Theory of TMDs

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Plan of the lectures

✓ Review the idea of structure functions for DIS and introduce them for semi-inclusive DIS

✓ Introduce the idea of quark-quark correlation functions

- Parametrize correlation functions in terms of PDFs or Transverse Momentum Distributions (TMDs)
- Obtain the expression of structure functions for semi-inclusive DIS in terms of TMDs
- Discuss concept of TMD factorization and TMD evolution
- Discuss a bit of phenomenology

Analysis of quark-quark correlation functions

General form of correlation function

Available vectors $p, \; P$

see, e.g., Mulders, Tangerman, NPB 461 (96)
Goeke, Metz, Schlegel, PLB 618 (05)

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Available Dirac matrices $\mathbf{1},\ \gamma_5,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ i\sigma^{\mu\nu}\gamma_5$ $$\sigma^{\mu\nu} \equiv \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right].$$

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Constraints

Hermiticity: $\Phi(p,P,S) = \gamma^0\, \Phi^\dagger(p,P,S)\, \gamma^0,$ (1a)

parity: $\Phi(p,P,S) = \gamma^0\, \Phi(\tilde{p},\tilde{P},-\tilde{S})\, \gamma^0$ (1b)

$$\tilde{p}^\nu = \delta^{\nu\mu} p_\mu$$

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Parton correlation function for unpolarized hadron

$$\Phi(p,P) = M A_1 + \not{P} A_2 + \not{p} A_3 + \frac{i}{2M}[\,\not{P},\,\not{p}]\,A_4$$

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Parton Distribution Function, or standard PDF, or collinear PDF…
It comes from the decomposition of the correlation function integrated over – and transverse components

Dirac matrices representation

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

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$$\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\not{n}_\pm = \gamma^\mp = \frac{1}{\sqrt{2}}(\gamma^0 \mp \gamma^3)$$

Good/bad and right/left projectors

$$\mathcal{P}^+ = \gamma^-\gamma^+/2, \qquad \mathcal{P}_R = (1+\gamma_5)/2,$$

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Probabilistic interpretation of PDFs

$$\Phi(x) = f_1(x)\frac{\gamma^-}{2}$$

$$\Phi\gamma^0 = \begin{pmatrix} f_1 & 0 & 0 & 0 \\ 0 & f_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \psi_i \big|P\big\rangle\big\langle P\big| \psi_j^\dagger$$

f1 represents the probability of finding a good-right quark or a good-left quark in an unpolarized target.

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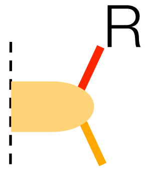

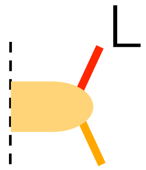

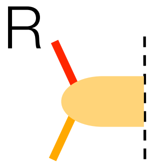

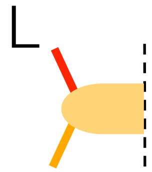

$$\Phi\gamma^0 = \begin{array}{c|cccc} & R & L & & \\ \hline R & f_1 & 0 & 0 & 0 \\ L & 0 & f_1 & 0 & 0 \\ & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 \end{array} \sim \psi_i \big|P\big\rangle\big\langle P\big| \psi_j^\dagger$$

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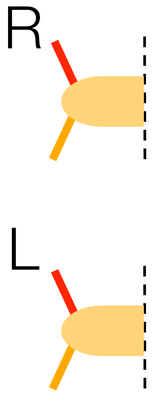

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Including twist 3 (just to see the difference)

$$\Phi(x) = f_1(x)\frac{\gamma^-}{2} + \frac{M}{2P^+}e(x)$$

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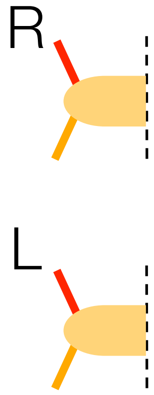

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Results for structure functions in inclusive DIS

$$2MW^{\mu\nu} = \frac{1}{x}\left[-g_{\perp}^{\mu\nu} F_{UU,T} + \hat{t}^{\mu}\hat{t}^{\nu} F_{UU,L}\right]$$

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Note: this would be zero even at twist 3

SIDIS: Transverse Momentum Distributions

If we keep only the leading terms in 1/P+ (leading twist)

(Same as before) $\Phi(p,P) \approx P^+ \left(A_2 + xA_3\right) \not{n}_+ + P^+ \frac{i}{2M} \left[\not{n}_+ , \not{p}_T \right] A_4,$

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Where we introduced the TMDs

$$f_1(x,p_T^2) = 2P^{+}\int dp^{-}\,(A_2 + xA_3)\,, \qquad h_1^{\perp}(x,p_T^2) = 2P^{+}\int dp^{-}\,(-A_4)\,.$$

Results for structure functions in SIDIS

$$2MW^{\mu\nu}(q,P,S) = \frac{2z_h}{x_B}\Big[-g_\perp^{\mu\nu} F_{UU,T}(x_B,z_h,P_{h\perp}^2,Q^2) + \hat{t}^\mu \hat{t}^\nu F_{UU,L}(x_B,z_h,P_{h\perp}^2,Q^2)$$

$$+ \left(\hat{t}^\mu \hat{h}^\nu + \hat{t}^\nu \hat{h}^\mu\right) F_{UU}^{\cos\phi_h}(x_B,z_h,P_{h\perp}^2,Q^2) + \left(\hat{h}^\mu \hat{h}^\nu + g_\perp^{\mu\nu}\right) F_{UU}^{\cos 2\phi_h}(x_B,z_h,P_{h\perp}^2,Q^2)\Big]$$

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$$2MW^{\mu\nu}(q,P,S,P_h) = \frac{2z_h}{x_B}\,\mathcal{C}\Big[\mathrm{Tr}(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu)\Big]$$

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$$\Phi(x,p_T) = \frac{1}{2}\left\{ f_1\, \not{n}_+ + i h_1^\perp \frac{[\not{p}_T, \not{n}_+]}{2M} \right\}.$$

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$$\Phi(x,p_T) = \frac{1}{2}\left\{ f_1\,\not{n}_+ + i h_1^\perp \frac{[\not{p}_T,\not{n}_+]}{2M}\right\}. \qquad \Delta(z,k_T) = \frac{1}{2}\left\{ D_1\,\not{n}_- + i H_1^\perp \frac{[\not{k}_T,\not{n}_-]}{2M_h}\right\}$$

Results for structure functions in SIDIS

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

$$F_{UU,L} = 0,$$

$$F_{UU}^{\cos\phi_h} = 0,$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\,h_1^{\perp} H_1^{\perp}\right],$$

$$\mathcal{C}\big[w f D\big] = \sum_a x\,e_a^2 \int d^2\boldsymbol{p}_T\,d^2\boldsymbol{K}_T\,\delta^{(2)}\big(z\boldsymbol{p}_T - \boldsymbol{K}_T - \boldsymbol{P}_{h\perp}\big)\,w(\boldsymbol{p}_T,\boldsymbol{K}_T)\,f^a(x,p_T^2)\,D^a(z,K_T^2),$$

TMDs and PDFs

TMDs and PDFs

At “parton model” level, the following connections hold

$$f_1(x) = \int d^2\boldsymbol{p}_T \; f_1(x, \boldsymbol{p}_T^2)$$

$$D_1(z) = \int d^2\boldsymbol{K}_T \; D_1(z, \boldsymbol{K}_T^2) = z^2 \int d^2\boldsymbol{k}_T \; D_1(z, z^2\boldsymbol{k}_T^2)$$

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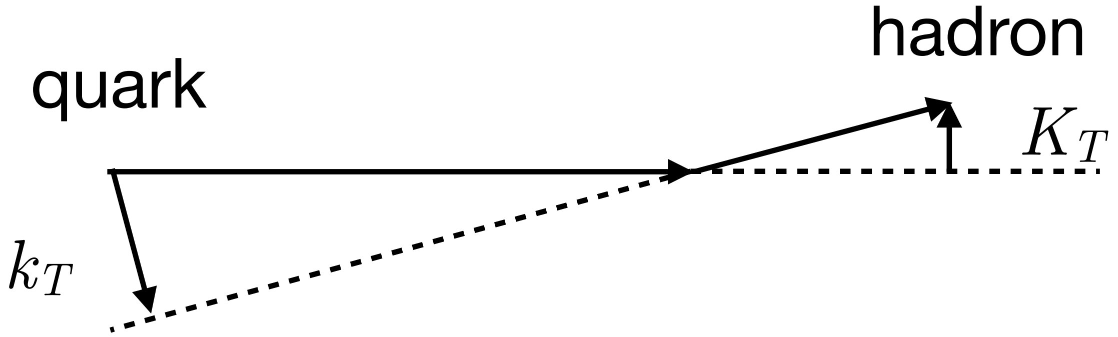

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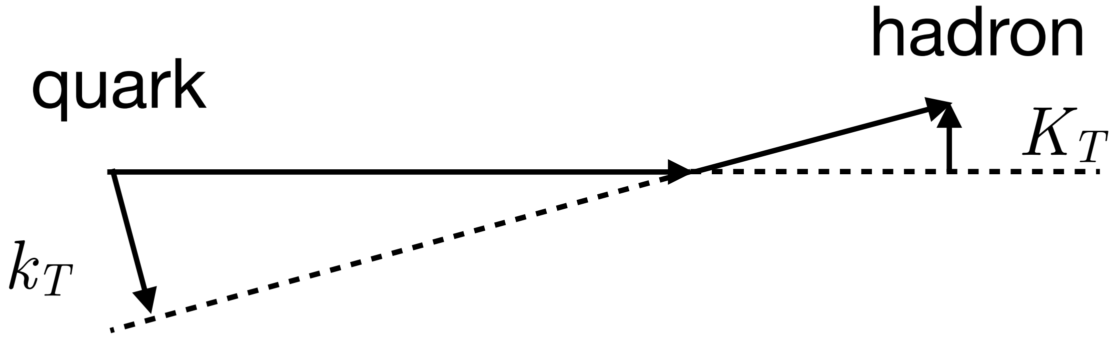

Beware, sometimes we use an imprecise notation

$$D_1(z, z^2\boldsymbol{k}_T^2) \equiv D_1(z, \boldsymbol{k}_T^2)$$

Probabilistic interpretation

$$\Phi\gamma^0 = \begin{pmatrix} f_1 & i\frac{(p_x+ip_y)}{M}h_1^\perp & 0 & 0 \\ -i\frac{(p_x-ip_y)}{M}h_1^\perp & f_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \psi_i \left|P\right\rangle\left\langle P\right| \psi_j^\dagger$$

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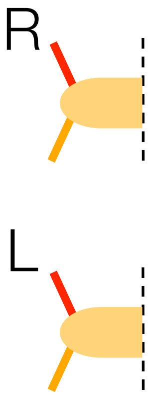

$$\Phi\gamma^0 = \begin{array}{c} \\ \text{R} \\ \text{L} \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} \text{R} & \text{L} & & \end{array} \\ \begin{pmatrix} f_1 & i\frac{(p_x+ip_y)}{M}h_1^\perp & 0 & 0 \\ -i\frac{(p_x-ip_y)}{M}h_1^\perp & f_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \sim \psi_i \, \big|P\big\rangle\big\langle P\big| \, \psi_j^\dagger$$

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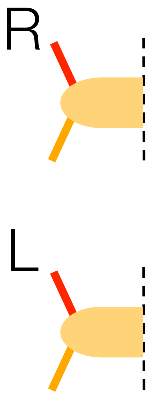

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- Probabilistic interpretation

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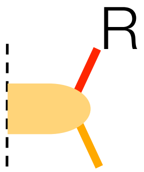

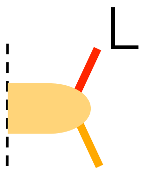

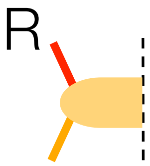

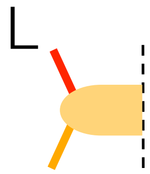

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- Probabilistic interpretation
- Positivity bounds

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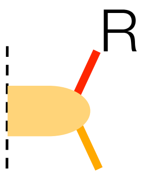
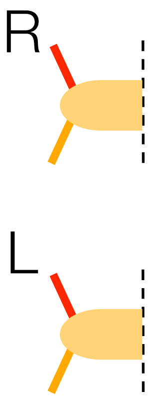
Probabilistic interpretation





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- Probabilistic interpretation
- Positivity bounds
- Need of orbital angular momentum

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b_T space

$$F_{UU,T}(x,z,\boldsymbol{P}_{h\perp}^2,Q^2) = \sum_a x e_a^2 \int d^2\boldsymbol{b}_T e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}/z}\,\hat{f}_1^a\left(x,b_T^2\right)\,\hat{D}_1^{a\to h}\left(z,b_T^2\right)$$

$$= \sum_a x e_a^2 \int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|/z)\hat{f}_1^a\left(x,b_T^2\right)\,\hat{D}_1^{a\to h}\left(z,b_T^2\right)$$

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$$\hat{D}_1^{a\to h}\left(z,b_T^2\right) = \int d^2\boldsymbol{K}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{K}_T/z}\, D_1^{a\to h}\left(x,K_T^2\right)$$

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Fourier transforms

$$\begin{aligned}\hat{f}_1^a\big(x, b_T^2\big) &= \int d^2\boldsymbol{p}_T \, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T} \, f_1^a\big(x, p_T^2\big) \\ &= 2\pi \int_0^\infty dp_T \;\, p_T J_0(b_T p_T) \, f_1^a\big(x, p_T^2\big)\end{aligned}$$

Fourier transforms

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$$= 2\pi \int_0^\infty dp_T \;\, p_T J_0(b_T p_T) \, f_1^a\big(x, p_T^2\big)$$

Beware that there can be different definitions of the FT. With this choice, for b_T=0

$$\hat{f}_1^a\big(x, 0\big) = \int d^2\boldsymbol{p}_T f_1^a\big(x, p_T^2\big) = f_1(x)$$

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Beware that for the fragmentation functions

$$z^2 \hat{D}_1^{a\to h}\big(z, 0\big) = \int d^2\boldsymbol{K}_T \, D_1^{a\to h}\big(z, K_T^2\big) = D_1^{a\to h}(z)$$

Gaussian ansatz

$$F_{UU,T}(x,z,\boldsymbol{P}_{h\perp}^2,Q^2) = \sum_a x e_a^2 \int d^2\boldsymbol{b}_T e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}/z} f_1^a(x) D_1^{a\to h}(z) \frac{1}{z^2} e^{-\frac{g_1}{4}b_T^2} e^{-\frac{g_3}{4z^2}b_T^2}$$

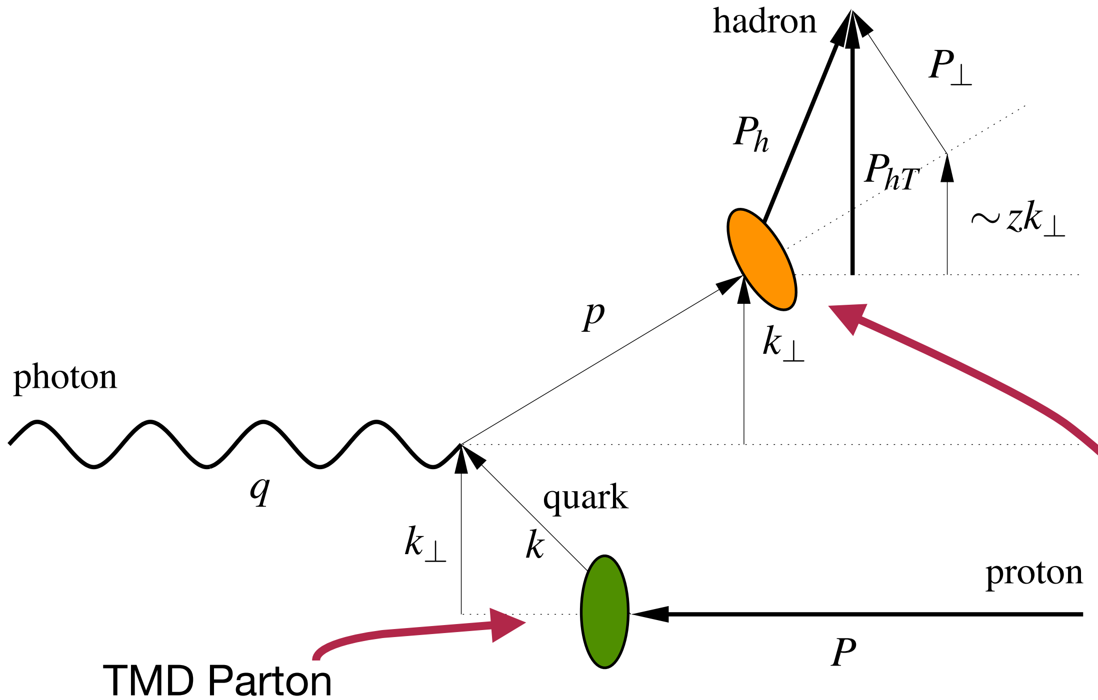
$$= \sum_a x e_a^2 \, f_1^a(x) D_1^{a\to h}(z) \frac{1}{\pi\left(z^2 g_1 + g_3\right)} e^{-\frac{P_{h\perp}^2}{z^2 g_1 + g_3}}$$

$$g_1 = \langle p_T^2\rangle, \qquad g_3 = \langle K_T^2\rangle, \qquad z^2 g_1 + g_3 = \langle P_{h\perp}^2\rangle$$

Beware of different notations…

Amsterdam	Seattle (arXiv:1108.1713)	
p	k	momentum of parton in distribution function
$\boldsymbol{p}_T$	$\boldsymbol{k}_\perp$	parton transverse momentum in distribution function
k	p	momentum of fragmenting parton
$\boldsymbol{k}_T$	$\boldsymbol{p}_\perp$	trans. momentum of fragmenting parton w.r.t. final hadron
$\boldsymbol{K}_T$	$\boldsymbol{P}_\perp$	trans. momentum of final hadron w.r.t. fragmenting parton
$\boldsymbol{P}_{h\perp}$	$\boldsymbol{P}_{hT}$	transverse momentum of final hadron w.r.t. virtual photon

TMDs in semi-inclusive DIS (SIDIS)



TMD Parton
Distribution Functions

TMD Parton
Fragmentation Functions

$$
\begin{aligned}
F_{UU,T}(x,z,P_{hT}^2,Q^2) &= \mathcal{C}\big[f_1 D_1\big] \\
&= x \sum e_a^2 \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{p}_\perp \, \delta^{(2)}\big(\boldsymbol{k}_\perp - \boldsymbol{K}_\perp/z - \boldsymbol{P}_{hT}/z\big) f_1^a(x,k_T^2)\, D_1^a(z,K_T^2)
\end{aligned}
$$

Key points

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- The correlation function for SIDIS can be decomposed in terms of TMDs

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- The structure functions can be written as transverse-momentum convolutions of TMD PDF and FF

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- The correlation function for SIDIS can be decomposed in terms of TMDs
- The structure functions can be written as transverse-momentum convolutions of TMD PDF and FF
- The TMDs have a probabilistic interpretation at parton-model level

What about polarization?

Polarized PDFs and TMDs at leading twist

$$\Phi(x) = \frac{1}{2}\left\{ f_1 \, \not{n}_+ + S_L \, g_1 \gamma_5 \, \not{n}_+ + h_1 \, \frac{\left[\not{S}_T , \, \not{n}_+\right]\gamma_5}{2} \right\}$$

Polarized PDFs and TMDs at leading twist

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$$\begin{aligned}\Phi(x, p_T) = \frac{1}{2}\Bigg\{ & f_1\, \not{n}_+ - f_{1T}^{\perp}\, \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M}\, \not{n}_+ + S_L\, g_{1L} \gamma_5\, \not{n}_+ - g_{1T}\, \frac{p_T \cdot S_T}{M} \gamma_5\, \not{n}_+ \\ & + h_{1T}\, \frac{\left[\not{S}_T,\, \not{n}_+\right]\gamma_5}{2} + S_L\, h_{1L}^{\perp}\, \frac{\left[\not{p}_T,\, \not{n}_+\right]\gamma_5}{2M} \\ & - h_{1T}^{\perp}\, \frac{p_T \cdot S_T}{M}\, \frac{\left[\not{p}_T,\, \not{n}_+\right]\gamma_5}{2M} + i\, h_1^{\perp}\, \frac{\left[\not{p}_T,\, \not{n}_+\right]}{2M} \Bigg\}\end{aligned}$$

Polarized PDFs and TMDs at leading twist

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$$\begin{aligned}\Phi(x, p_T) = \frac{1}{2}\Bigg\{ & f_1 \not{n}_+ - f_{1T}^{\perp} \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} \not{n}_+ + S_L\, g_{1L} \gamma_5 \not{n}_+ - g_{1T} \frac{p_T \cdot S_T}{M} \gamma_5 \not{n}_+ \\ & + h_{1T} \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} + S_L\, h_{1L}^{\perp} \frac{[\not{p}_T, \not{n}_+]\gamma_5}{2M} \\ & - h_{1T}^{\perp} \frac{p_T \cdot S_T}{M} \frac{[\not{p}_T, \not{n}_+]\gamma_5}{2M} + i\, h_1^{\perp} \frac{[\not{p}_T, \not{n}_+]}{2M} \Bigg\}\end{aligned}$$

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Polarized PDFs and TMDs at leading twist

$$f_1(x)$$

$$\Phi(x) = \frac{1}{2}\left\{ f_1\, \slashed{n}_+ + S_L\, g_1 \gamma_5\, \slashed{n}_+ + h_1 \frac{\left[\slashed{S}_T,\, \slashed{n}_+\right]\gamma_5}{2} \right\}$$

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$$+ h_{1T} \frac{\left[\slashed{S}_T,\, \slashed{n}_+\right]\gamma_5}{2} + S_L\, h_{1L}^{\perp} \frac{\left[\slashed{p}_T,\, \slashed{n}_+\right]\gamma_5}{2M}$$
$$- h_{1T}^{\perp} \frac{p_T \cdot S_T}{M} \frac{\left[\slashed{p}_T,\, \slashed{n}_+\right]\gamma_5}{2M} + i\, h_1^{\perp} \frac{\left[\slashed{p}_T,\, \slashed{n}_+\right]}{2M} \Bigg\}$$

Polarized PDFs and TMDs at leading twist

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$f_1(x, p_T^2)$

$$\begin{aligned}\Phi(x, p_T) = \frac{1}{2}\Bigg\{ & f_1 \not{n}_+ - f_{1T}^{\perp} \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} \not{n}_+ + S_L\, g_{1L} \gamma_5 \not{n}_+ - g_{1T} \frac{p_T \cdot S_T}{M} \gamma_5 \not{n}_+ \\ & + h_{1T} \frac{\left[\not{S}_T,\, \not{n}_+\right]\gamma_5}{2} + S_L\, h_{1L}^{\perp} \frac{\left[\not{p}_T,\, \not{n}_+\right]\gamma_5}{2M} \\ & - h_{1T}^{\perp} \frac{p_T \cdot S_T}{M} \frac{\left[\not{p}_T,\, \not{n}_+\right]\gamma_5}{2M} + i\, h_1^{\perp} \frac{\left[\not{p}_T,\, \not{n}_+\right]}{2M} \Bigg\}\end{aligned}$$

Table of twist-2 TMDs

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

Table of twist-2 TMDs

helicity

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

Mulders-Tangerman, NPB 461 (96)
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helicity

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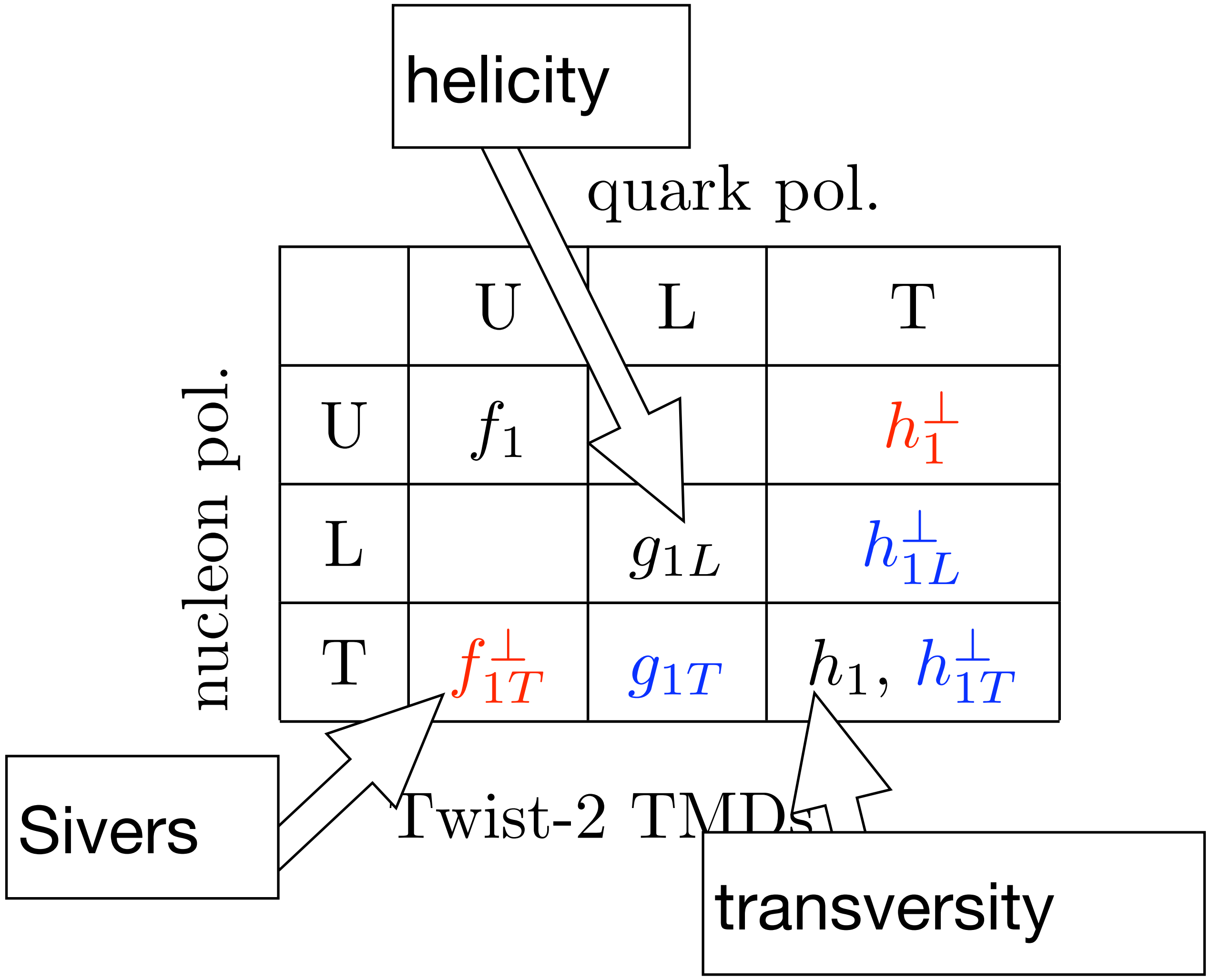
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

transversity

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

Table of twist-2 TMDs



helicity

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

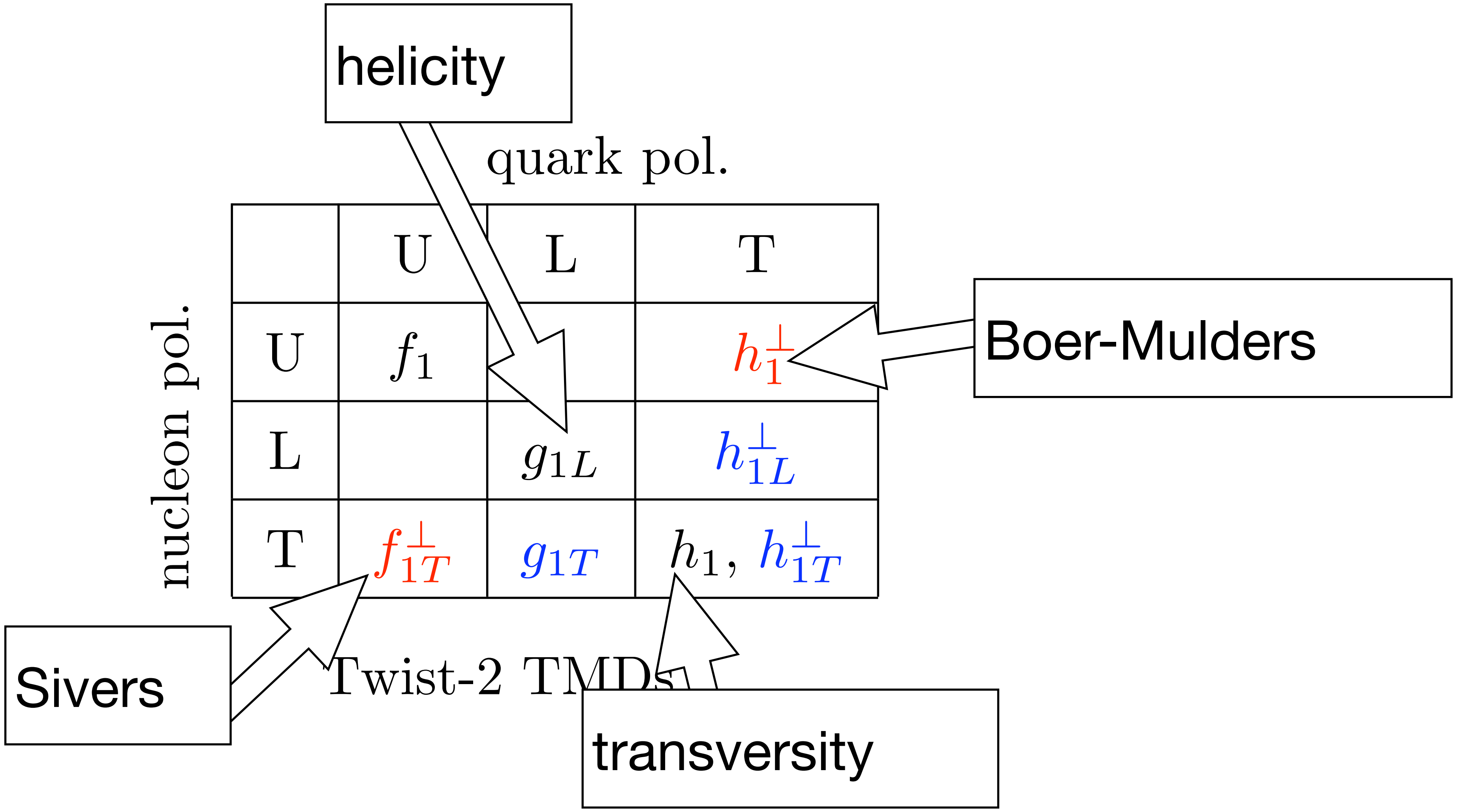
Twist-2 TMDs

Sivers

transversity

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

Table of twist-2 TMDs


helicity

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Boer-Mulders

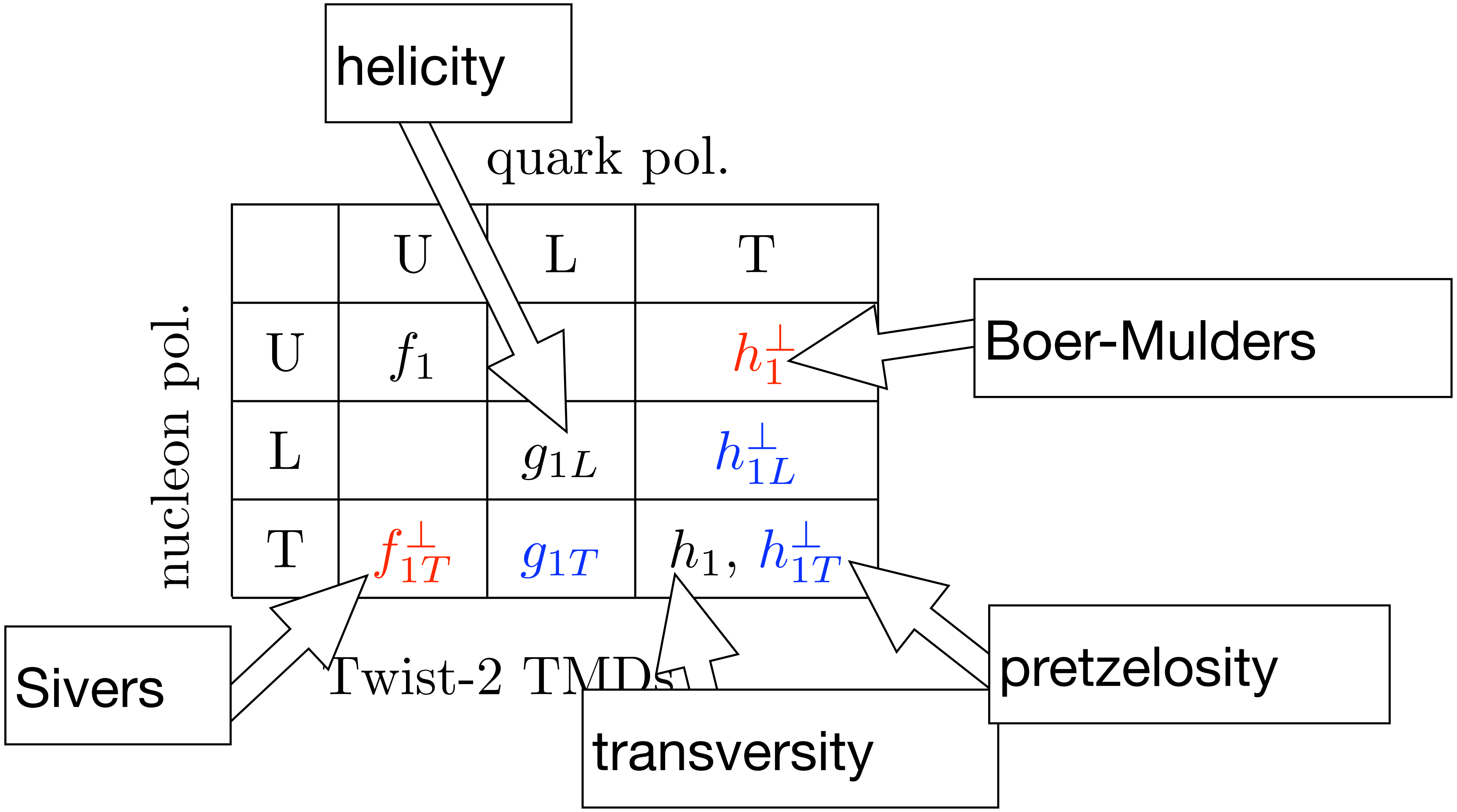
Sivers

Twist-2 TMDs

transversity

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

Table of twist-2 TMDs



quark pol.

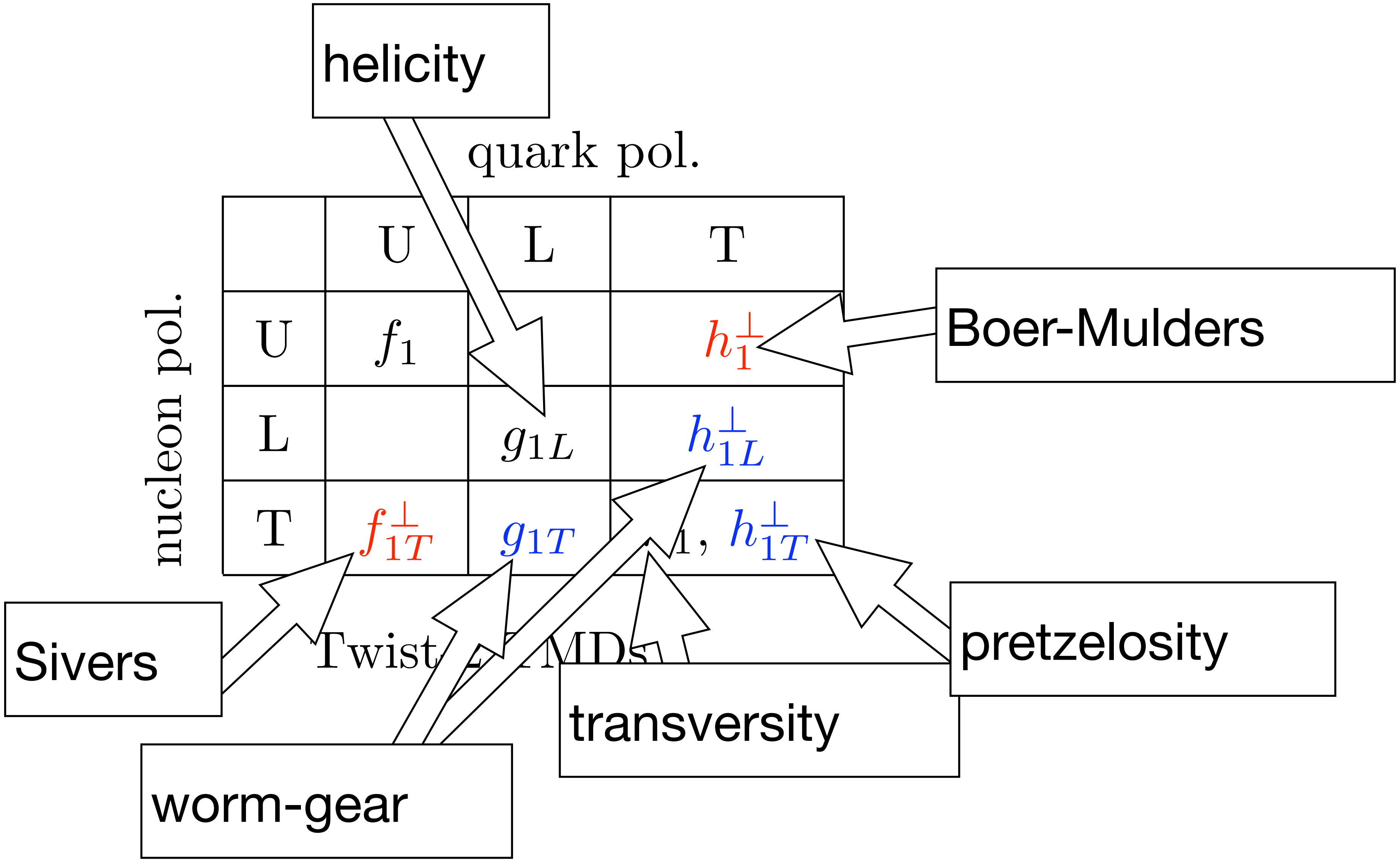
nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

Table of twist-2 TMDs



	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$1, h_{1T}^{\perp}$

Twist-2 TMDs

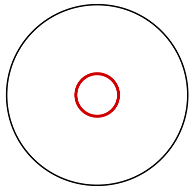
TMDs in black survive transverse-momentum integration

TMDs in red are T-odd

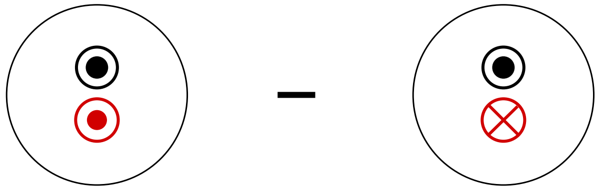
Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

Interpretation of polarized TMDs

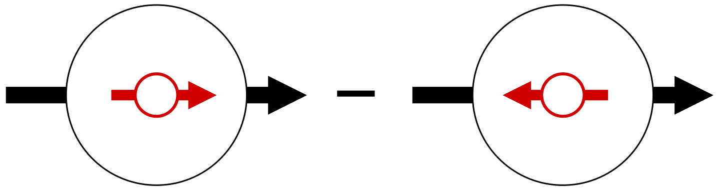
$f_1 =$



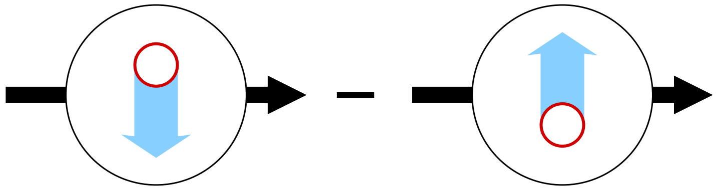
$g_1 =$



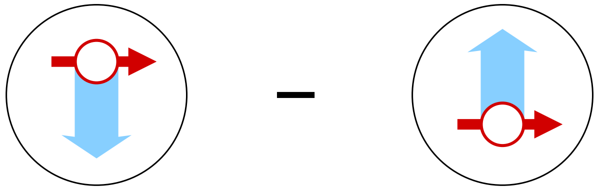
$h_1 =$



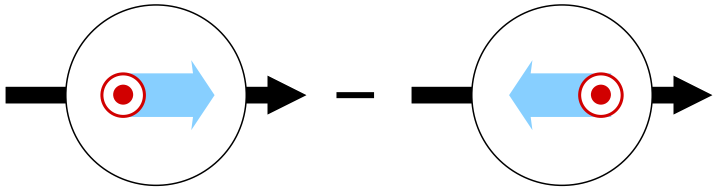
$f_{1T}^{\perp} =$



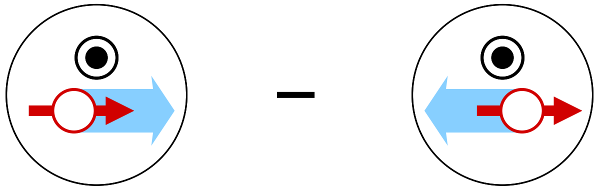
$h_1^{\perp} =$



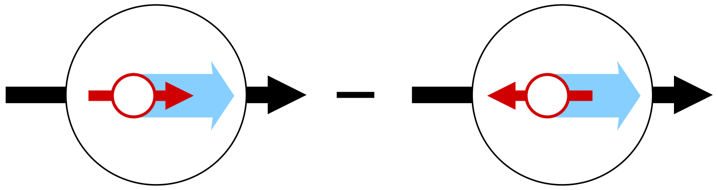
$g_{1T} =$



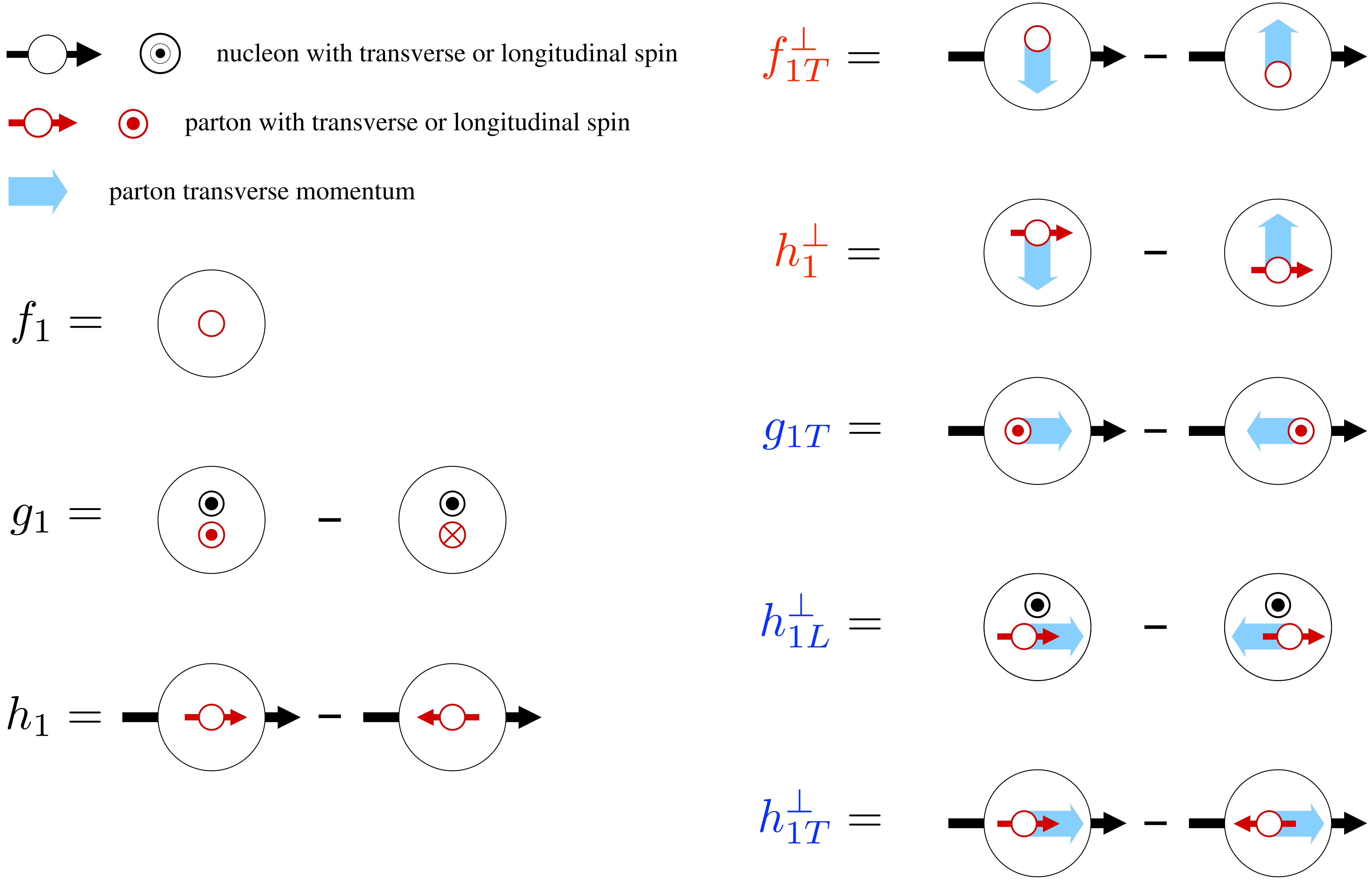
$h_{1L}^{\perp} =$



$h_{1T}^{\perp} =$



Interpretation of polarized TMDs



nucleon with transverse or longitudinal spin

parton with transverse or longitudinal spin

parton transverse momentum

$f_1 =$

$g_1 =$ –

$h_1 =$ –

$f_{1T}^{\perp} =$ –

$h_1^{\perp} =$ –

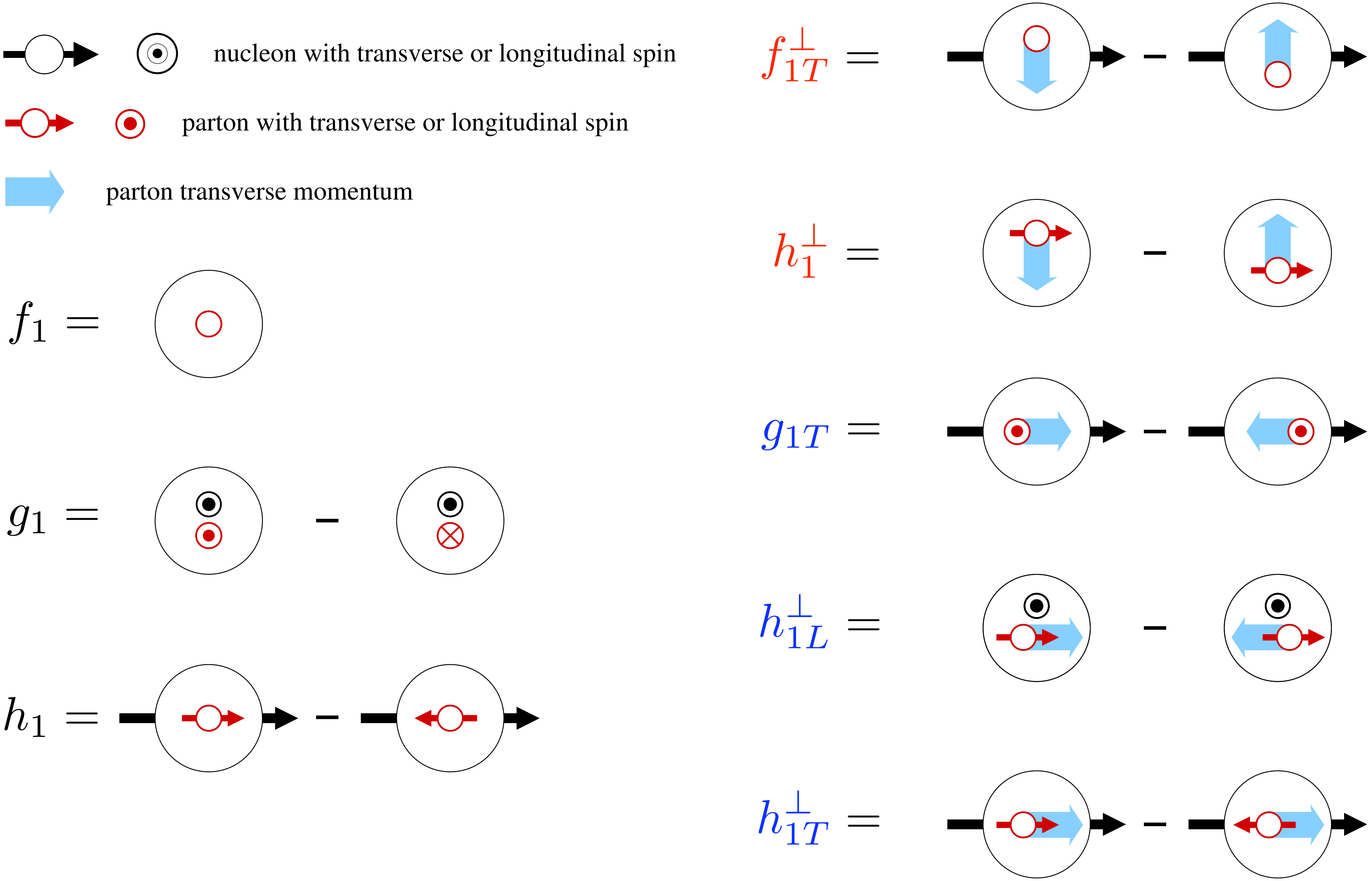
$g_{1T} =$ –

$h_{1L}^{\perp} =$ –

$h_{1T}^{\perp} =$ –

Interpretation of polarized TMDs

Proton goes out of the screen/ photon goes into the screen



nucleon with transverse or longitudinal spin

parton with transverse or longitudinal spin

parton transverse momentum

$f_1 =$

$g_1 = \quad - \quad$

$h_1 = \quad - \quad$

$f_{1T}^{\perp} = \quad - \quad$

$h_1^{\perp} = \quad - \quad$

$g_{1T} = \quad - \quad$

$h_{1L}^{\perp} = \quad - \quad$

$h_{1T}^{\perp} = \quad - \quad$

Results for polarized SIDIS

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right. \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}\right. \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad \left. + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\right] + S_T\lambda_e\left[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}\right. \\
&\qquad \left.\left. + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\right\}
\end{aligned}
$$

Unpolarized sector

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

$$F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\tilde{H}}{z}\right)\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^\perp H_1^\perp\right],$$

Longitudinally pol. beam or/and target

$$F_{LU}^{\sin\phi_h} = \mathcal{O}\left(\frac{M}{Q}\right),$$

$$F_{UL}^{\sin\phi_h} = \mathcal{O}\left(\frac{M}{Q}\right),$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}h_{1L}^{\perp}H_1^{\perp}\right],$$

$$F_{LL} = \mathcal{C}\left[g_{1L}D_1\right],$$

$$F_{LL}^{\cos\phi_h} = \mathcal{O}\left(\frac{M}{Q}\right)$$

Transversely polarized target

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)+\boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)-4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)}{2M^2M_h} h_{1T}^{\perp} H_1^{\perp}\right],$$

$$F_{UT}^{\sin\phi_S} = \mathcal{O}\left(\frac{M}{Q}\right),$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M}{Q}\right)$$

Trasversely pol. target and long. pol. beam

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\right],$$

$$F_{LT}^{\cos\phi_S} = \mathcal{O}\left(\frac{M}{Q}\right),$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M}{Q}\right)$$