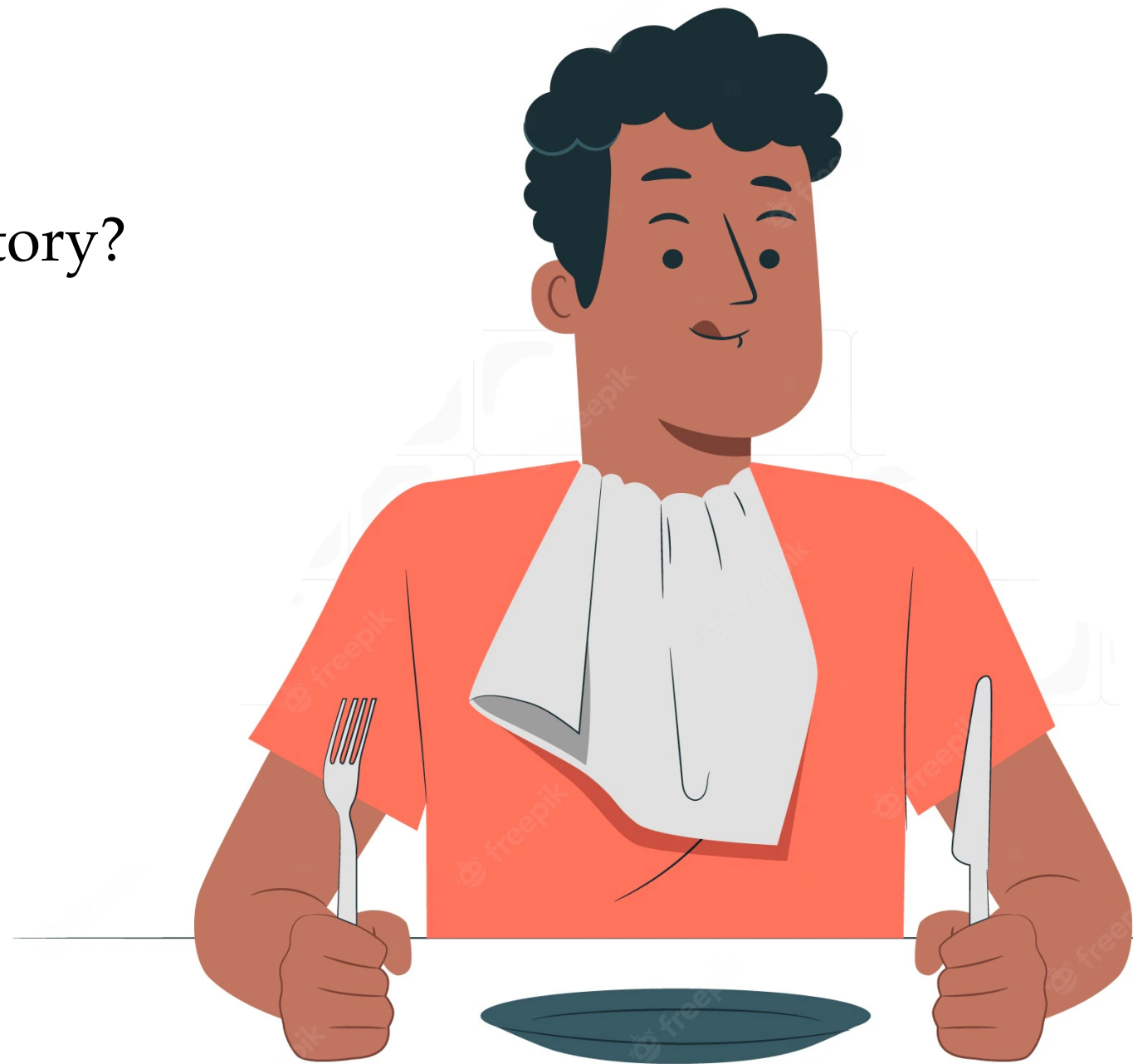


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EXERCISE:

Is it useful TMD formalism?

Are LO calculations satisfactory?



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The CSS formalism and TMD evolution are necessary for a good simultaneous description of high-energy DY and low-energy SIDIS experimental data



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