Istituto Nazionale di Fisica Nucleare

HAS QCD
HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

UNIVERSITÀ DI PAVIA

Cooking show: How to extract a TMD from a global fit

Matteo Cerutti — EIC_Net Summer School —- Corigliano Calabro - June 20th, 2023

Rules of the cooking show

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- A cooking show is NOT a lecture

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- I am going to USE formulas already discussed

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- I am going to USE formulas already discussed
- I am going to discuss WITH YOU such formulas

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DON'T GIVE UP!!

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- The aim of this cooking show is to discuss TOGETHER how to solve a "simple" exercise

Table of twist-2 TMDs

(A. Bacchetta lectures)

Quark Polarization

Nucleon Pol.	U	L	T
U			
L			
T			

Table of twist-2 TMDs

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Quark Polarization

Nucleon Pol.

	U	L	T
U	f_1		
L			
T			

Table of twist-2 TMDs

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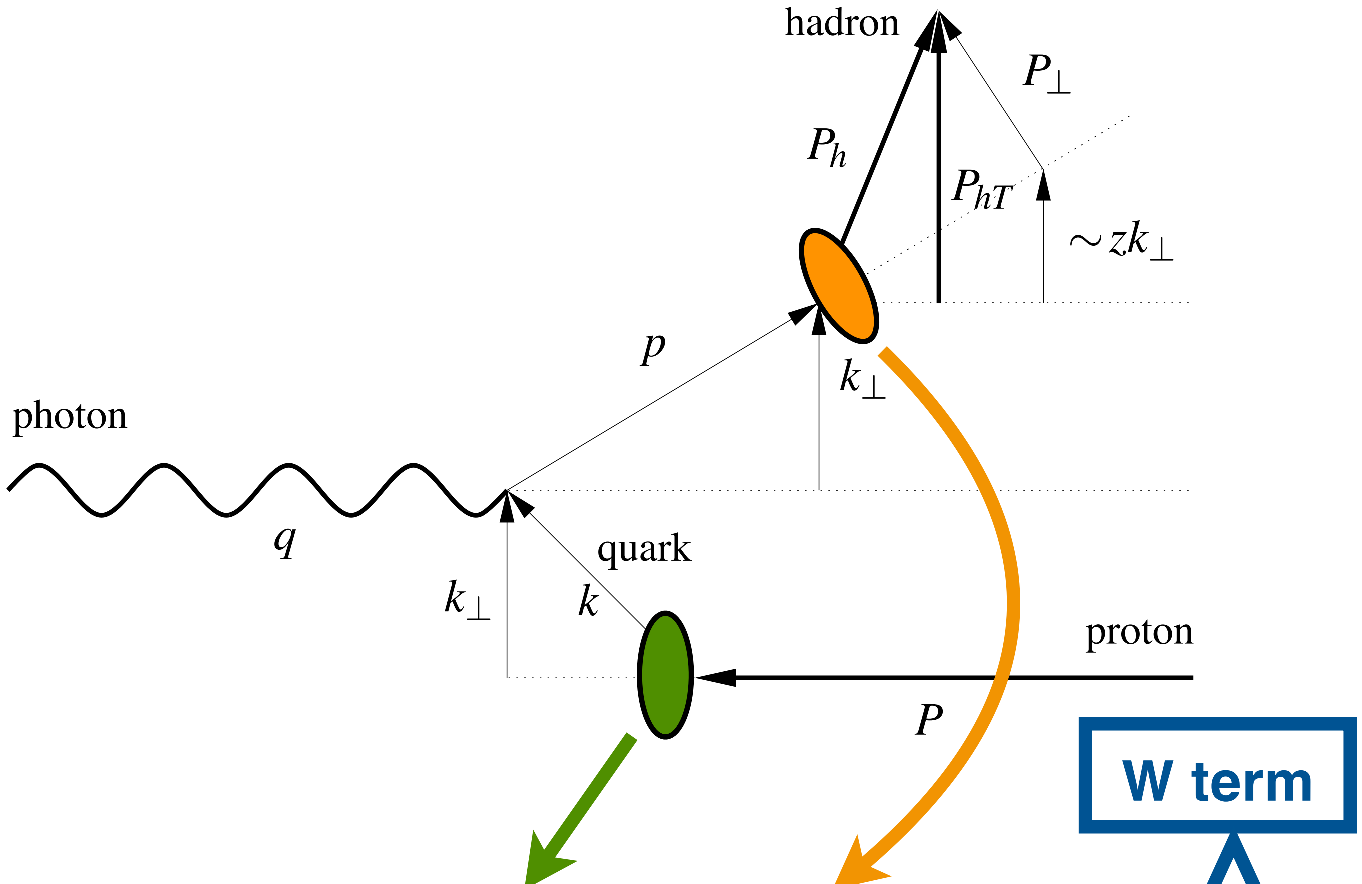
Quark Polarization

Nucleon Pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

TMD Factorization - SIDIS

(A. Bacchetta lectures)



$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

$$= x \sum_a \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

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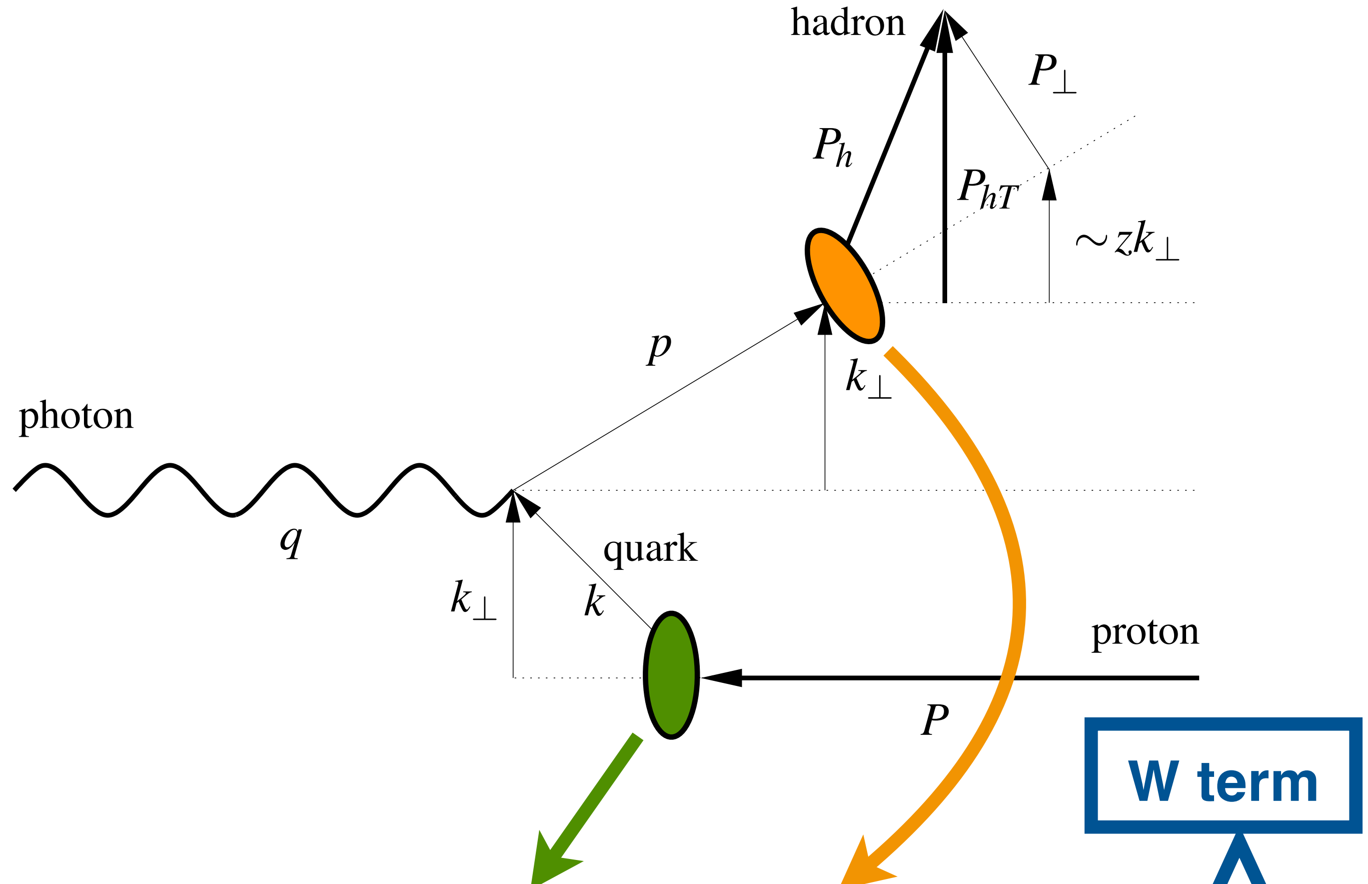

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$$= x \sum_a \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\boldsymbol{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

- The W term dominates in the region where qT ≪Q

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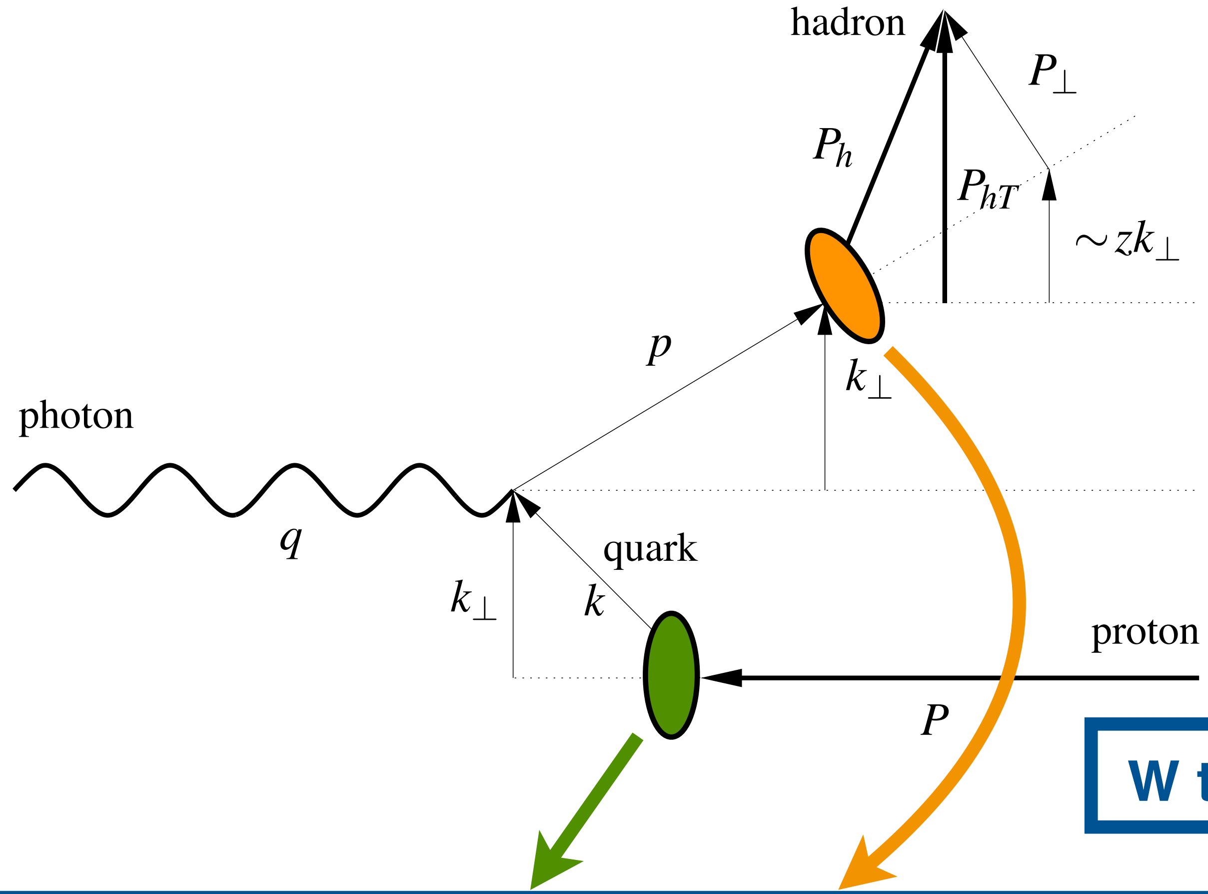


W term

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

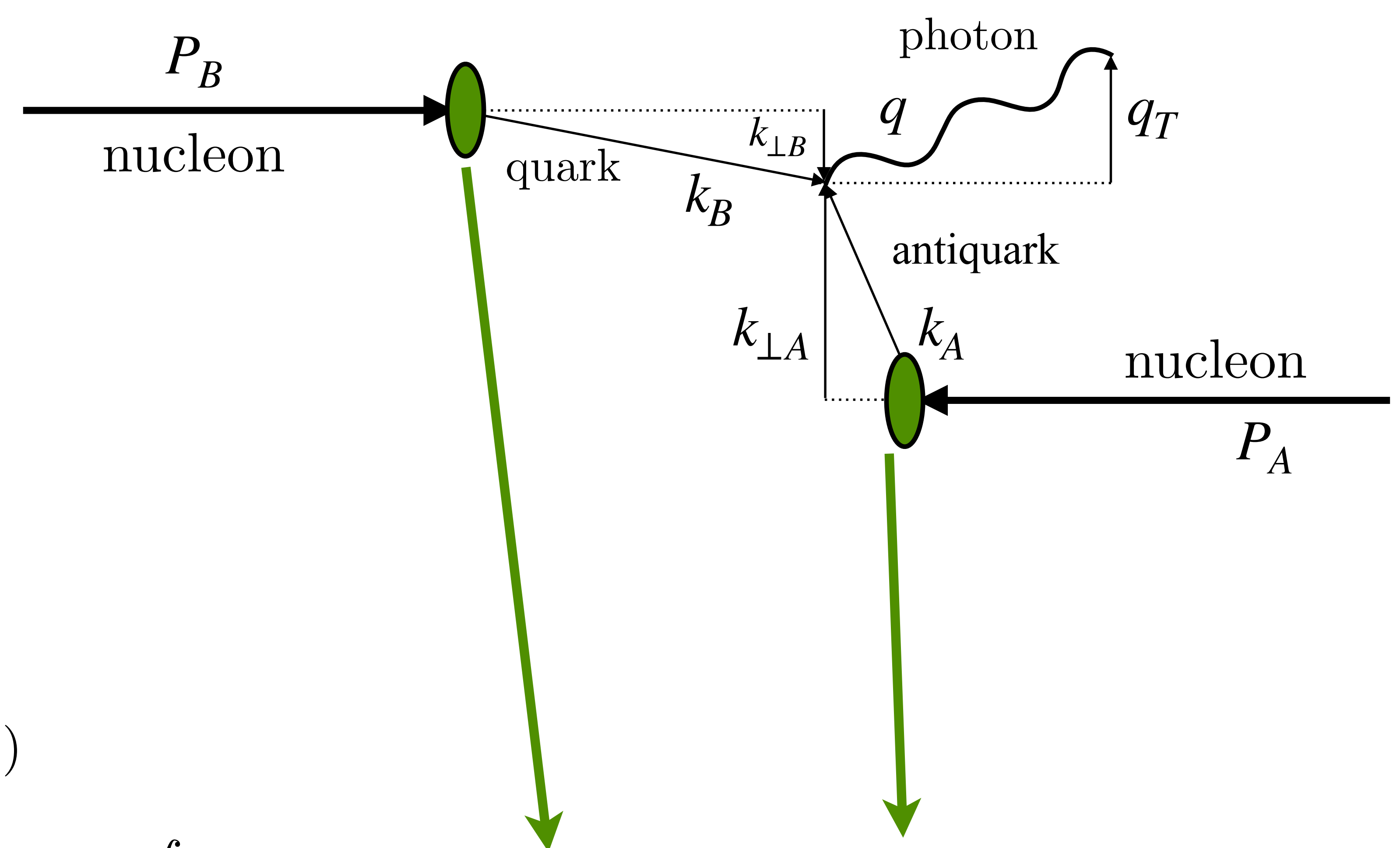
$$= x \sum_a \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

- The W term dominates in the region where qT ≪Q
- Y term is usually excluded in TMD analyses

TMD Factorization - Drell—Yan

(A. Bacchetta lectures)



$$F_{UU}^{1}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a\big(x_A, \boldsymbol{k}_{\perp A}^2; \mu^2\big)\, f_1^{\bar{a}}\big(x_B, \boldsymbol{k}_{\perp B}^2; \mu^2\big)\, \delta^{(2)}\big(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}\big)$$

$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{q}_T|) \hat{f}_1^q\big(x_A, b_T^2; \mu^2\big)\, \hat{f}_1^{\bar{q}}\big(x_B, b_T^2; \mu^2\big)$$

Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

Structure of a TMD

(A. Bacchetta lectures)

Fourier Transform in b_T-space

$$\tilde{f}_1(x, b_T^2; Q^2) = \int \frac{d^2 k_T}{(2\pi)^2} e^{i b_T \cdot k_T} f_1(x, k_T^2; Q^2)$$

J. Collins, “Foundation of Perturbative QCD”

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How to model a TMD distribution?

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$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_{b_*}) e^{S_{\rm pert}(b_*; \mu_{b_*}, \mu, \zeta)} e^{S_{\rm NP}(b_T^2; \zeta)} \hat{f}_{NP}^q(x, b_T^2)$$

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Matching coefficients (perturbatively calculable)

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Collinear PDF

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Collinear PDF

LHAPDF

https://lhapdf.hepforge.org

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(E. Tassi lectures)

J. Collins, "Foundation of Perturbative QCD"

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Sudakov form factor
(Perturbatively calculable)

J. Collins, "Foundation of Perturbative QCD"

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Structure of a TMD - TMD evolution

(A. Bacchetta lectures)

Evolution in 2 dimensions

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(\mu) \qquad \frac{\partial K(\mu)}{\partial \ln \mu} = -\gamma_K(\alpha_S(\mu))$$

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \mu} = \gamma(\mu, \zeta) \qquad \frac{\partial \gamma(\mu, \zeta)}{\partial \ln \sqrt{\zeta}} = -\gamma_K(\alpha_S(\mu))$$

Collins and Soper, Nucl. Phys. B193 (1981)
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Sudakov form factor = solution of evolution equations

$$\hat{f}_1(x, b_T; \mu, \zeta) = e^{S(b_T; \mu, \zeta)} \hat{f}_1(x, b_T; \mu_0, \zeta_0)$$

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J. Collins, "Foundation of Perturbative QCD"

$$e^{S(b_T;\mu,\zeta)} = \exp\left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(\alpha_S(\mu')) - \gamma_K(\alpha_S(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\}$$

$$\gamma_F(\alpha_S(\mu)) = \gamma(\mu, \mu^2)$$

Structure of a TMD - TMD evolution

(A. Bacchetta lectures)

$$\mu_0 = \sqrt{\zeta_0} = \mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

Structure of a TMD - TMD evolution

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Stewart, Tackmann, Walsh and Zuberi, Phys. Rev. D89 (2014)
Ebert and Tackmann, JHEP 02 (2017)

$$S_{\mathrm{pert}}(\mu_b, Q) = 1 + \sum_{k=0}^{\infty} R_{\mathrm{N}^k\mathrm{LL}}$$

$$S_{\mathrm{pert}}(\mu_b, Q) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(Q)}{4\pi}\right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n,2n-k)} \qquad L = \ln\left(\frac{Q^2}{\mu_b^2}\right)$$

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)

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Accuracy	H and C	K and γ_F	γ_K	α_S evol.
LL	0	-	1	-
NLL	0	1	2	NLO
NLL’	1	1	2	NLO
NNLL	1	2	3	NNLO
NNLL’	2	2	3	NNLO
N^3LL	2	3	4	N^3LO
N^3LL’	3	3	4	N^3LO

Structure of a TMD

(A. Bacchetta lectures)

Fourier Transform in b_T-space

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How to model a TMD distribution?

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$$\mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$$

Model-dependent function

J. Collins, "Foundation of Perturbative QCD"

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Model-dependent function

FIT to data

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T} \xrightarrow{b_T \gg 1} 0$$

Structure of a TMD - NP content

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$$\mu_b = \frac{2e^{-\gamma_E}}{b_T} \xrightarrow{b_T \gg 1} 0 \qquad \boxed{\alpha_S(\mu_b) \to +\infty}$$

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\infty \xleftarrow{b_T \ll 1} \mu_b = \frac{2e^{-\gamma_E}}{b_T} \xrightarrow{b_T \gg 1} 0 \qquad \boxed{\alpha_S(\mu_b) \to +\infty}$$

Structure of a TMD - NP content

(A. Bacchetta lectures)

$$\boxed{\mu_b > Q} \quad \infty \quad \xleftarrow{b_T \ll 1} \quad \mu_b = \frac{2e^{-\gamma_E}}{b_T} \quad \xrightarrow{b_T \gg 1} \quad 0 \quad \boxed{\alpha_S(\mu_b) \to +\infty}$$

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b_*-prescription

Structure of a TMD - NP content

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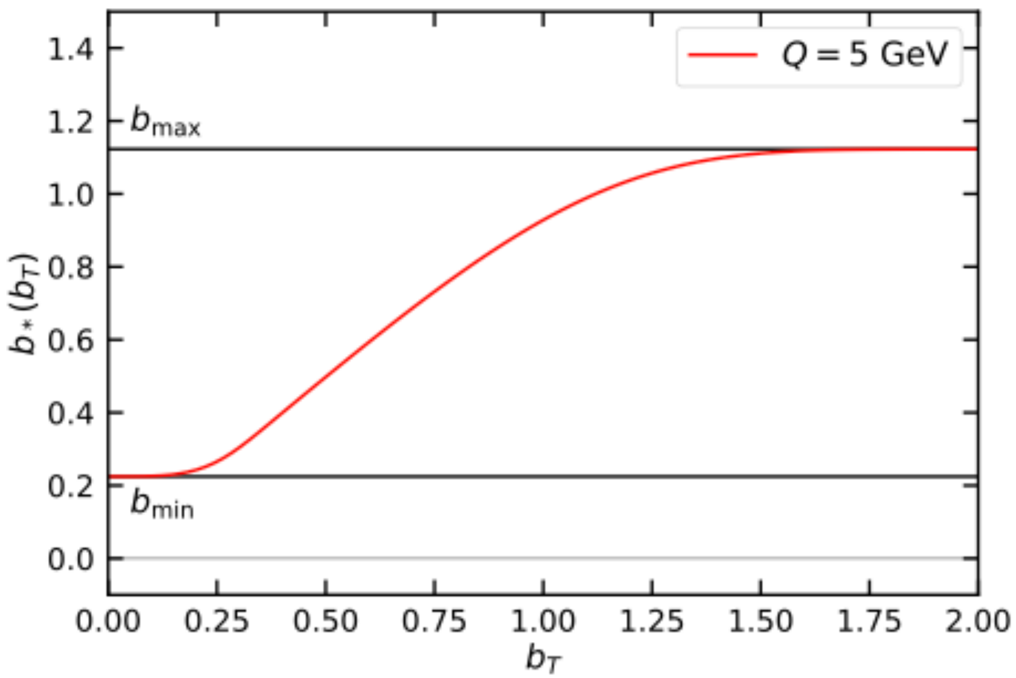
$$b_*(b_T) = b_{\max}\left(\frac{1-\exp\left(-\frac{b_T^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b_T^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{Q} \qquad b_{\max} = 2e^{-\gamma_E}$$



J.C.Collins, D.E.Soper,and G.F.Sterman, Nucl.Phys. **B250**,199 (1985)

J.C.Collins, L.Gamberg, et al., Phys.Rev.D 94 (2016)

A.Bacchetta, M.G.Echevarria, P.J.G.Mulders, M.Radici, and A.Signori, JHEP **11**, 076 (2015)

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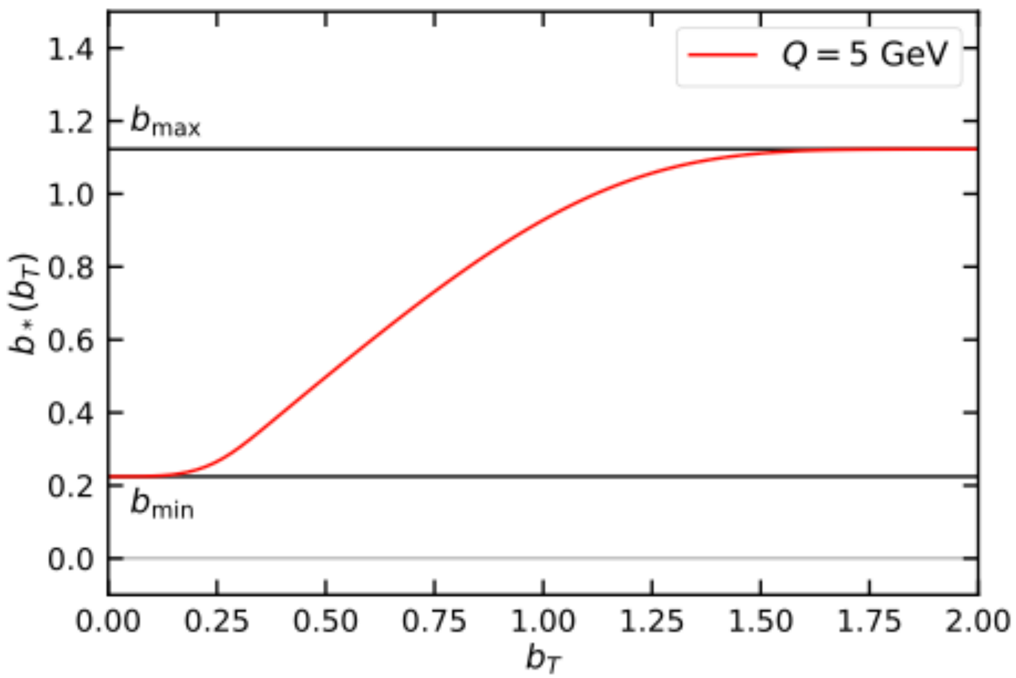

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A.Bacchetta, M.G.Echevarria, P.J.G.Mulders, M.Radici, and A.Signori, JHEP ***11****, 076 (2015)*

$$\hat{f}_1(x, b_T; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T); \mu, \zeta)}\right]\hat{f}_1(x, b_*(b_T); \mu, \zeta) \equiv f_{\rm NP}(x, b_T; \zeta)\hat{f}_1(x, b_*(b_T); \mu, \zeta)$$

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$$\boxed{\mu_b > Q} \quad \infty \xleftarrow{b_T \ll 1} \mu_b = \frac{2e^{-\gamma_E}}{b_T} \xrightarrow{b_T \gg 1} 0 \quad \boxed{\alpha_S(\mu_b) \to +\infty}$$

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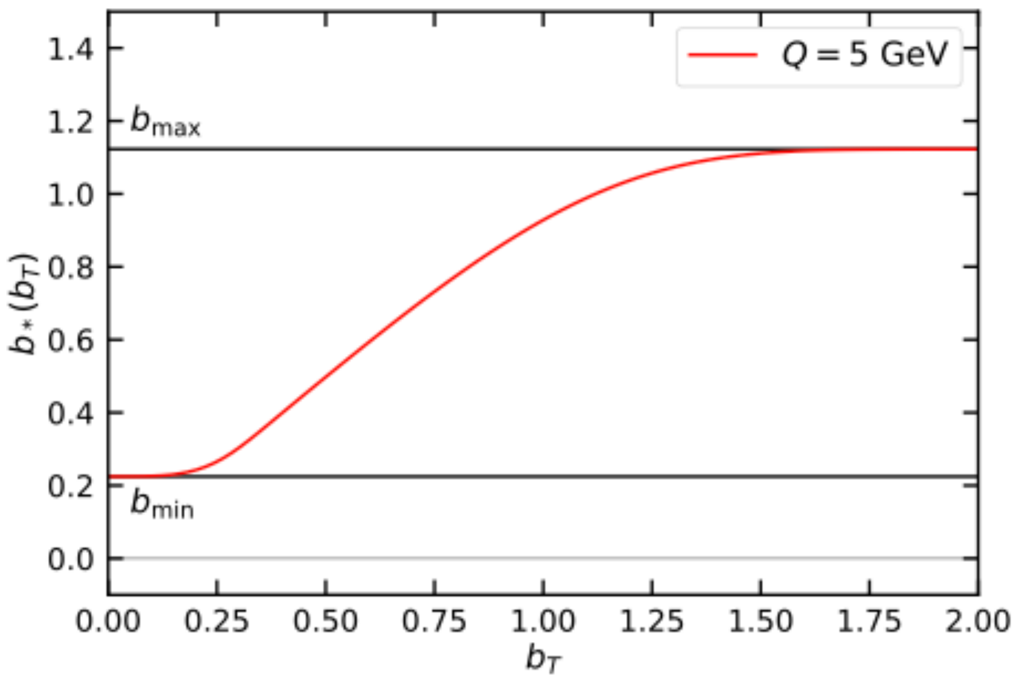

J.C.Collins, D.E.Soper,and G.F.Sterman, Nucl.Phys. ***B250****,199 (1985)*

J.C.Collins, L.Gamberg, et al., Phys.Rev.D 94 (2016)

A.Bacchetta, M.G.Echevarria, P.J.G.Mulders, M.Radici, and A.Signori, JHEP ***11****, 076 (2015)*

$$\hat{f}_1(x, b_T; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T); \mu, \zeta)}\right]\hat{f}_1(x, b_*(b_T); \mu, \zeta) \equiv \boxed{f_{\mathrm{NP}}(x, b_T; \zeta)}\hat{f}_1(x, b_*(b_T); \mu, \zeta)$$

GLOBAL FIT

Is it worth it??!

CSS formalism (NLL)

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$$\hat{f}_1(x, b_T^2; \mu, \zeta) = f_1(x, \mu_{b_*}) e^{S_{\mathrm{pert}}(b_*; \mu_{b_*}, \mu, \zeta)} \times e^{S_{\mathrm{NP}}(b_T^2; \zeta)} \hat{f}^q_{NP}(x, b_T^2)$$

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Parton Model (LO)

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CSS formalism (NLL)

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SIDIS unpolarized structure function

CSS formalism (NLL)

(A. Bacchetta lectures)

SIDIS unpolarized structure function

$$F_{UU,T}(x,z,P_{hT},Q) = x \sum_q e_q^2 \int db_T b_T J_0(b_T\, P_{hT}) \hat{f}_{1\,q/N}(x, b_T^2; Q, Q^2)$$

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CSS formalism (NLL)

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$$\hat{D}_1(z,b_T^2;\mu,\zeta) = D_1(z,\mu_{b_*}) e^{S_{\rm pert}(b_*;\mu_{b_*},\mu,\zeta)} \times e^{S_{\rm NP}(b_T^2;\zeta)} \hat{D}_{NP}(z,b_T^2)$$

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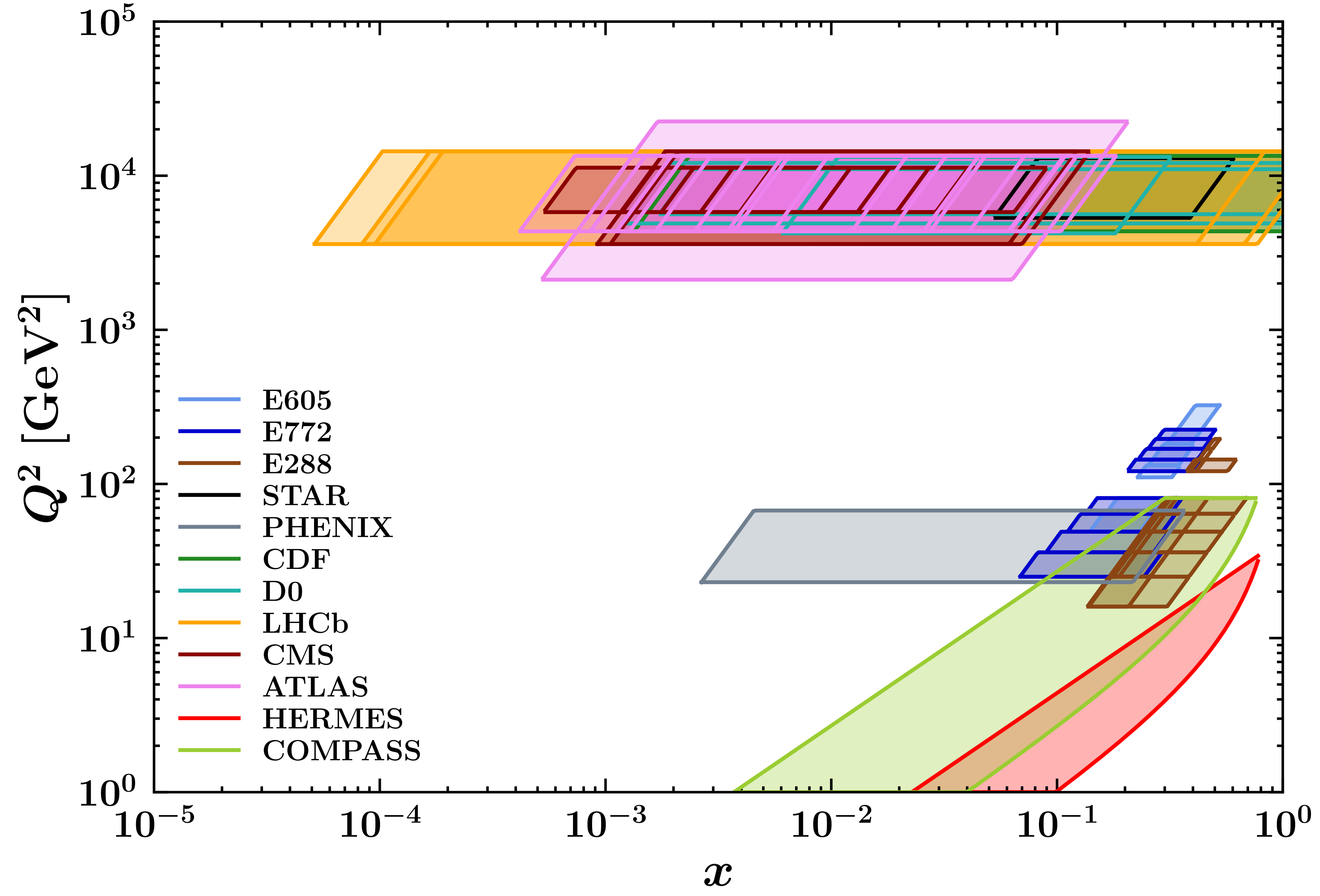
Phenomenology

Experimental data



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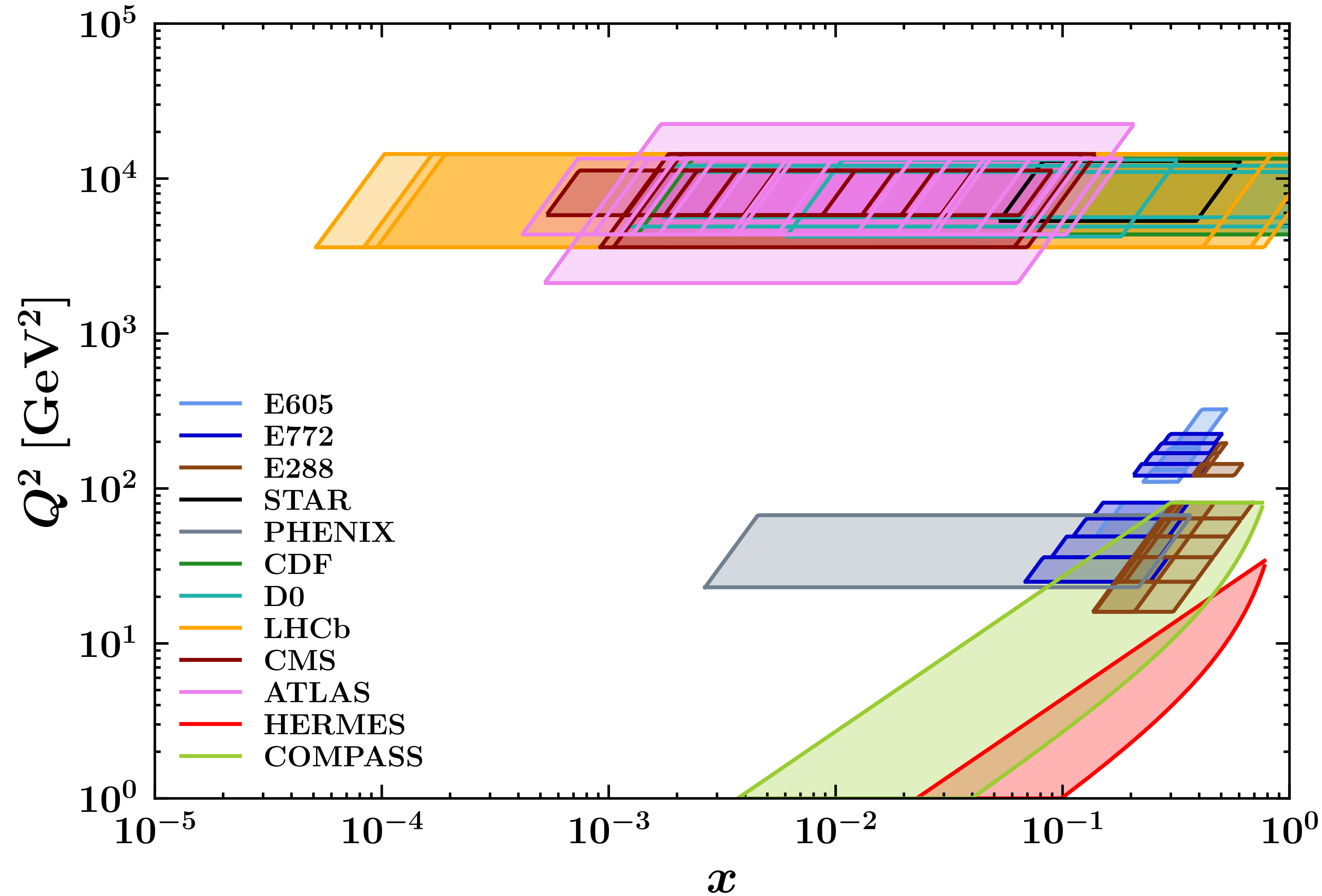
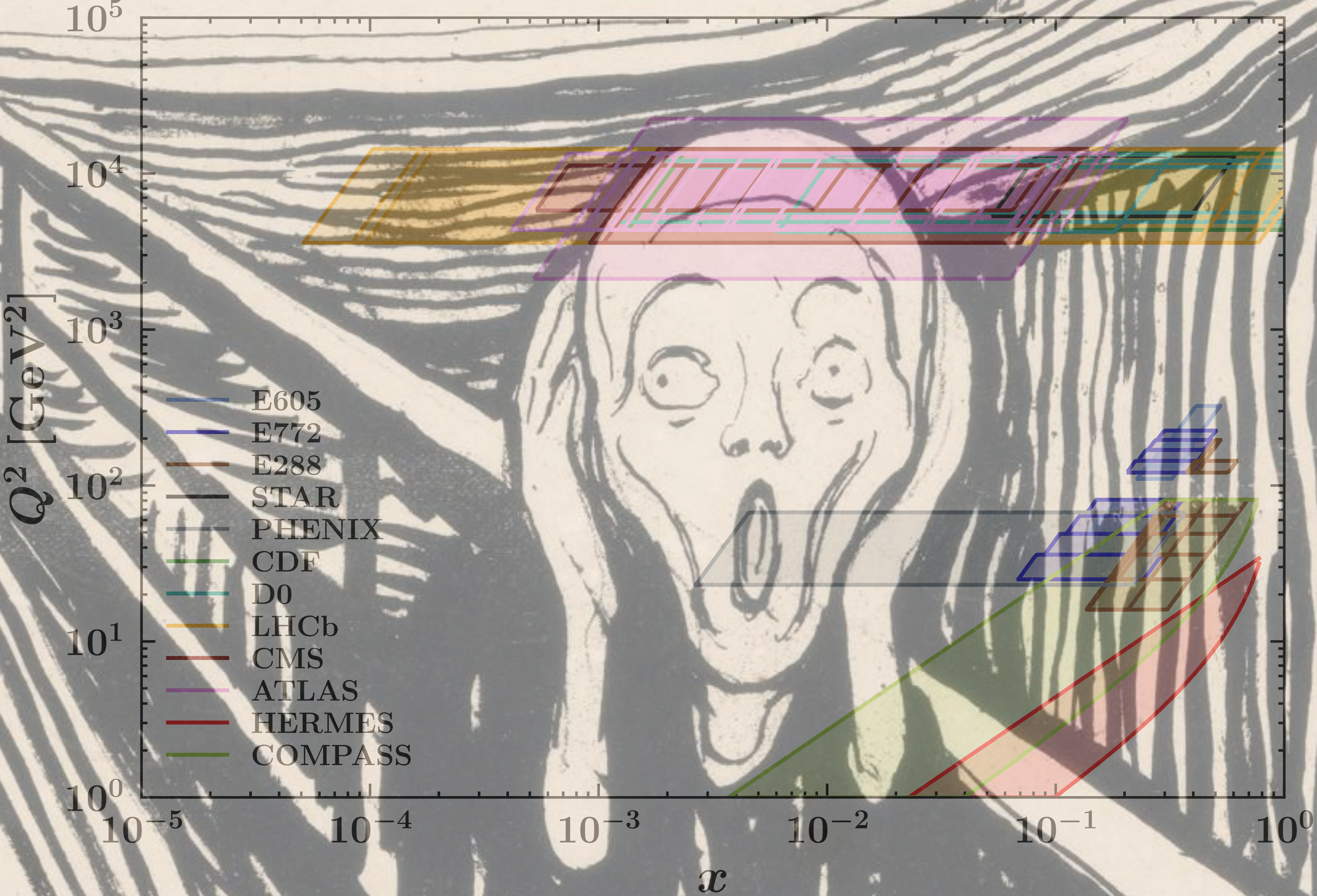


484 (DY) + 1547 (SIDIS)
2031 fitted experimental points

Experimental data



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2031 fitted experimental points

Nonperturbative parameterizations

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

A. Bacchetta, F. Conti and M. Radici, Phys. Rev. D 78 (2008) 074010

A. Bacchetta, L.P. Gamberg, G.R. Goldstein and A. Mukherjee, Phys. Lett. B 659 (2008) 234

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11 parameters for TMD PDF
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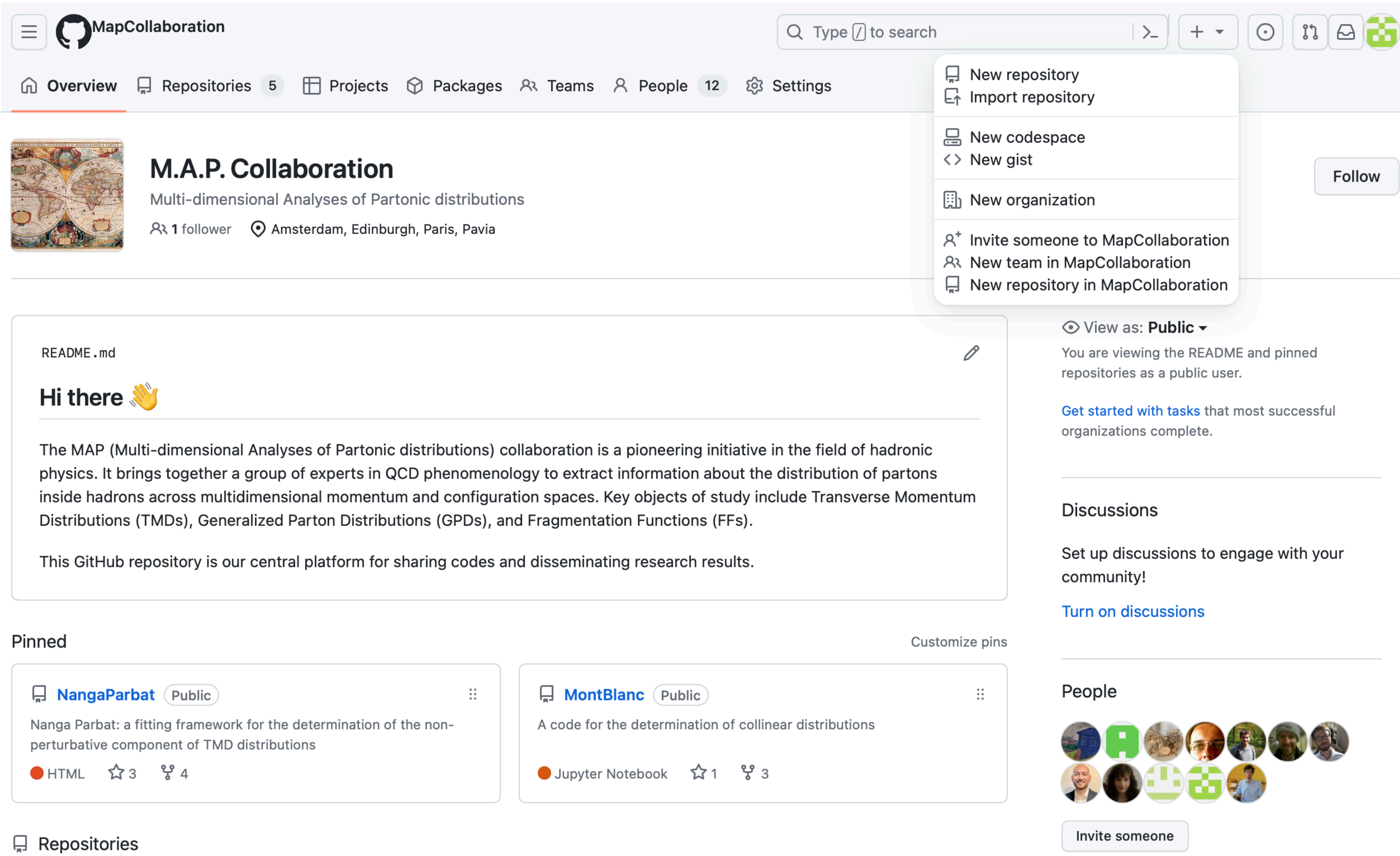
KEEP
CALM
AND
CODE
ON


HOW?

Ask the MAP Collaboration

https://github.com/MapCollaboration



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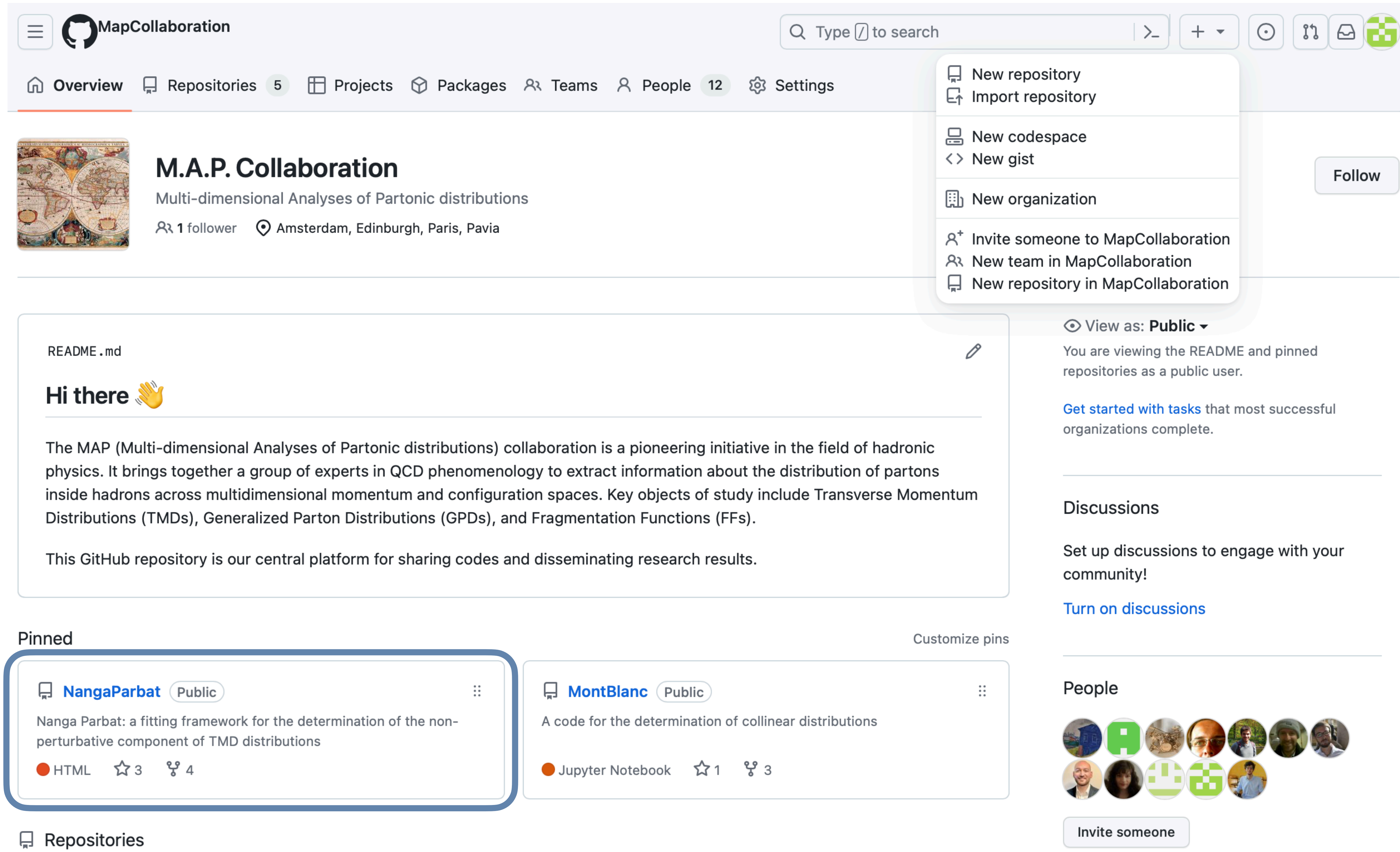


NangaParbat fitting framework

https://github.com/MapCollaboration/NangaParbat

README.md

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

NangaParbat fitting framework

git clone

Dependencies

In order to install the code a number of external but relatively standard libraries are required. Here is the list:

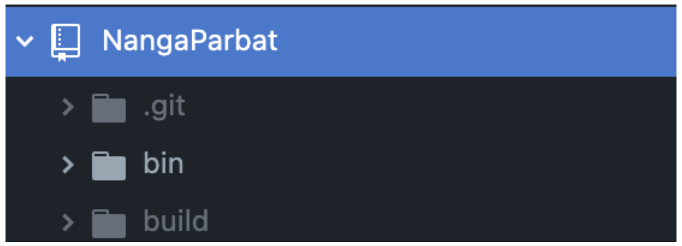
- `yaml-cpp`,
- `eigen3`,
- `ceres-solver`,
- `GSL`,
- `ROOT` with `Minuit2`,
- `LHAPDF6`,
- `APFEL++`.

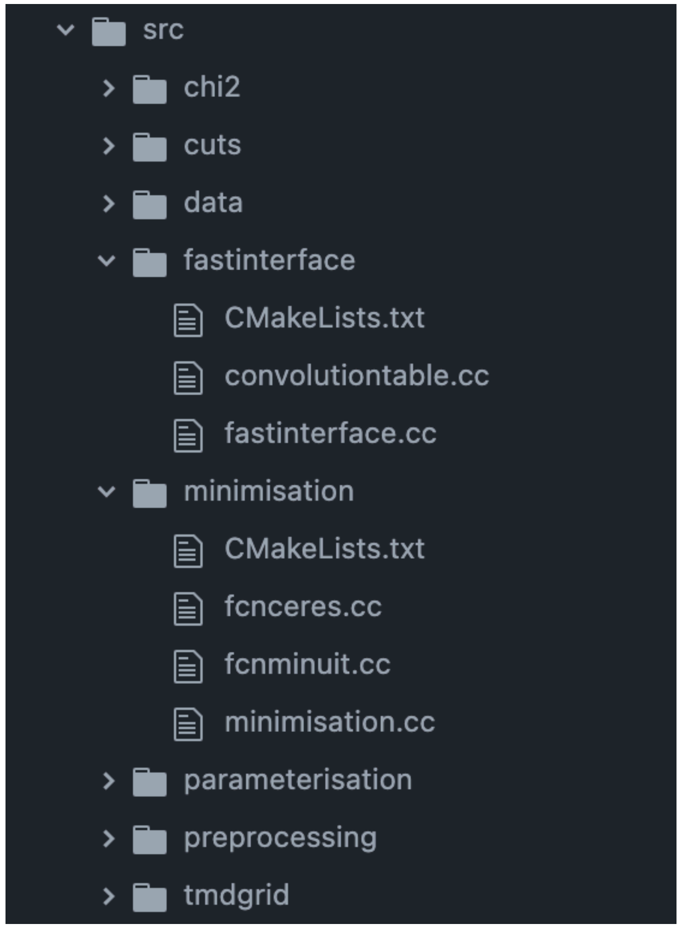
Most of these libraries can be installed through standard package managers such as `Homebrew` on MacOS and `apt-get` on Linux.

NangaParbat also has a **lite version**, that does not require all the dependencies listed above. The mandatory dependencies are `yaml-cpp`, `eigen3`, `LHAPDF6`, `APFEL++`.

During the installation, NangaParbat detects if one or more libraries among `GSL`, `ROOT` or `ceres-solver` are missing and installs and compiles only the part of the code that is possible to run.

Implemented strategy to perform a fit





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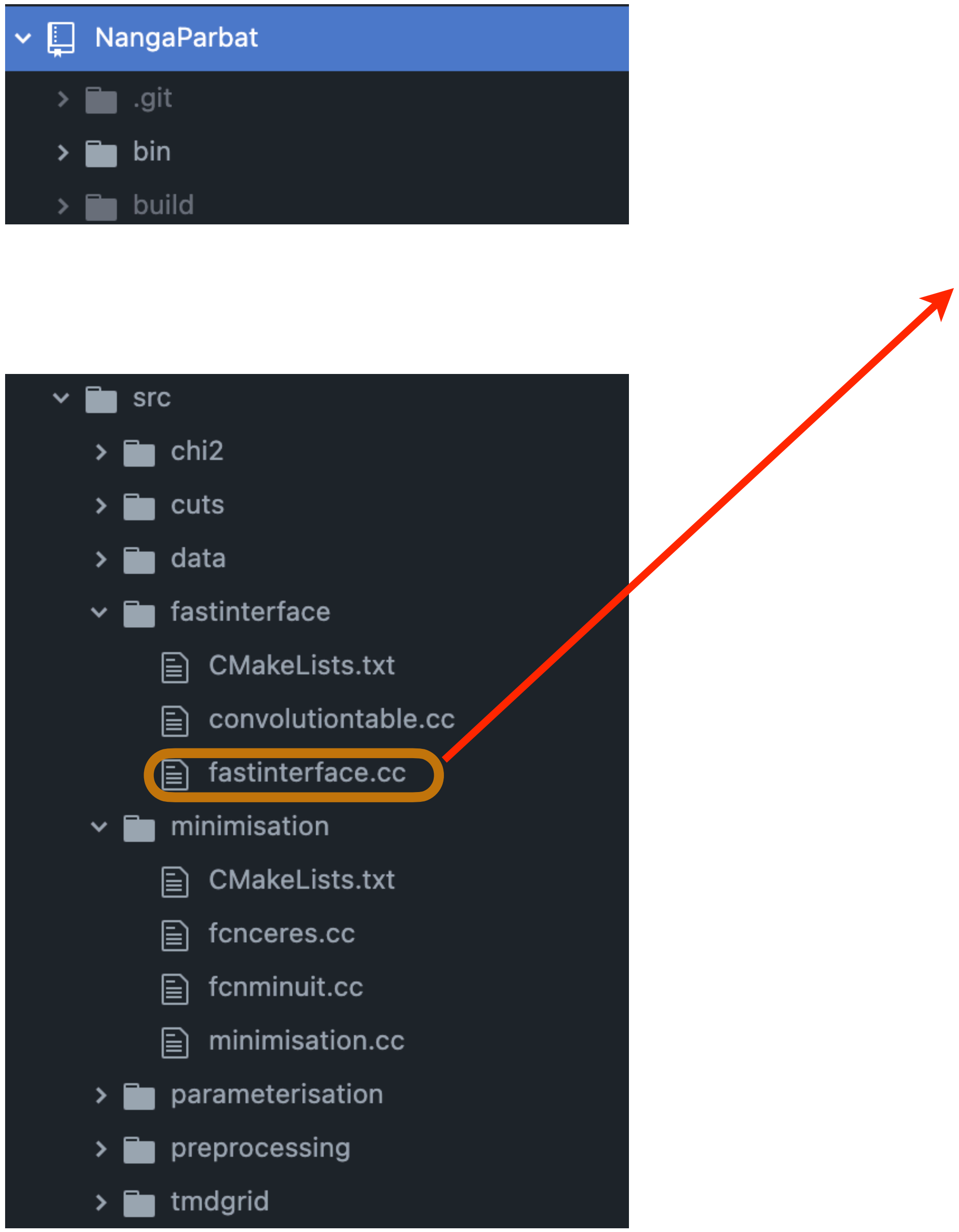
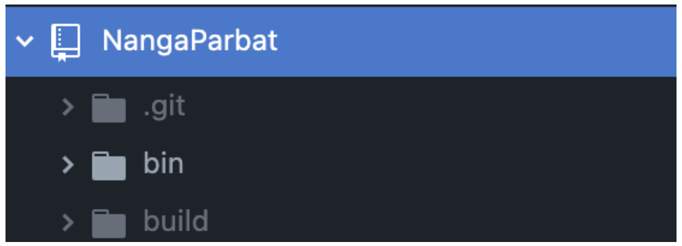
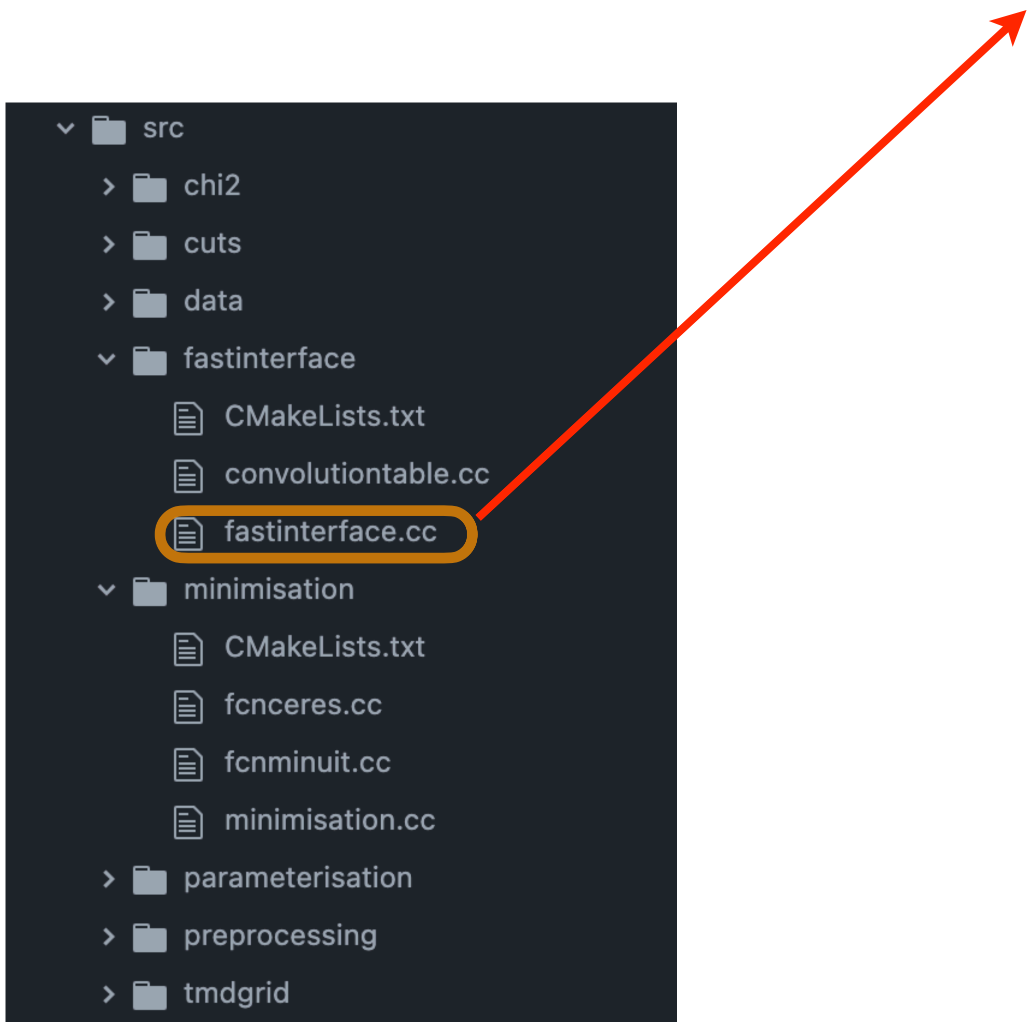


Generation of interpolation tables

Implemented strategy to perform a fit





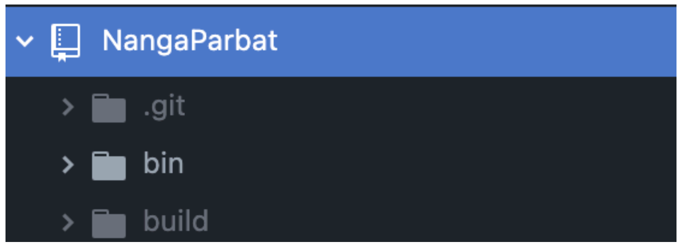
Generation of interpolation tables

Kinematical prefactors

Collinear distributions and evolution

TMD evolution

Implemented strategy to perform a fit



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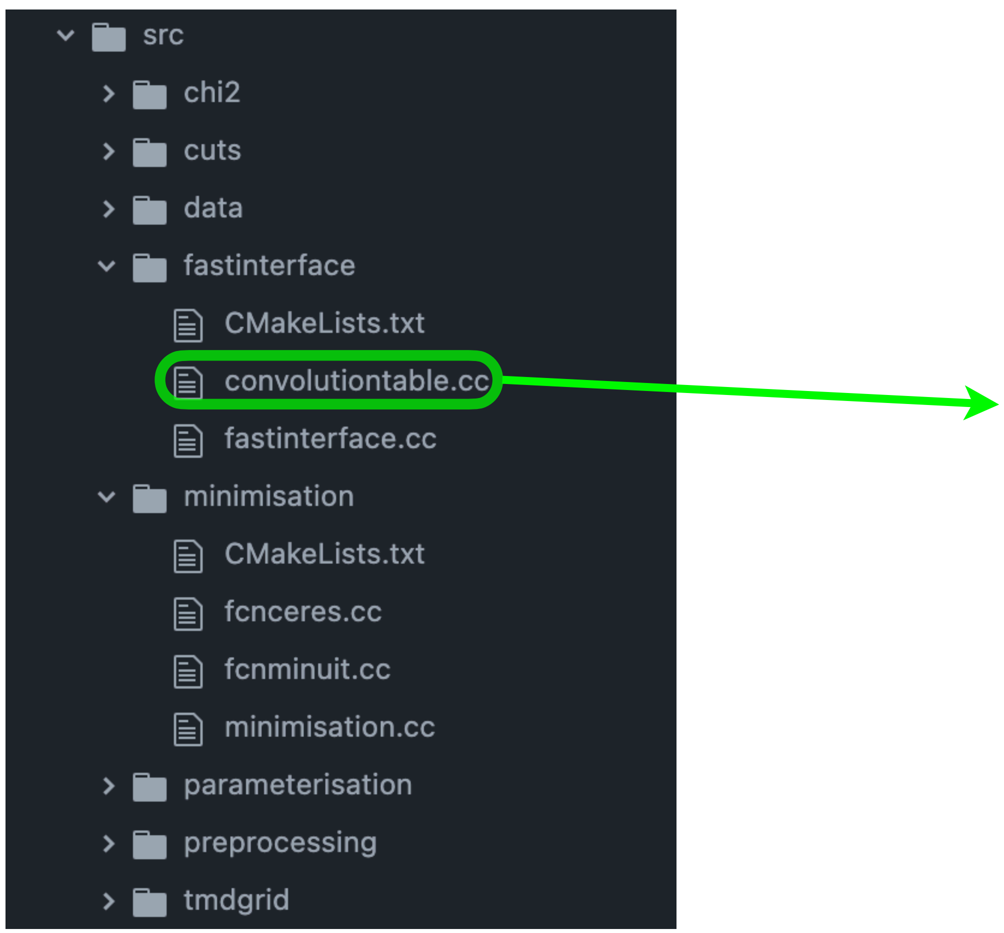
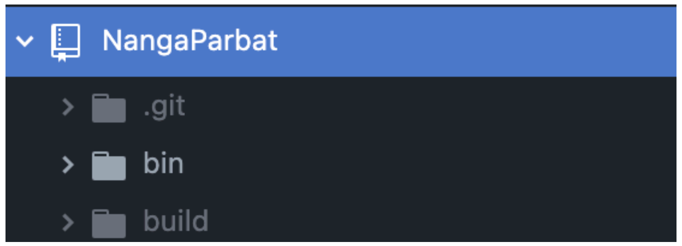
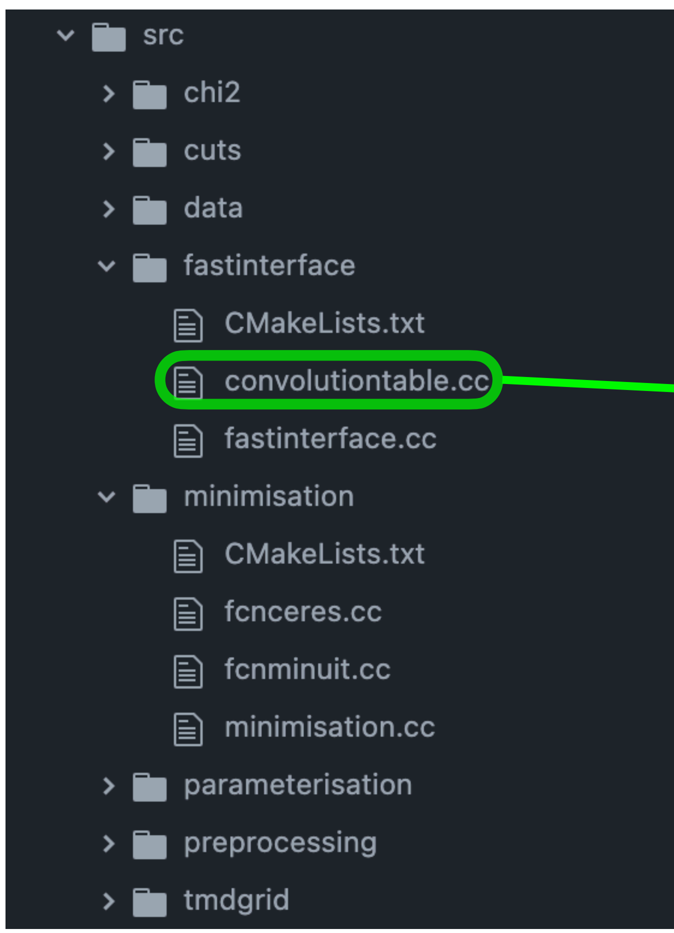


Calculation of the prediction

Implemented strategy to perform a fit





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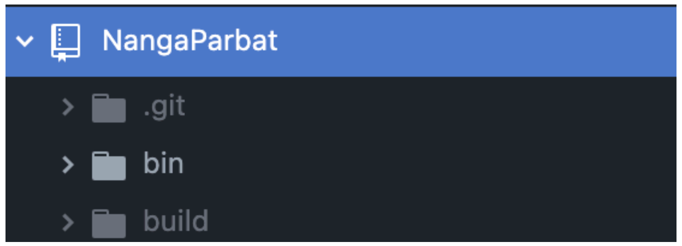

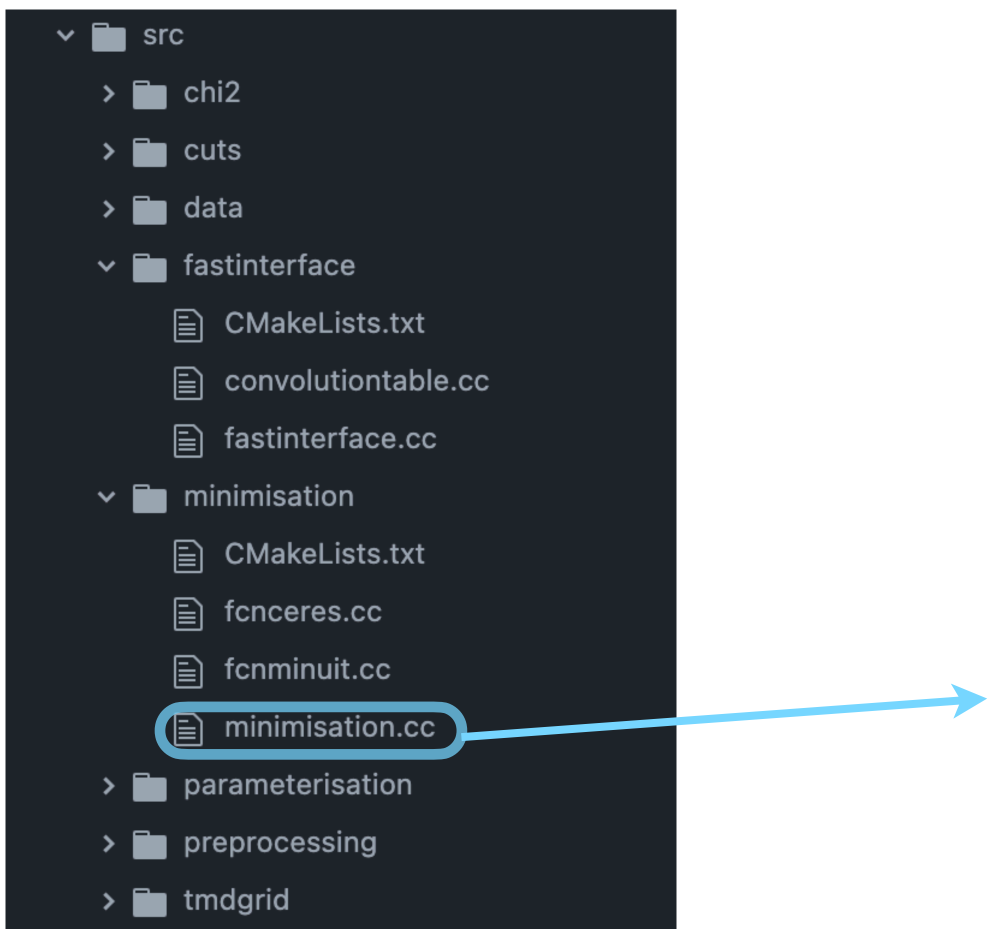

Generation of interpolation tables

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Collinear distributions and evolution

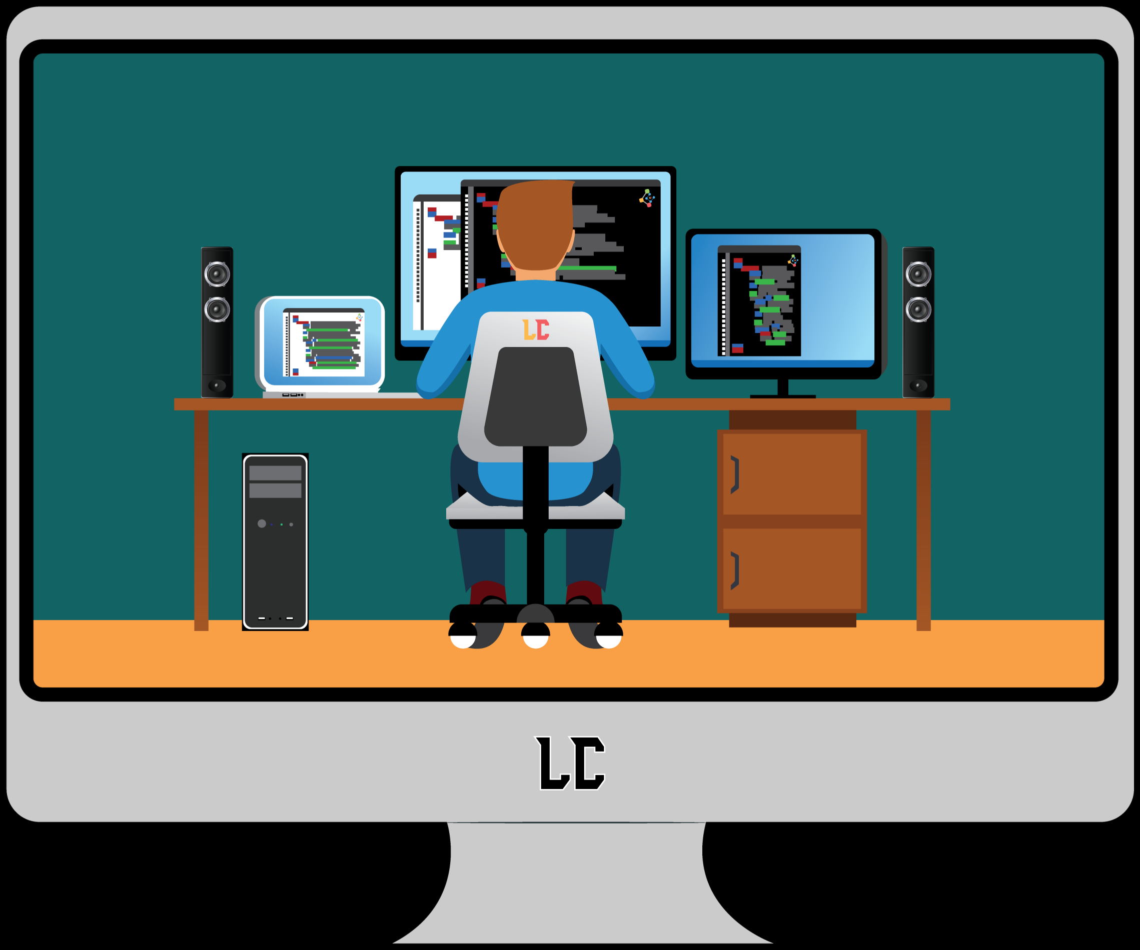
TMD evolution

Calculation of the prediction

NP model

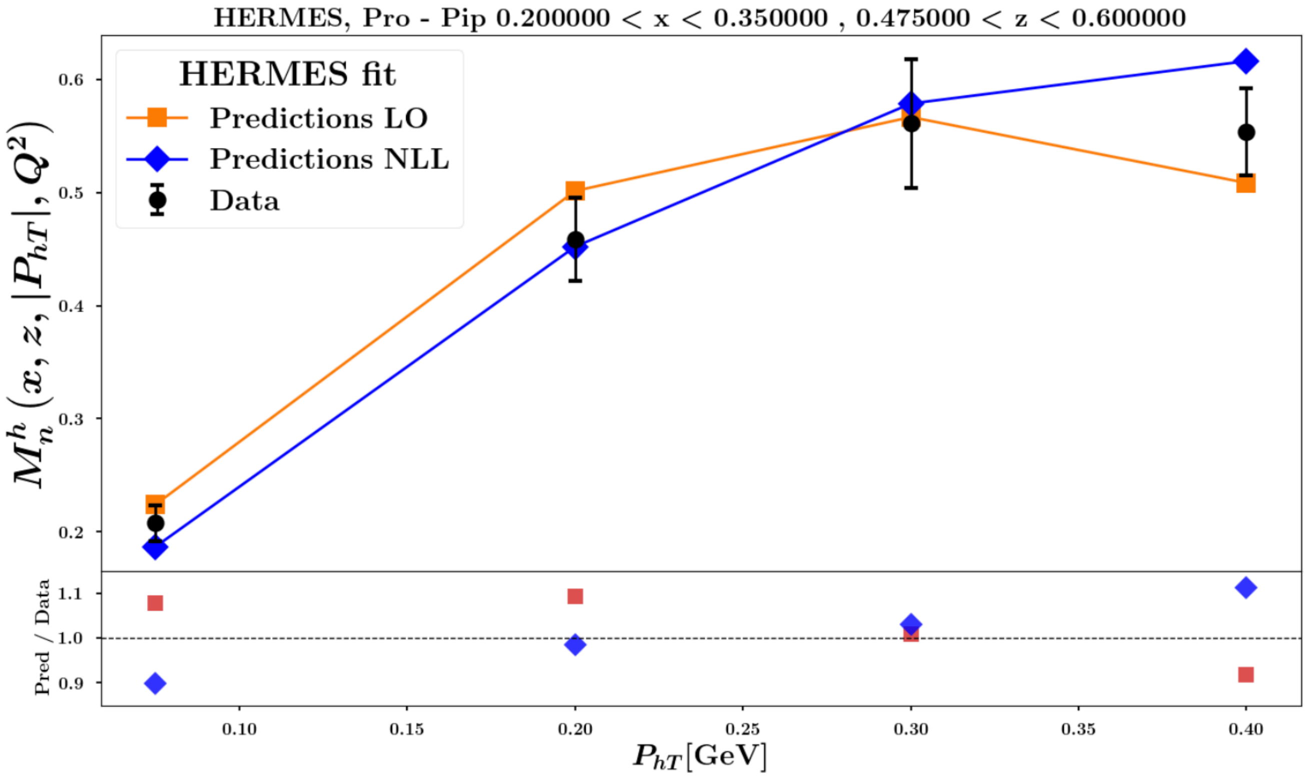
Minimization of chi-squared function

LET'S
</CODE>




Results

It was a cooking show, wasn't it?



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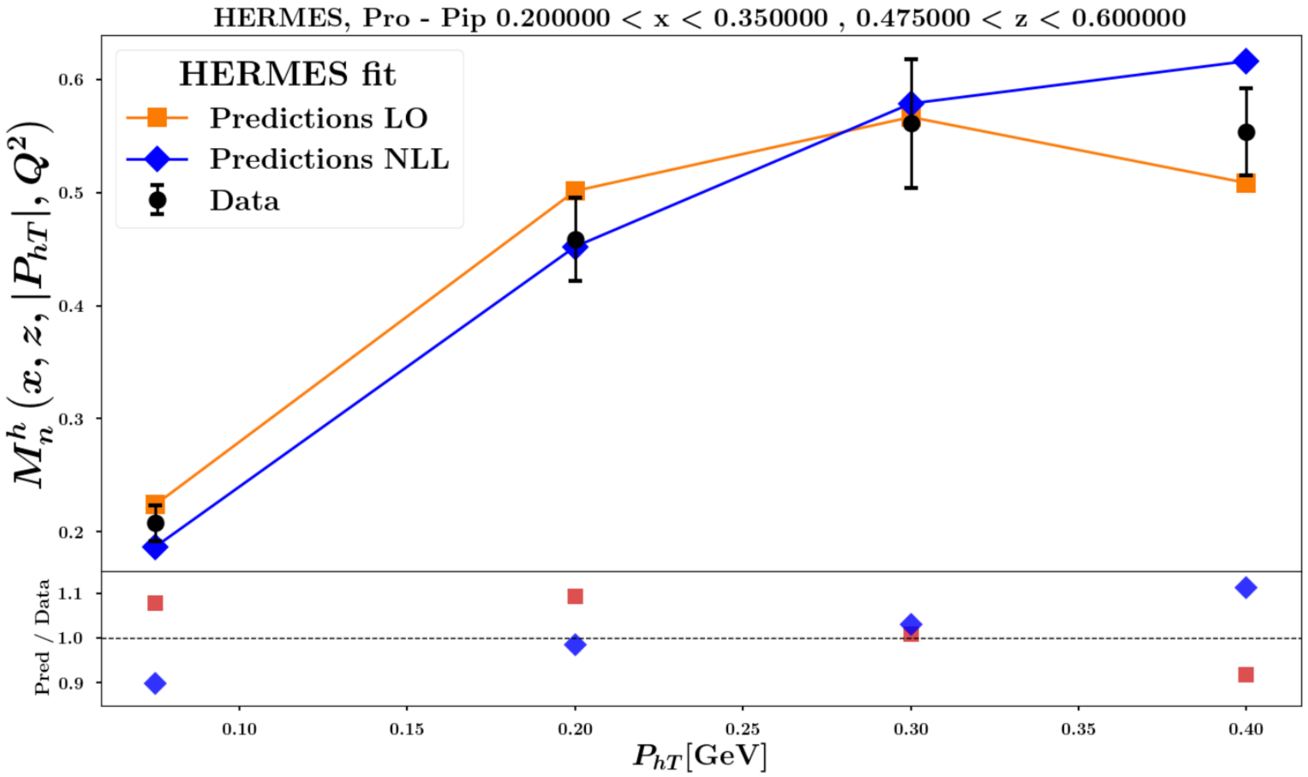
At LO we can fit HERMES multiplicities



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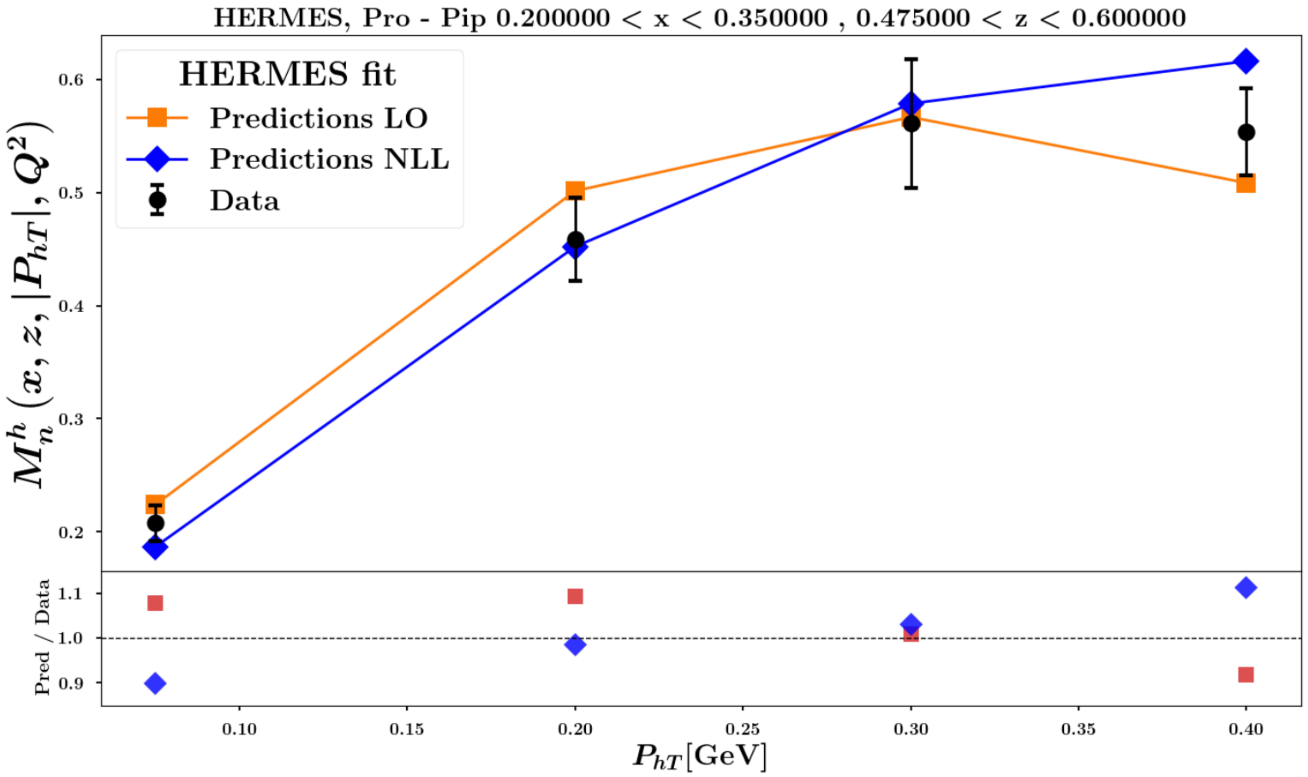

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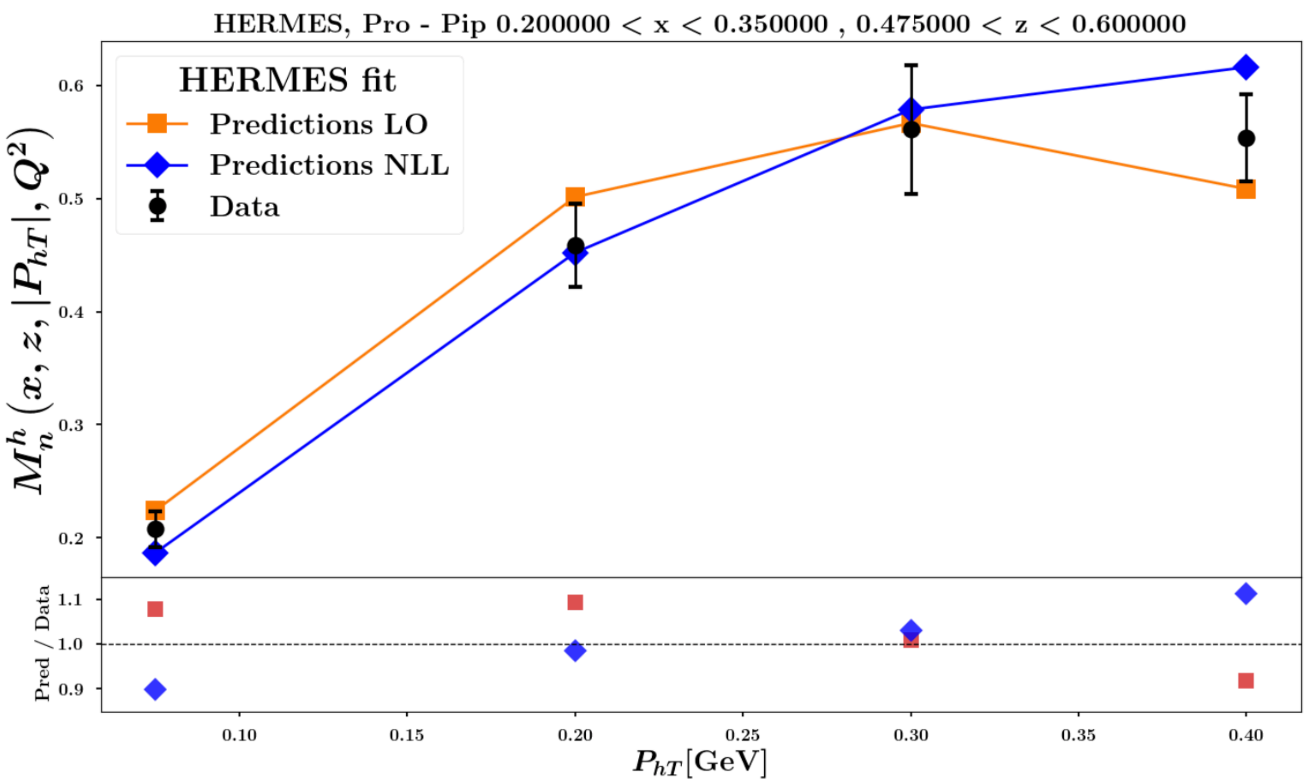

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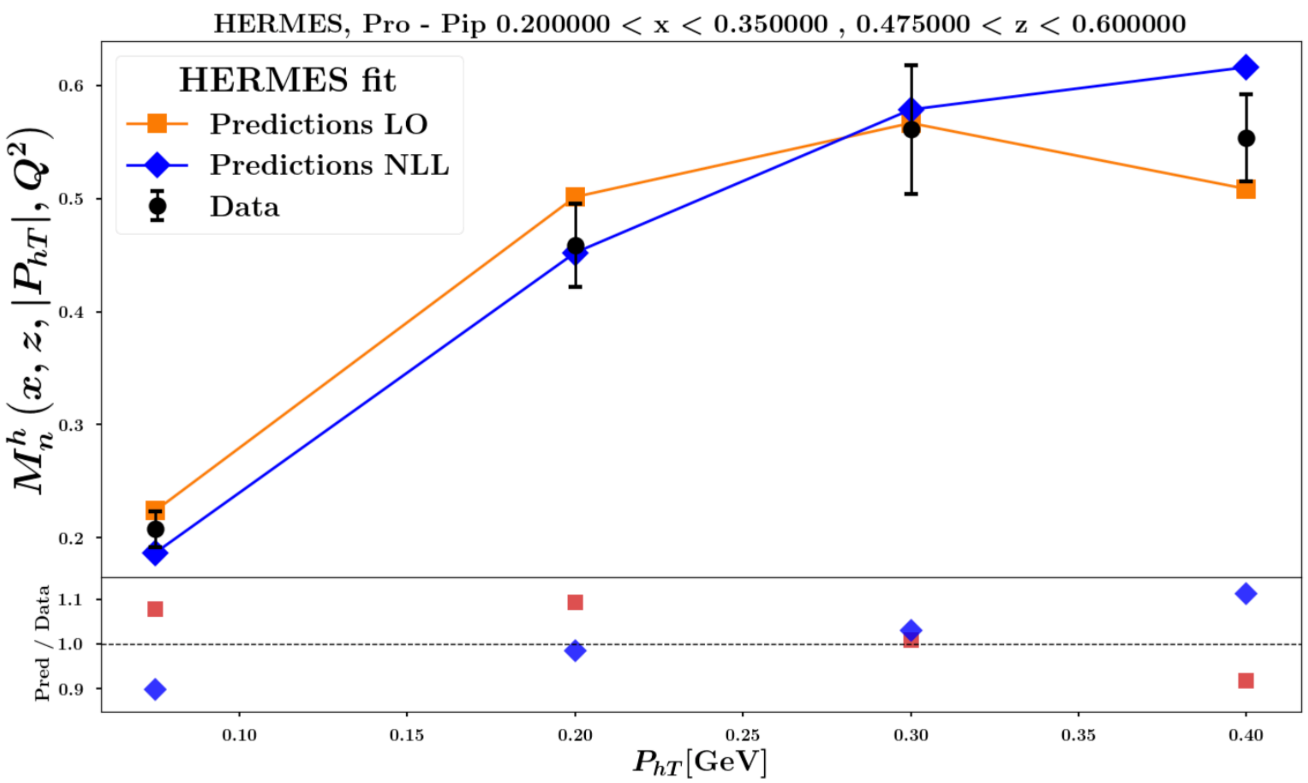

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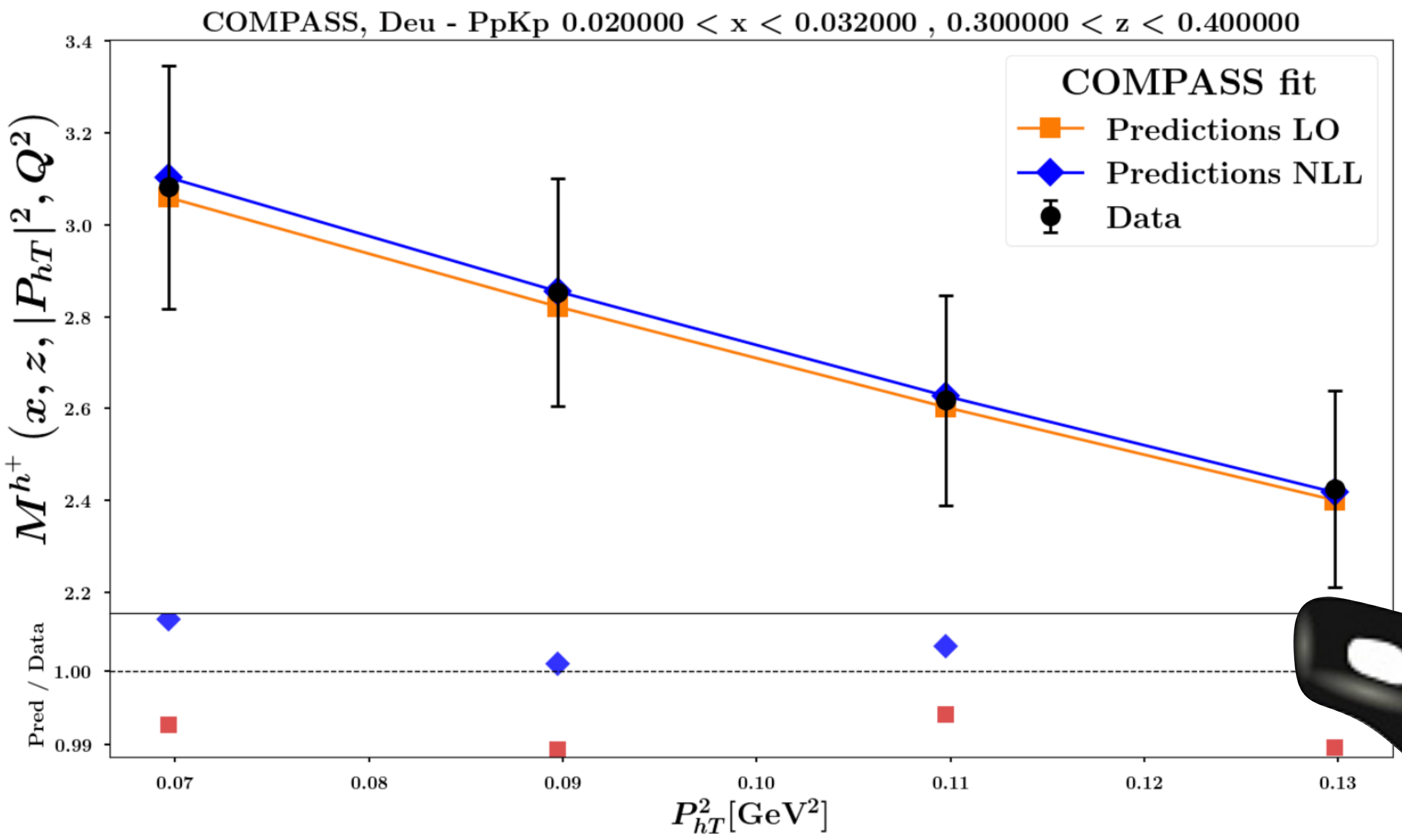

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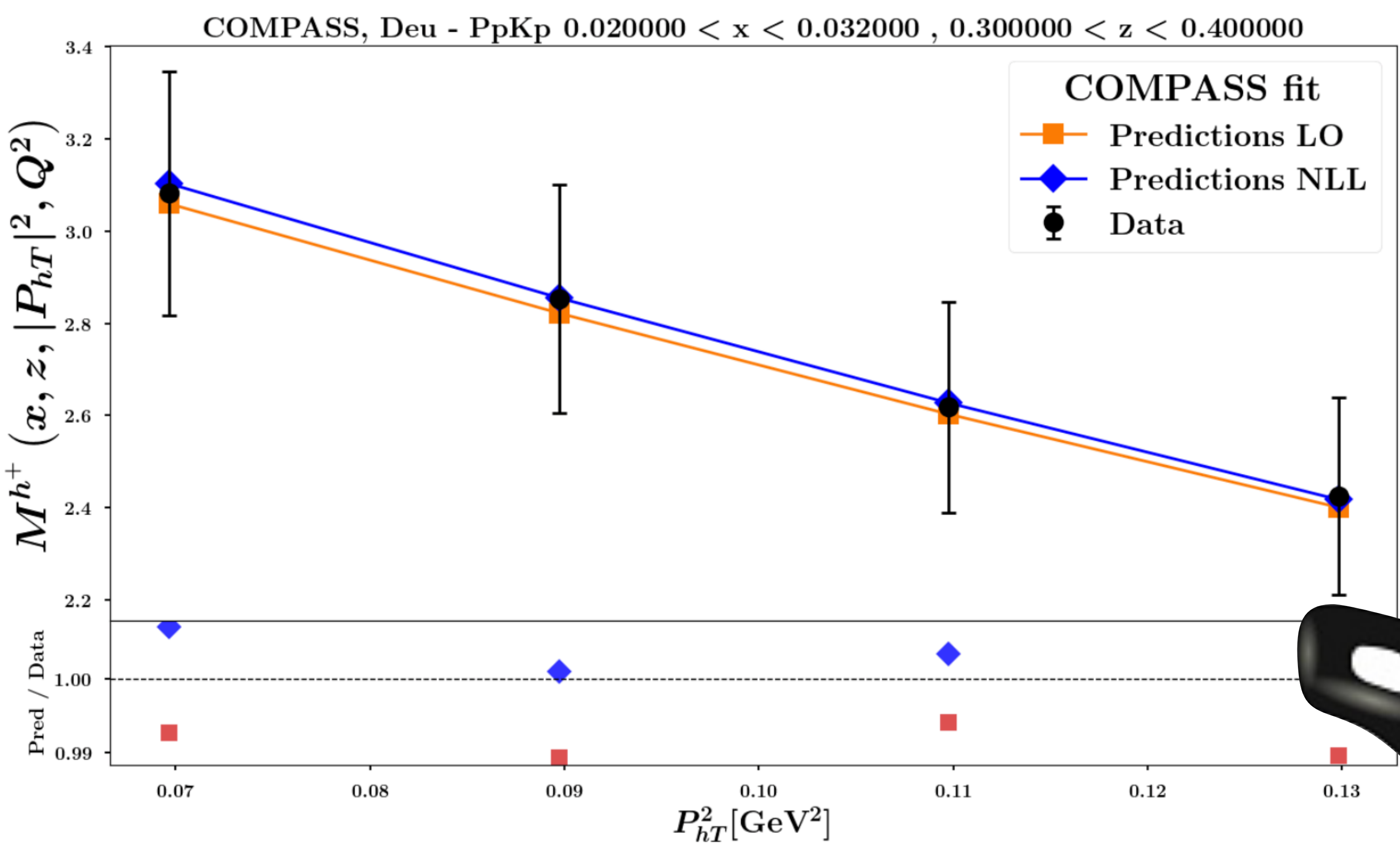

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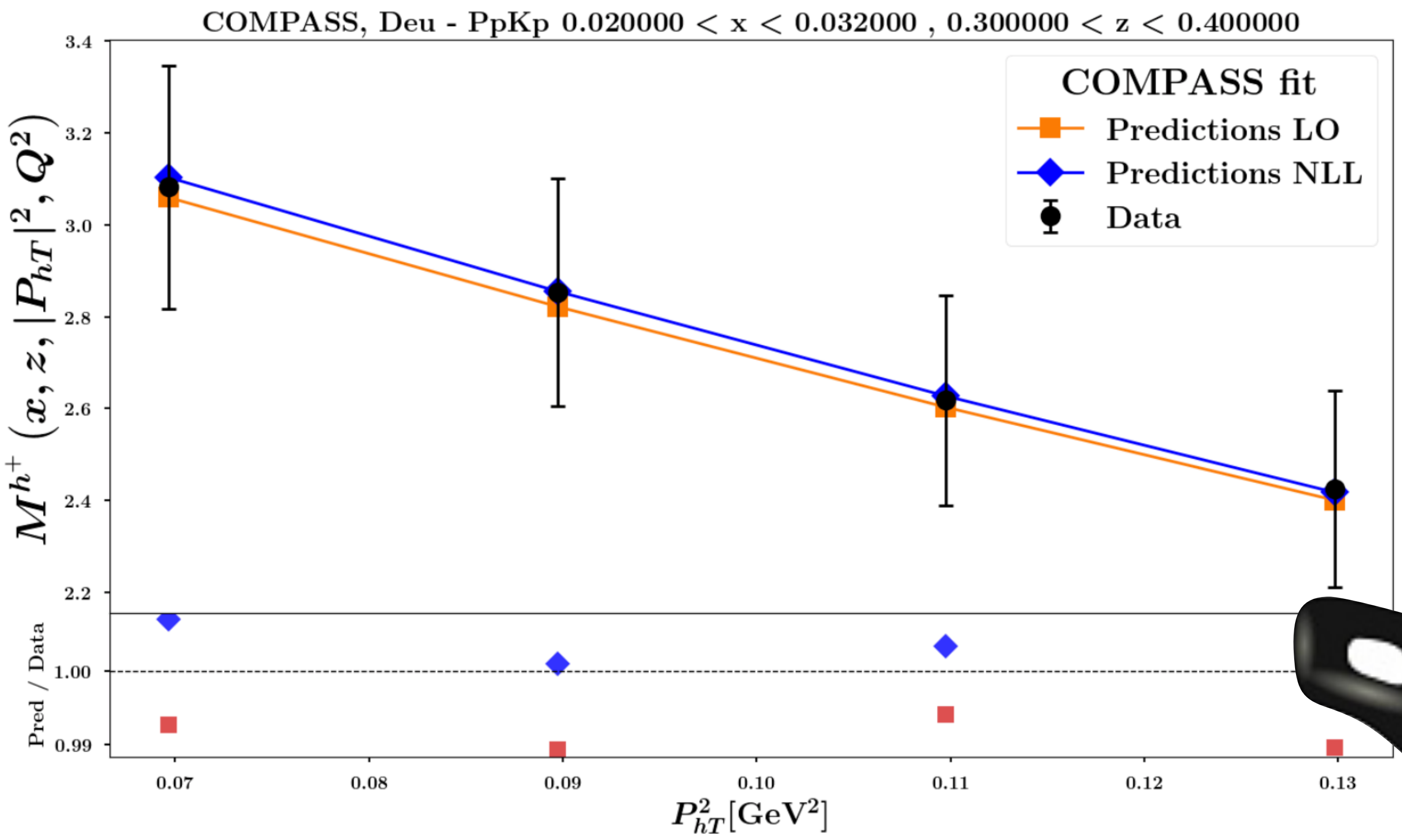

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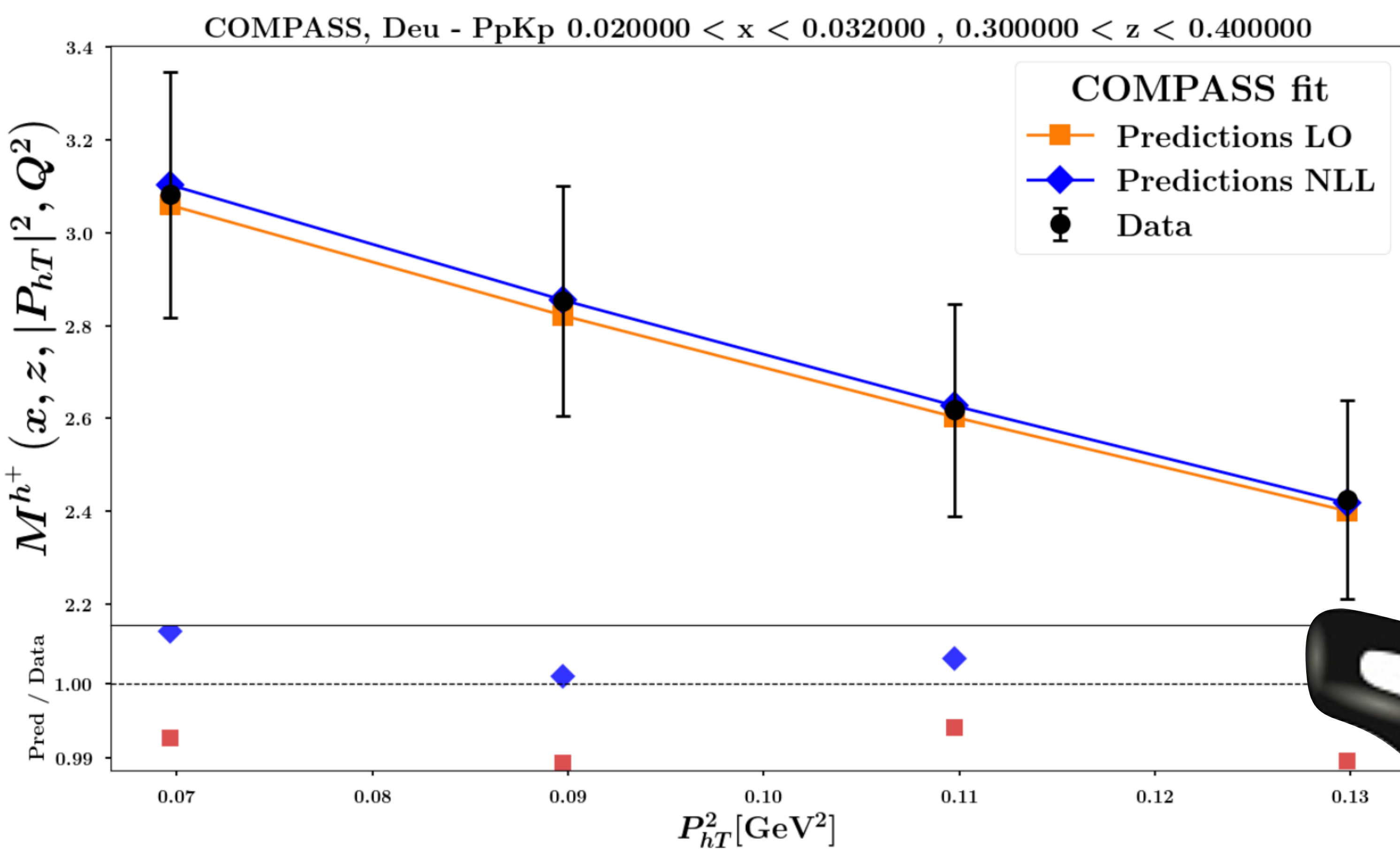

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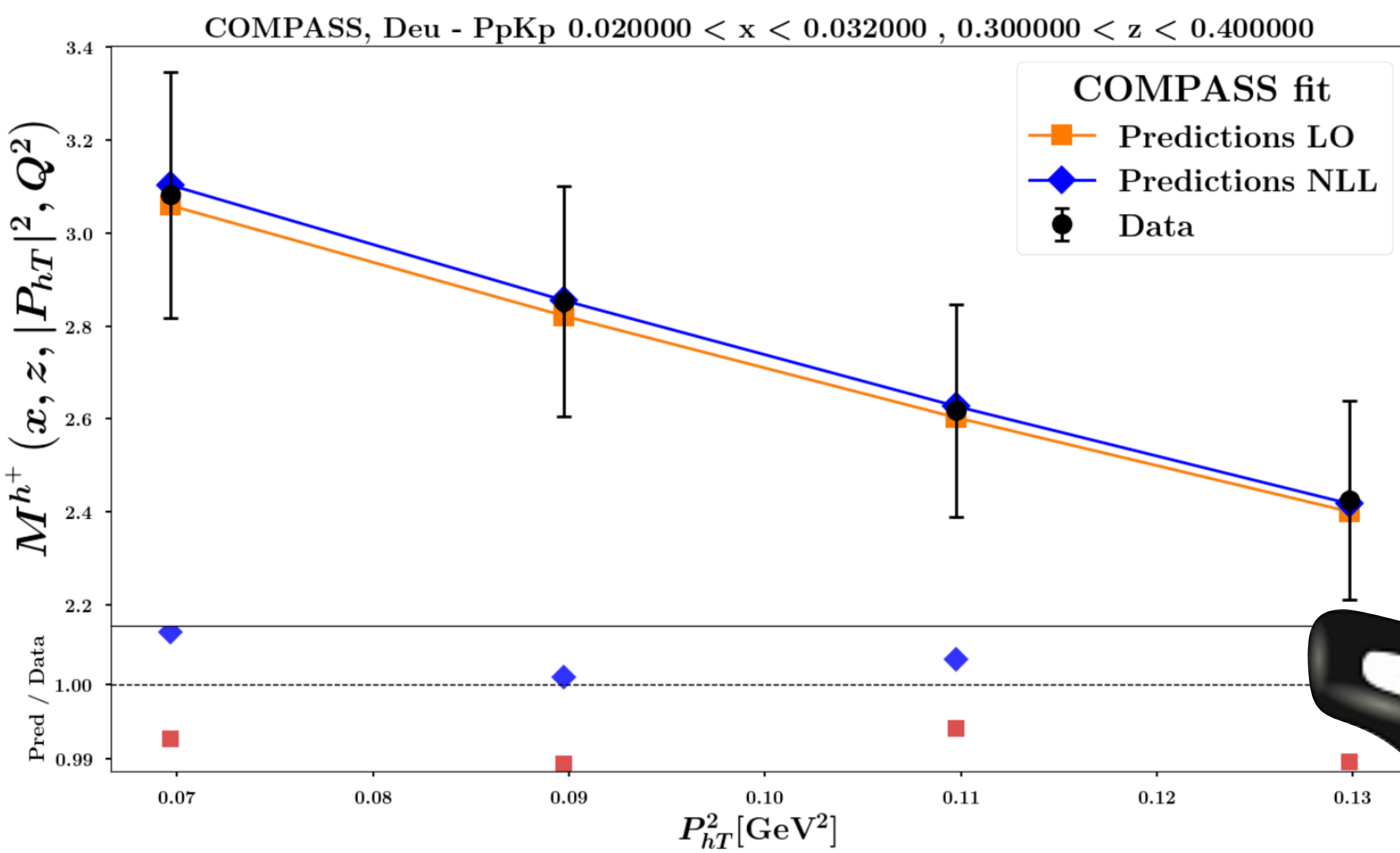

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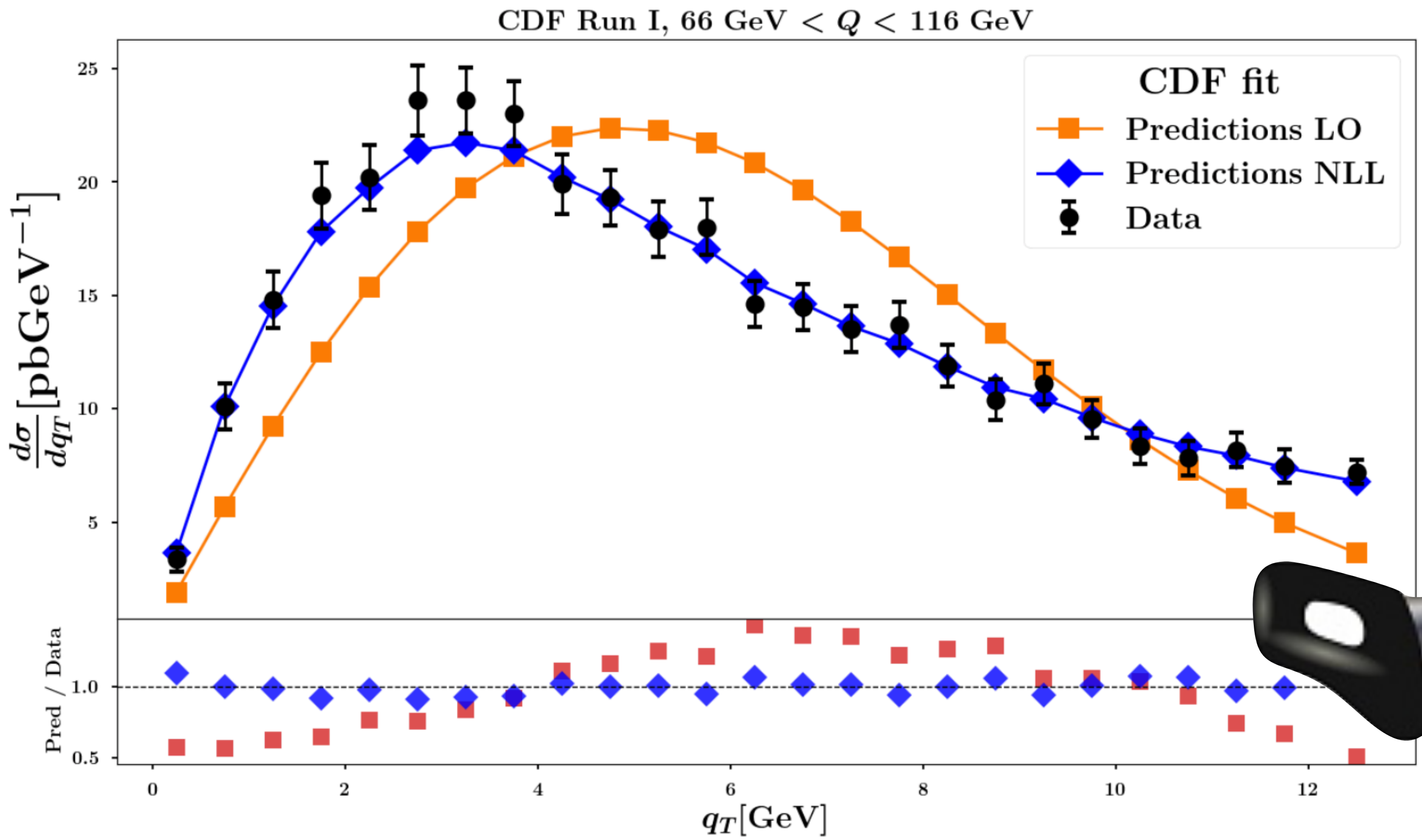

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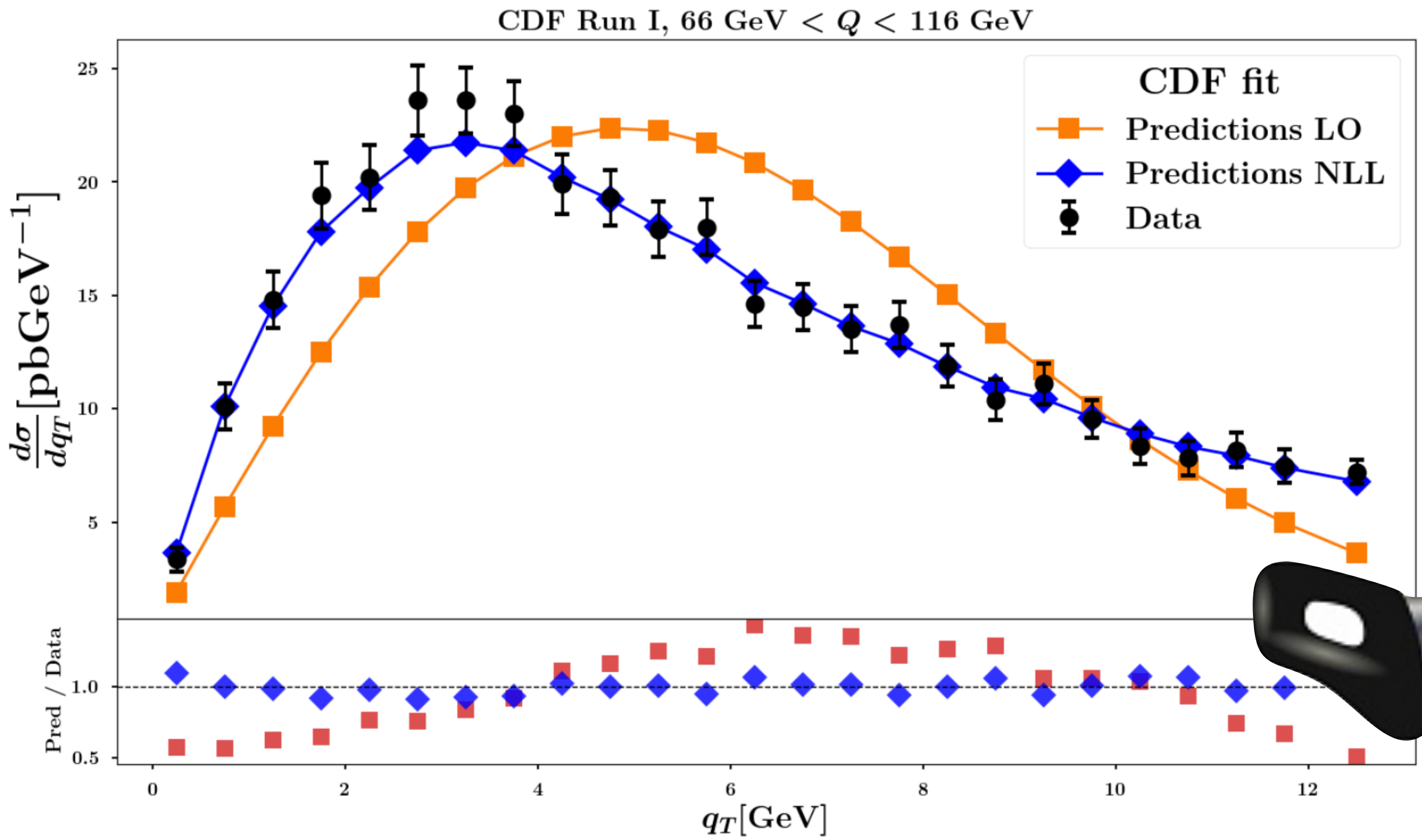
At LO we canNOT fit CDF q_T-distribution



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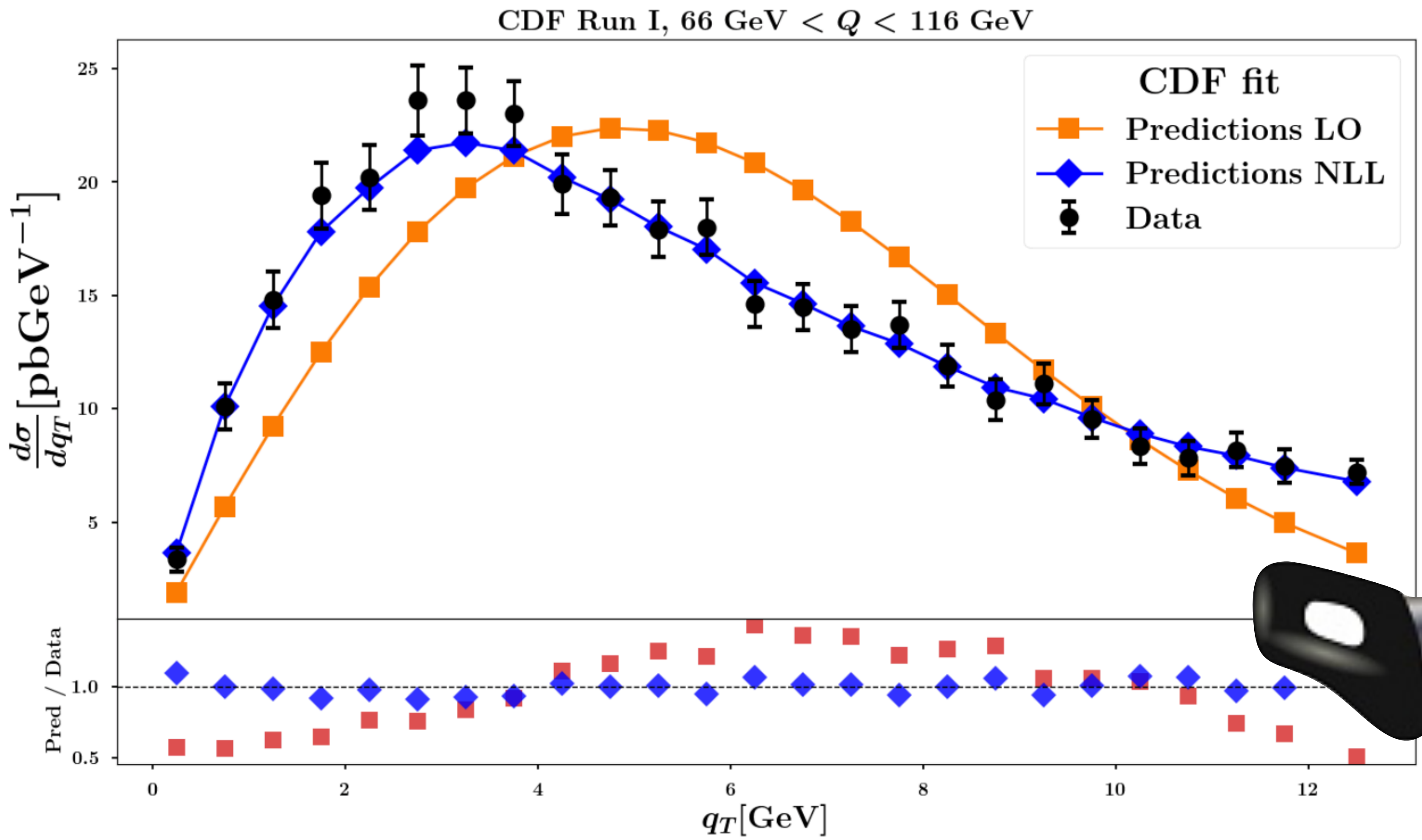

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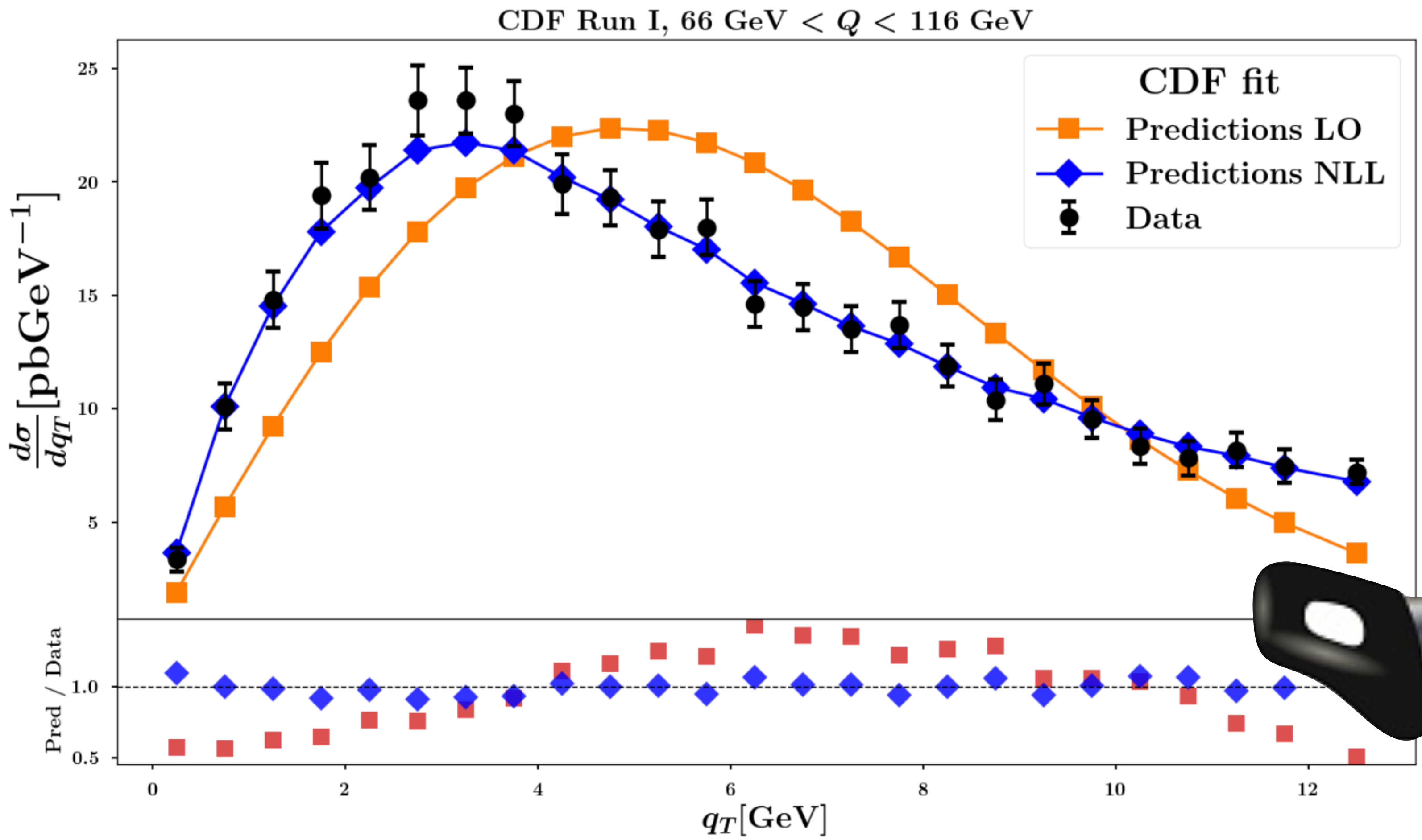

At NLL we can fit CDF q_T-distribution

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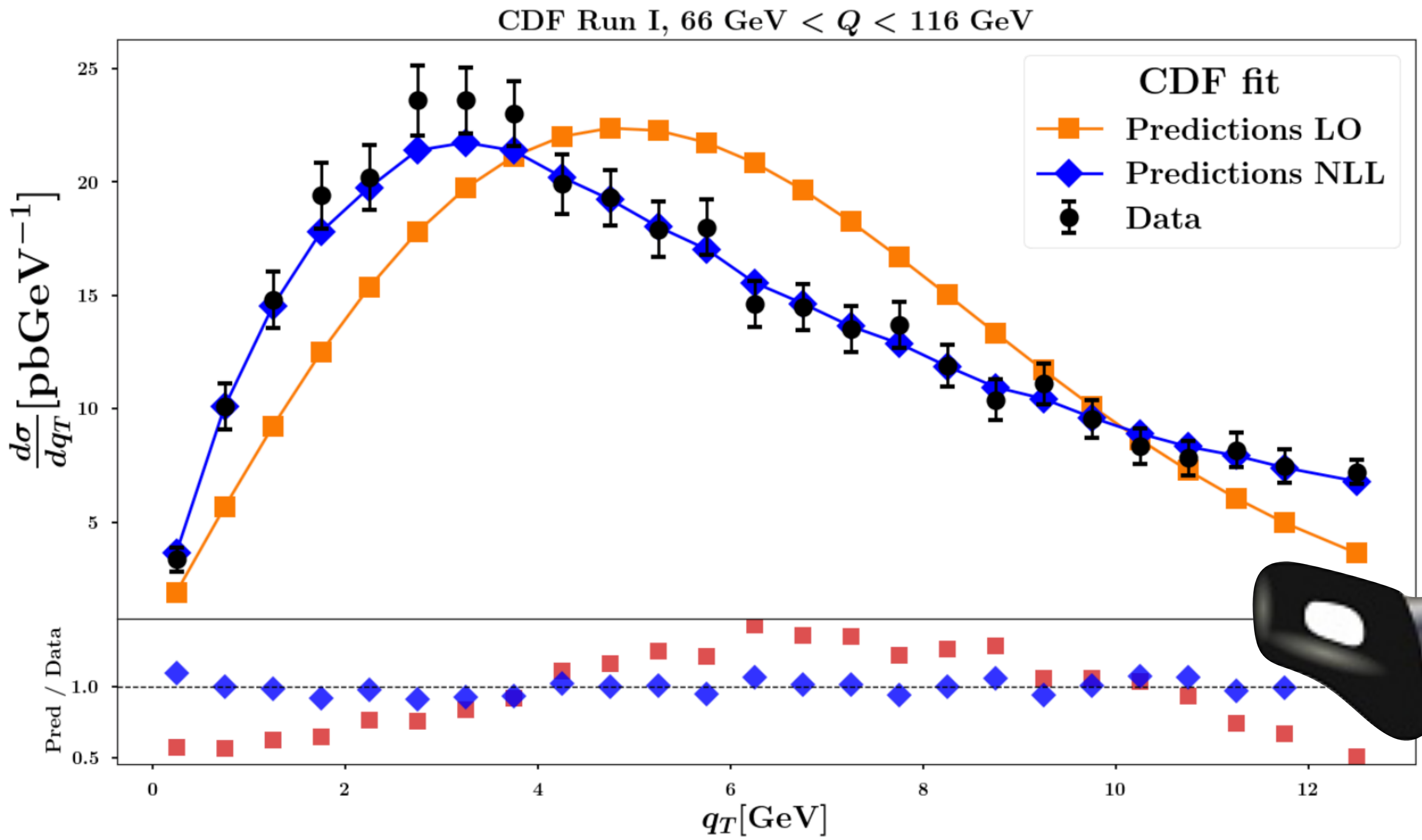

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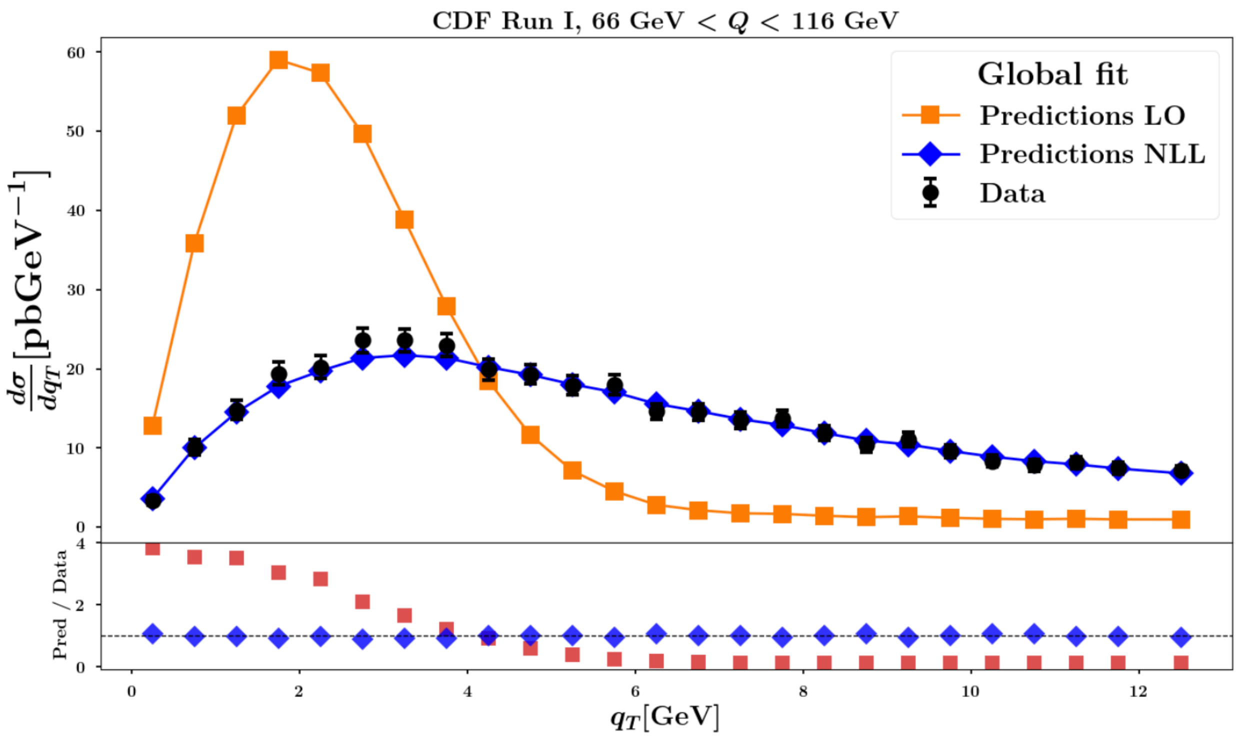

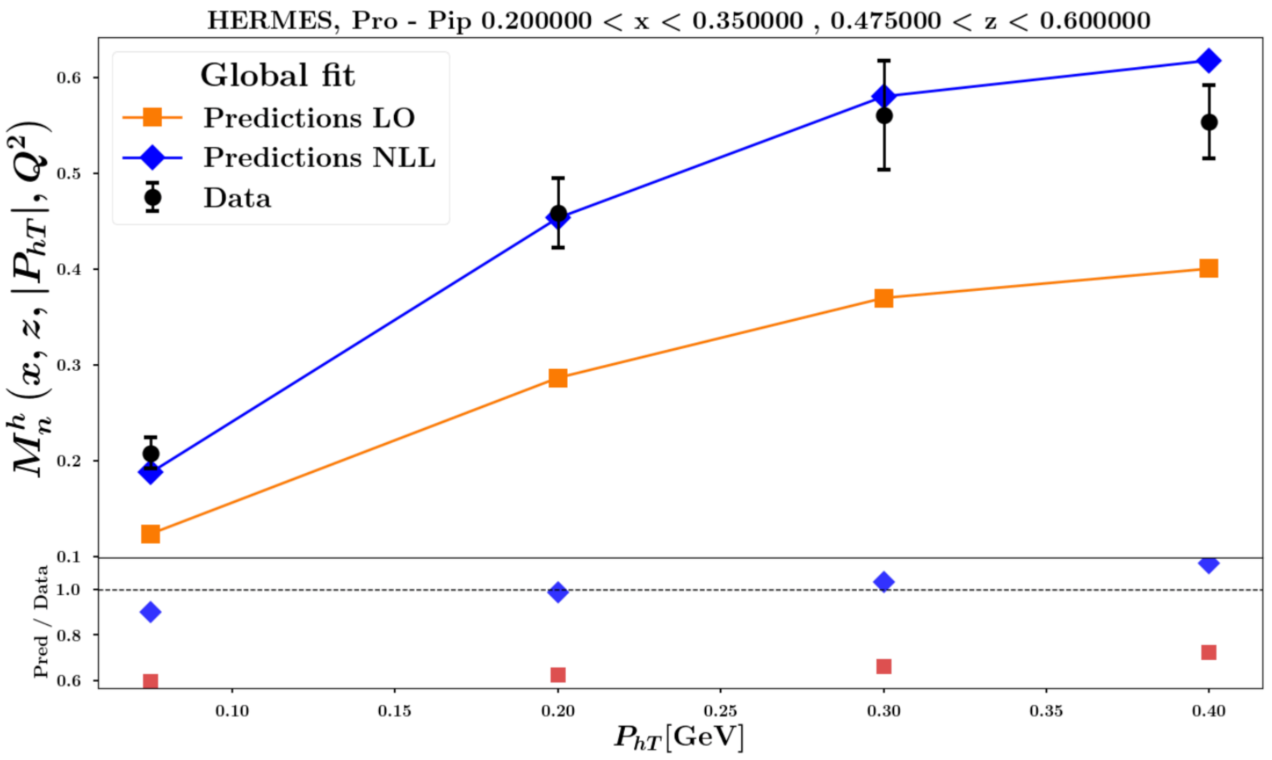

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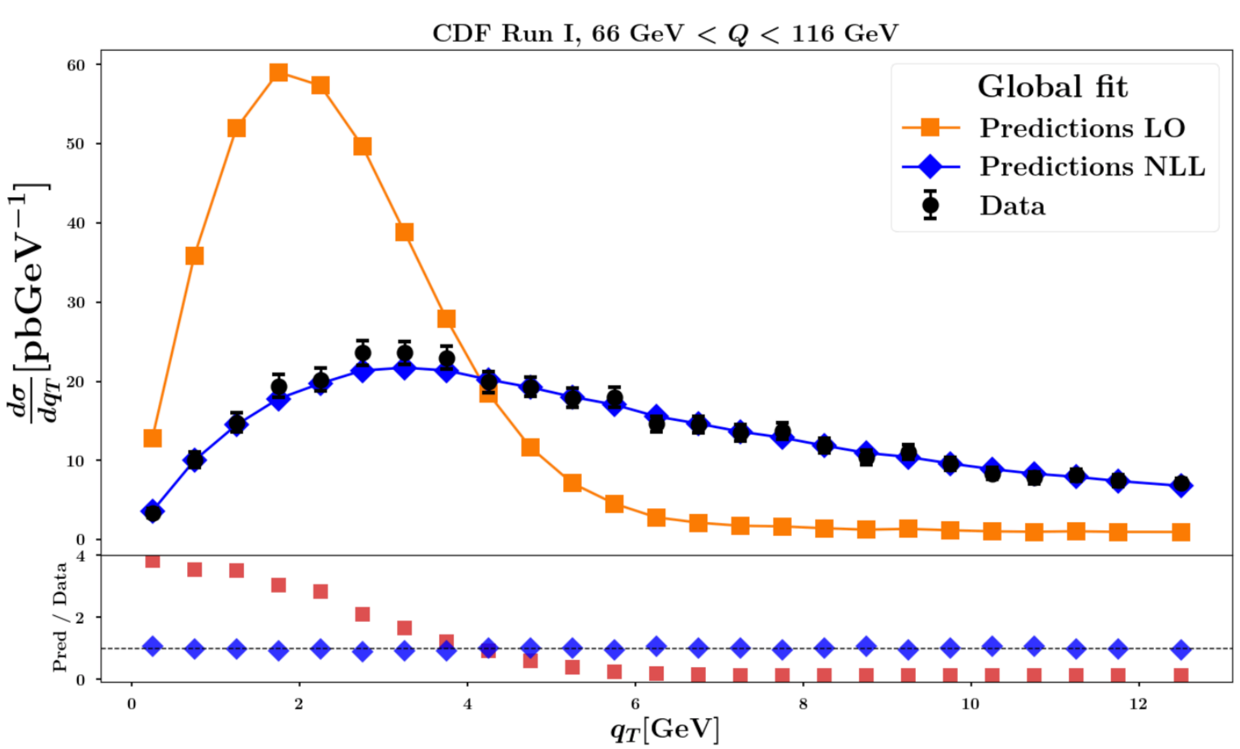
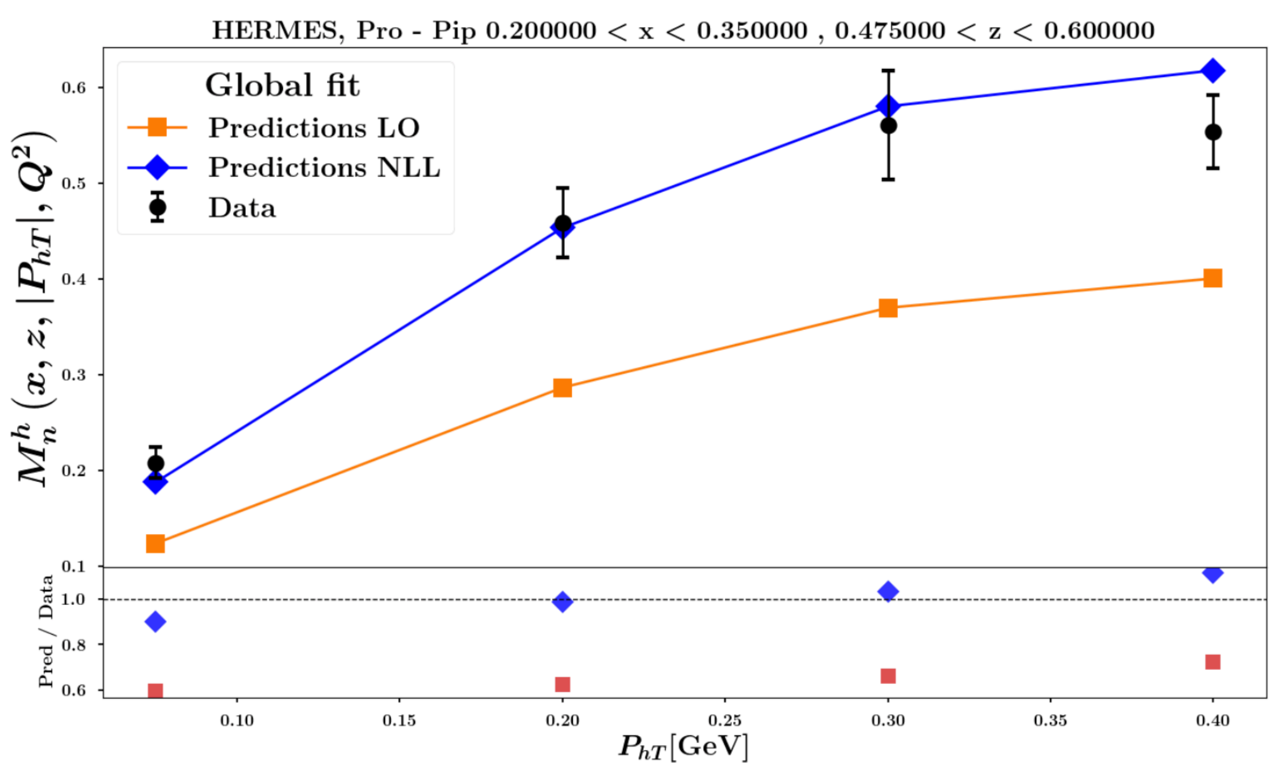
At LO we canNOT fit simultaneously CDF q_T-distribution, HERMES and COMPASS multiplicities





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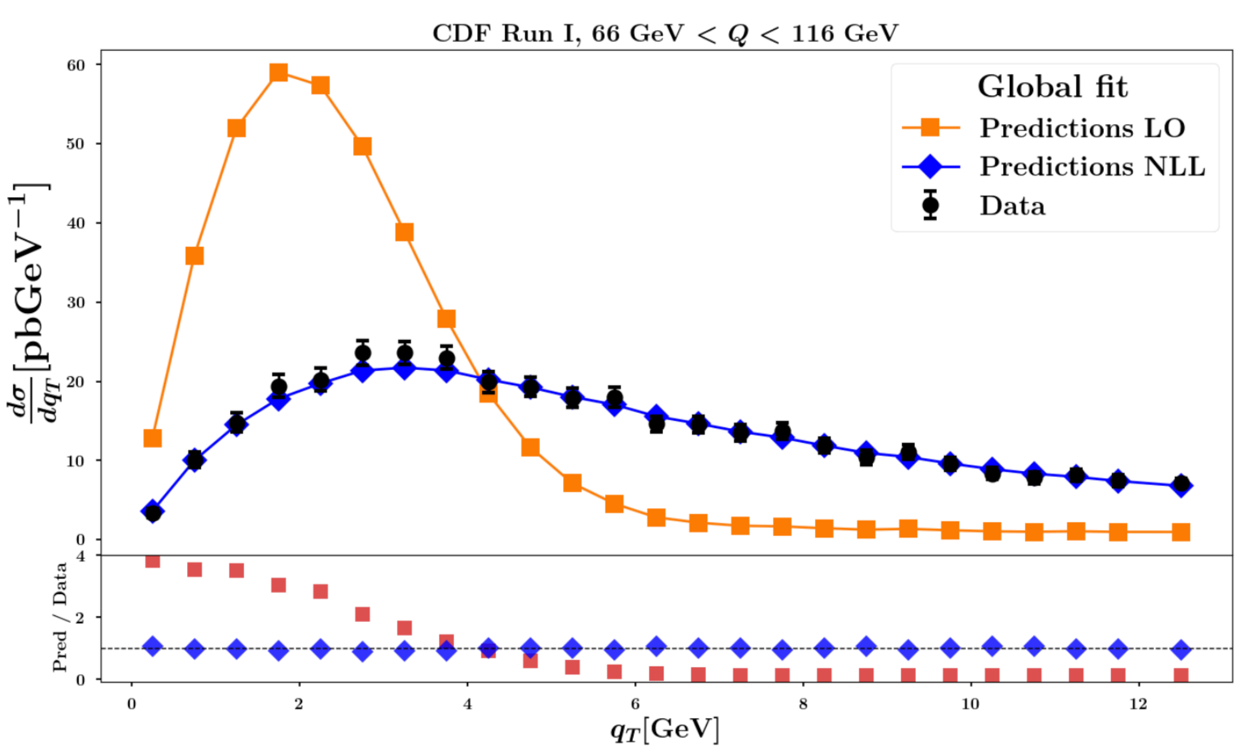

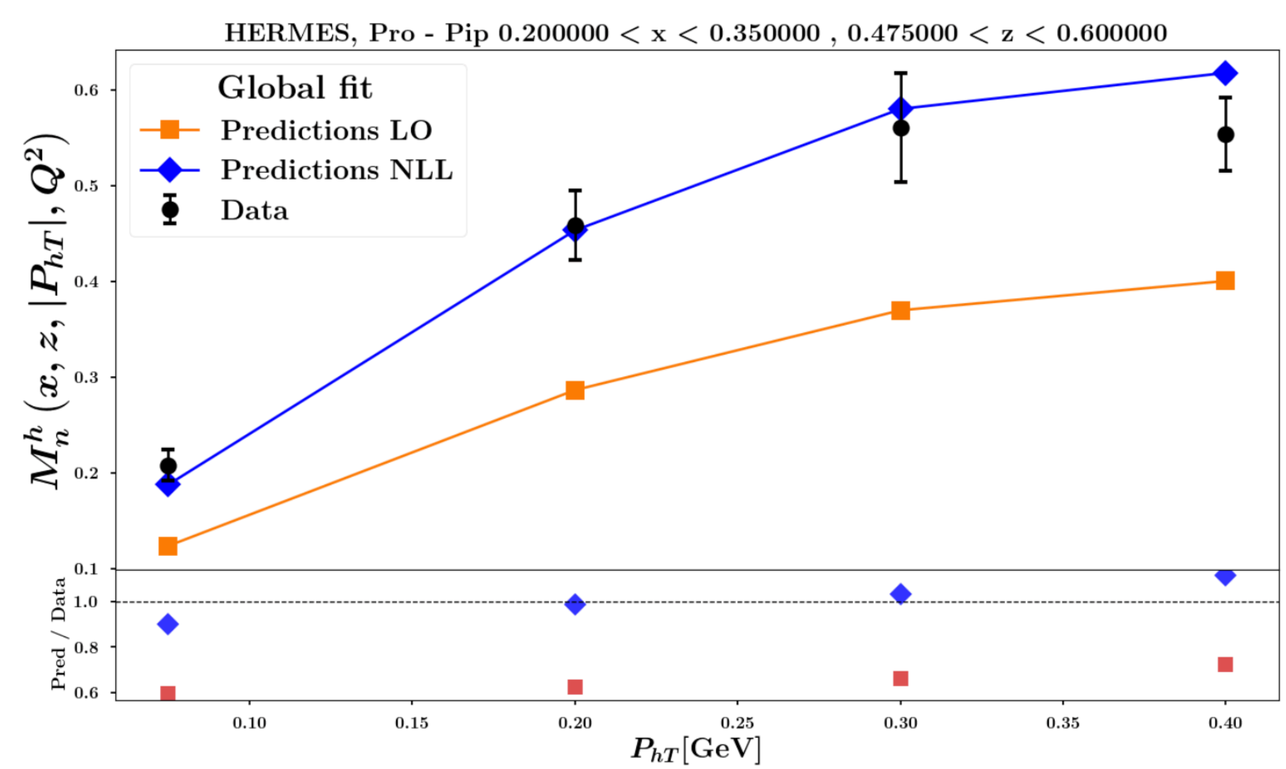

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Are LO calculations satisfactory?

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RESULT:
The CSS formalism and TMD evolution are necessary for a good simultaneous description of high-energy DY and low-energy SIDIS experimental data

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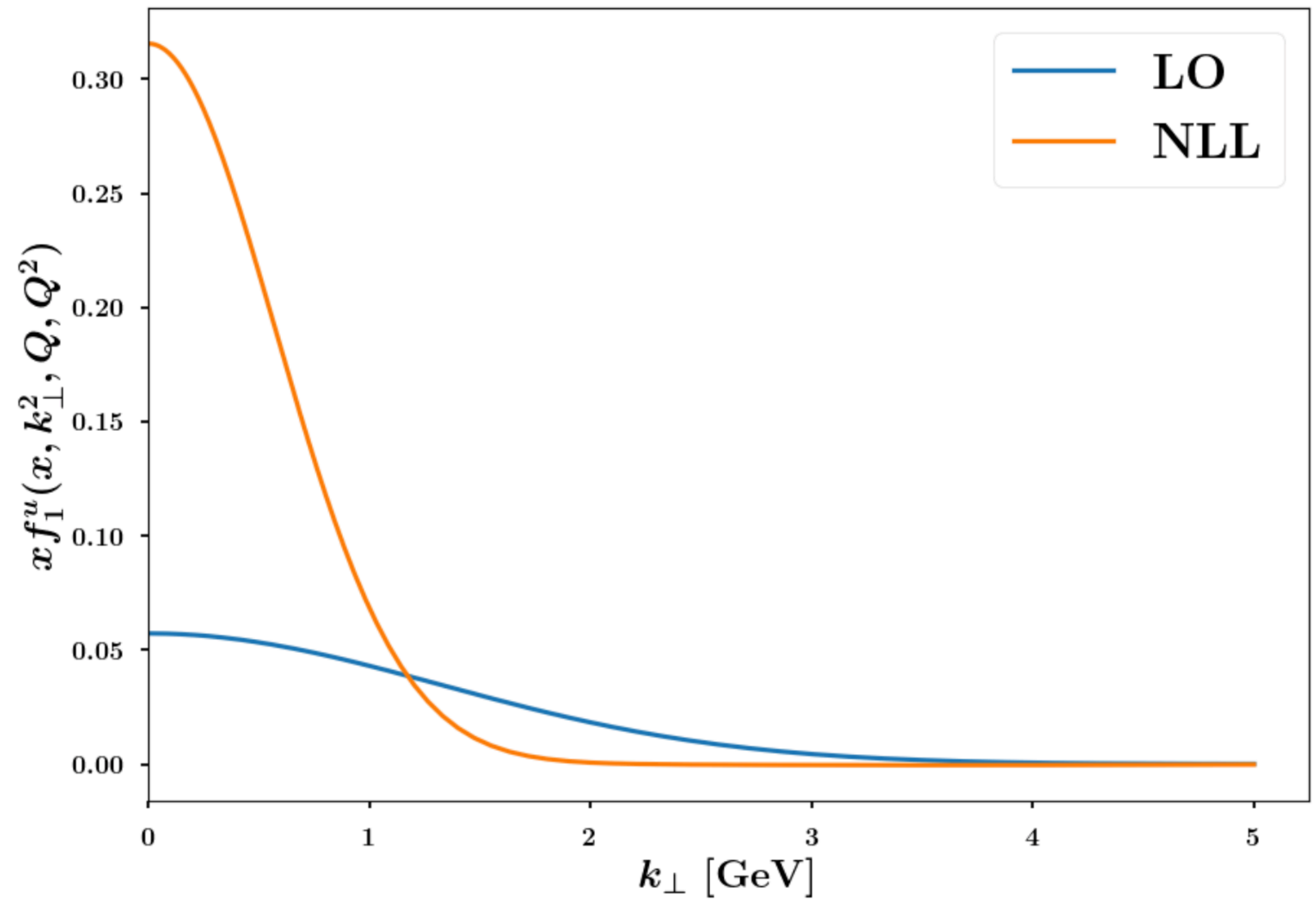
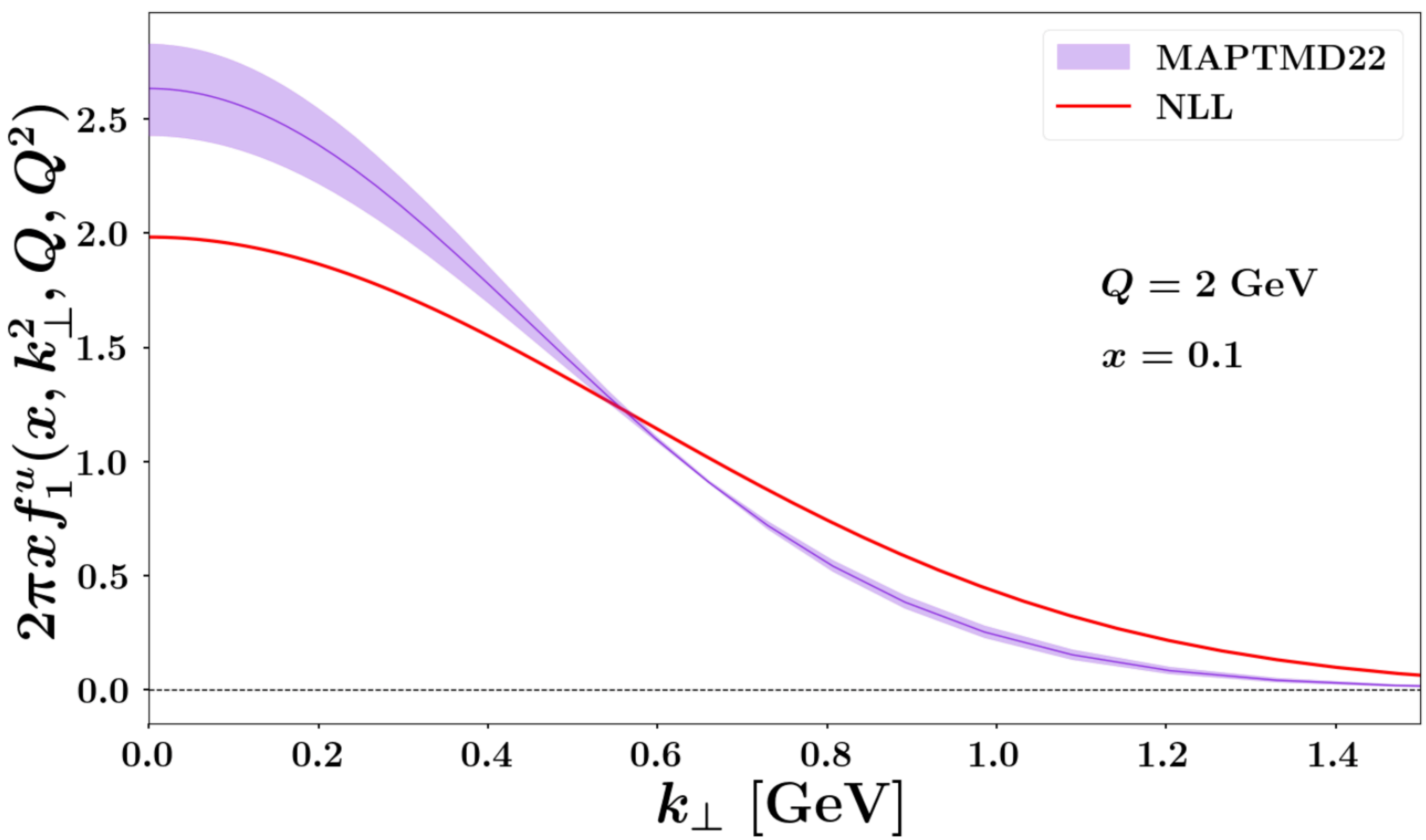

LO
NLL
$xf_1^u(x, k_\perp^2, Q, Q^2)$
$k_\perp$ [GeV]


MAPTMD22
NLL
$Q = 2$ GeV
$x = 0.1$
$2\pi x f_1^u(x, k_\perp^2, Q, Q^2)$
$k_\perp$ [GeV]