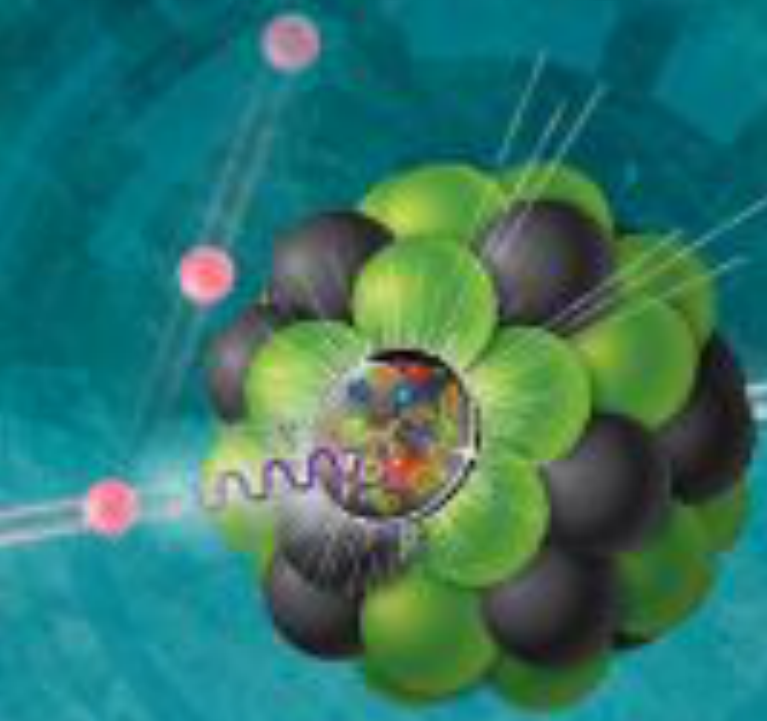


# TMD GLUON DISTRIBUTIONS

Francesco Giovanni Celiberto, UAH Madrid



1<sup>ST</sup> EUROPEAN SCHOOL ON  
THE PHYSICS OF THE  
ELECTRON-ION COLLIDER

18–22 Jun 2023  
Corigliano-Rossano, Italy



Madrid  
**UAH**



**talento**

cm

Programa de atracción  
de talento investigador  
Comunidad de Madrid



ANIVERSARIO  
PATRIMONIO  
MUNDIAL



## Gluon TMD PDFs and process dependence

1.

Hors d'œuvre - TMD process dependence - Gluon TMD process dependence  
Factorization and universality - Gluon TMD PDFs at twist-2



## 1. Gluon TMD PDFs and process dependence

Hors d'œuvre - TMD process dependence - Gluon TMD process dependence  
Factorization and universality - Gluon TMD PDFs at twist-2

## 2. Modeling gluon TMDs

Spectator-model gluon TMD PDFs - Unpolarized gluon PDF & TMD PDF  
3D proton imaging via gluon TMDs - Gluon TMD FFs at twist-2



# TMD gluon distributions

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## 3. Building T-odd gluon TMDs

Generation T-odd structures - T-odd gluon TMDs at twist-2  
f-type Sivers gluon TMD



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f-type Sivers gluon TMD

## 4. Phenomenology

Gold channels for gluon TMDs @EIC - Gluon TMD PDFs & quarkonia  
TMD PDFs, LDMEs & shape functions



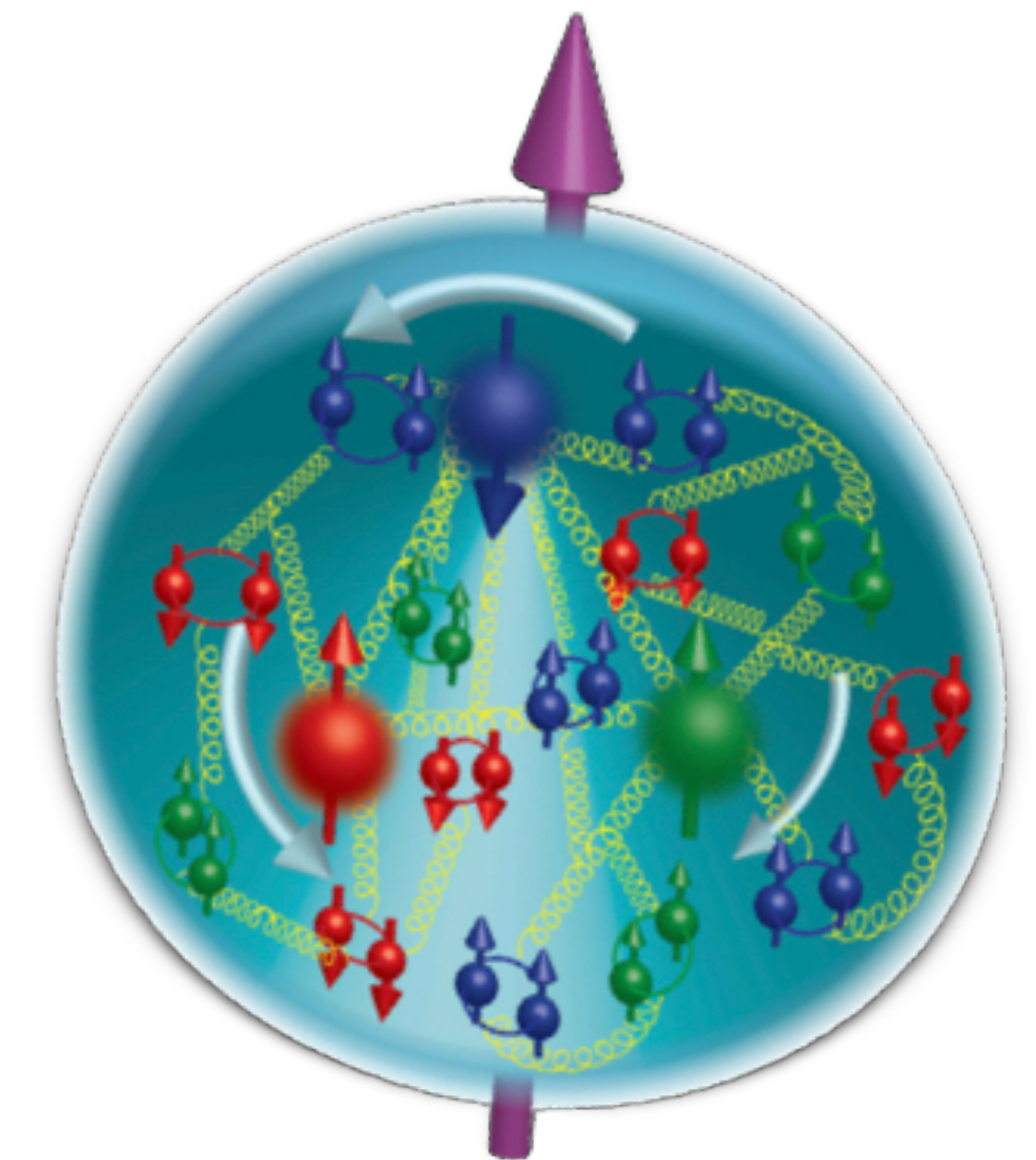
The background features a stylized, semi-transparent illustration of a nucleon (proton or neutron) with a complex internal structure. It shows a central core with various colored spheres (red, blue, green) and arrows, surrounded by a network of yellow wavy lines representing gluons. The overall color palette is light blue and green with some white highlights.

# 1. Gluon TMD PDFs & process dependence



# Parton densities: Hors d'œuvre

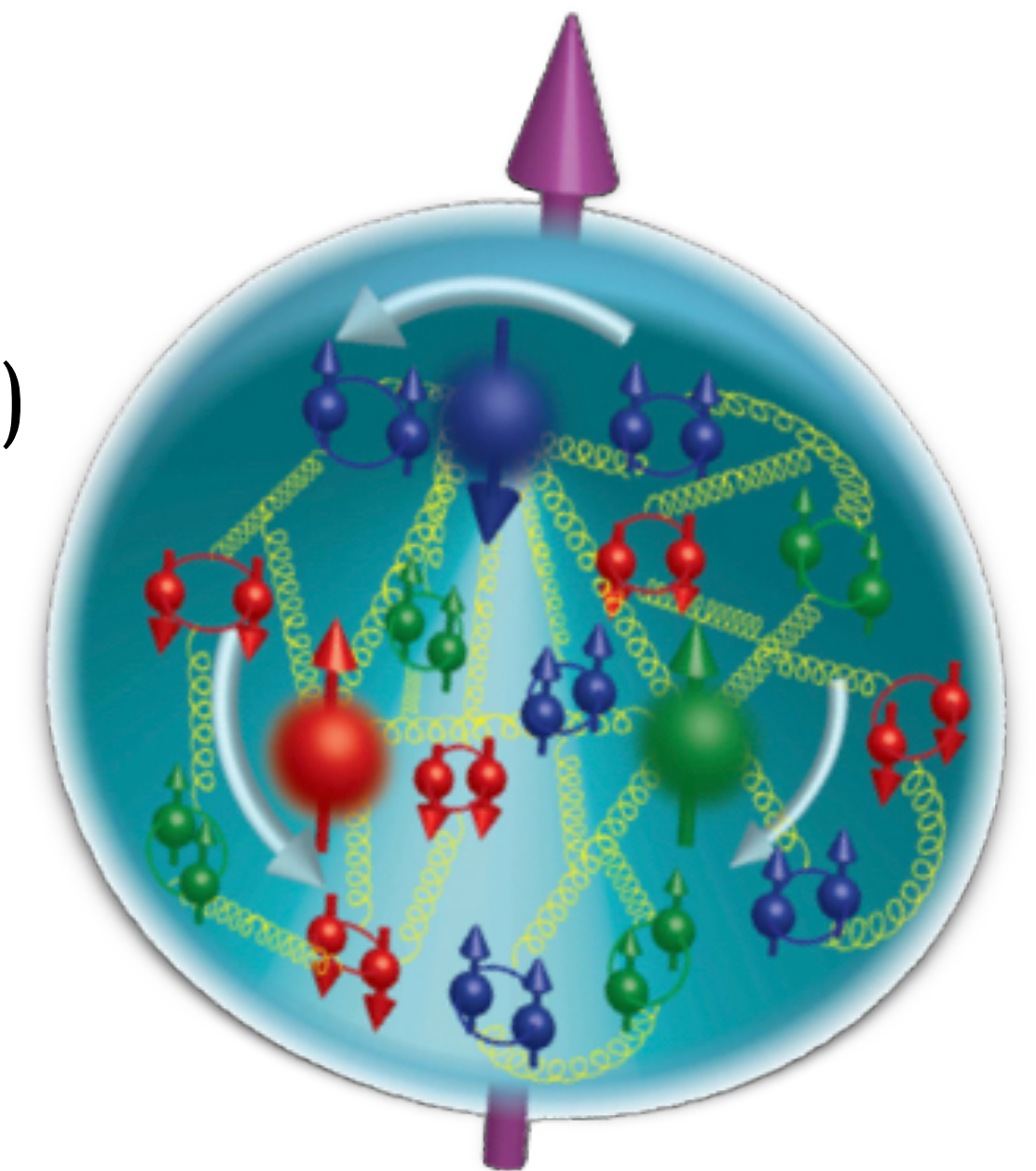
- **Parton densities** → relevant for the search of **New Physics**...
  - ...crucial role in the understanding and exploration of **QCD**
- Describe the internal structure of the nucleon in terms of its elementary constituents (quarks and gluons)
- **Nonperturbative** objects that enter the expression of cross sections
- Can be *extracted* from experiments via *global fits*





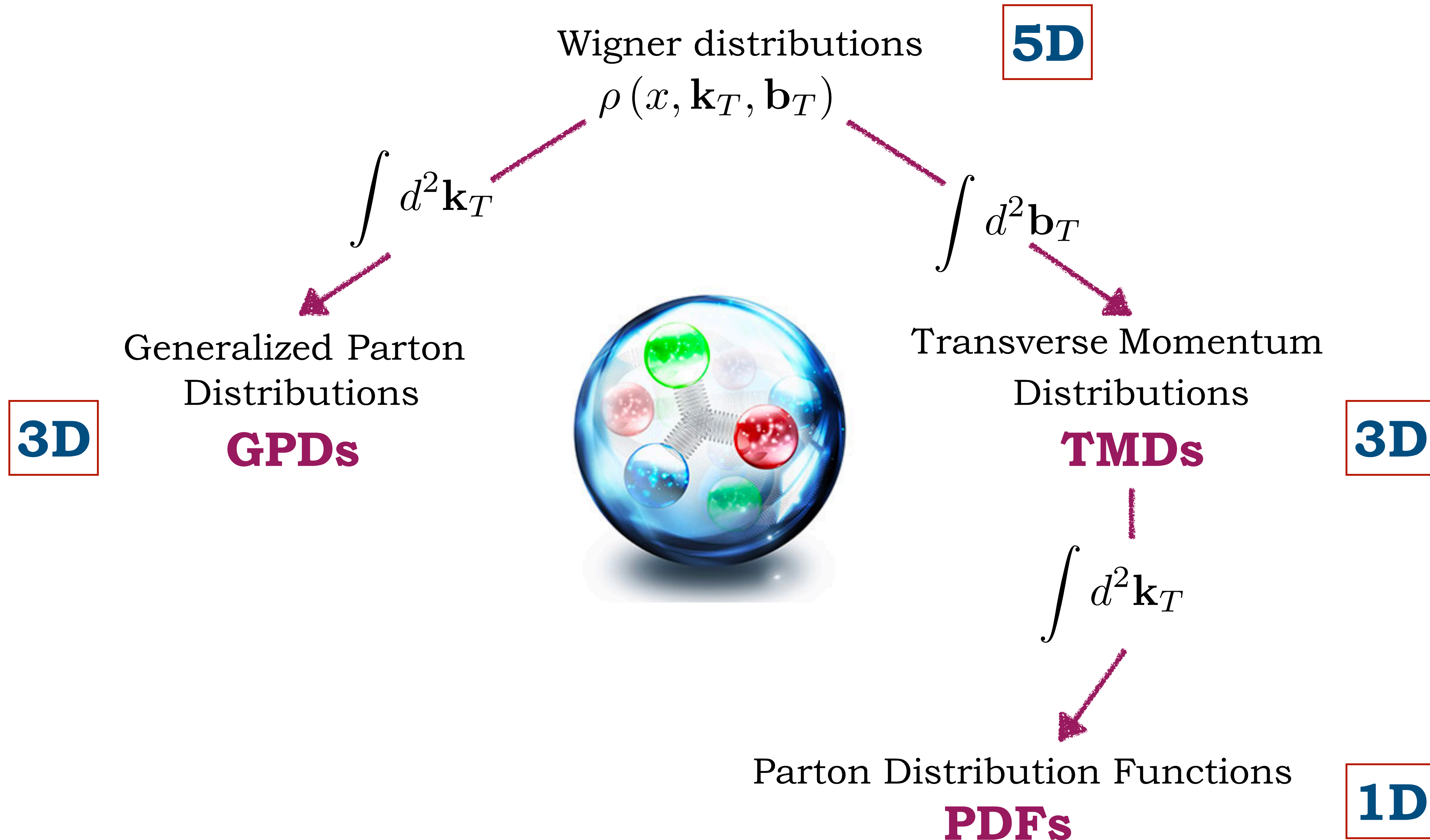
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- Describe the internal structure of the nucleon in terms of its elementary constituents (quarks and gluons)
- **Nonperturbative** objects that enter the expression of cross sections
- Can be *extracted* from experiments via *global fits*
- Several types of distributions (1D collinear, **3D TMD**, **3D GPD**, ...)
  - Follow from different **factorization theorems**
  - Exhibit peculiar **universality properties**
  - Obey distinct **evolution equations**



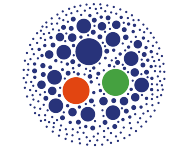


# An incomplete family tree





# Gluon TMD PDFs: A largely unexplored territory



Theory: different gauge-link structures...

...more diversified kind of modified universality!



Pheno: golden channels for extraction

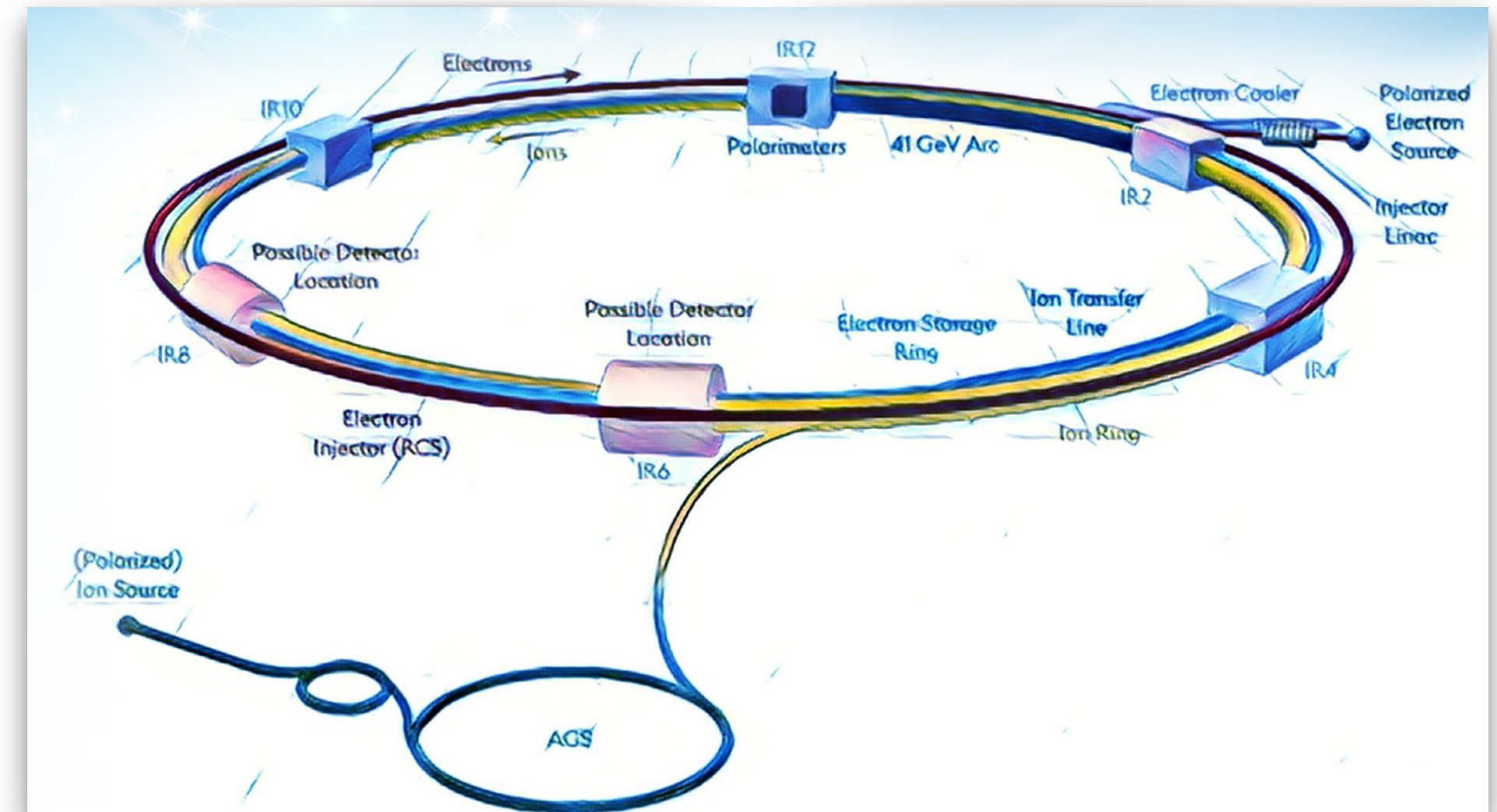
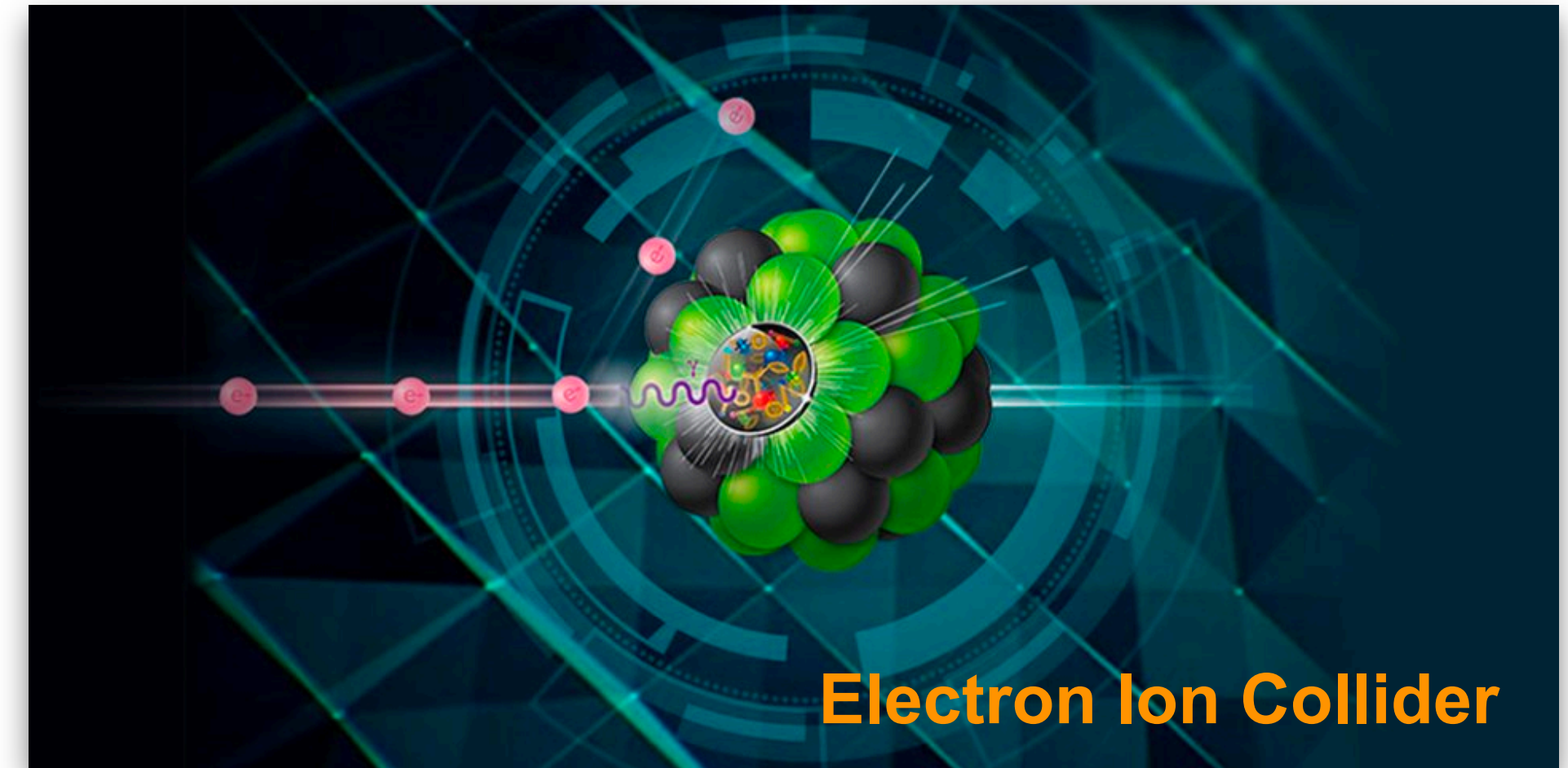
of quark TMDs are subleading for gluon TMDs

# Gluon TMD PDFs: A largely unexplored territory

- **Theory:** different gauge-link structures...  
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- **Pheno:** golden channels for extraction of quark TMDs are subleading for gluon TMDs

## 3D proton imaging

- Gluon TMD PDFs  $\Rightarrow$  core sector of EIC studies
- Need for a flexible model, suited to pheno



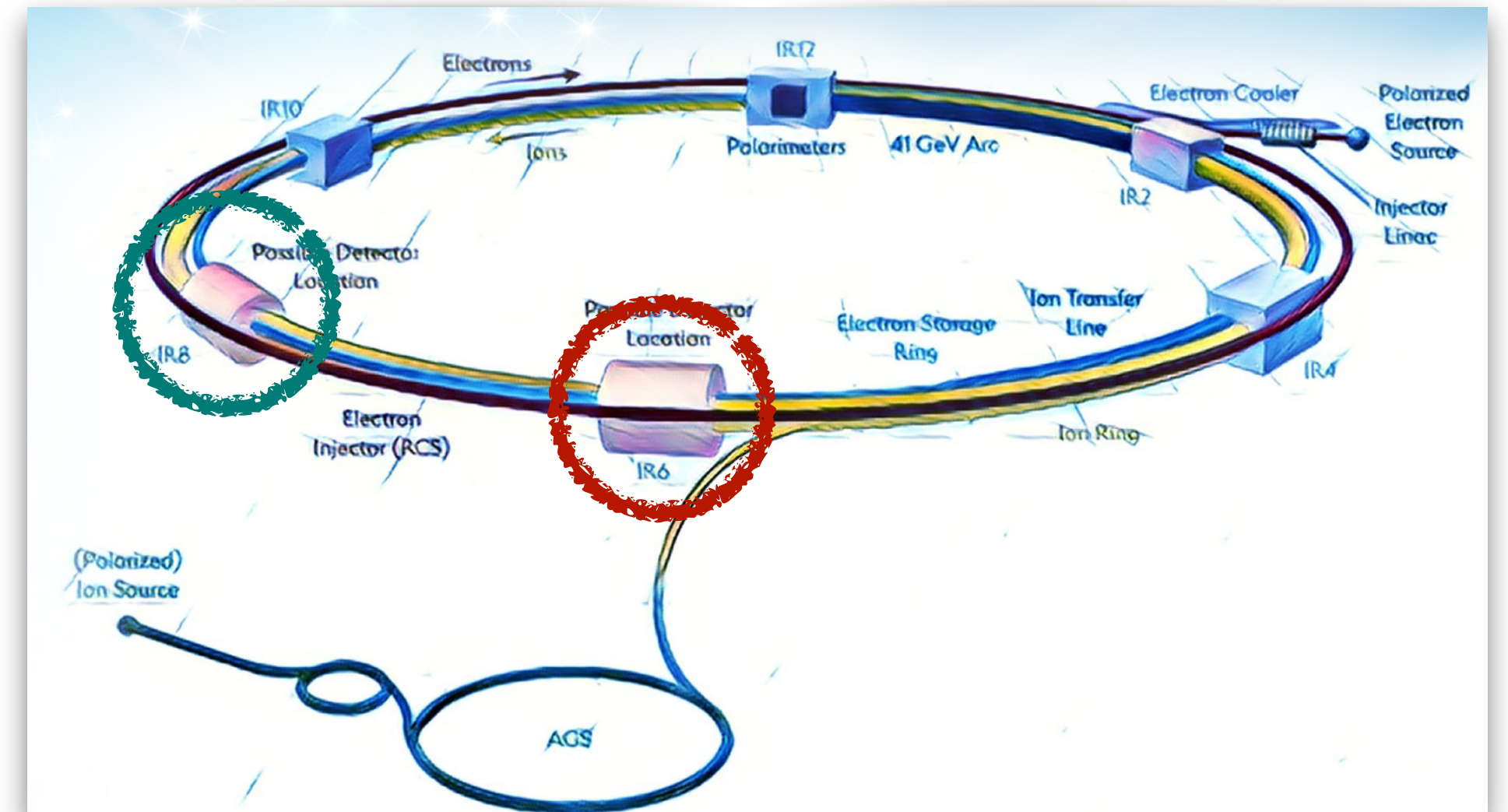
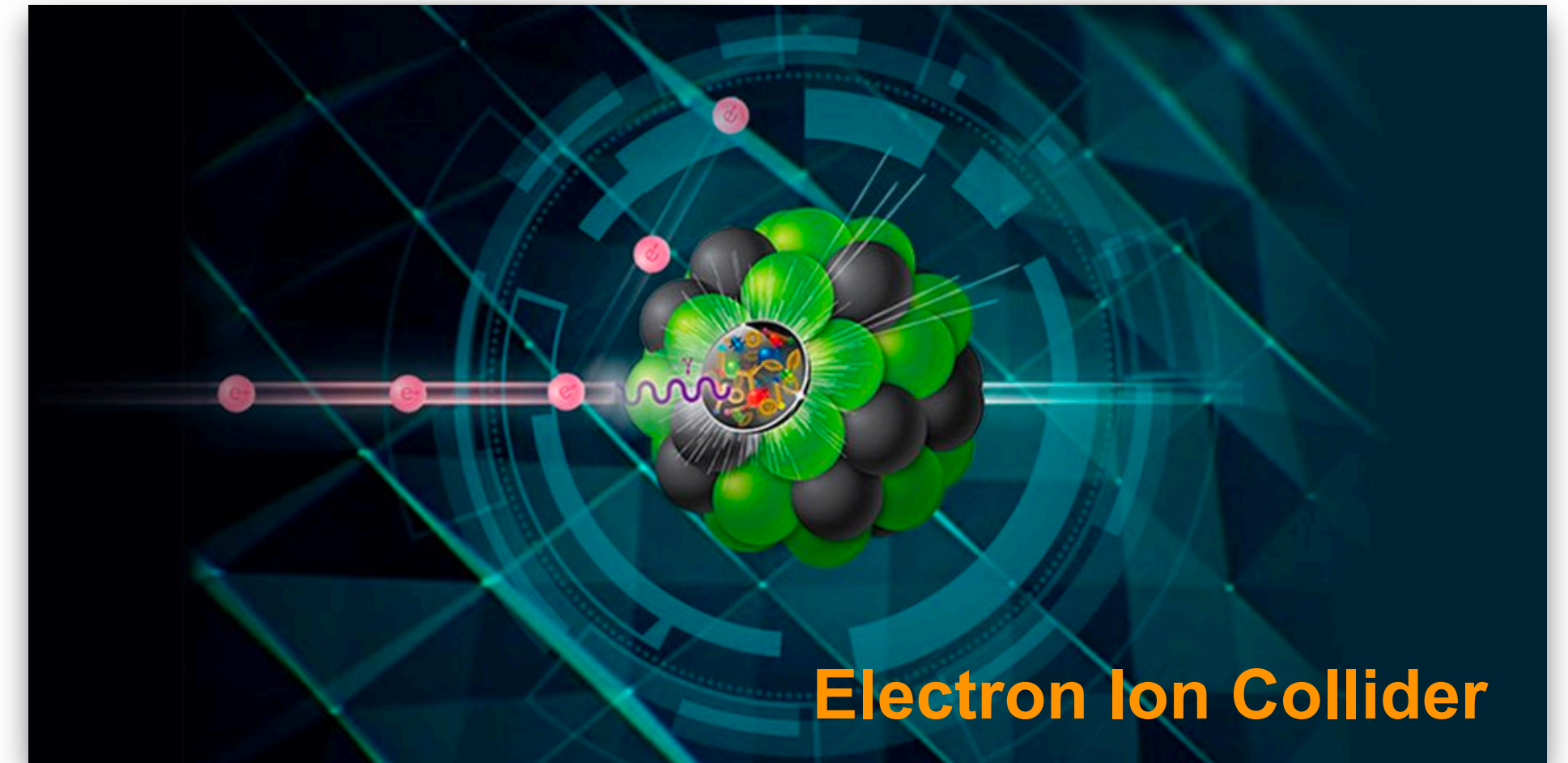


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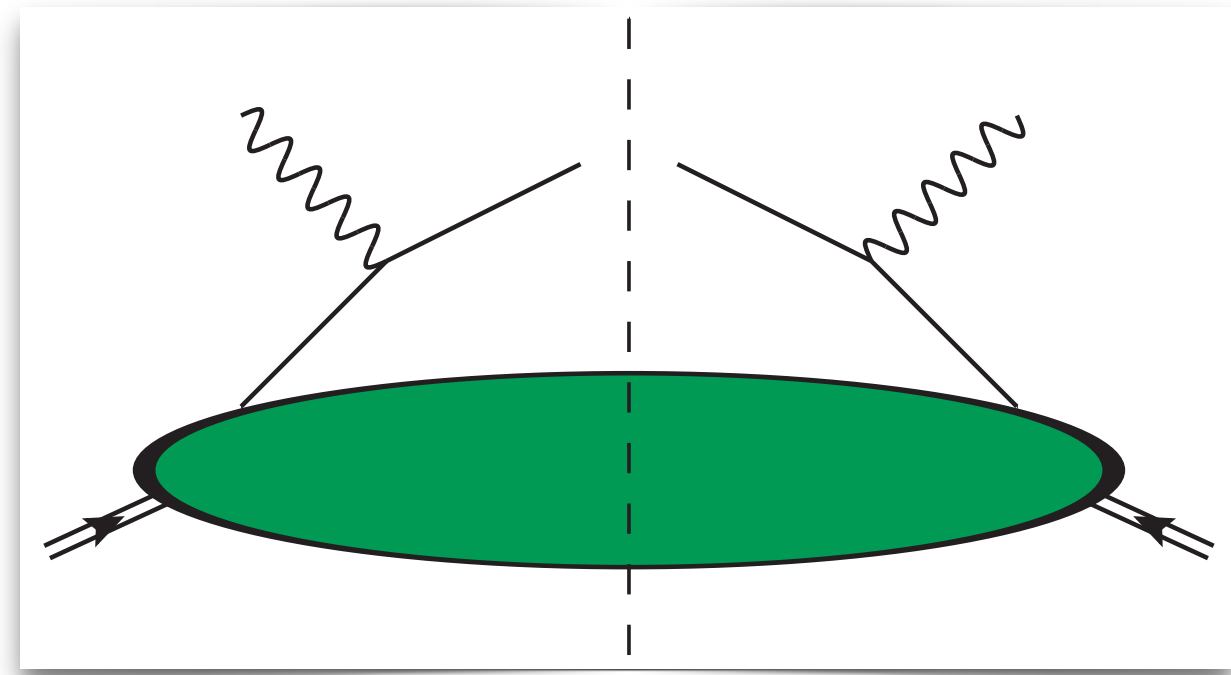
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## 3D proton imaging

- Gluon TMD PDFs  $\Rightarrow$  core sector of EIC studies
- Need for a flexible model, suited to pheno
- Gluon and nucleon polarization at twist-2
- Window of opportunities at ePIC & 2<sup>nd</sup> detector

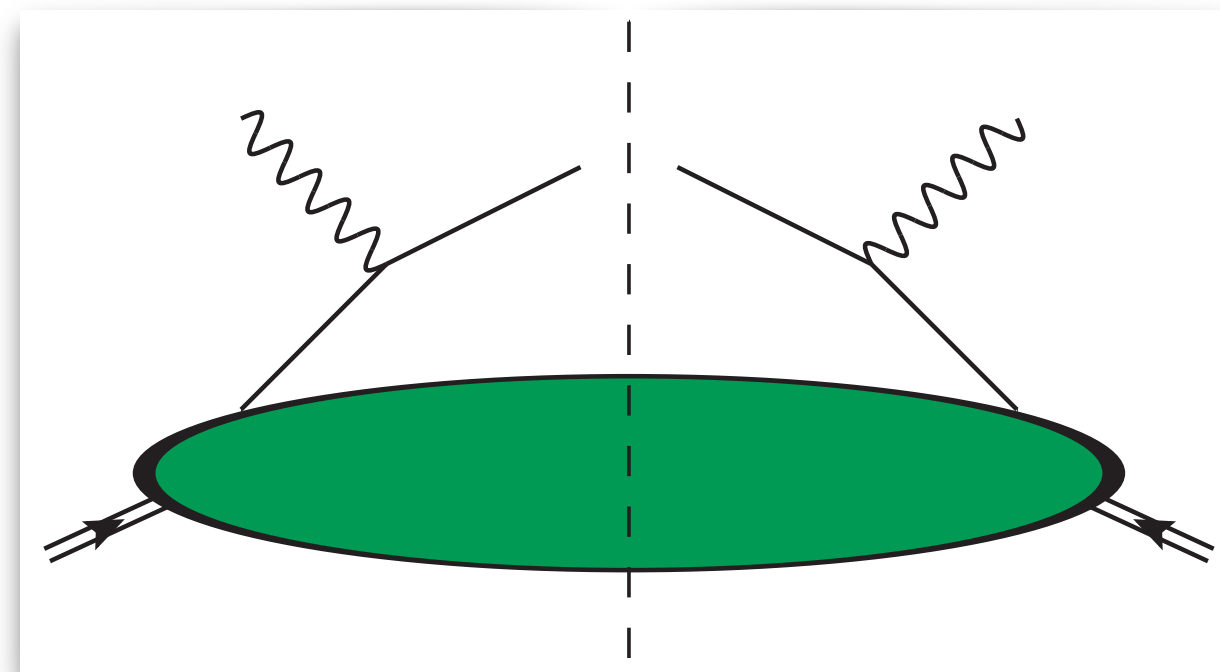


# Gauge links and processes dependence





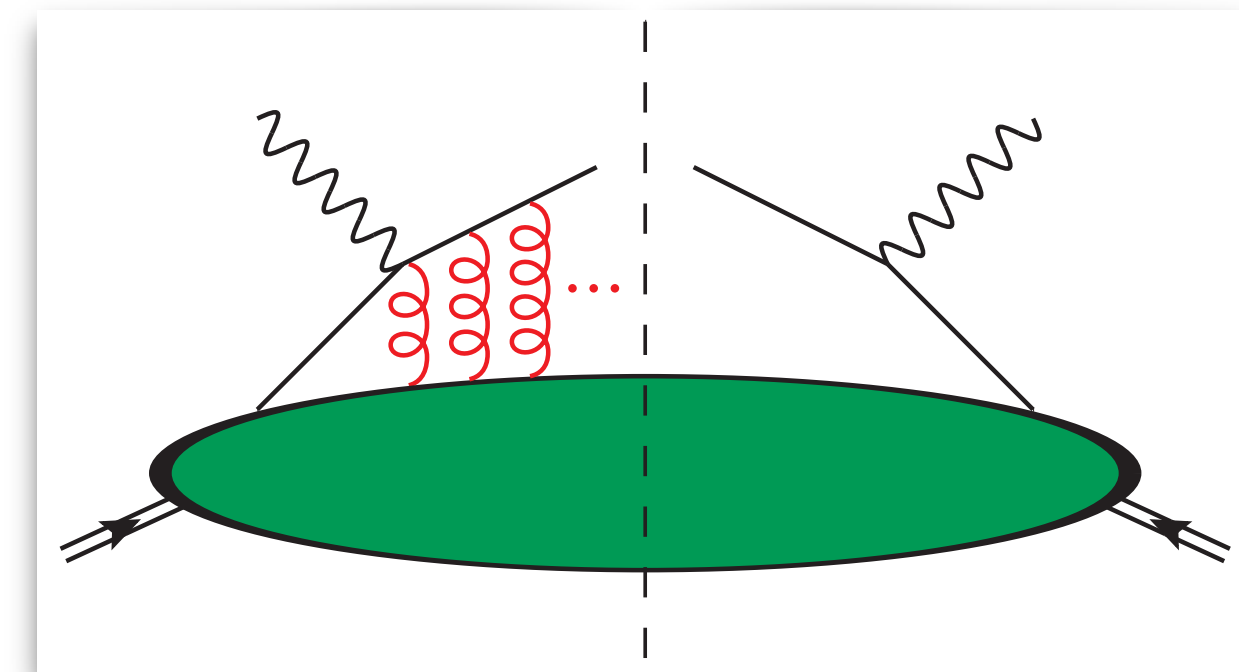
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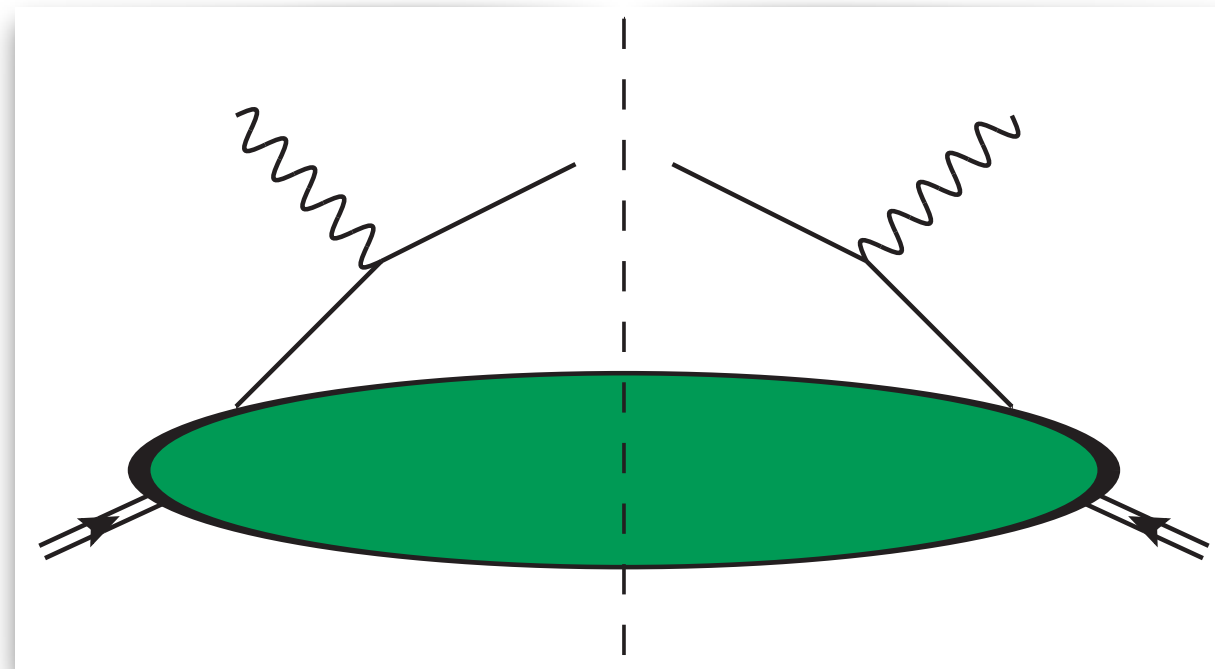
$$\bar{\psi}_j(0) U(0, \xi) \psi_i(\xi)$$



**Gauge link (Wilson line)**  
**Resummation** of (calculable)  
**infinite gluon emissions**



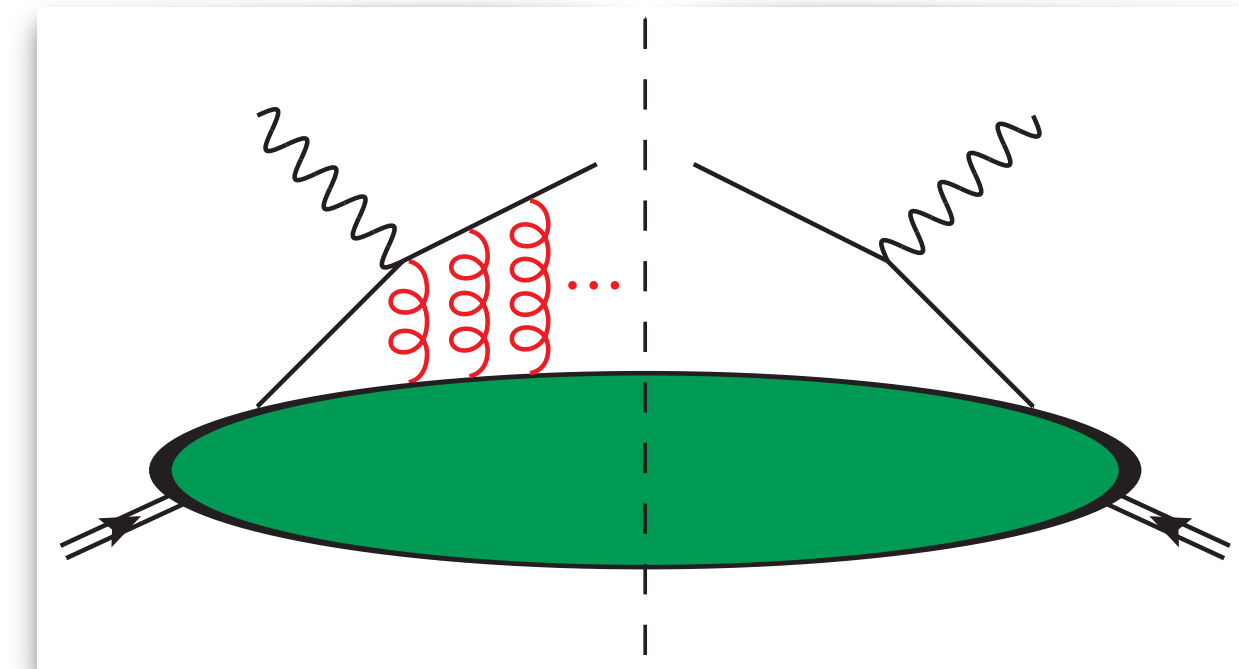
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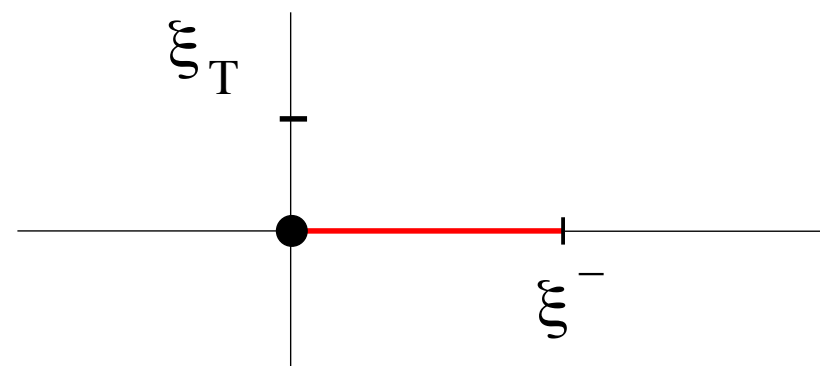


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## Collinear PDFs

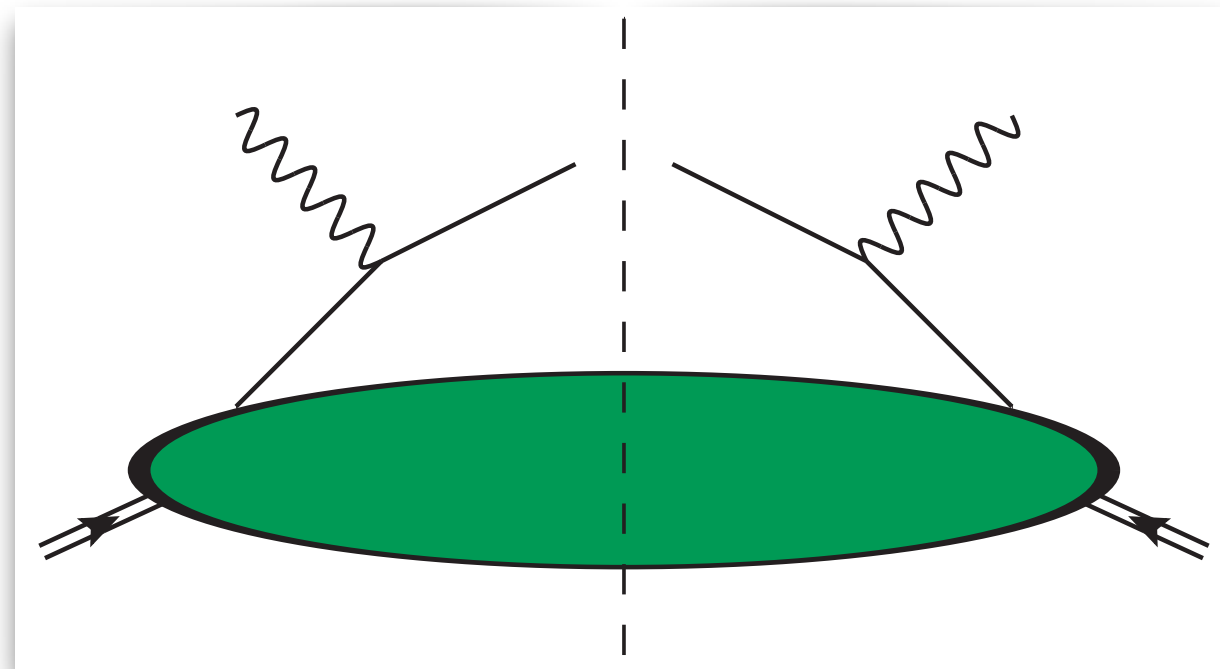
$$\Phi_{ij}(x) \doteq \int d^2 \mathbf{p}_T \Phi_{ij}(x, \mathbf{p}_T) = \int \frac{d\xi^-}{2\pi} e^{ip \cdot \xi} \langle P | \bar{\psi}_j(0) \psi_i(\xi) | P \rangle |_{\xi^+ = 0, \xi_T = 0}$$



- Light-cone:  $\xi^+ = 0, \xi = 0$
- **Straight** gauge link (unique!)
- ( $A^+ = 0$ ) light-cone: WL =  $\hat{1}$
- ✓ **Universality warranted**



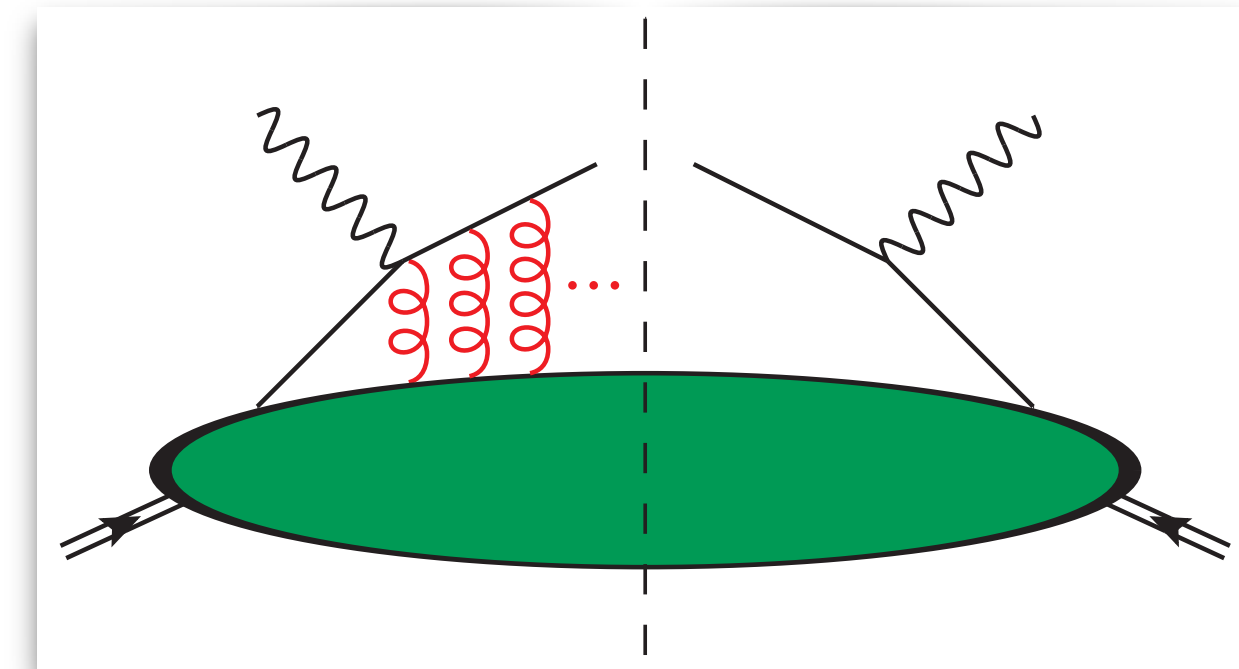
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$$\bar{\psi}_j(0) U(0, \xi) \psi_i(\xi)$$



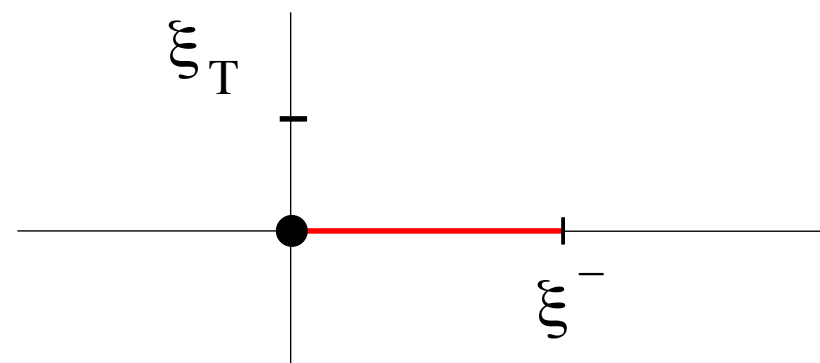
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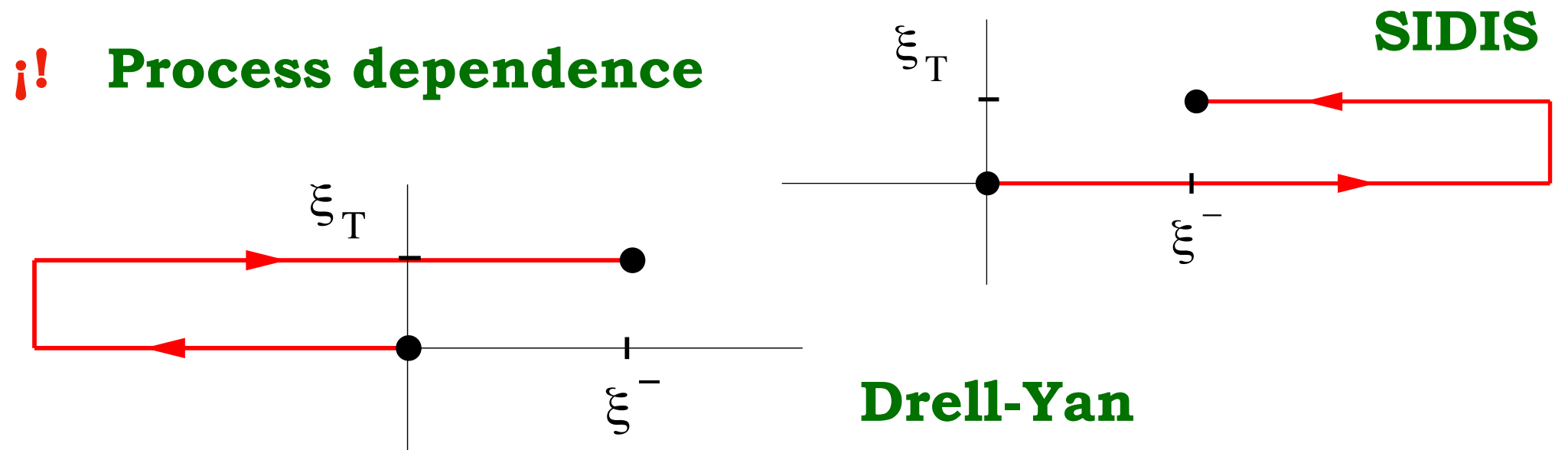
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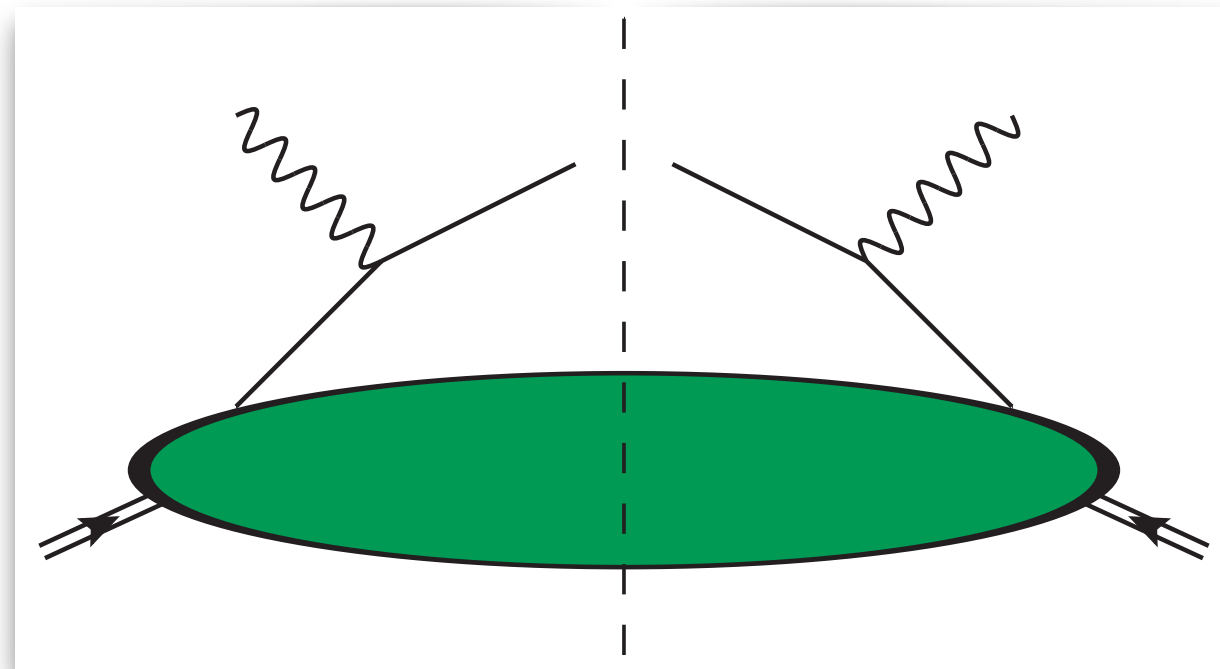
## TMD PDFs

- *Transverse* gauge link not eliminated by gauge choice
- **Staple-like** gauge link (not unique!)

⚠ **Process dependence**



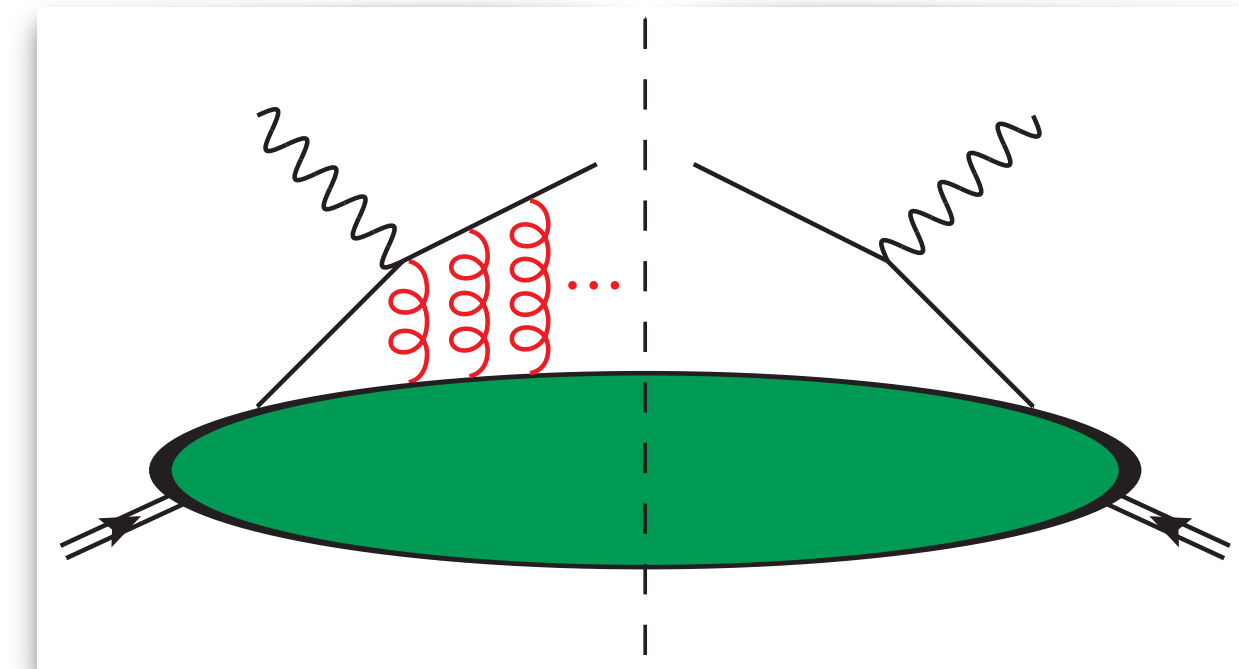
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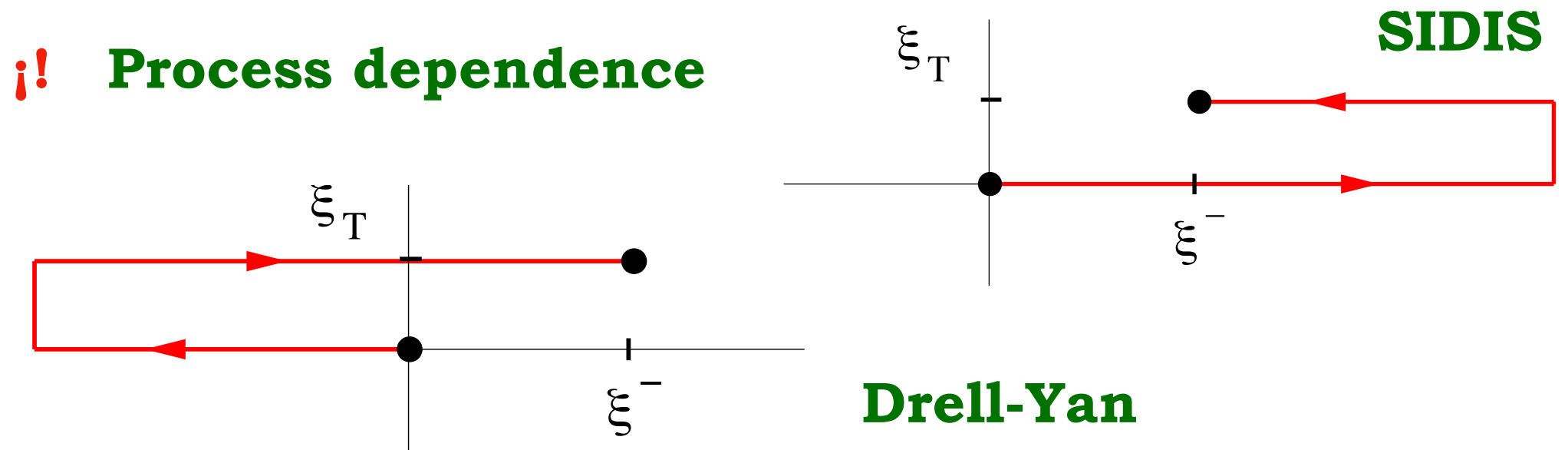
## Link with precision QCD

- *Subtraction* techniques
- WL → factorized **soft & collinear emissions**
- **Infrared-safe** observables
- 😊 **Hadron structure** and **precision QCD**  
*speak the same language*

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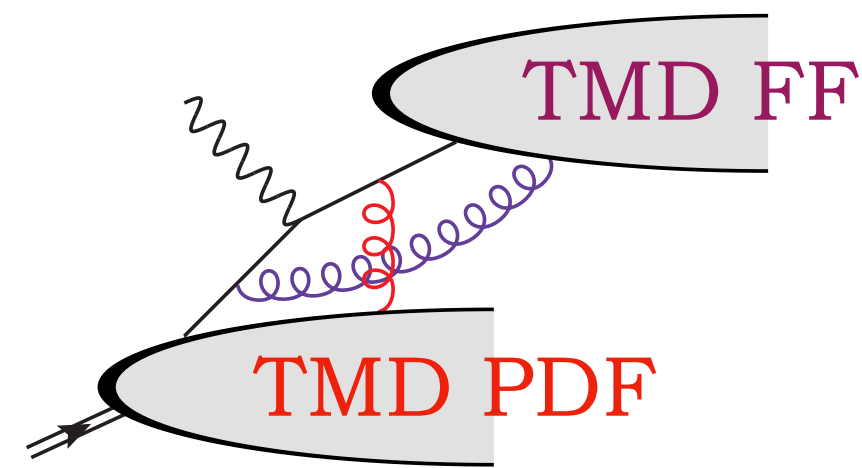
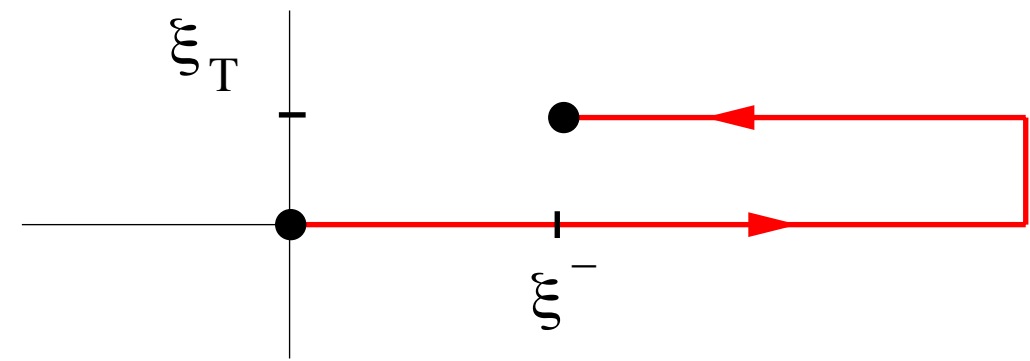




# Process dependence of quark TMD PDFs & FFs

## SIDIS

[ + ] staple link

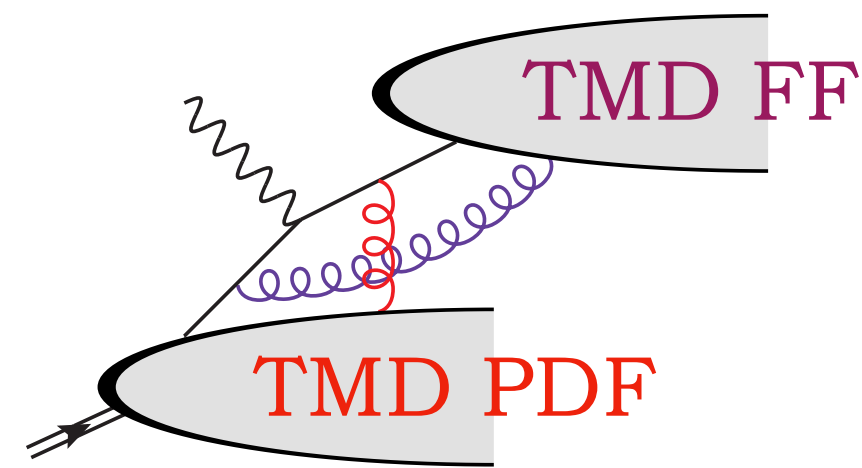
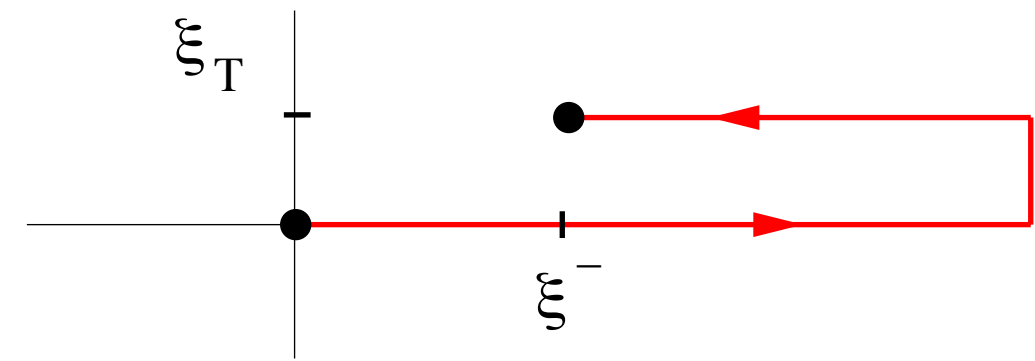


- \* PDF  $\rightarrow$  color flow annihilated within final state
- \* FF  $\rightarrow$  color flow from final to initial state

# Process dependence of quark TMD PDFs & FFs

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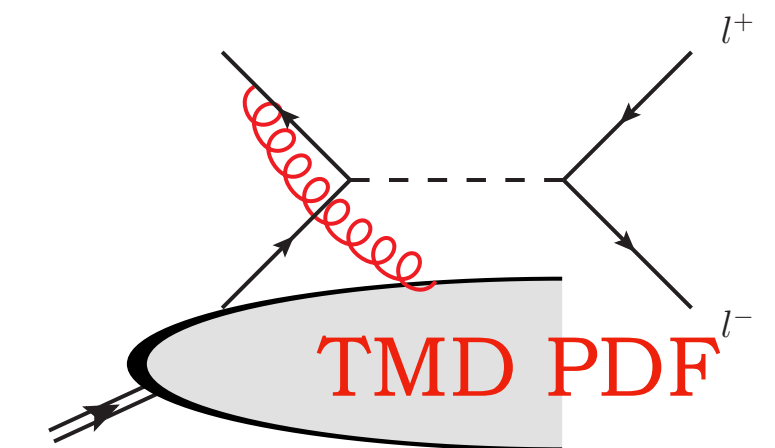
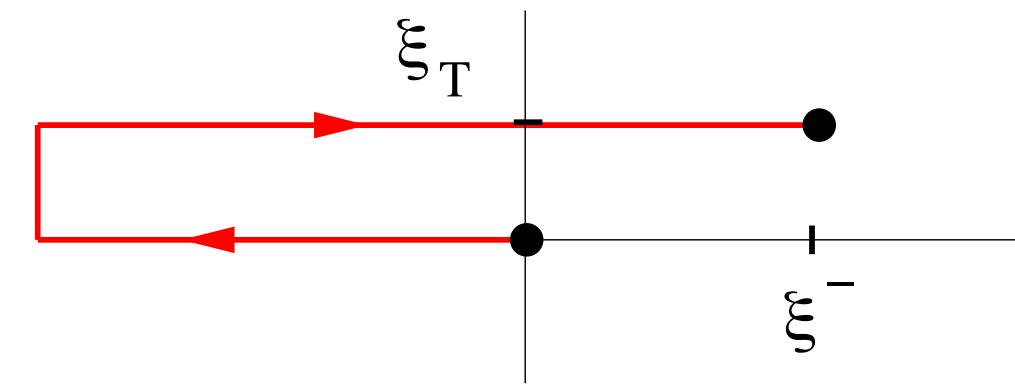
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## Drell-Yan

[ - ] staple link



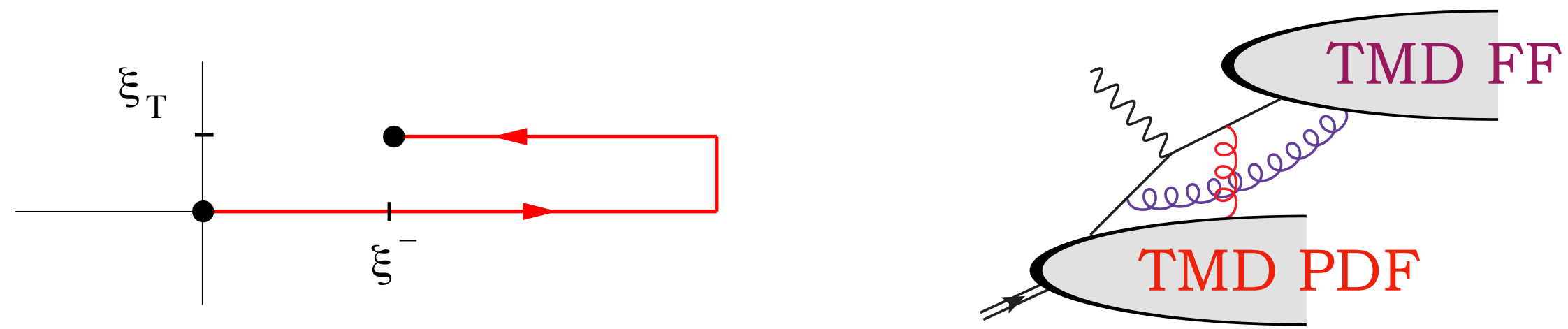
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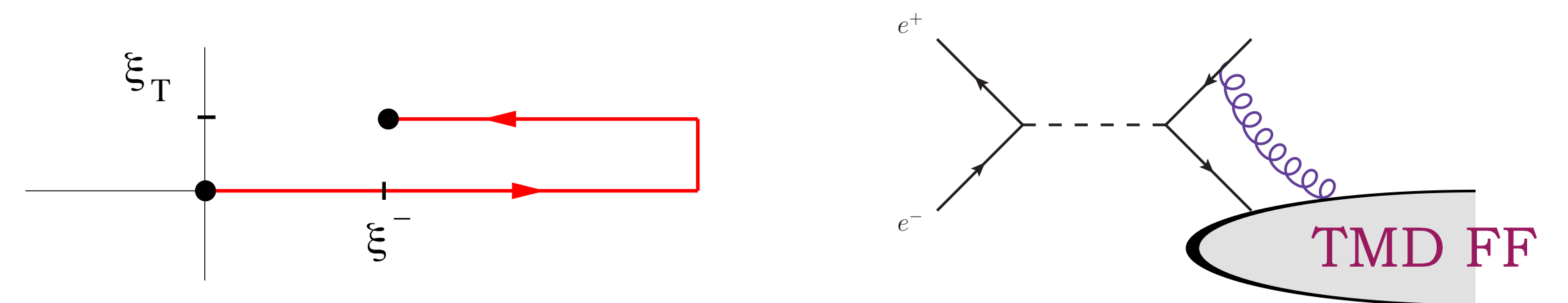
[ - ] staple link



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## $e^+ + e^- \rightarrow$ hadrons

[ + ] staple link



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## Modified universality

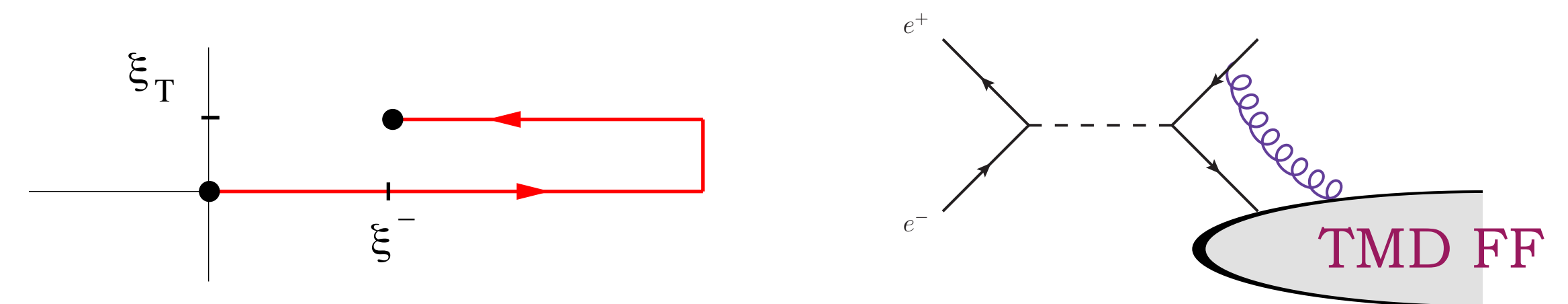
- \* PDFs  $\rightarrow$  change of sign in T-odd densities

$$f_{1T}^\perp [\text{SIDIS}] \equiv f_{1T}^\perp [ + ] = -f_{1T}^\perp [ - ] \equiv -f_{1T}^\perp [\text{DY}]$$

- \* FFs  $\rightarrow$  standard universality preserved

## $e^+ + e^- \rightarrow \text{hadrons}$

[ + ] staple link



- \* FF  $\rightarrow$  color flow annihilated within final state



# Gluon TMD PDFs: Gauge links & modified universality

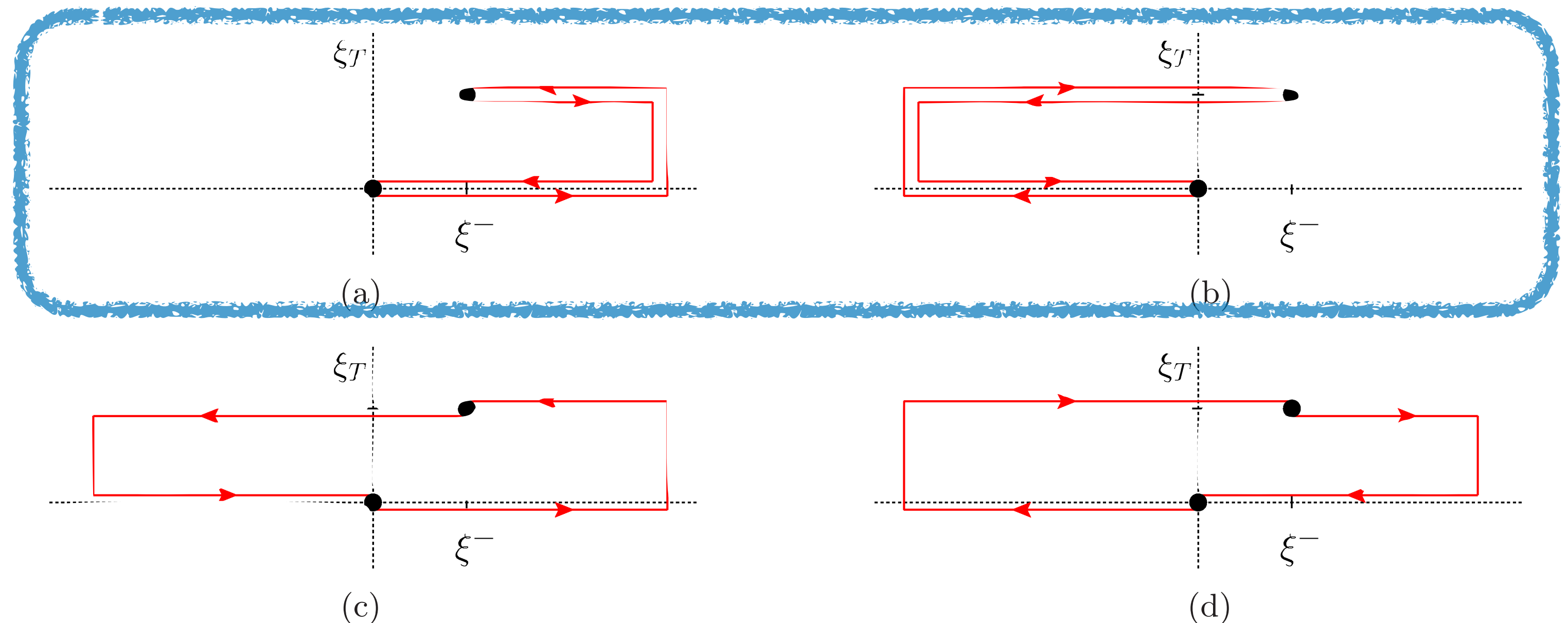
- \* Gluon TMDs → more complicated structure with respect to quark staple links
- \* Factorization-preserving processes → two main kinds of modified universality
- \* Different classes of processes → distinct gluon TMDs, not related to each other

# Gluon TMD PDFs: Gauge links & modified universality

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## *f*-type (WW)

(a) [ + , + ] or (b) [ - , - ]



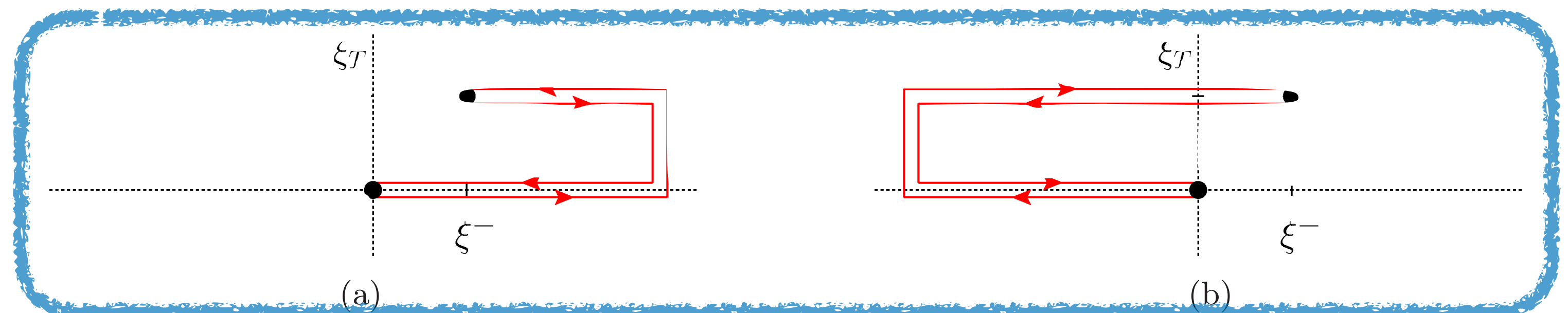


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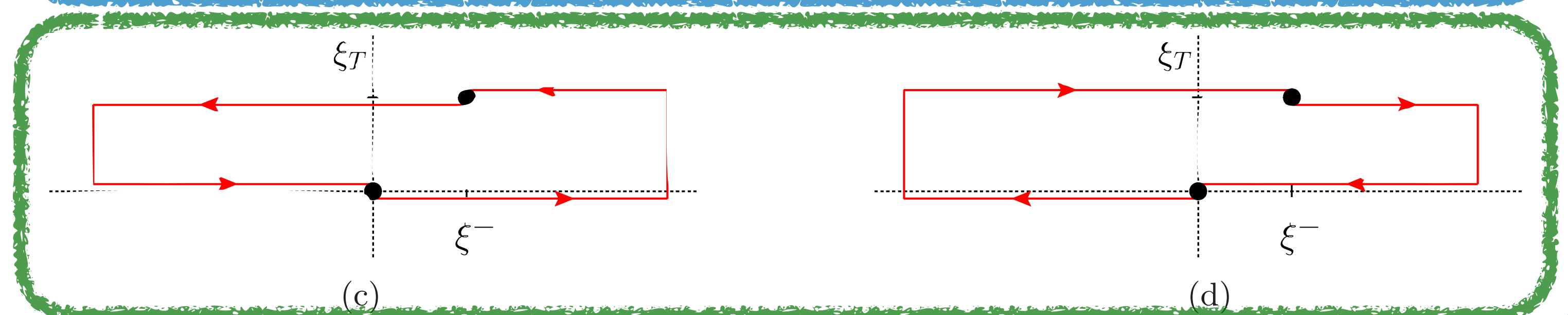
## *f*-type (WW)

(a) [ + , + ] or (b) [ - , - ]



## *d*-type (dipole)

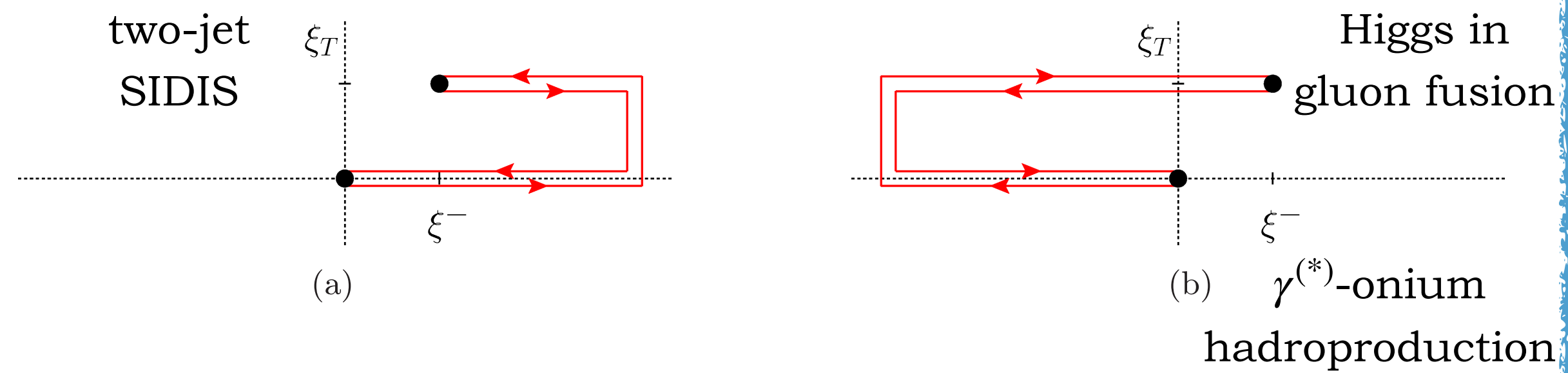
(c) [ + , - ] or (d) [ - , + ]



# Accessing f-type and d-type gluon TMD PDFs

## *f*-type (WW)

(a) [ + , + ] or (b) [ - , - ]



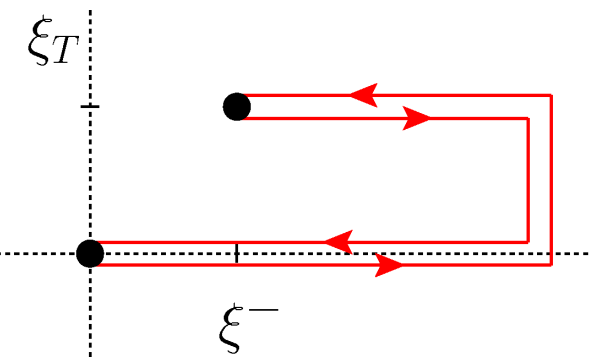


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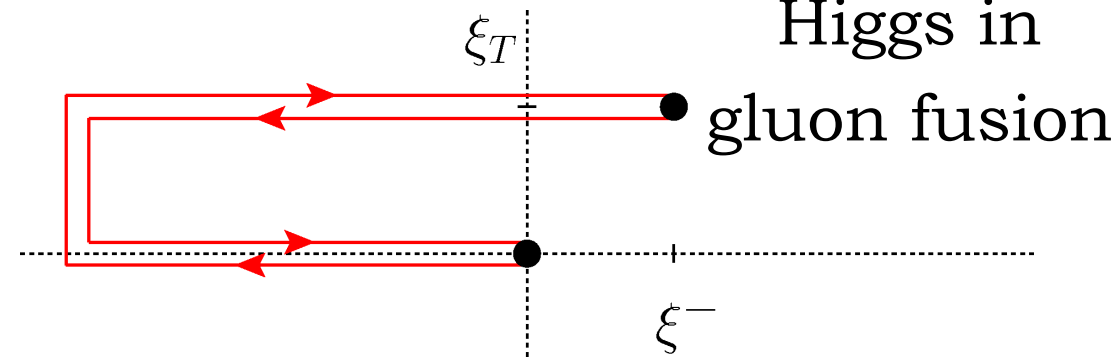
(a) [ + , + ] or (b) [ - , - ]

two-jet  
SIDIS



(a)

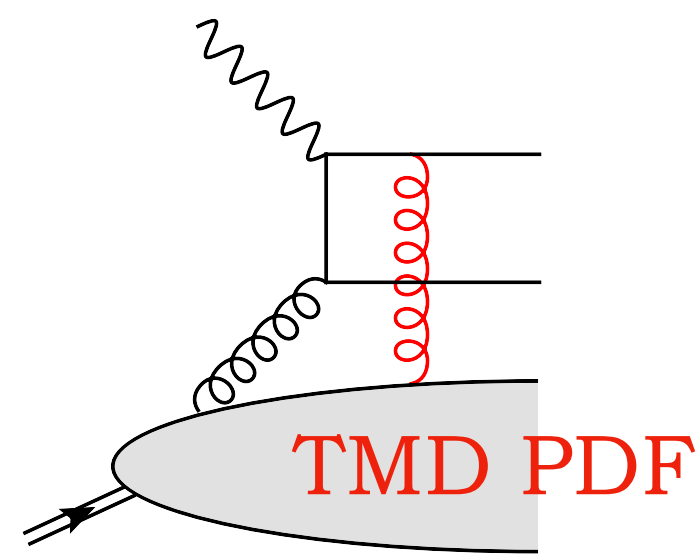
Higgs in  
gluon fusion



(b)

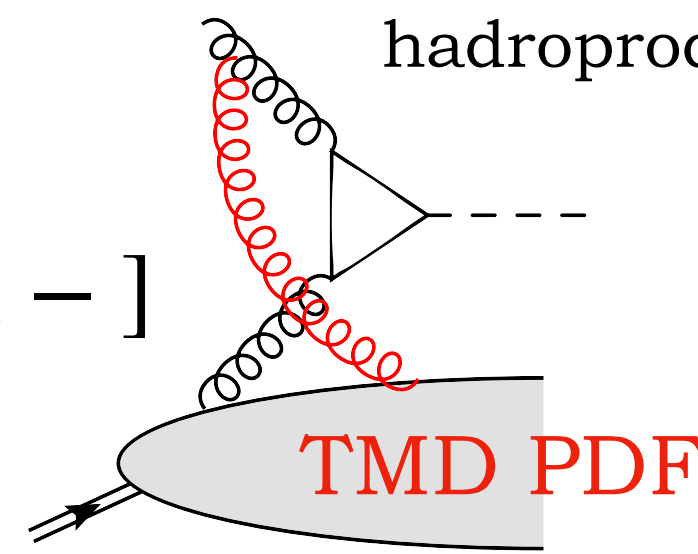
$\gamma^{(*)}$ -onium  
hadroproduction

[ +



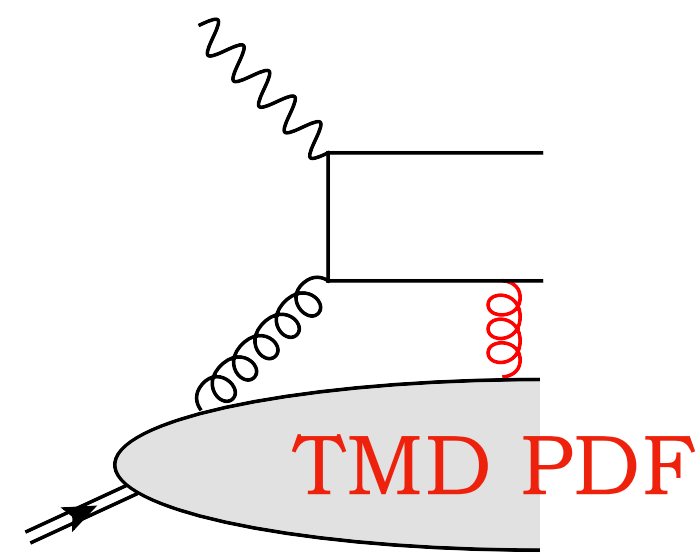
TMD PDF

[ - , - ]



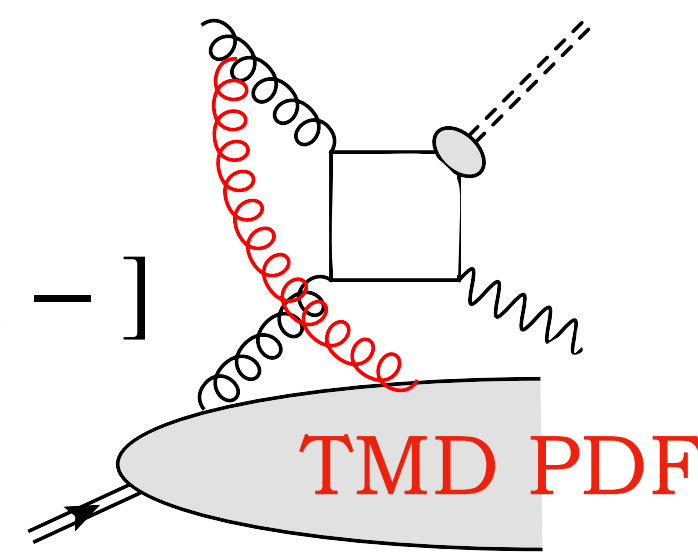
TMD PDF

+ ]



TMD PDF

[ - , - ]



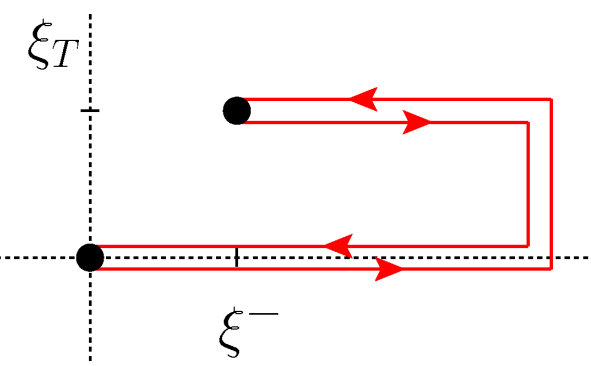
TMD PDF

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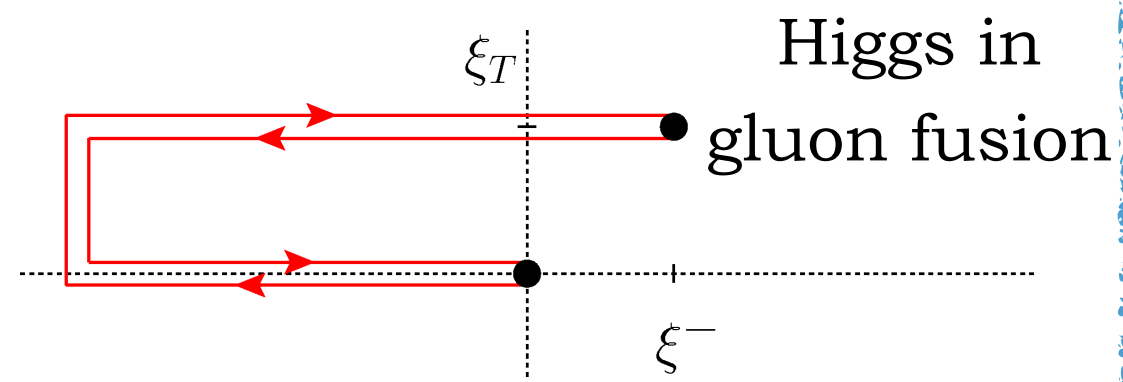
## *f*-type (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

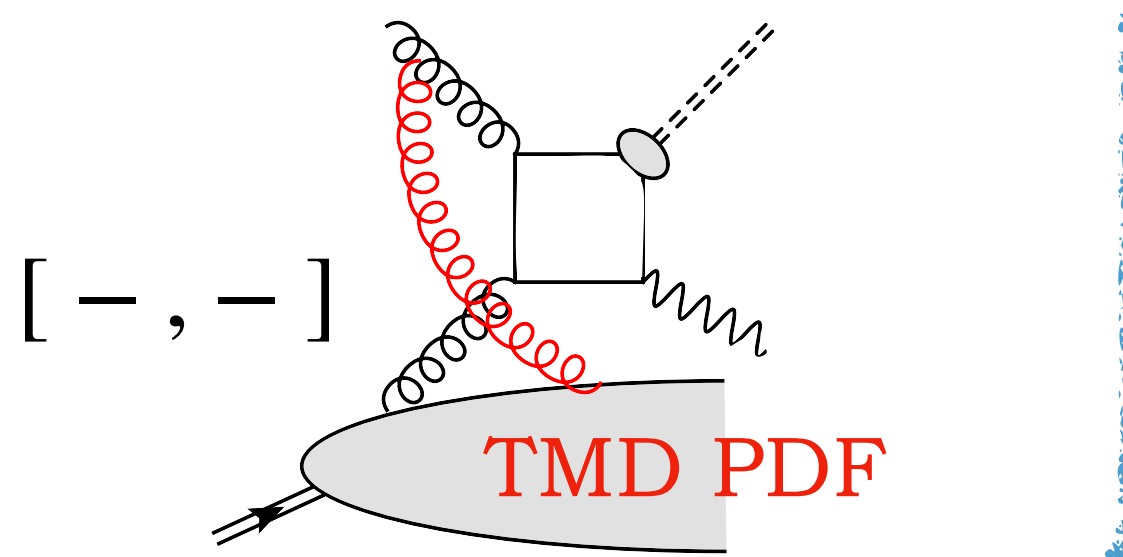
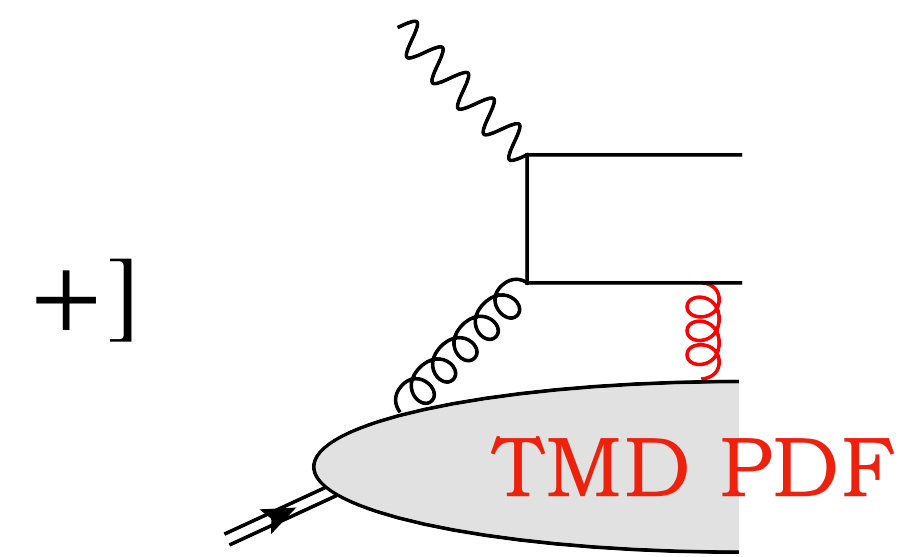
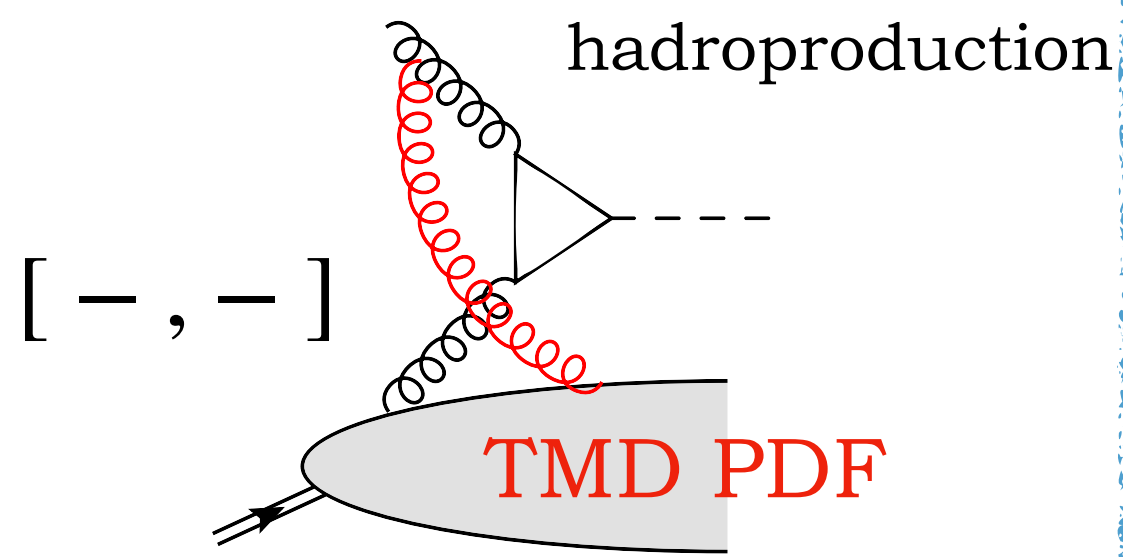
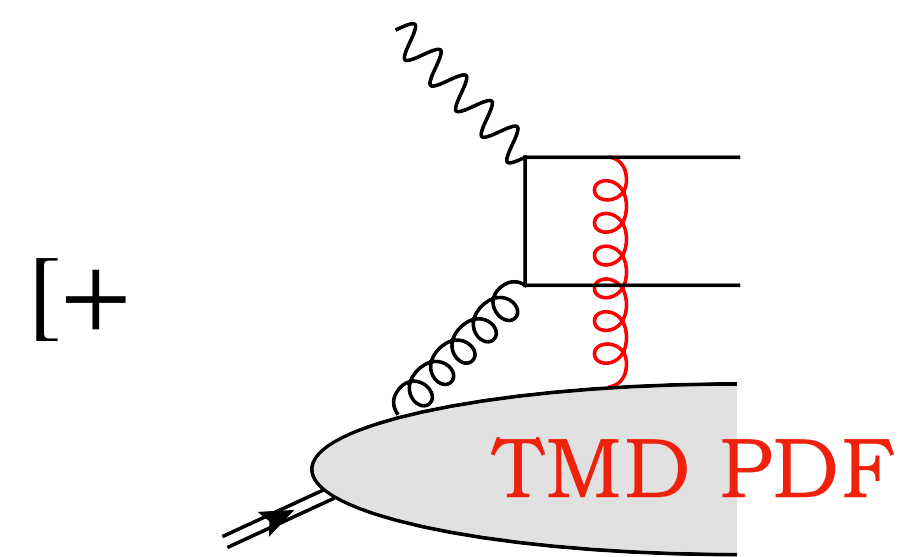
two-jet  
SIDIS



(a)

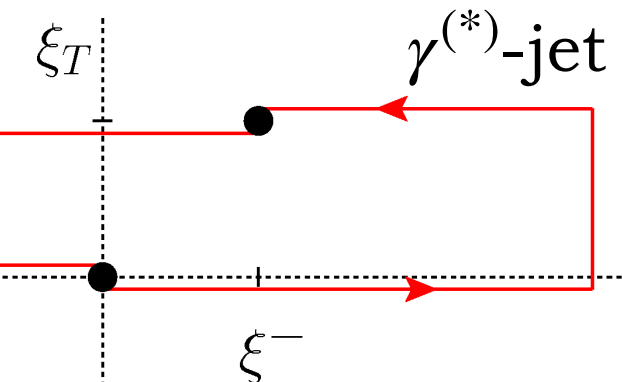


(b)

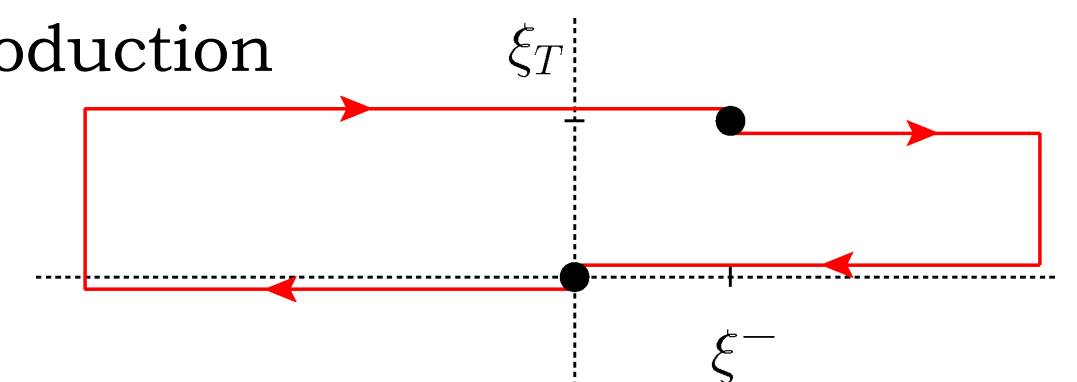


## *d*-type (DP)

(c)  $[+, -]$  or (d)  $[-, +]$

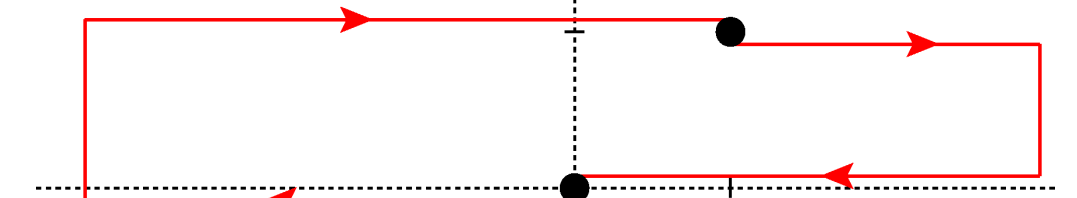
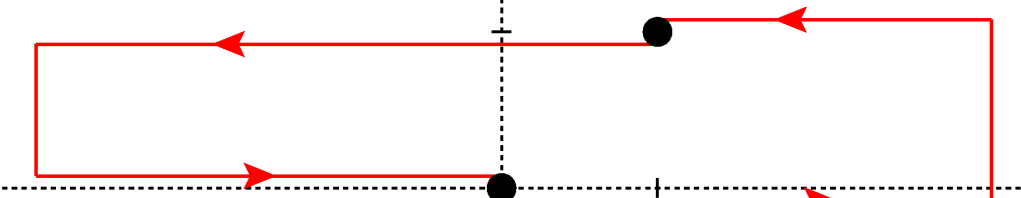


(c)



(d)

$\gamma^{(*)}$ -jet hadroproduction



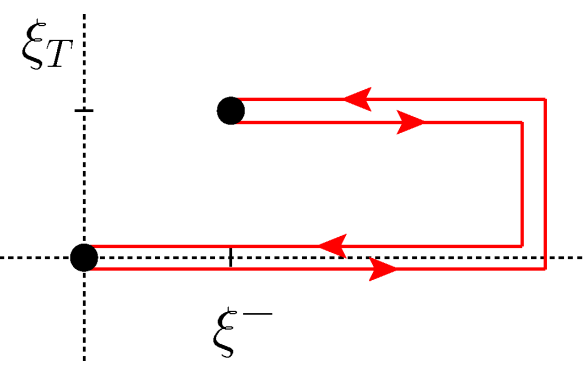


# Accessing f-type and d-type gluon TMD PDFs

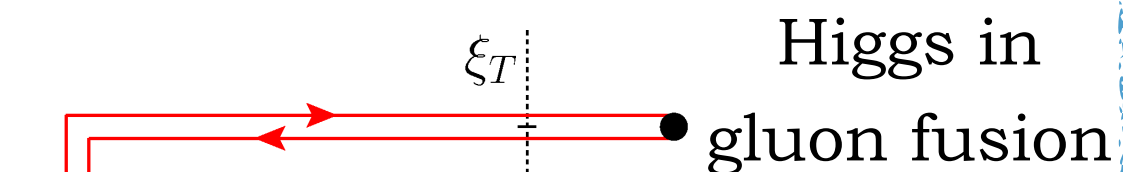
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two-jet  
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(a)

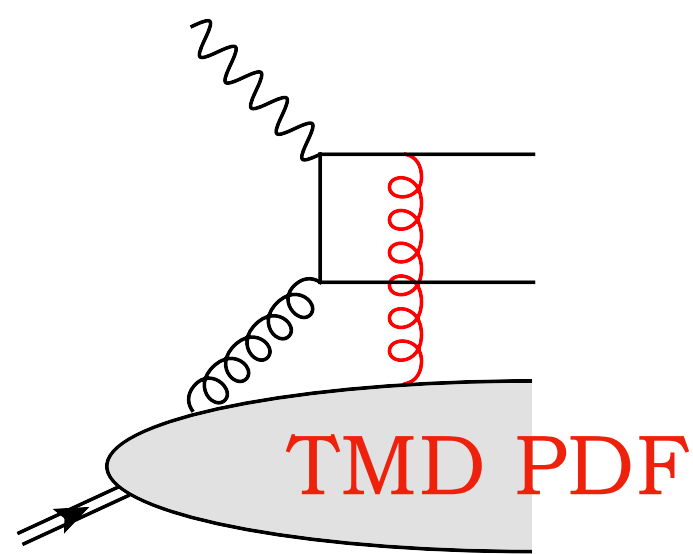


(b)

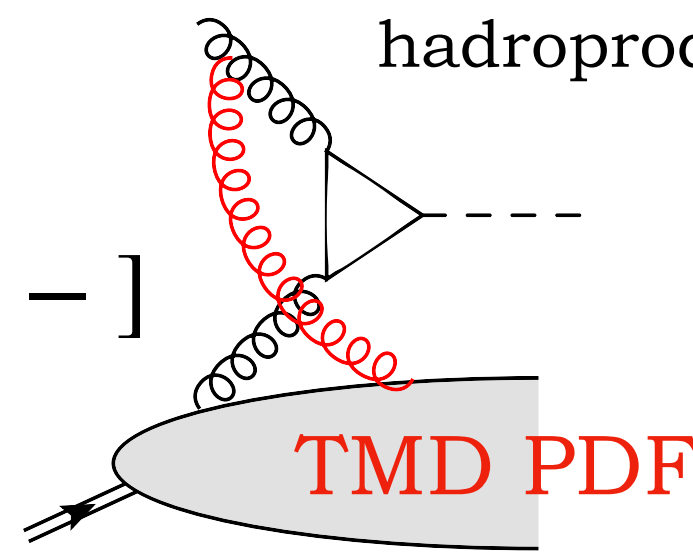
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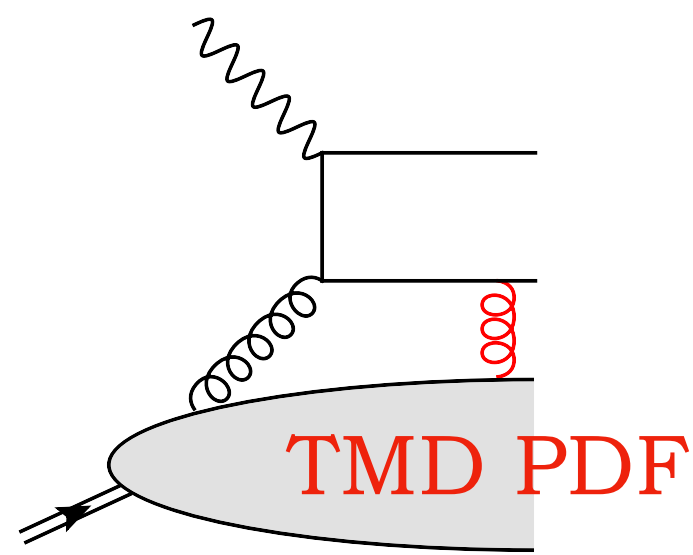
[+]



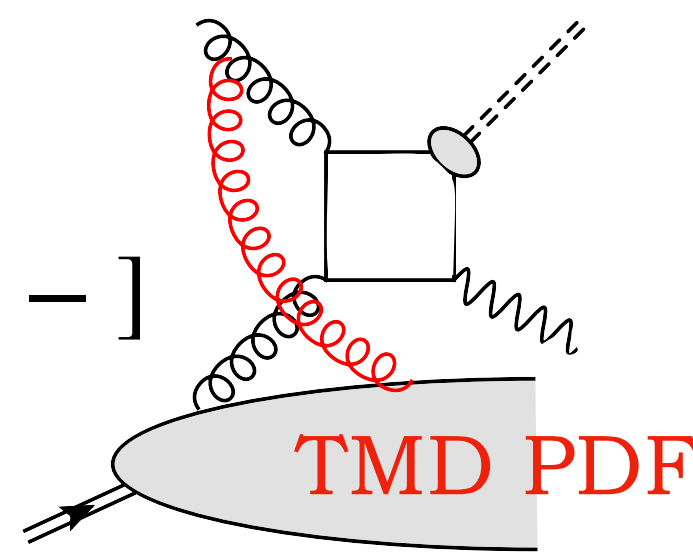
[-, -]



[+]



[-, -]

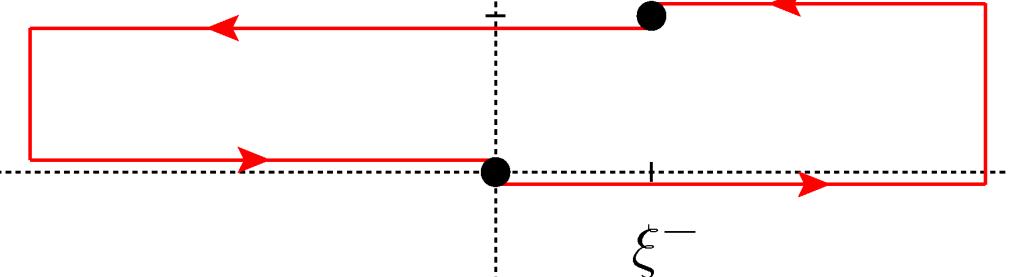


## *d*-type (DP)

(c)  $[+, -]$  or (d)  $[-, +]$

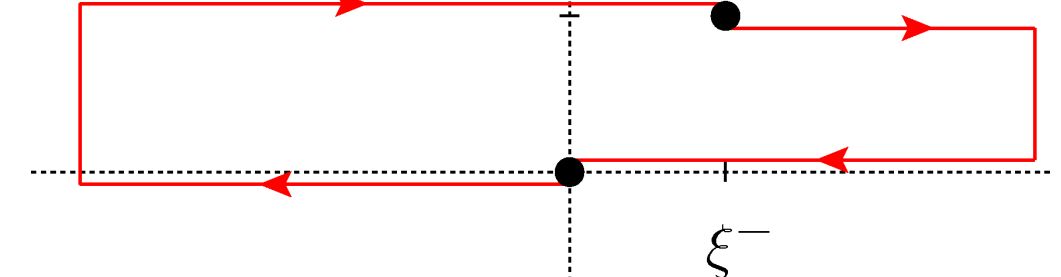
$\xi_T$

$\gamma^{(*)}$ -jet hadroproduction



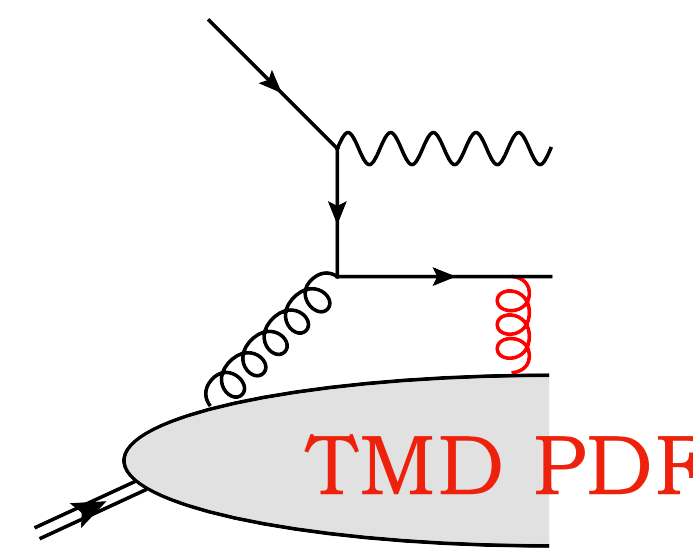
(c)

$\xi_T$

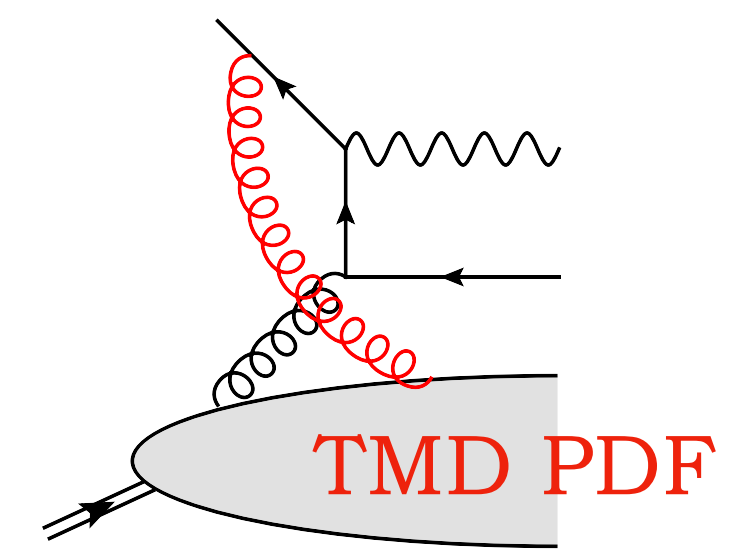


(d)

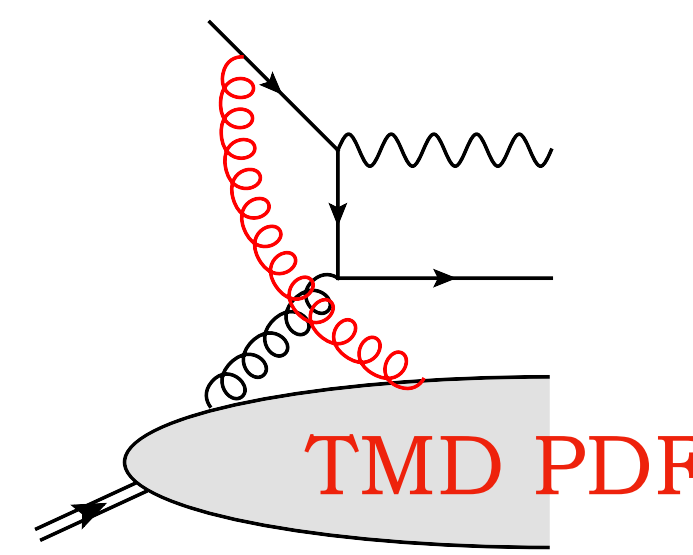
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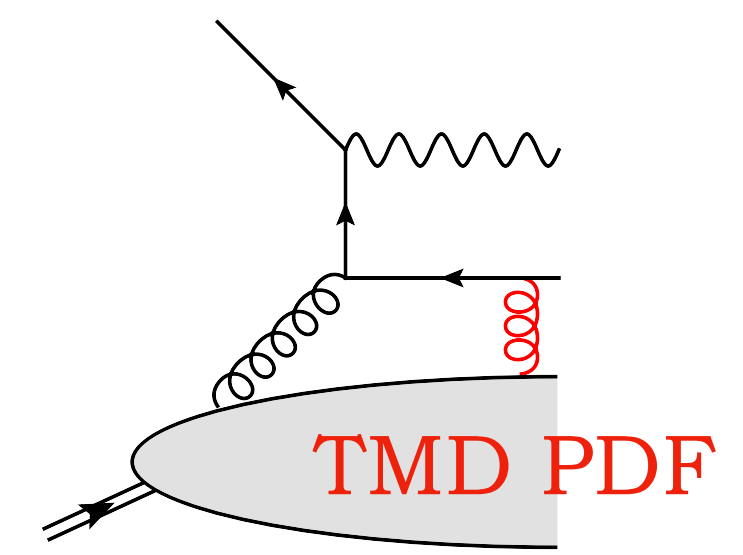
[-]



-]



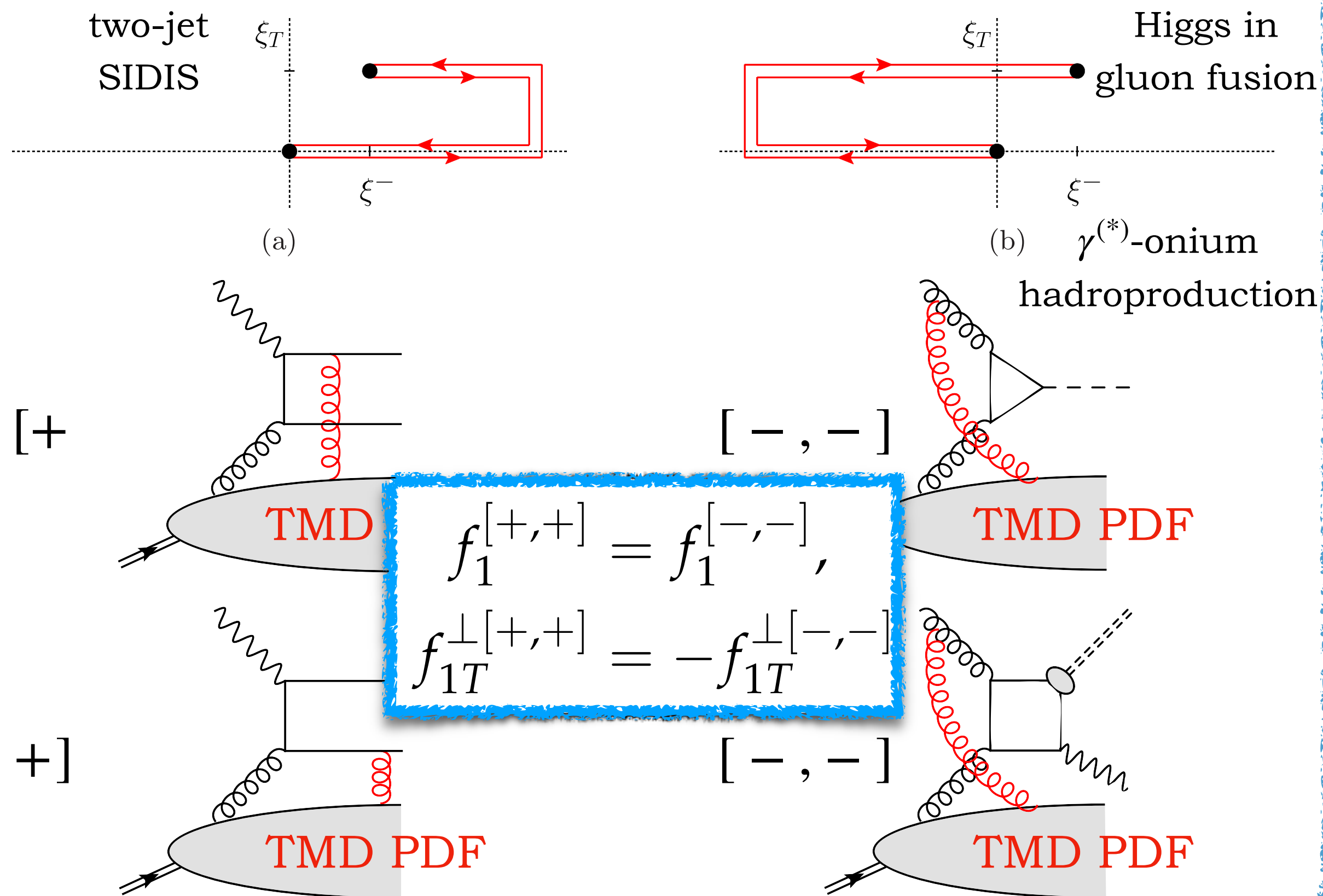
[+]



# Accessing f-type and d-type gluon TMD PDFs

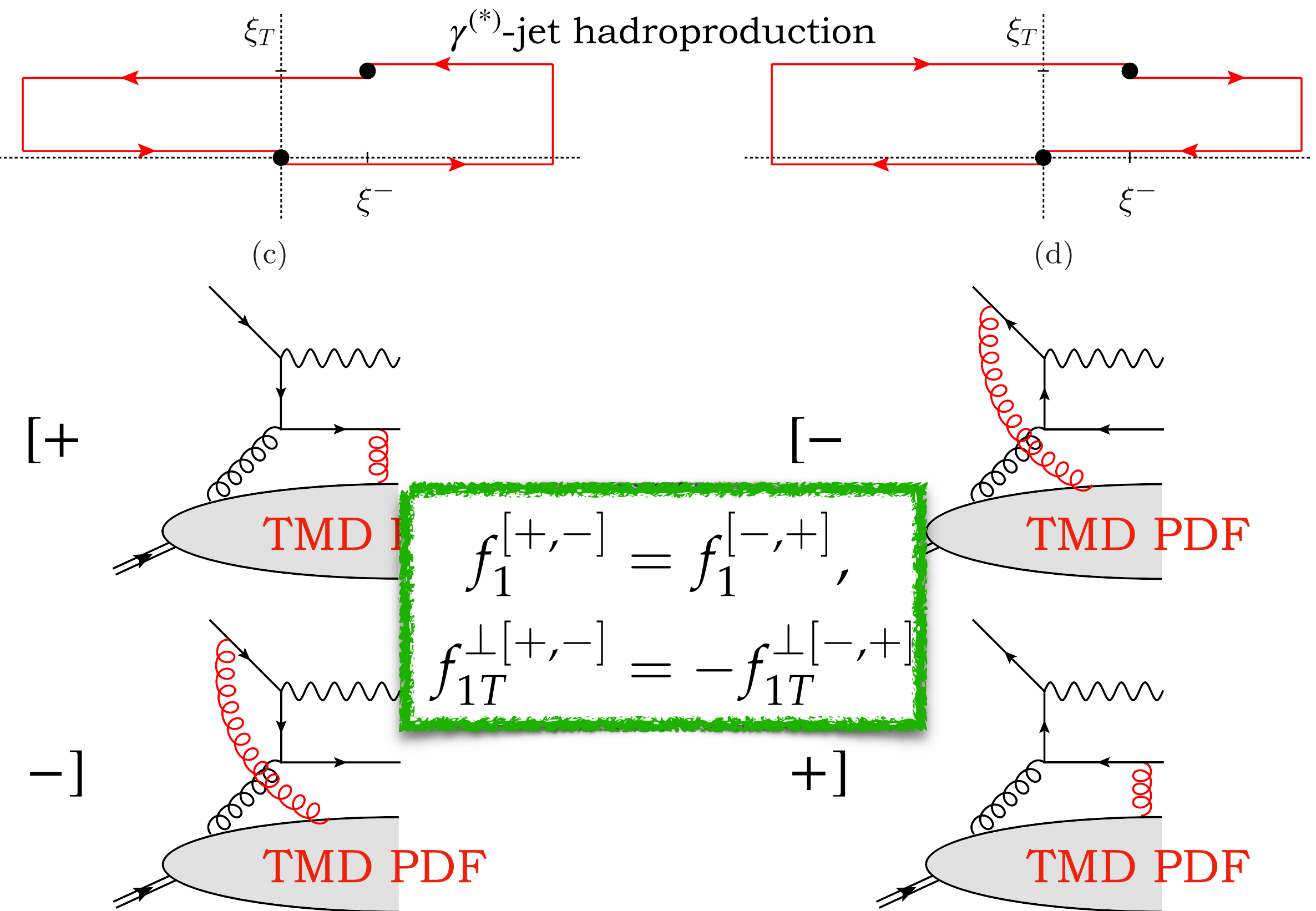
## f-type (WW)

(a) [ + , + ] or (b) [ - , - ]



## d-type (DP)

(c) [ + , - ] or (d) [ - , + ]



! Gauge link  $\rightarrow$  two main independent sets of TMD PDFs, **not related** to each other !

# Dihadron hadroproduction and factorization breaking

\* Proof of factorization violation [\[T. J. Rogers, P. J. Mulders \(2010\)\]](#)

\* Assumed factorization in SCET and CGC

\* Significance of low- $x$  studies

\* Size of factorization-breaking effects small?

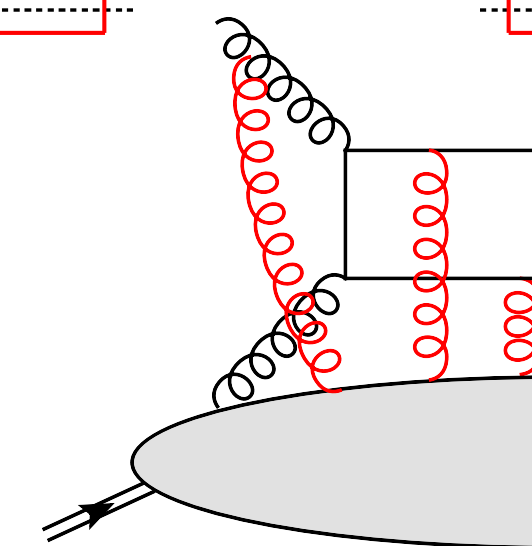
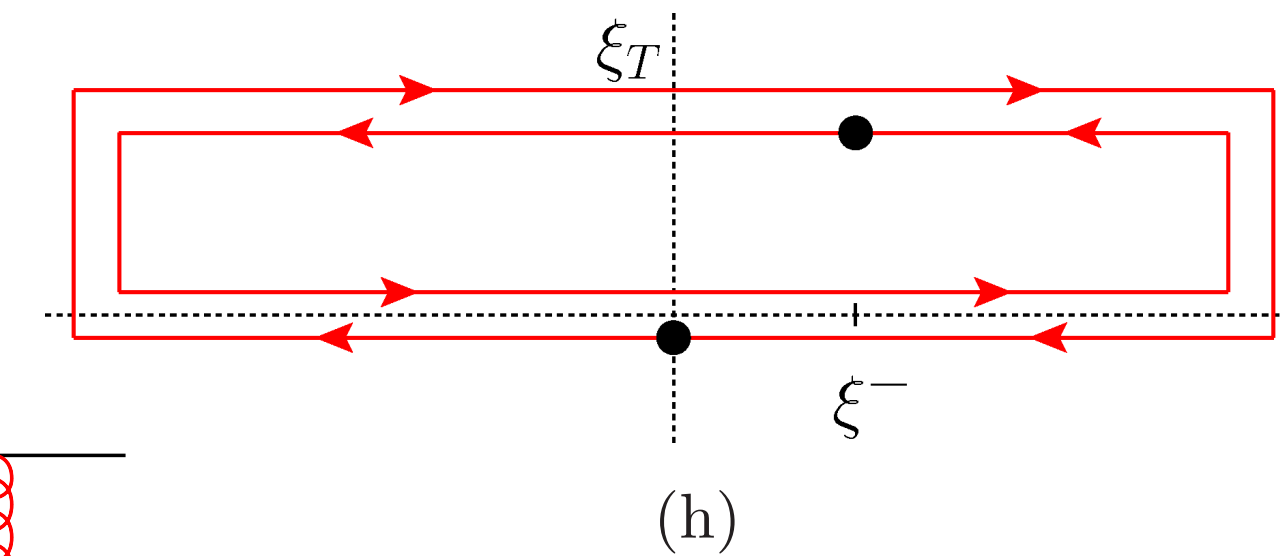
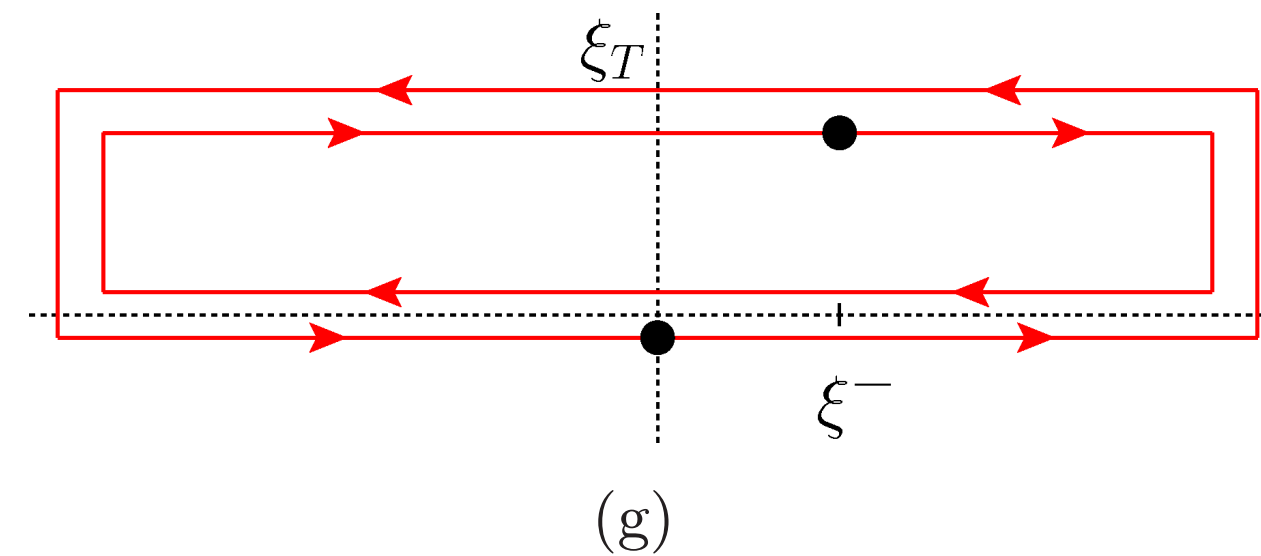
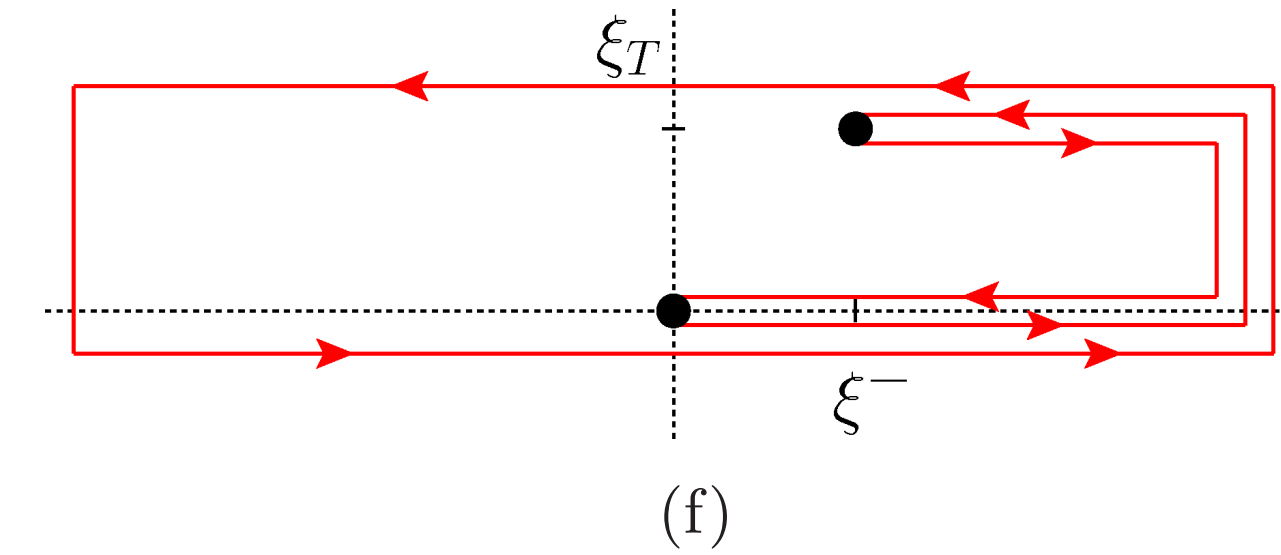
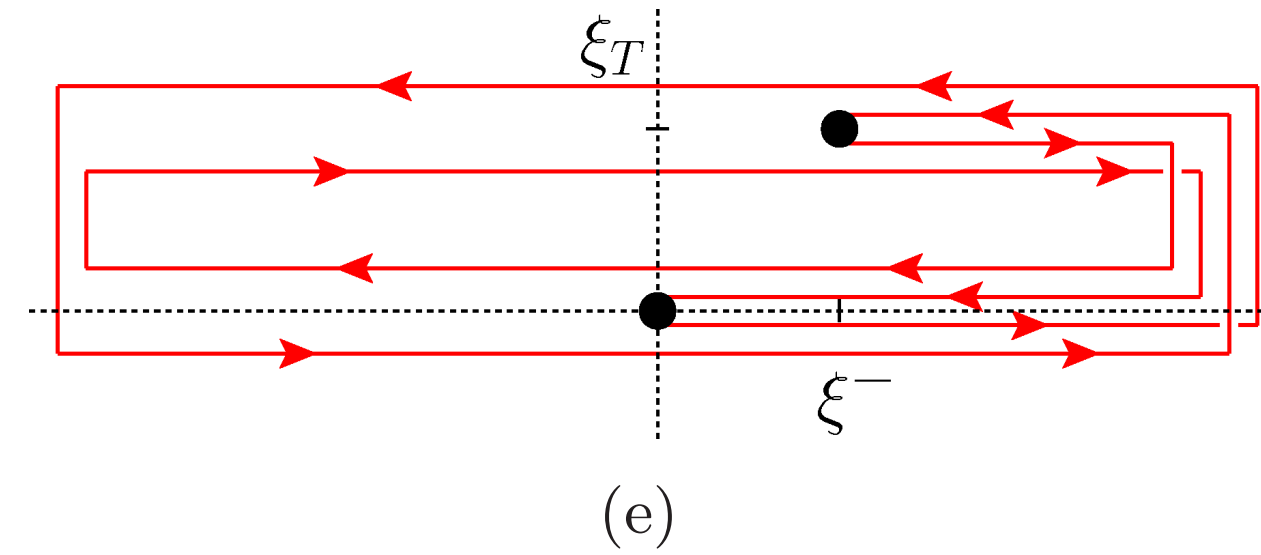
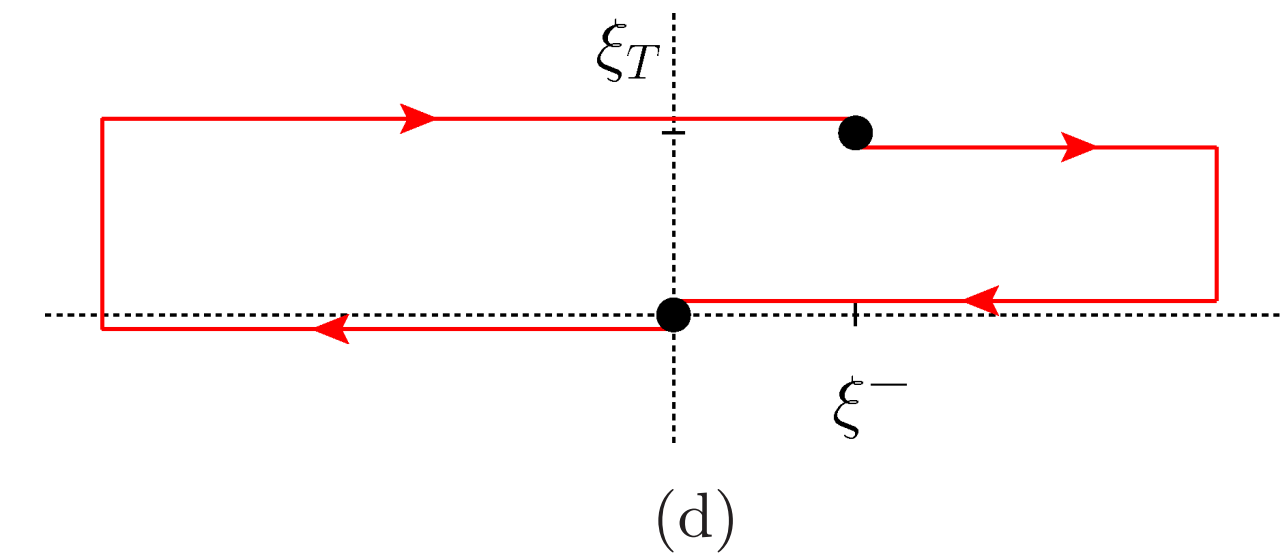
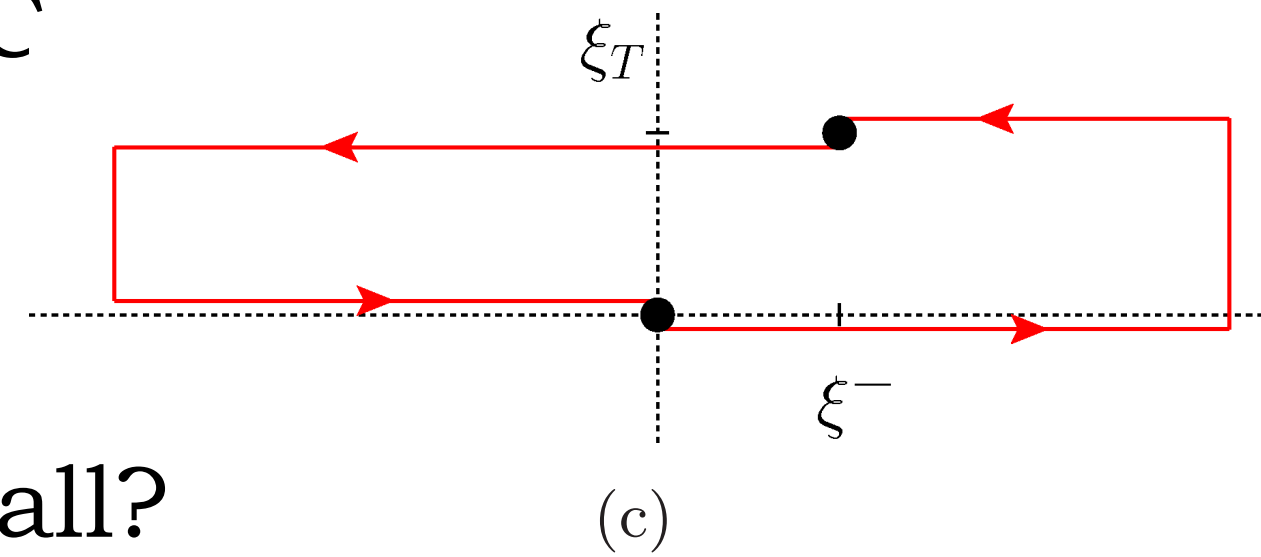
\* DP TMDs:

(c)  $[+, -]$  and (d)  $[-, +]$

\* Appearance of new gauge **loop links**:

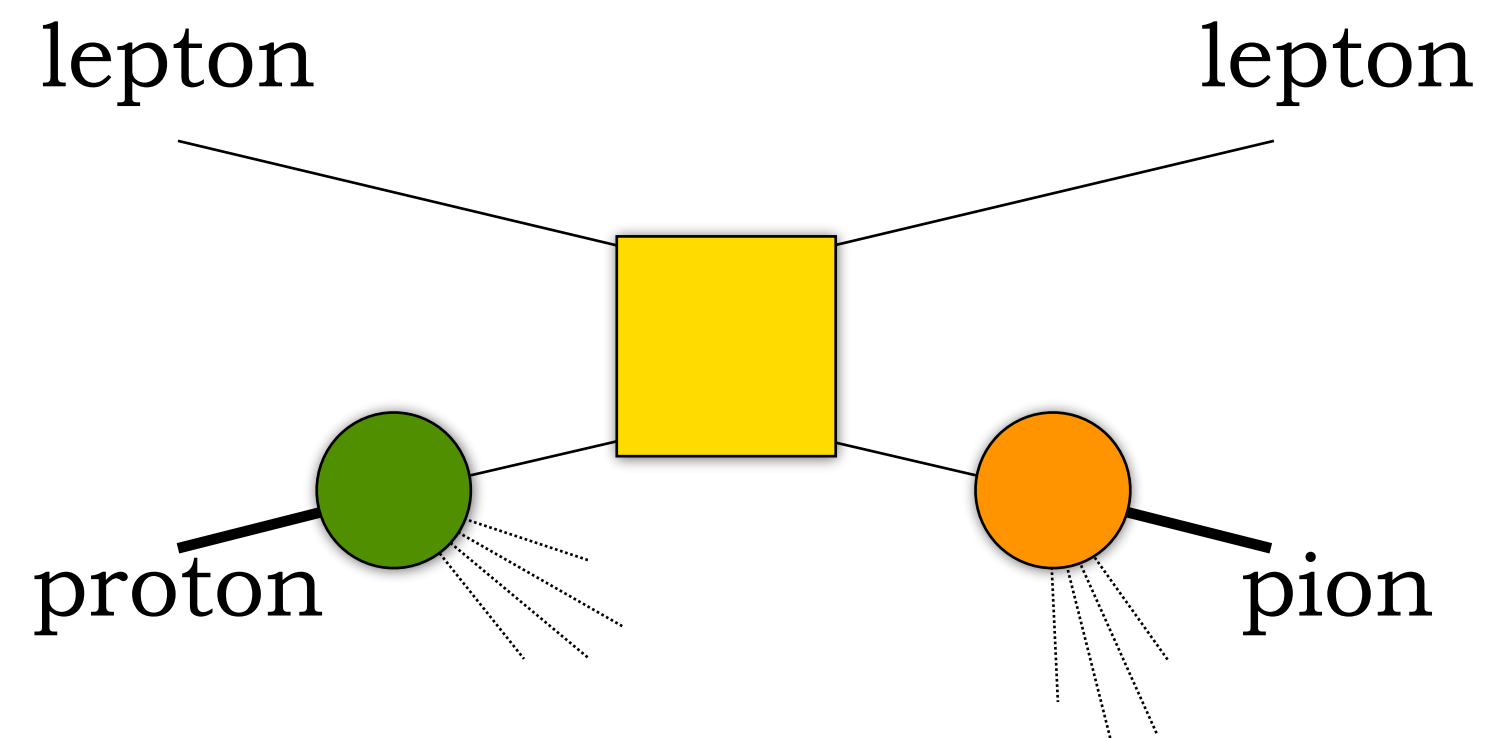
(e)  $[+\square, +\square]$ , (f)  $[+, +\square]$ ,

(g)  $[\square, \square]$ , and (h)  $[\square, \square]$



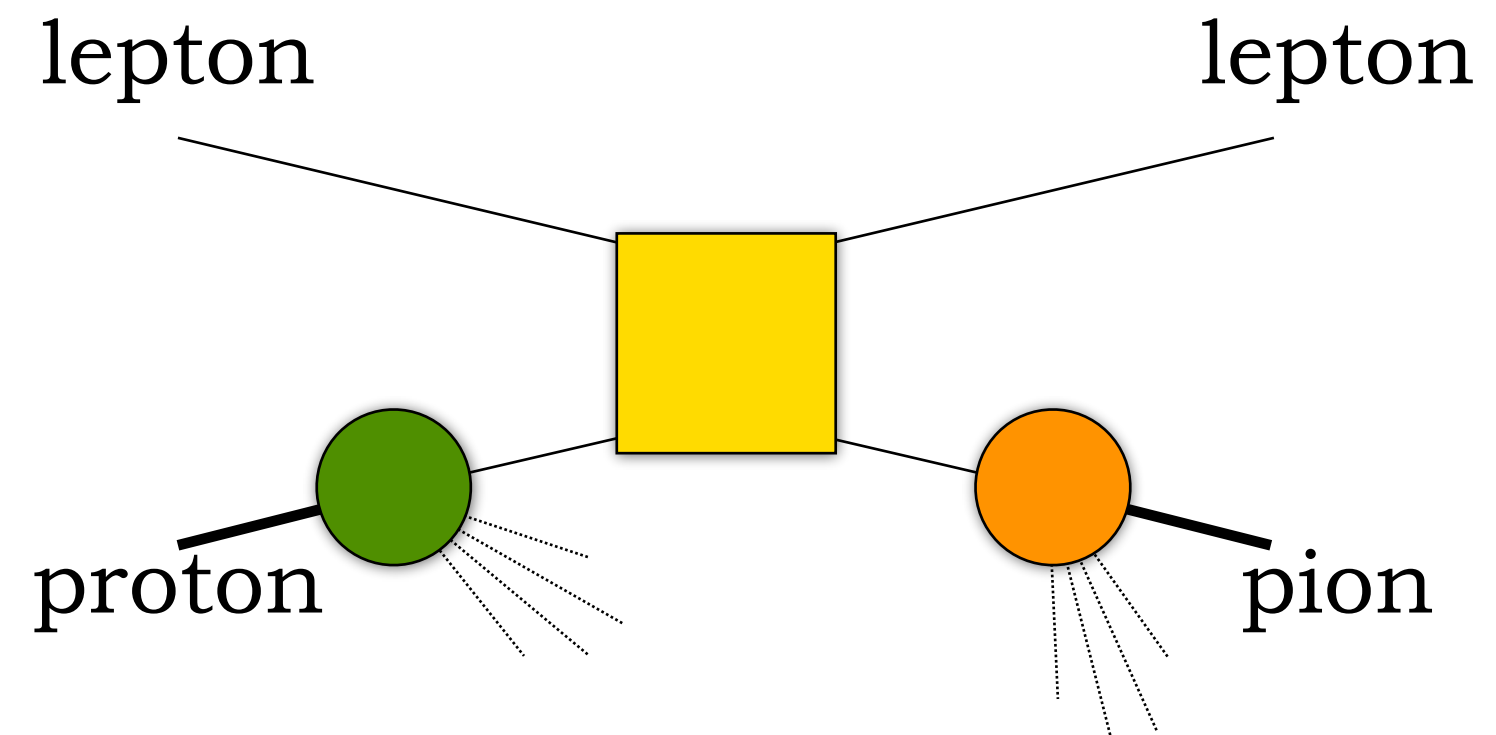


## SIDIS

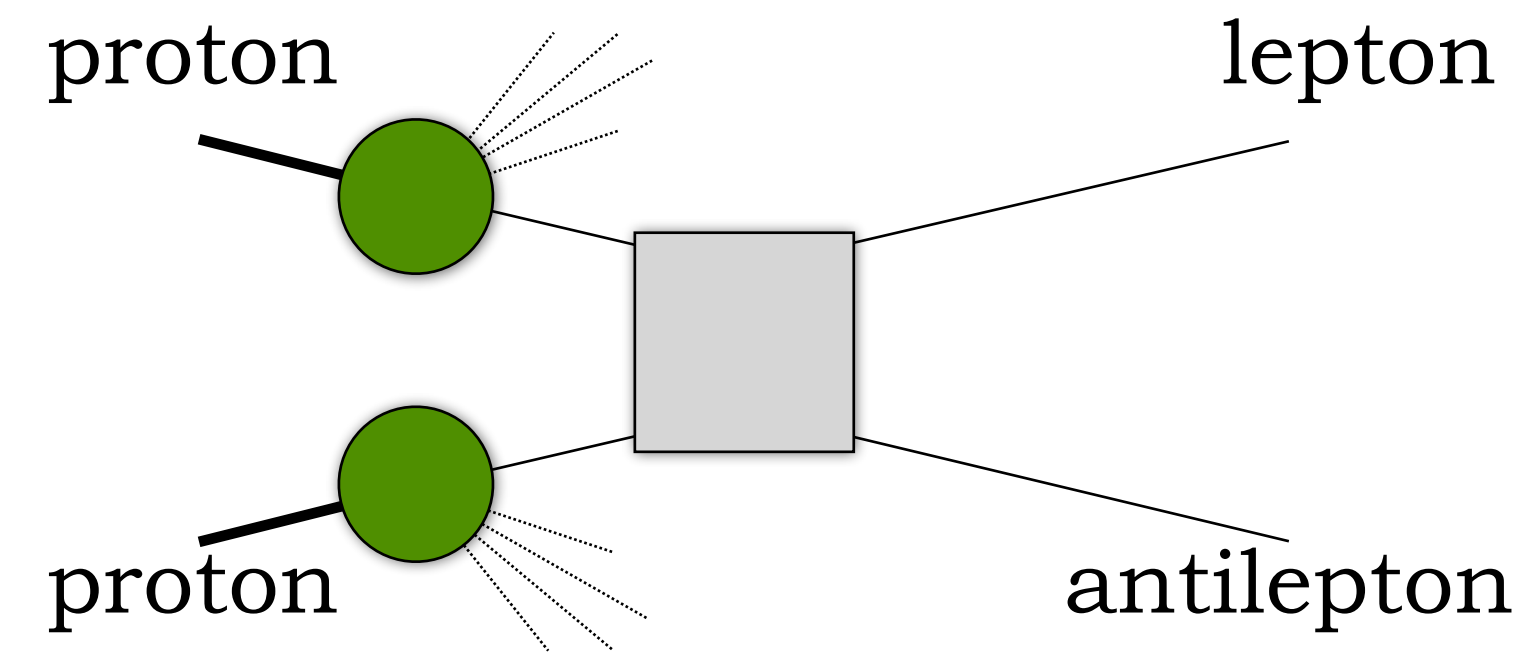


# Factorization and universality

## SIDIS

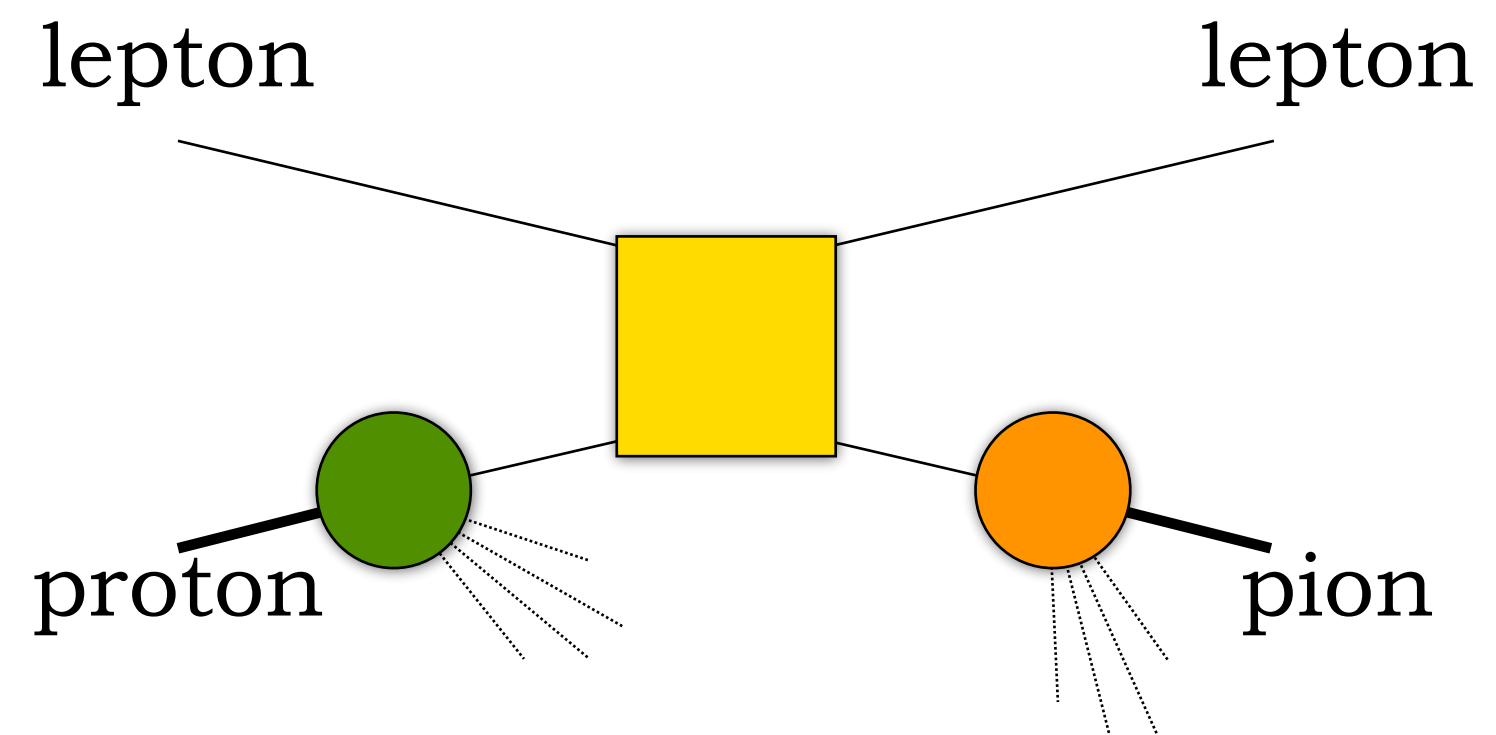


## Drell-Yan

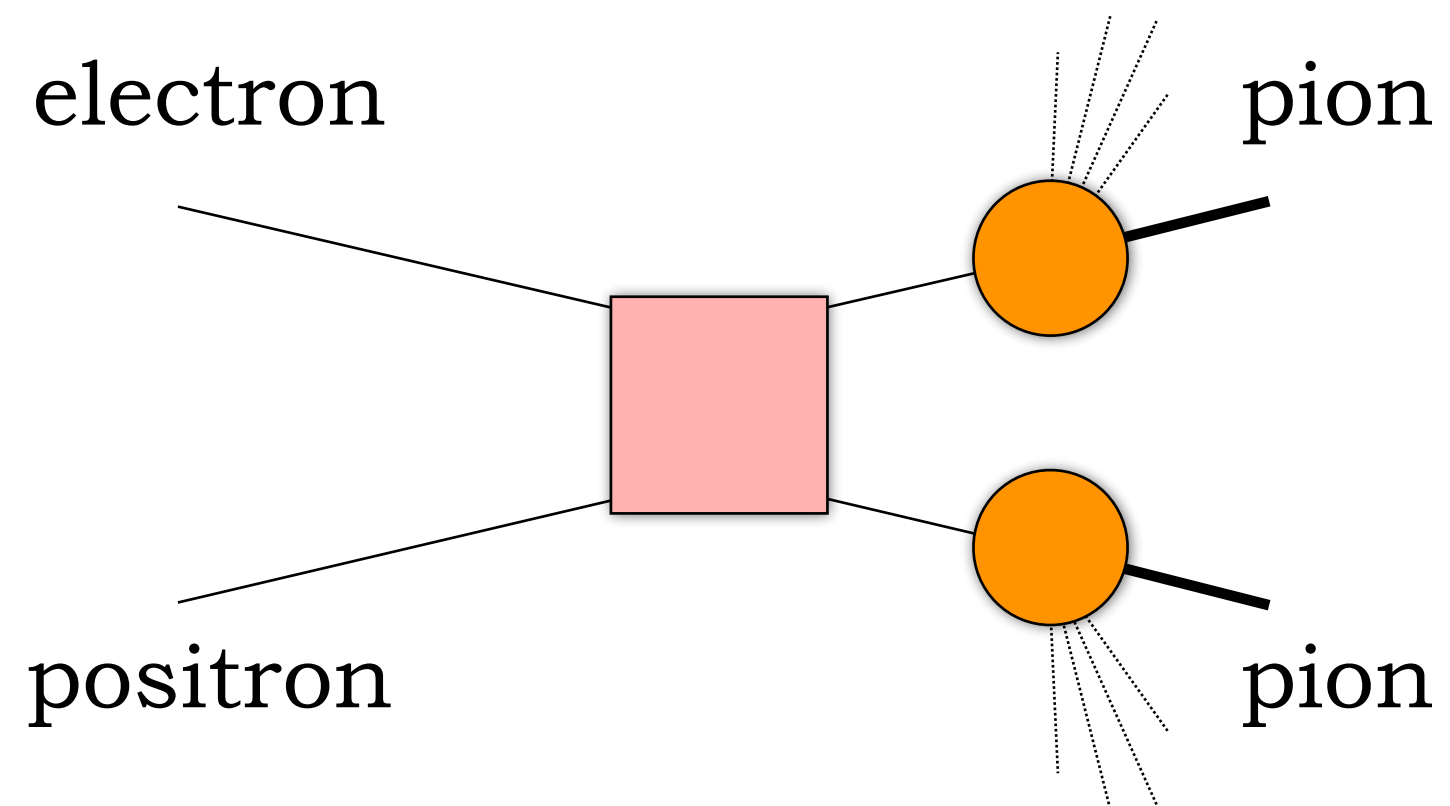
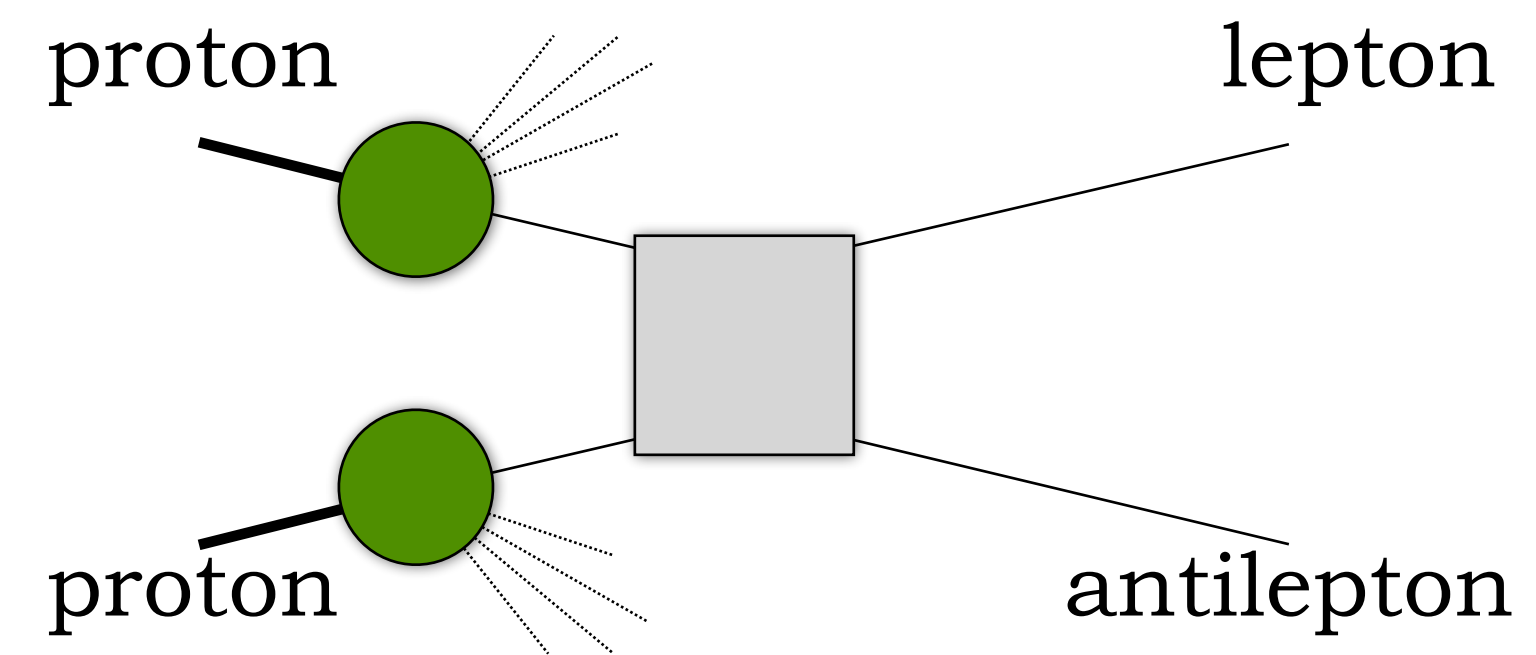


# Factorization and universality

## SIDIS



## Drell-Yan



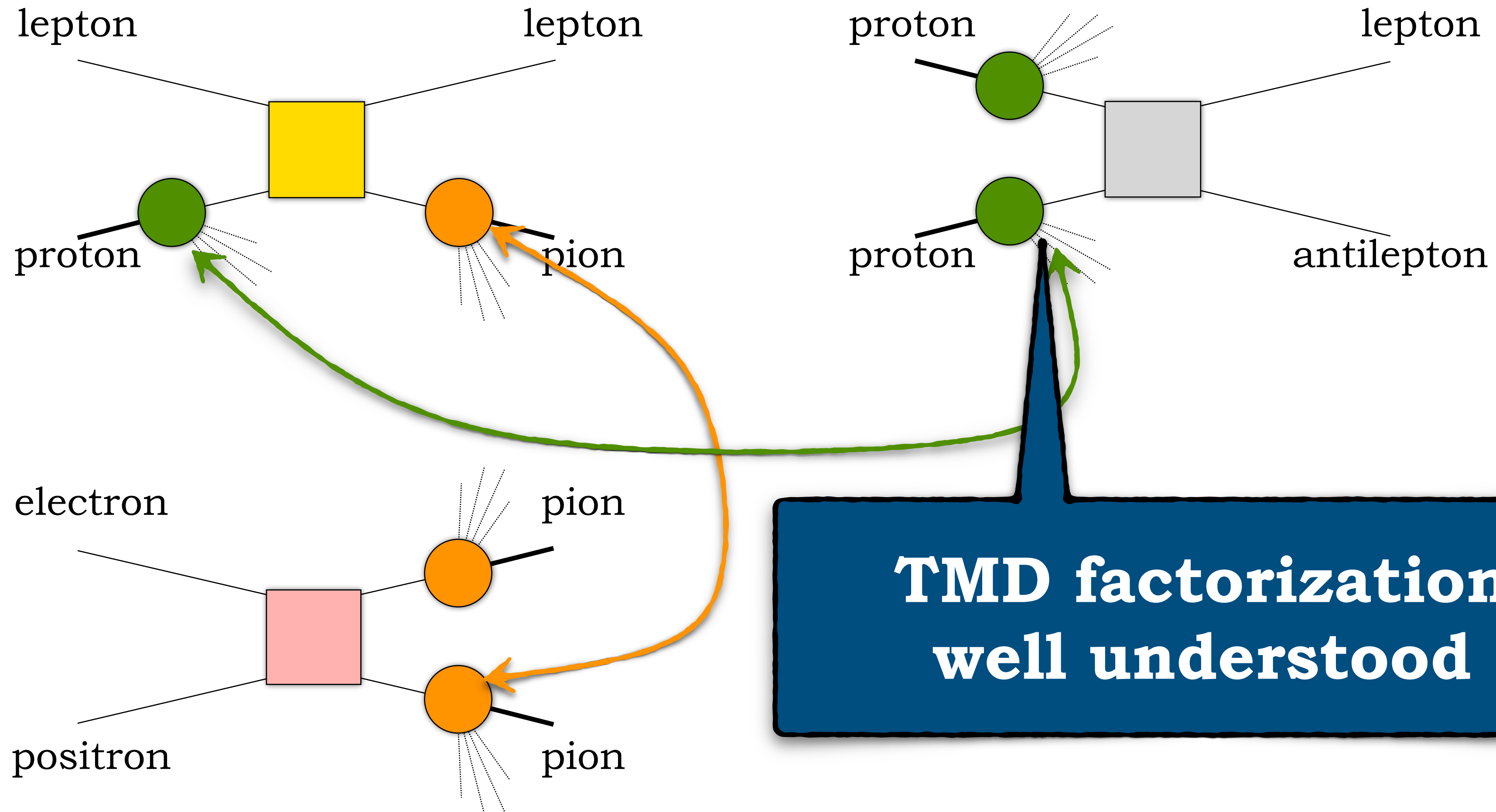
$$e^+ + e^- \rightarrow \mathbf{hadrons}$$



# Factorization and universality

**SIDIS**

**Drell-Yan**



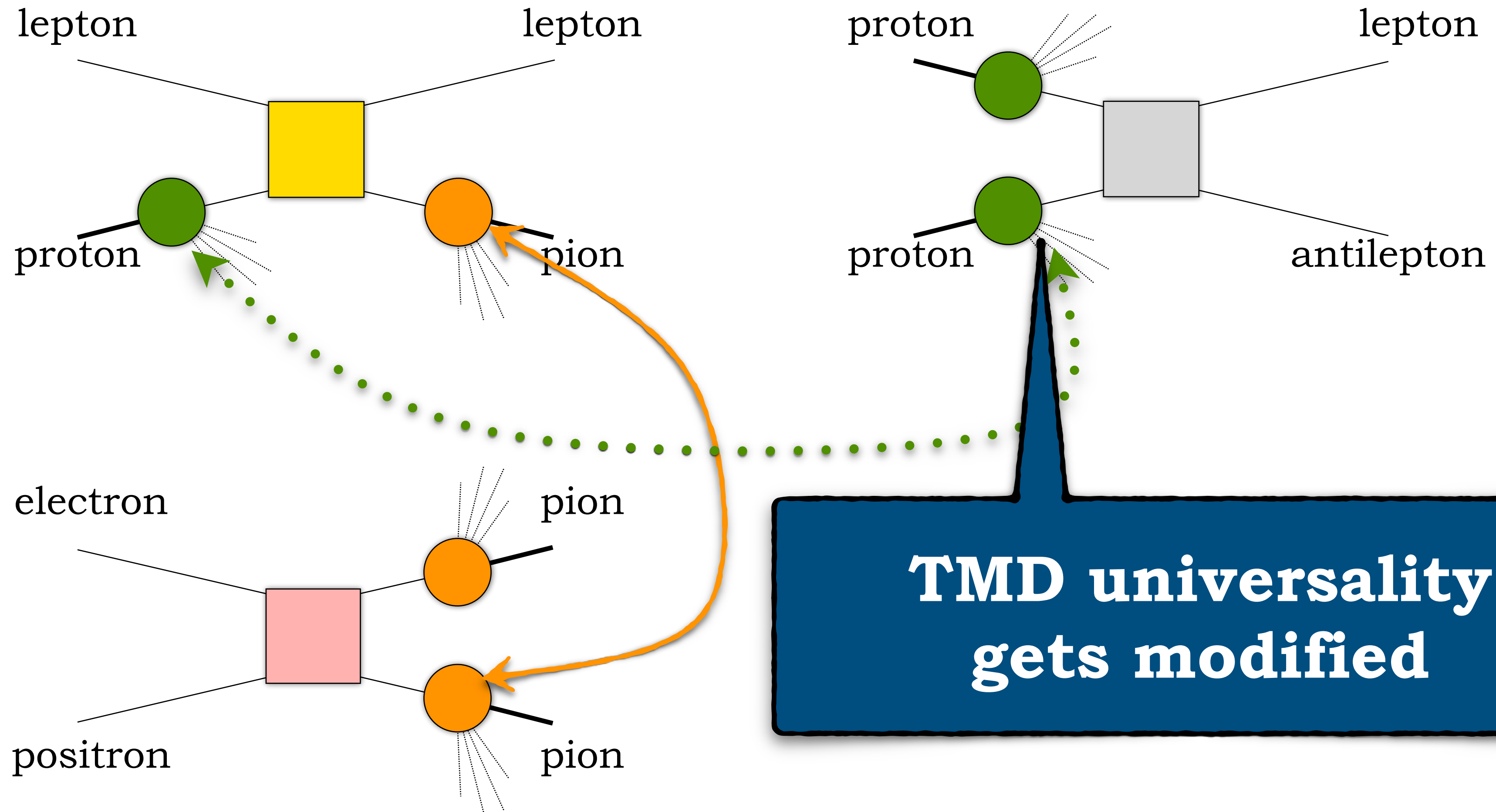
**TMD factorization  
well understood**

$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

# Factorization and universality

**SIDIS**

**Drell-Yan**

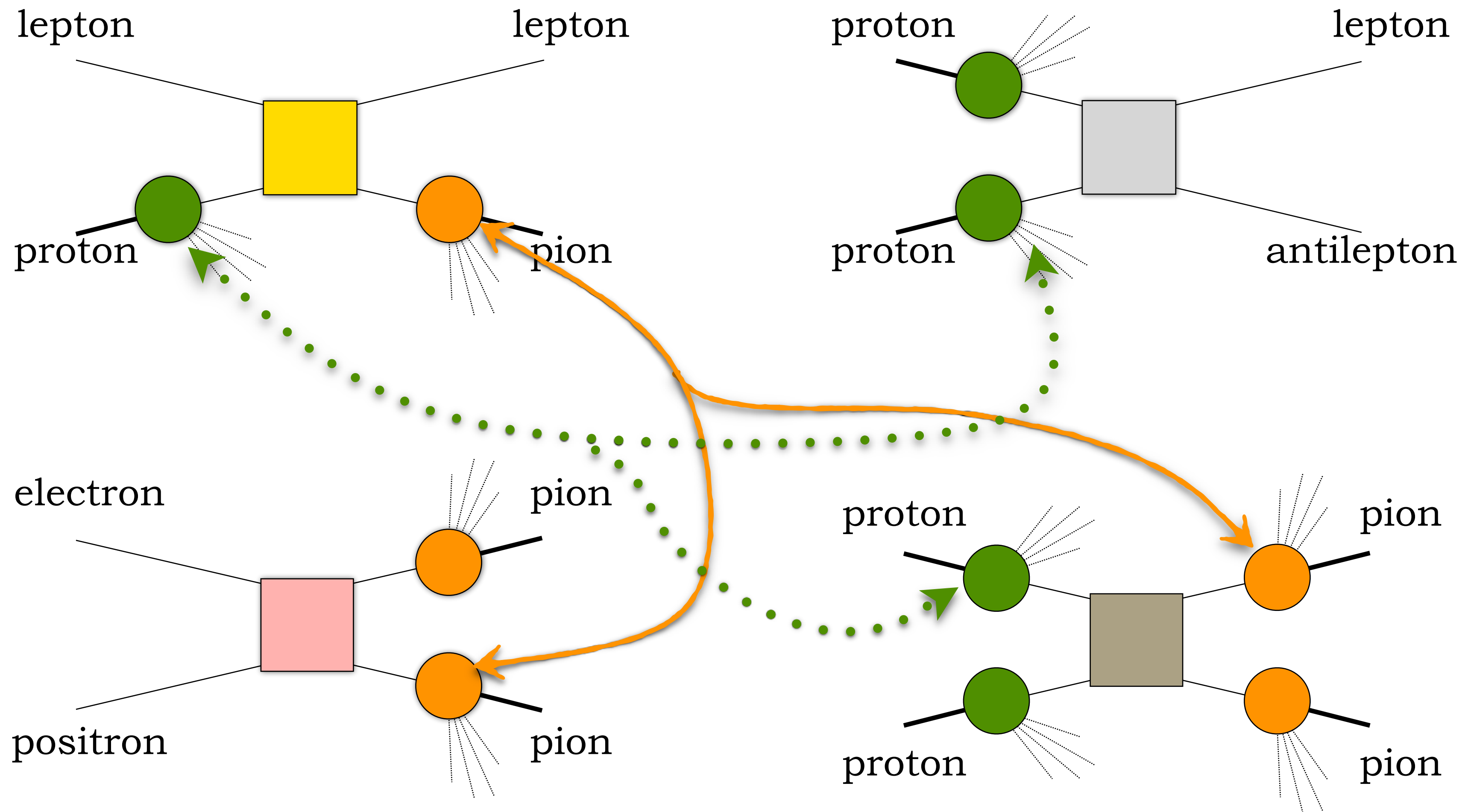


$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

# Factorization and universality

## SIDIS

## Drell-Yan



$$e^+ + e^- \rightarrow \text{hadrons}$$

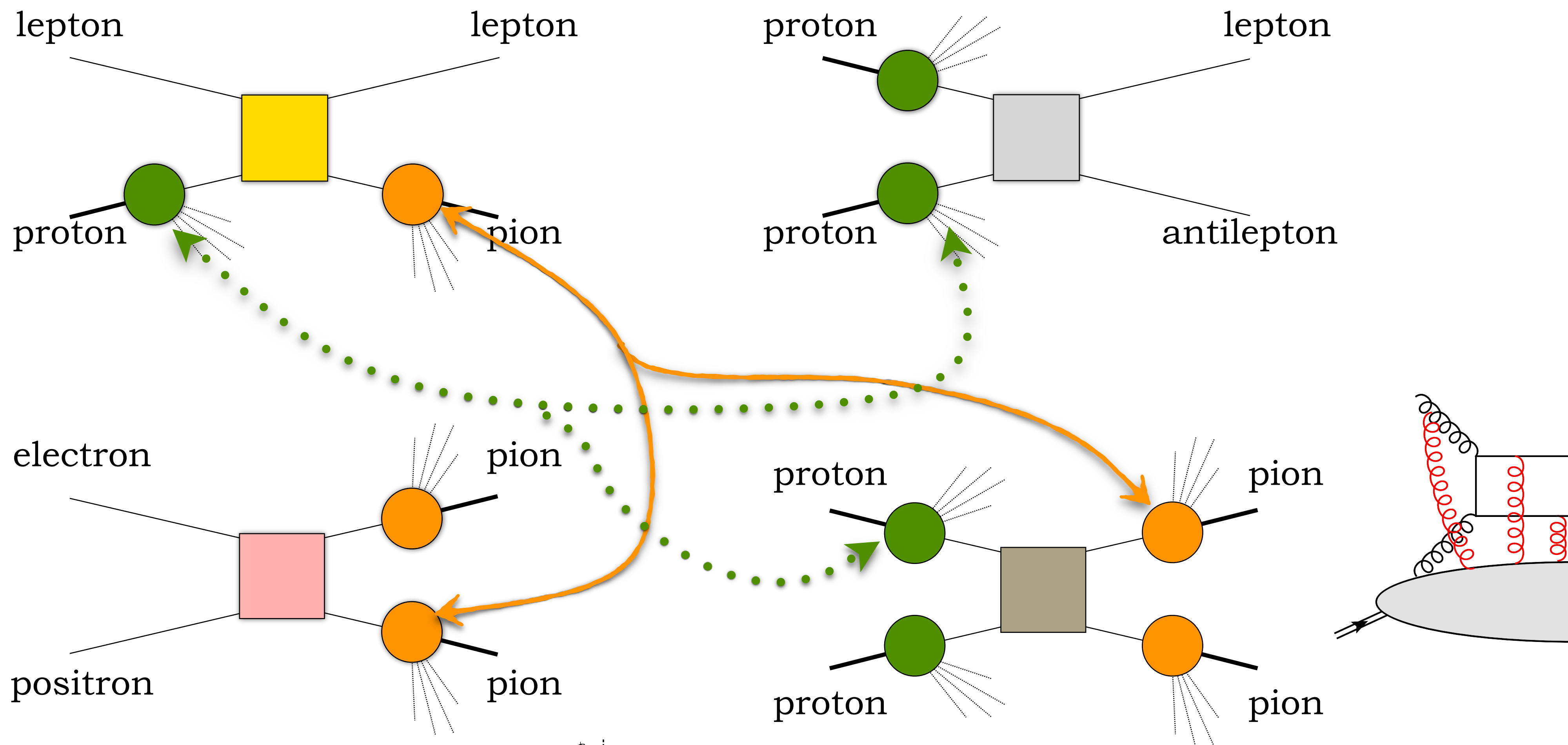
$$p + p \rightarrow \text{hadrons}$$



# Factorization and universality

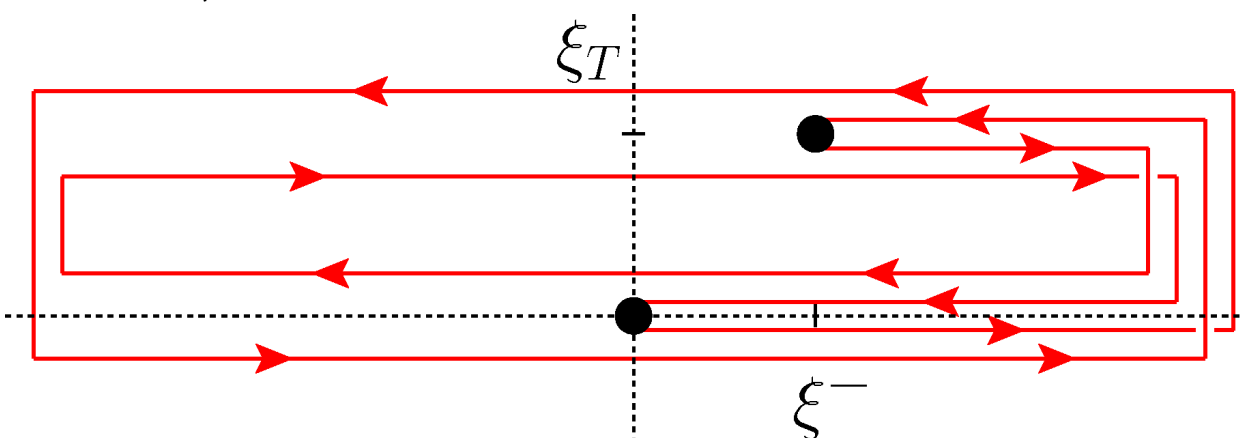
**SIDIS**

**Drell-Yan**



$e^+ + e^- \rightarrow \mathbf{hadrons}$

$p + p \rightarrow \mathbf{hadrons}$

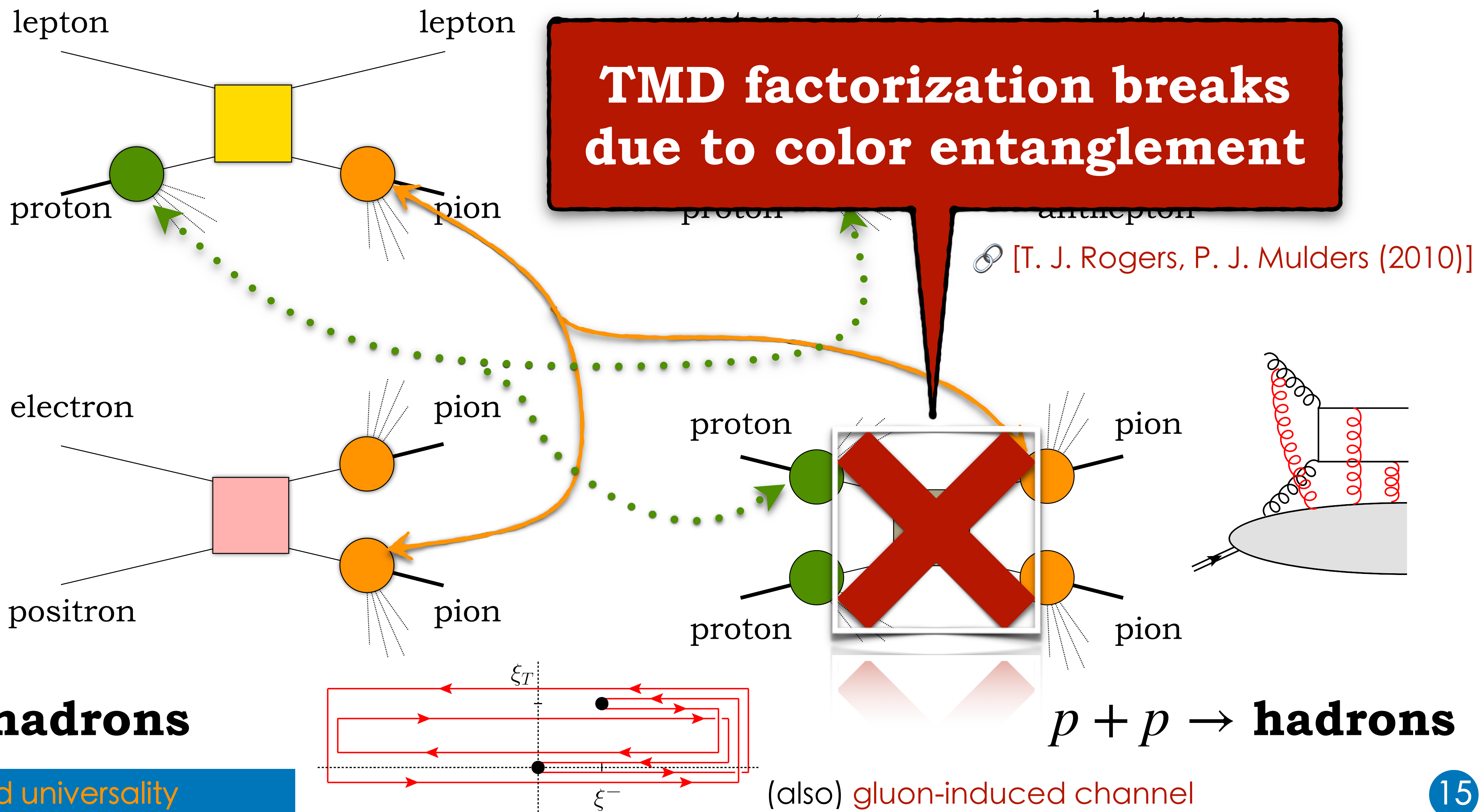


(also) gluon-induced channel

# Factorization and universality

**SIDIS**

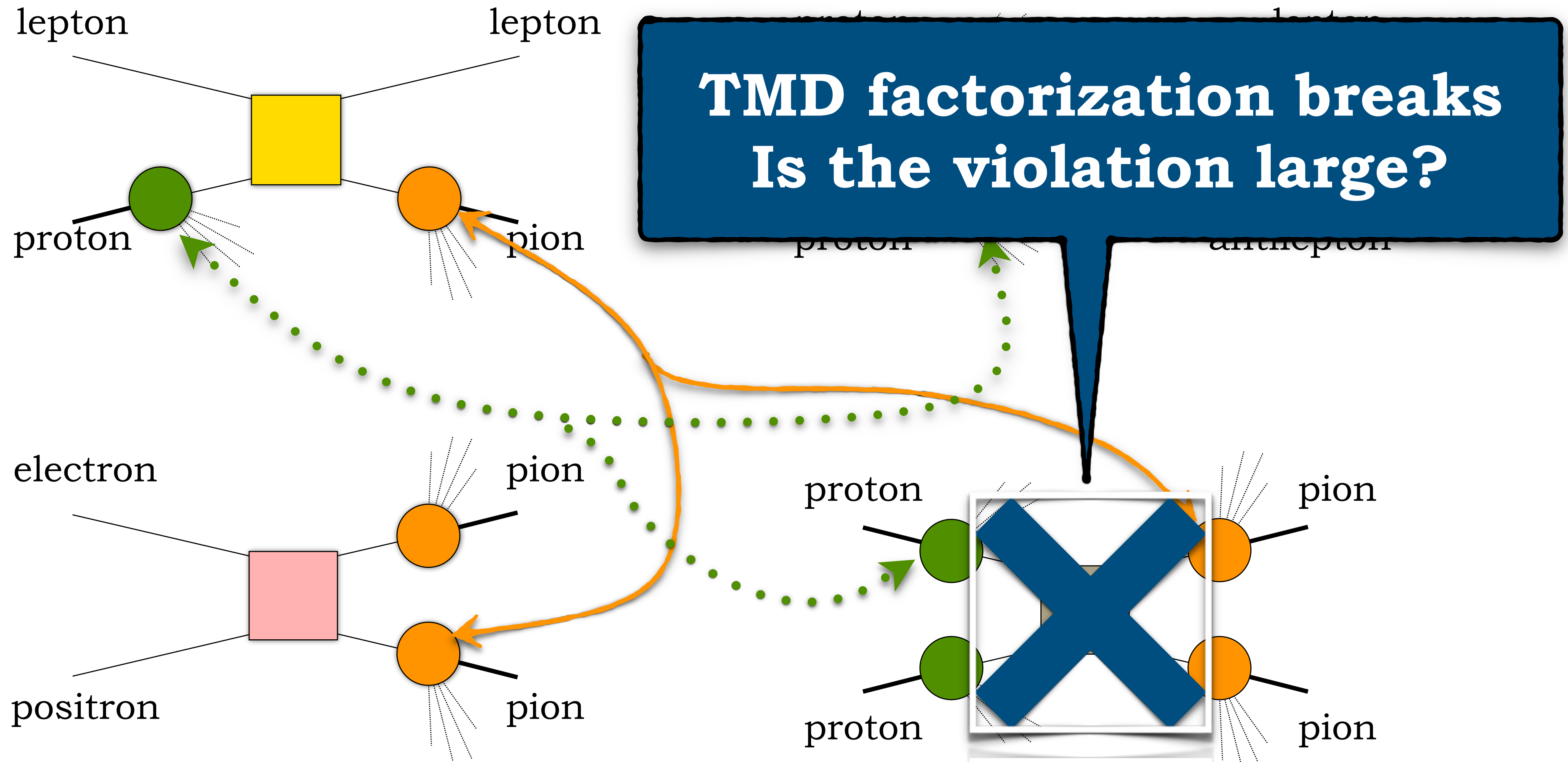
**Drell-Yan**



# Factorization and universality

**SIDIS**

**Drell-Yan**



$$e^+ + e^- \rightarrow \mathbf{hadrons}$$

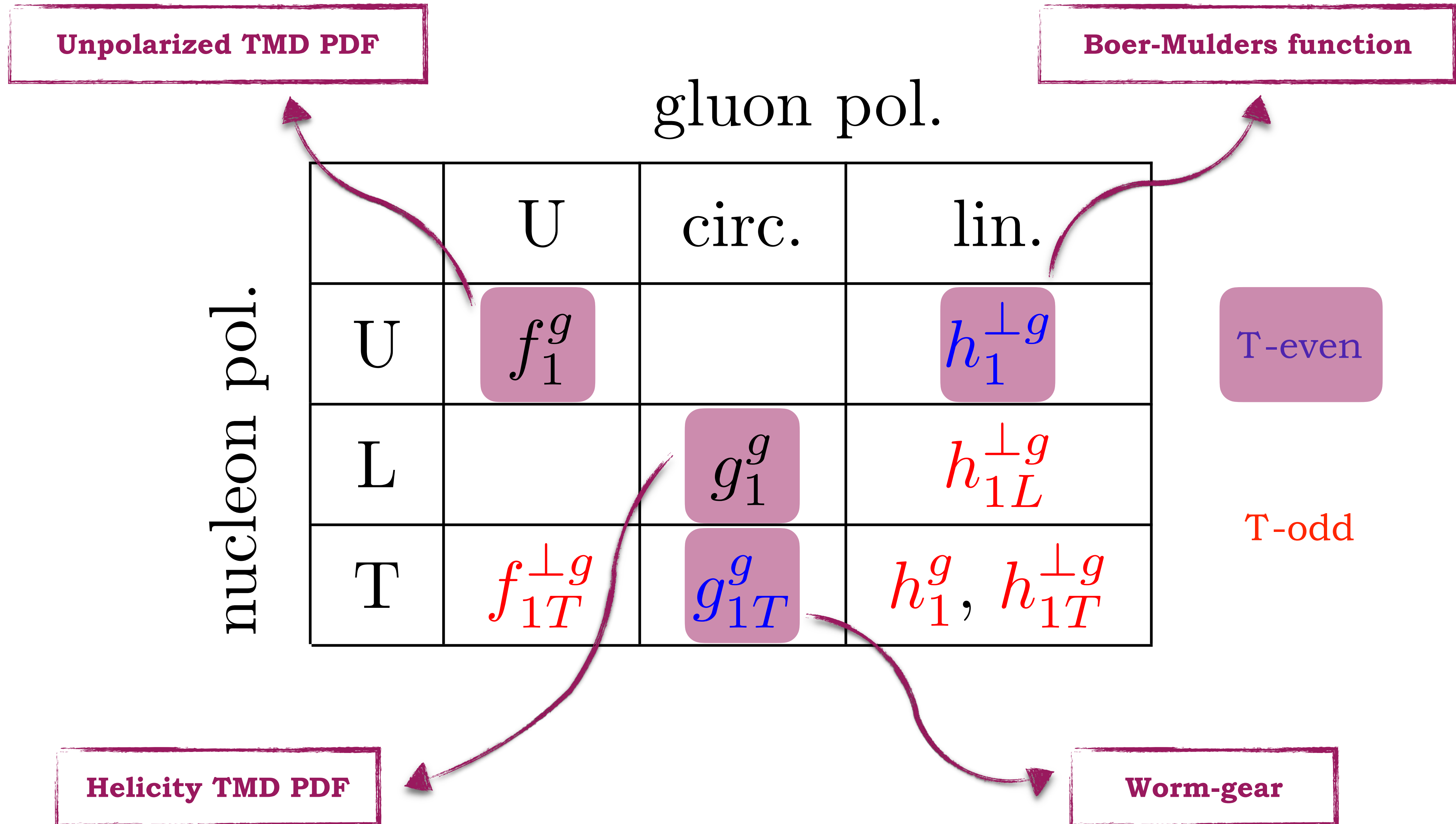
$$p + p \rightarrow \mathbf{hadrons}$$



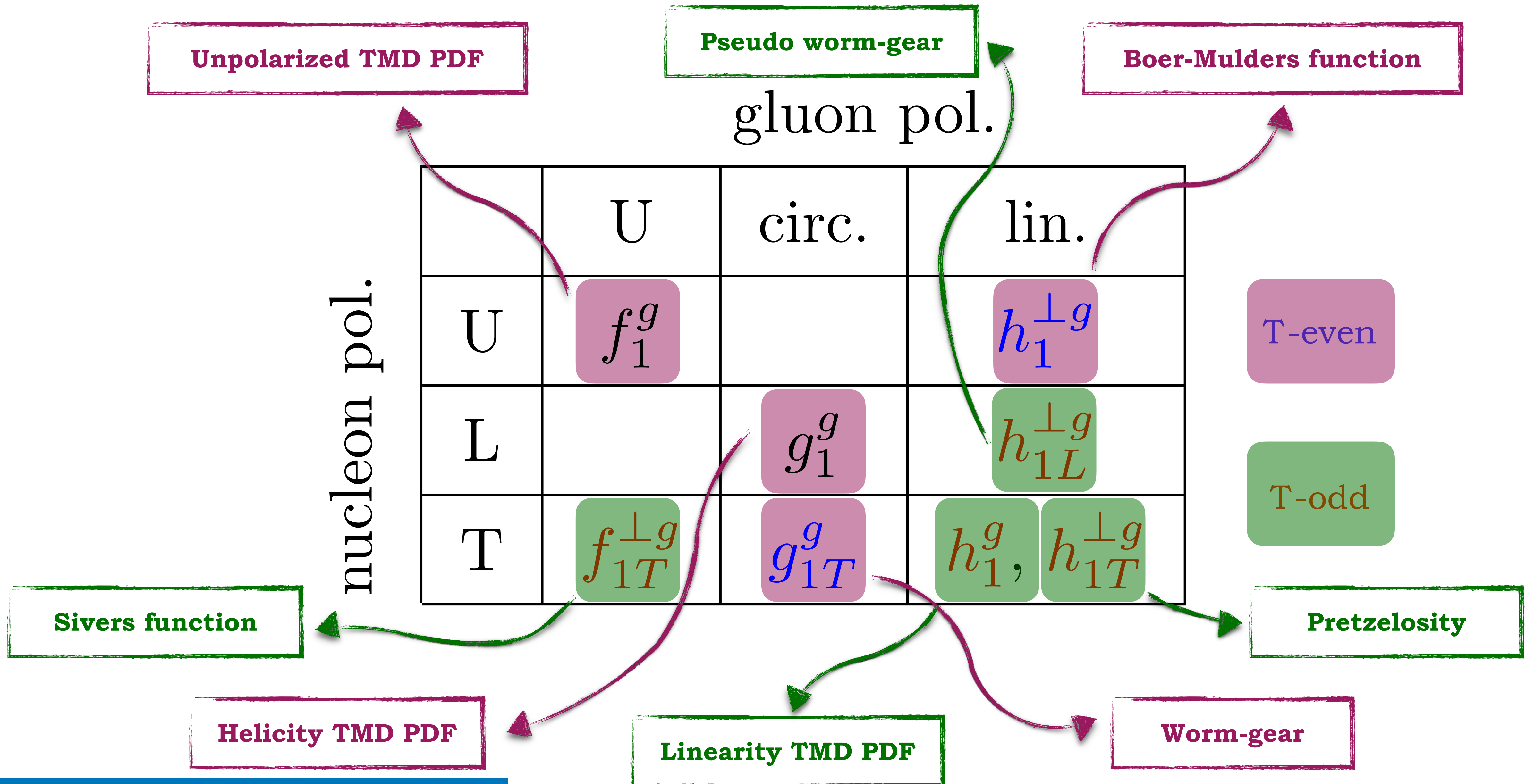
# Gluon TMD PDFs at leading twist

		gluon pol.			
		U	circ.	lin.	
nucleon pol.	U	$f_1^g$		$h_1^{\perp g}$	T-even
	L		$g_1^g$	$h_{1L}^{\perp g}$	T-odd
	T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$	

# Gluon TMD PDFs at leading twist



# Gluon TMD PDFs at leading twist





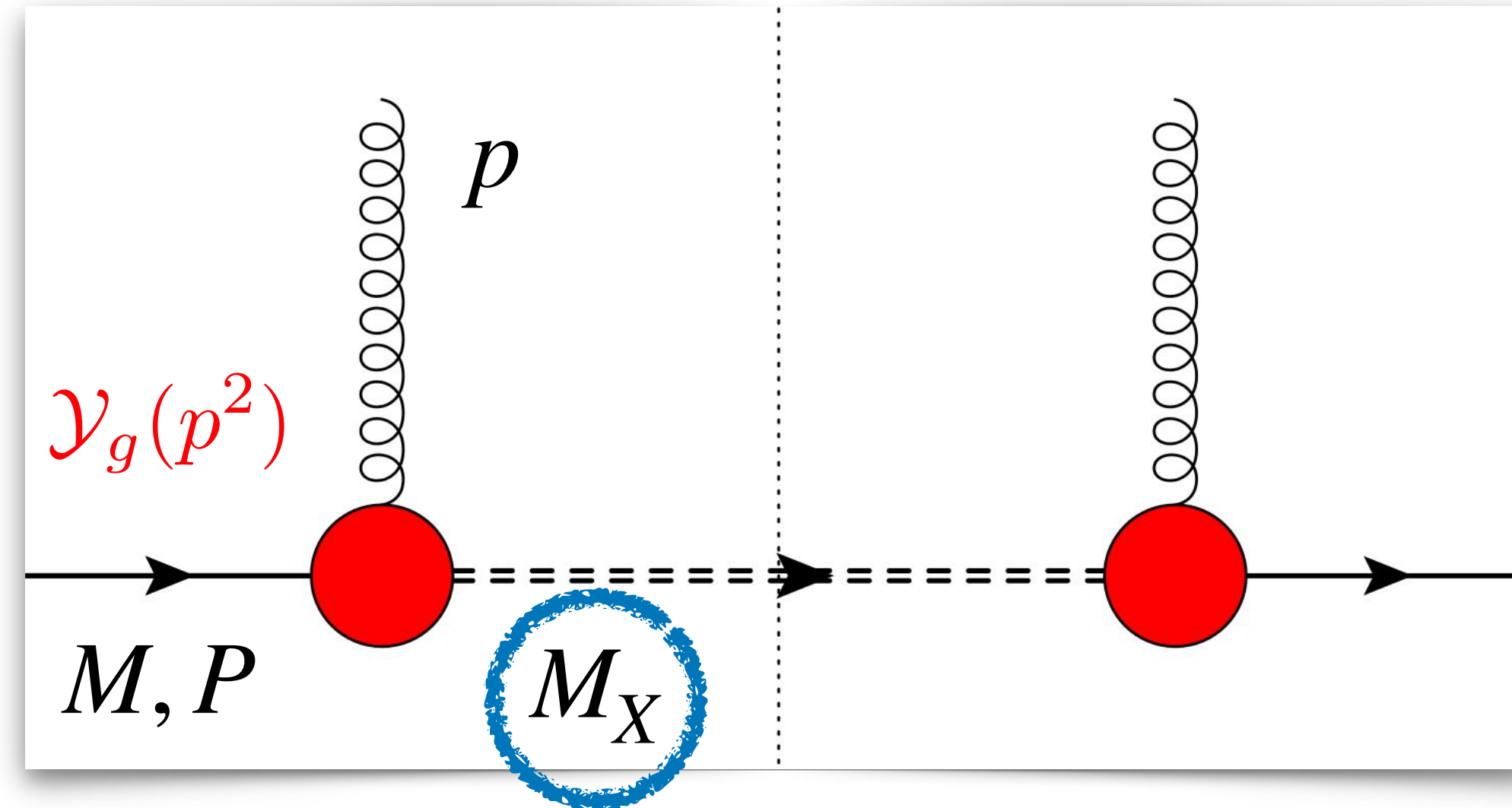
## 2. Modeling gluon TMDs

The background features a repeating pattern of circular diagrams, each representing a gluon Transverse Momentum Distribution (TMD). These diagrams are rendered in a semi-transparent, light blue style. Each diagram shows a central gluon, depicted as a red sphere with a small red arrow pointing upwards, surrounded by a yellow wavy line representing the gluon's transverse momentum distribution. The diagrams are arranged in a grid-like pattern, with some overlapping, creating a sense of depth and repetition.



## Spin-1/2 spectator

Lowest Fock state:  
**tri-quark** spectator  
on-shell and  
with mass  $M_X$



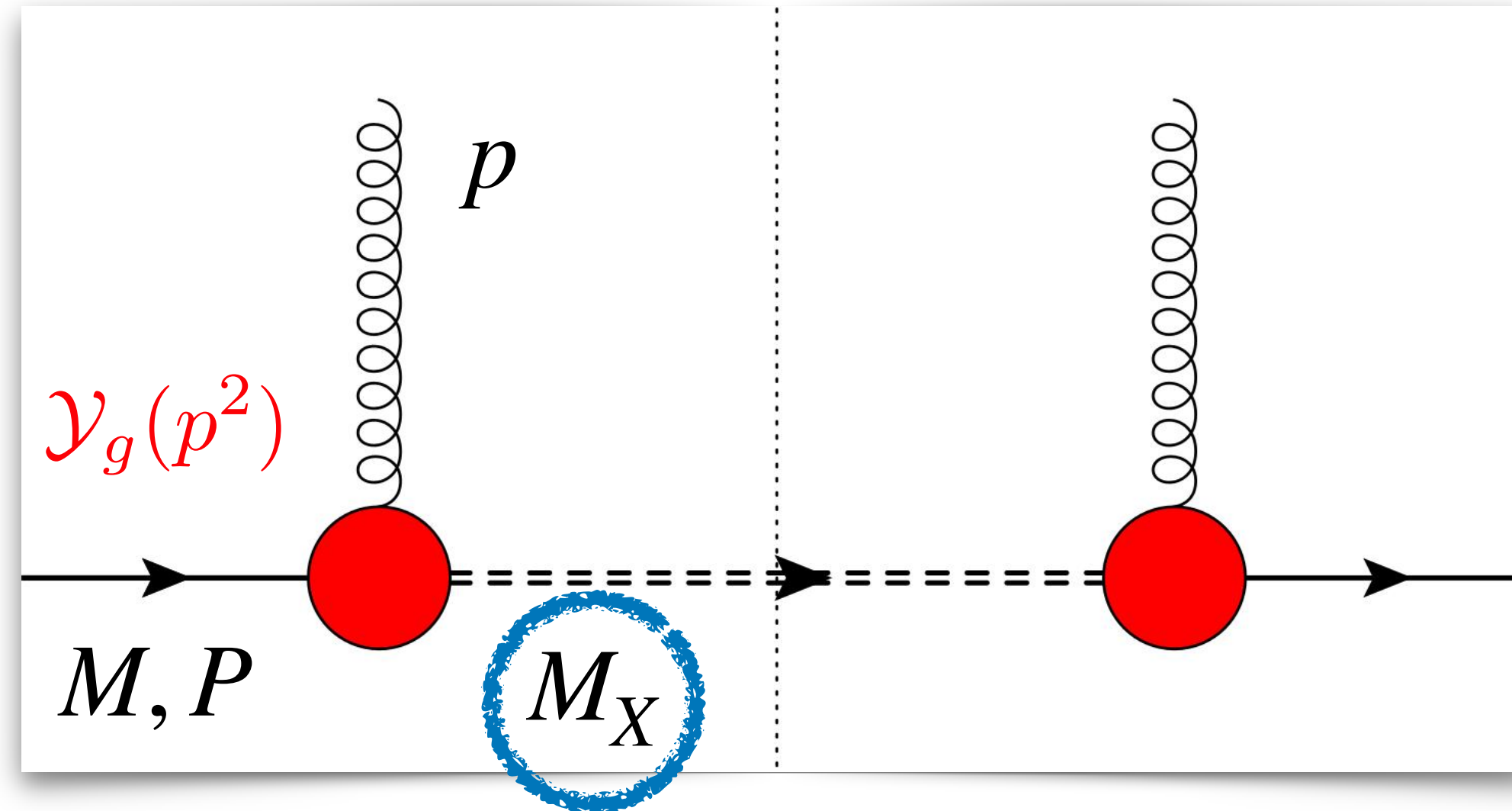


# Spectator-model gluon TMD PDFs



## Spin-1/2 spectator

Lowest Fock state:  
**tri-quark** spectator  
 on-shell and  
 with mass  $M_X$



## Nucleon-gluon-spectator vertex

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} \text{Tr} \left[ (\not{P} + M) \frac{1 + \gamma^5 \not{\xi}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M) \right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$

mimics proton form factors  
 (conserved EM current  
 of a free nucleon)







## Link with collinear factorization

1.  $p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution
2. TMDs and PDFs *decouple* due to evolution



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1.  $p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution
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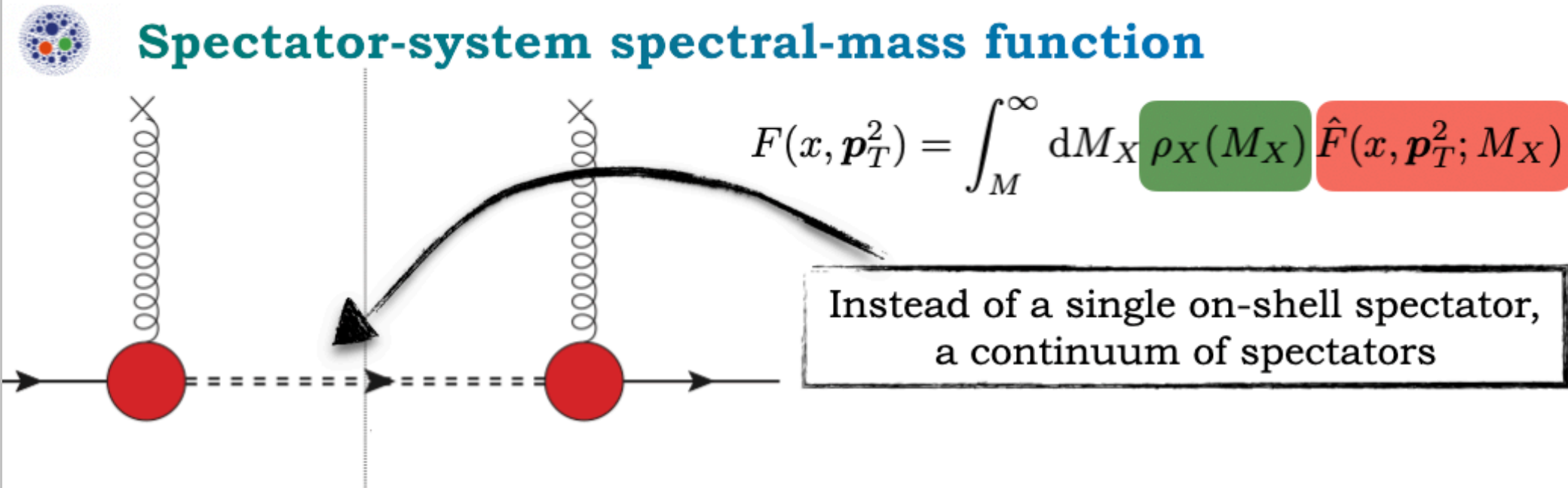


## Dipolar form factor(s)

1. Cancels singularity of gluon propagator
2. Suppresses effects of high  $p_T$
3. Compensates log divergences arising from  $p_T$ -integration
4. Adds three more parameters:  $\kappa_{1,2}$  and  $\Lambda_X$

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

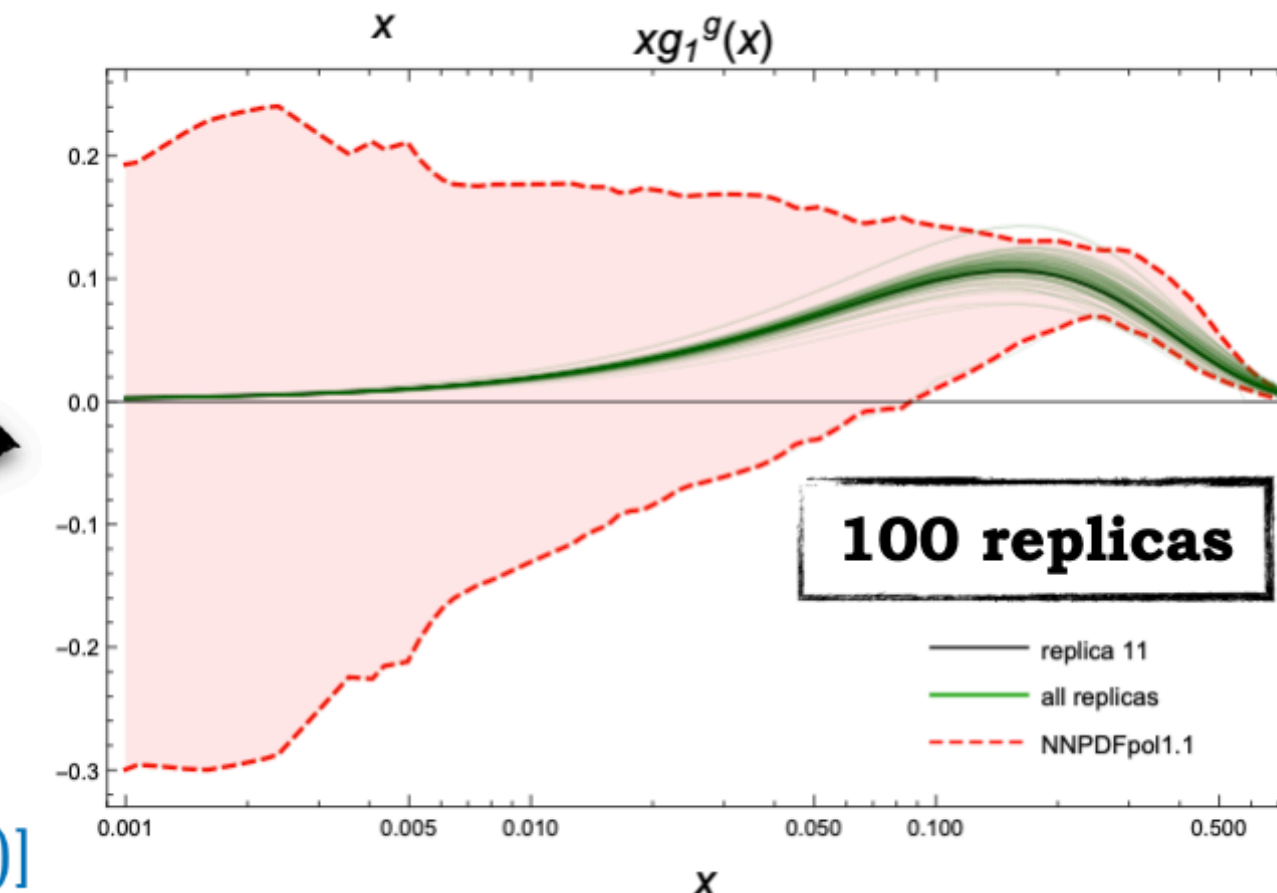
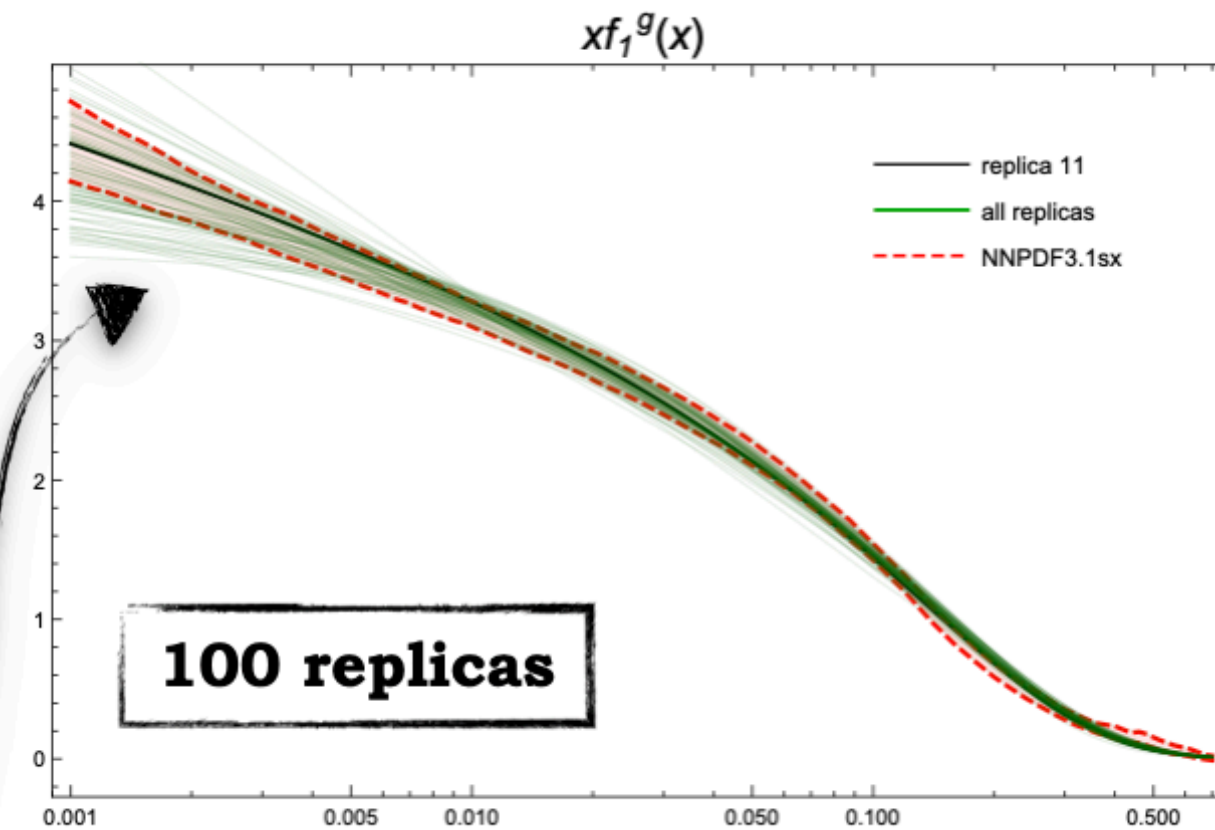
## Our model at a glance



**Link with collinear factorization**

$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution

Spectral function **learns** small- and moderate- $x$  info encoded in **NNPDF** collinear parametrizations (NNPDF3.1sx + NNPDFpol1.1)

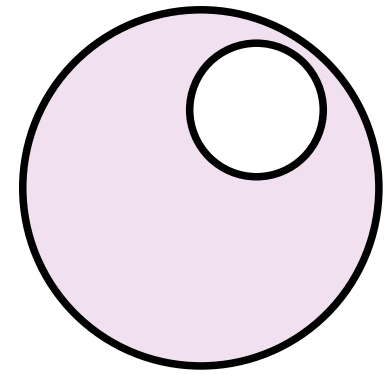


- ✓ Simultaneous fit of  $f_1$  and  $g_1$  PDFs
- ✓ Inclusion of small- $x$  resummation effects (**BFKL**)
- ✓ Calculation of all leading-twist T-even gluon TMDs

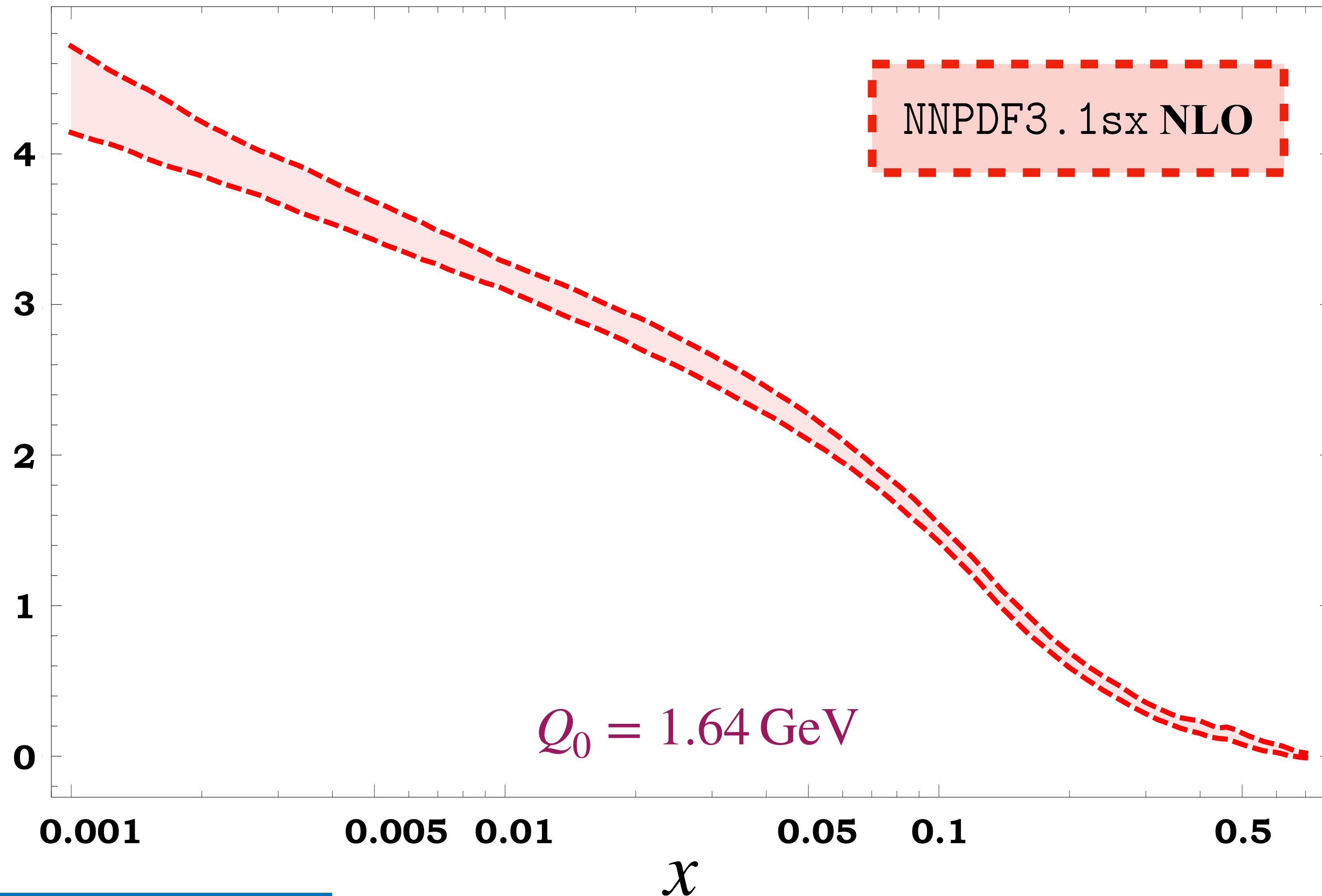
**2.3 Modeling gluon TMDs** [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]



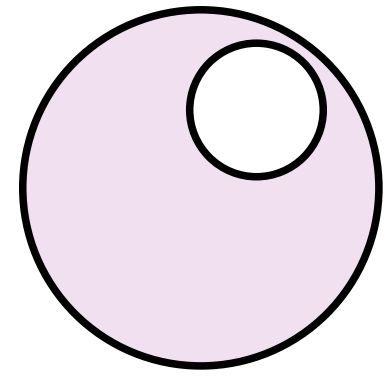
# Unpolarized gluon collinear PDF



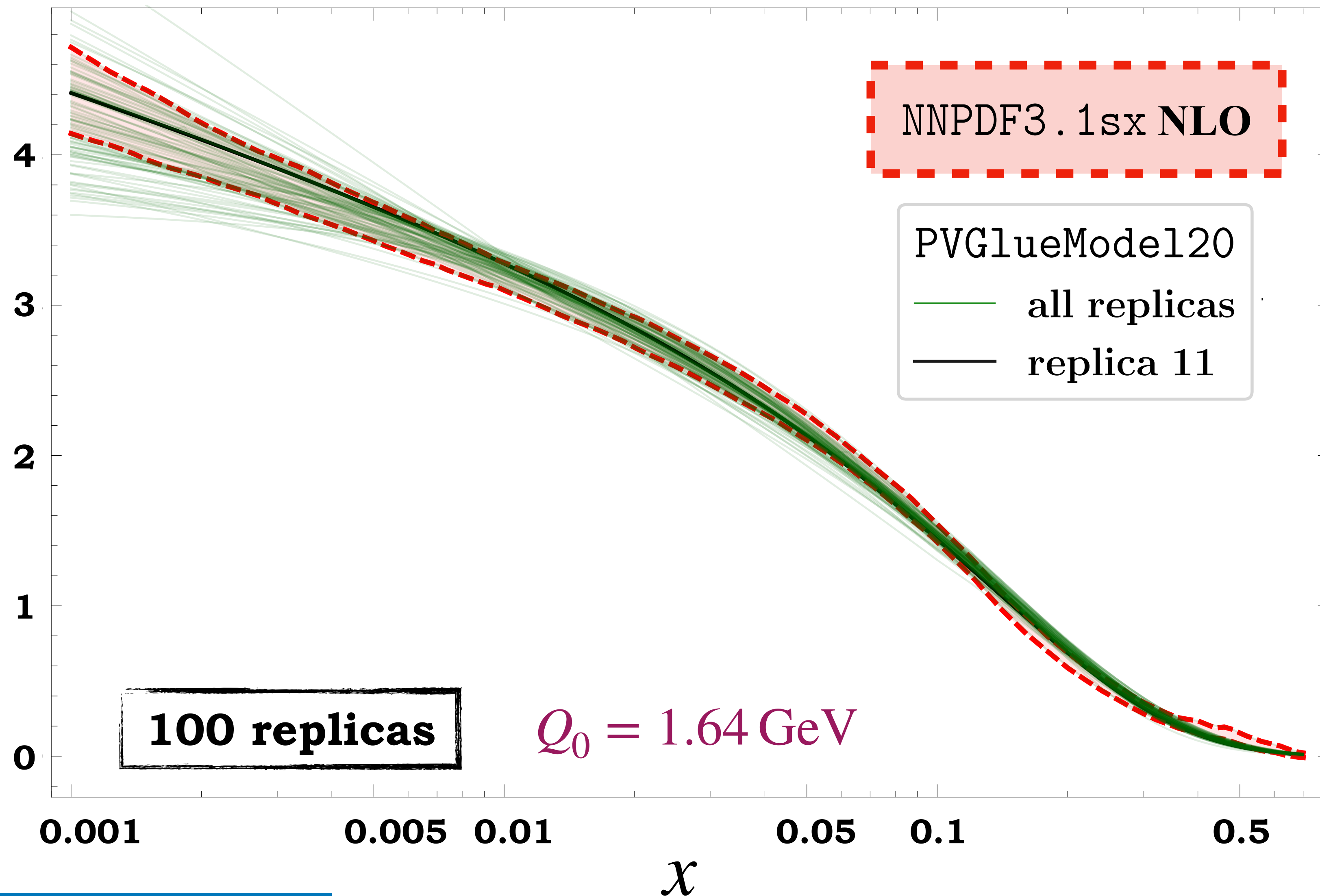
$$x f_1(x)$$



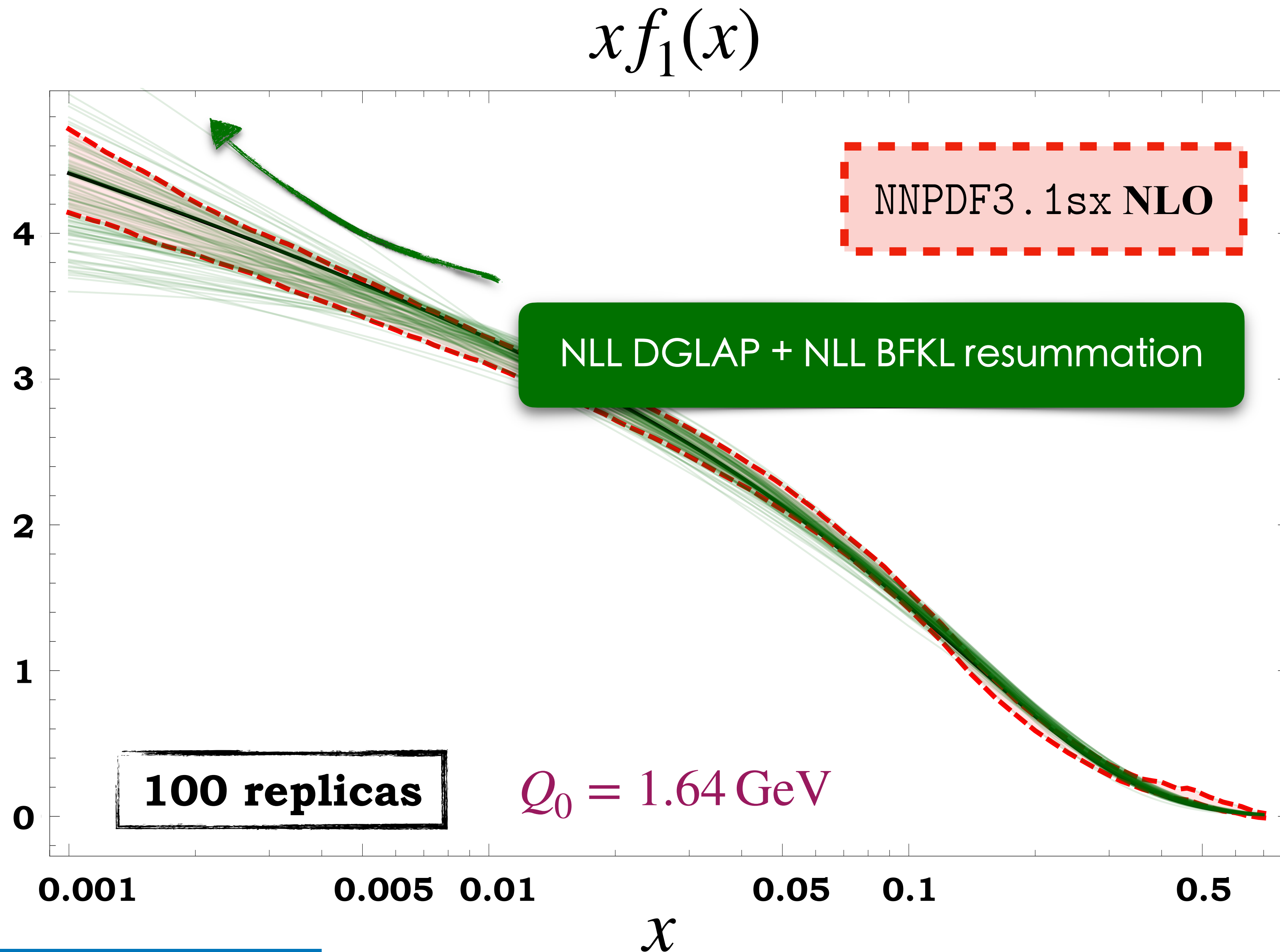
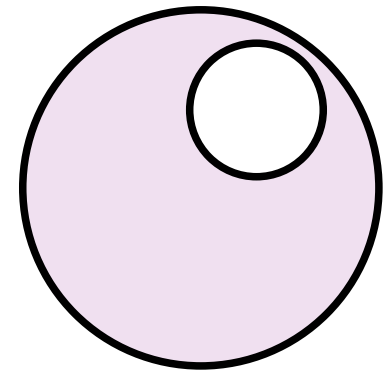
# Unpolarized gluon collinear PDF



$$x f_1(x)$$

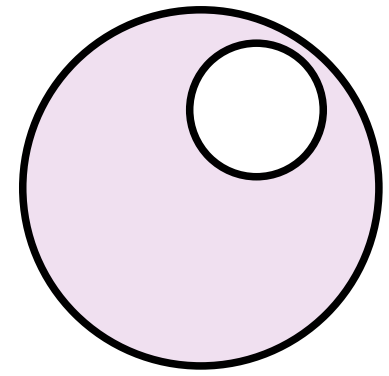


# Unpolarized gluon collinear PDF

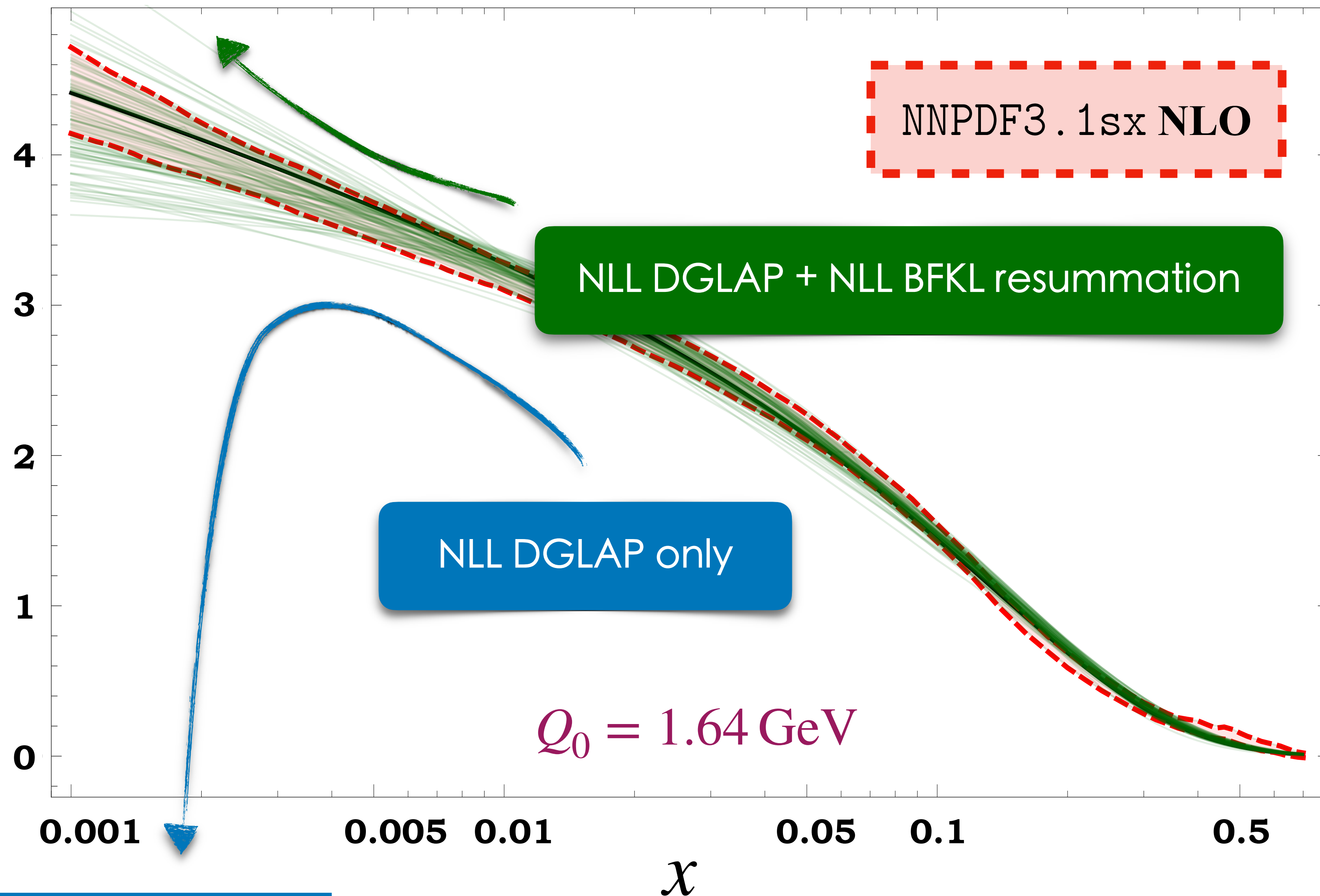


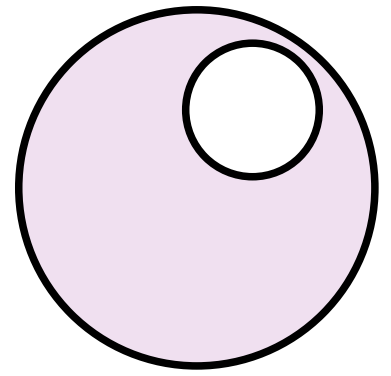


# Unpolarized gluon collinear PDF

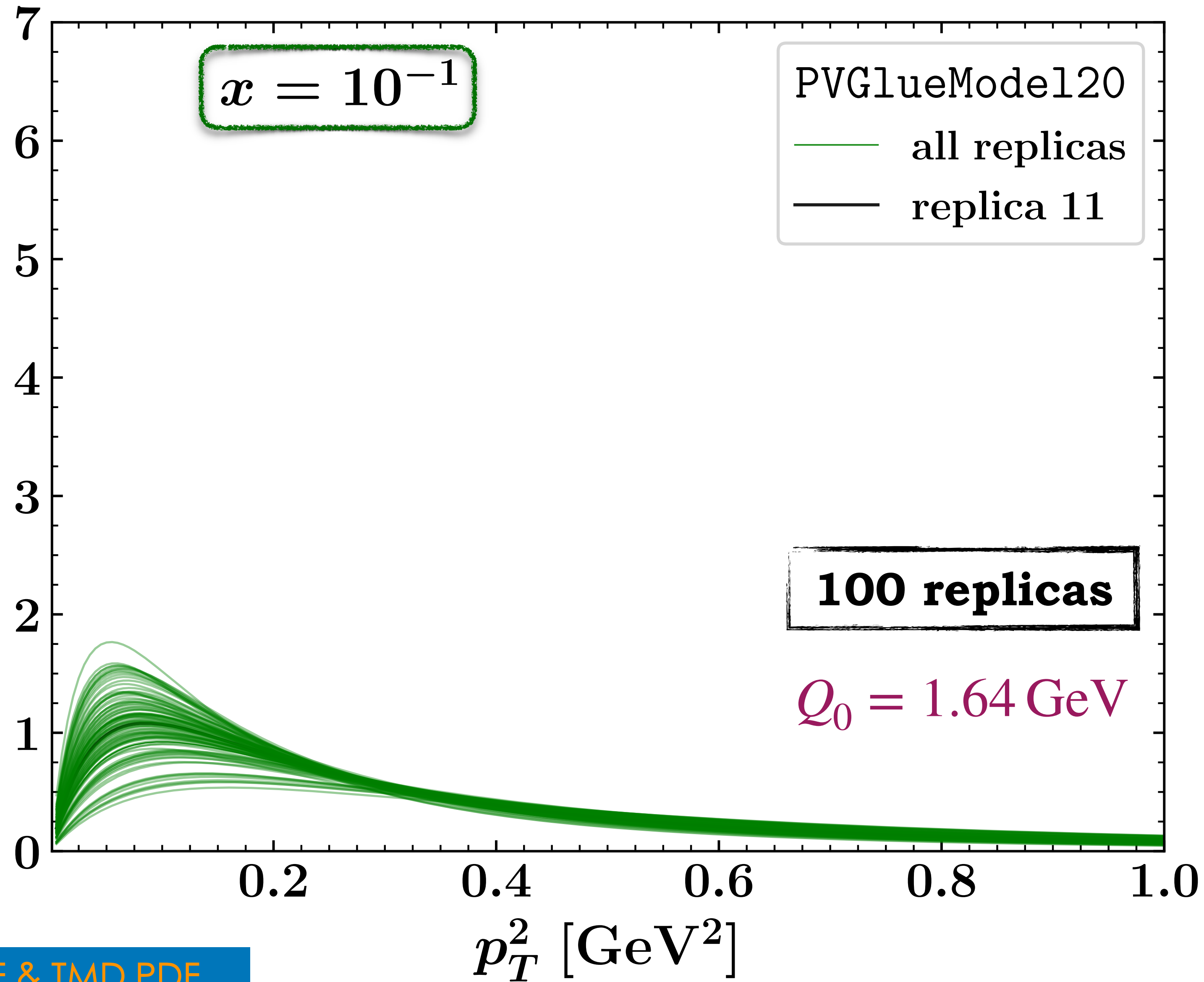


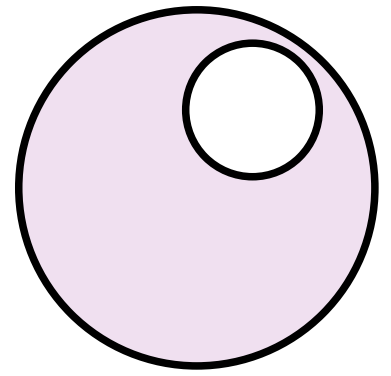
$$x f_1(x)$$



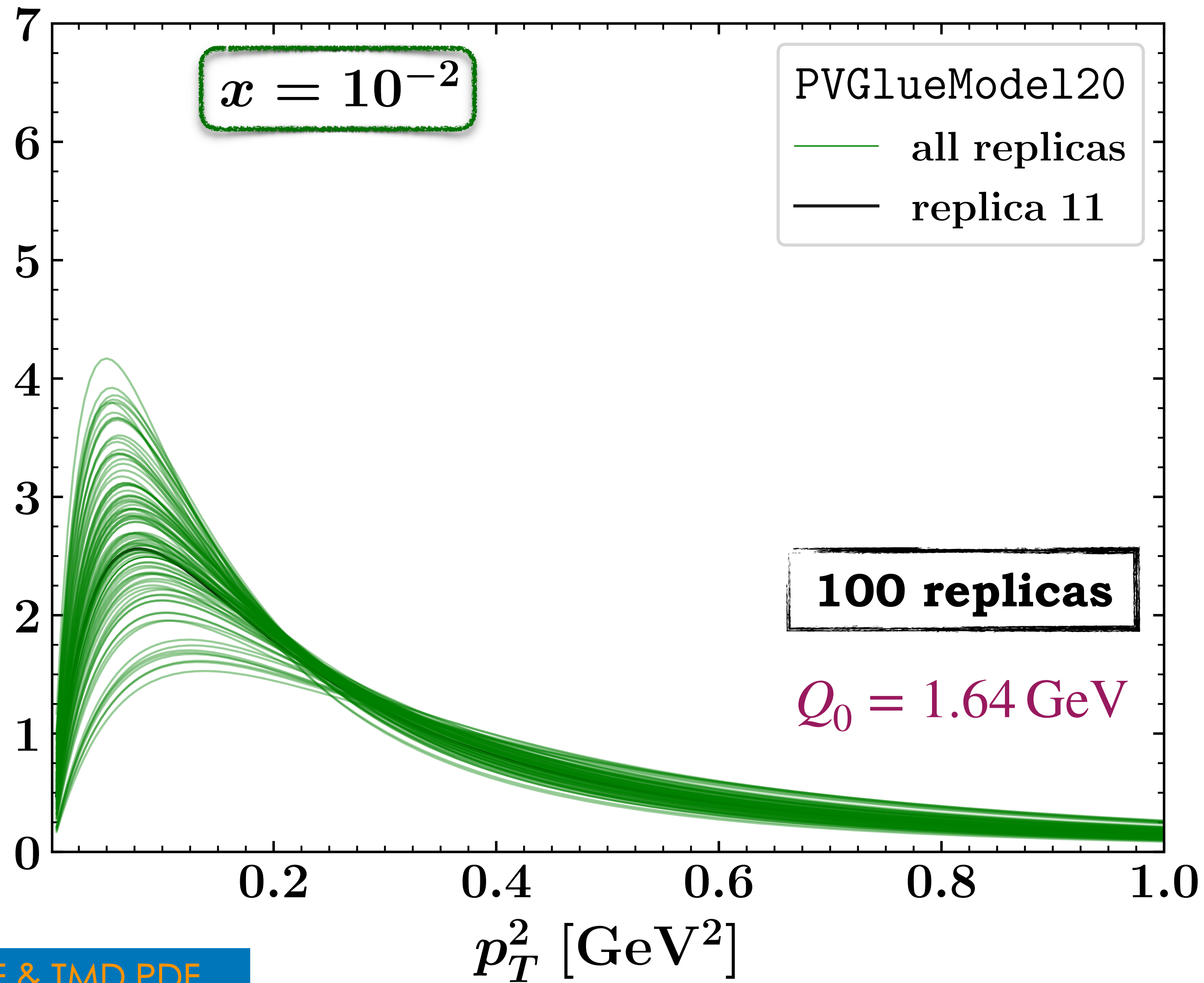


$$x f_1(x, p_T^2)$$

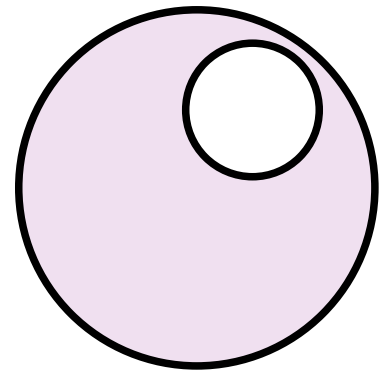




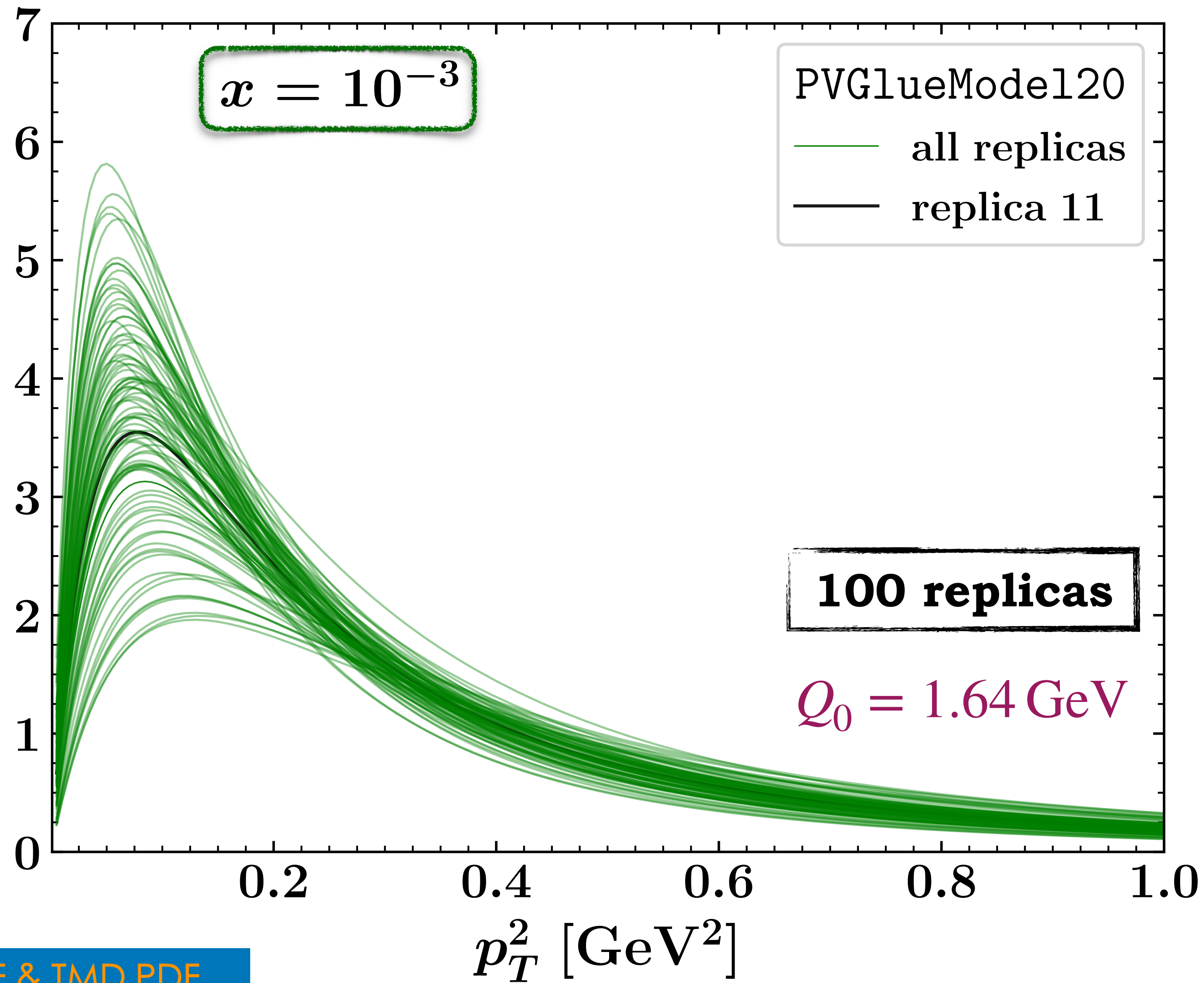
$$x f_1(x, p_T^2)$$







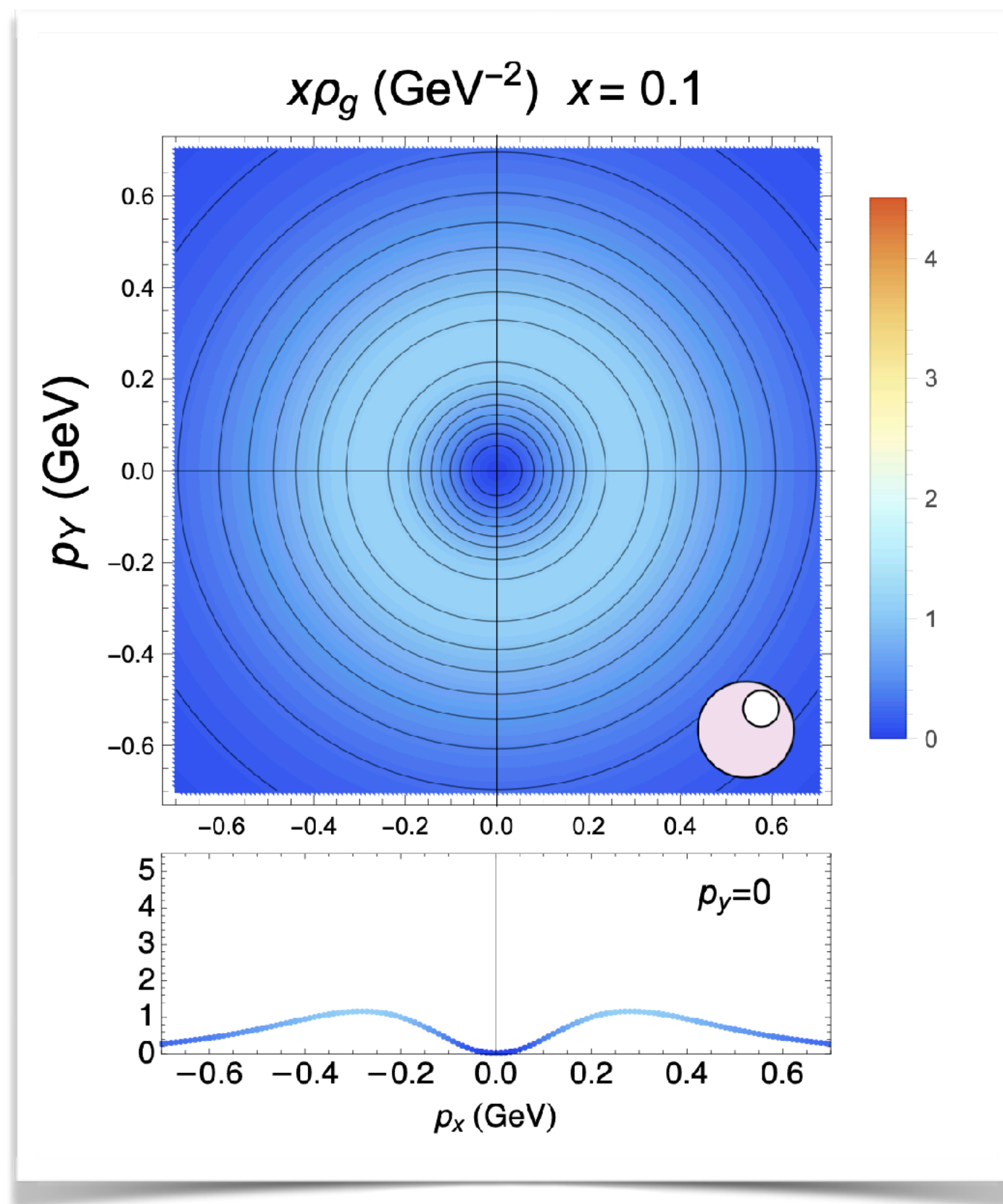
$$x f_1(x, p_T^2)$$



# 3D proton imaging: Tomographic reconstruction & TMDs

[A. Bacchetta, F.G.C., M. Radici, P. Tael, Eur. Phys. J. C 80 (2020) no.8]

## Unpolarized



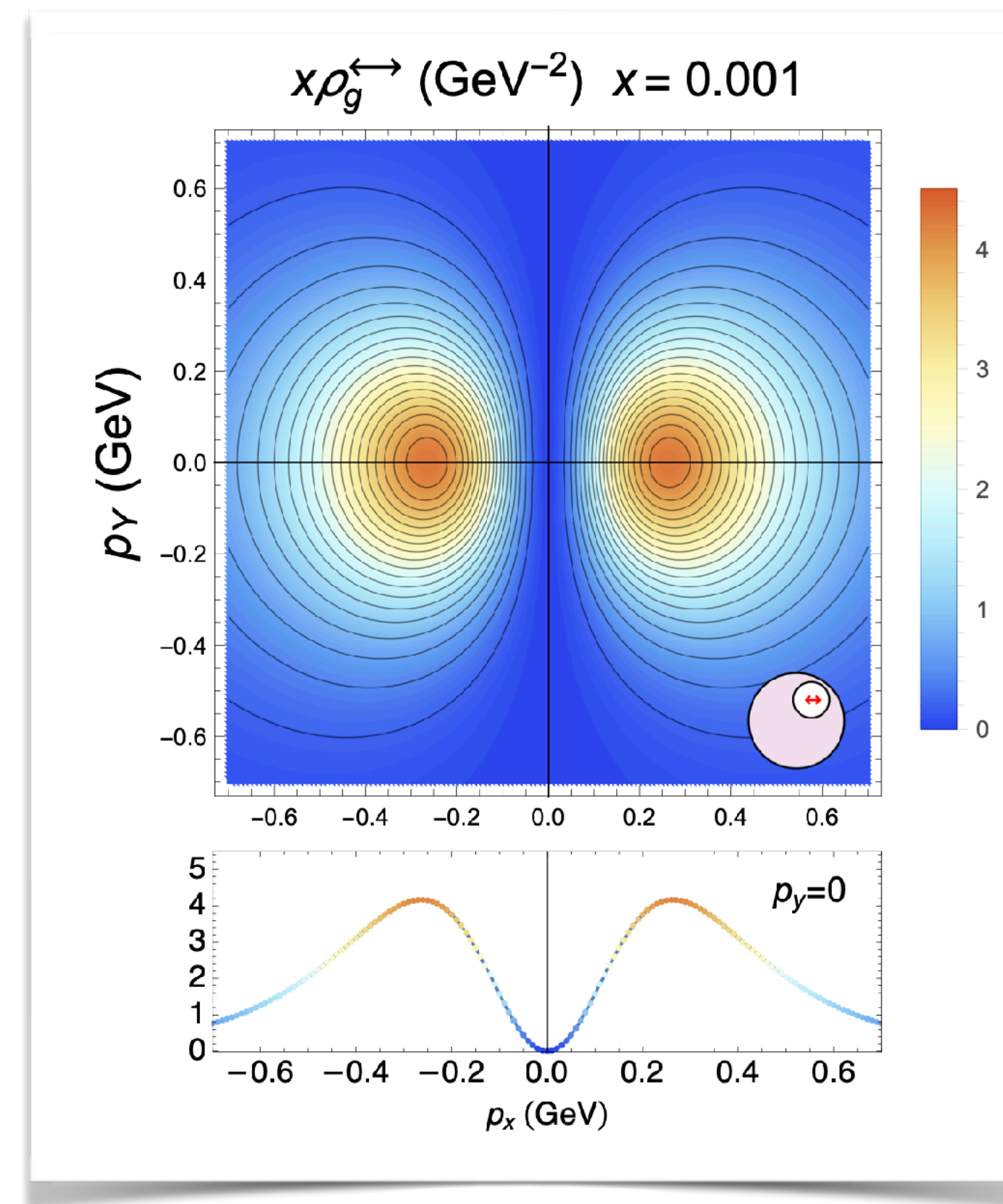
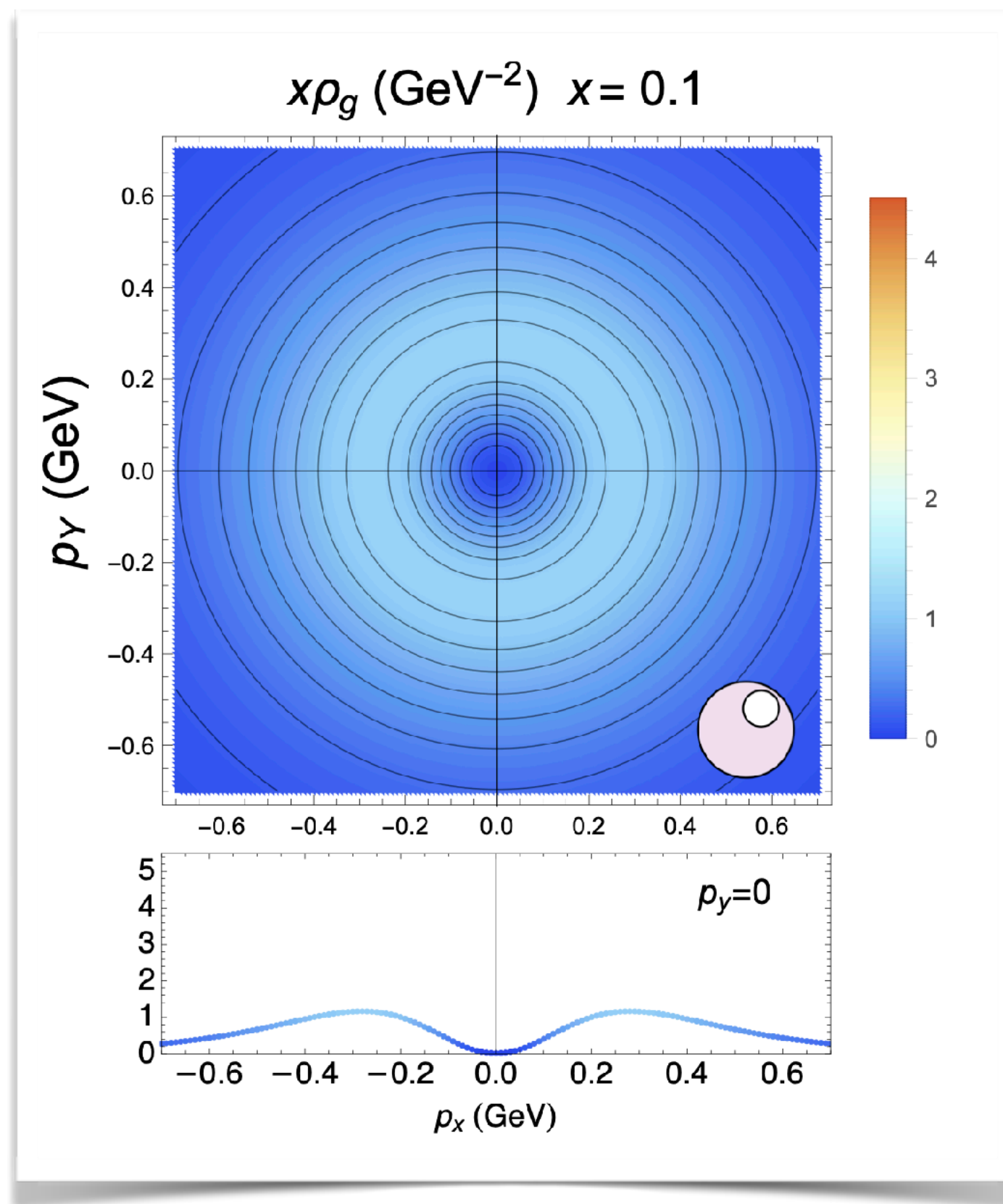


# 3D proton imaging: Tomographic reconstruction & TMDs

[A. Bacchetta, F.G.C., M. Radici, P. Tael, Eur. Phys. J. C 80 (2020) no.8]

## Unpolarized

## Boer-Mulders



$$x\rho^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2} \left[ x f_1^g(x, \mathbf{p}_T^2) + \frac{p_x^2 - p_y^2}{2M^2} x h_1^{\perp g}(x, \mathbf{p}_T^2) \right]$$



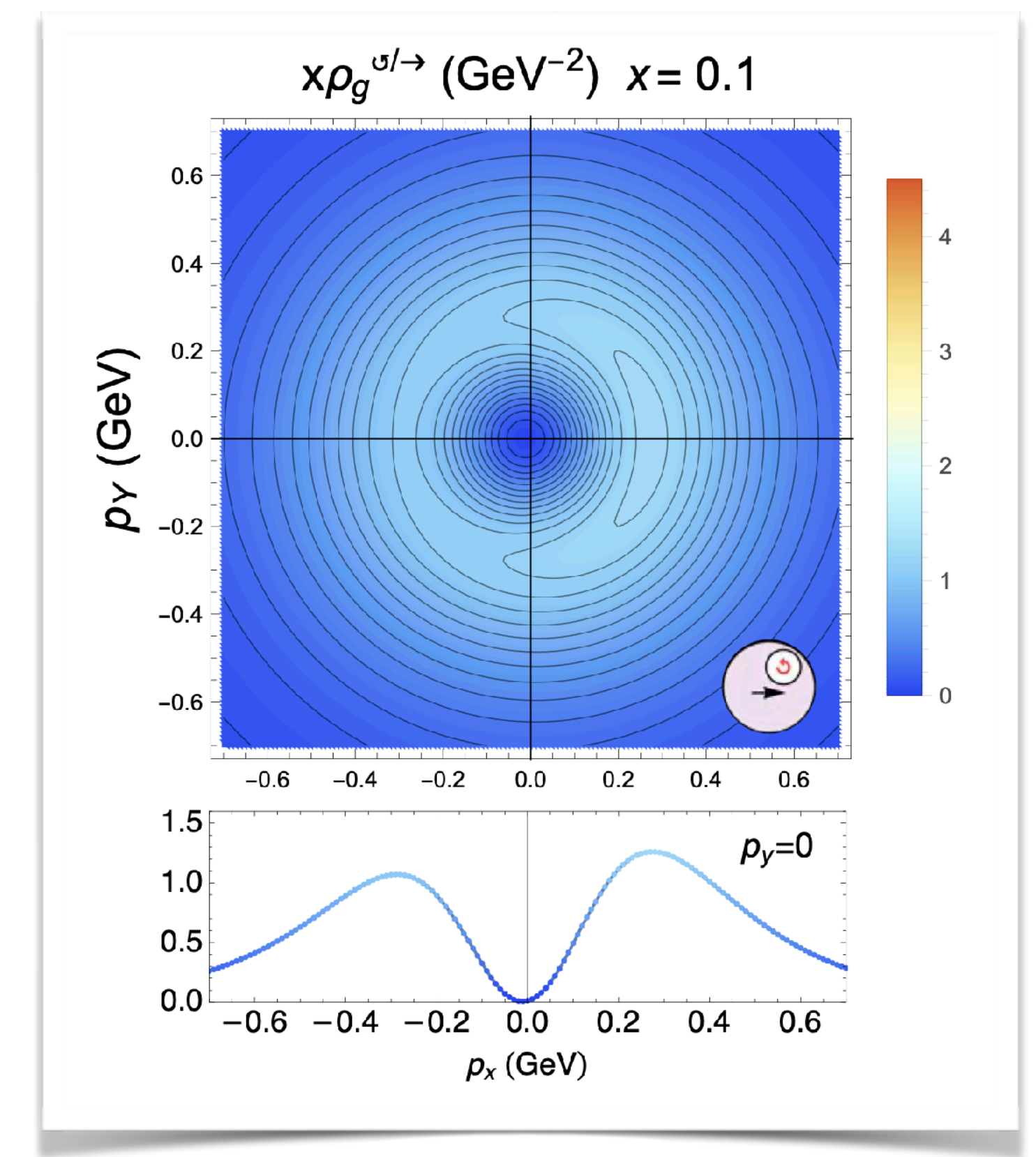
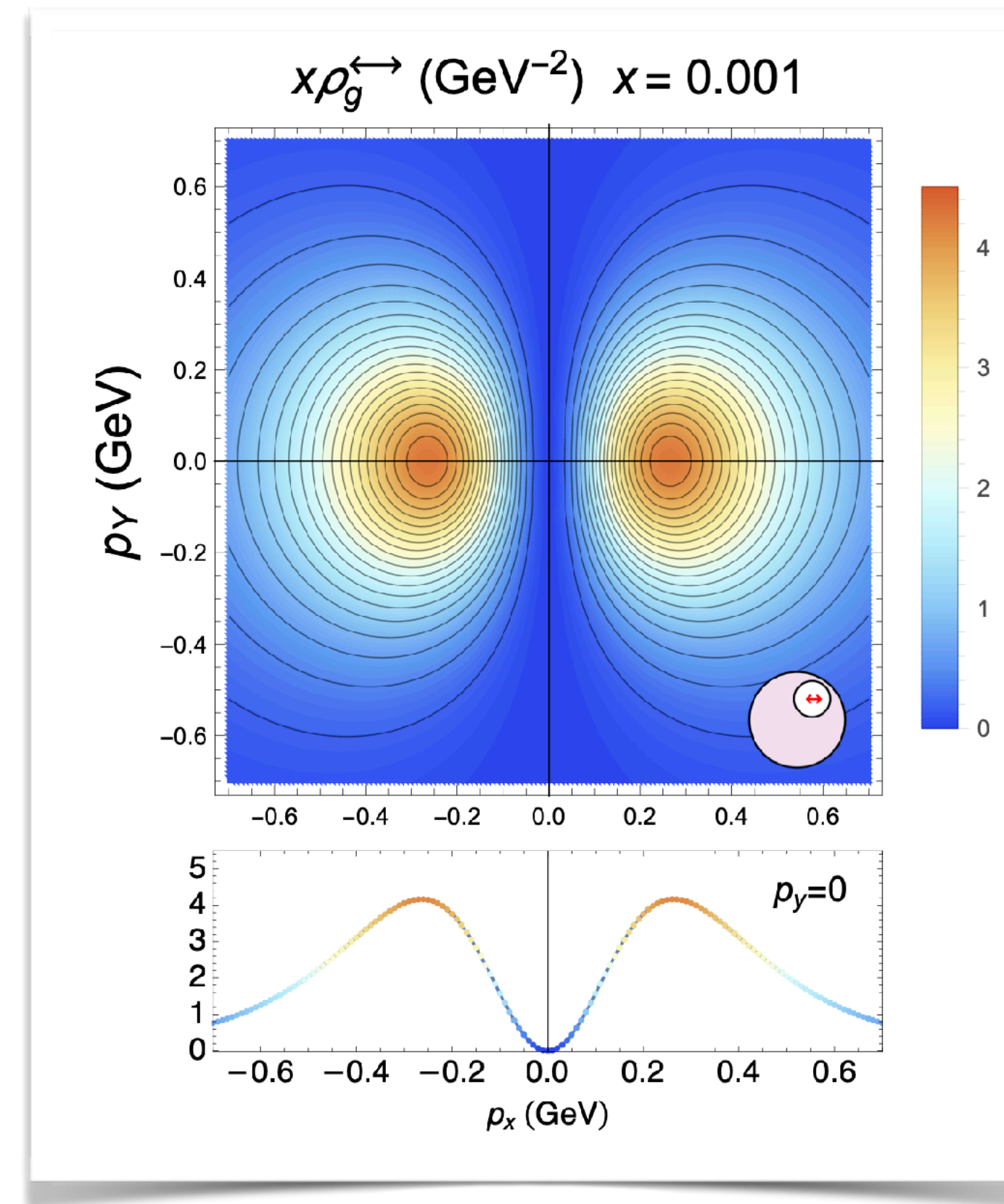
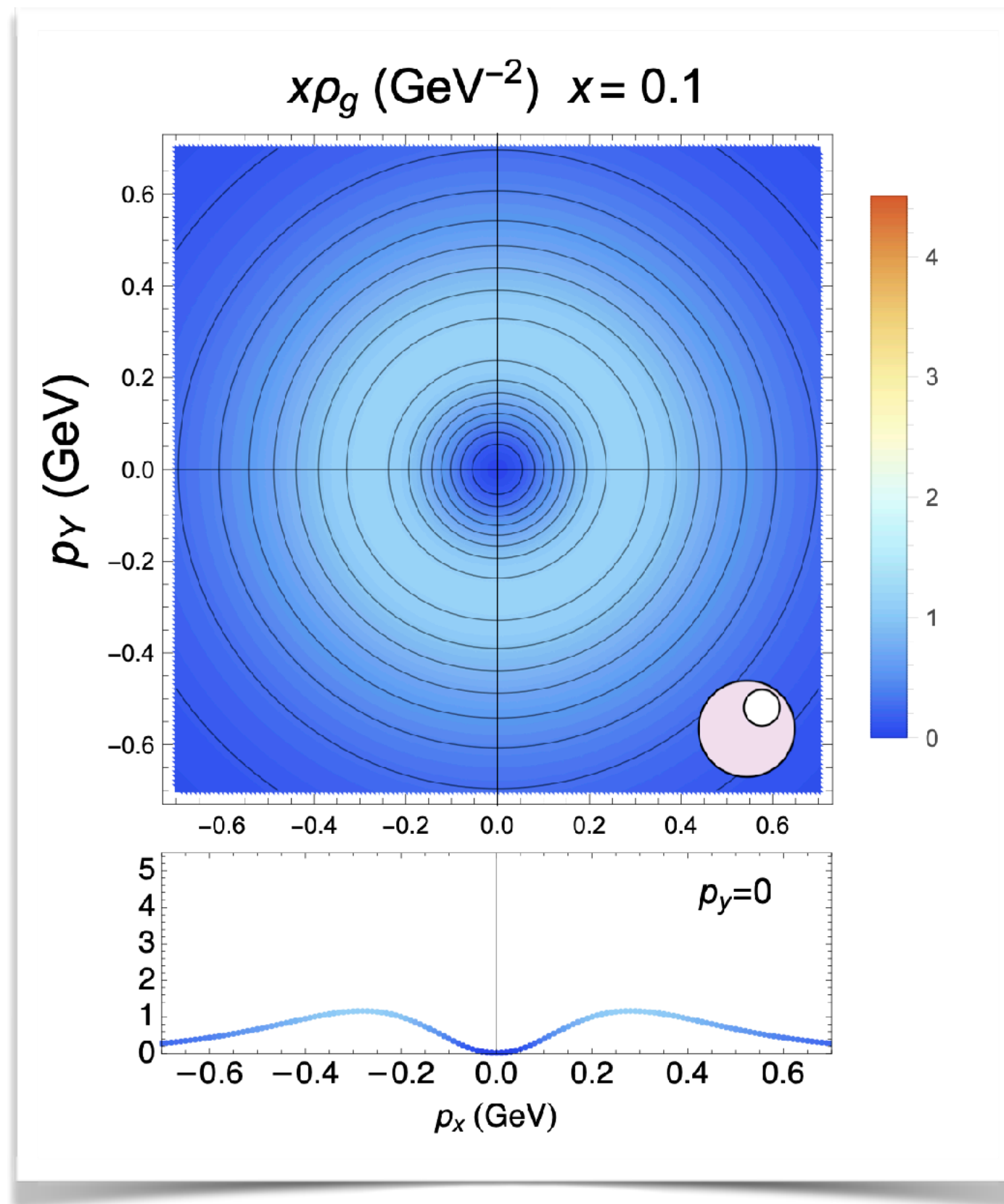
# 3D proton imaging: Tomographic reconstruction & TMDs

[A. Bacchetta, F.G.C., M. Radici, P. Tael, Eur. Phys. J. C 80 (2020) no.8]

## Unpolarized

## Boer-Mulders

## Worm-gear



$$x\rho^{\leftarrow\rightarrow}(x, p_x, p_y) = \frac{1}{2} \left[ x f_1^g(x, \mathbf{p}_T^2) + \frac{p_x^2 - p_y^2}{2M^2} x h_1^{\perp g}(x, \mathbf{p}_T^2) \right]$$

$$x\rho^{\ominus/\rightarrow}(x, p_x, p_y) = x f_1^g(x, \mathbf{p}_T^2) - \frac{p_x}{M} x g_{1T}^g(x, \mathbf{p}_T^2)$$

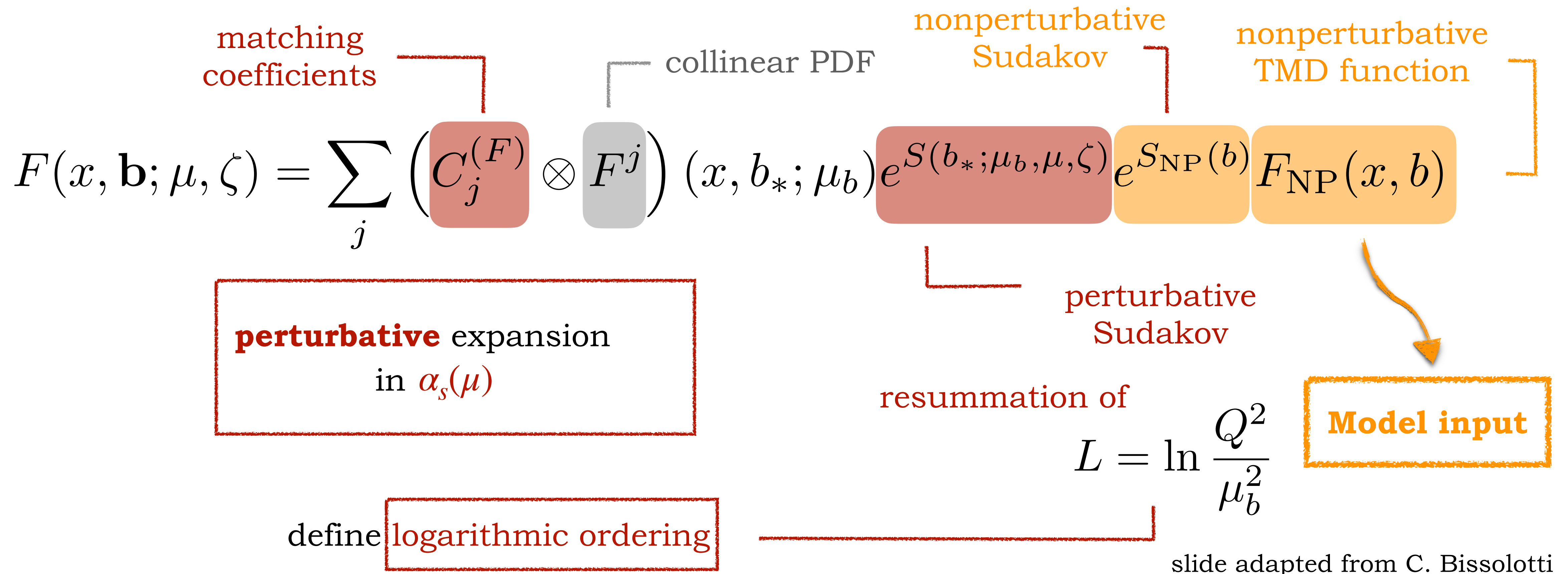
# Anatomy of gluon TMD PDFs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left( C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients  
 collinear PDF  
 nonperturbative Sudakov  
 nonperturbative TMD function  
 perturbative Sudakov  
 resummation of  
 $L = \ln \frac{Q^2}{\mu_b^2}$   
 define logarithmic ordering  
 slide adapted from C. Bissolotti

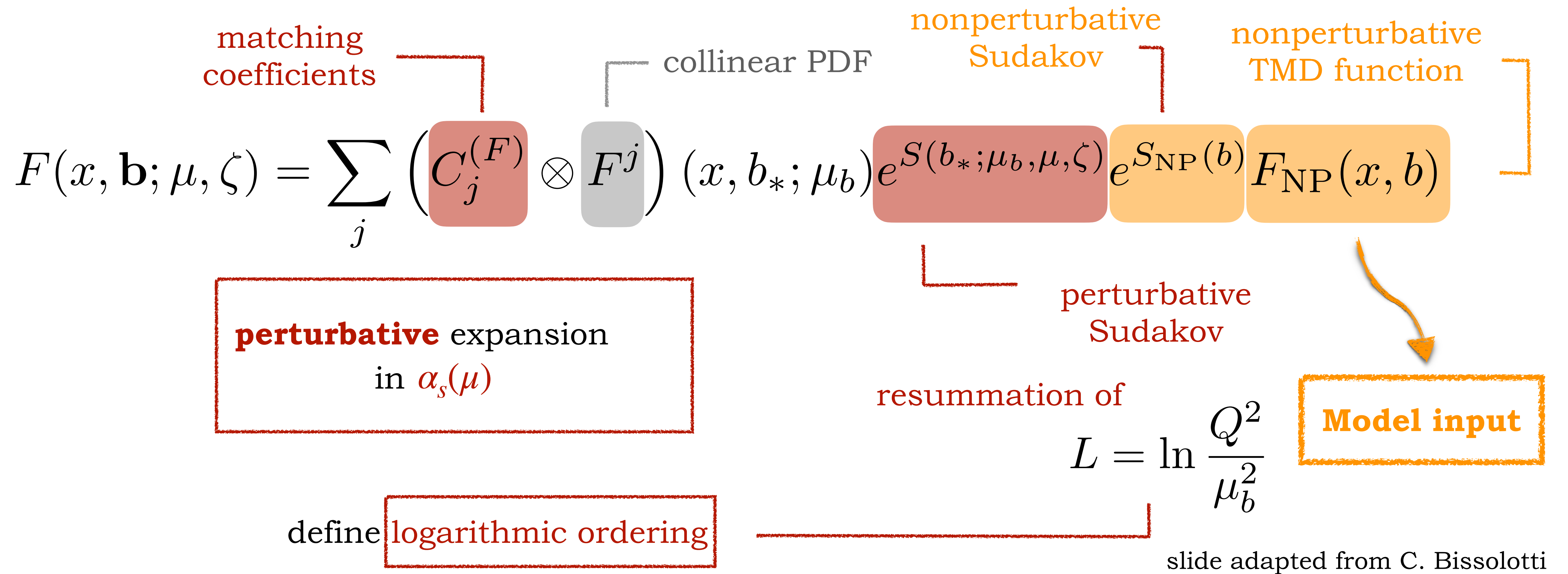


# Anatomy of gluon TMD PDFs



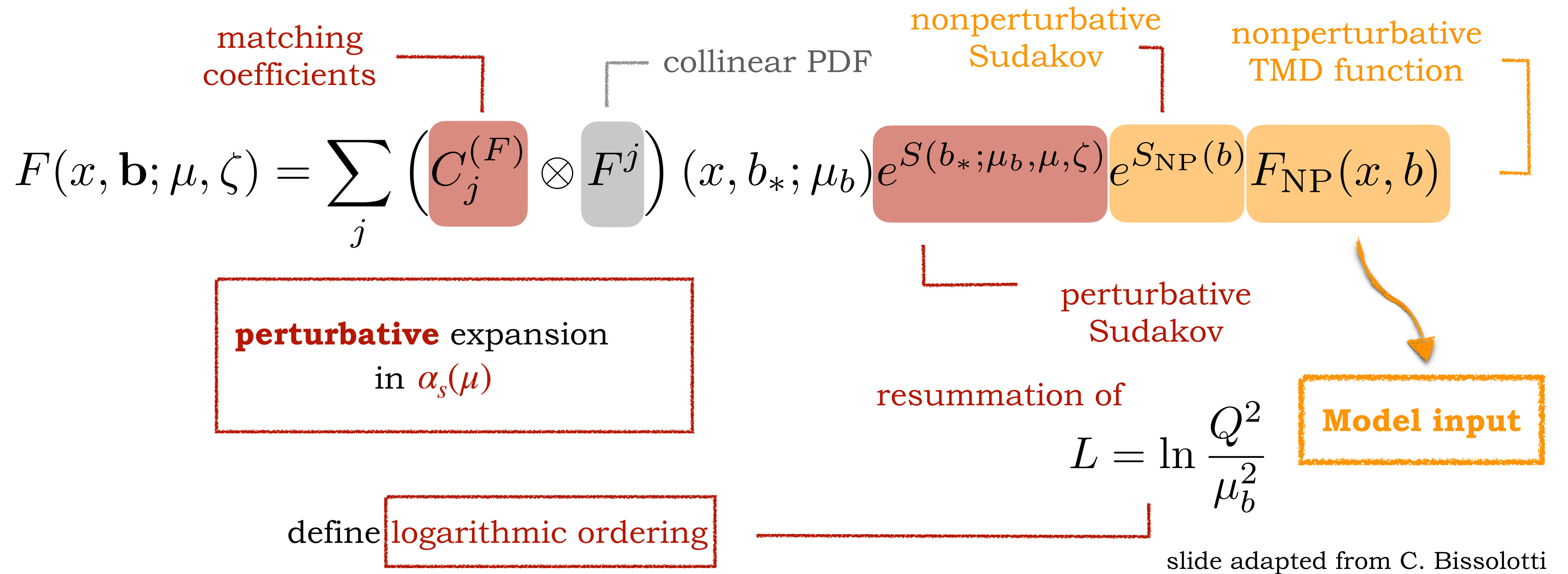


# Anatomy of gluon TMD PDFs



$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

# Anatomy of gluon TMD PDFs



$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

$$h_1^\perp(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(h_1^\perp)} \otimes f_1^j = [\mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

⚠ **Suppression of genuine NP effects ?** ←

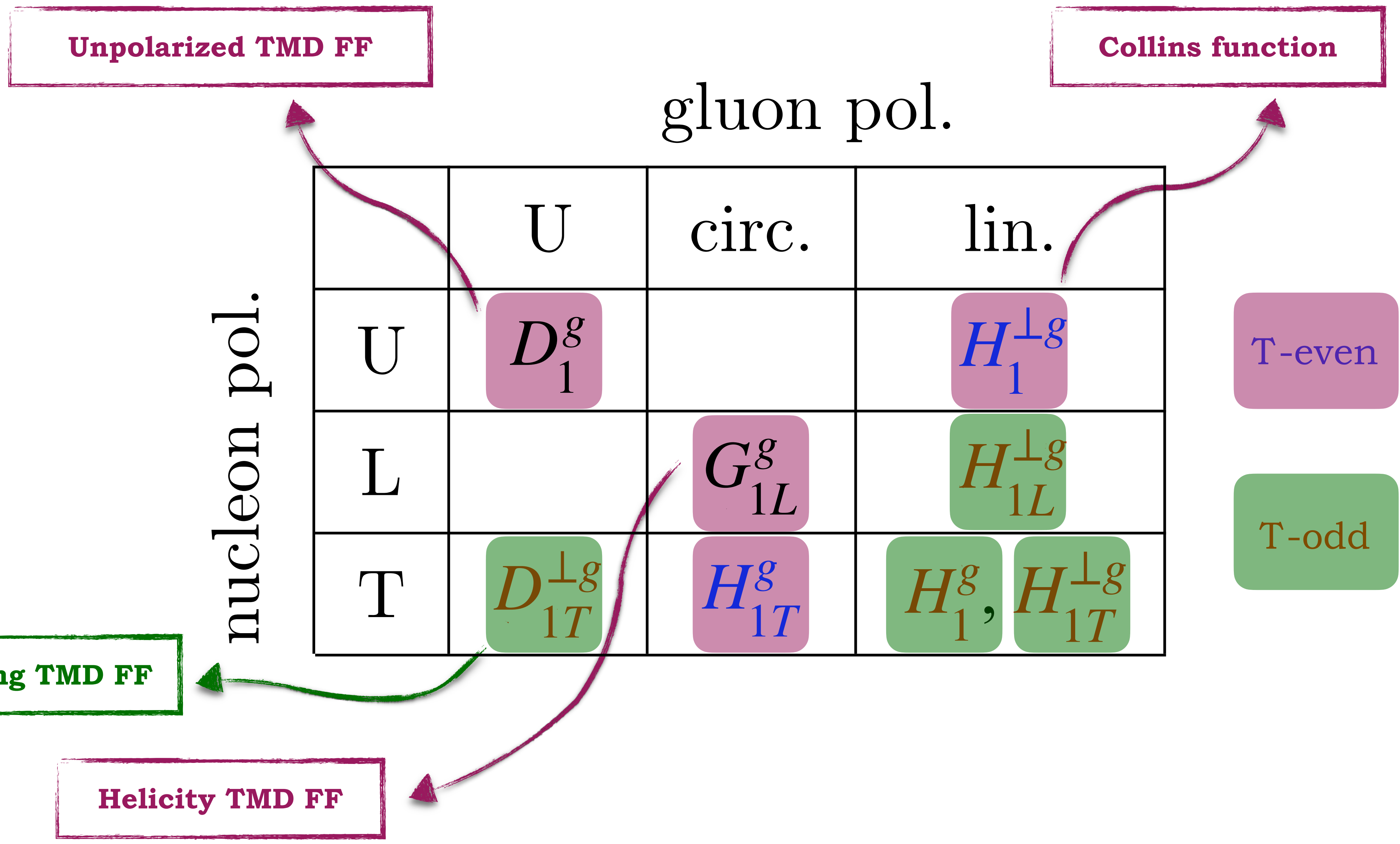
# Gluon TMD FFs at leading twist

gluon pol.

	U	circ.	lin.		
nucleon pol.	U	$D_1^g$		$H_1^{\perp g}$	T-even
	L		$G_{1L}^g$	$H_{1L}^{\perp g}$	T-odd
	T	$D_{1T}^{\perp g}$	$H_{1T}^g$	$H_1^g, H_{1T}^{\perp g}$	



# Gluon TMD FFs at leading twist



# Gluon TMD FFs at leading twist

Unpolarized TMD FF

Collins function

gluon pol.

	U	circ.	lin.
U	$D_1^g$		$H_1^{\perp g}$
L		$G_{1L}^g$	$H_{1L}^{\perp g}$
T	$D_{1T}^{\perp g}$	$H_{1T}^g$	$H_1^g, H_{1T}^{\perp g}$

nucleon pol.

T-even

T-odd

Polarizing TMD FF

Helicity TMD FF

*i*  $D_{1T}^{\perp[+,+]} = D_{1T}^{\perp[-,-]}$  and  $D_{1T}^{\perp[+,-]} = D_{1T}^{\perp[-,+]}$ , **but**  $D_{1T}^{\perp[+,+]} \neq D_{1T}^{\perp[+,-]}$  !





# 3. Building T-odd gluon TMDs

The background features a repeating pattern of circular diagrams, each representing a gluon Transverse Momentum Distribution (TMD). Each diagram shows a central gluon (a red sphere with a red arrow) surrounded by a cloud of other particles, including quarks (blue and green spheres with arrows) and gluons (yellow wavy lines). The diagrams are arranged in a grid-like pattern, with some overlapping, and are set against a light blue and green background with a subtle grid and starburst effects.

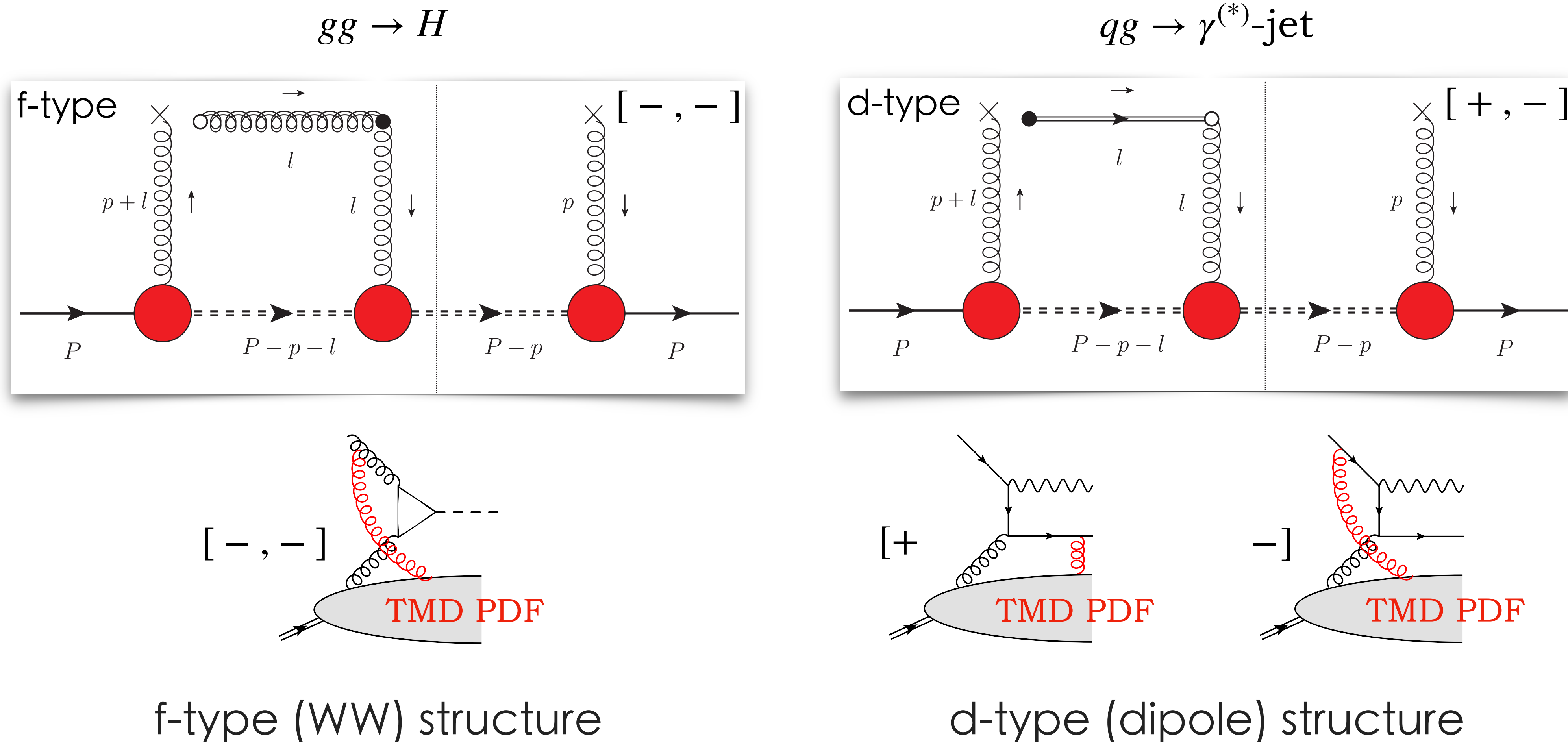


# T-odd gluon TMDs in a spectator model

-  No residual gluon-spectator interaction at tree level
-  Interference with one-gluon exchange (eikonal)

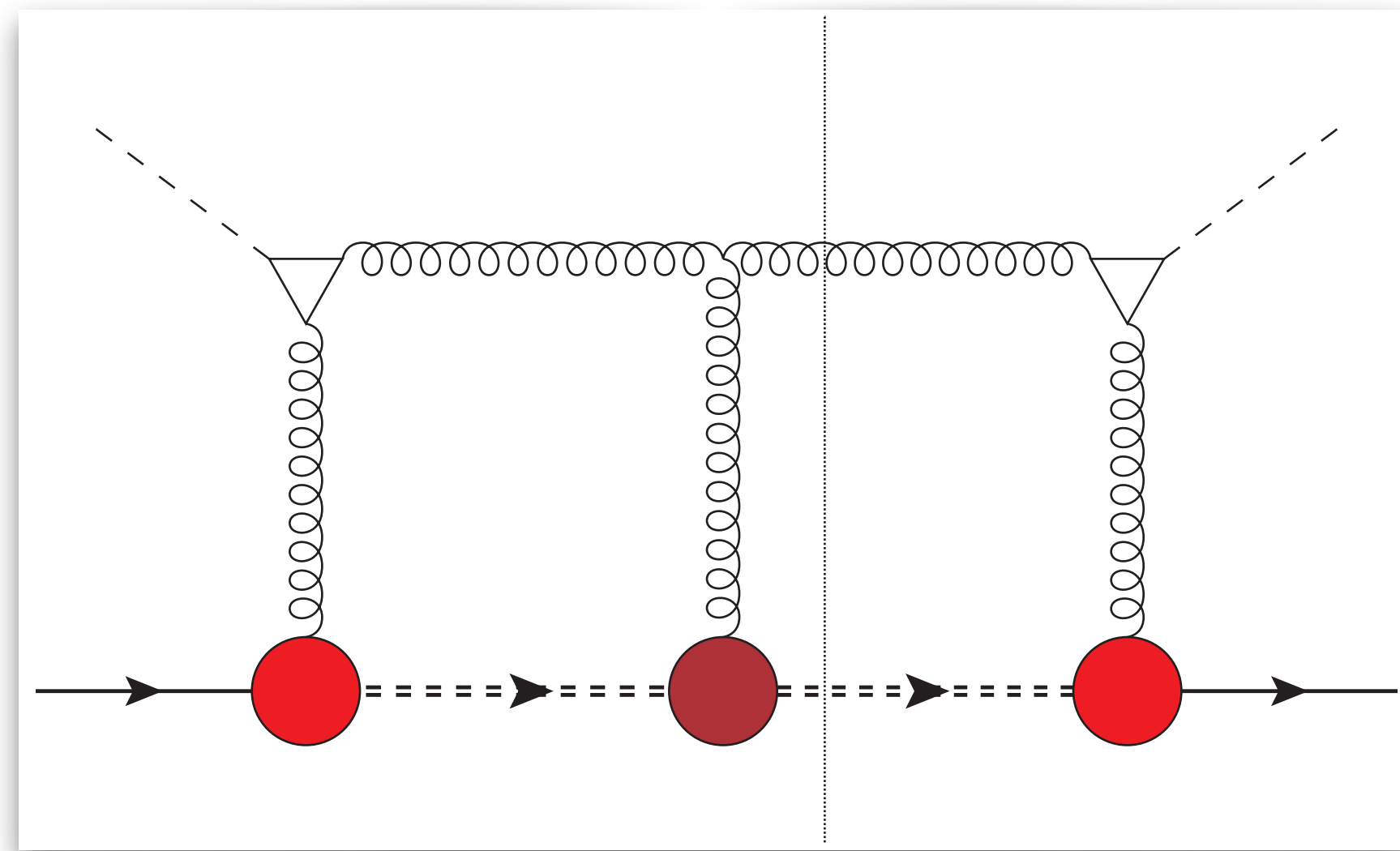
# T-odd gluon TMDs in a spectator model

- No residual gluon-spectator interaction at tree level
- Interference with one-gluon exchange (eikonal)

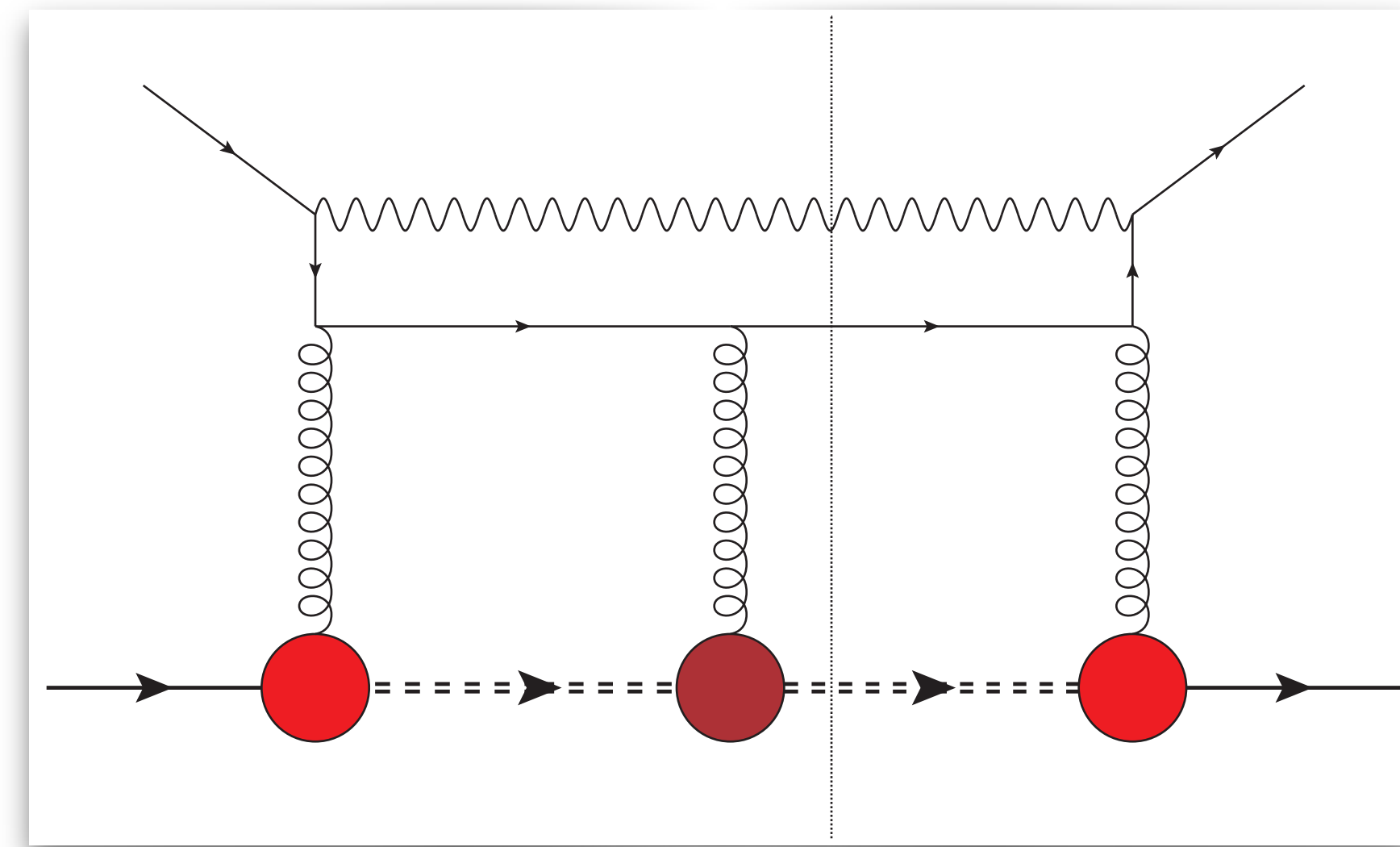


# T-odd gluon TMDs in a spectator model

Higgs-gluon fusion  $\Rightarrow$  f-type [ + , + ]



Photon-jet emission  $\Rightarrow$  d-type [ + , - ]

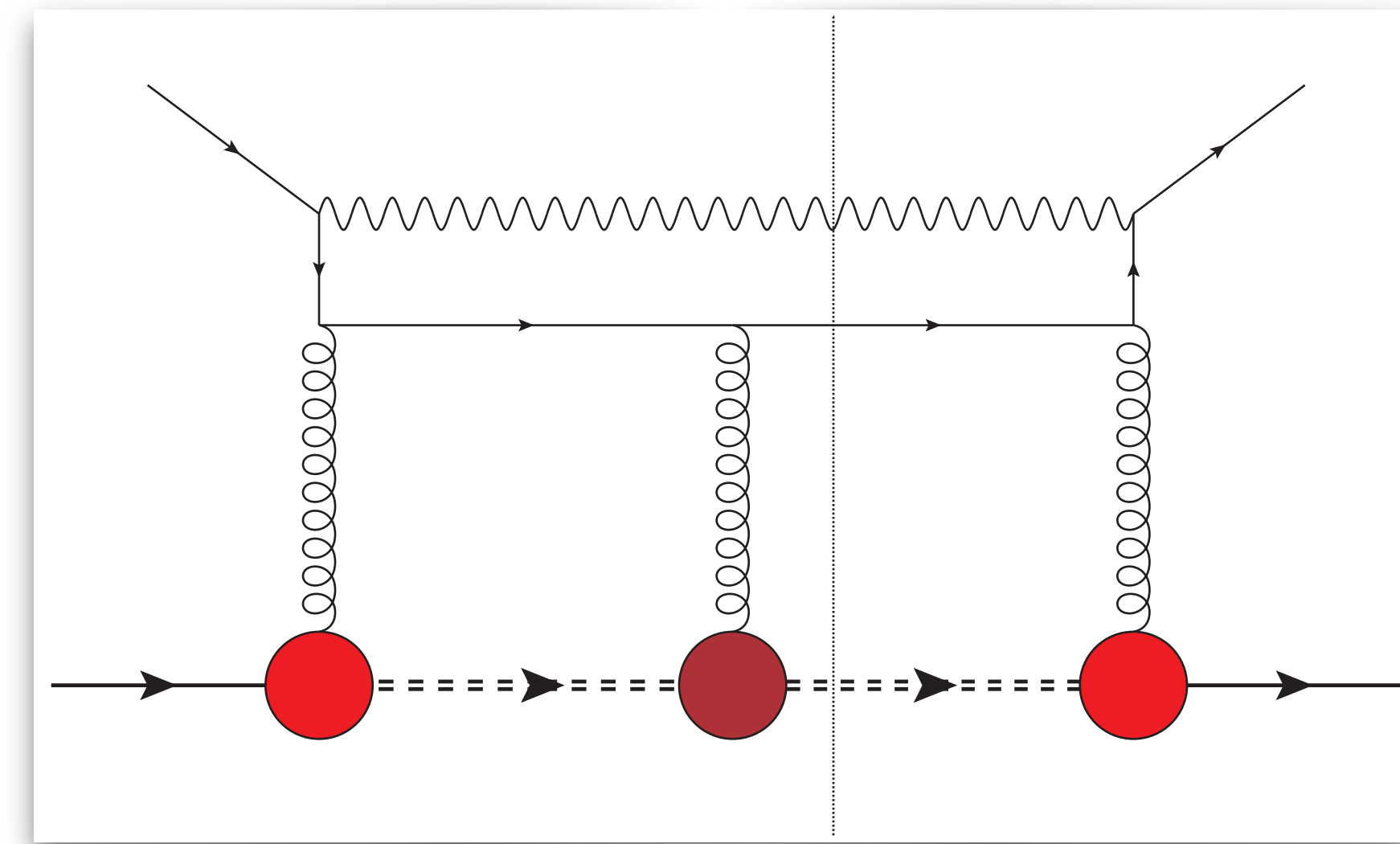
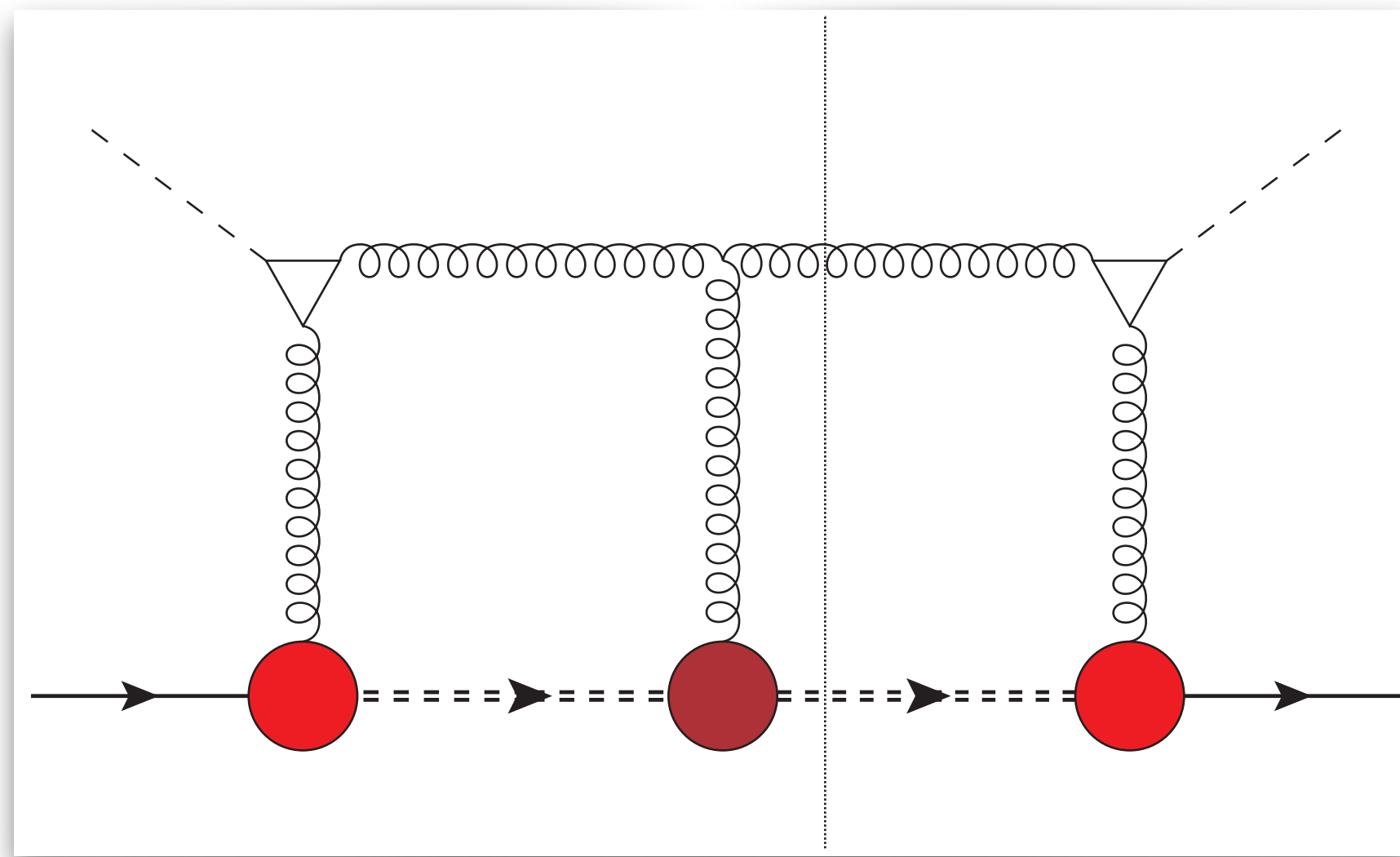




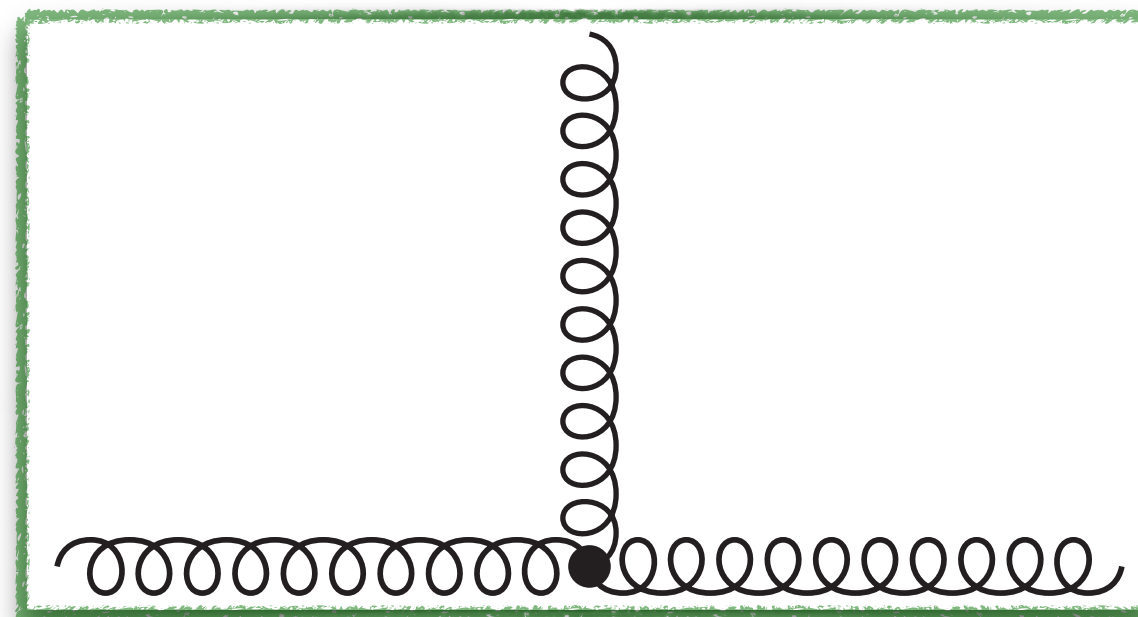
# T-odd gluon TMDs in a spectator model

Higgs-gluon fusion  $\Rightarrow$  f-type [ + , + ]

Photon-jet emission  $\Rightarrow$  d-type [ + , - ]



\* If the model were pQCD, say a gluon-target model...

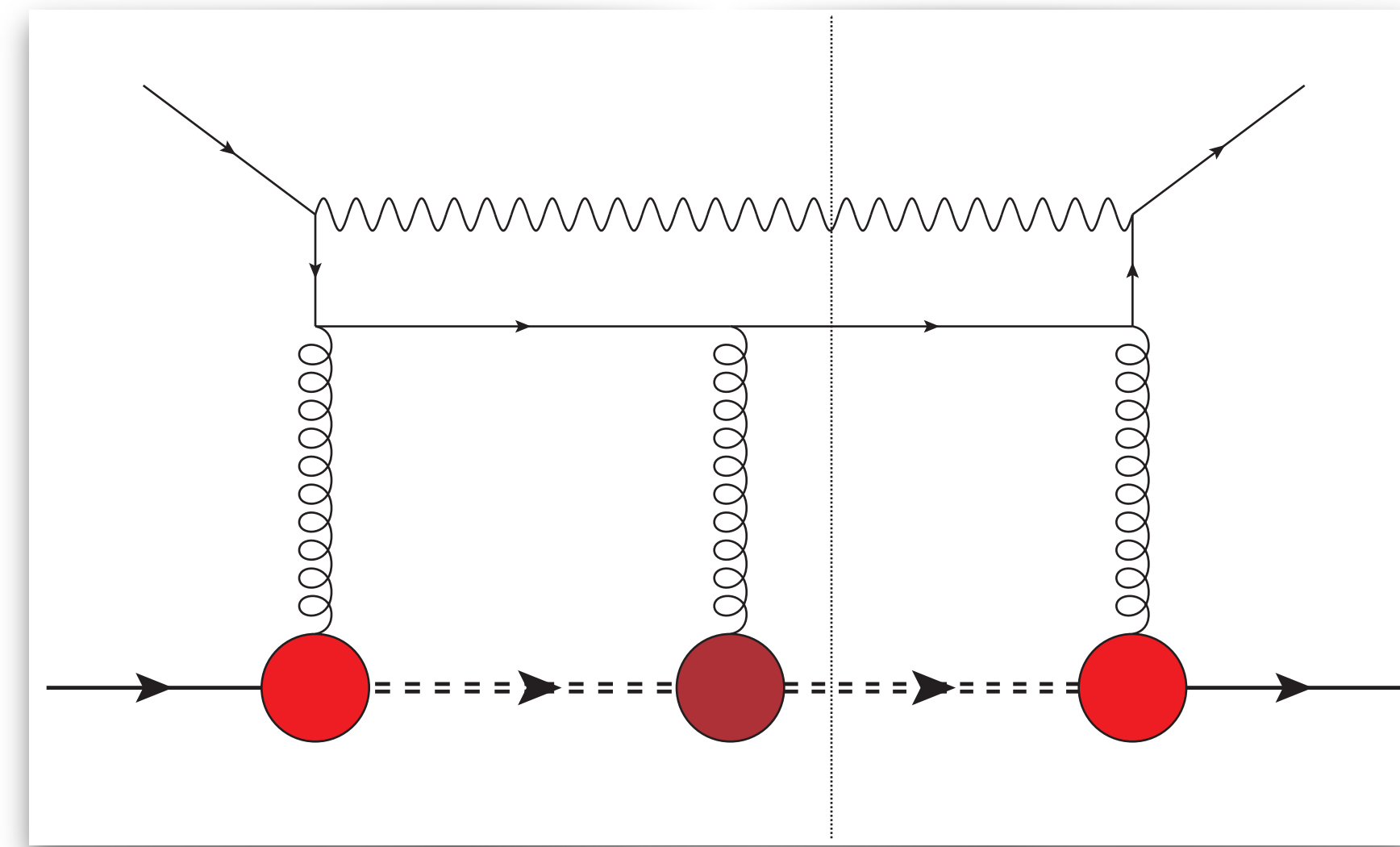
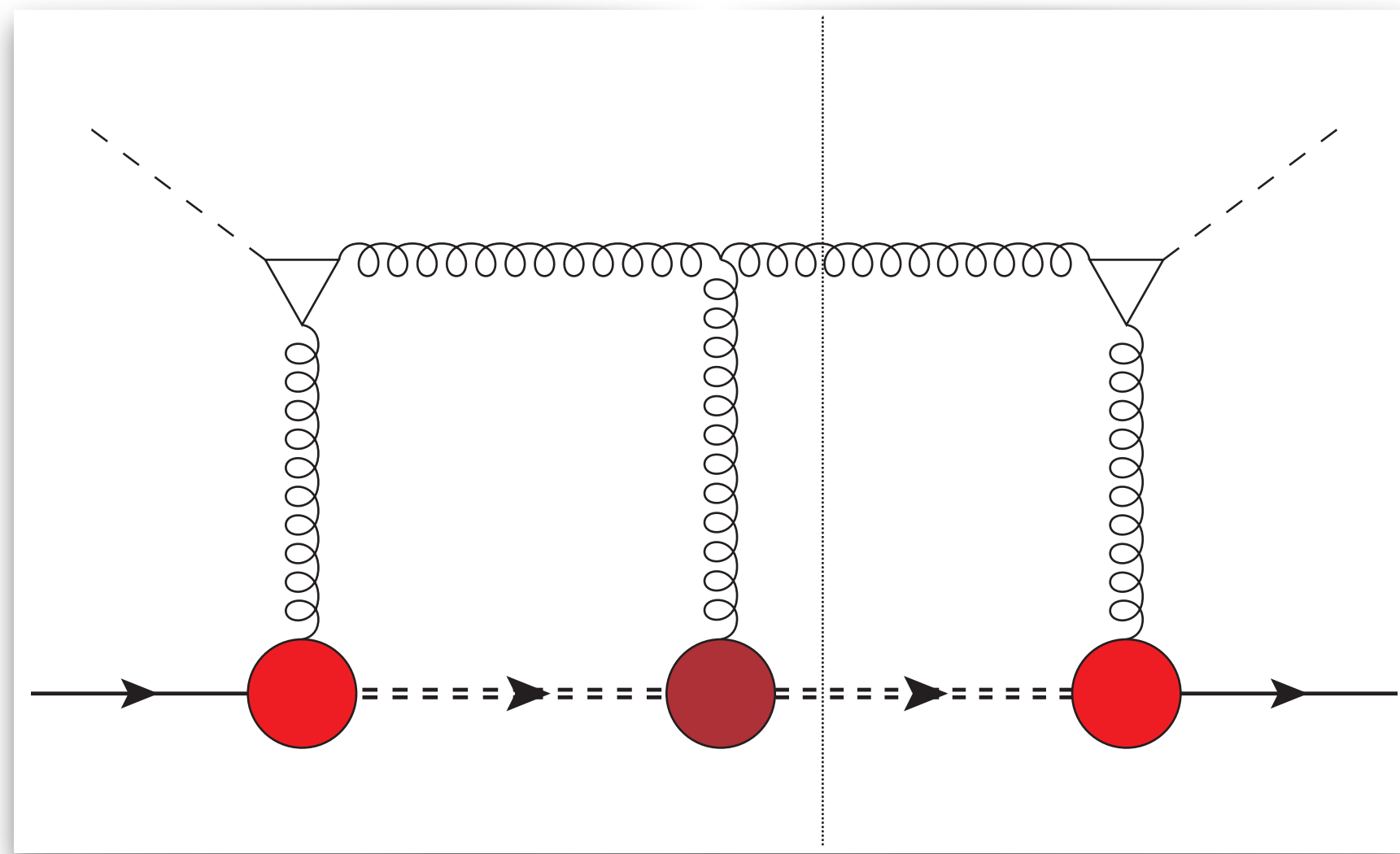


$$- g_s f_{abc}$$

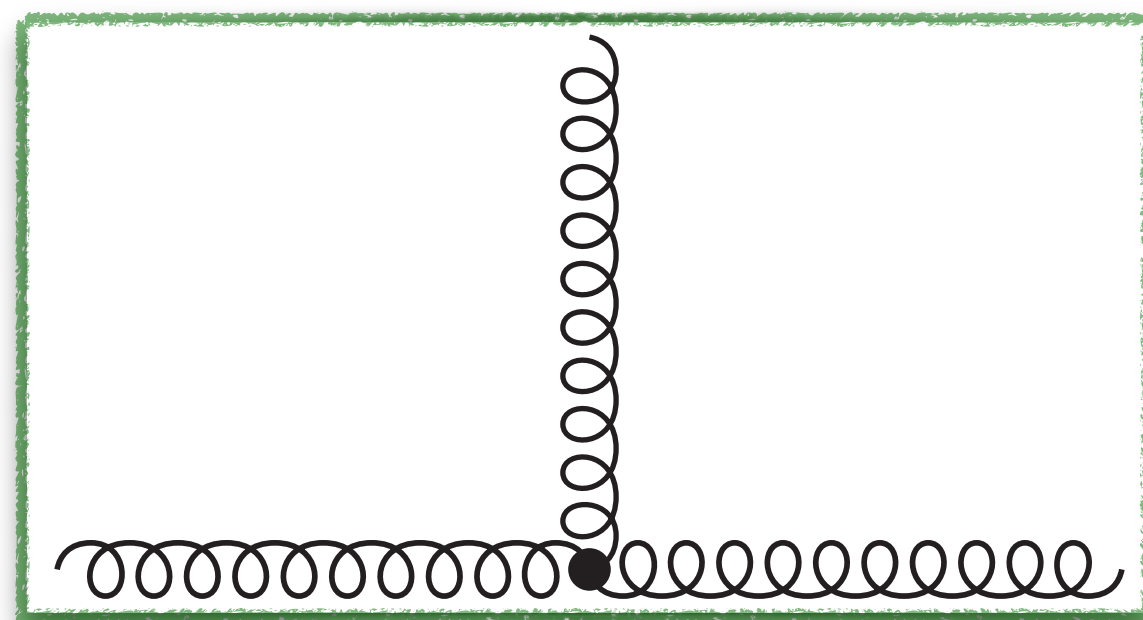
# T-odd gluon TMDs in a spectator model

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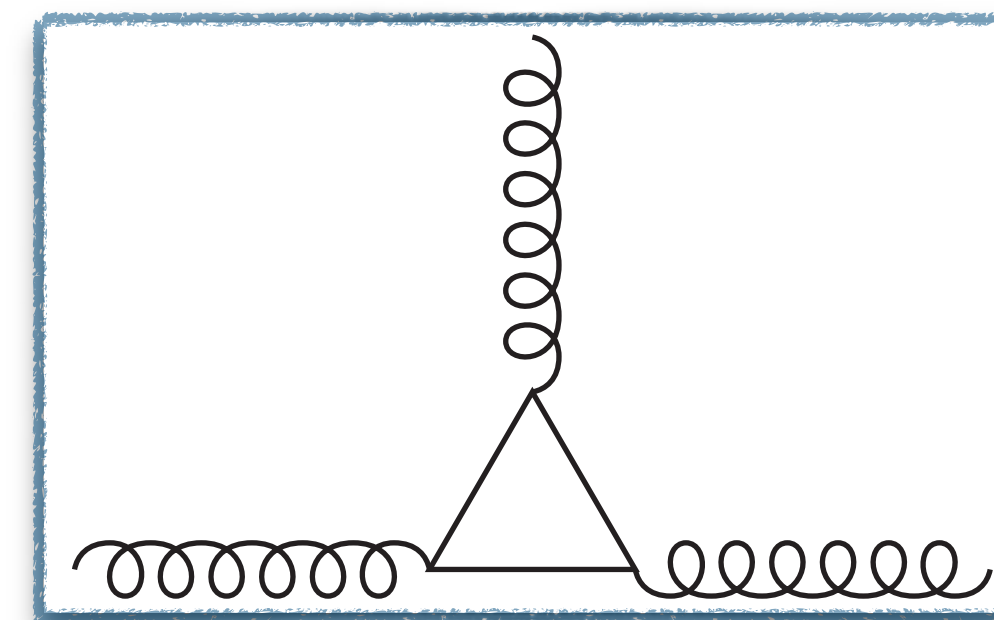


\* If the model were pQCD, say a gluon-target model...



$$-g_s f_{abc}$$

$$i g_s^3 d_{abc}$$



$$-i t_c$$

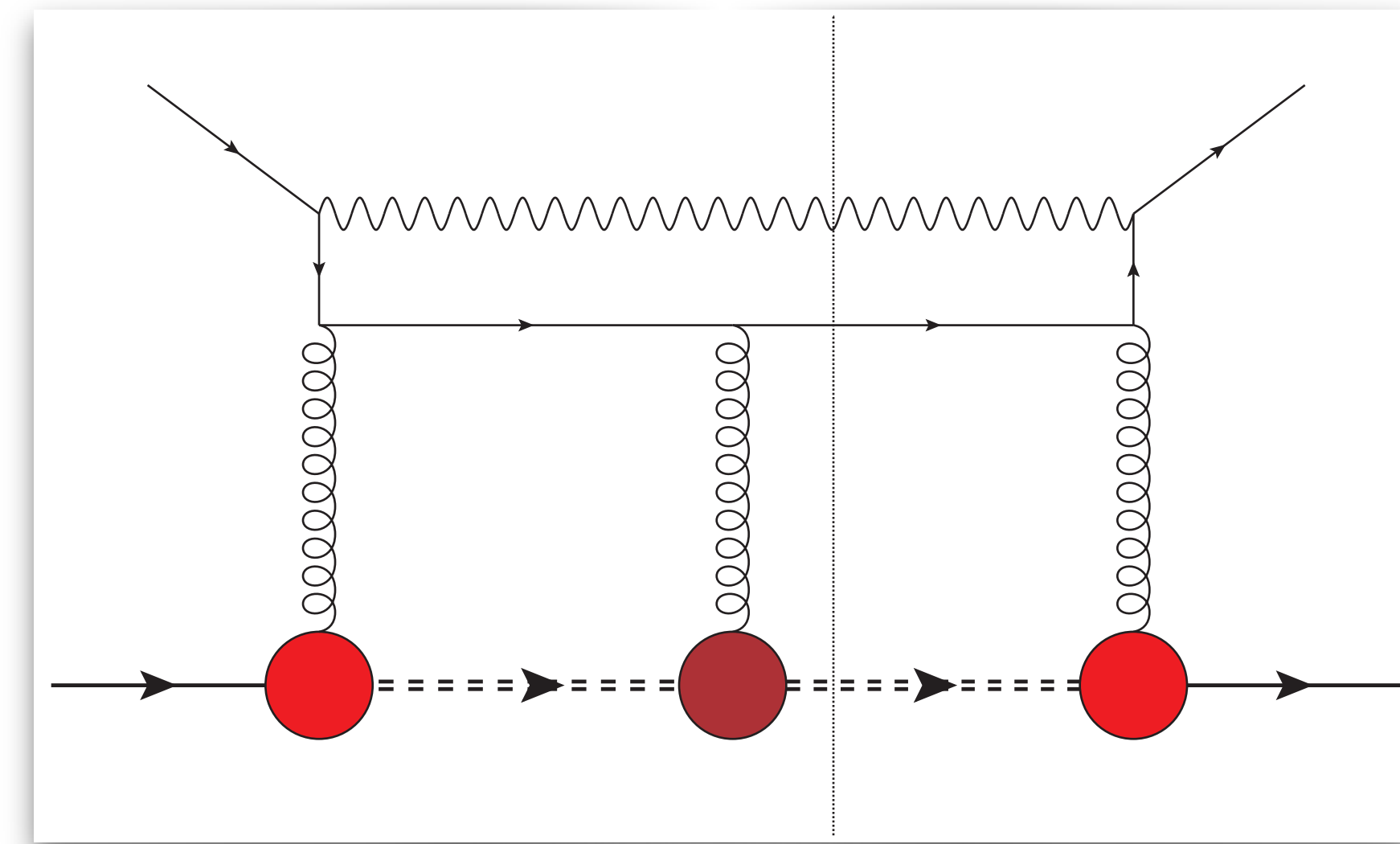
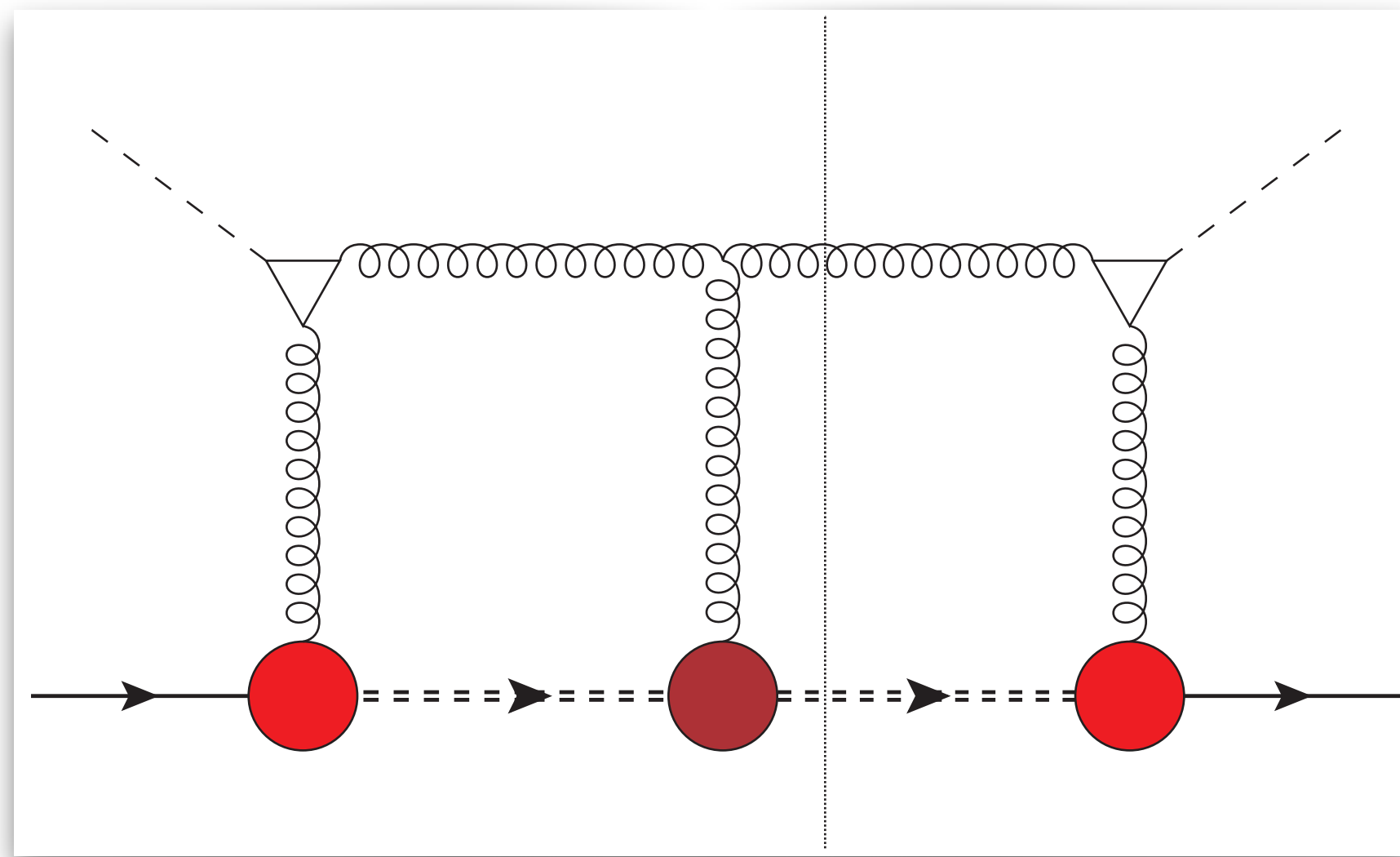
$$-i t_a$$

$$-i t_b$$

# T-odd gluon TMDs in a spectator model

Higgs-gluon fusion  $\Rightarrow$  f-type [ + , + ]

Photon-jet emission  $\Rightarrow$  d-type [ + , - ]



\* If the model were pQCD, say a gluon-target model...

! ...but the model is not pQCD !

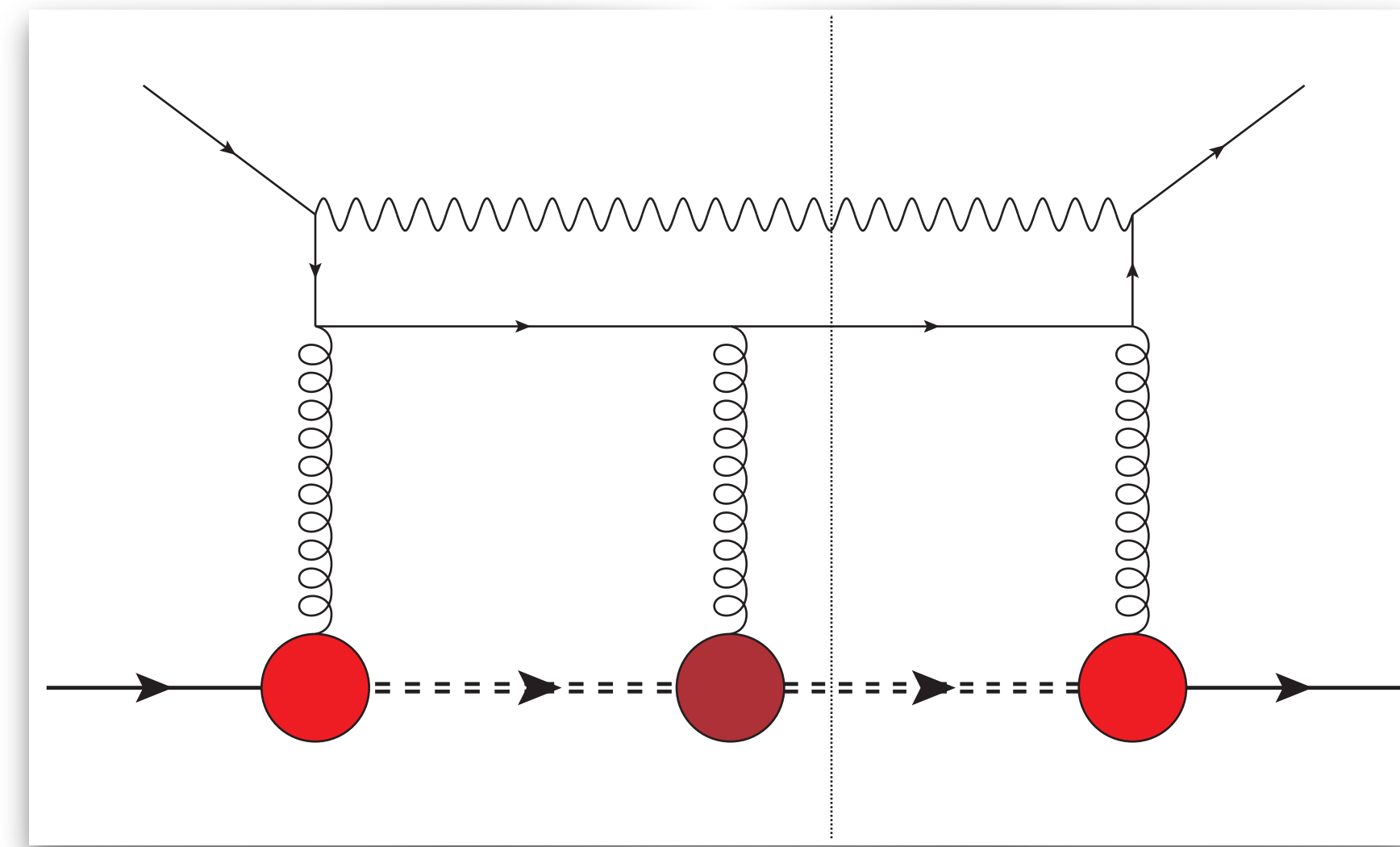
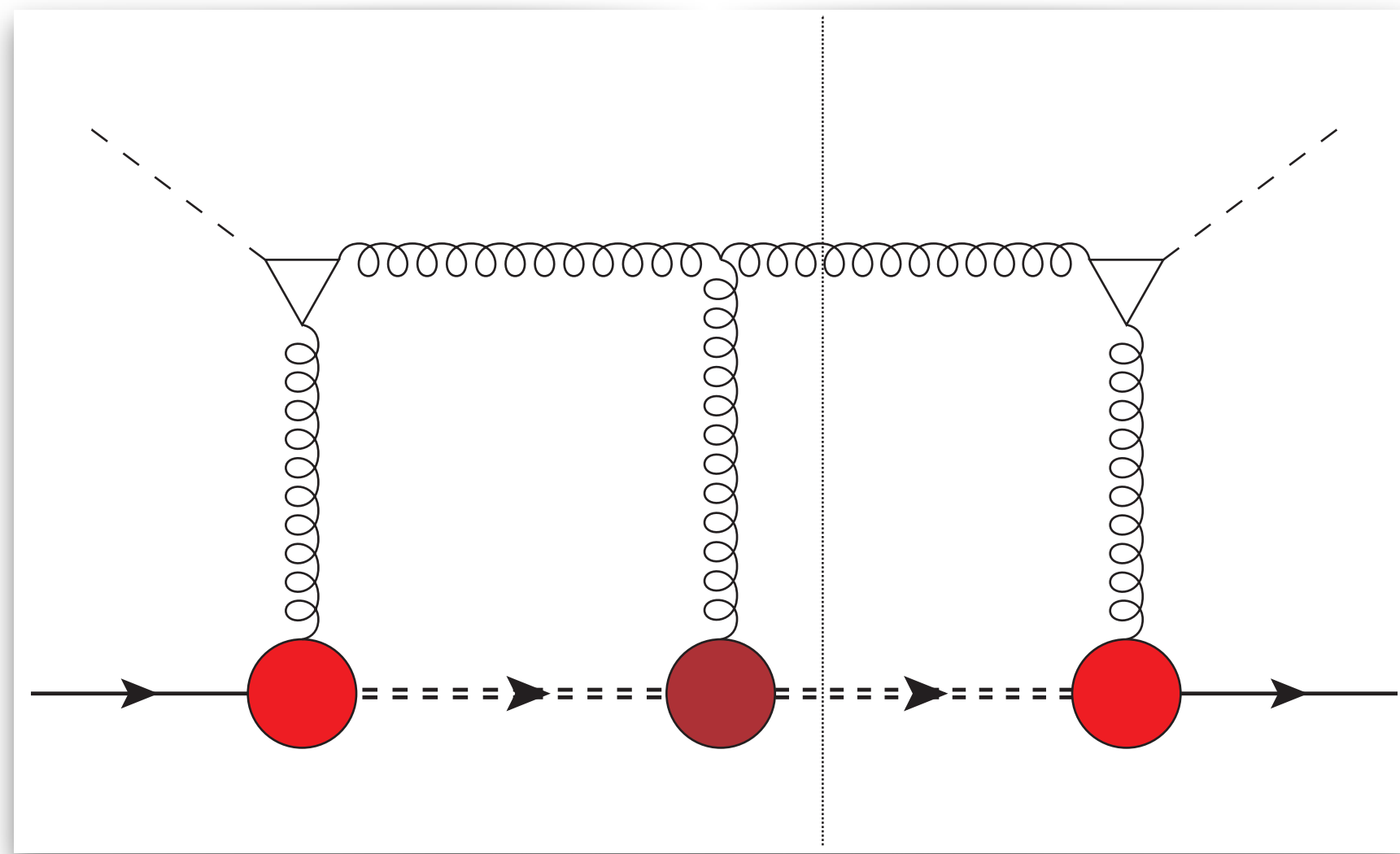
We want to model the nonperturbative content of T-odd TMD PDFs



# T-odd gluon TMDs in a spectator model

Higgs-gluon fusion  $\Rightarrow$  f-type [ + , + ]

Photon-jet emission  $\Rightarrow$  d-type [ + , - ]



 nucleon-gluon-spectator

 spectator-gluon-spectator

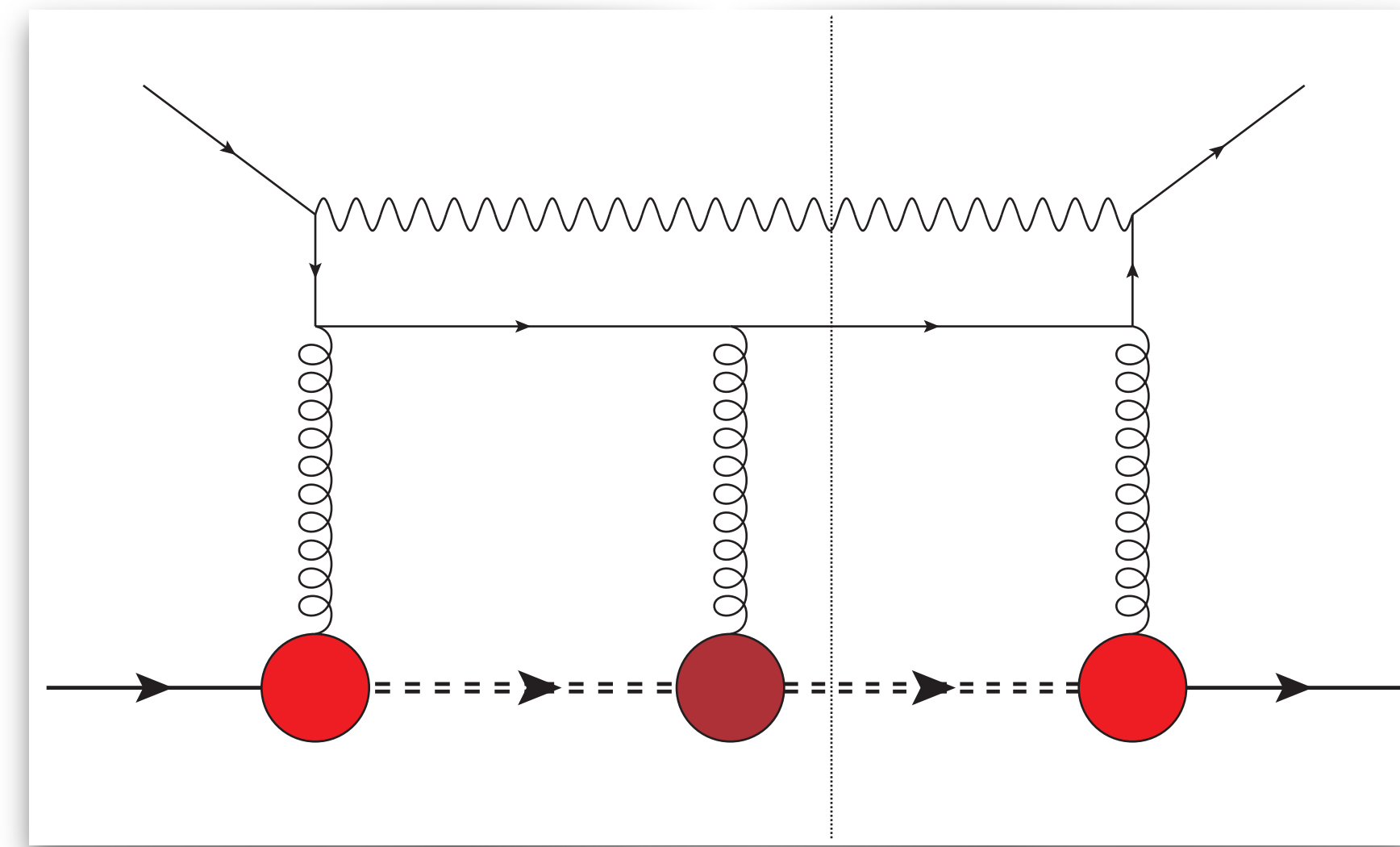
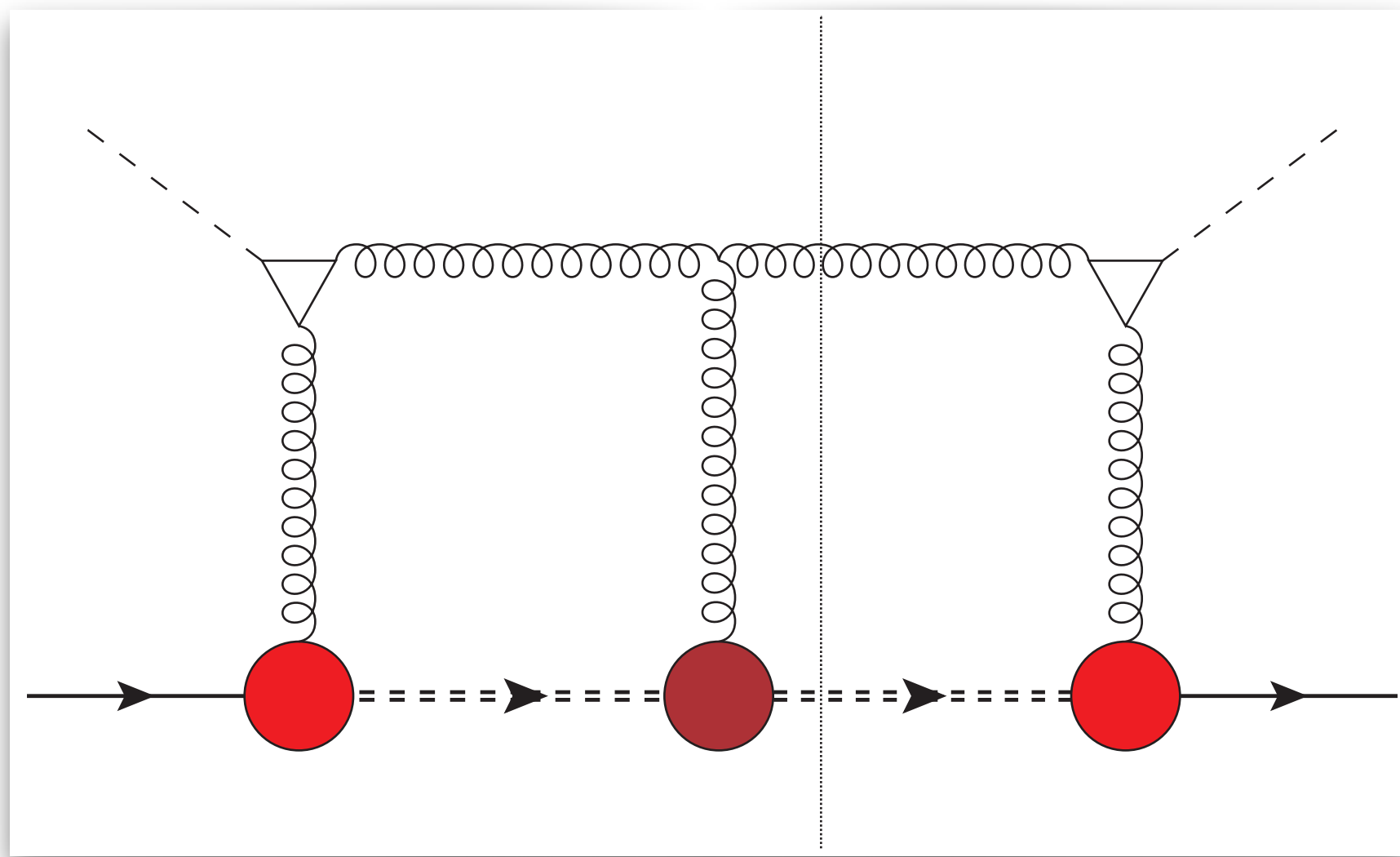
$$\mathcal{Y}_{bc}^{\mu}(p^2) = \delta_{bc} \left[ g_1(p^2) \gamma_{\mu} + g_2(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right]$$

$$\mathcal{X}_{abc}^{\mu}(p^2) = f^{abc} \left[ g_1^f(p^2) \gamma^{\mu} + g_2^f(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right] - i d^{abc} \left[ g_1^d(p^2) \gamma^{\mu} + g_2^d(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right]$$

# T-odd gluon TMDs in a spectator model

Higgs-gluon fusion  $\Rightarrow$  f-type [ + , + ]

Photon-jet emission  $\Rightarrow$  d-type [ + , - ]



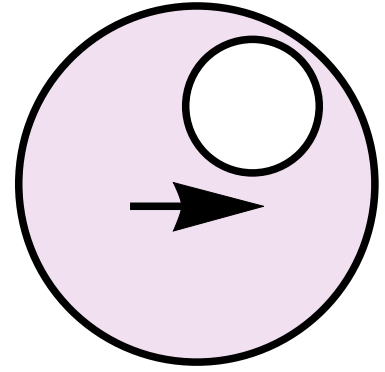
 nucleon-gluon-spectator

 spectator-gluon-spectator

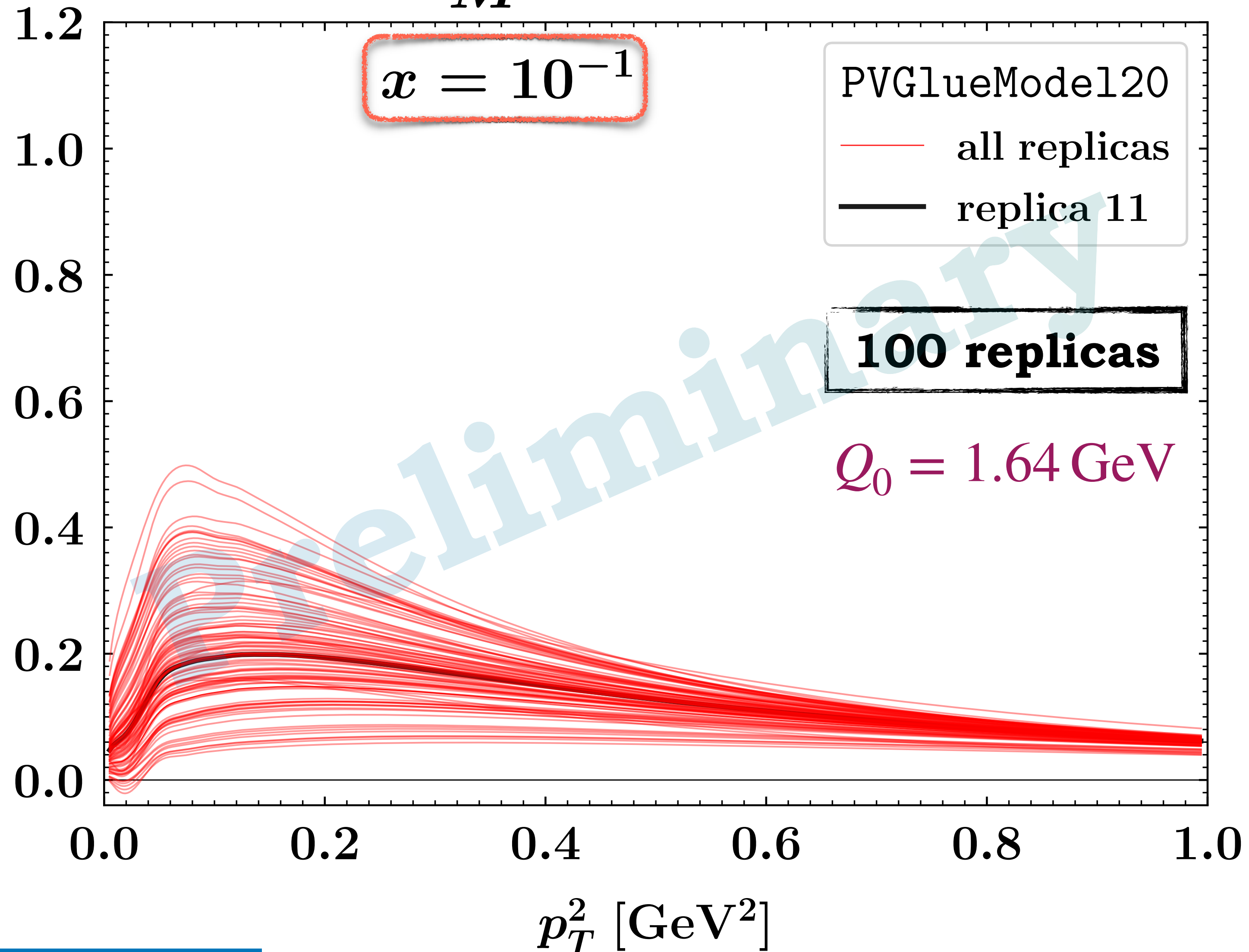
$$\mathcal{Y}_{bc}^{\mu}(p^2) = \delta_{bc} \left[ g_1(p^2) \gamma_{\mu} + g_2(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right]$$

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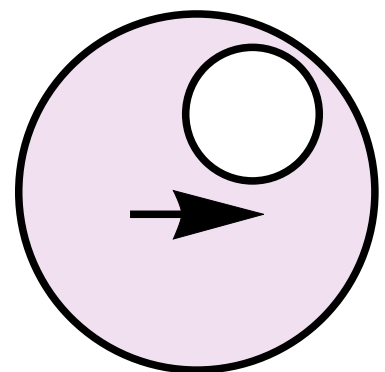
Assumption:  $g_{1,2}^d(p^2) = g_{1,2}^f(p^2) \equiv g_{1,2}(p^2) \quad \Leftrightarrow \quad f_{1T}^{\perp[+,-]} = \frac{c_{[+,-]}}{c_{[+,+]}} f_{1T}^{\perp[+,+]} \equiv \frac{10}{18} f_{1T}^{\perp[+,+]}$



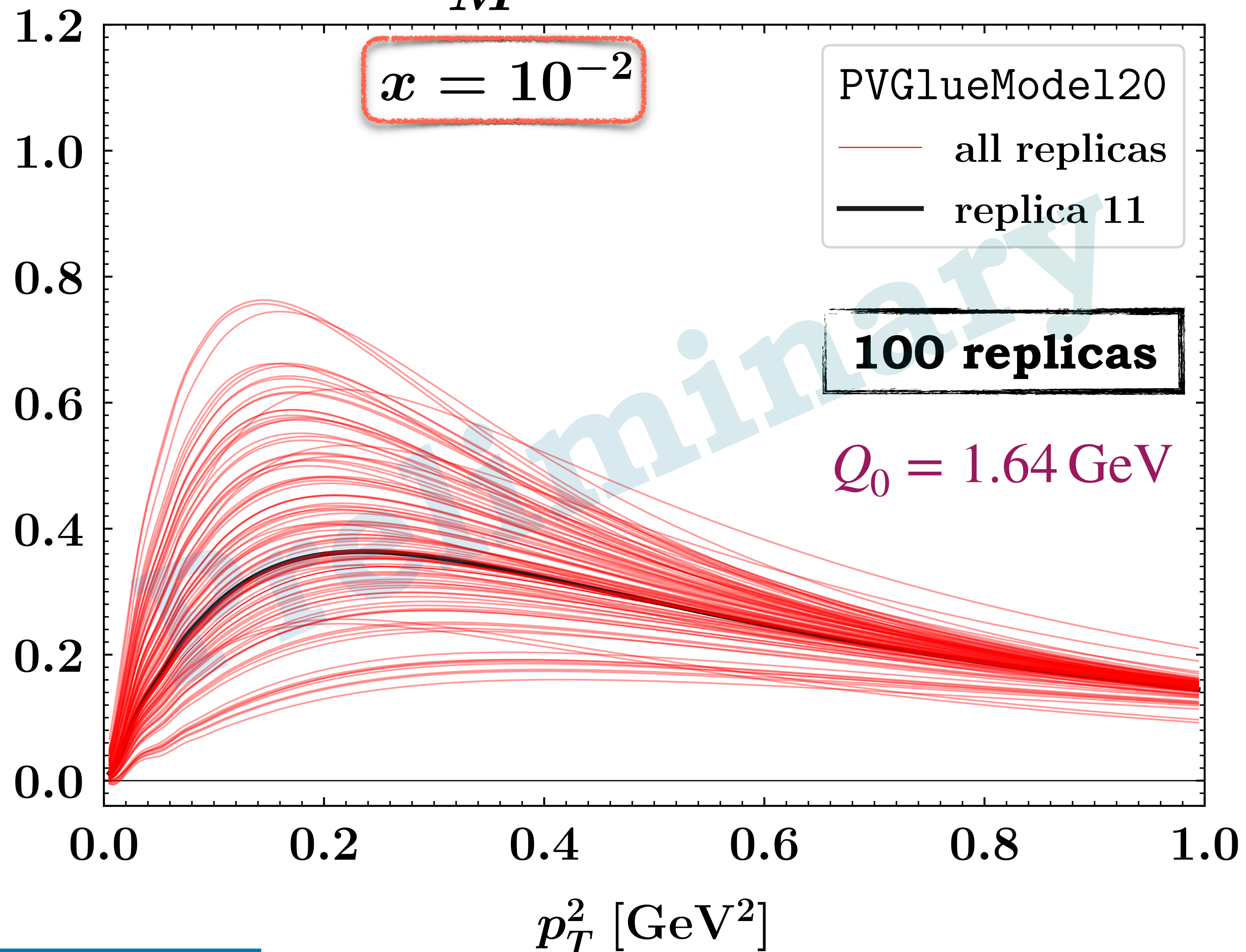
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

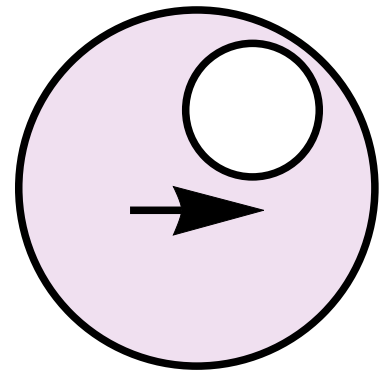




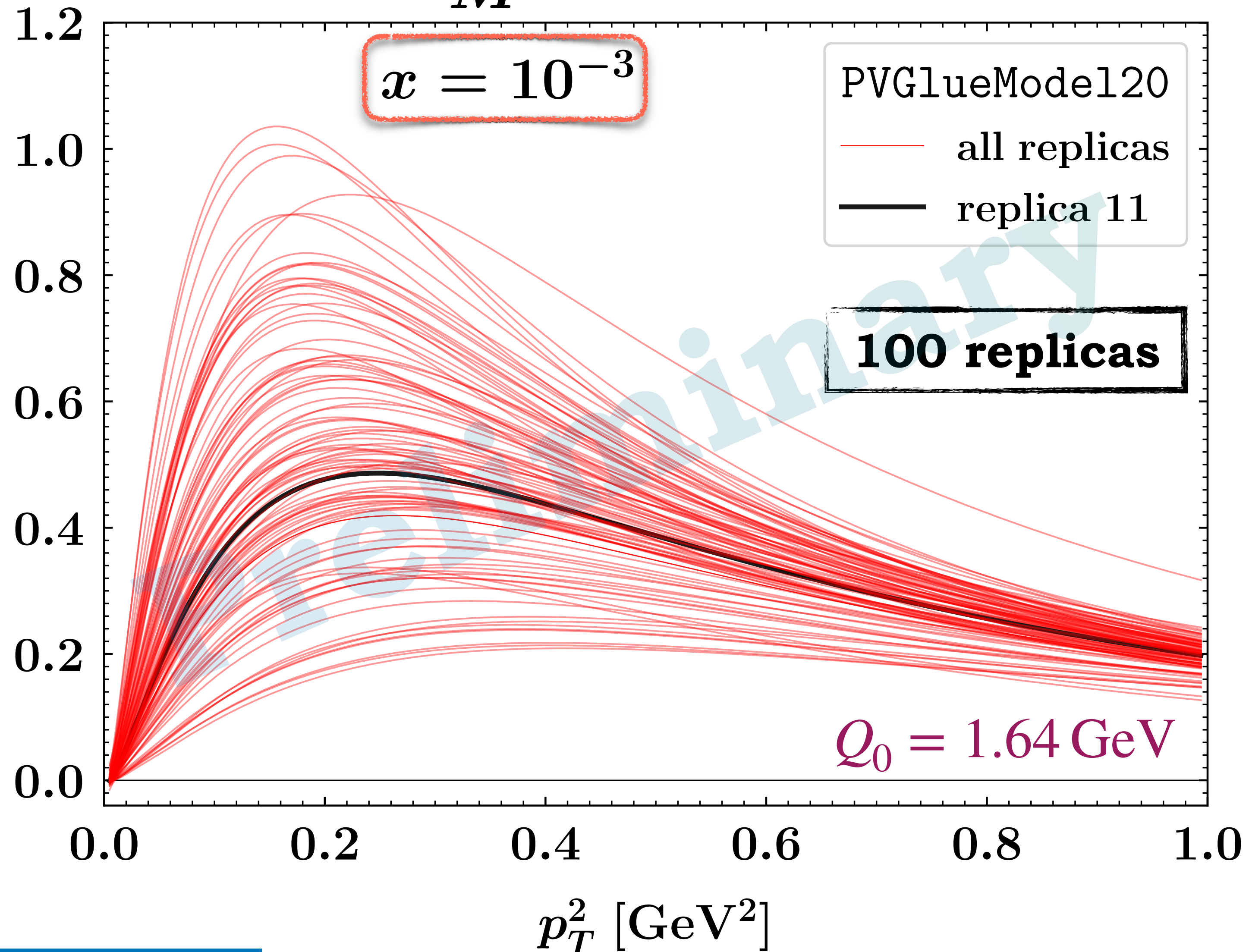


$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

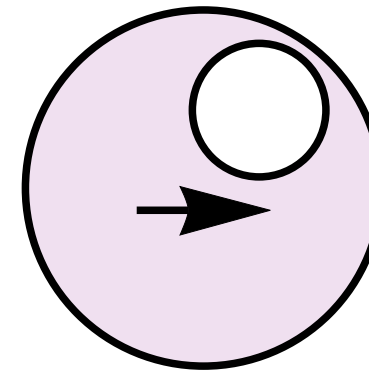




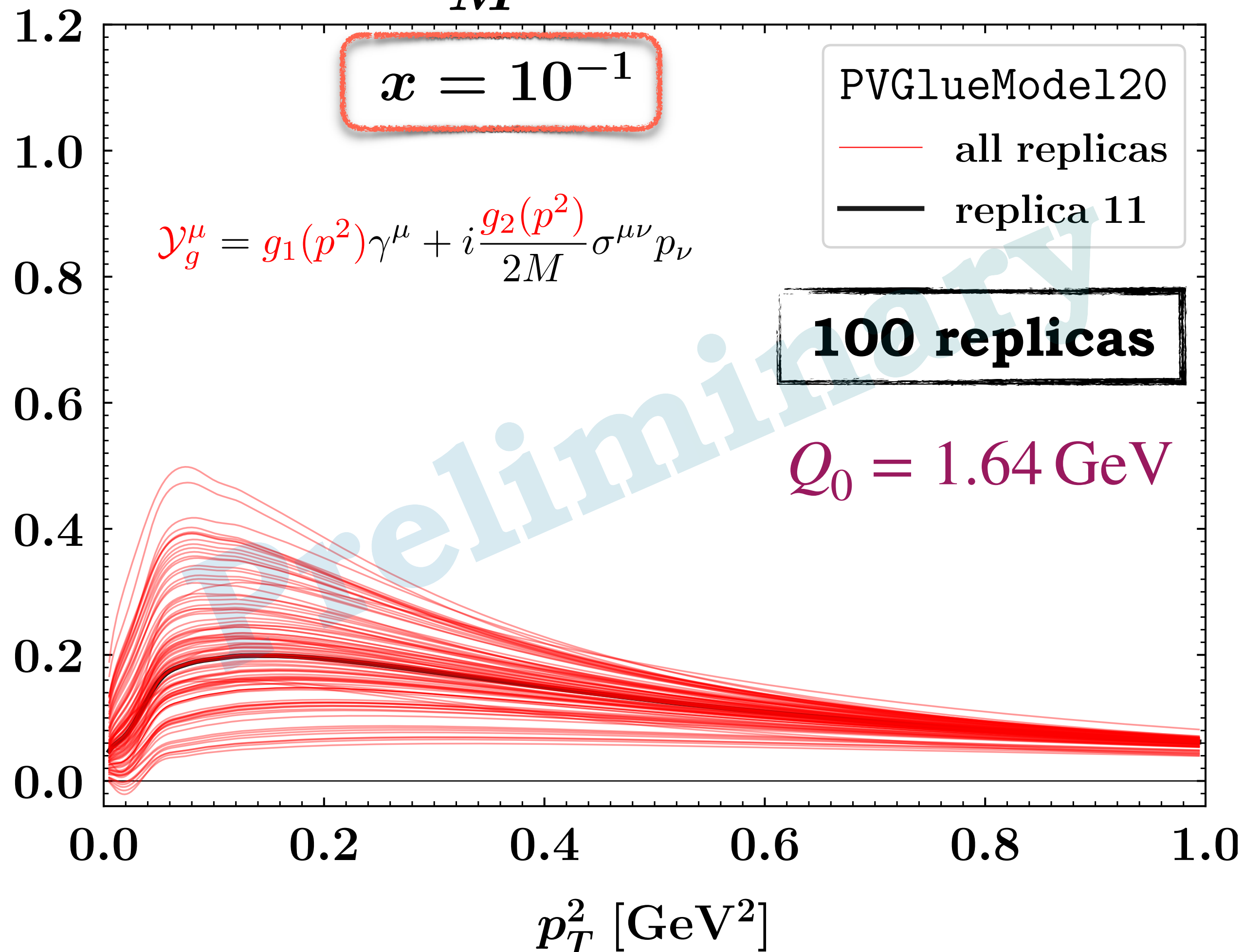
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



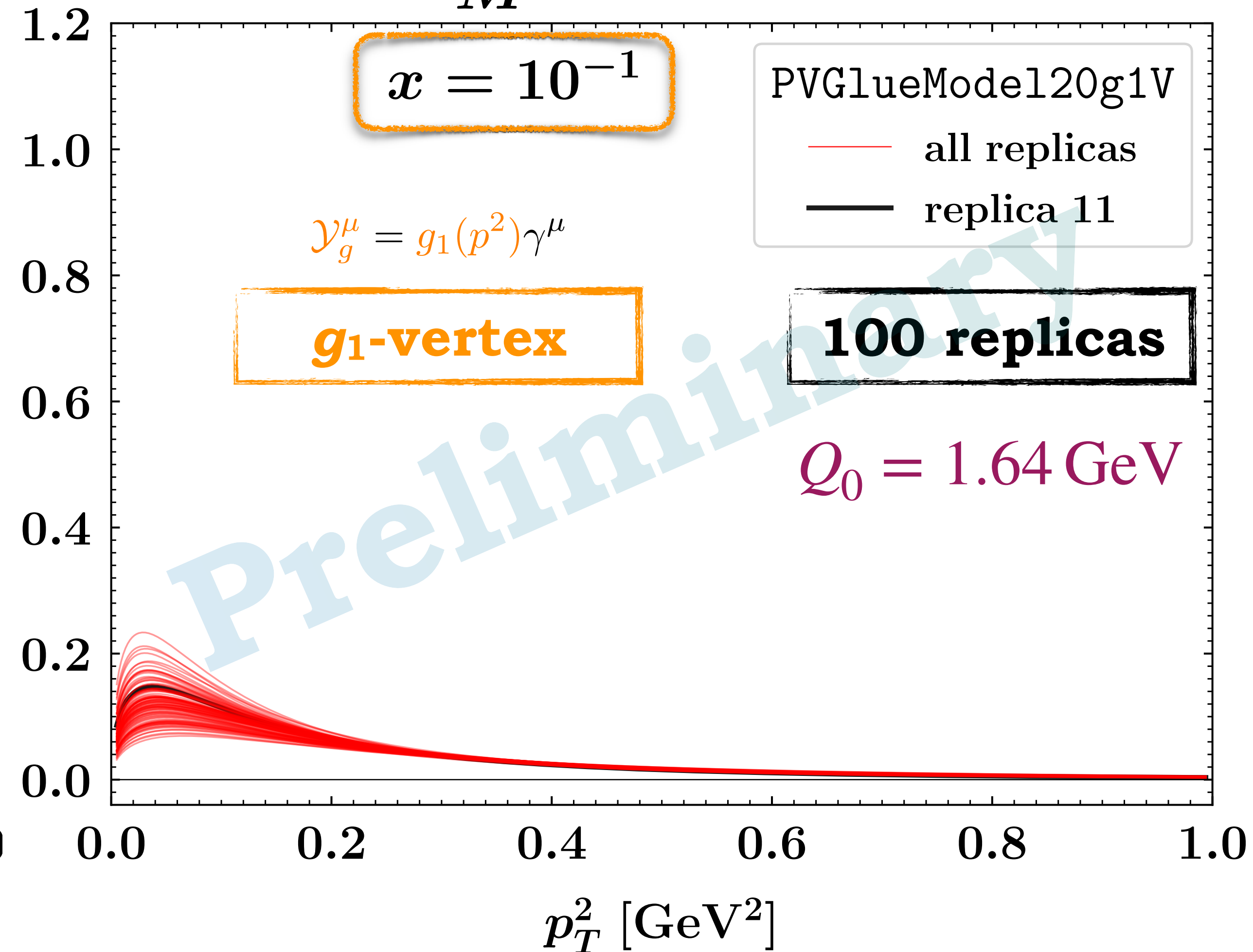




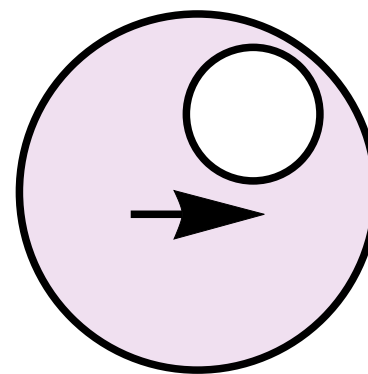
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



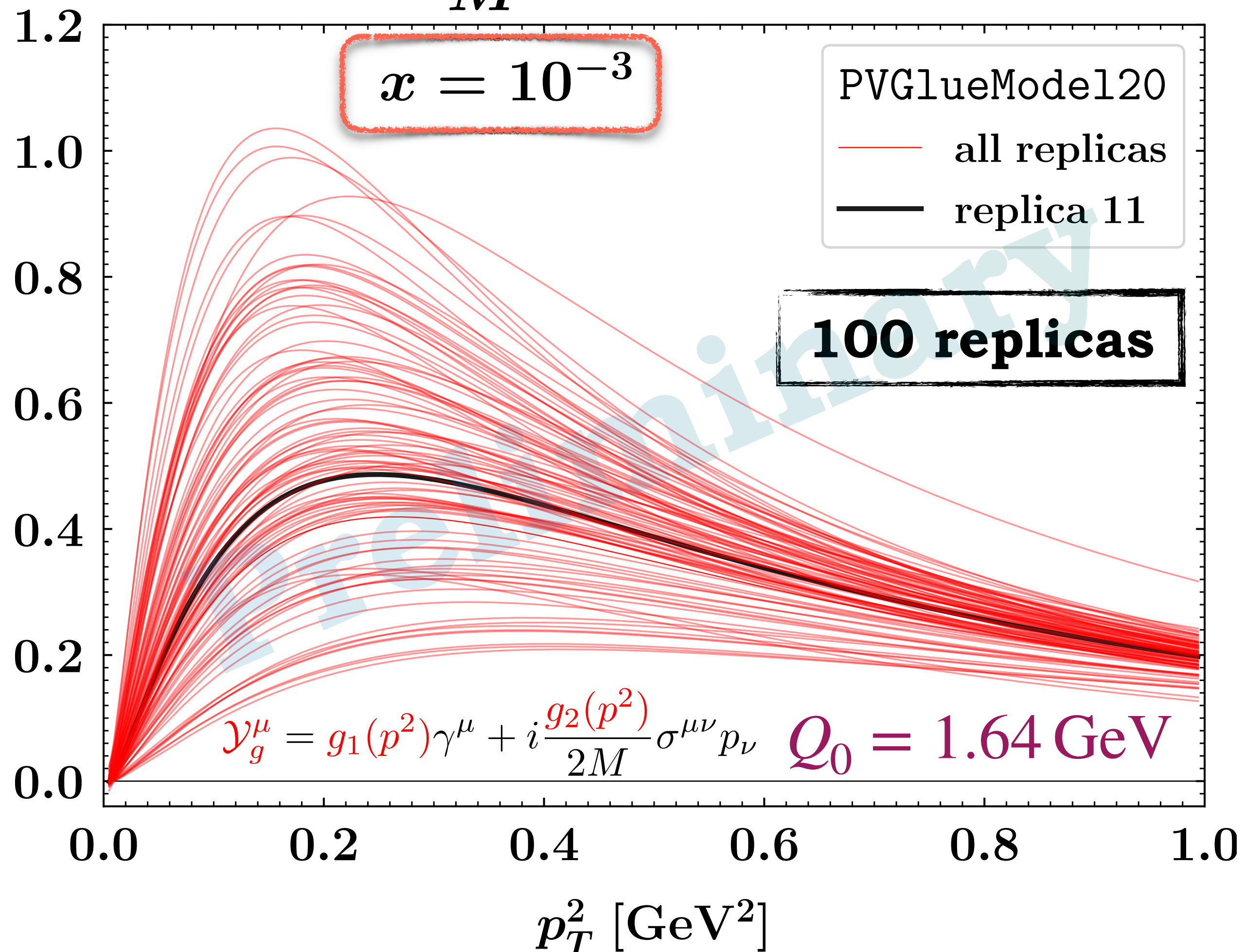
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



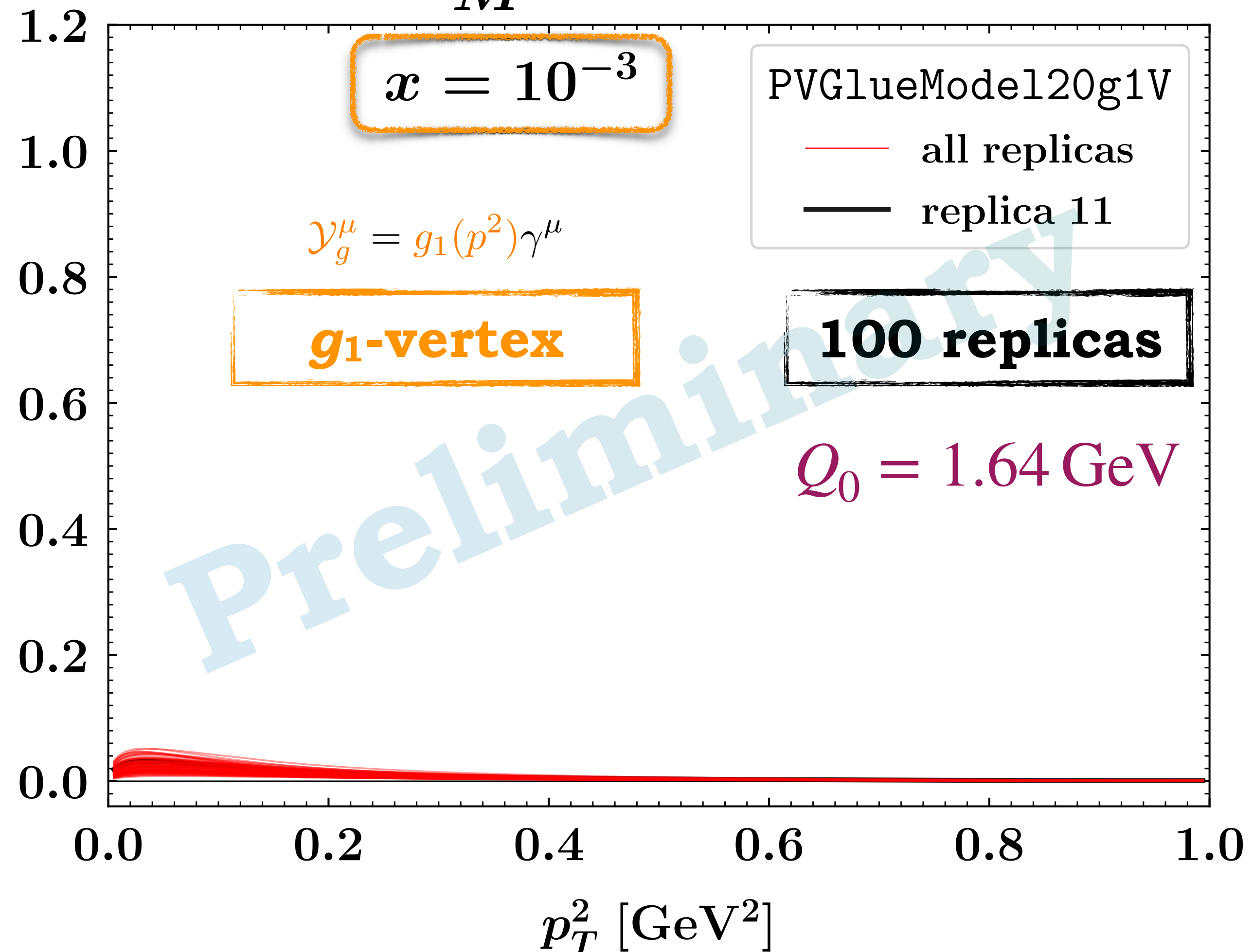




$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

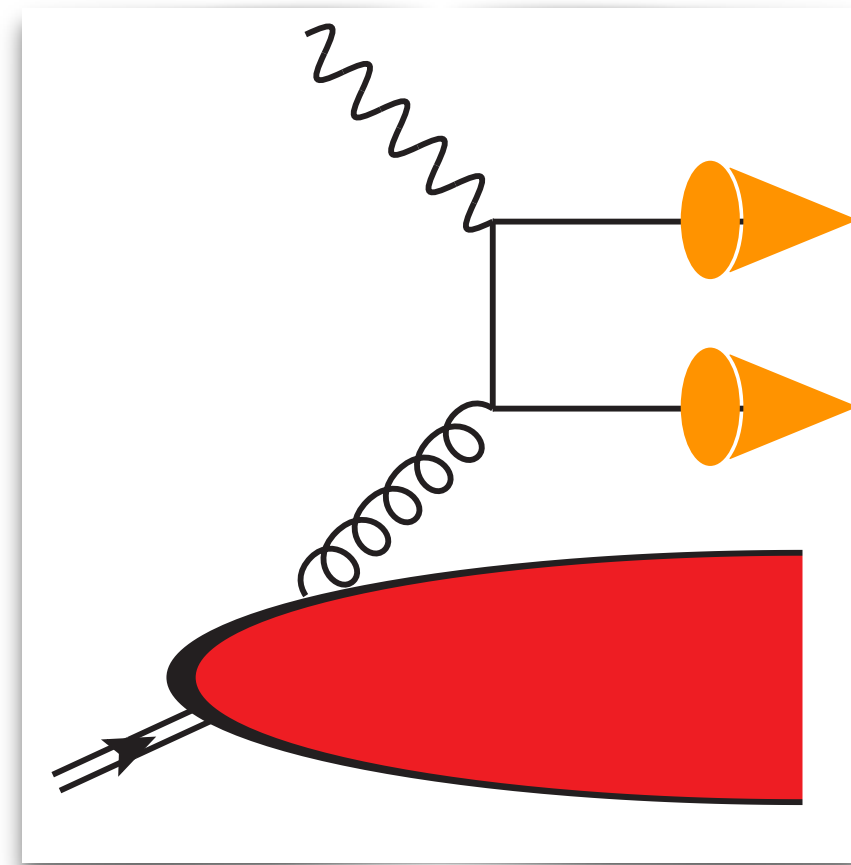




The background features a stylized illustration of a human brain in shades of light blue and green. The brain is shown from a slightly elevated perspective, with various regions highlighted. Overlaid on the brain are numerous small, colorful icons representing neural activity or cognitive processes. These icons include yellow coiled lines, red spheres with arrows, blue spheres with arrows, and green spheres with arrows, all set against a backdrop of soft, glowing light rays and starburst effects.

# 4. Phenomenology

## Two-jet SIDIS



jet function

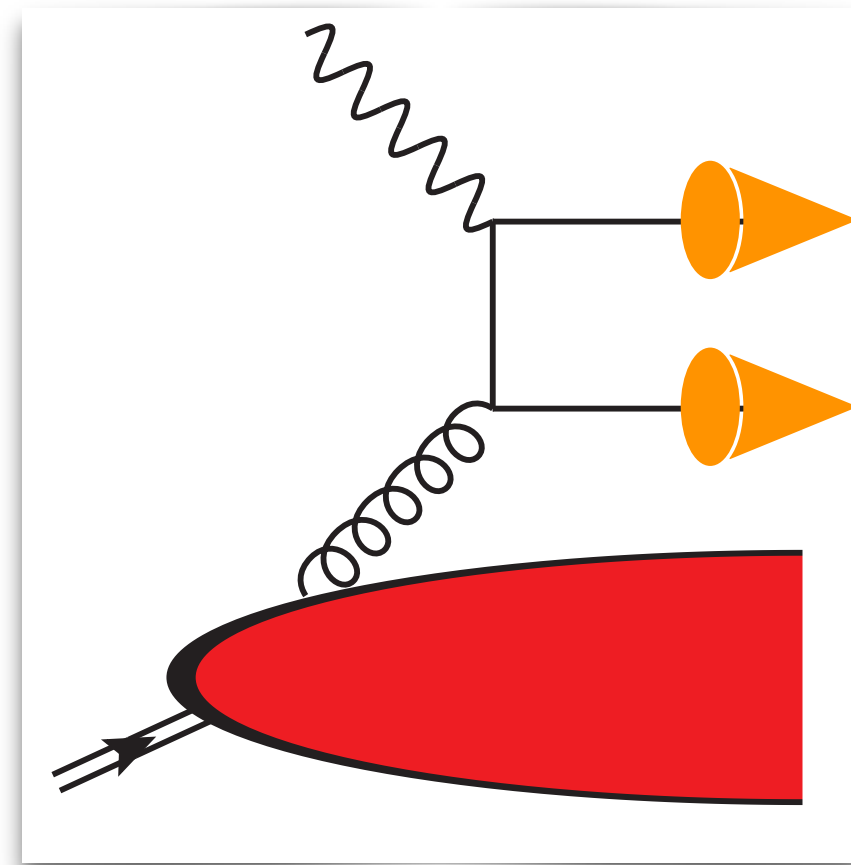
jet function

TMD PDF



# Golden channels for gluon TMD PDFs @EIC

## Two-jet SIDIS

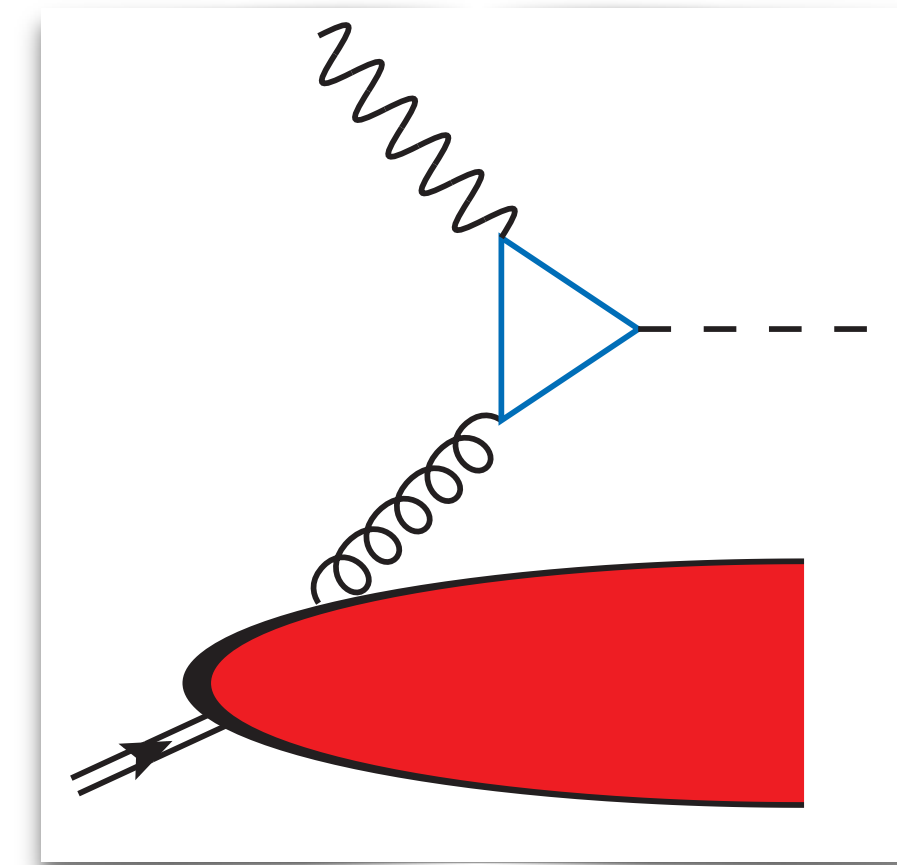


jet function

jet function

TMD PDF

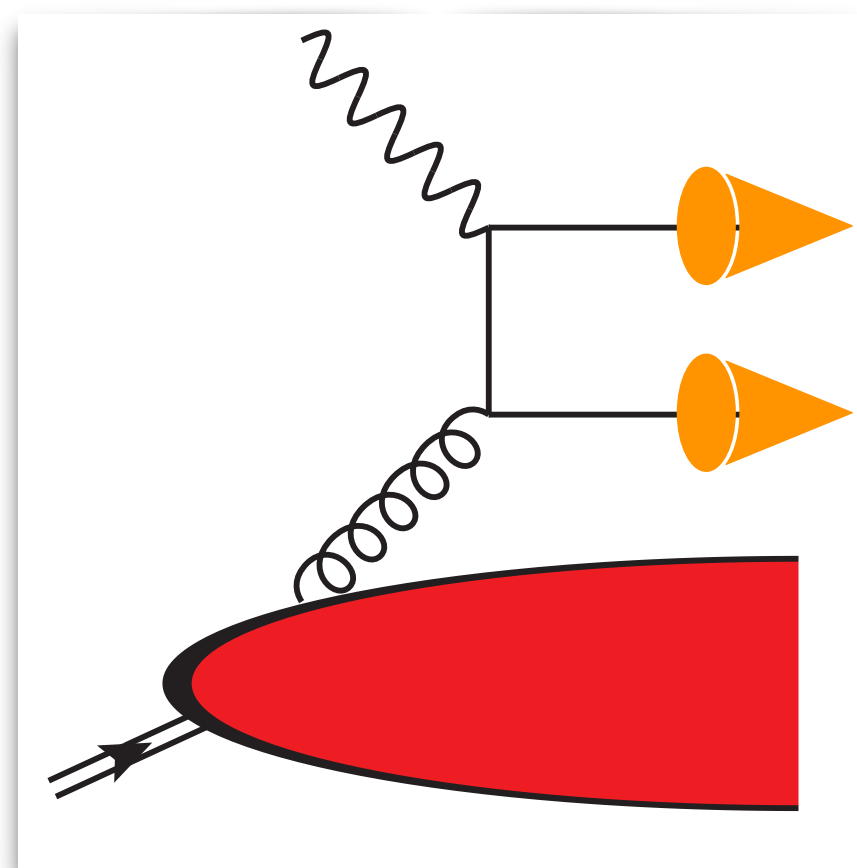
## Higgs in ep collisions



TMD PDF

# Golden channels for gluon TMD PDFs @EIC

## Two-jet SIDIS

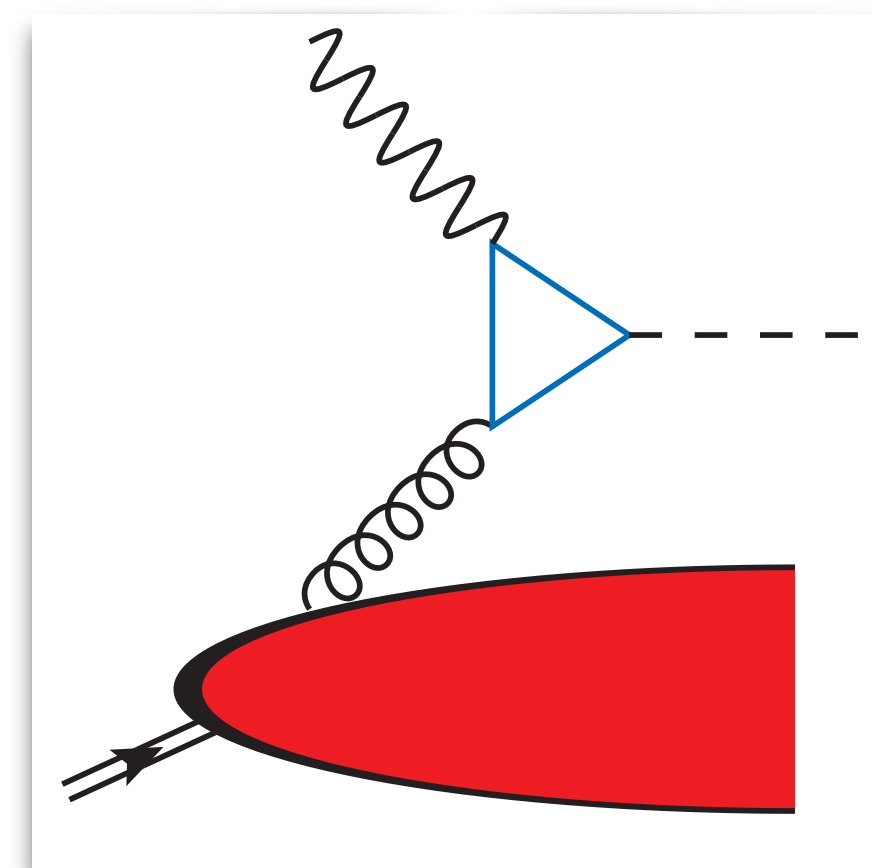


jet function

jet function

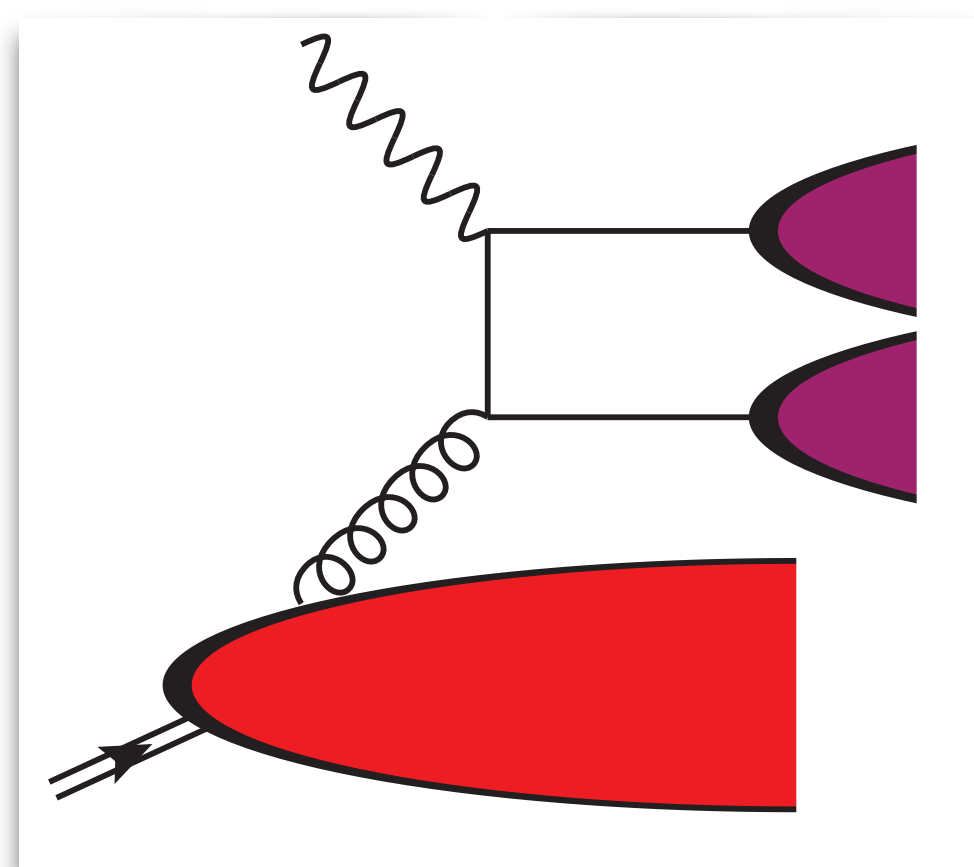
TMD PDF

## Higgs in ep collisions



TMD PDF

## Double D meson



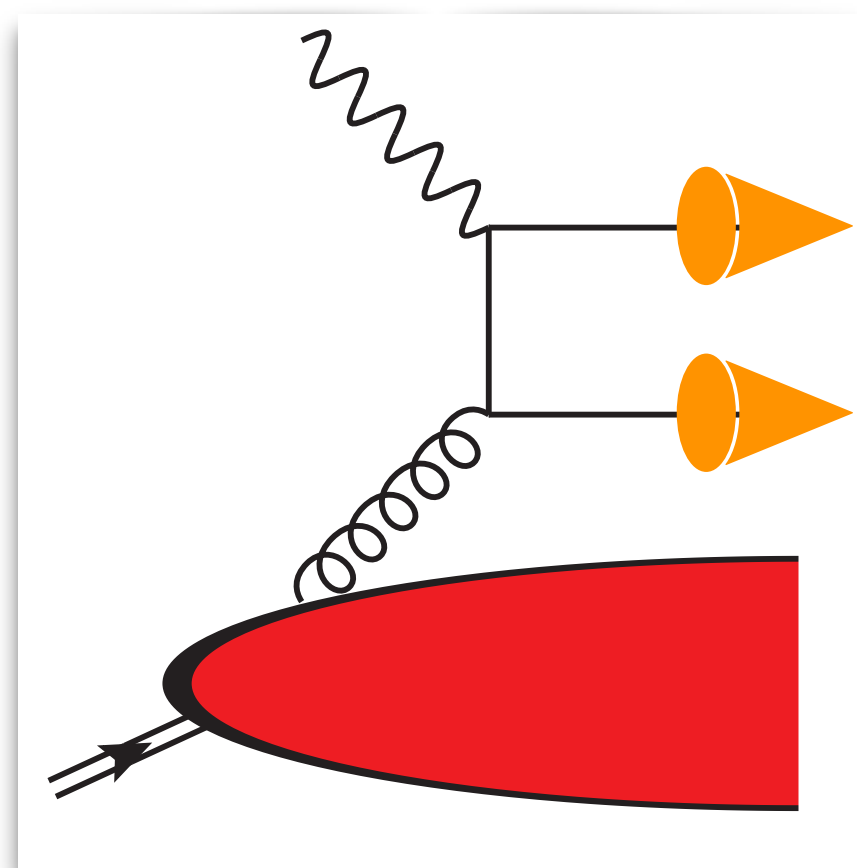
TMD FF

TMD FF

TMD PDF

# Golden channels for gluon TMD PDFs @EIC

## Two-jet SIDIS

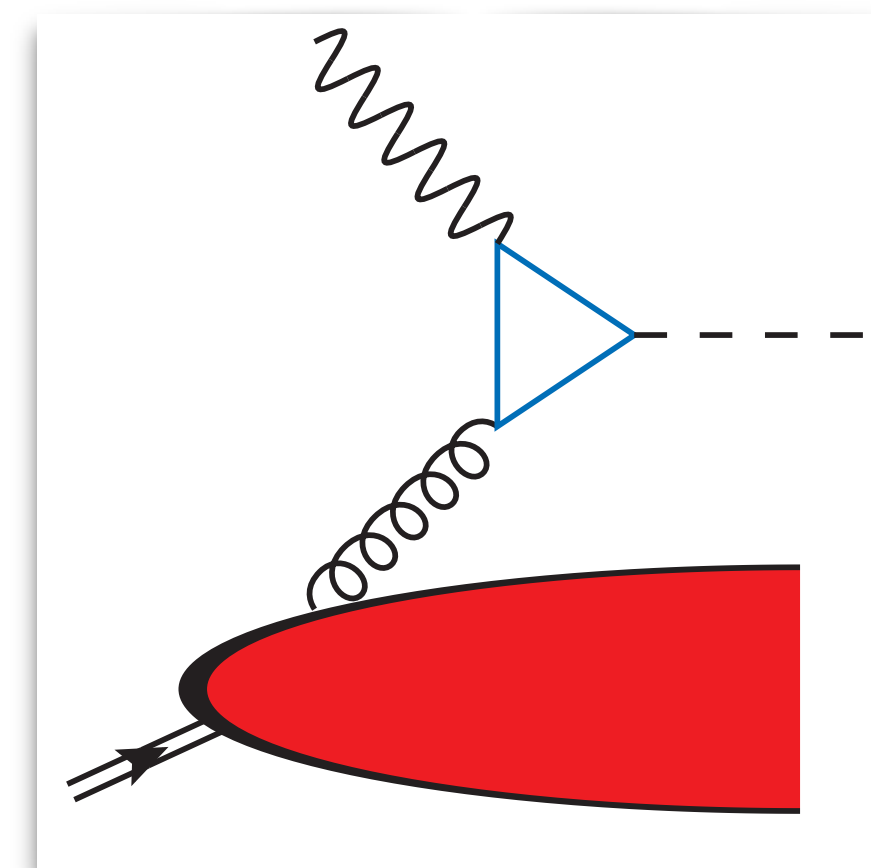


jet function

jet function

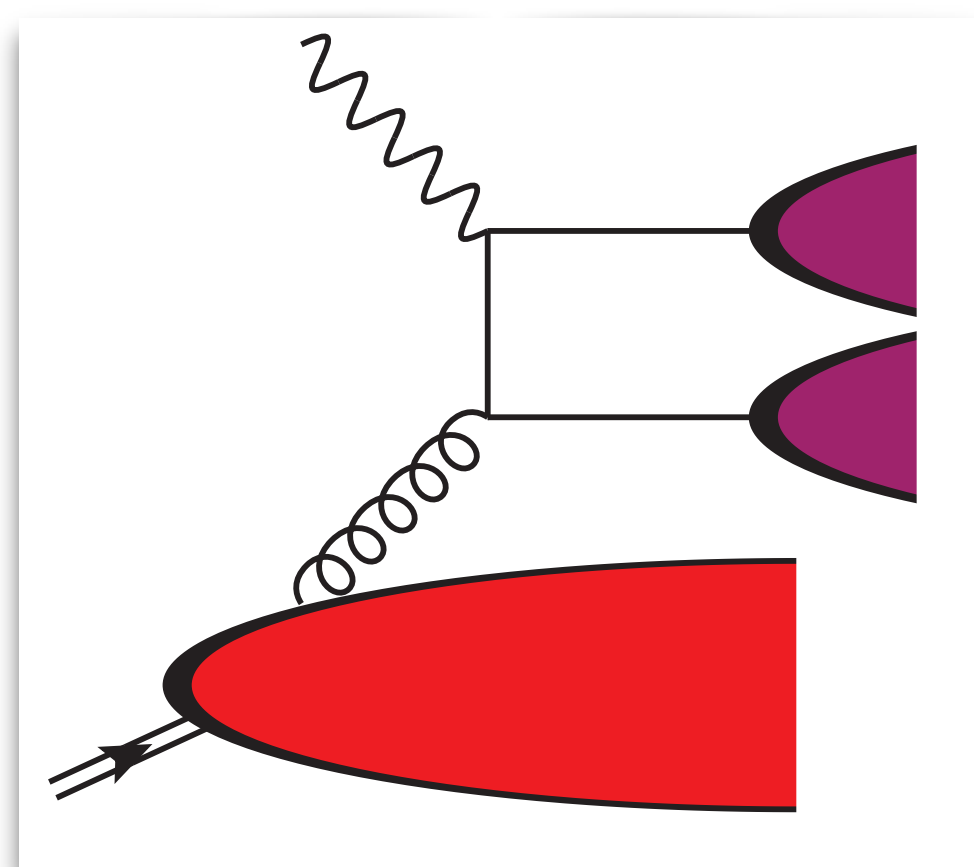
TMD PDF

## Higgs in ep collisions



TMD PDF

## Double D meson

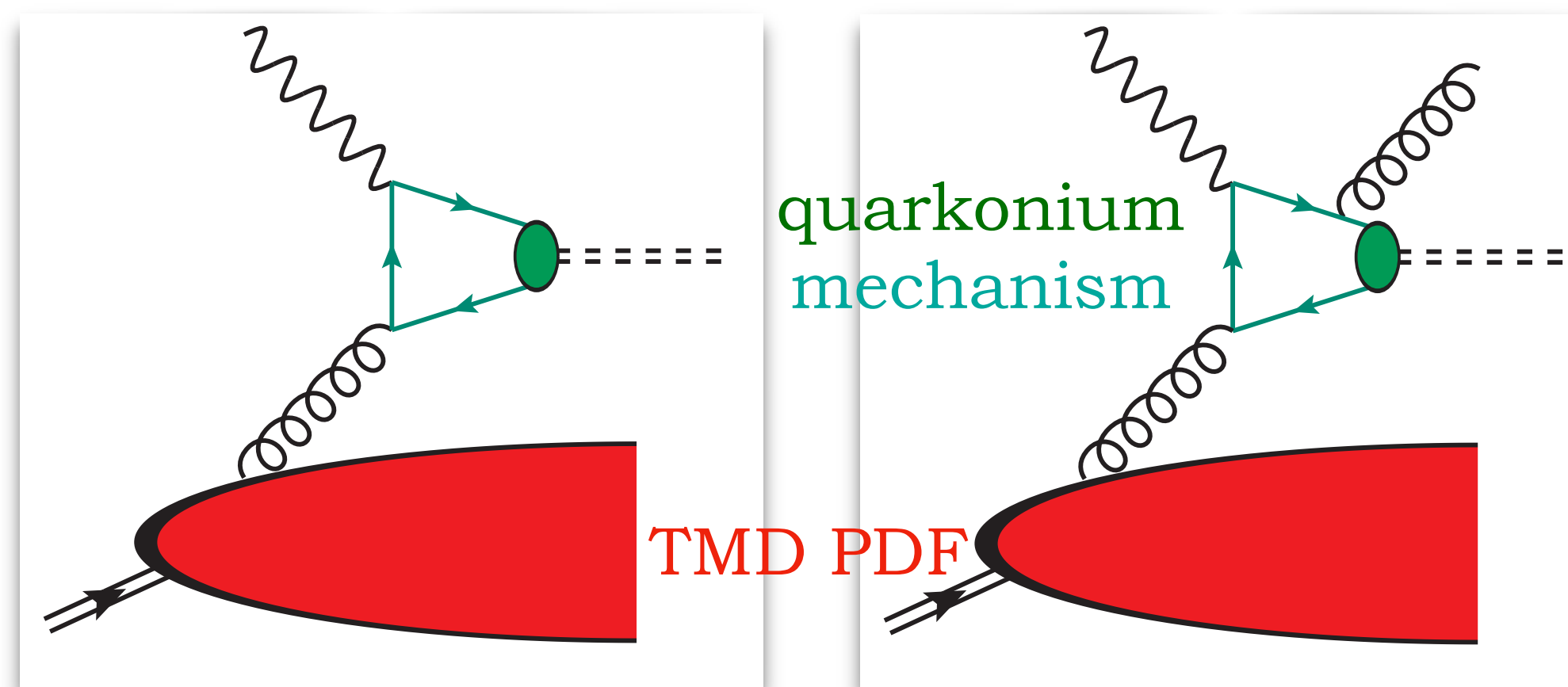


TMD FF

TMD FF

TMD PDF

## Quarkonia



TMD PDF



# Quarkonia: Assets & challenges

## Assets

 Onia  $\Rightarrow$  clean channels of f-type gluon TMDs

Initial-state color flow  $\Rightarrow$   $[-, -]$  gauge link

(overview)  [D. Boer (2017)]

Sivers	$ep^\dagger \rightarrow e' Q \bar{Q} X$ $ep^\dagger \rightarrow e' j_1 j_2 X$
$f_{1T}^{\perp g[-,-]}$	✓
$f_{1T}^{\perp g[+,-]}$	×

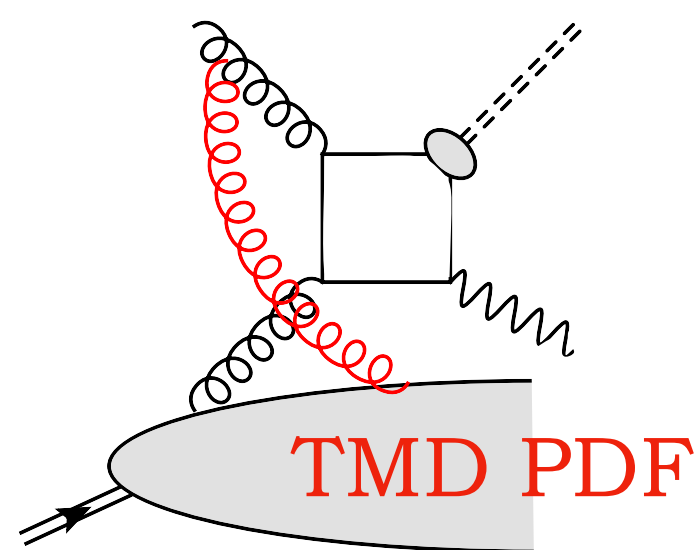
Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
$h_1^{\perp g[-,-]}(\text{WW})$	✓
$h_1^{\perp g[+,-]}(\text{DP})$	×

## Challenges

# Quarkonia: Assets & challenges

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Onia  $\Rightarrow$  clean channels of f-type gluon TMDs



Initial-state color flow  $\Rightarrow$   $[-, -]$  gauge link

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$f_{1T}^{\perp g[-,-]}$	✓
$f_{1T}^{\perp g[+,-]}$	×

(overview) [\[D. Boer \(2017\)\]](#)

Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
$h_1^{\perp g[-,-]}(WW)$	✓
$h_1^{\perp g[+,-]}(DP)$	×

$\eta_{c,b}$   $\Rightarrow$  LHC complementarity, TMD factorization

$$\frac{d\sigma}{dq_T} \sim \text{at low transverse momentum for [pseudo]scalar state}$$

$$\sim \mathcal{C} \left[ \begin{array}{cc} f_1^{g/A} & f_1^{g/B} \end{array} \right] \pm \mathcal{C} \left[ \begin{array}{cc} h_1^{\perp g/A} & h_1^{\perp g/B} \end{array} \right]$$

unpolarized gluons
lin. polarized gluons

(factorization) [\[M. García Echevarría \(2019\)\]](#)

(pheno) [\[A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. \(to appear\)\]](#)

## Challenges

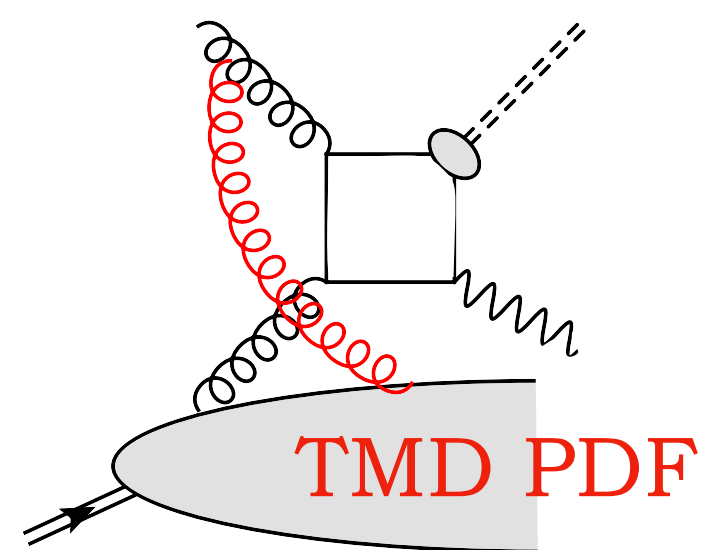
Precision TMD  $\Leftrightarrow$  production mechanism(s)

(production mechanisms, LHC pheno) [\[J.-P. Lansberg \(2020\)\]](#)

# Quarkonia: Assets & challenges

## Assets

Onia  $\Rightarrow$  clean channels of f-type gluon TMDs



Initial-state color flow  $\Rightarrow$   $[-, -]$  gauge link

Sivers	$ep^\uparrow \rightarrow e' Q \bar{Q} X$ $ep^\uparrow \rightarrow e' j_1 j_2 X$
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(overview) [\[D. Boer \(2017\)\]](#)

Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
$h_1^{\perp g[-,-]}(WW)$	✓
$h_1^{\perp g[+,-]}(DP)$	×

$\eta_{c,b}$   $\Rightarrow$  LHC complementarity, TMD factorization

$$\frac{d\sigma}{dq_T} \sim \text{at low transverse momentum for (pseudo)scalar state}$$

$$\sim \mathcal{C} \left[ \begin{matrix} f_1^{g/A} & f_1^{g/B} \\ \text{unpolarized gluons} \end{matrix} \right] \pm \mathcal{C} \left[ \begin{matrix} h_1^{\perp g/A} & h_1^{\perp g/B} \\ \text{lin. polarized gluons} \end{matrix} \right]$$

(factorization) [\[M. García Echevarría \(2019\)\]](#)

(pheno) [\[A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. \(to appear\)\]](#)

## Challenges

Precision TMD  $\Leftrightarrow$  production mechanism(s)

(production mechanisms, LHC pheno) [\[J.-P. Lansberg \(2020\)\]](#)

- Color Evaporation Model**  
 $(Q\bar{Q})$  decorrelated from onium, semi-soft gluon emissions  
 Overshoots data at large  $p_T$
- Color Singlet Model**  
 $(Q\bar{Q})$  to onium, no gluon emissions  
 Fails at large  $p_T$ , improves at NLO
- NRQCD and Color Octet**  
 Higher Fock states, soft gluon emissions  
 Problems at low  $p_T$ , fails on polarization



## TMD PDFs & shape functions

 NRQCD  $\Rightarrow$  double expansion:  $\alpha_s \oplus v$


 NRQCD  $\Rightarrow$   $d\sigma(|Q\rangle) \propto \mathcal{H} \otimes \text{LDME}$

$$|Q\rangle = \mathcal{O}(1) |Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v) |Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^1S_0^{(8)}g]\rangle \\ + \mathcal{O}(v^2) |Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \dots$$

S-wave quarkonium wave function

## TMD PDFs & shape functions

 NRQCD  $\Rightarrow$  double expansion:  $\alpha_s \oplus v$

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S-wave quarkonium wave function

 TMD  $\Rightarrow$  from LDMEs to shape functions (ShFs)

 2 mechanisms: bound state + soft-gluon

(factorization)  [M. Garcia Echevarria (2019)]

(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

(unpol.  $J/\psi$ )  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Taelis (2020)]


(pol.  $J/\psi$ )  [D. Boer, U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, R. Sangem (2022)]

(unpol.  $J/\psi$ )  [D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

# Quarkonia: A path toward precision

## TMD PDFs & shape functions

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S-wave quarkonium wave function

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(unpol.  $J/\psi$ )  [D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

## Revised TMD shape function in SIDIS


$$\Delta^{[n]}(\kappa_T^2) \propto \frac{\alpha_s}{2\pi^2 \kappa_T^2} C_A \left( 1 + \ln \frac{M_Q^2}{M_Q^2 + Q^2} \right) \langle O[n] \rangle$$



# Quarkonia: A path toward precision

## TMD PDFs & shape functions

 NRQCD  $\Rightarrow$  double expansion:  $\alpha_s \oplus v$

 NRQCD  $\Rightarrow$   $d\sigma(|Q\rangle) \propto \mathcal{H} \otimes \text{LDME}$

$$|Q\rangle = \mathcal{O}(1) |Q\bar{Q} [^3S_1^{(1)}]\rangle + \mathcal{O}(v) |Q\bar{Q} [^3P_J^{(8)} g]\rangle + \mathcal{O}(v^2) |Q\bar{Q} [^1S_0^{(8)} g]\rangle + \mathcal{O}(v^2) |Q\bar{Q} [^3S_1^{(1,8)} gg]\rangle + \mathcal{O}(v^2) |Q\bar{Q} [^3D_J^{(1,8)} gg]\rangle + \dots$$

S-wave quarkonium wave function

 TMD  $\Rightarrow$  from LDMEs to shape functions (ShFs)

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(unpol.  $J/\psi$ )  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Taelis (2020)]

(pol.  $J/\psi$ )  [D. Boer, U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, R. Sangem (2022)]

(unpol.  $J/\psi$ )  [D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

## Revised TMD shape function in SIDIS

$$\Delta^{[n]}(\kappa_T^2) \propto \frac{\alpha_s}{2\pi^2 \kappa_T^2} C_A \left( 1 + \ln \frac{M_Q^2}{M_Q^2 + Q^2} \right) \langle O[n] \rangle$$

 2 mechanisms: bound state + soft-gluon

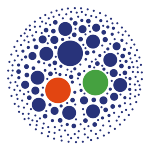
 Perturbative tail  $\otimes$  LDME

 ShFs and TMD FFs exhibit different divergences

# Quarkonia & Gluon TMDs: a path toward precision

## TMD PDFs & shape functions

 **NRQCD**  $\Rightarrow$  double expansion:  $\alpha_s \oplus v$

 **NRQCD**  $\Rightarrow$   $d\sigma(|Q\rangle) \propto \mathcal{H} \otimes \text{LDME}$

$$|Q\rangle = \mathcal{O}(1) |Q\bar{Q} [^3S_1^{(1)}]\rangle + \mathcal{O}(v) |Q\bar{Q} [^3P_J^{(8)} g]\rangle + \mathcal{O}(v^2) |Q\bar{Q} [^1S_0^{(8)} g]\rangle + \mathcal{O}(v^2) |Q\bar{Q} [^3S_1^{(1,8)} gg]\rangle + \mathcal{O}(v^2) |Q\bar{Q} [^3D_J^{(1,8)} gg]\rangle + \dots$$

S-wave quarkonium wave function

 **TMD**  $\Rightarrow$  from LDMEs to shape functions (ShFs)

 2 mechanisms: bound state + soft-gluon

(factorization)  [M. Garcia Echevarria (2019)]

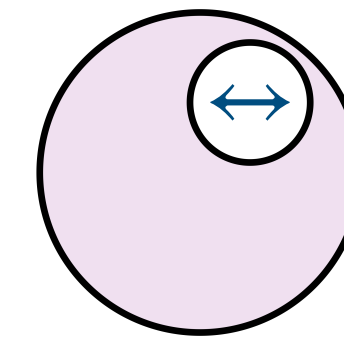
(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

(unpol.  $J/\psi$ )  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Tael (2020)]

(pol.  $J/\psi$ )  [D. Boer, U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, R. Sangem (2022)]

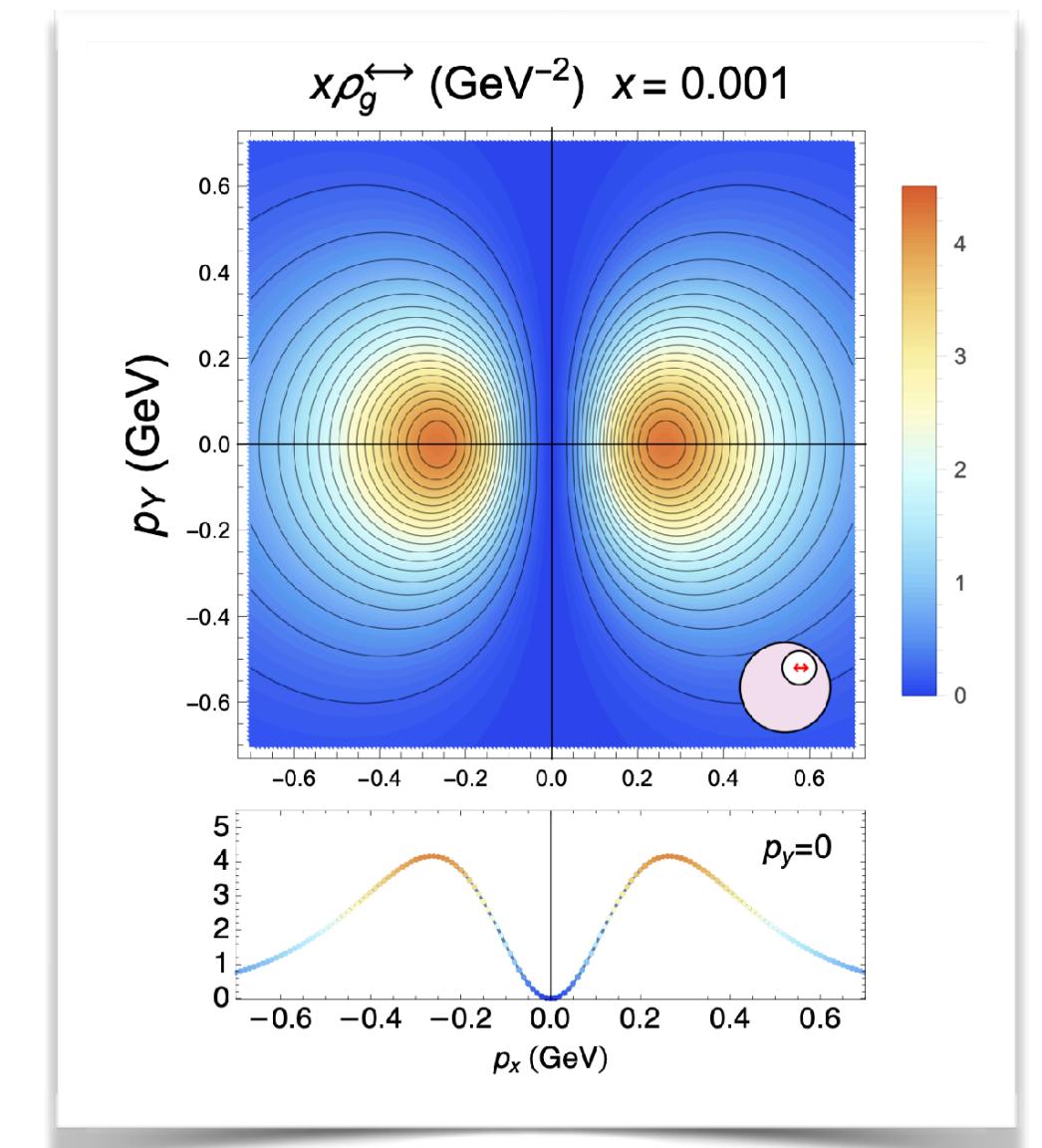
(unpol.  $J/\psi$ )  [D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

## 3D proton imaging: LHC & EIC



EIC, LHCb, FT@LHC

Boer-Mulders



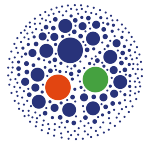
 [A. Bacchetta, F.G. C., M. Radici, P. Tael (2020)]



# Quarkonia & Gluon TMDs: a path toward precision

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S-wave quarkonium wave function

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(factorization)  [M. Garcia Echevarria (2019)]

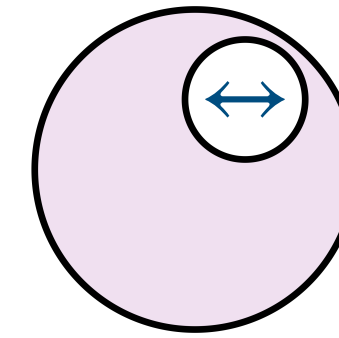
(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

(unpol.  $J/\psi$ )  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Tael (2020)]

(pol.  $J/\psi$ )  [D. Boer, U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, R. Sangem (2022)]

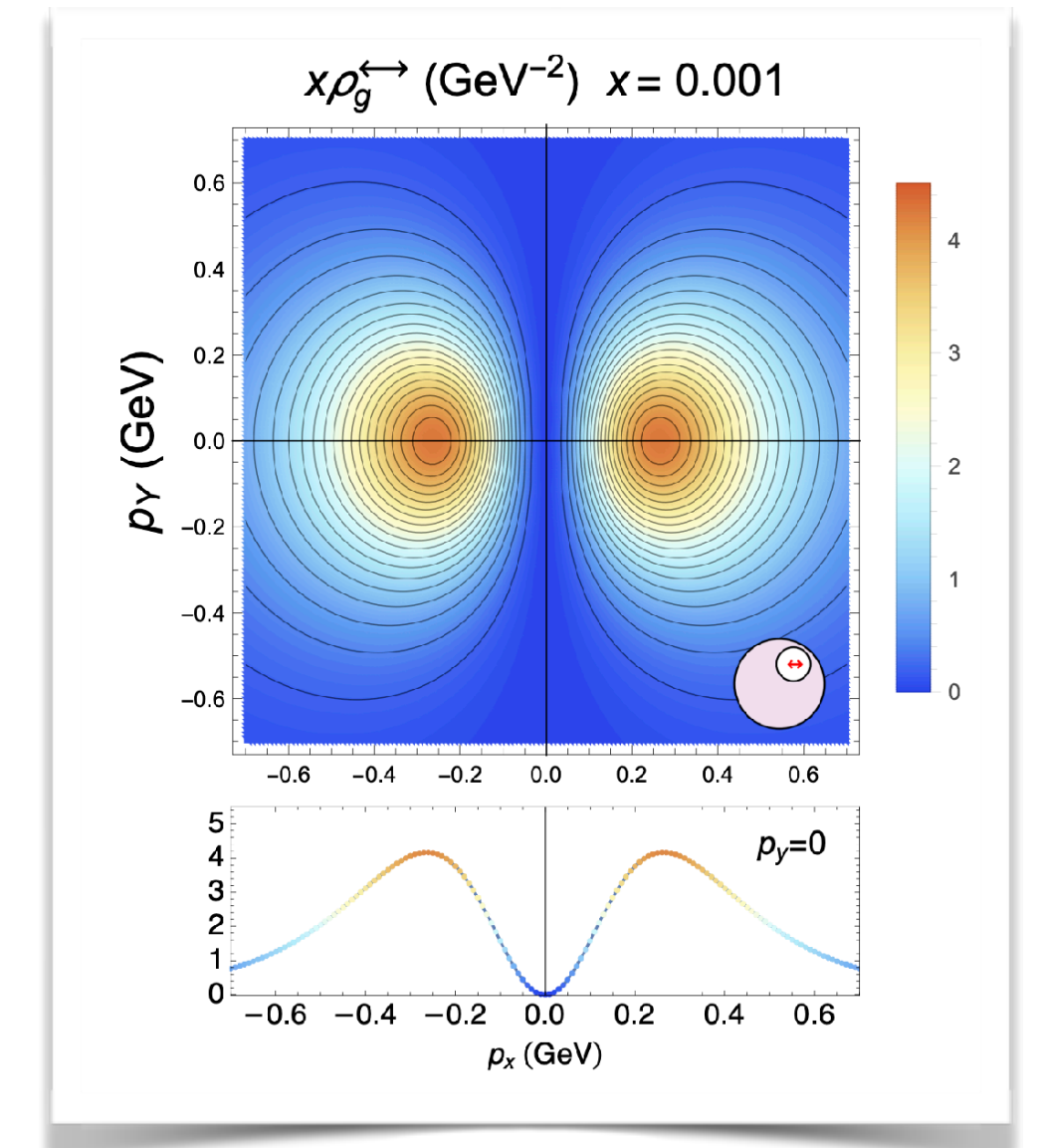
(unpol.  $J/\psi$ )  [D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan (2023)]

## 3D proton imaging: LHC & EIC



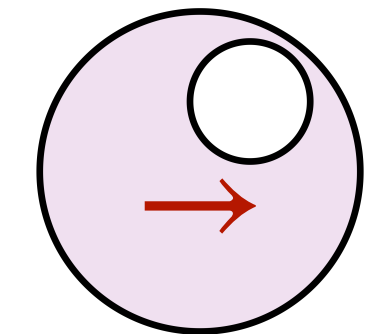
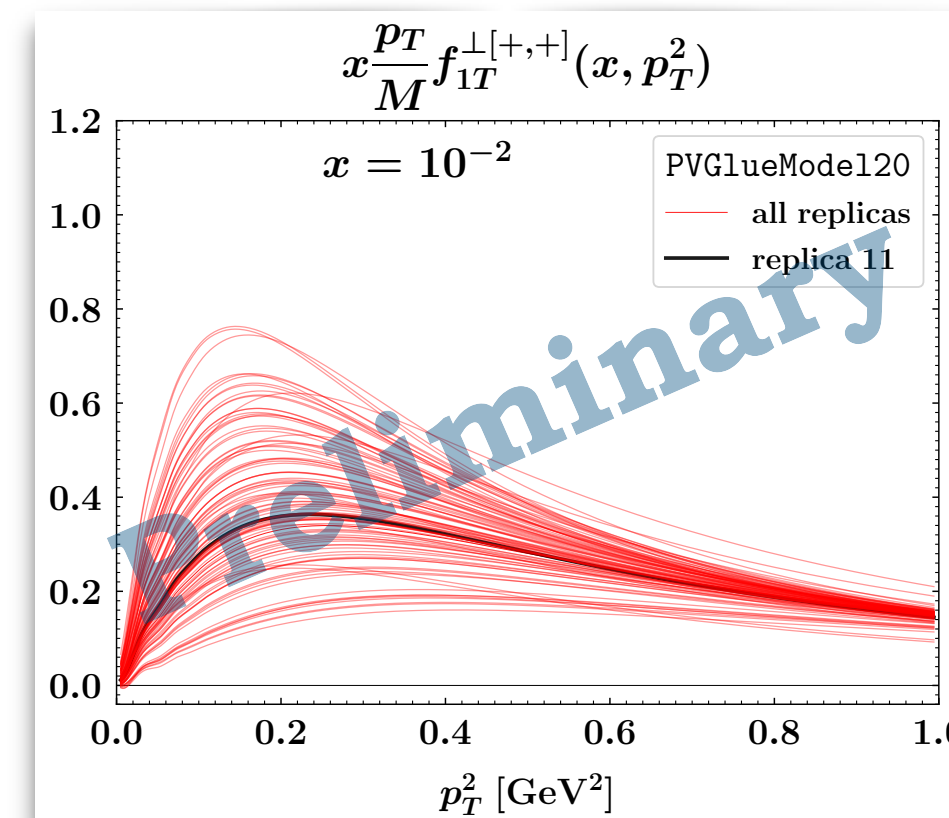
EIC, LHCb, FT@LHC

Boer-Mulders



 [A. Bacchetta, F.G. C., M. Radici, P. Tael (2020)]

[A. Bacchetta, F.G. C., M. Radici (to appear)]

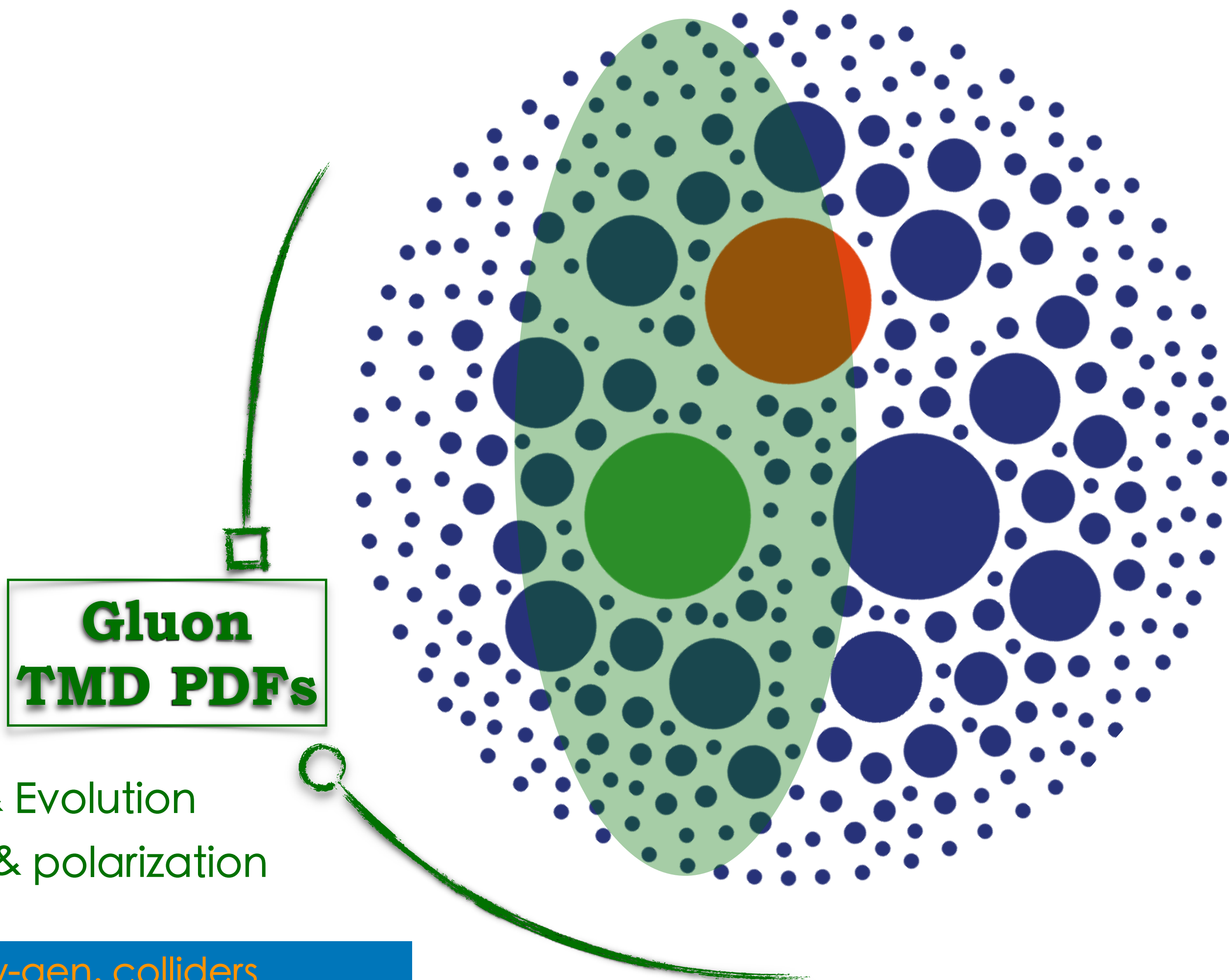


EIC, LHCspin

Sivers



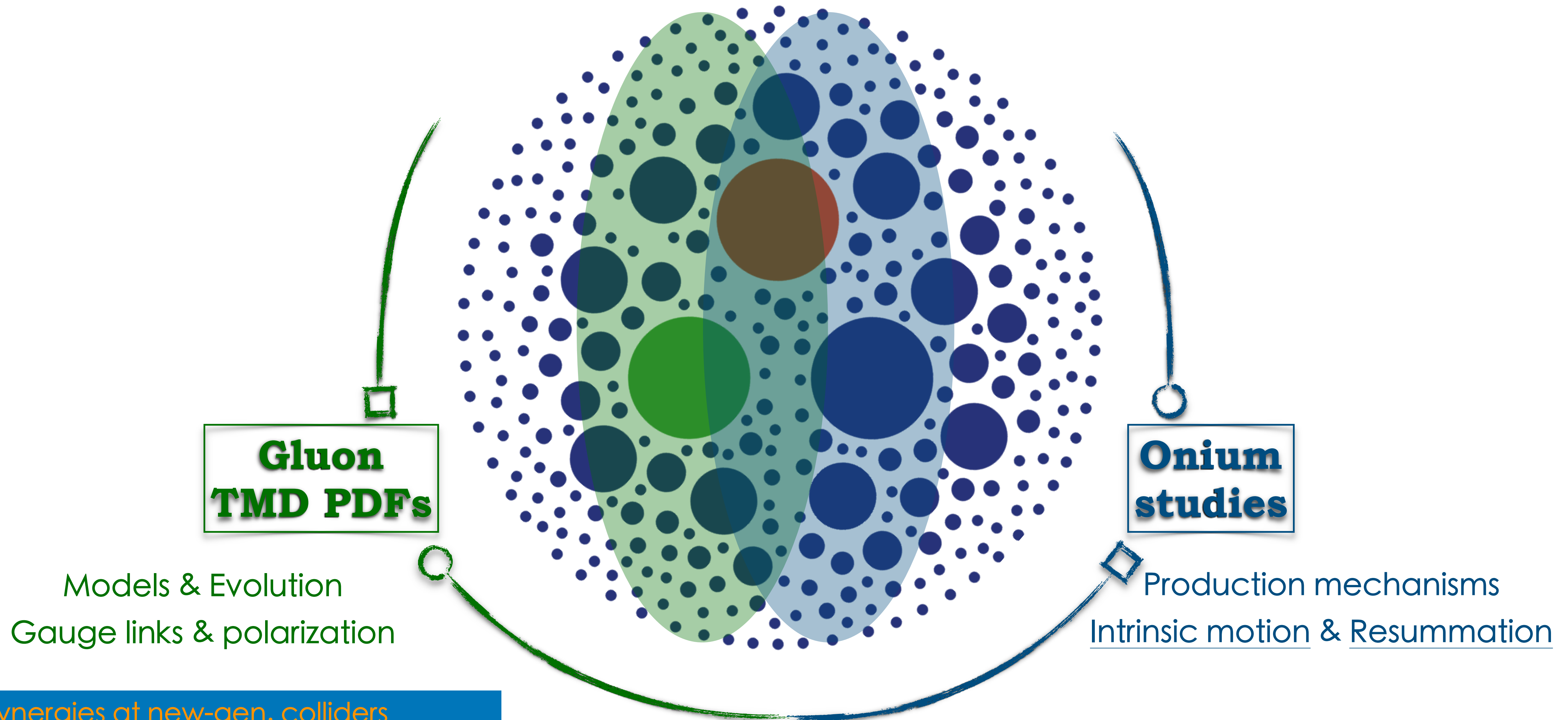
# Gluon TMD PDFs @EIC: A win-win strategy



**Gluon  
TMD PDFs**

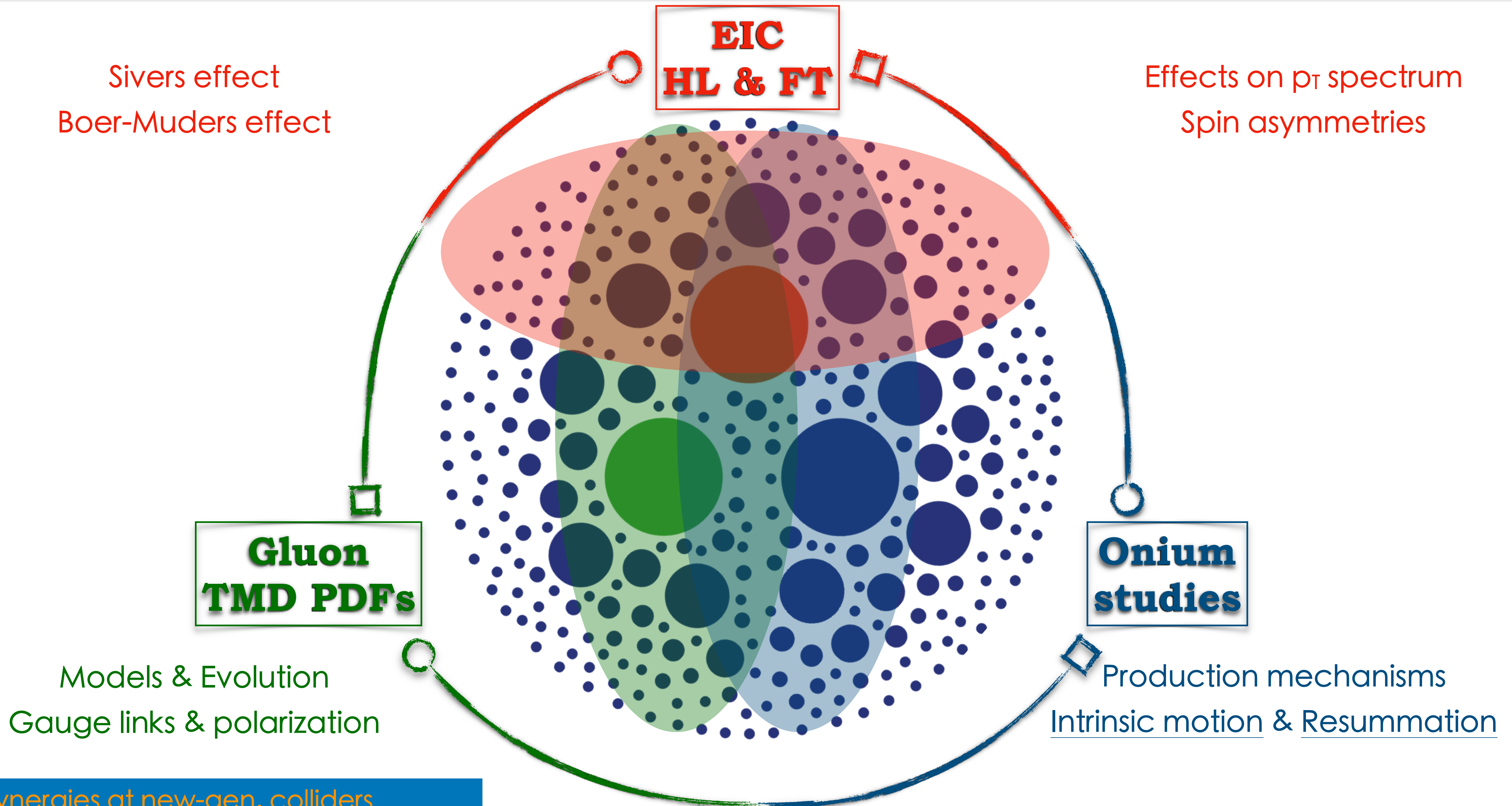
Models & Evolution  
Gauge links & polarization

# Gluon TMD PDFs @EIC: A win-win strategy





# Gluon TMD PDFs @EIC: A win-win strategy







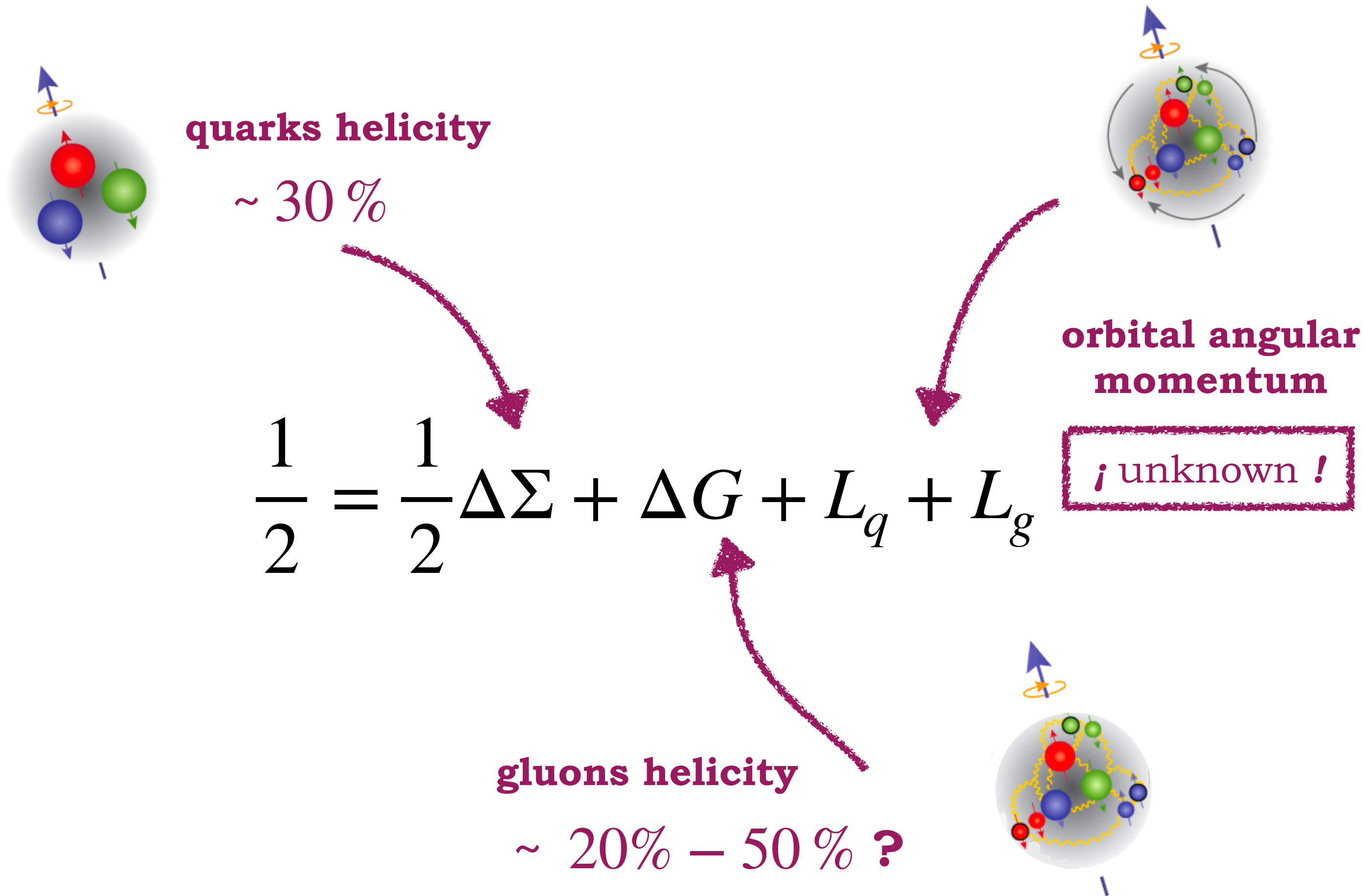
# Extras





# TMD factorization

# The proton spin crisis



**Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...**

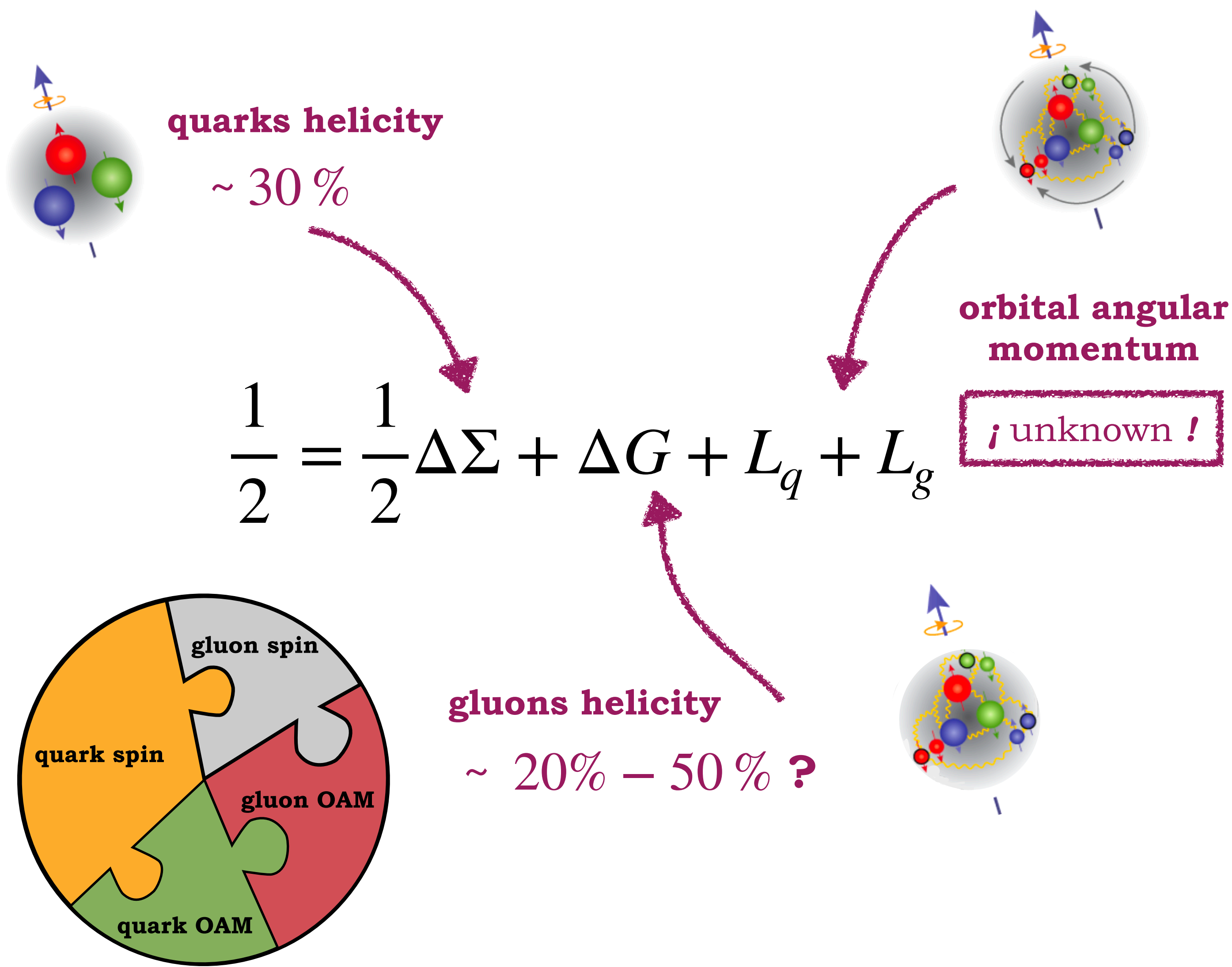
(proton spin crisis) [EMC Collaboration, CERN (1987)]

slide adapted from C. Bissolotti

**Backup**



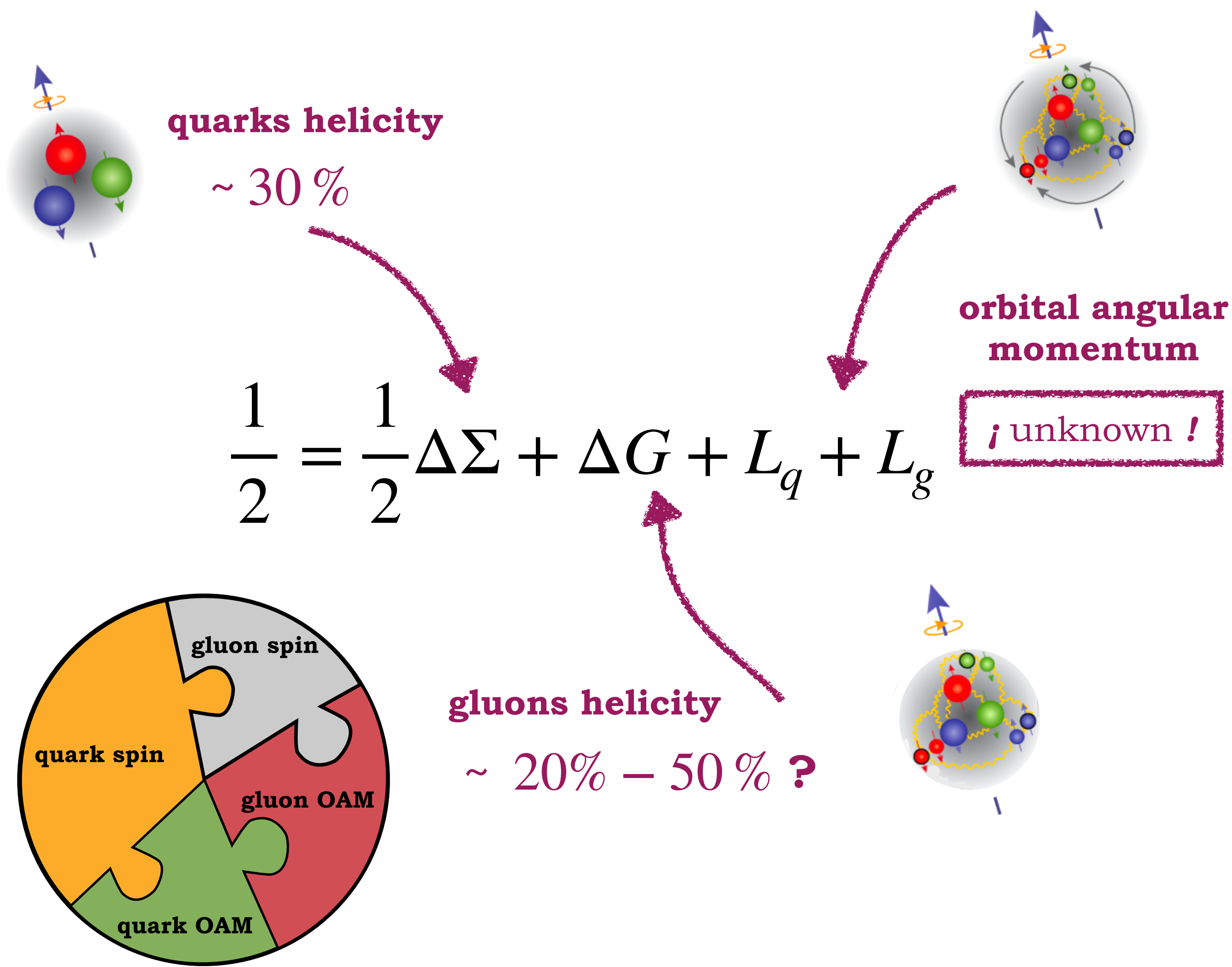
# The proton spin crisis



**Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...**

(proton spin crisis) [EMC Collaboration, CERN (1987)]

# The proton spin crisis



...many other effects in hadronic interactions cannot be understood in the purely collinear approach

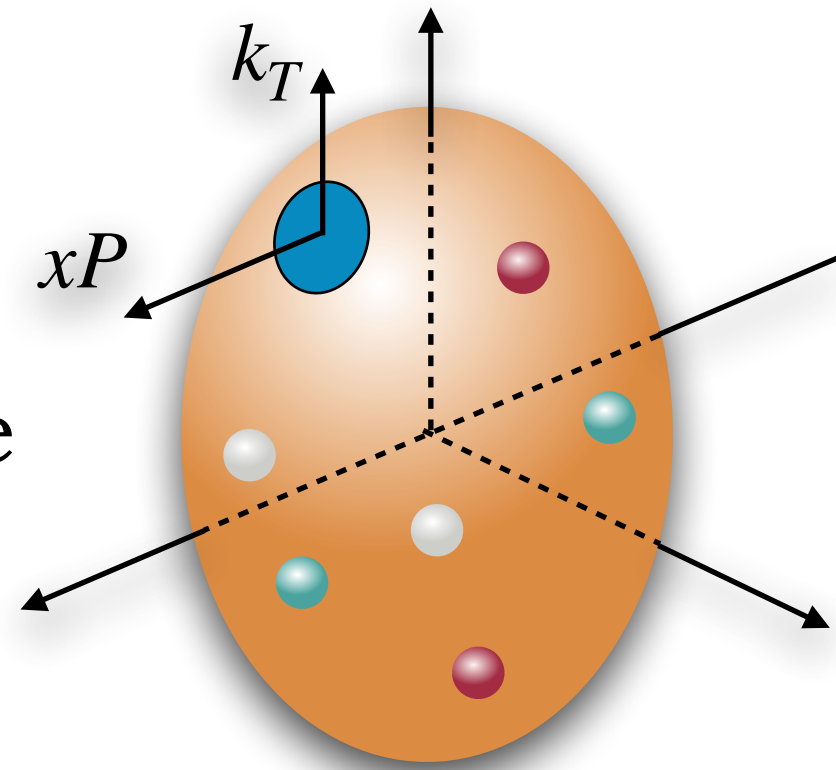
Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

(proton spin crisis) [EMC Collaboration, CERN (1987)]

# Parton densities: an incomplete family tree

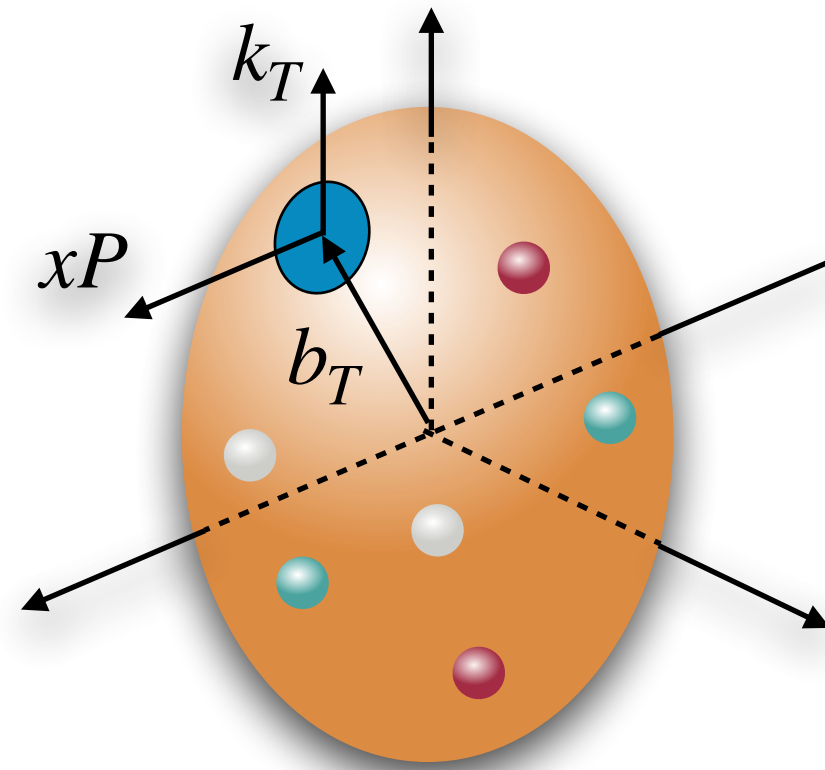
**3D**

TMDs  
(semi-)inclusive



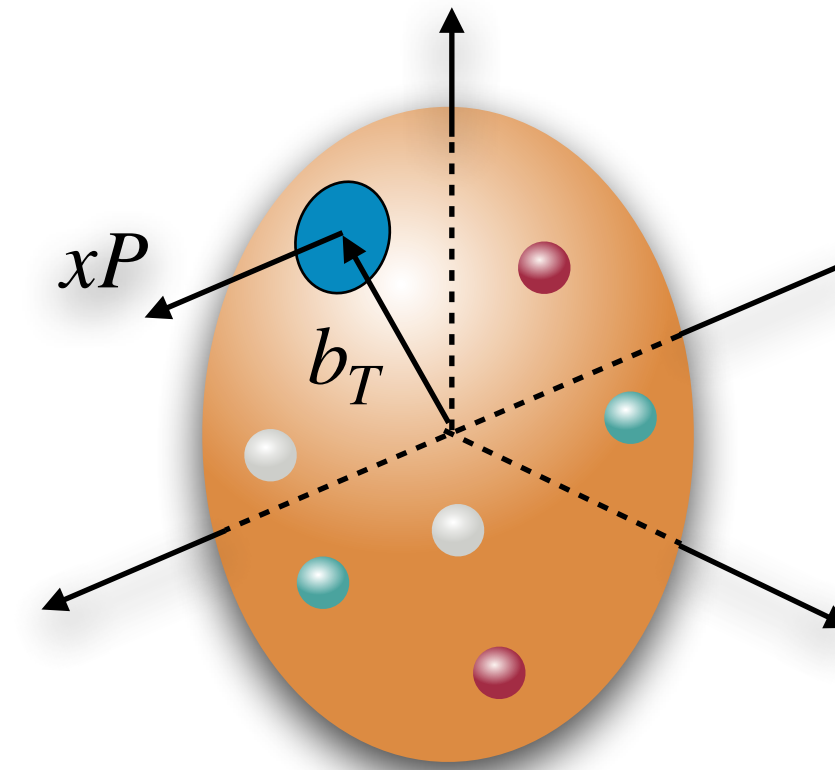
**5D**

Wigner distributions



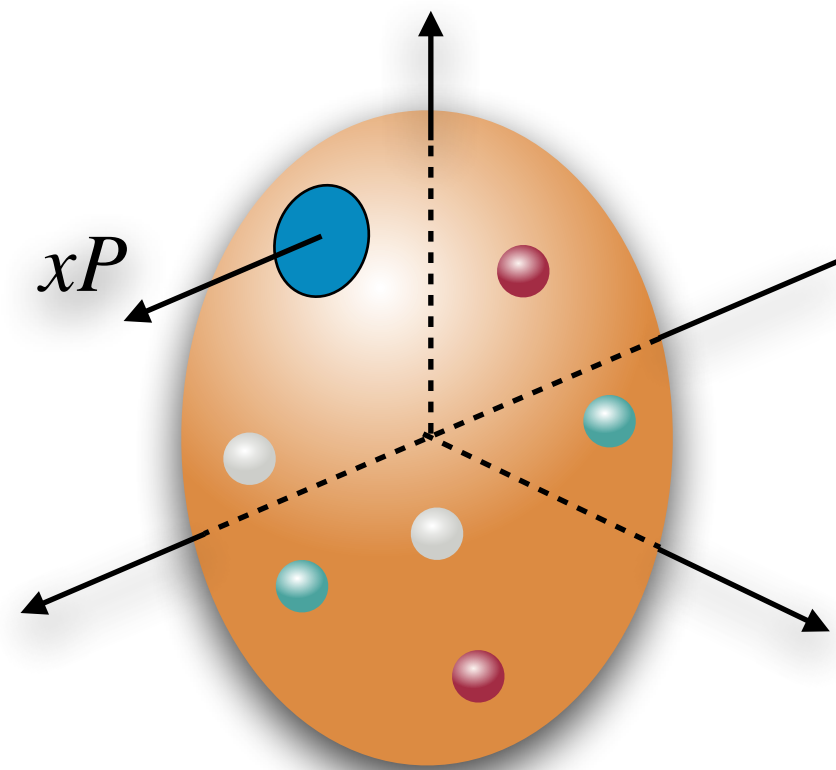
**3D**

FT of GPDs  
exclusive

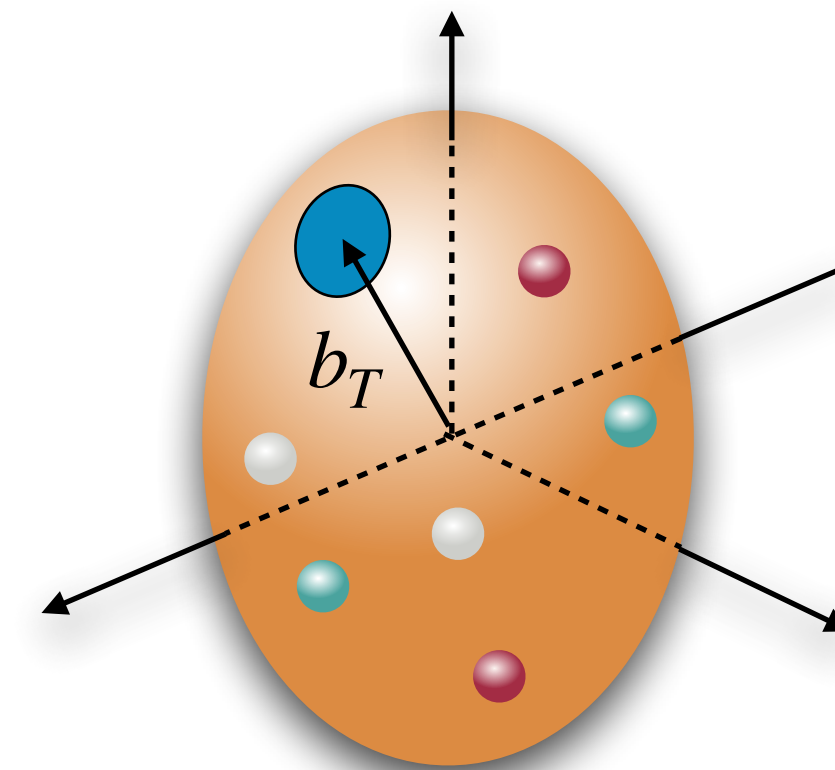


PDFs

(semi-)inclusive



FT of Form Factors



**1D**

→  $\vec{b}_\perp$  dependence

⋯→  $\vec{k}_\perp$  dependence

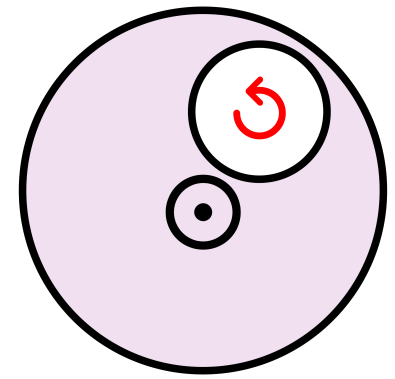


these two variables are NOT Fourier conjugate

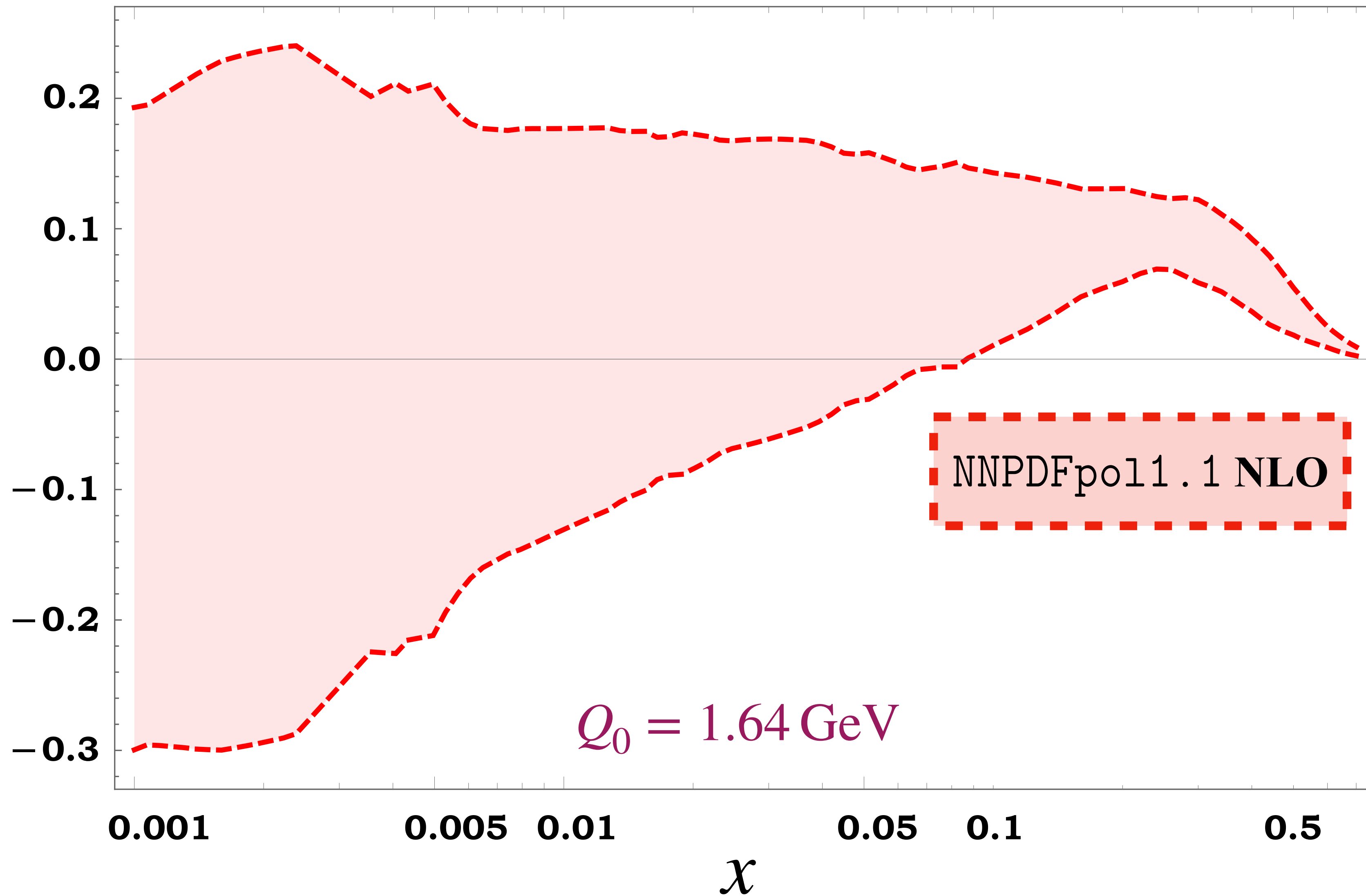
**2D**



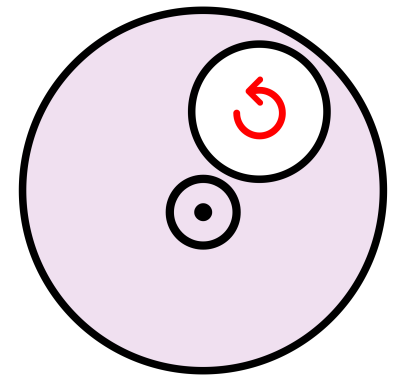
# Helicity gluon collinear PDF



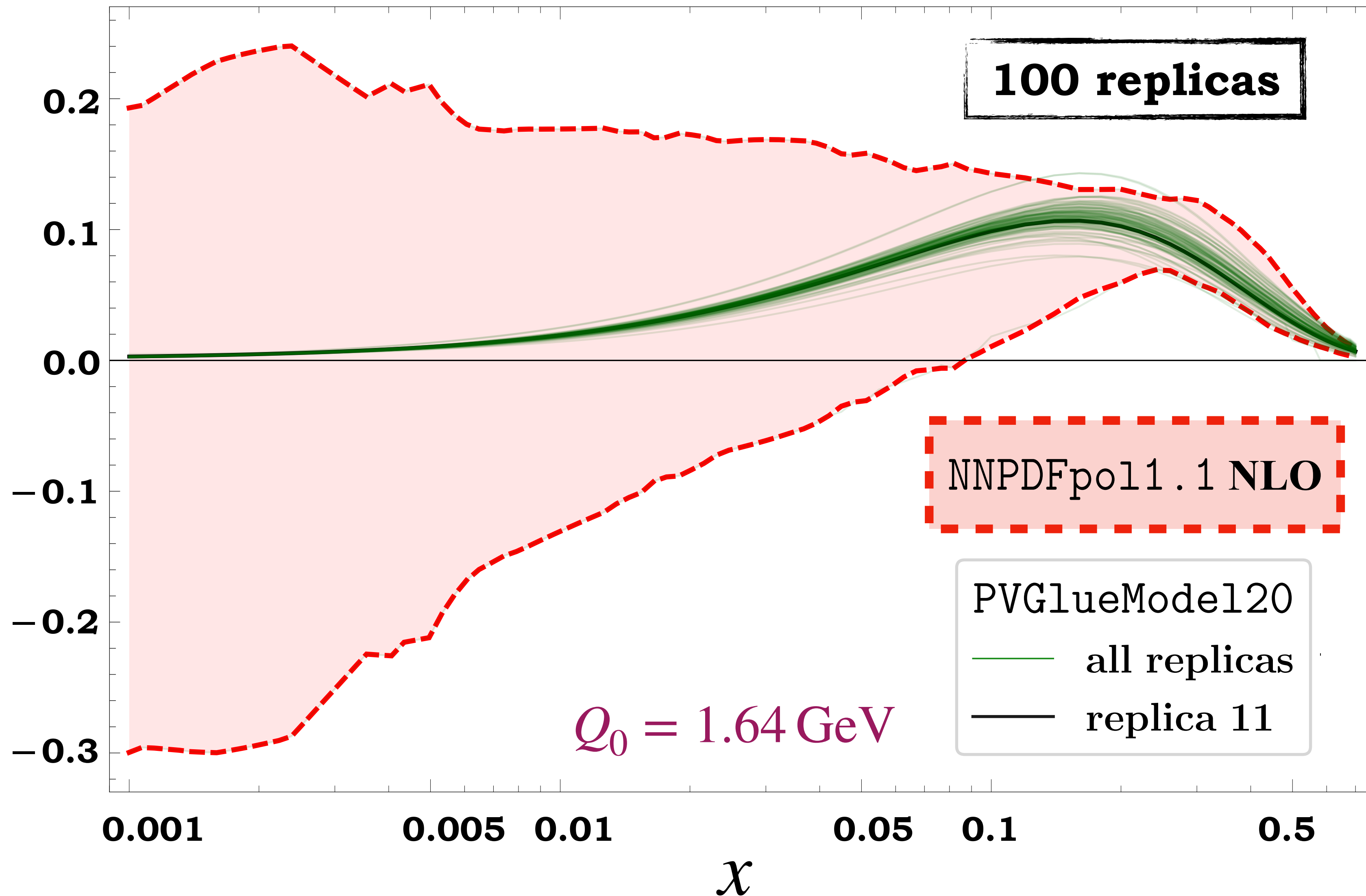
$$x g_1(x)$$



# Helicity gluon collinear PDF



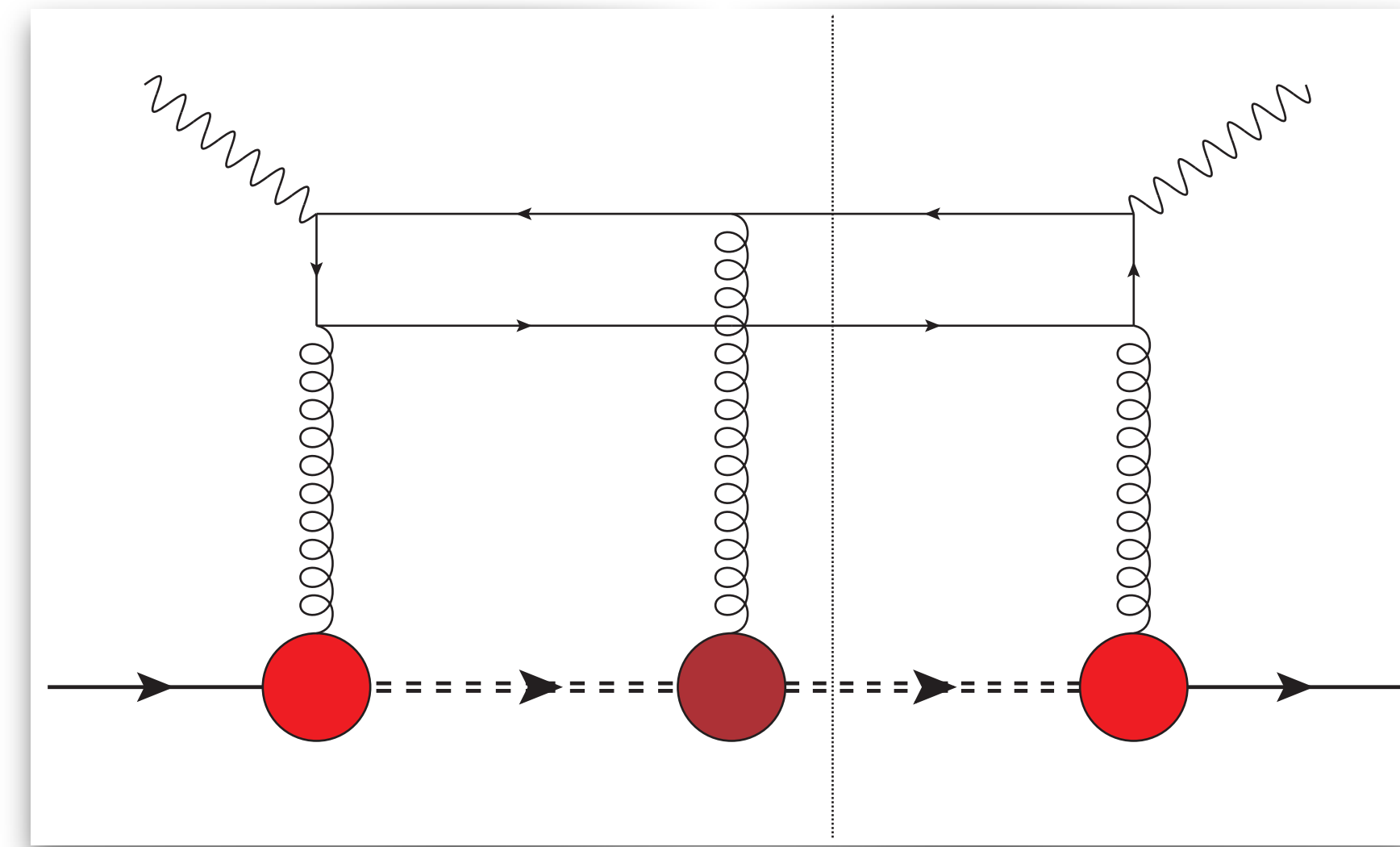
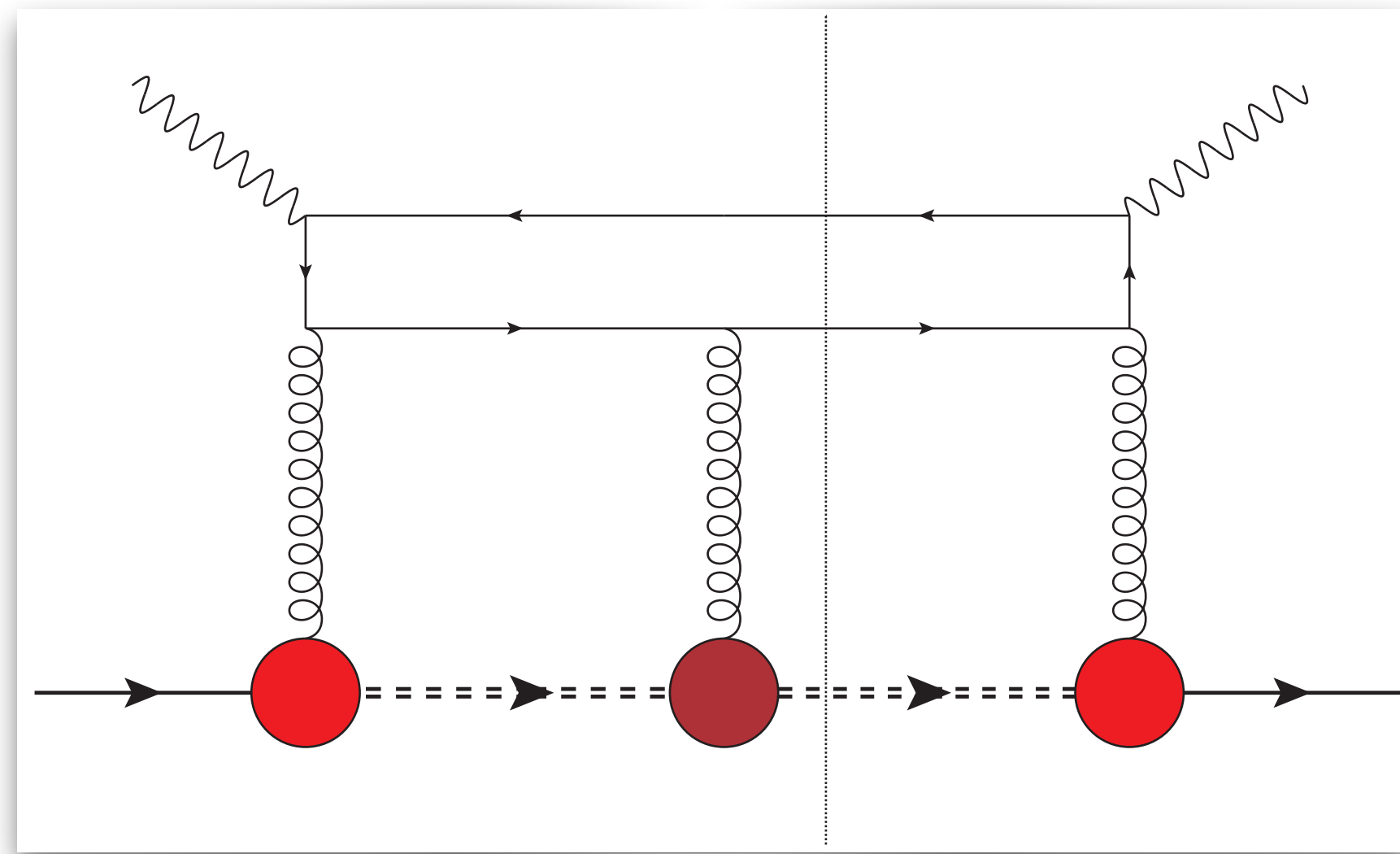
$$x g_1(x)$$

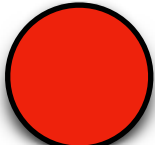
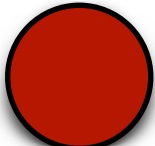


Backup

# Analytic structure of T-odd gluon TMDs

Two-jet SIDIS  $\Rightarrow$  f-type [ + , + ]

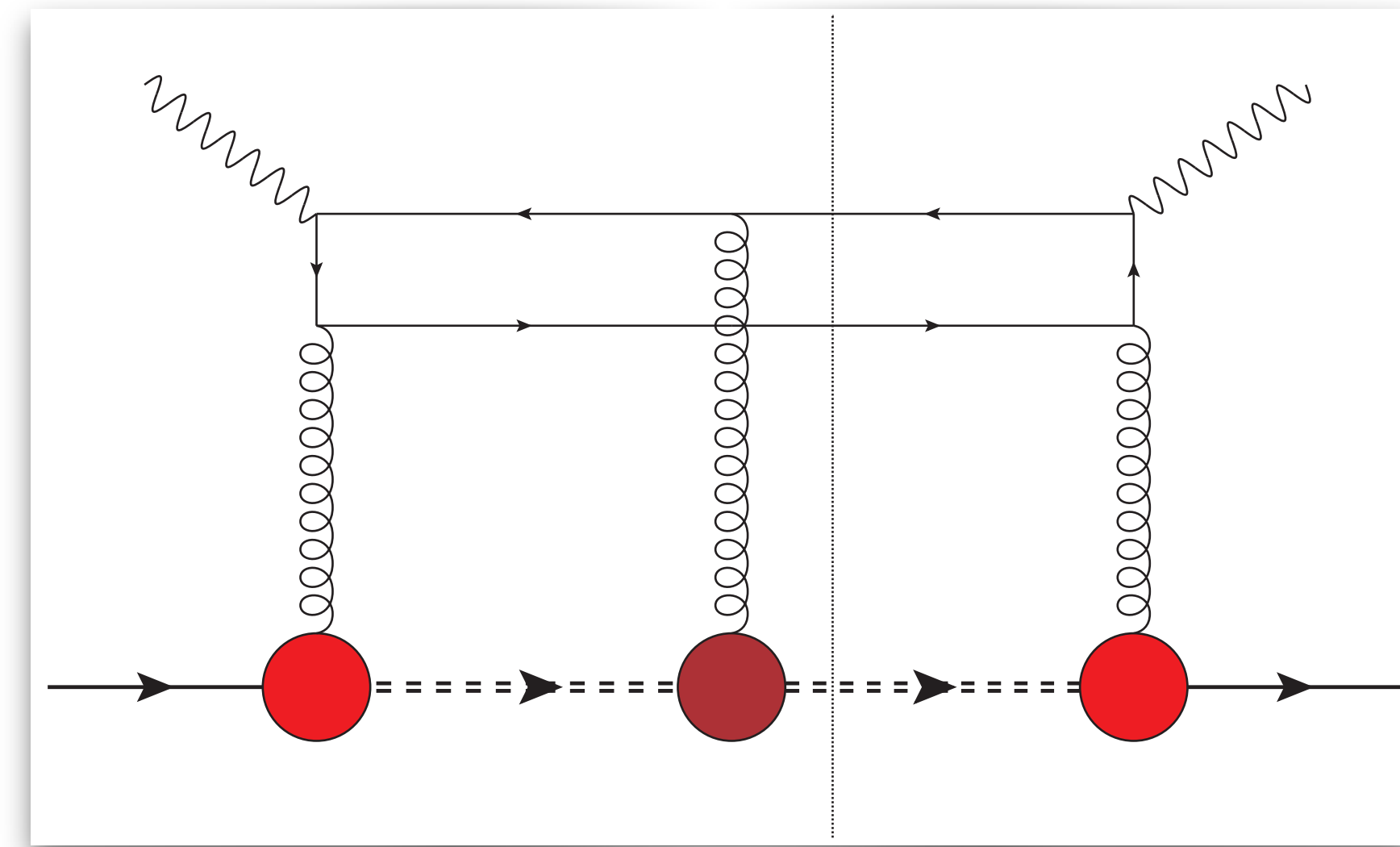
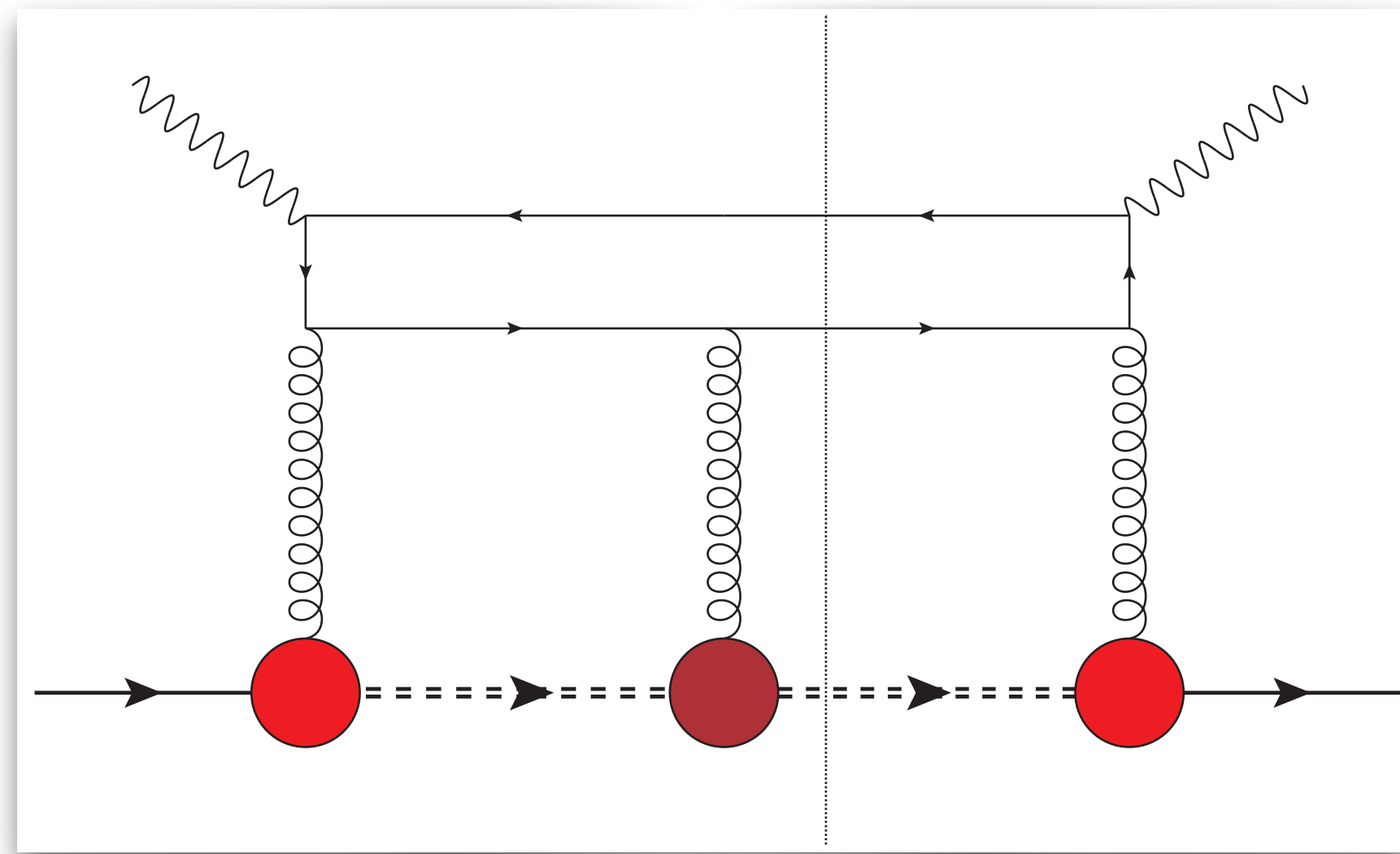


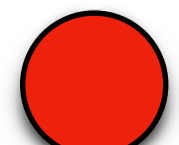
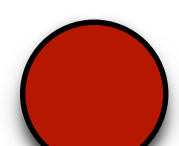
-  nucleon-gluon-spectator
-  spectator-gluon-spectator

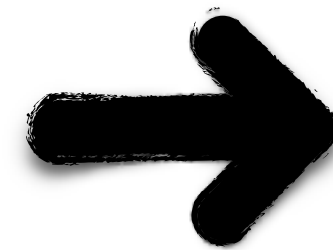


# Analytic structure of T-odd gluon TMDs

Two-jet SIDIS  $\Rightarrow$  f-type [ + , + ]



-  nucleon-gluon-spectator
-  spectator-gluon-spectator



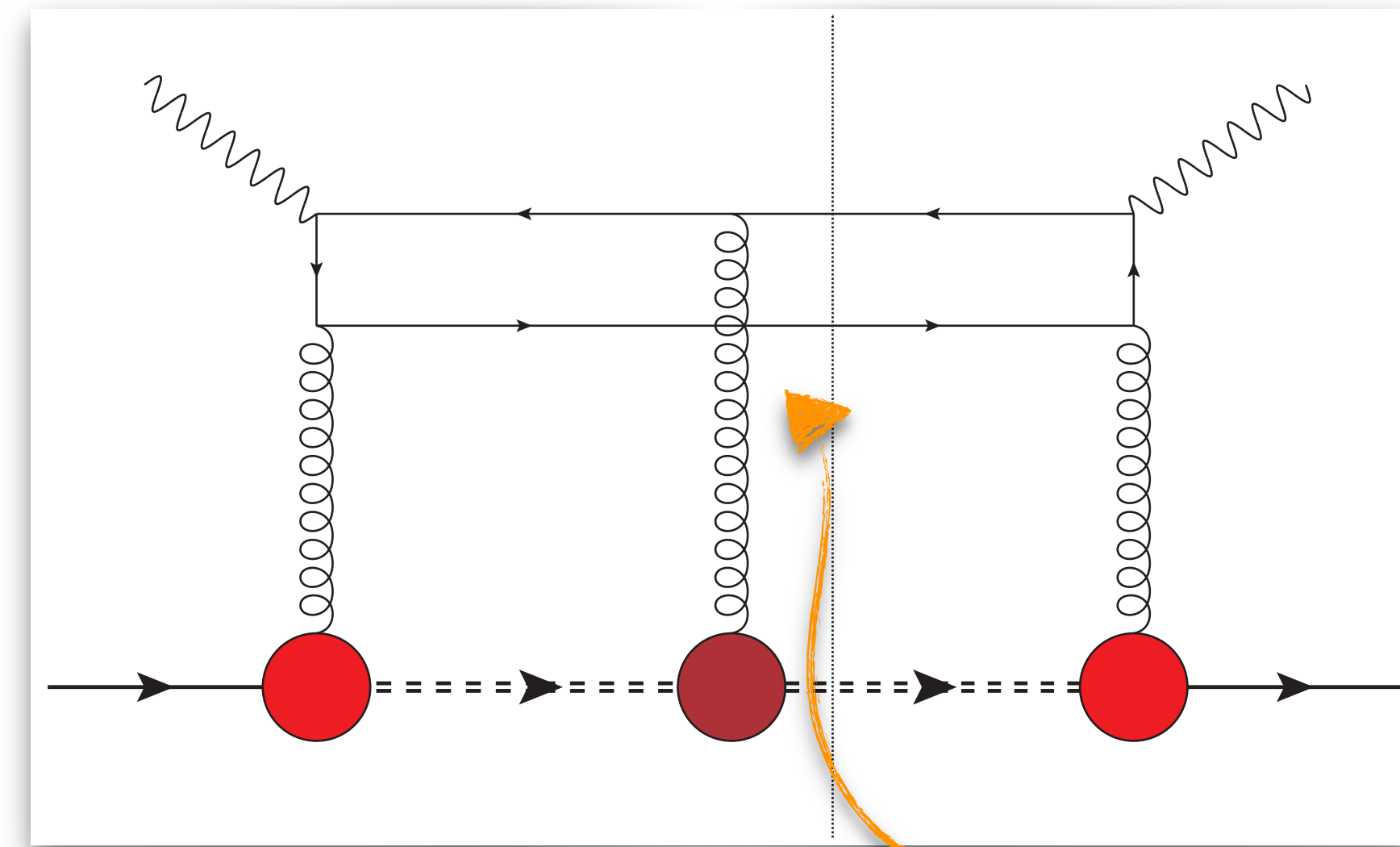
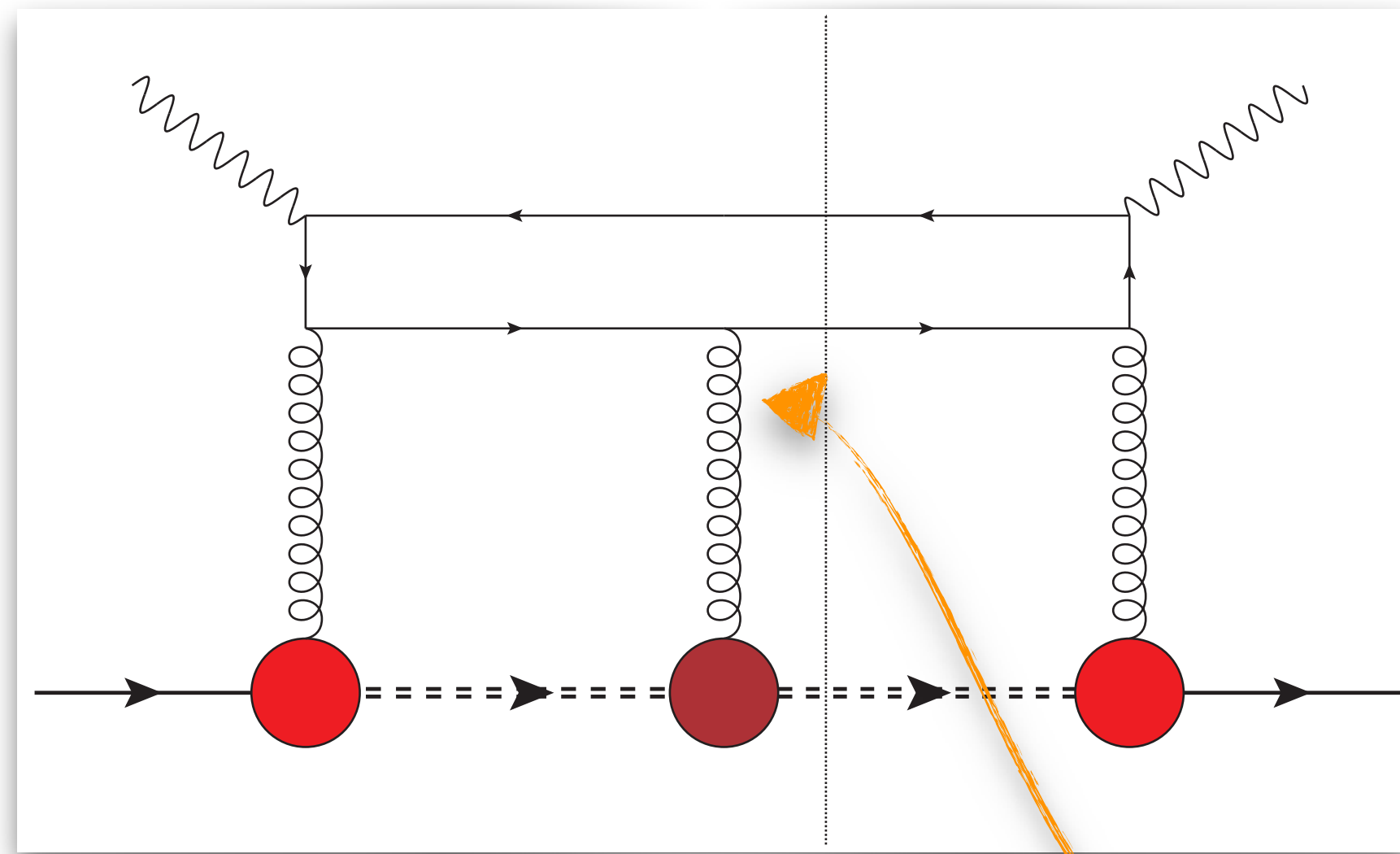
$$8 \times 8 \times 4$$

$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

$$C_{ijk}^{(F)}(x, \mathbf{p}_T^2) = \sum_{l=1}^7 C_{ijk}^{(F),l}(x, \mathbf{p}_T^2) \mathcal{D}_l(x, \mathbf{p}_T^2)$$

# Analytic structure of T-odd gluon TMDs

Two-jet SIDIS  $\Rightarrow$  f-type [ + , + ]



- nucleon-gluon-spectator
- spectator-gluon-spectator

➔

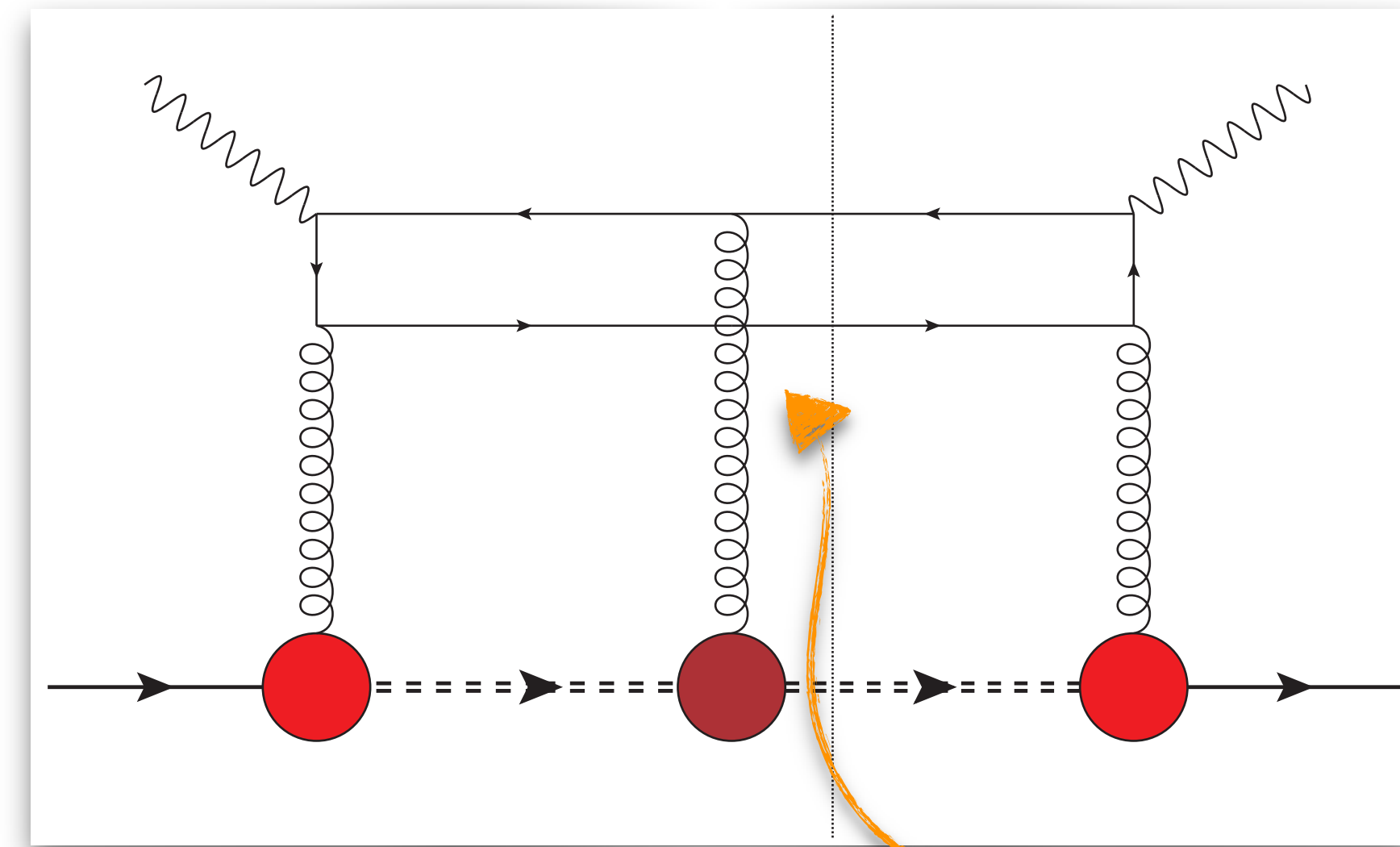
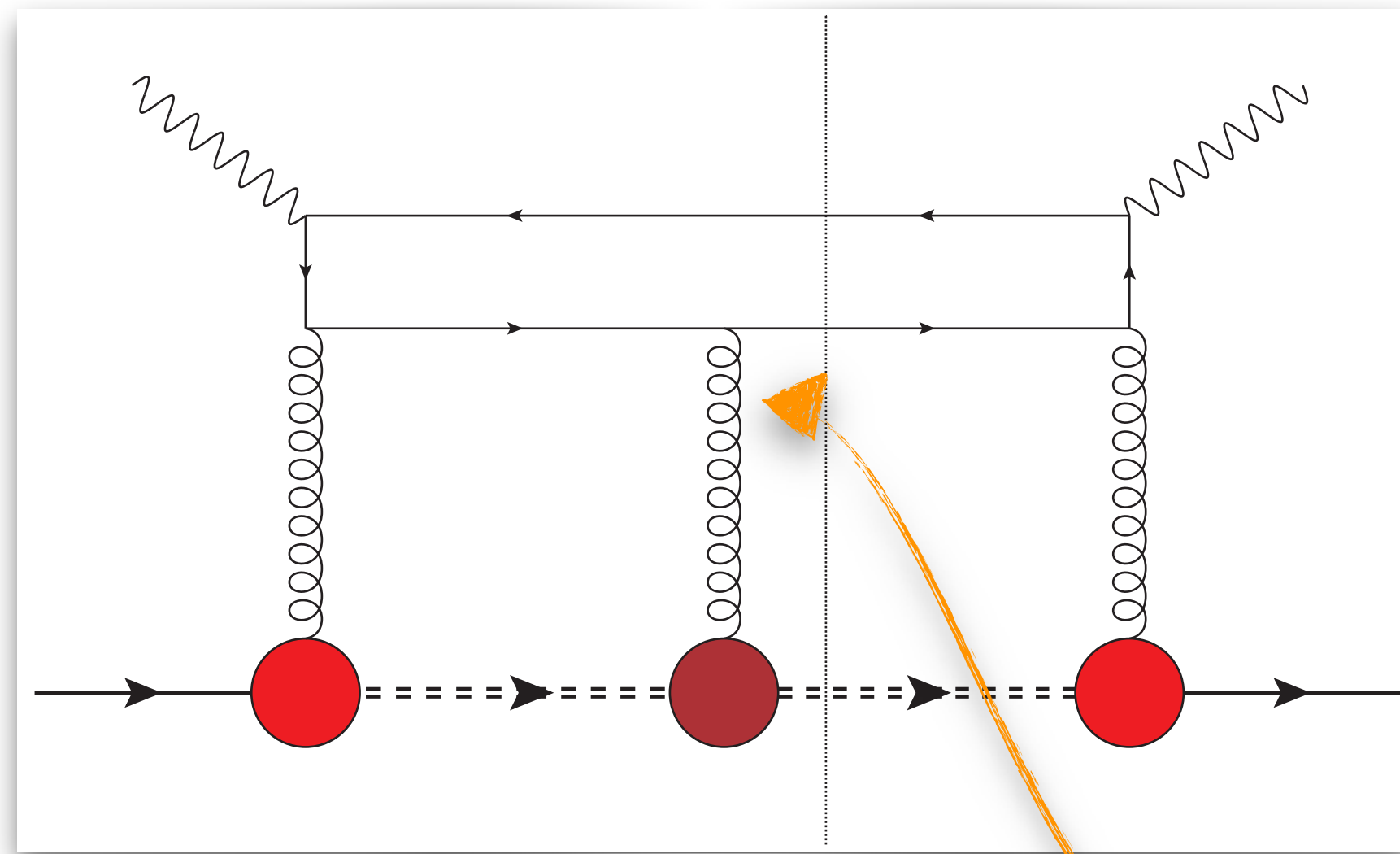
$$8 \times 8 \times 4$$

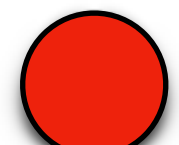
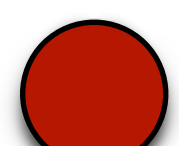
$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

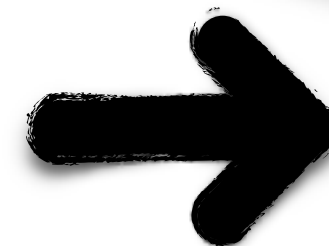
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Two-jet SIDIS  $\Rightarrow$  f-type [ + , + ]



-  nucleon-gluon-spectator
-  spectator-gluon-spectator



$$8 \times 8 \times 4$$

256 coeff. functions

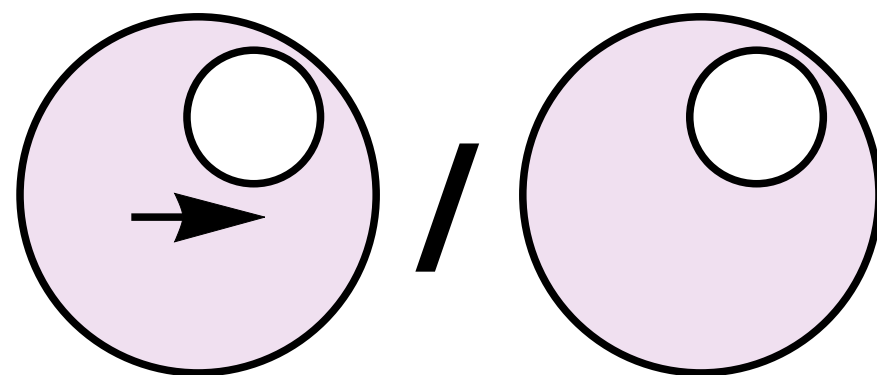
$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

$$C_{ijk}^{(F)}(x, \mathbf{p}_T^2) = \sum_{l=1}^7 C_{ijk}^{(F),l}(x, \mathbf{p}_T^2) \mathcal{D}_l(x, \mathbf{p}_T^2)$$



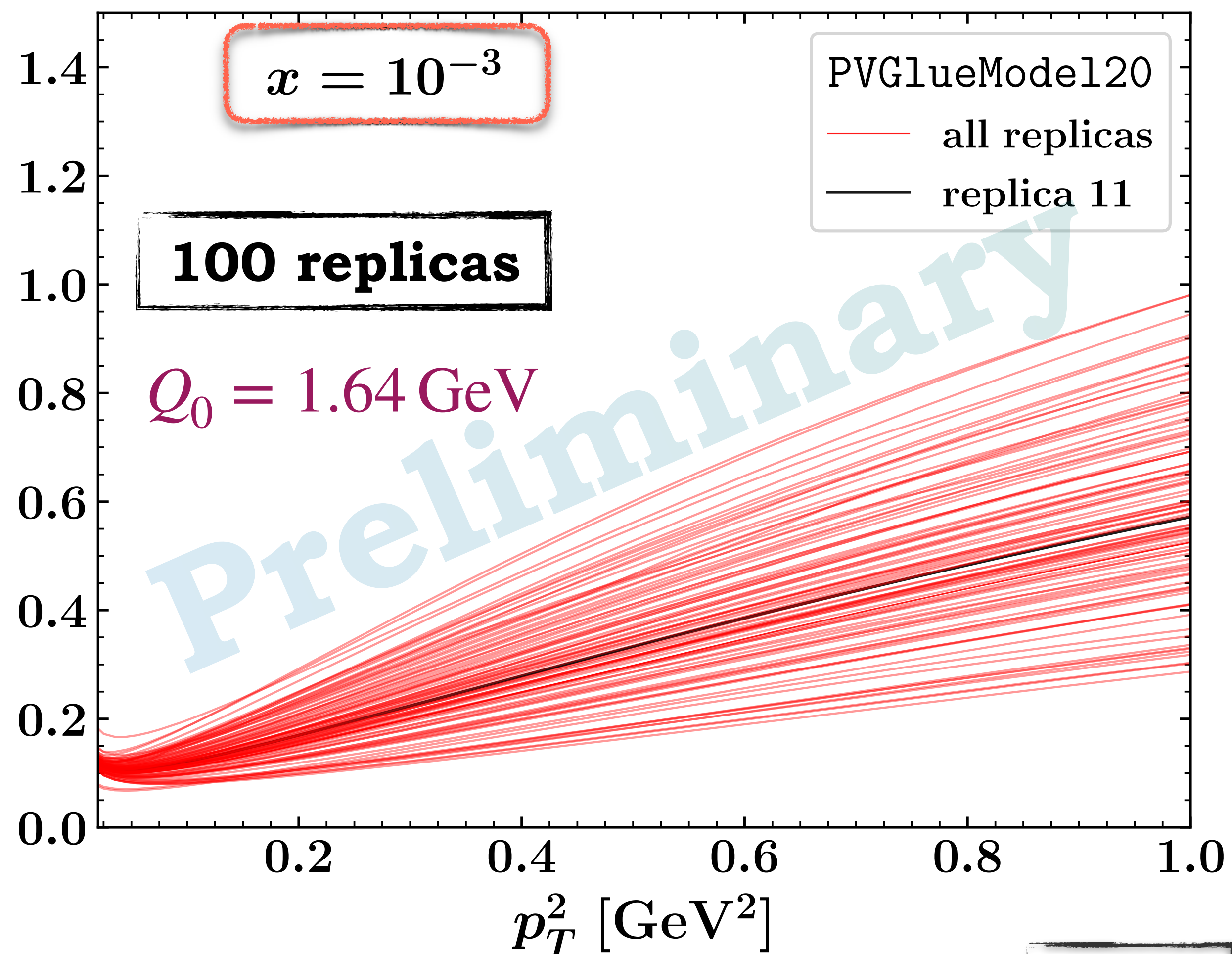
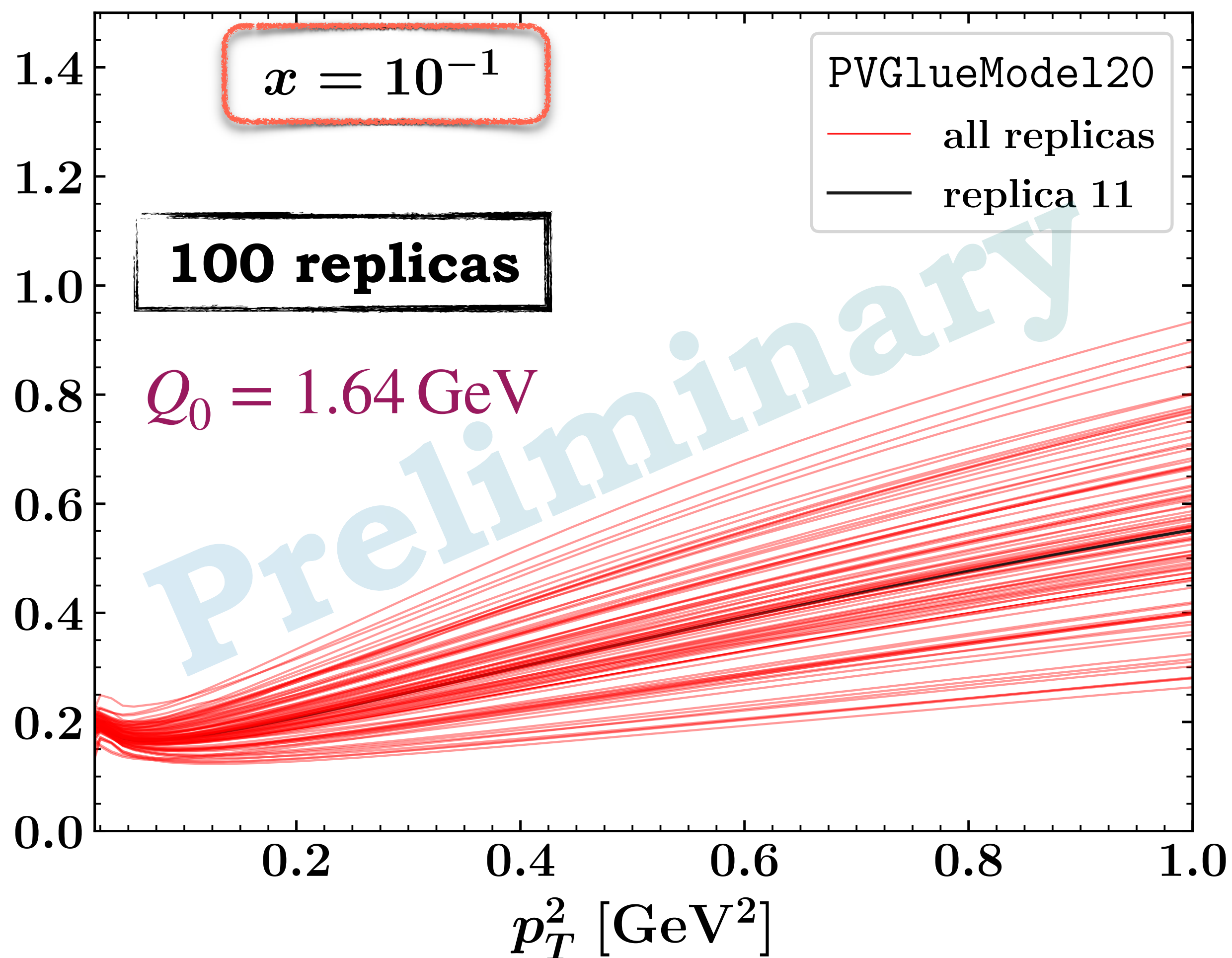
# $f$ -type Sivers/unpol.

A. Bacchetta, F.G. C., M. Radici (to appear)



$$\frac{p_T}{M} f_{1T}^{\perp[+,+]} / f_1$$

$$\frac{p_T}{M} f_{1T}^{\perp[+,+]} / f_1$$



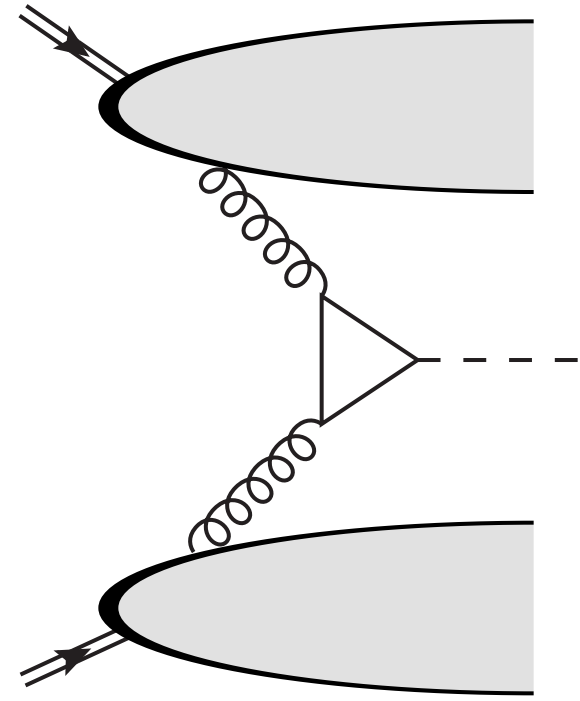
**Backup**



The background features a stylized, semi-transparent illustration of a proton. It is depicted as a large, light blue sphere containing several smaller, colorful spheres representing quarks (red, blue, green) and yellow wavy lines representing gluons. The proton is shown in a slightly tilted, three-dimensional perspective. The overall background is a light, pastel blue with some faint, abstract patterns and a soft glow.

# Accessing gluon TMDs @LHC

## Higgs in gluon fusion



No color entanglement  
TMD factorization

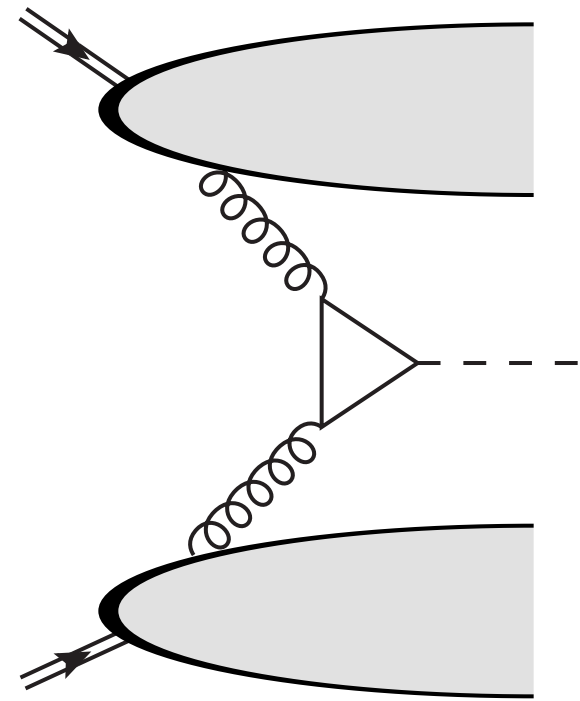


Large low- $p_T$  bin @CMS  
More data @HL-LHC



# Golden channels for gluon TMD PDFs @LHC

## Higgs in gluon fusion

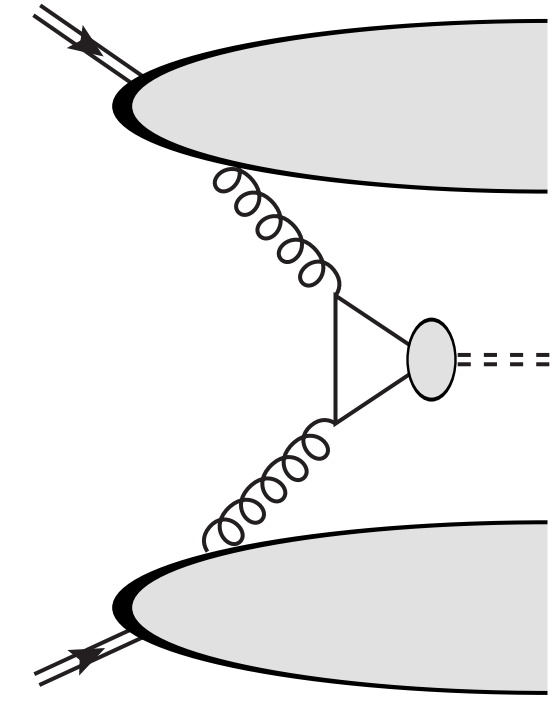


No color entanglement  
TMD factorization



Large low- $p_T$  bin @CMS  
More data @HL-LHC

## Single quarkonium



$\eta_{c,b}$   $J/\psi, \Upsilon$

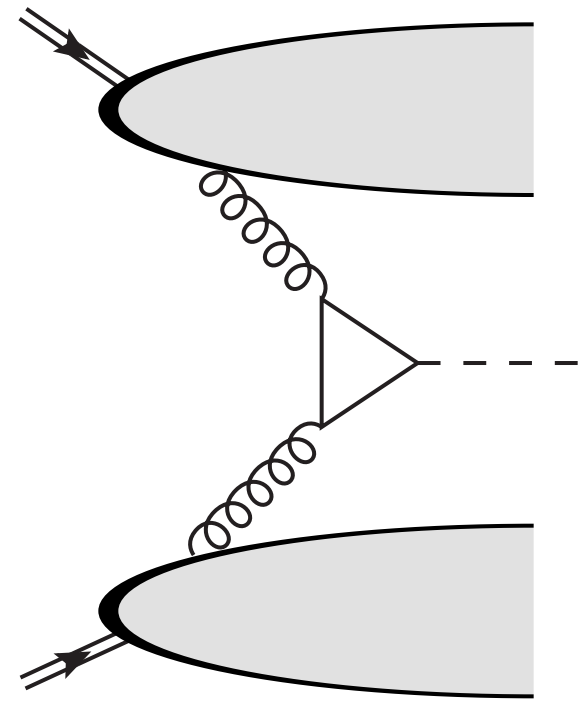
TMD factorization  
C-parity selection rules



Large- $p_T$  data @LHCb  
More data @FT-LHC

# Golden channels for gluon TMD PDFs @LHC

## Higgs in gluon fusion

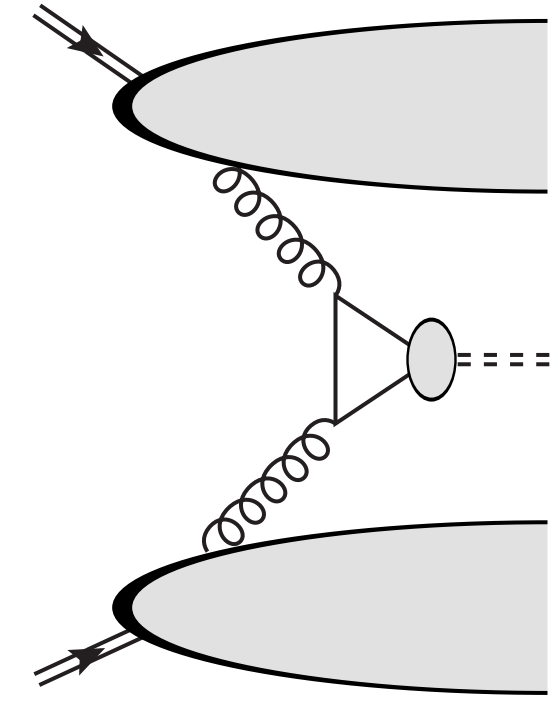


No color entanglement  
TMD factorization



Large low- $p_T$  bin @CMS  
More data @HL-LHC

## Single quarkonium



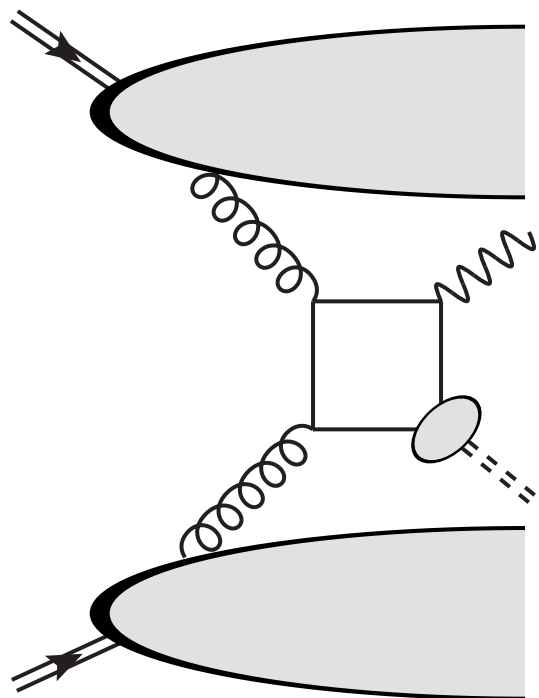
TMD factorization  
C-parity selection rules

$\eta_{c,b}$   $J/\psi, \Upsilon$



Large- $p_T$  data @LHCb  
More data @FT-LHC

## $J/\psi + \gamma^{(*)}$



Color entanglement  
Potential TMD violation (CO)  
Back-to-back suppresses CO

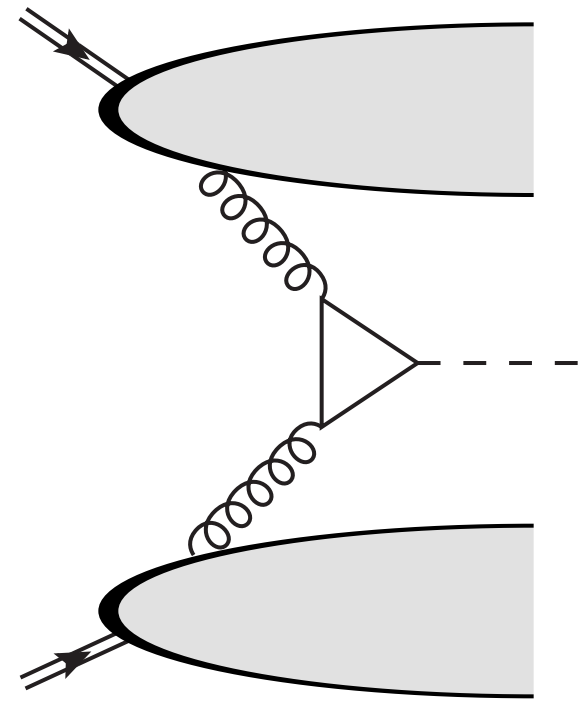


Possible studies @HL-LHC  
Currently no low- $p_T$  data

**Backup**

# Golden channels for gluon TMD PDFs @LHC

## Higgs in gluon fusion

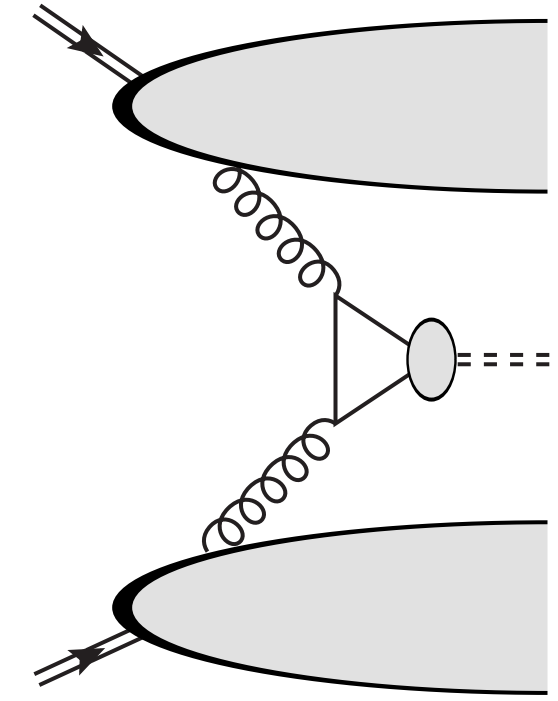


No color entanglement  
TMD factorization



Large low- $p_T$  bin @CMS  
More data @HL-LHC

## Single quarkonium



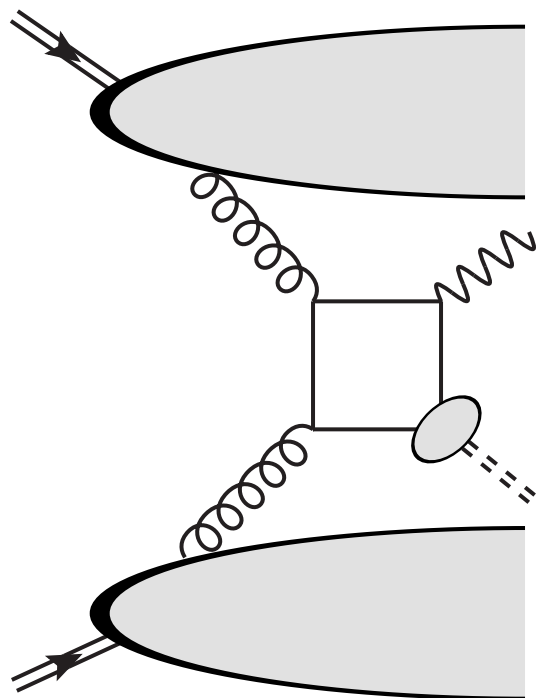
$\eta_{c,b}$   $J/\psi, \Upsilon$

TMD factorization  
C-parity selection rules



Large- $p_T$  data @LHCb  
More data @FT-LHC

## $J/\psi + \gamma^{(*)}$

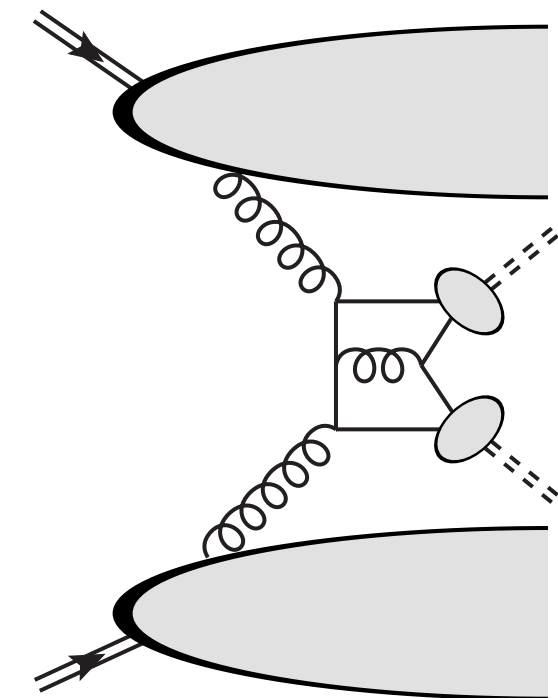


Color entanglement  
Potential TMD violation (CO)  
Back-to-back suppresses CO



Possible studies @HL-LHC  
Currently no low- $p_T$  data

## $J/\psi + J/\psi$



No color entanglement  
TMD factorization (CSM)



Low- $p_T$  data @LHCb  
Opportunities @HL- & @FT-LHC

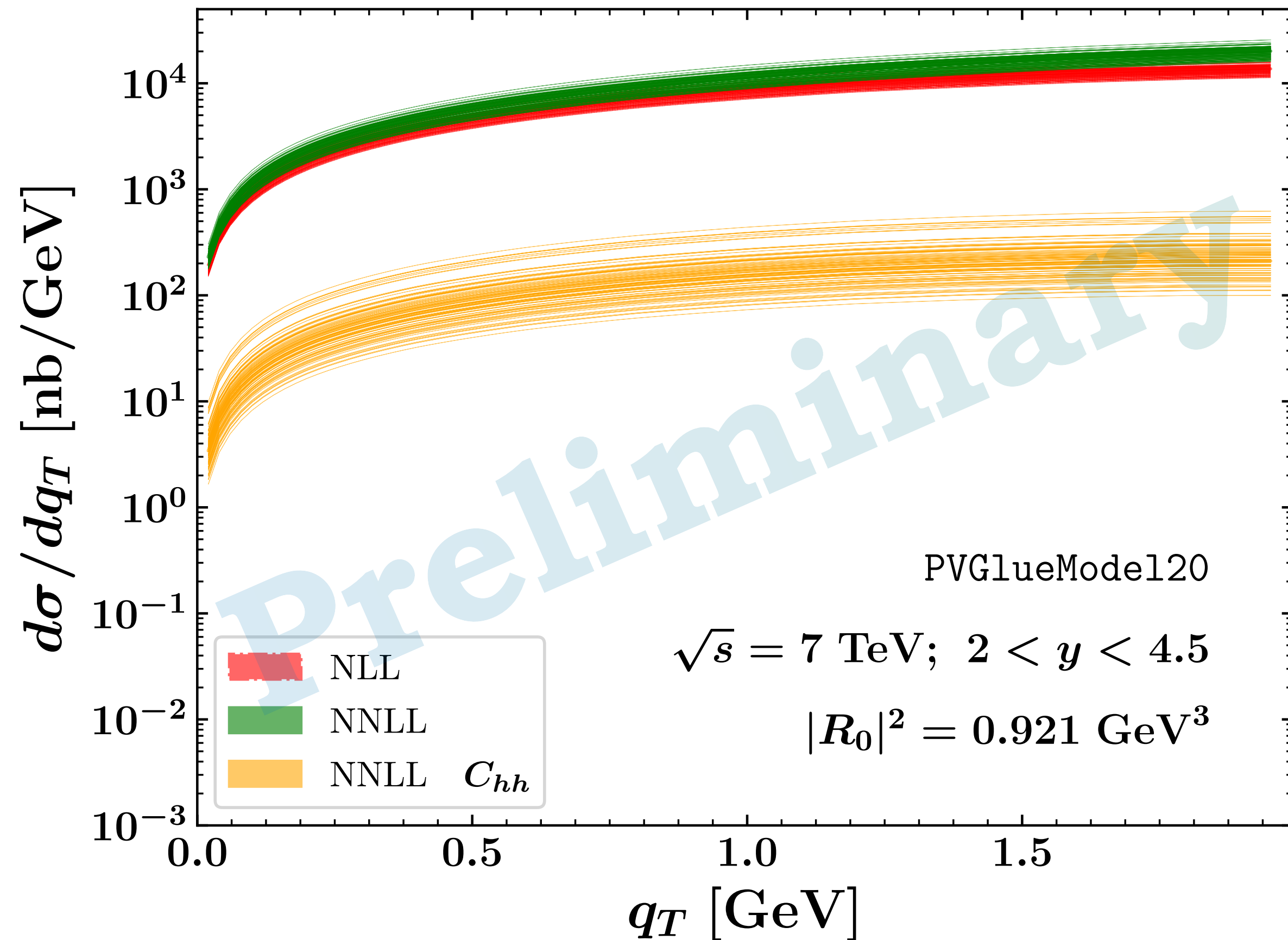
Backup



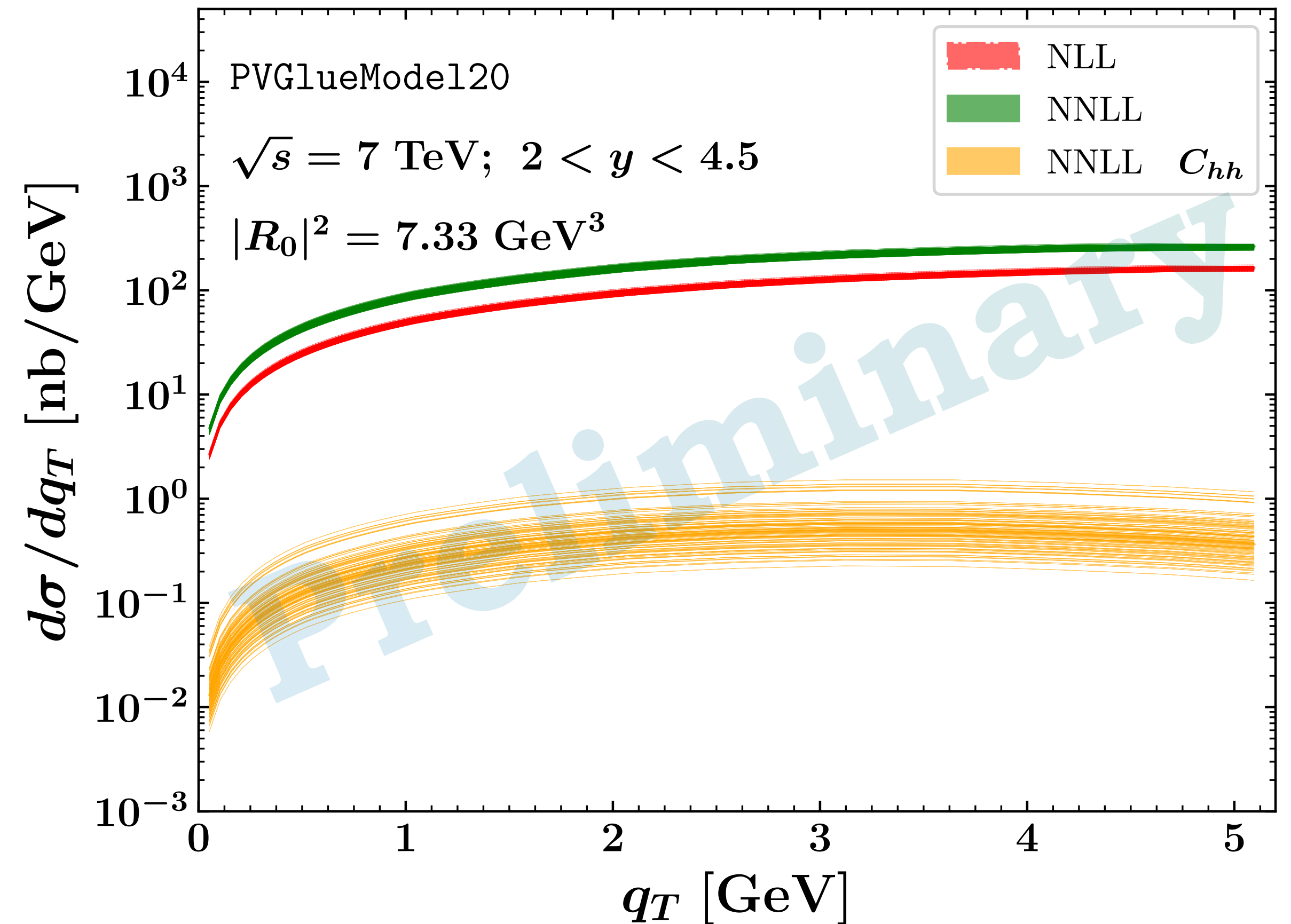
# $\eta_{c,b}$ production @ 7TeV LHCb


 Perturbative scales fixed, NP-evolution parameters fixed, TMD 100-replica analysis, [NRQCD](#) w/o [ShFs](#)

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$



$$p(P_1) + p(P_2) \rightarrow \eta_b(q_T)$$

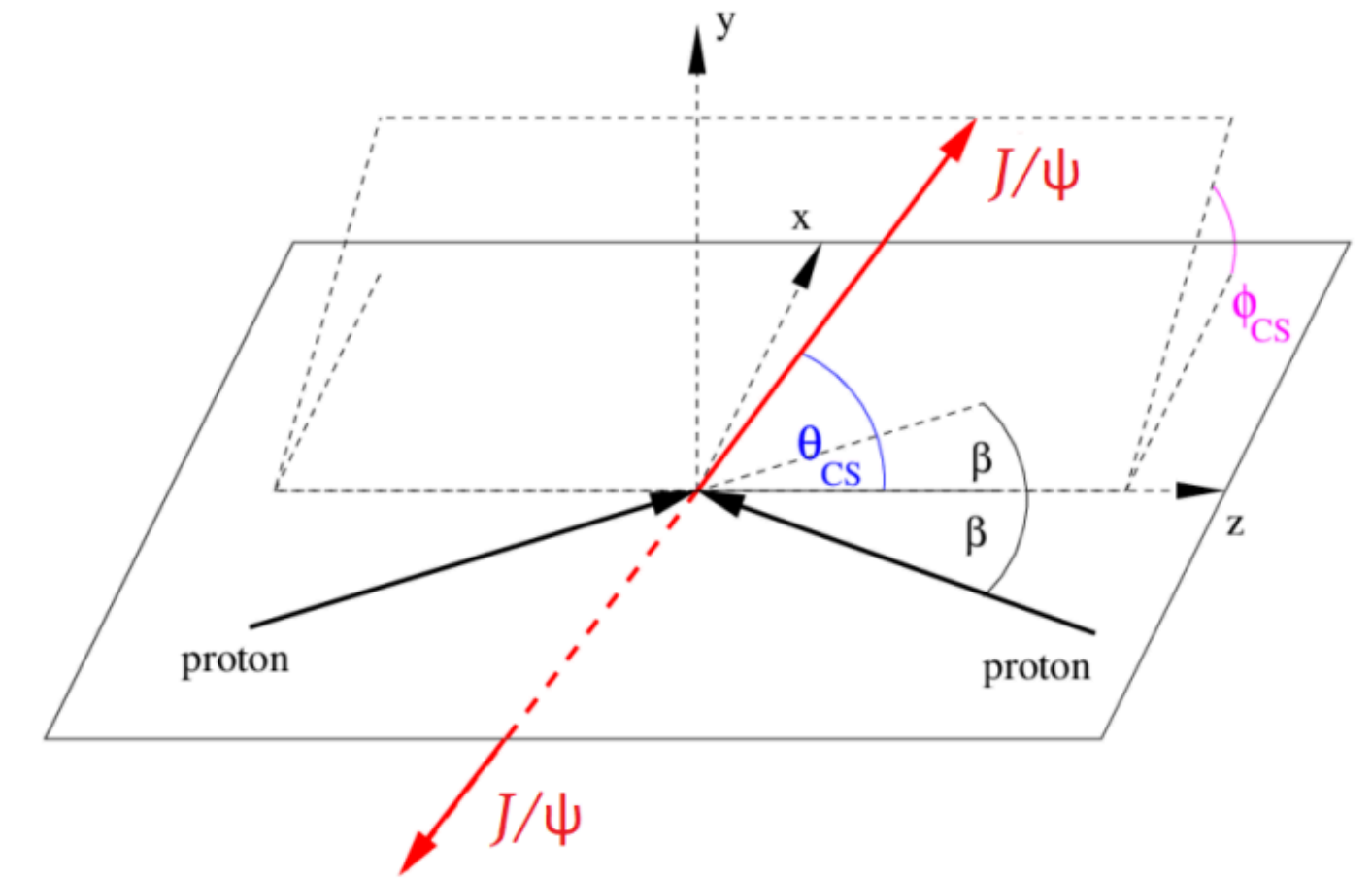


[A. Bacchetta, F.G.C., M.G. Echevarria, J.-P. Lansberg, M. Ozelik, M. Radici, A. Signori (to appear)]

# Double $J/\psi$ production @ (HL-)LHC

 More spin asymmetries, measurable @HL-LHC

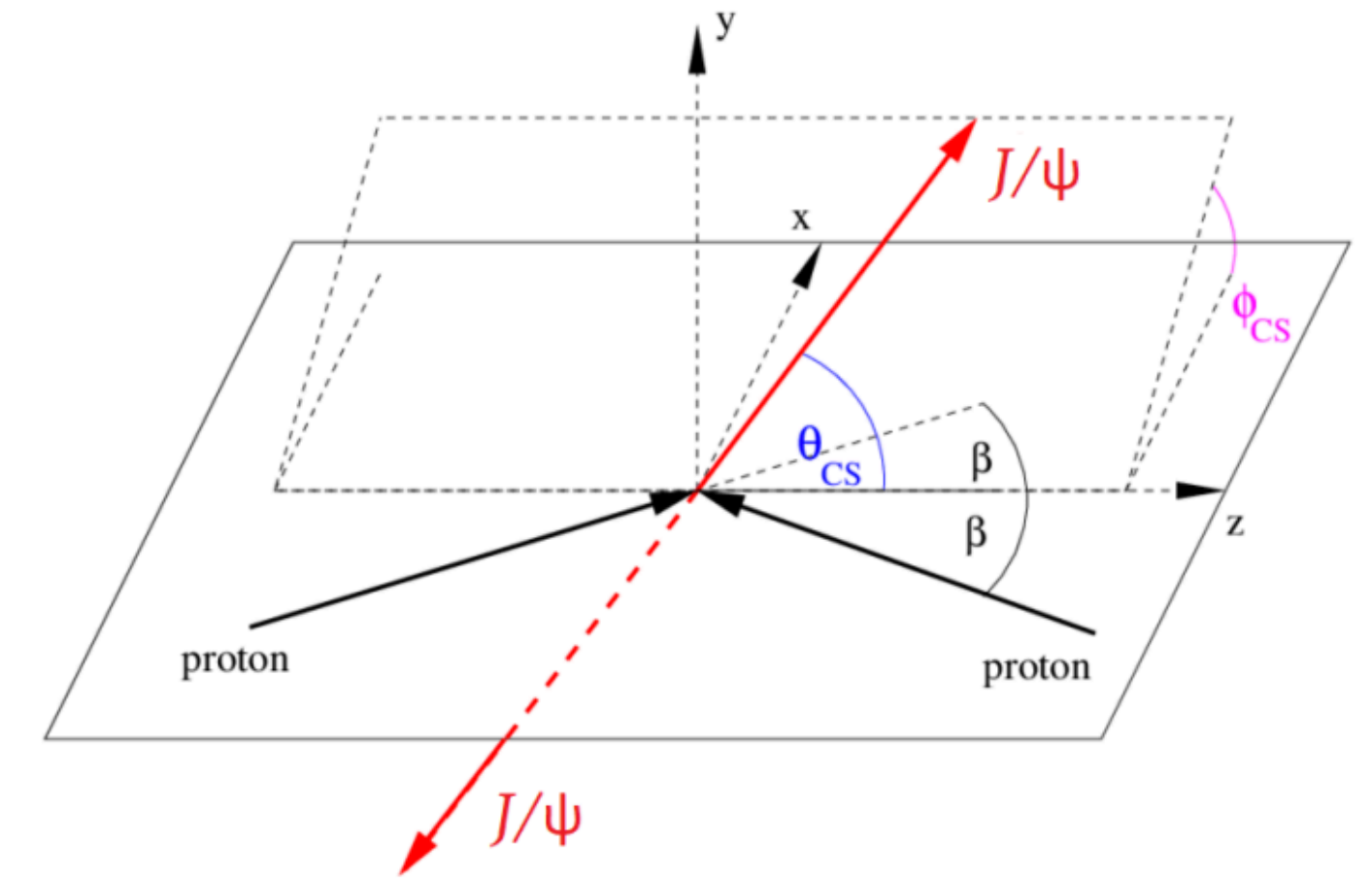
$$\frac{d\sigma}{dM_{QQ}dY_{QQ}d^2P_{QQT}d\Omega} = \frac{\sqrt{Q^2 - 4M_Q^2}}{(2\pi)^2 8s Q^2} \left\{ F_1 C[f_1^g f_1^g] \right. \\ \left. + F_2 C[w_2 h_1^{\perp g} h_1^{\perp g}] + \cos 2\phi_{CS} \left( F_3 C[w_3 f_1^g h_1^{\perp g}] \right. \right. \\ \left. \left. + F'_3 C[w'_3 h_1^{\perp g} f_1^g] \right) + \cos 4\phi_{CS} F_4 C[w_4 h_1^{\perp g} h_1^{\perp g}] \right\},$$



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TMD Models [\[J.-P. Lansberg et al. \(2018\)\]](#)

$$f_1^g(x, \mathbf{k}_T^2, \mu) = \frac{g(x, \mu)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{\mathbf{k}_T^2}{\langle k_T^2 \rangle}\right)$$

$$\text{? } f_1^g / h_1^{\perp g} (p_T \rightarrow 0) \text{ ?} \quad \Rightarrow \quad |h_1^{\perp g}| \leq f_1^g$$

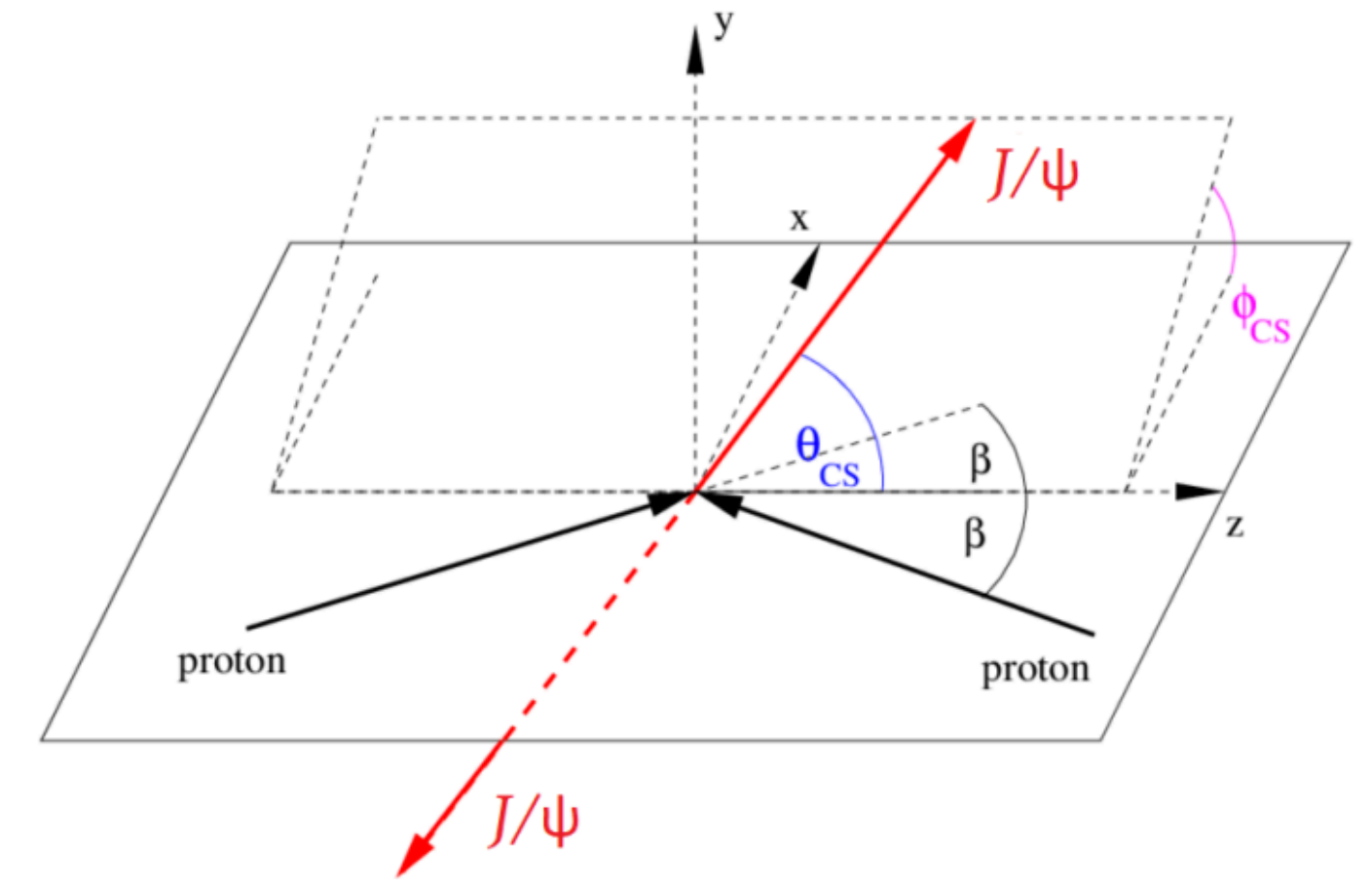
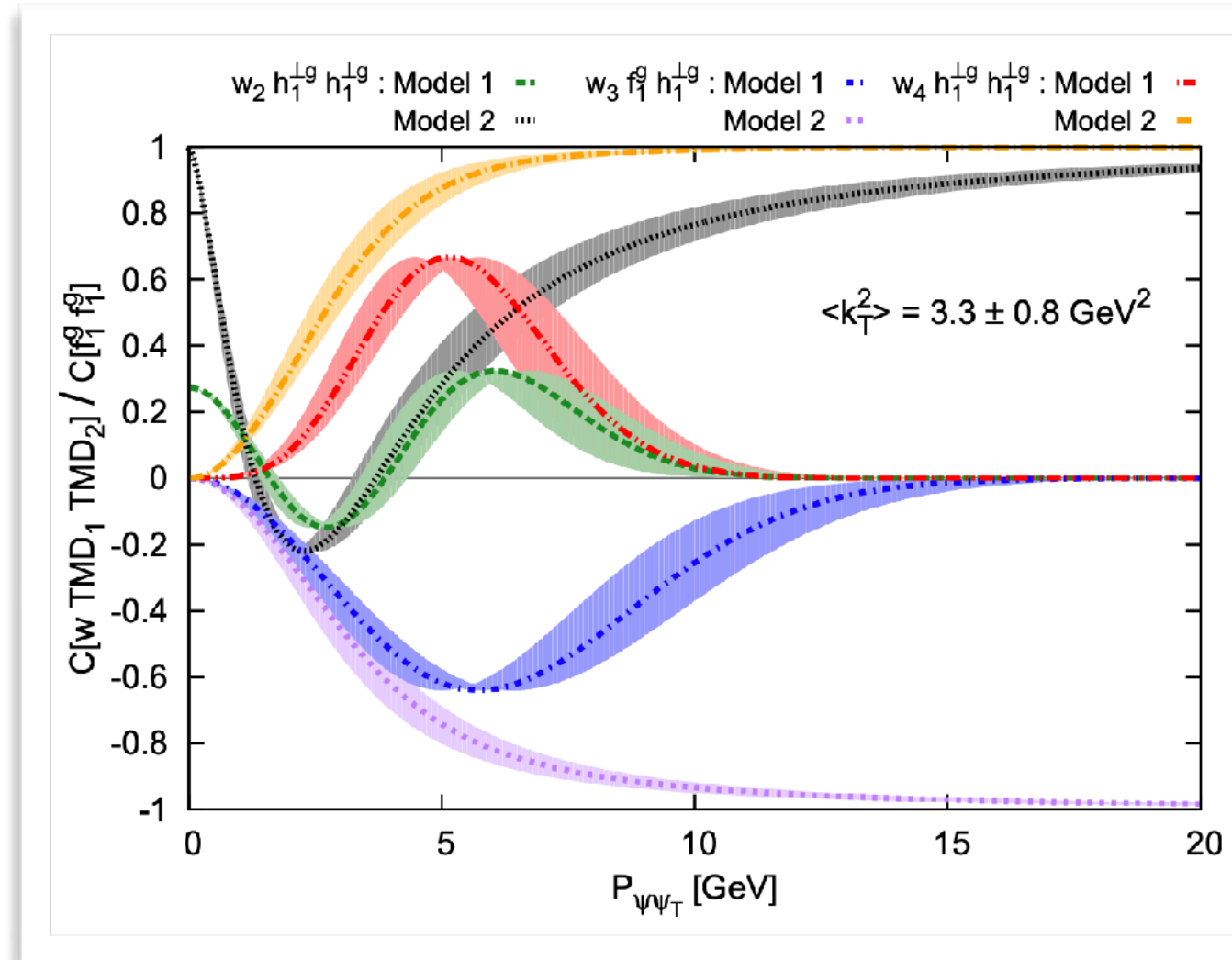
$$h_1^{\perp g}(x, \mathbf{k}_T^2, \mu) = \frac{2M_p^2}{\langle k_T^2 \rangle} \frac{(1-r)}{r} \frac{g(x, \mu)}{\pi \langle k_T^2 \rangle} \exp\left(1 - \frac{\mathbf{k}_T^2}{r \langle k_T^2 \rangle}\right)$$



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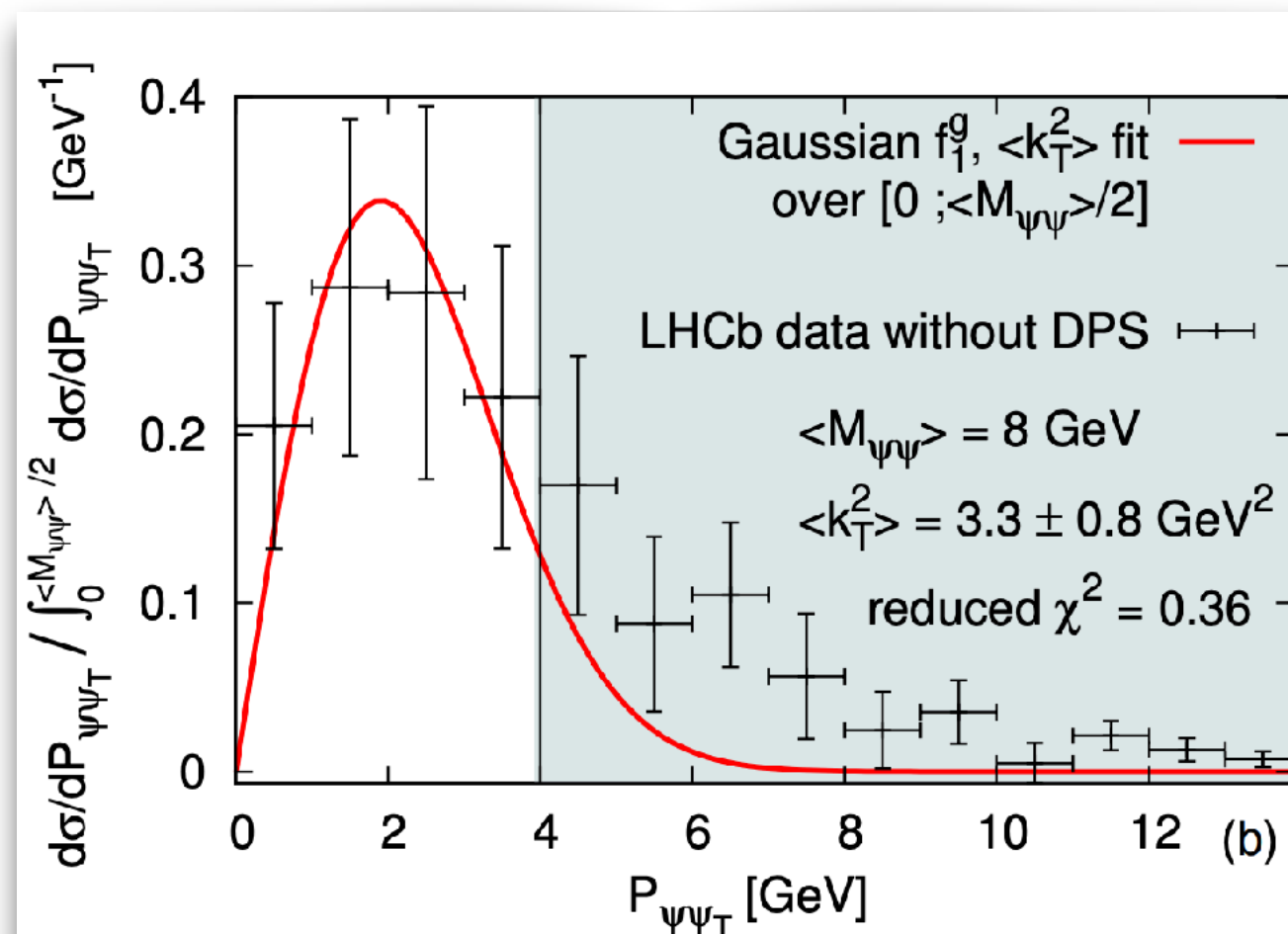


TMD Models [\[J.-P. Lansberg et al. \(2018\)\]](#)

$$f_1^g(x, k_T^2, \mu) = \frac{g(x, \mu)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right)$$

$$f_1^g / h_1^{\perp g} (p_T \rightarrow 0) ? \Rightarrow |h_1^{\perp g}| \leq f_1^g$$

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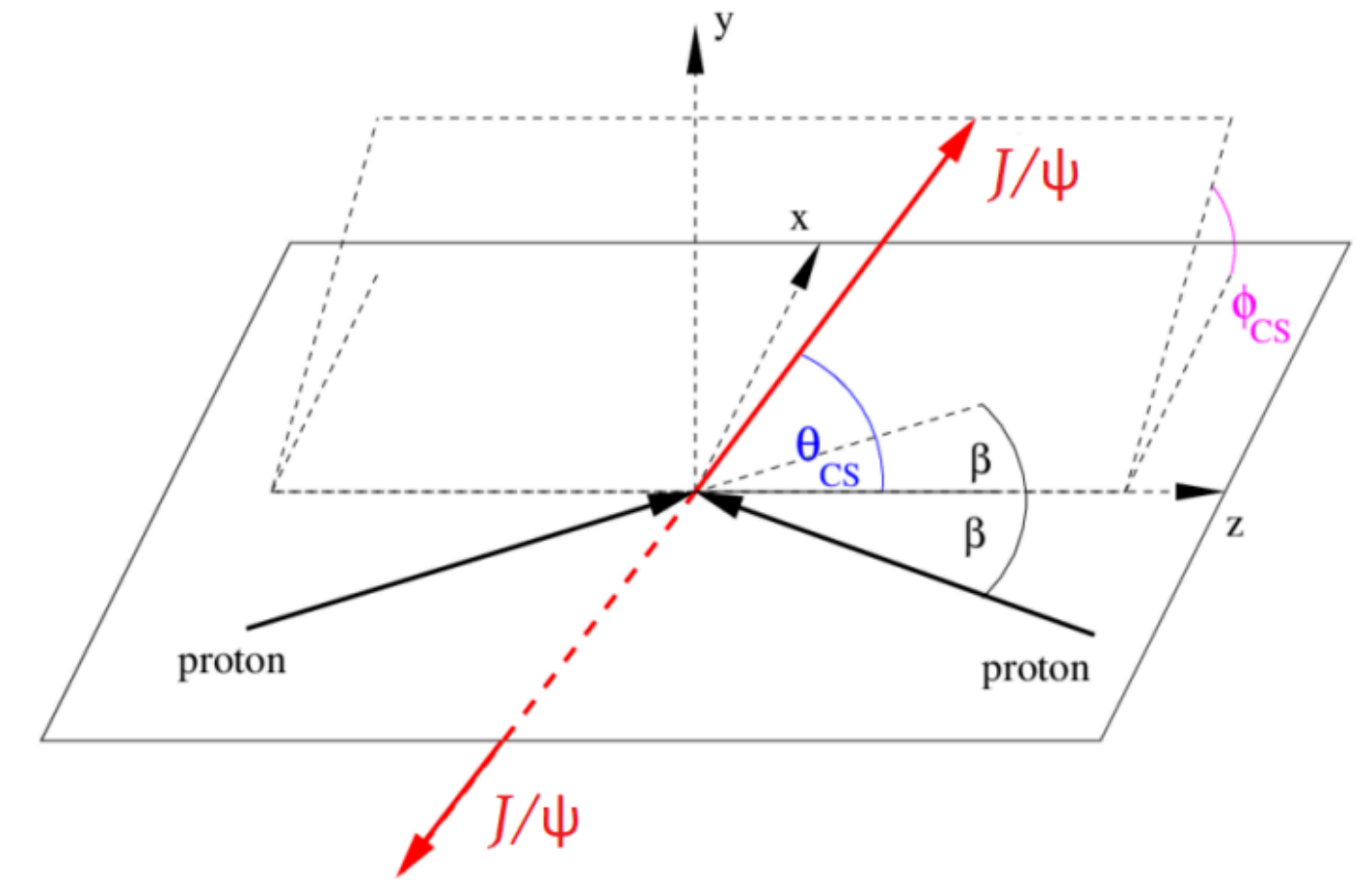
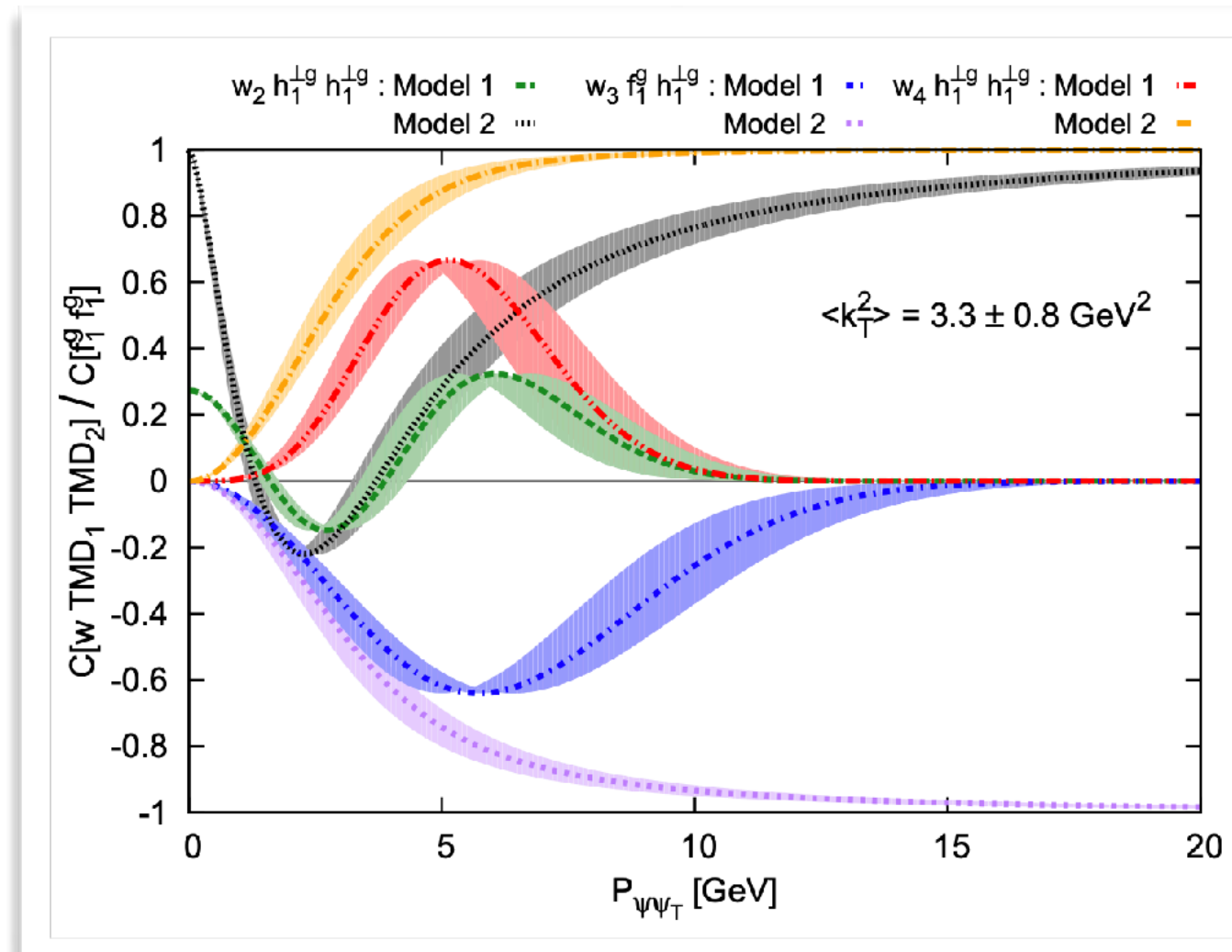
[Model-dependent fit on 13 TeV LHCb data]

Backup

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$$\frac{d\sigma}{dM_{QQ}dY_{QQ}d^2P_{QQT}d\Omega} = \frac{\sqrt{Q^2 - 4M_Q^2}}{(2\pi)^2 8s Q^2} \left\{ F_1 C[f_1^g f_1^g] + F_2 C[w_2 h_1^{\perp g} h_1^{\perp g}] + \cos 2\phi_{CS} \left( F_3 C[w_3 f_1^g h_1^{\perp g}] + F'_3 C[w'_3 h_1^{\perp g} f_1^g] \right) + \cos 4\phi_{CS} F_4 C[w_4 h_1^{\perp g} h_1^{\perp g}] \right\},$$



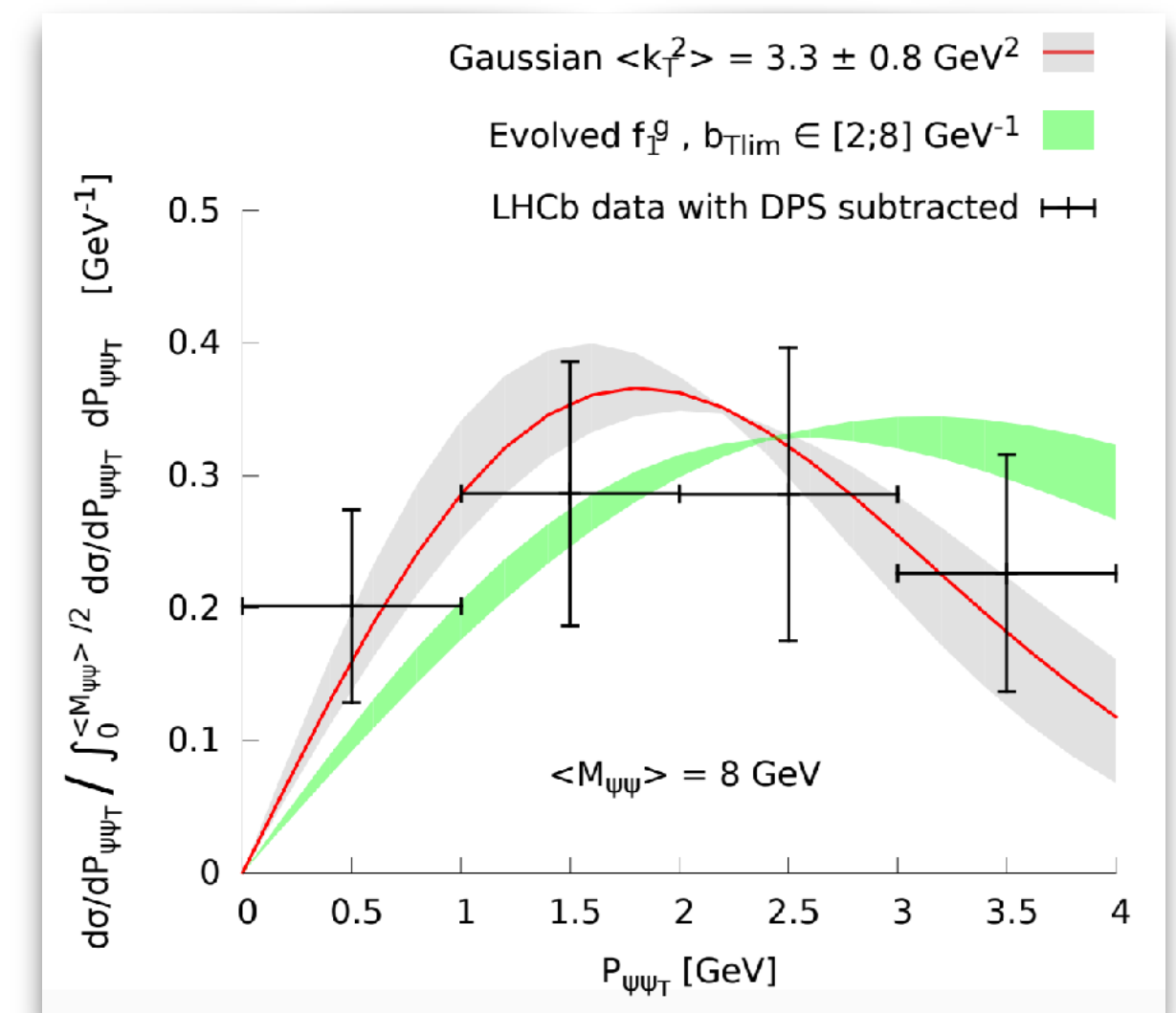
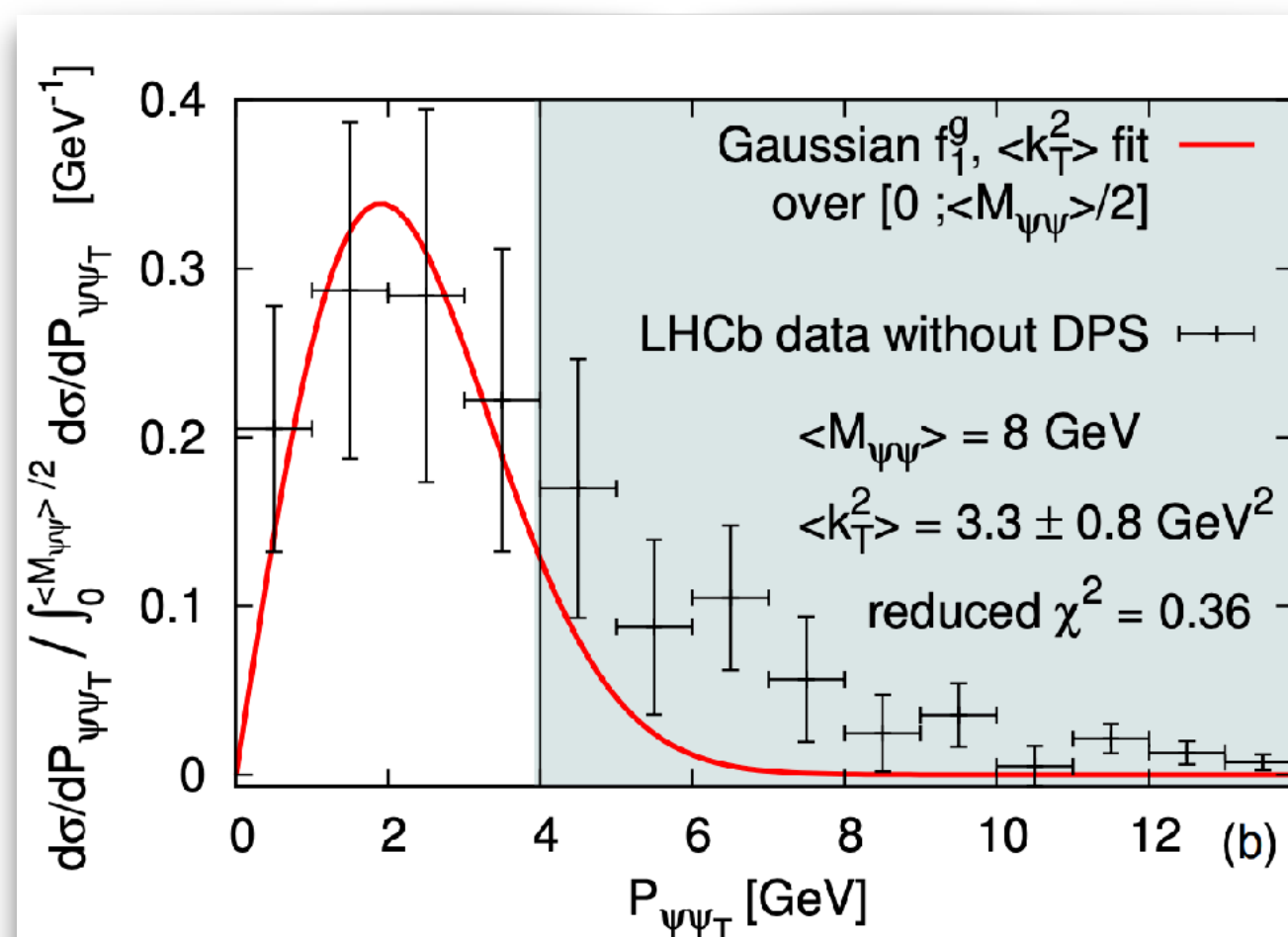
NP + TM resummation

TMD Models [J.-P. Lansberg et al. (2018)]

$$f_1^g(x, k_T^2, \mu) = \frac{g(x, \mu)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right)$$

$$f_1^g / h_1^{\perp g} (p_T \rightarrow 0) ? \Rightarrow |h_1^{\perp g}| \leq f_1^g$$

$$h_1^{\perp g}(x, k_T^2, \mu) = \frac{2M_p^2}{\langle k_T^2 \rangle} \frac{(1-r)}{r} \frac{g(x, \mu)}{\pi \langle k_T^2 \rangle} \exp\left(1 - \frac{k_T^2}{r \langle k_T^2 \rangle}\right)$$



[F. Scarpa et al. (2020)]

[Model-dependent fit on 13 TeV LHCb data]

Backup

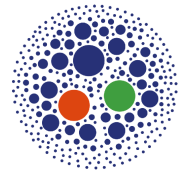


# Spectator-model gluon TMDs

The background features a repeating pattern of circular diagrams illustrating the spectator model for gluon Transverse Momentum Distributions (TMDs). Each diagram shows a central gluon (represented by a red sphere with a red arrow) interacting with a quark (represented by a blue sphere with a blue arrow) within a nucleon. The gluon's transverse momentum is shown as a red arrow pointing away from the quark. The diagrams are arranged in a grid, with some overlapping, and are set against a light blue background with a subtle pattern of wavy lines and small starburst effects.



# Assumptions of the model



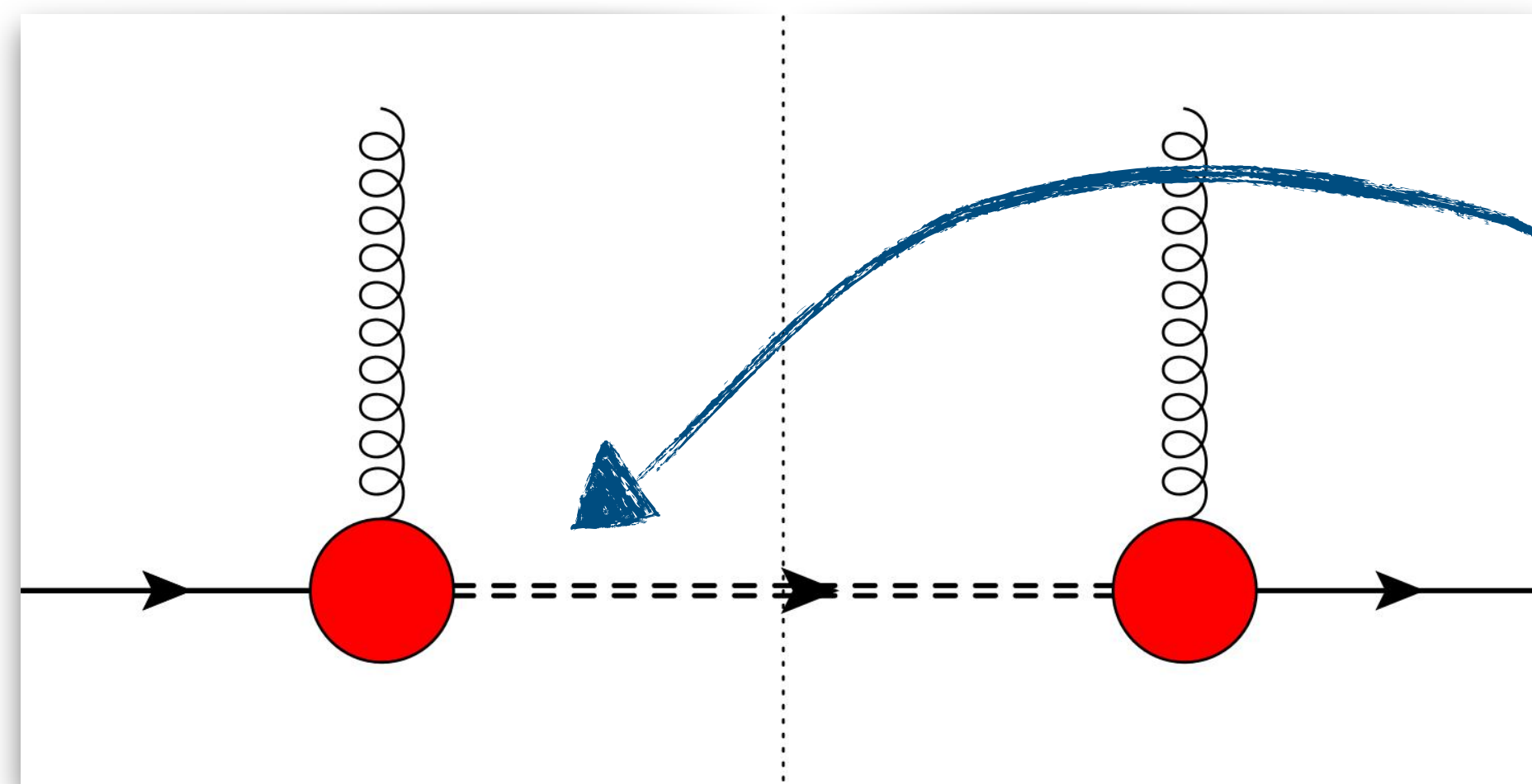
## Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



Instead of a single on-shell spectator, a continuum of spectators

$\mathcal{V}_g(p^2)$

# Assumptions of the model



## Spectator-system spectral-mass function

**spectral-mass function**

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

**spectator-model TMD**

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X \left( M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\} \right) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

**low- $x$  (high- $\mu^2$ ) tail**  $\propto (a - b)$

$q\bar{q}$  contributions energetically available at large  $M_X$

$$\mu^2 = M_X^2 - M^2$$

**moderate- $x$  trend**

pure tri-quark contribution at low  $M_X$

# Glueon TMD correlator and T-odd glueon densities

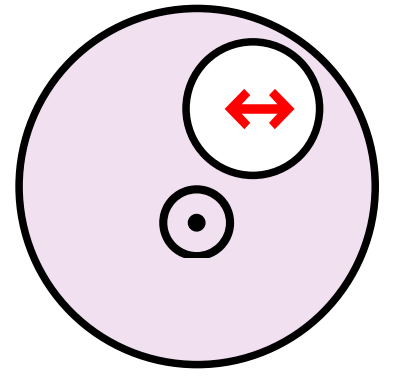
$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[ \delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[ i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[ \frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ \left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$



# Glueon TMD correlator and T-odd gluon densities

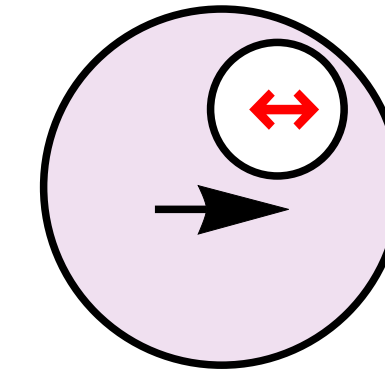


$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[ \delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[ i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right]$$

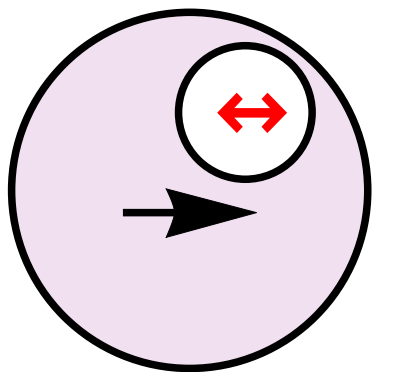
$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[ \frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

**pseudo worm-gear**

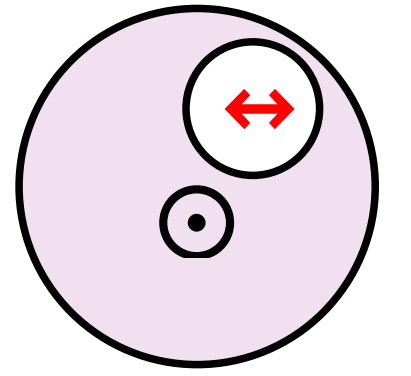


**linearity TMD**

**pretzelosity**



# Gluon TMD correlator and T-odd gluon densities



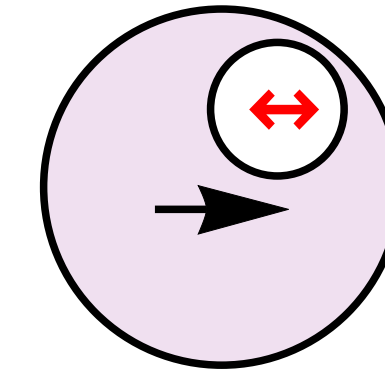
$$\Gamma_U^{ij}(x, k) = x \left[ \delta_T^{ij} f_1(x, k^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, k^2) \right]$$

$$\Gamma_L^{ij}(x, k) = x \left[ i\epsilon_T^{ij} S_L g_1(x, k^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, k^2) \right]$$

$$\Gamma_T^{ij}(x, k) = x \left[ \frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, k^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, k^2) \right.$$

$$\left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, k^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, k^2) \right]$$

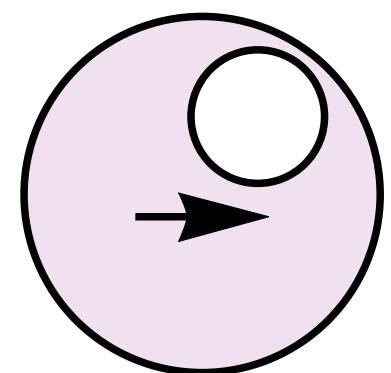
pseudo worm-gear



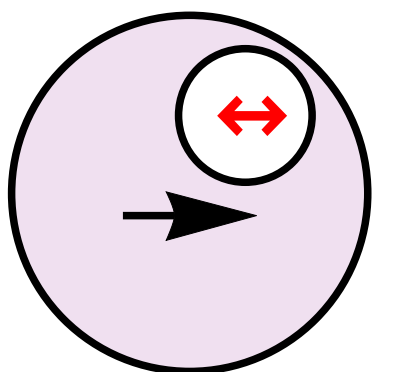
linearity TMD

pretzelosity

Sivers



$$\frac{\epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, k^2) = \frac{1}{2} \delta_{Tij} \Gamma_T^{ij}(x, k)$$



Backup

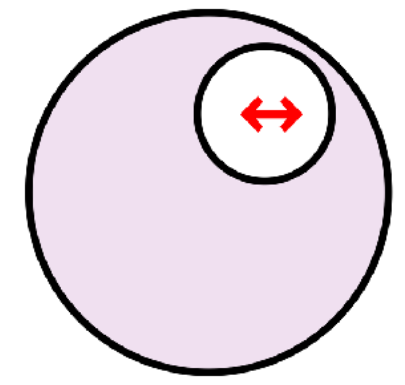


# Gluon TMD phenomenology

The background features a complex, multi-layered illustration of gluon transverse momentum distributions (TMDs). It consists of several overlapping, semi-transparent circular regions. Each region contains a network of yellow wavy lines representing gluons, with various colored spheres (red, blue, green) and arrows indicating interactions and spin. The overall aesthetic is scientific and abstract, with a light blue and white color palette and a subtle grid pattern.



# Boer-Mulders effect in unpolarized pp collisions

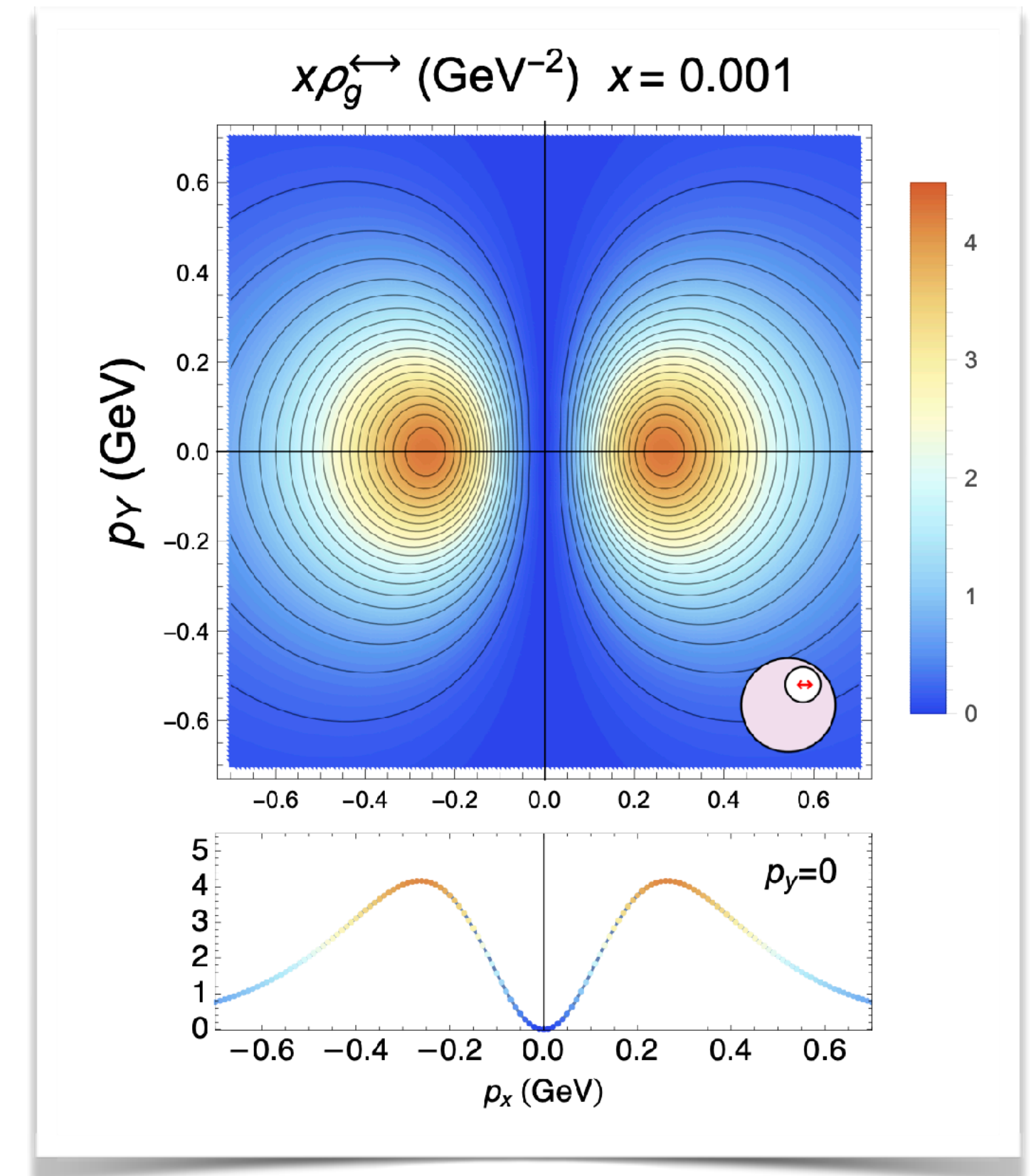


$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

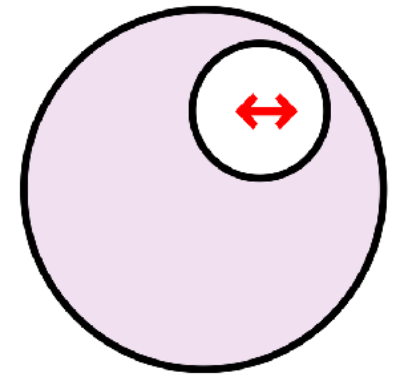
unpol.

Boer-Mulders



[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

# Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar  $p_T$ -distributions: Higgs,  $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

at low transverse momentum  
for (pseudo)scalar state

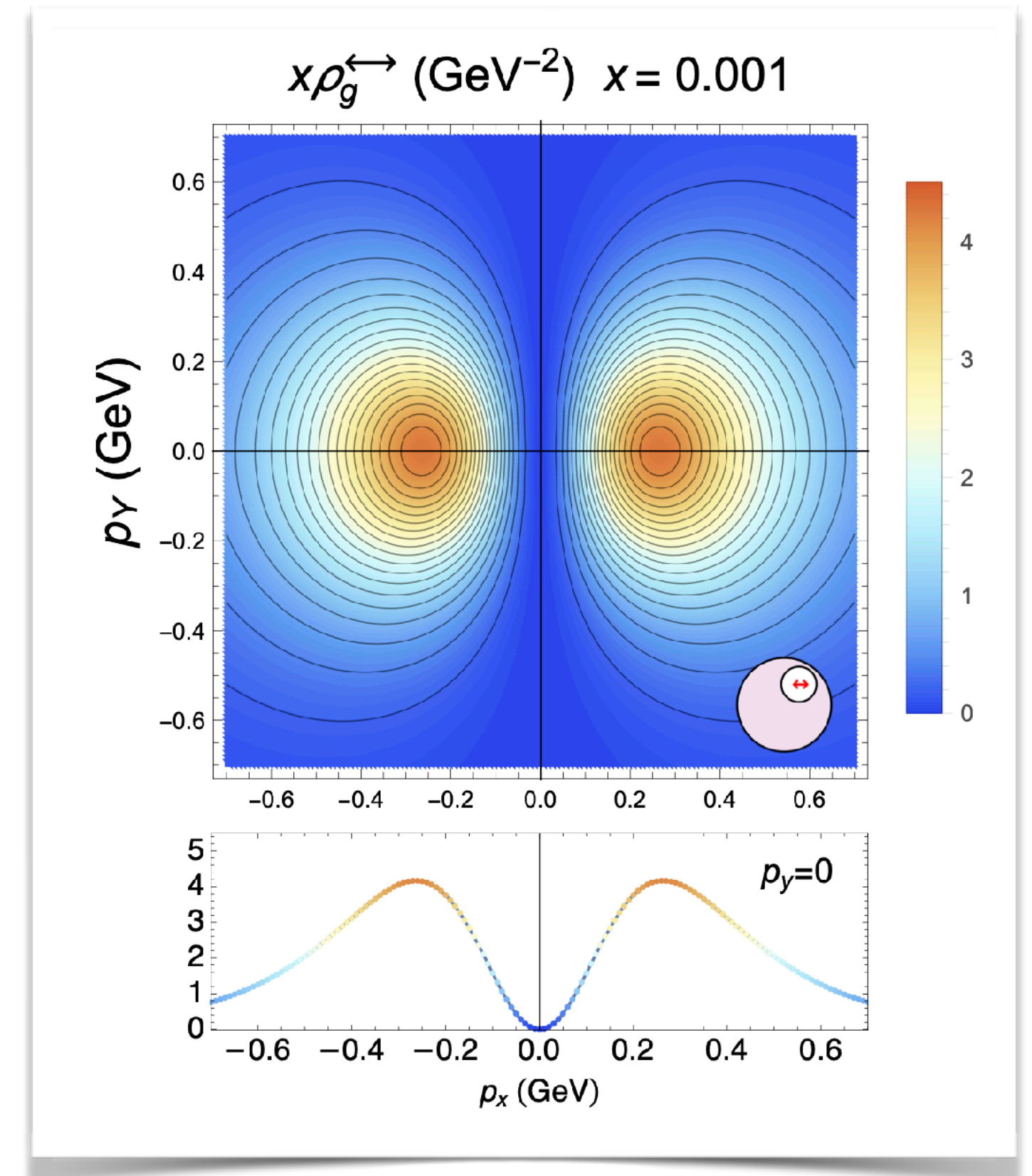
$$\sim \mathcal{C} \left[ \begin{matrix} f_1^{g/A} & f_1^{g/B} \end{matrix} \right] \pm \mathcal{C} \left[ \begin{matrix} h_1^{\perp g/A} & h_1^{\perp g/B} \end{matrix} \right]$$

unpolarized gluons

lin. polarized gluons

NRQCD

$$\frac{CS}{CO} \sim \frac{1}{v^4}$$



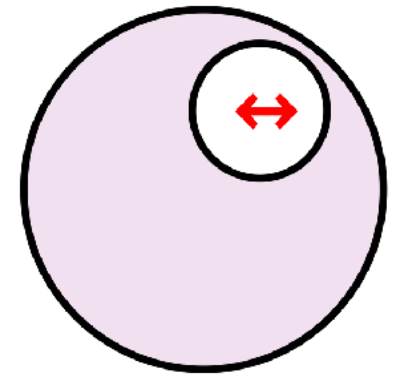
[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]  
(Higgs+jet angular distributions)

[D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]



# Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



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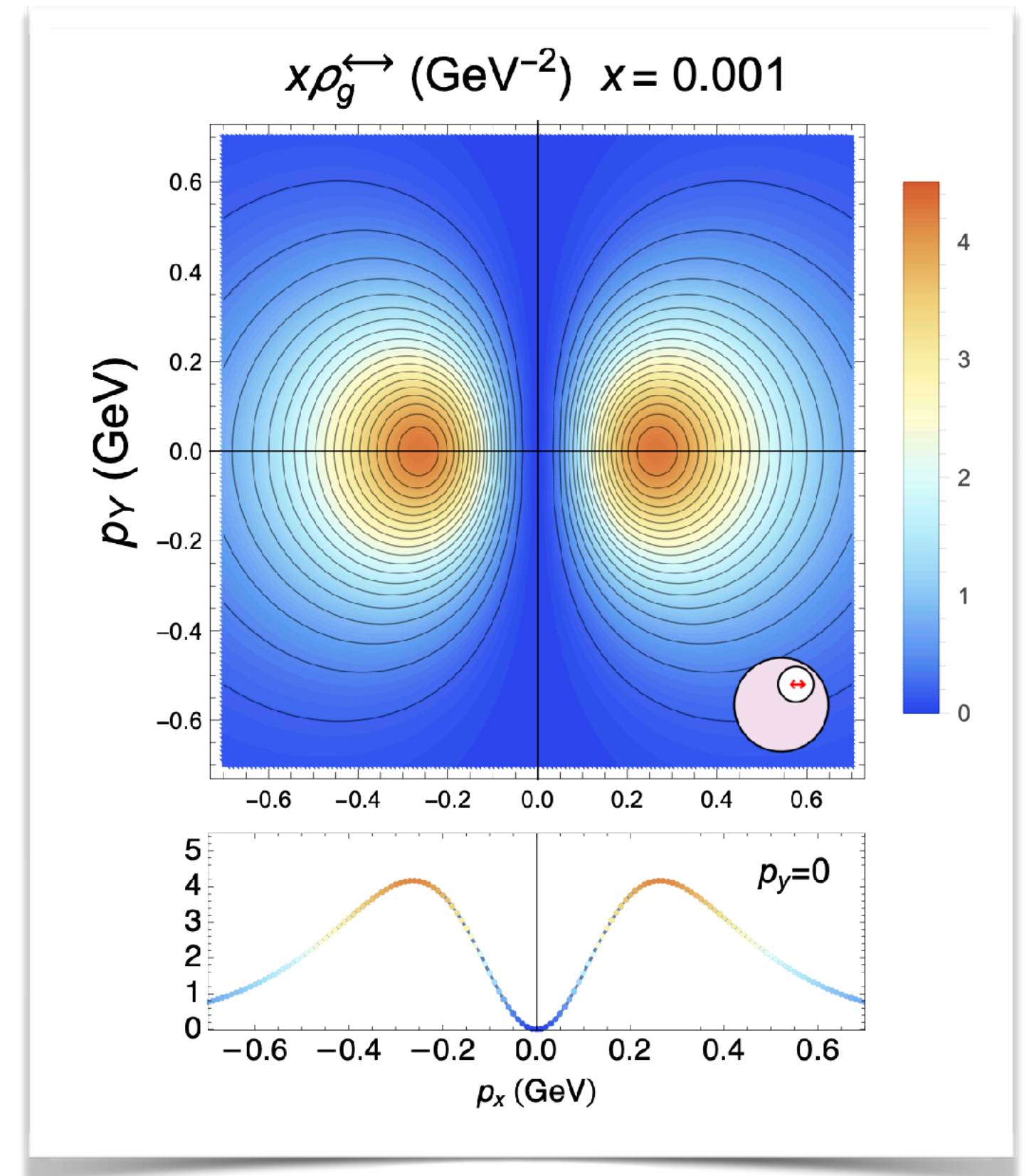
$$\sim \mathcal{C} \left[ \begin{array}{cc} f_1^{g/A} & f_1^{g/B} \end{array} \right] \pm \mathcal{C} \left[ \begin{array}{cc} h_1^{\perp g/A} & h_1^{\perp g/B} \end{array} \right]$$

unpolarized gluons

lin. polarized gluons

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[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]  
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[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

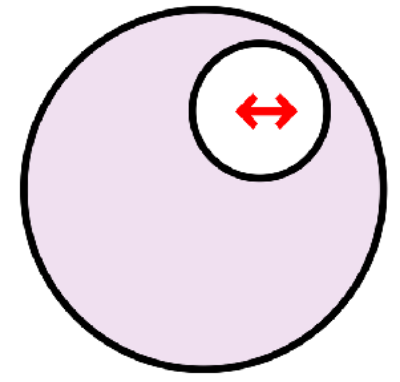


Model prediction at low x

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$



# Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar  $p_T$ -distributions: Higgs,  $\eta_{c,b}$

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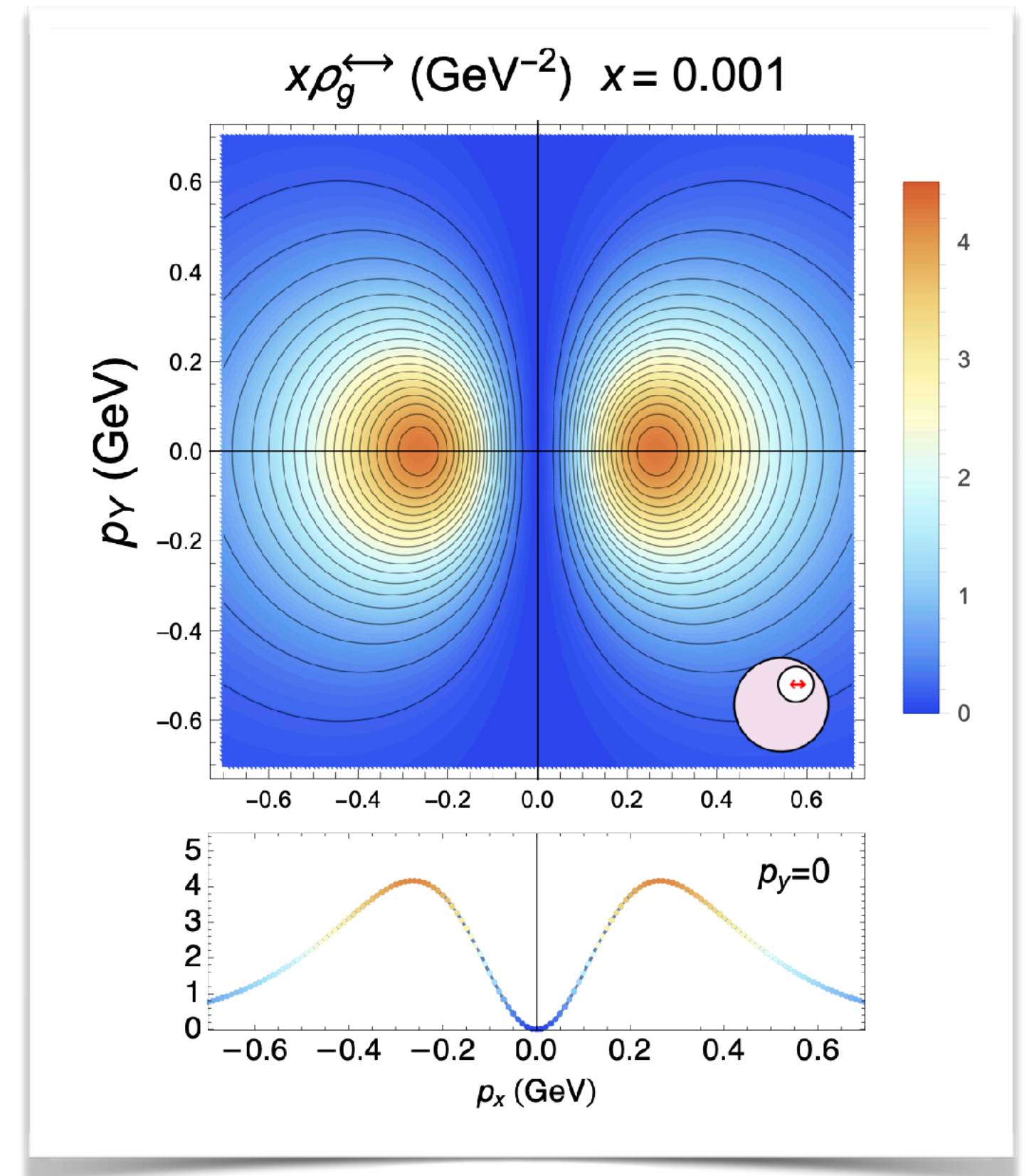
$$\sim \mathcal{C} \begin{bmatrix} f_1^{g/A} & f_1^{g/B} \end{bmatrix} \pm \mathcal{C} \begin{bmatrix} h_1^{\perp g/A} & h_1^{\perp g/B} \end{bmatrix}$$

unpolarized gluons

lin. polarized gluons

NRQCD

$$\frac{CS}{CO} \sim \frac{1}{v^4}$$



[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]  
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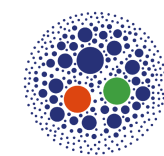
[D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]



Model prediction at low x

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$



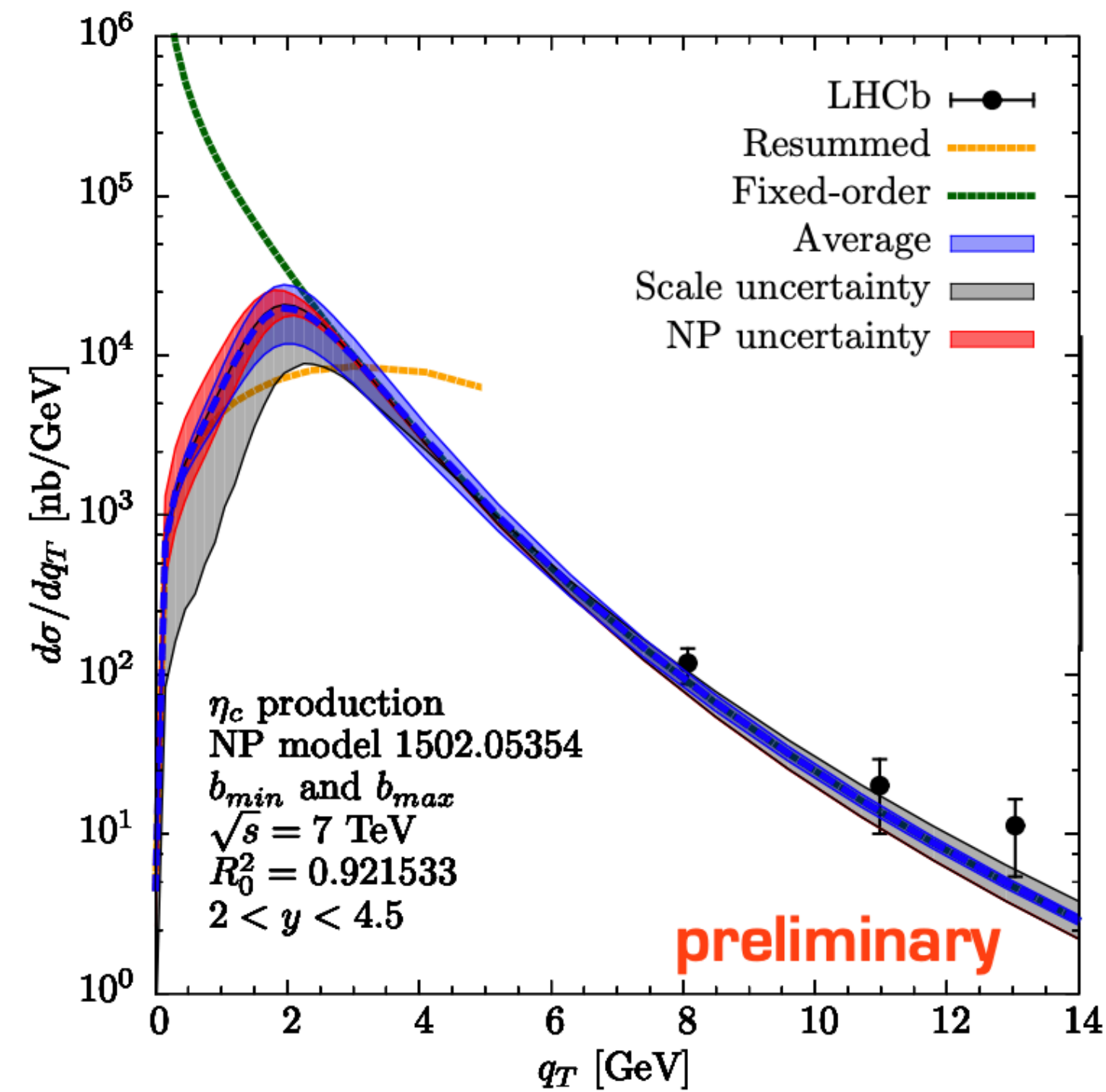
BFKL regime (linear low-x evolution)



$$f_1^g(x, p_T^2) = h_1^{\perp g}(x, p_T^2) + \text{higher twist}$$

## $\eta_c$ production at LHC

full transverse momentum spectrum:  
low  $q_T$  matched with high  $q_T$  region



**blue band:** uncertainty from matching

**grey band:** scale uncertainty

**red band:** nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = - \left[ \frac{a_1}{2} + \frac{a_2}{2} \ln Q^2 \right] \bar{b}_T^2$$

$a_i = 0.5 \text{ GeV}^2$ , var. 50%, envelope

both for unpolarized and  
linearly polarized distributions

**the formalism is in good shape!**  
we need the data at low  $q_T$

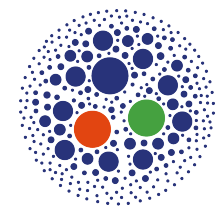




# HEF and the UGD

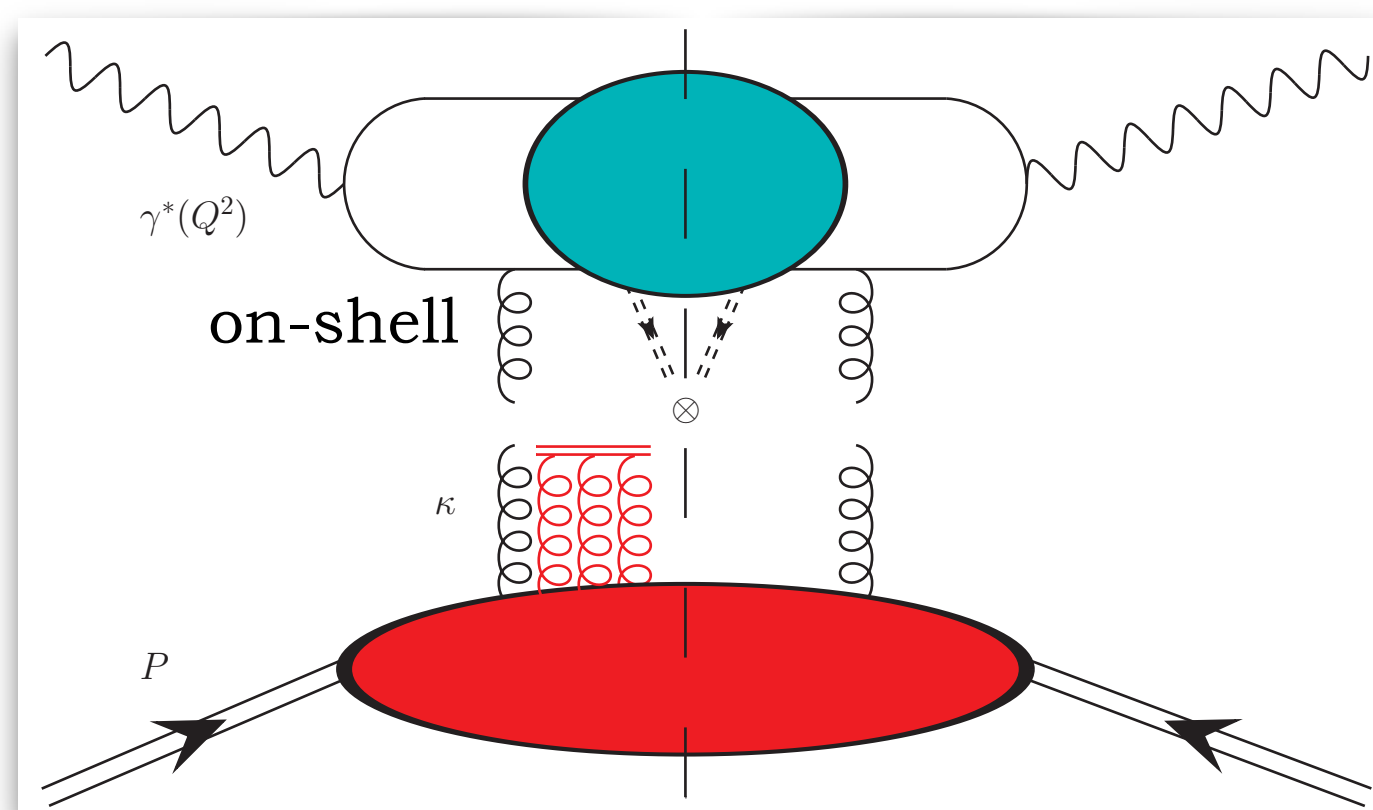


# TMD versus high-energy factorization



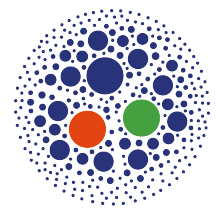
## TMD

- \* Semi-inclusive processes
- \*  $\kappa_T \ll$  hardest scale
- \* Language of **parton correlators**
- \* Diagram: SIDIS onium



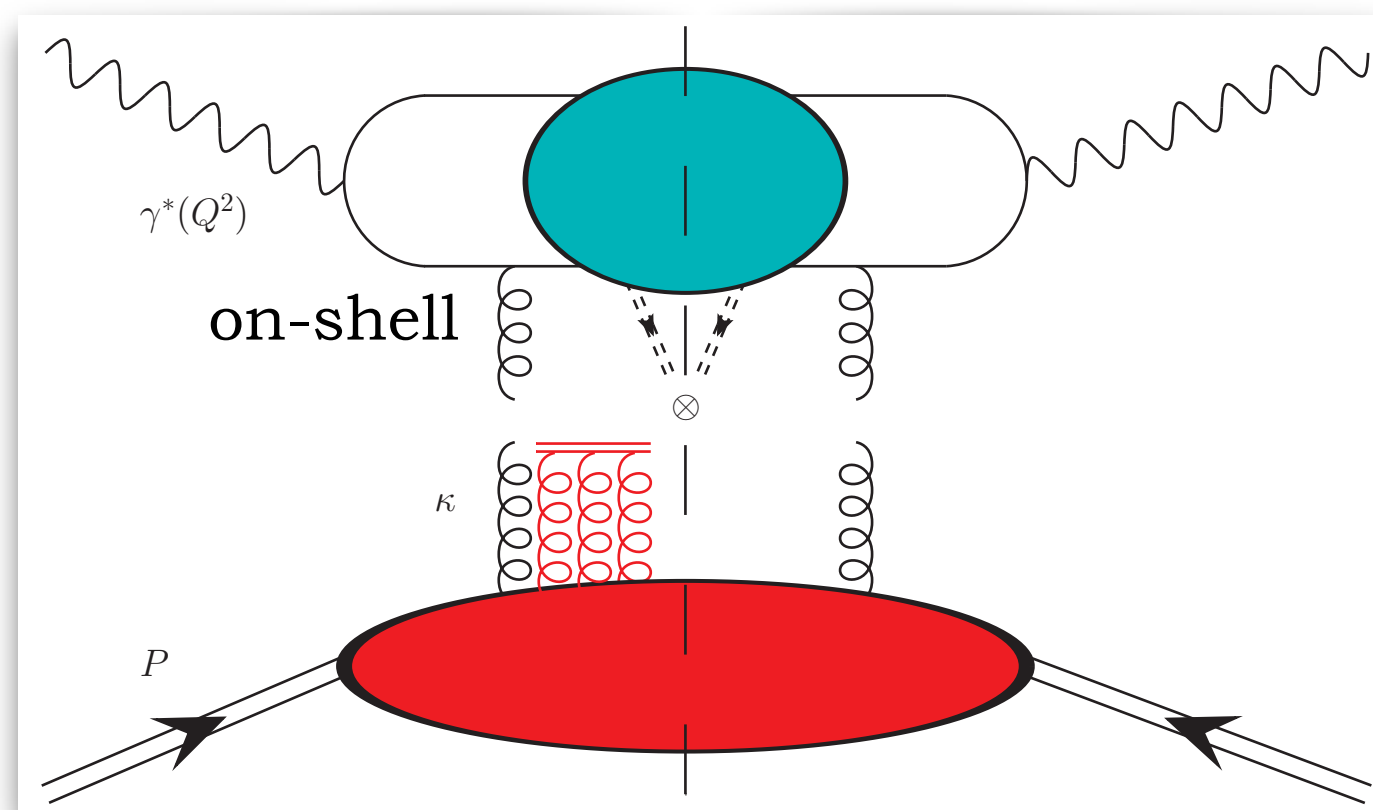
TMD  
PDF

# TMD versus high-energy factorization

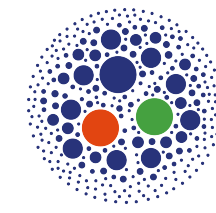


## TMD

- \* Semi-inclusive processes
- \*  $\kappa_T \ll$  hardest scale
- \* Language of **parton correlators**
- \* Diagram: SIDIS onium

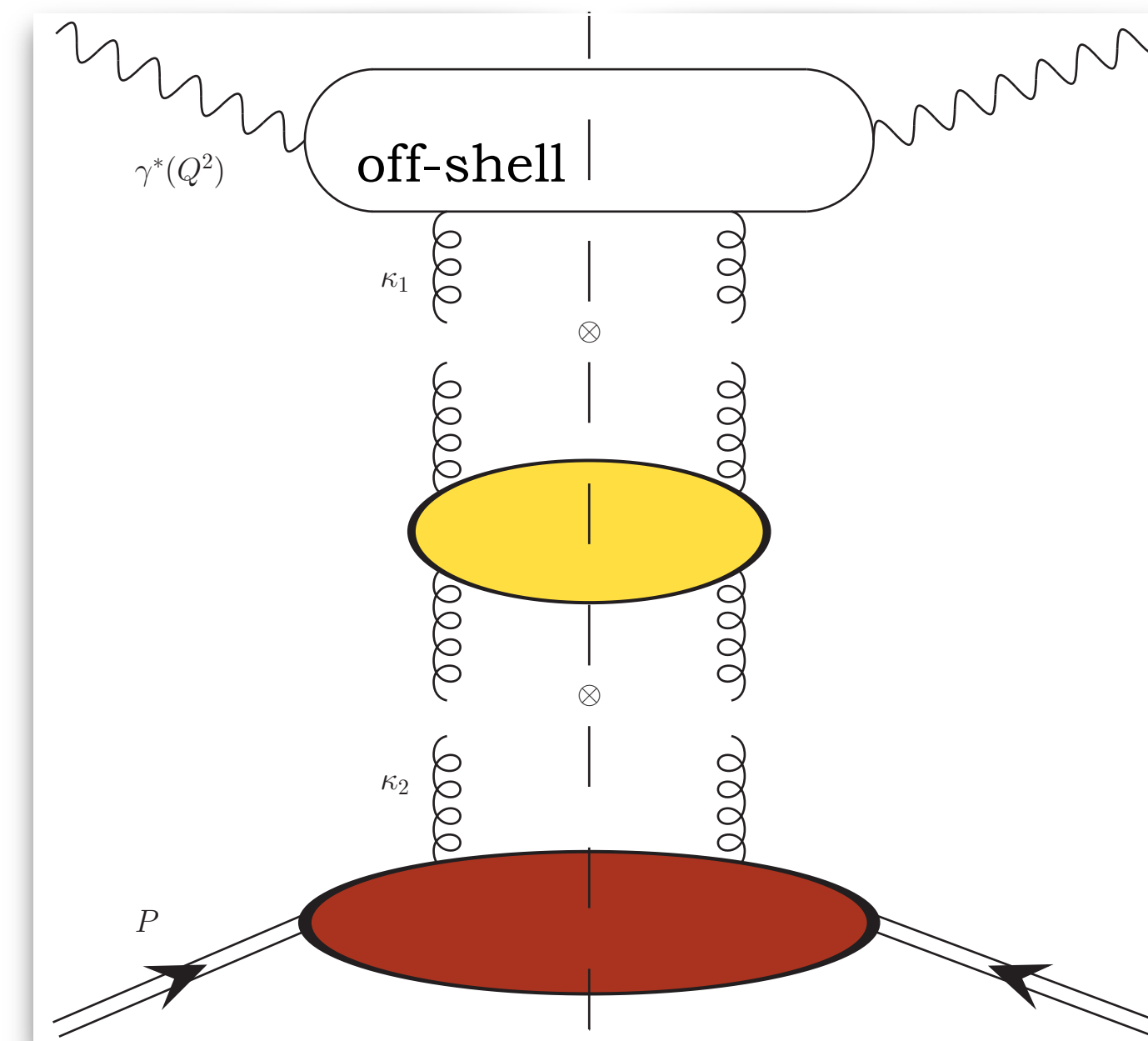


TMD  
PDF



## HEF

- \* Inclusive or exclusive processes (!)
- \* Small  $x$ , large  $\kappa_T$
- \* Language of **Reggeized gluons**
- \* Diagram: DIS



$\Phi \gamma^* \rightarrow \gamma^*$



$\mathcal{G}_{\text{BFKL}}$



$\Phi^P_{[\text{NP}]}$

# TMD versus high-energy factorization

TMD

IR-safe colorless  $\{\Phi^{i \rightarrow 0}\}$

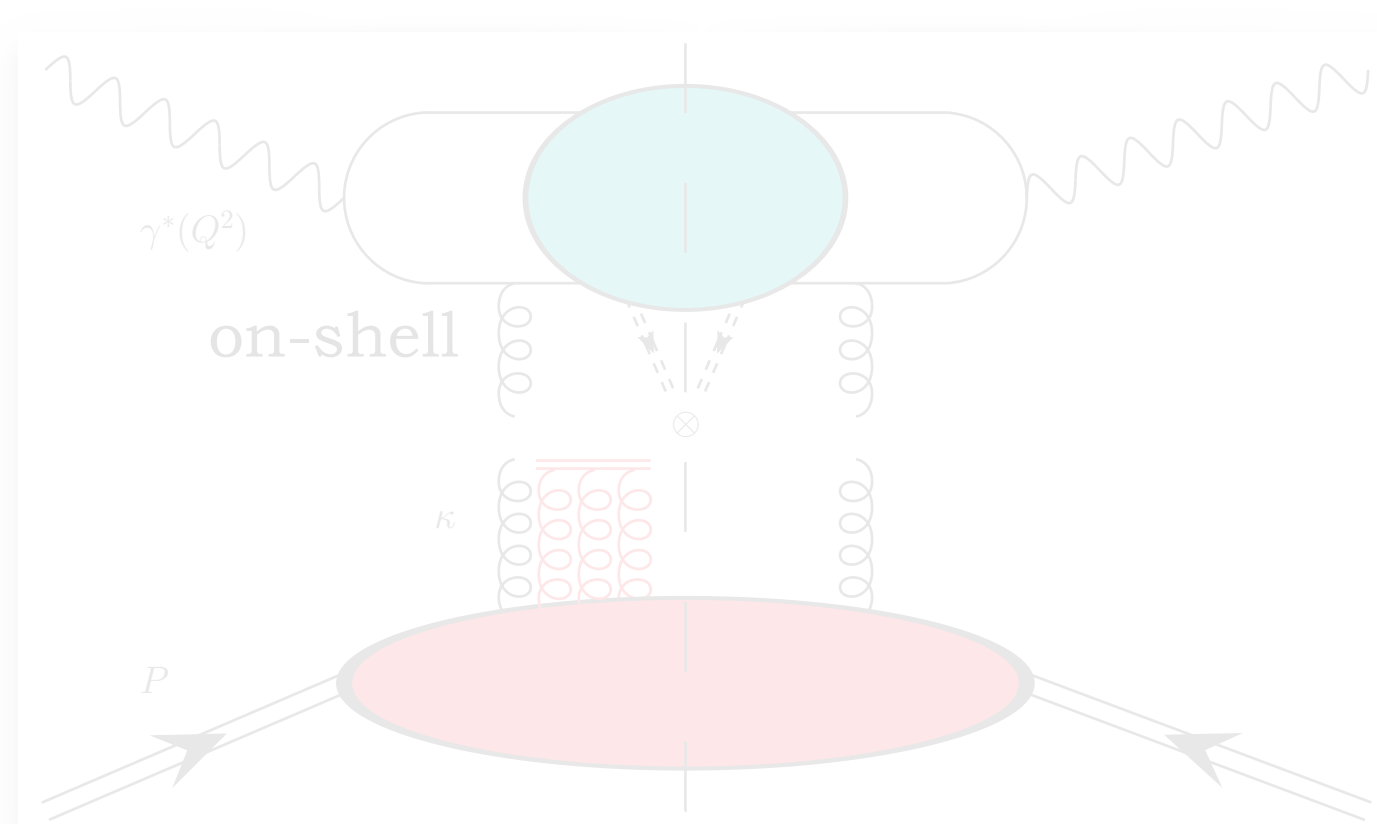
(Fadin-Martin theorem)

\* Semi-inclusive processes  
 [V.S. Fadin, A.D. Martin (1999)]

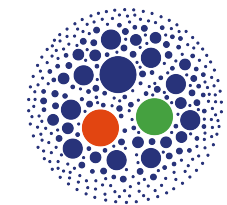
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\* Diagram: SIDIS onium



TMD  
PDF



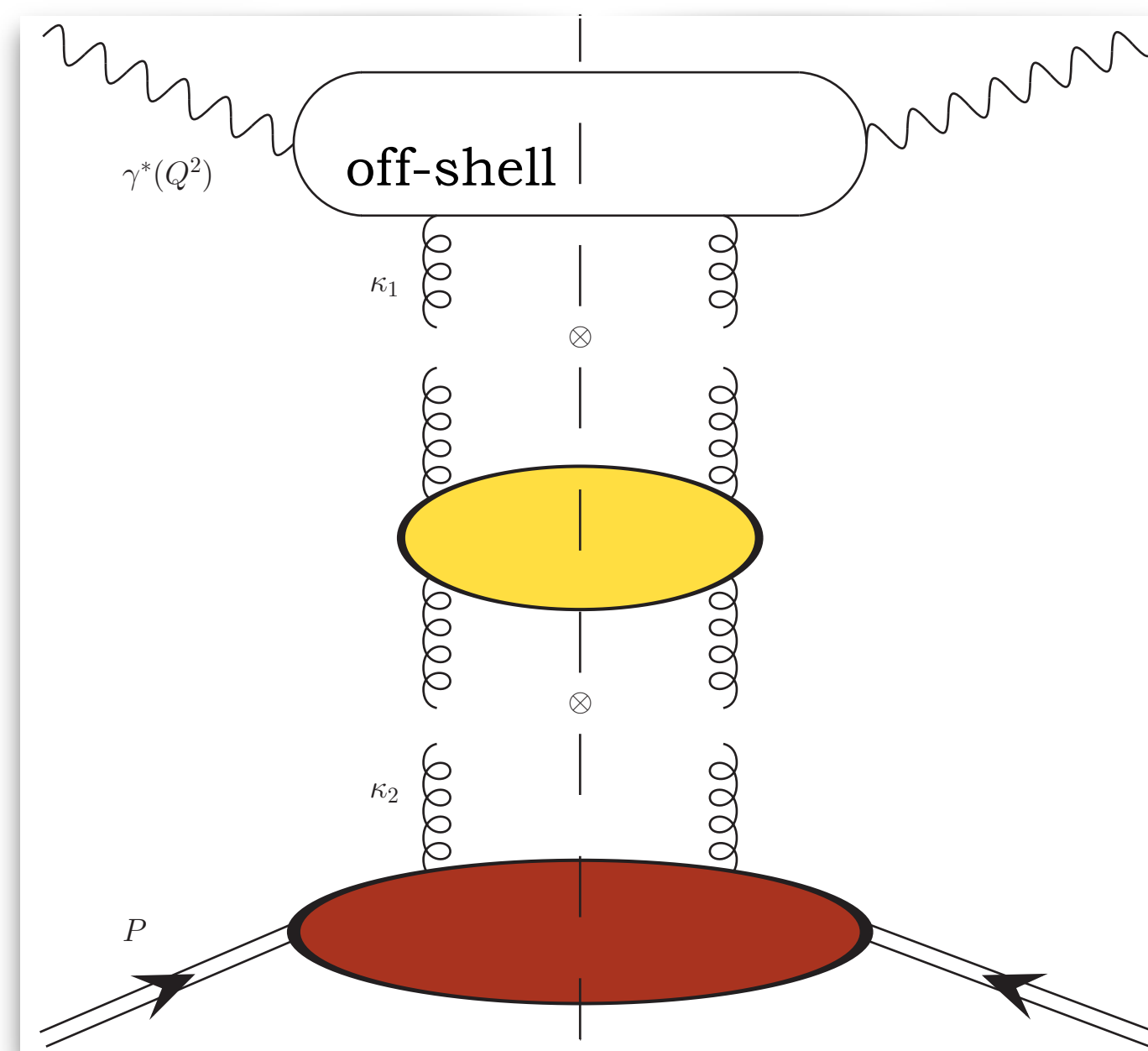
HEF

\* Inclusive or exclusive processes (!)

\* Small  $x$ , large  $\kappa_T$

\* Language of Reggeized gluons

\* Diagram: DIS



$\Phi^{\gamma^* \rightarrow \gamma^*}$



$\mathcal{G}_{\text{BFKL}}$

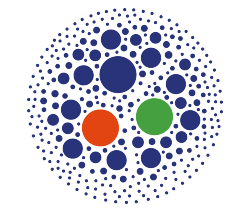


$\Phi^P_{[\text{NP}]}$

Backup

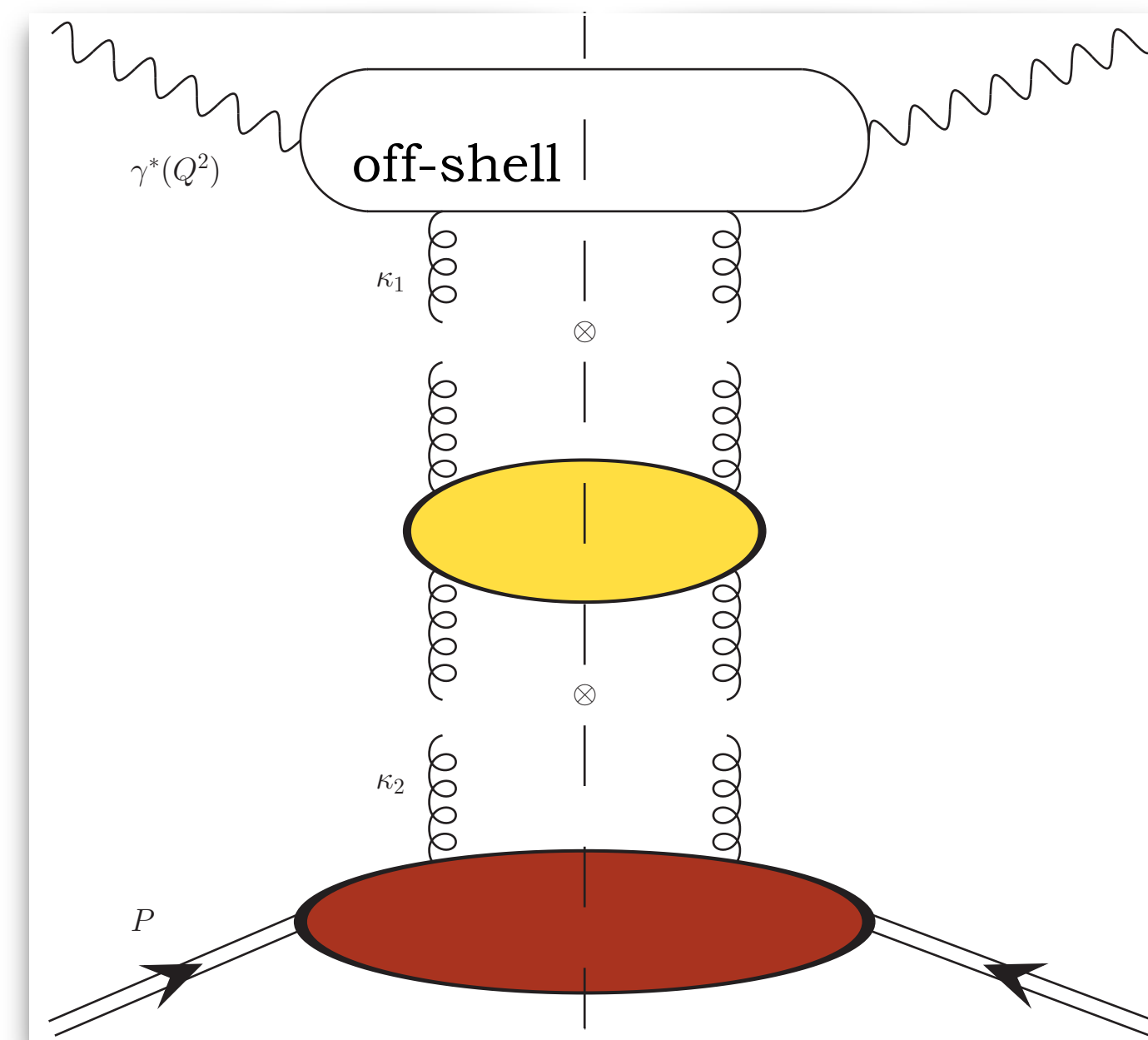


# TMD versus high-energy factorization



**HEF**

- \* Inclusive or exclusive processes (!)
- \* Small  $x$ , large  $\kappa_T$
- \* Language of **Reggeized gluons**
- \* Diagram: DIS



$\Phi^{\gamma^* \rightarrow \gamma^*}$

$\otimes$

$\mathcal{G}_{\text{BFKL}}$

$\otimes$

$\Phi^P_{[\text{NP}]}$

**IR-safe colorless  $\{\Phi^{i \rightarrow 0}\}$**

(Fadin-Martin theorem)

- \* Semi-inclusive processes  
[V.S. Fadin, A.D. Martin (1999)]

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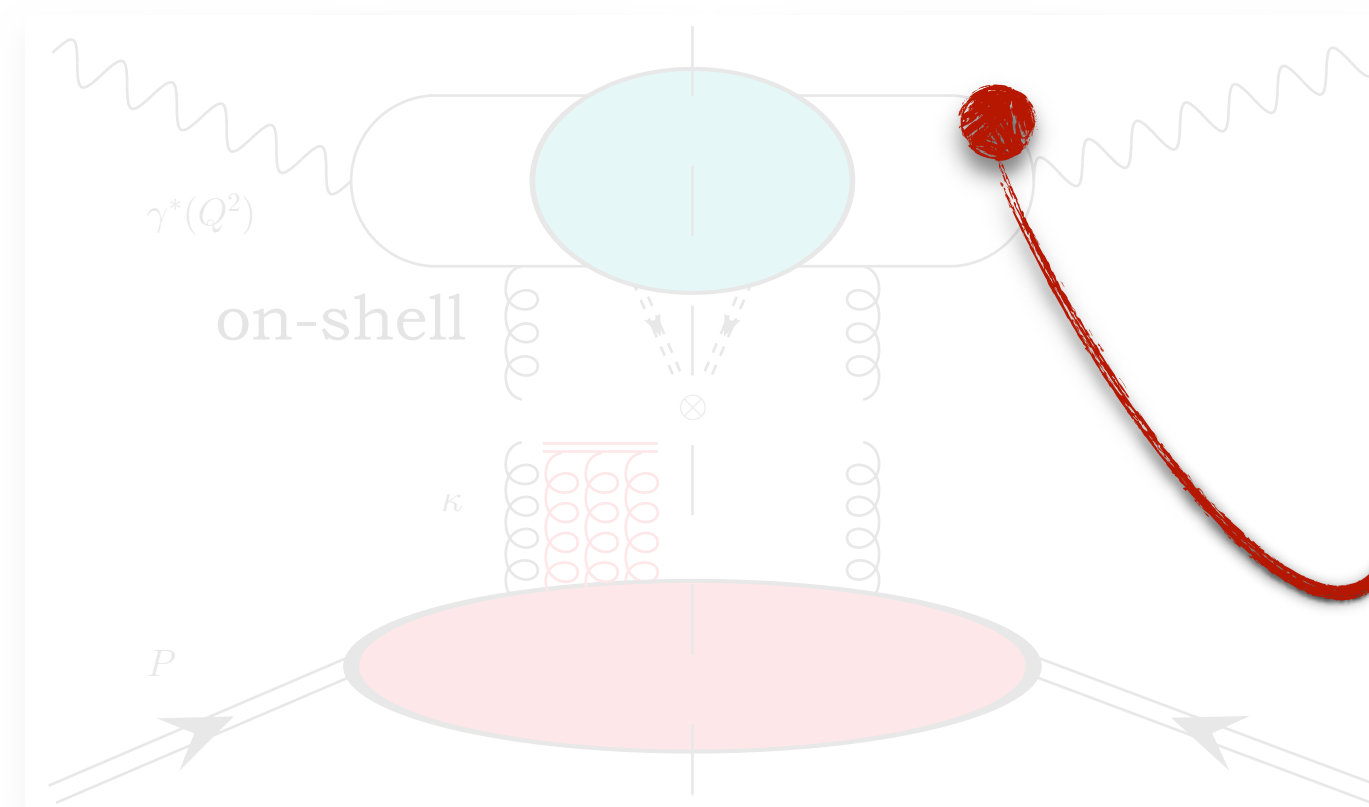
- \* Language of **parton correlators**

- \* Diagram: SIDISonium

**IR diffusion pattern**

(Bartels' cigar)

- [J. Bartels, H. Lotter (1993)]



$Q^2$

$\updownarrow$

$Q_0^2$

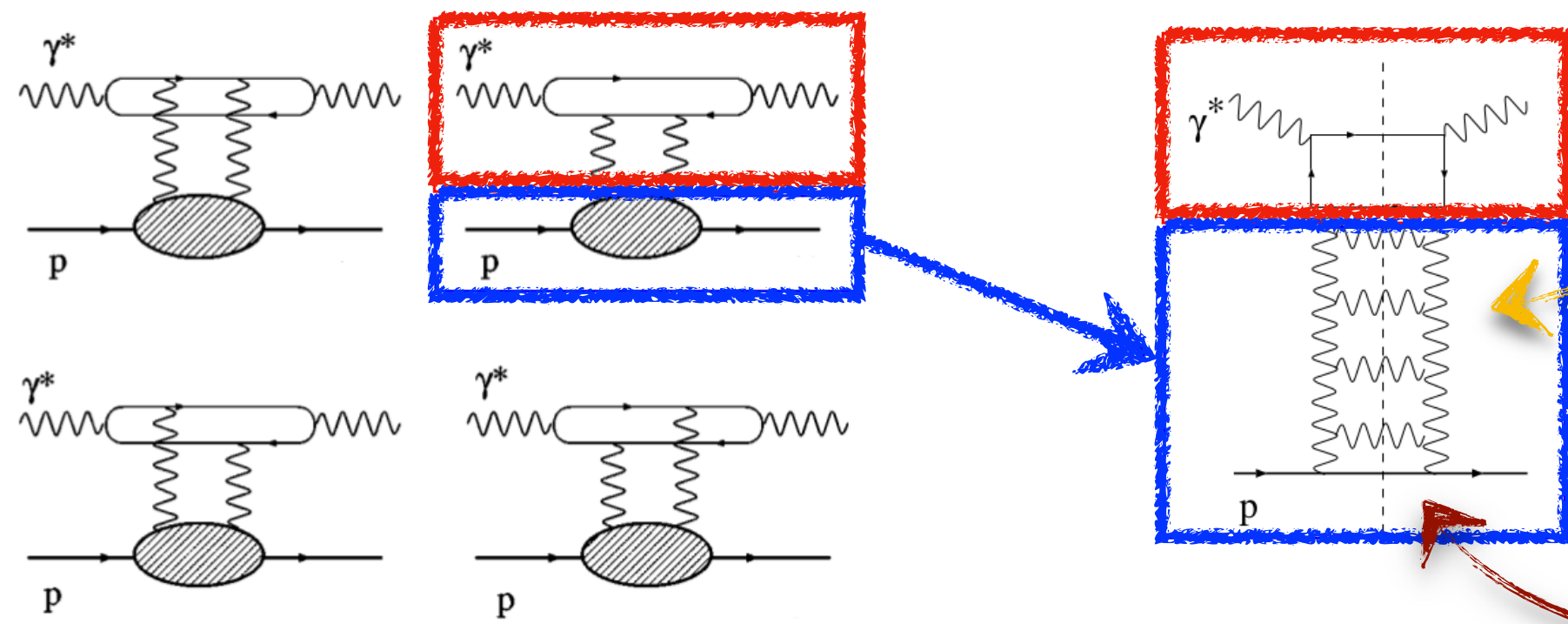
TMD  
PDF

# High-energy factorization and the UGD

- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\text{tot}}(\gamma^* p \rightarrow X) \propto \text{Im}_s \{ \mathcal{A}(\gamma^* p \rightarrow \gamma^* p) \} \equiv \Phi_{\gamma^* \rightarrow \gamma^*} \circledast \mathcal{F}(x, \kappa^2)$$

- ◇  $\mathcal{F}(x, \kappa^2)$  is the **unintegrated gluon distribution (UGD)** in the proton

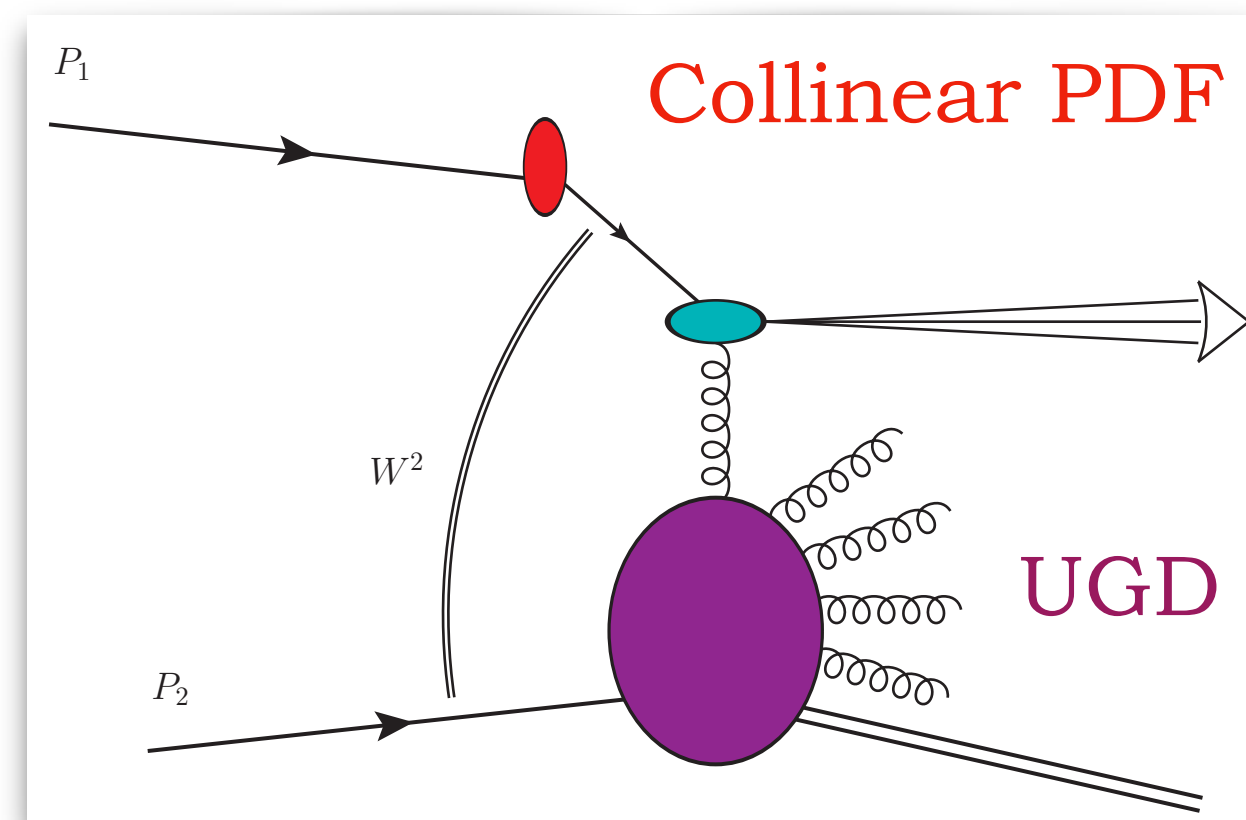


- ▶ Small- $x$  limit: **UGD** = [ **BFKL gluon ladder** ]  $\circledast$  [ **proton impact factor** ]
  - ◇ Takes into account the **resummation** of **high-energy logs**
  - ◇ Describes the **coupling** of the gluon Green's function to the **proton**
- ▶ Proton impact factor is non-perturbative  $\implies$  UGD needs to be modeled!

# Hybrid or pure factorization?

## Forward emissions

- \* *Asymmetric* config.  $\leftrightarrow$  fast parton + small- $x$  gluon
- \* Hybrid **high-energy** / **collinear** factorization



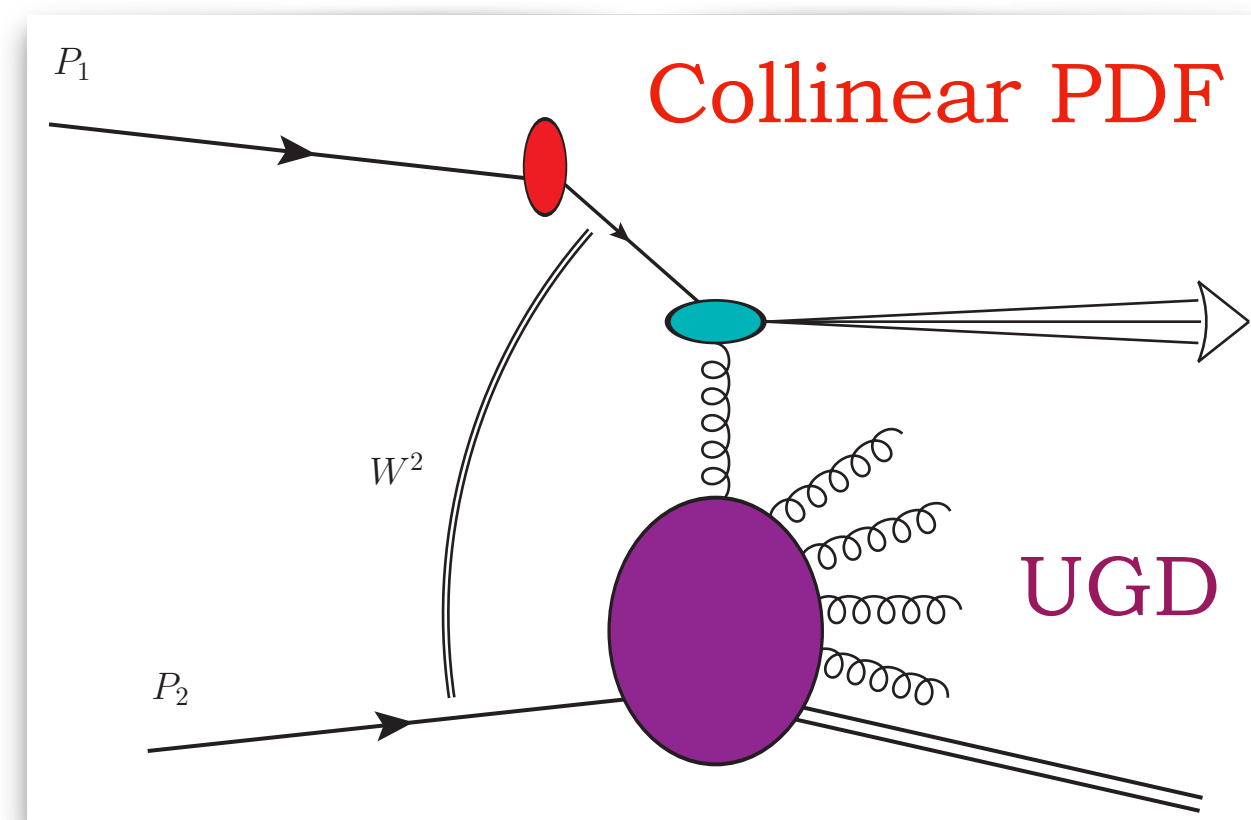
- \* *Distinctive signals* of small- $x$  dynamics **expected**
- \* Phenomenology:  
*forward jet, Drell-Yan, Higgs or vector meson*



# Hybrid or pure factorization?

## Forward emissions

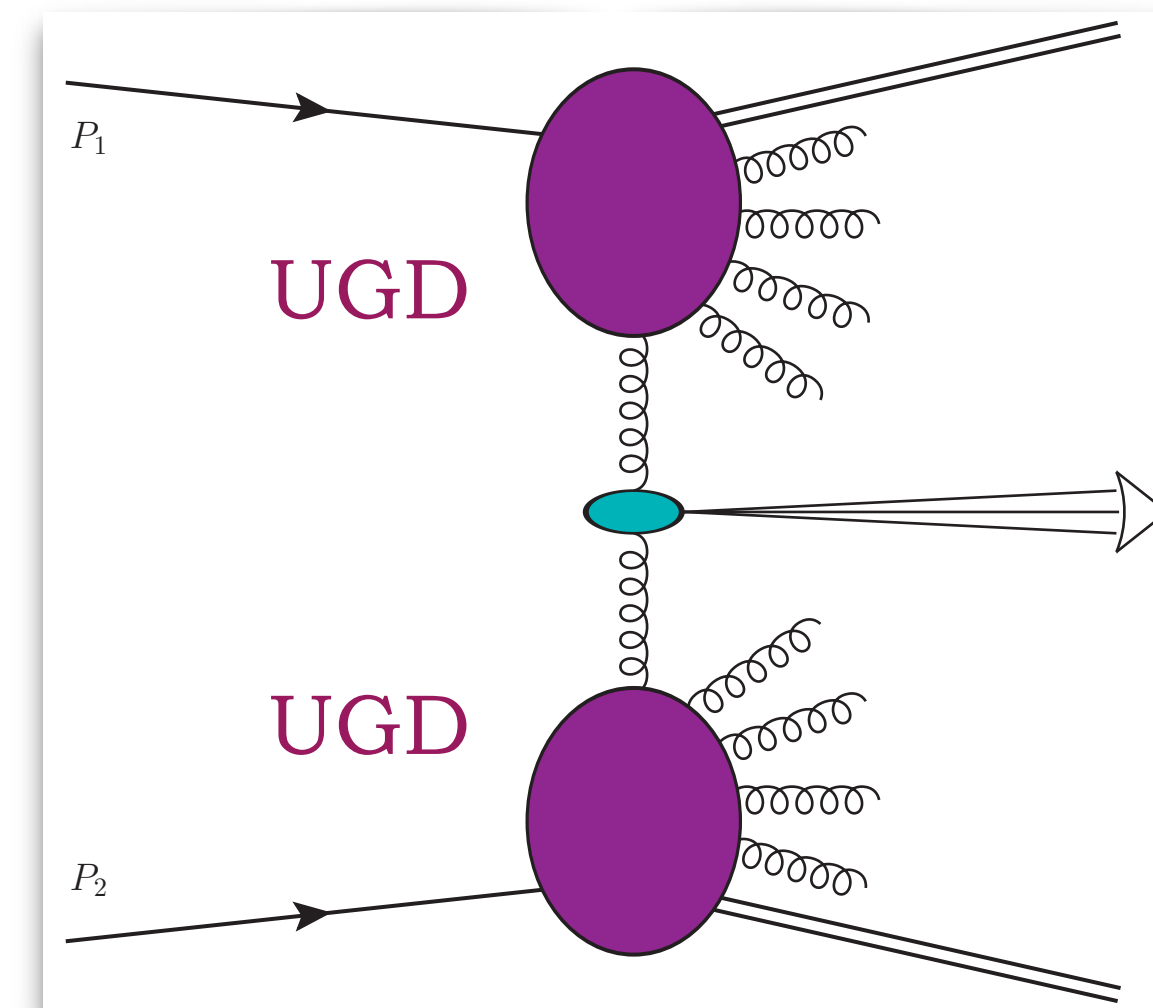
- \* *Asymmetric* config.  $\leftrightarrow$  fast parton + small- $x$  gluon
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- \* *Distinctive signals* of small- $x$  dynamics **expected**
- \* Phenomenology:  
*forward jet, Drell-Yan, Higgs or vector meson*

## Central emissions

- \* *Gluon induced*  $\leftrightarrow$  small- $x$  gluons
- \* Pure **high-energy** factorization

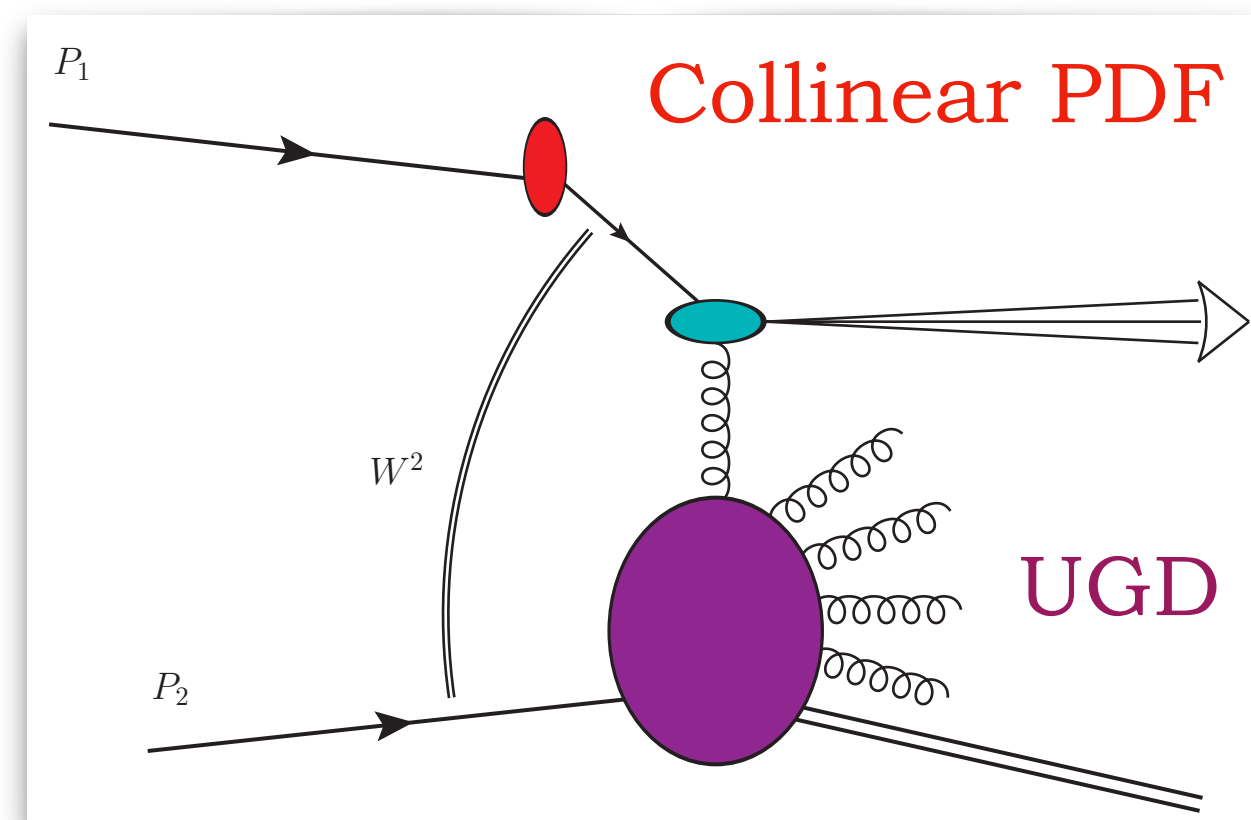


- \* Small- $x$  dynamics to **enhance** f.o. description
- \* Phenomenology:  
*central jet, Higgs or vector meson*

# Hybrid or pure factorization?

## Forward emissions

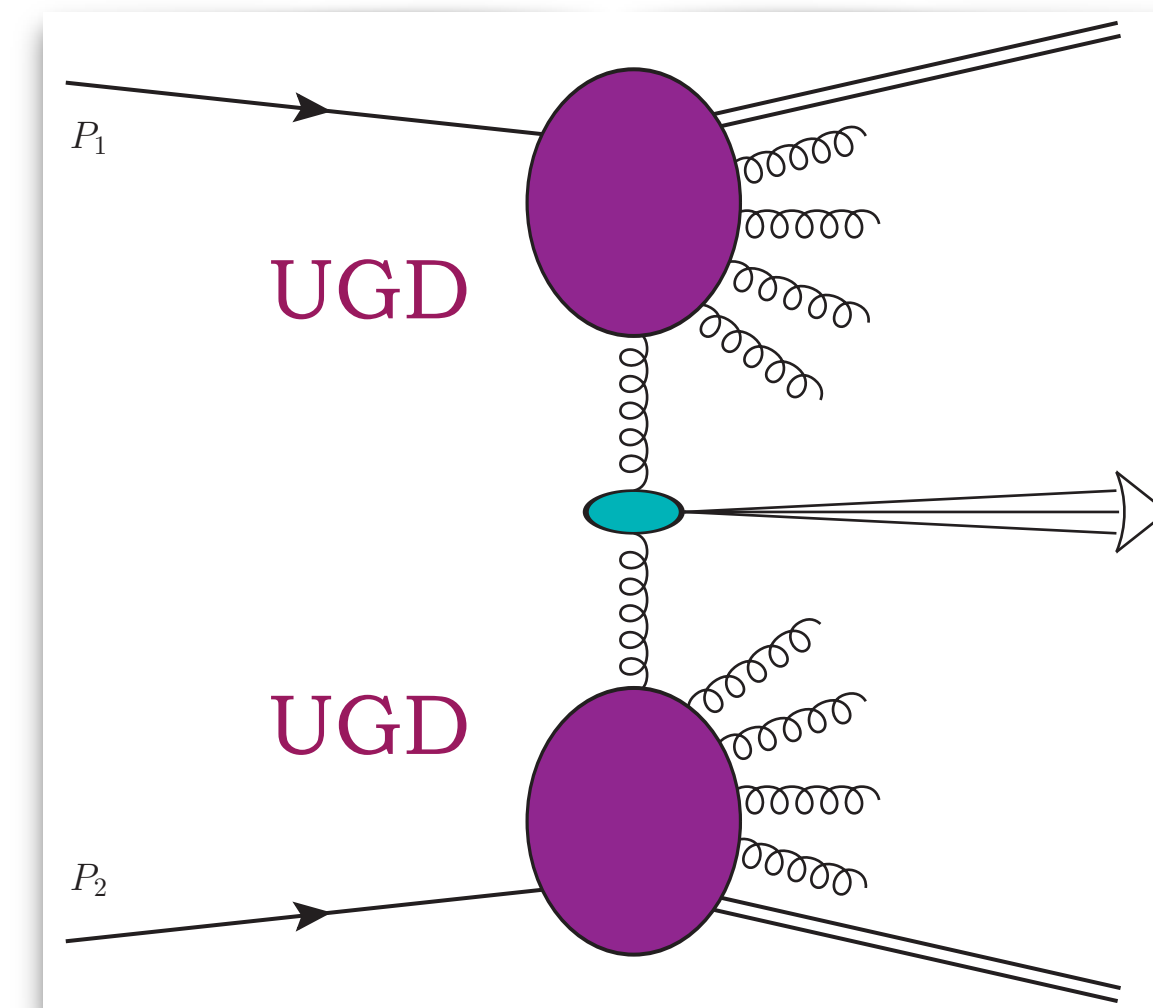
- \* *Asymmetric* config.  $\leftrightarrow$  fast parton + small- $x$  gluon
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## Central emissions

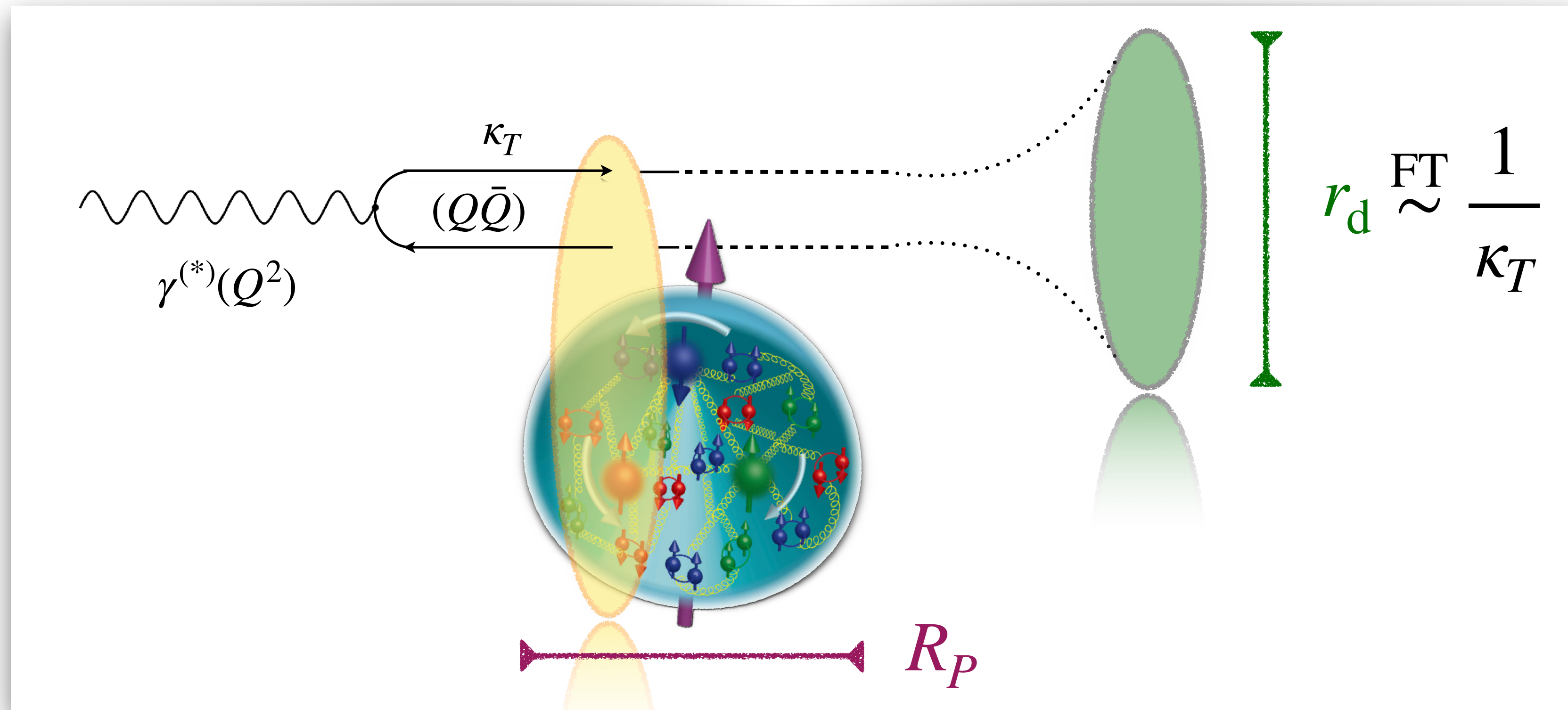
- \* *Gluon induced*  $\leftrightarrow$  small- $x$  gluons
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- \* Small- $x$  dynamics to **enhance** f.o. description
- \* Phenomenology:  
*central* jet, Higgs or vector meson

Table complemented by *exclusive* counterparts and *lepto-hadronic* channels

# Diffractive $\gamma^*P$ scatterings and color dipoles



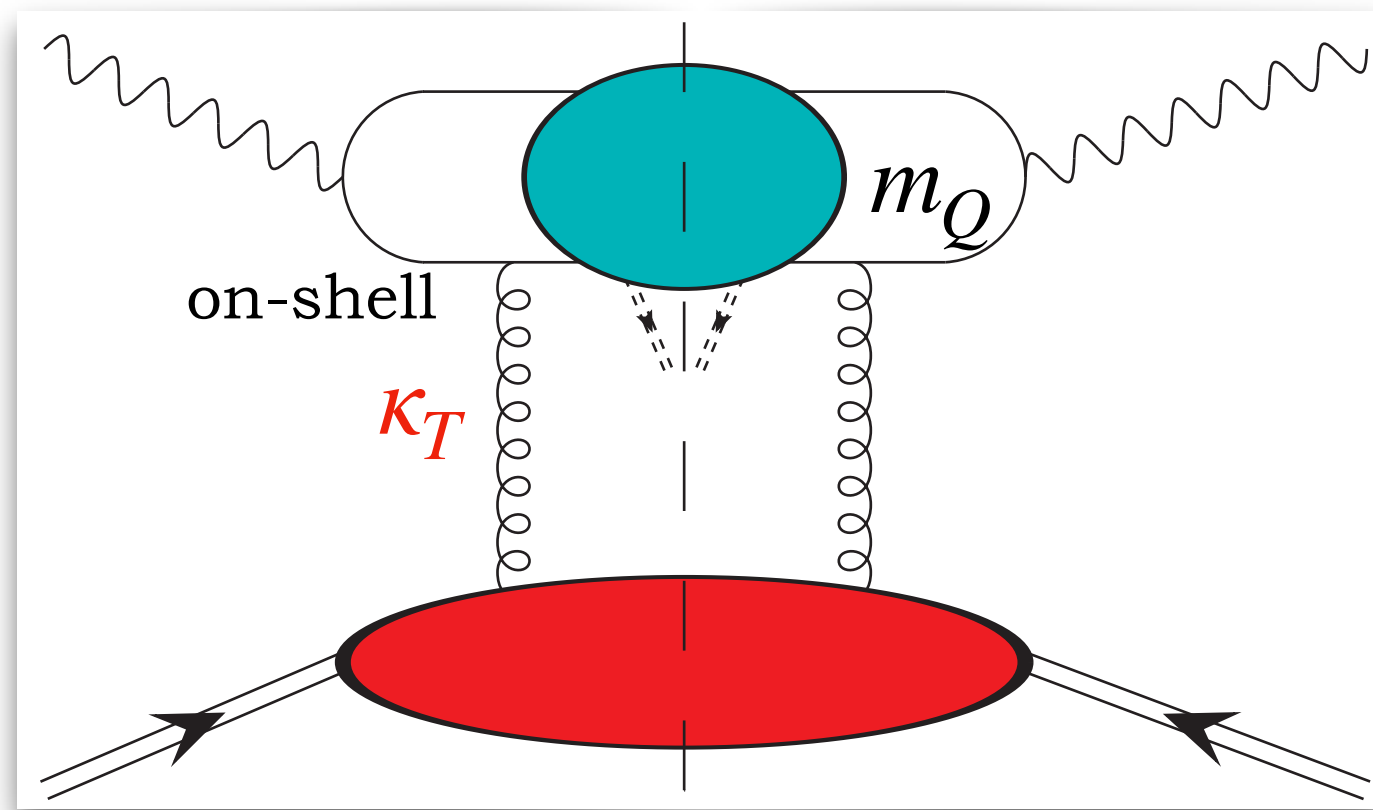
$$W_{\mu\nu} \propto \text{Im} \left\{ i \int d^4x e^{iq \cdot x} \langle P | T [J_\mu(x) J_\nu(0)] | P \rangle \right\}$$

- \* Small- $x \Rightarrow$  Ioffe time  $\gg R_P$
- \* At least one  $J_\mu$  outside proton...
- \* ...color dipole picture!



# Inclusive quarkonium production mechanisms

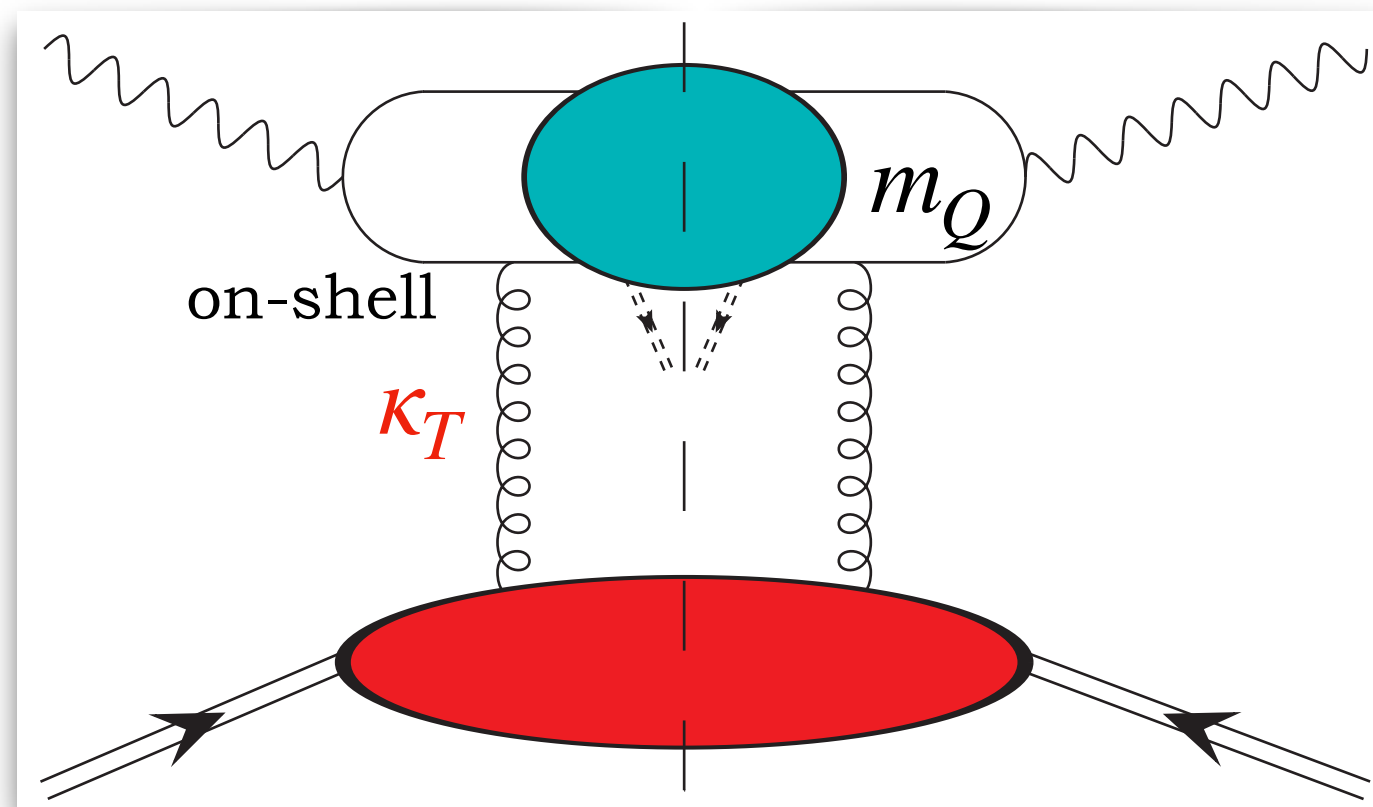
$$\kappa_T \ll Q$$



- \* Gluon TMD PDF
- \* Short-distance ( $Q\bar{Q}$ ) + ShFs

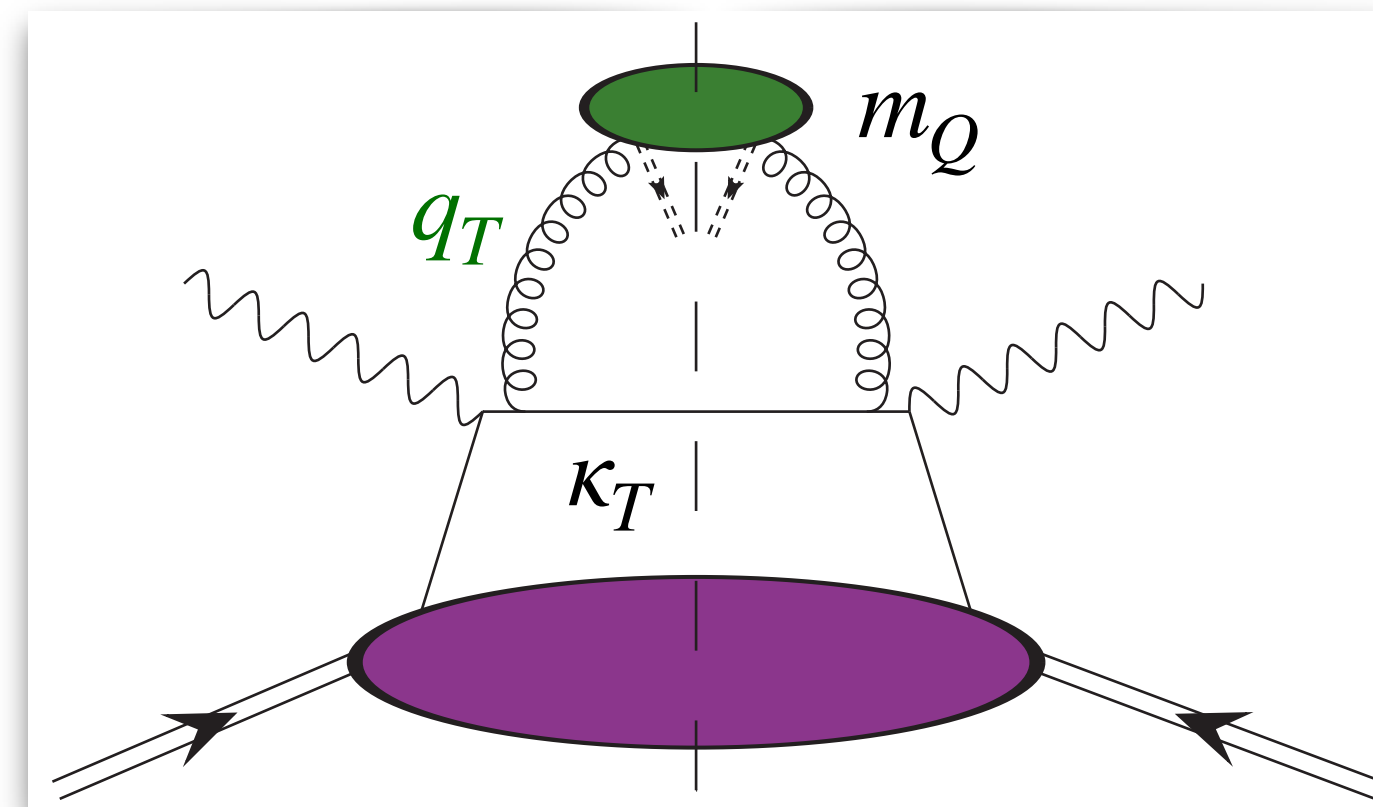
# Inclusive quarkonium production mechanisms

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- \* Gluon TMD PDF
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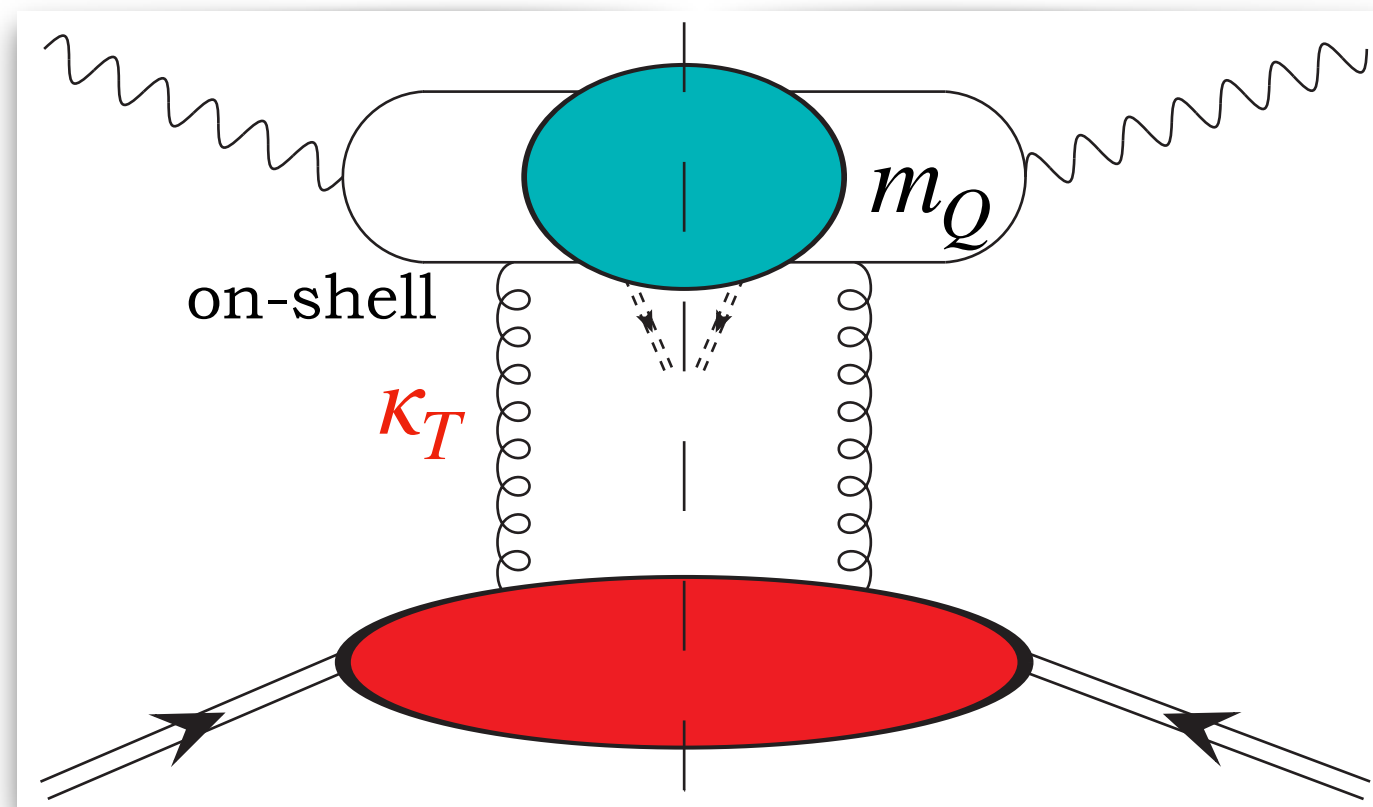
$$\kappa_T \gg m_Q$$



- \* Quark collinear PDF
- \* Onium in jet
- \* Single-quark TMD FF

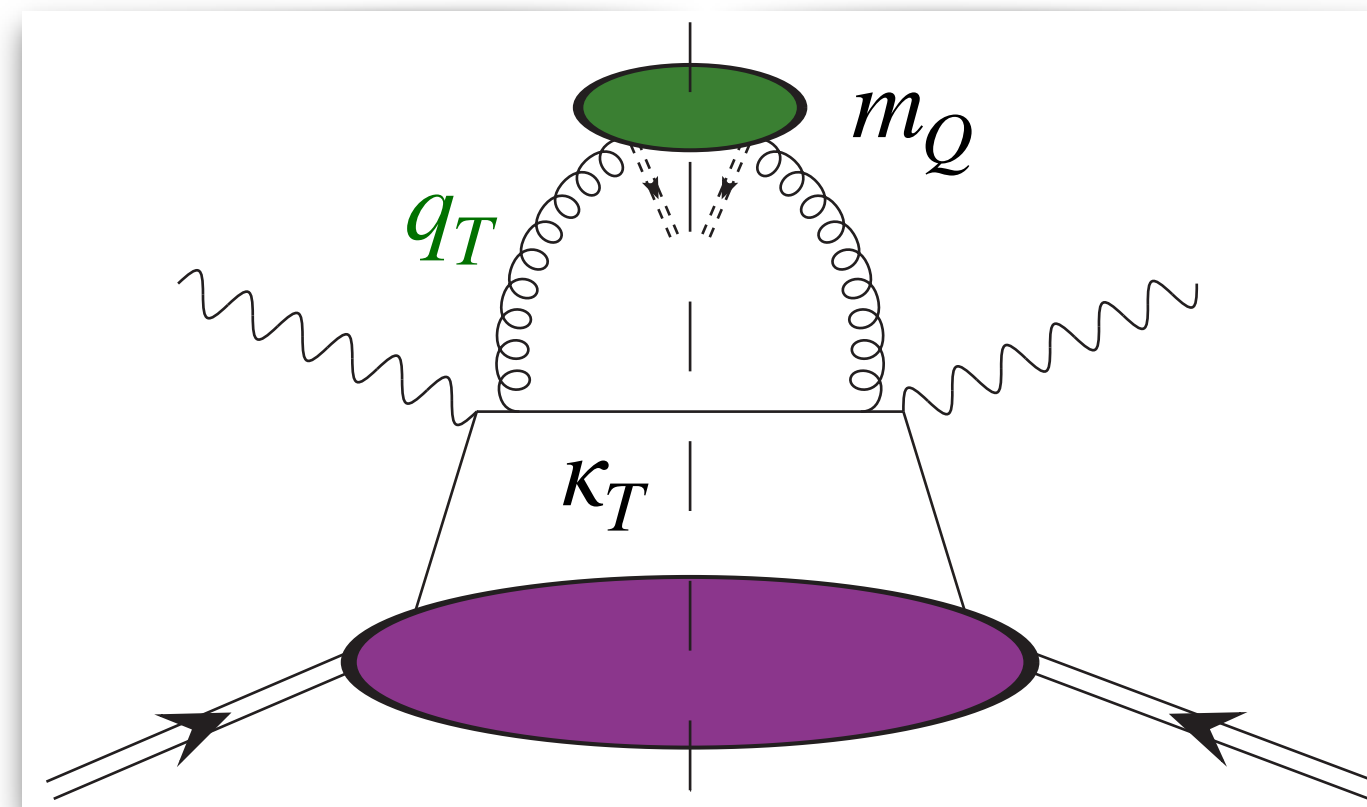
# Inclusive quarkonium production mechanisms

$$\kappa_T \ll Q$$



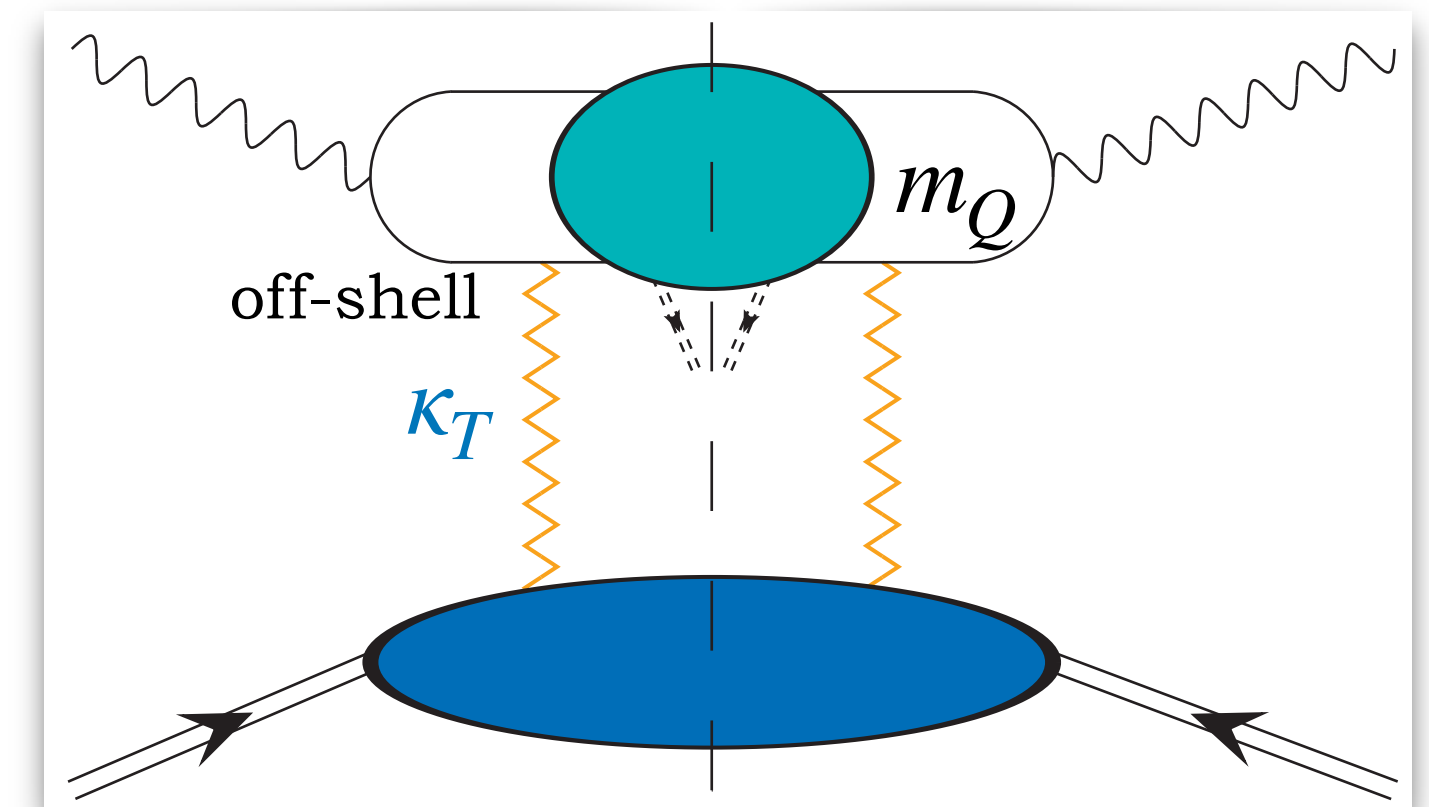
- \* Gluon TMD PDF
- \* Short-distance ( $Q\bar{Q}$ ) + ShFs

$$\kappa_T \gg m_Q$$



- \* Quark collinear PDF
- \* Onium in jet
- \* Single-quark TMD FF

**HEF**



- \* BFKL UGD
- \* Reggeized gluons
- \* Dipole mechanism



The background of the slide features a repeating pattern of Feynman diagrams. Each diagram is contained within a circular frame and depicts a particle interaction. A central vertex is connected to four external lines: two incoming lines from the left and two outgoing lines to the right. The lines are color-coded: red spheres, blue spheres, green spheres, and yellow wavy lines. Some lines have arrows indicating the direction of particle flow. The overall aesthetic is scientific and technical, with a light blue and green color palette.

# Exclusive forward $\rho$ meson leptonproduction

## Exclusive light VM: $\rho^0, \omega, \phi$

\* *Small-size* dipoles  $\Rightarrow$  large  $\kappa_T$

\* **Collinear** description: twist-2/-3 LVM NP **DAs**

$$\Phi^{\gamma^* \rightarrow \rho} \propto \int_0^1 dz T_H^{\gamma^* \rightarrow \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \phi^{\lambda_\rho}(z, \mu_F)$$

\* Significance of small  $\kappa_T$  under investigation...

\* HERA indication: no large- $r_d$  dynamics

\* **LVMs as tools**: discrimination among UGD models

\* **LVMs as tools**: UGD extraction  $\Leftarrow$  HERA + EIC fits



# Single forward emissions

## Exclusive light VM: $\rho^0, \omega, \phi$

- \* *Small-size* dipoles  $\Rightarrow$  large  $\kappa_T$
- \* **Collinear** description: twist-2/-3 LVM NP **DAs**
- $$\Phi^{\gamma^* \rightarrow \rho} \propto \int_0^1 dz T_H^{\gamma^* \rightarrow \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \phi^{\lambda_\rho}(z, \mu_F)$$
- \* Significance of small  $\kappa_T$  under investigation...
- \* HERA indication: no large- $r_d$  dynamics
- \* **LVMs as tools**: discrimination among UGD models
- \* **LVMs as tools**: UGD extraction  $\Leftarrow$  HERA + EIC fits

## Quarkonia

- \* Size of dipoles  $\Rightarrow$  wide range of  $\kappa_T$
- \* Description: **NRQCD** (combined with LFWFs)
- $$\left[ \text{LFWF} \otimes \mathcal{A}_{\text{dip.}} \right] \xleftrightarrow{\text{dilute}} \left[ \Phi^{\gamma^* \rightarrow J/\Psi} \otimes \text{UGD} \right]$$
- \* Validity of *small-size* dipoles questionable...
- \* NRQCD: large- $r_d$  dynamics for  $\Psi(2s)$  ( $\Upsilon(2s)$  ?)
- [\[K. Suzuki et al. \(2000\)\]](#); [\[J. Cepila et al. \(2019\)\]](#); [\[M. Hentschinski et al. \(2020\)\]](#)
- \* **Onia as tools**: scan of TMD/HEF intersection range