# History of Deep-Inelastic scattering
## (Collinear PDFs from past, present and future data)

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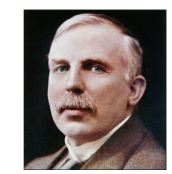
# Outline (Lecture 2)

Pre-history…

- Part I (Fixed-target Experiments)
  - Quark model
  - SLAC-MIT Coll.
  - Parton model
  - Gargamelle
  - pQCD and QCD-improved parton model
- Part II (HERA and LHC)
  - Highlights
  - PDFs and LHC
- Part III (EIC)
  - Studies on collinear PDFs

# Pre-History (1909-1960)

- Rutherford scattering
- Atomic nucleus
- Protons and neutrons
- Magnetic moments (p & n)
- Strong force
- Form factors





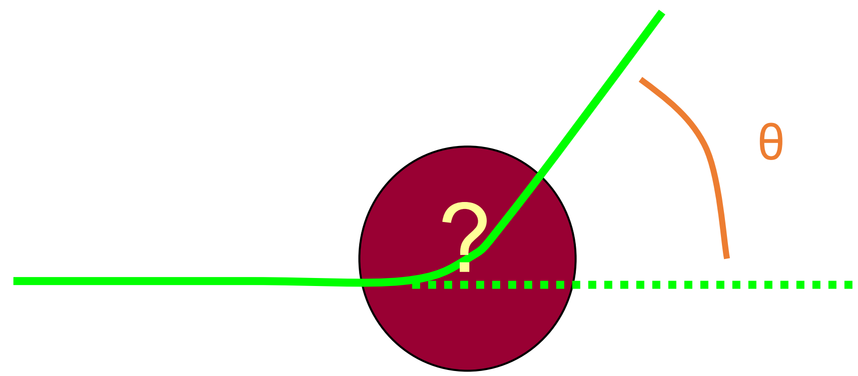
See supplementary material

Nobel prize 1908

# Rutherford Scattering

**"The most famous fixed-target experiment of all time:"**

Rutherford taught us the most important lesson:
the use of a scattering process to investigate the structure of matter



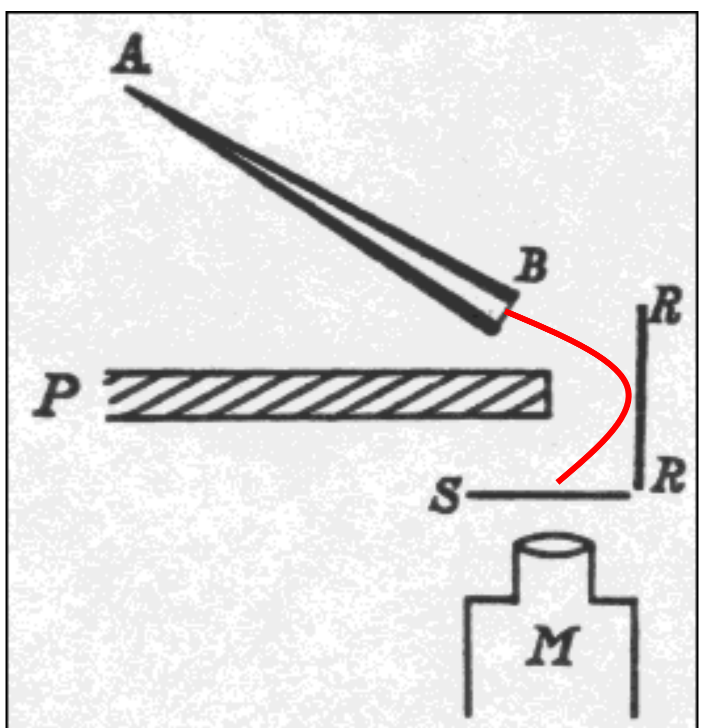




FIG. 1.

H. Geiger and E. Marsden observed the high-angle scattering ($\theta > 90^0$) alpha particles deflected by a thin gold foil.

Proc.Roy. Soc. **A 82**, 495,1909

Rutherford interpreted the results as due to the scattering of alpha particles from a massive central charge

$$\sigma(\theta) = \frac{z^2 Z^2 e^4}{16 E^2} \frac{1}{\sin^4 \frac{1}{2}\theta}$$

Phil. Mag. **21**,669,1911

# Ernest Rutherford


“In our laboratory today we live in an atmosphere deemed with the flying fragments of exploding atoms and on this occasion I wish to say a few words on the methods and ideas employed to break up atoms and realize…the old dream of alchemists of transmutation of one element into another…”

You tube Link. : https://www.youtube.com/watch?v=zBHD8ksx_Sg

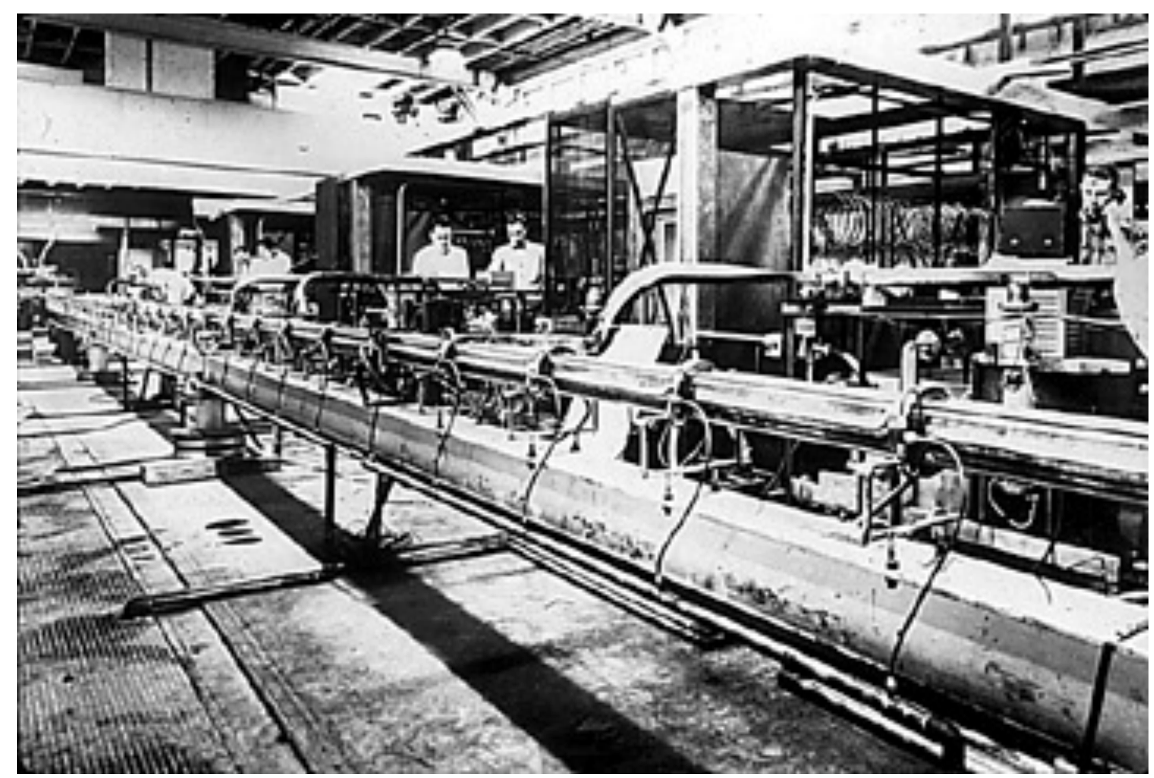
# Linacs at the Stanford University

In the mid-1930s, the Varian brothers (research assistants at the microwave department of Stanford University) developed the 'klystron' using a special electromagnetic cavity (Rhumbatron) invented by W. Hansen.

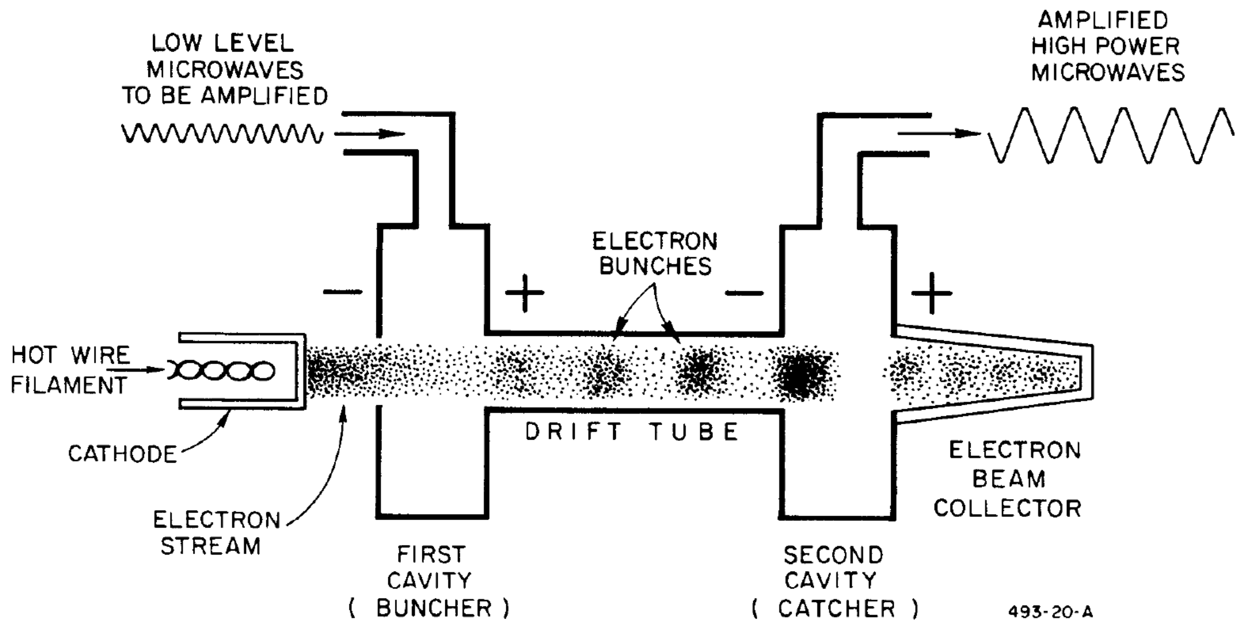
This device allowed Stanford's HEPL to play a leading role in the development of linear accelerators...

Under the direction of E. Ginzton HEPL began the construction of a series of 'small-scale' linacs (MARK I,II,III...).

MARKIII was to be fundamental for the realisation of R. Hofstadter's experiments on e-N and e-p eastic scattering.



# Klystron


LOW LEVEL MICROWAVES TO BE AMPLIFIED
AMPLIFIED HIGH POWER MICROWAVES
ELECTRON BUNCHES
HOT WIRE FILAMENT
CATHODE
ELECTRON STREAM
DRIFT TUBE
ELECTRON BEAM COLLECTOR
FIRST CAVITY ( BUNCHER )
SECOND CAVITY ( CATCHER )
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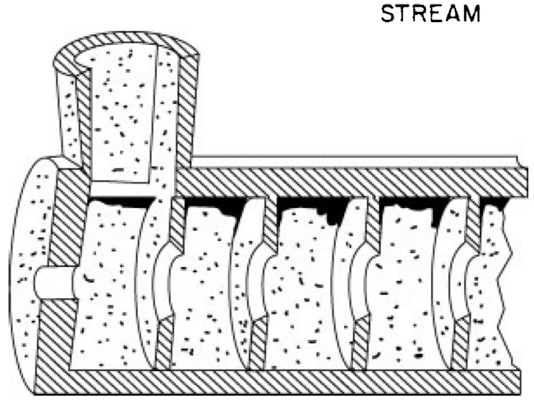
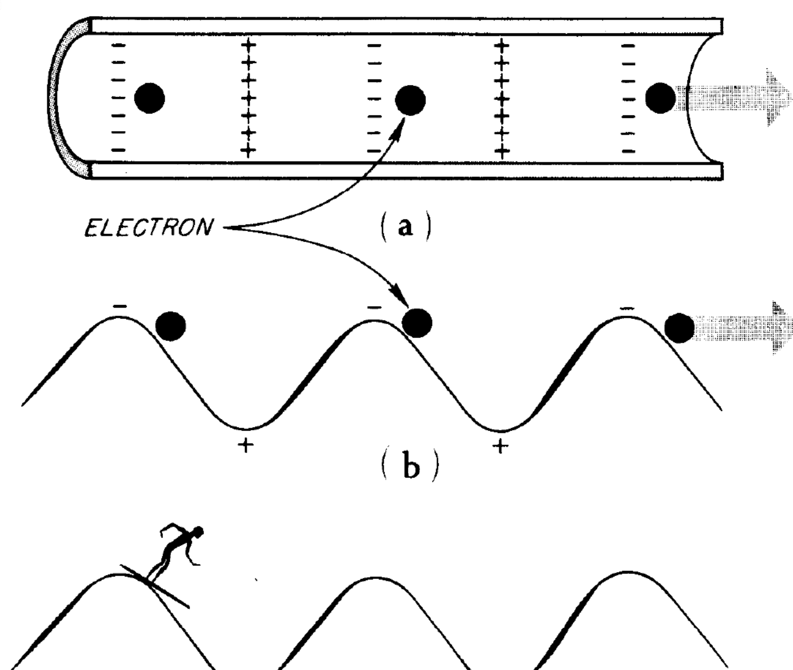
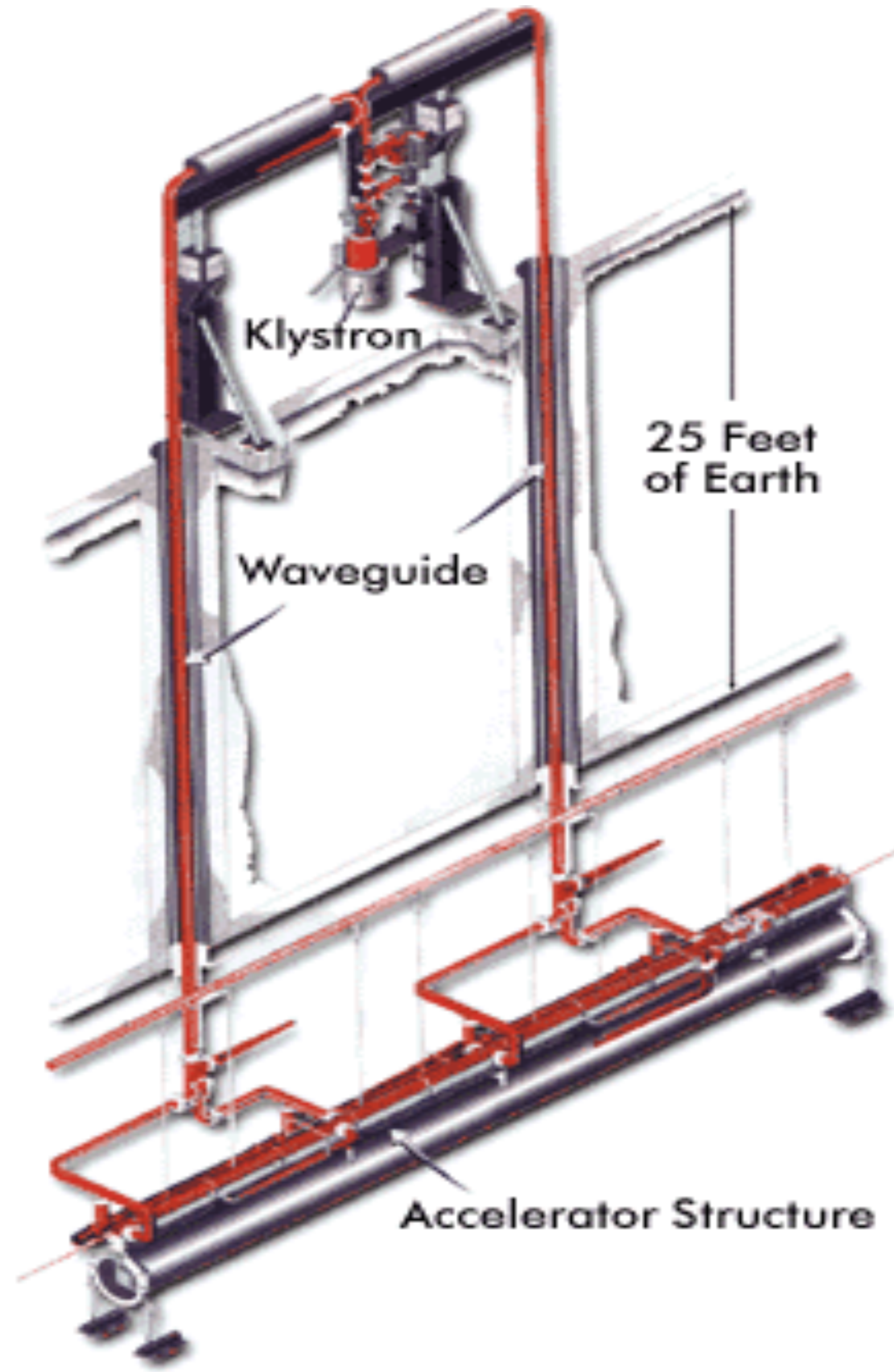



ELECTRON
( a )
( b )

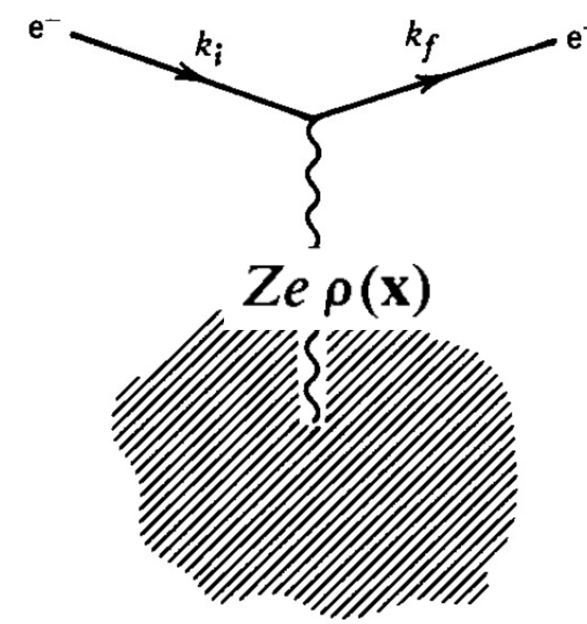

Klystron
25 Feet of Earth
Waveguide
Accelerator Structure

# Nuclear Form Factor

Stimulated by accelerators technology advances and fully muture QED various theoreticians (Rose (48), Elton(50)) started to calculate cross sections for <u>elastic electron-Nucleus scattering</u>

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q})|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}\, d^3x$$



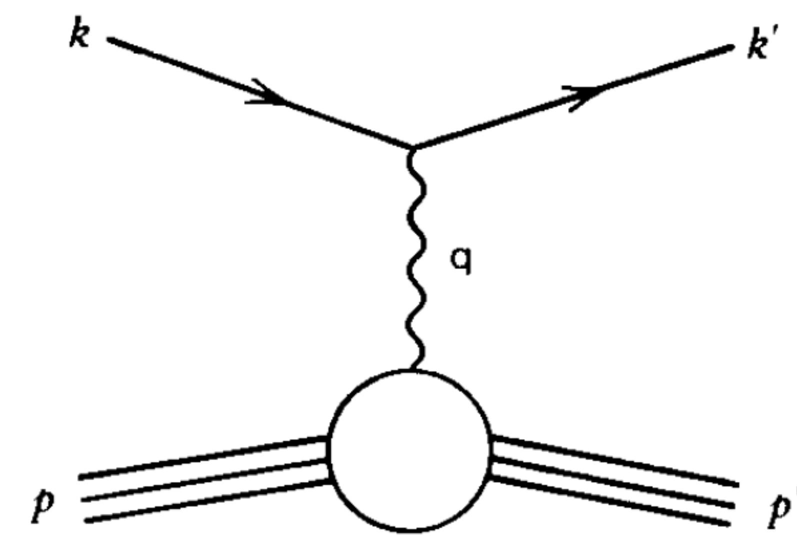

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 E^2}{4k^4 \sin^4 \frac{\theta}{2}} \left(1 - v^2 \sin^2 \frac{\theta}{2}\right)$$

$$\begin{aligned} F(\mathbf{q}) &= \int \left(1 + i\mathbf{q}\cdot\mathbf{x} - \frac{(\mathbf{q}\cdot\mathbf{x})^2}{2} + \cdots \right) \rho(\mathbf{x})\, d^3x \\ &= 1 - \frac{1}{6}|\mathbf{q}|^2 \langle r^2 \rangle + \cdots, \end{aligned}$$

$$r_{\text{m}}^2 = -6 \left.\frac{\mathrm{d}F(\mathbf{q})}{\mathrm{d}(|\mathbf{q}|^2)}\right|_{|\mathbf{q}|=0}.$$

# Nucleon Form Factors

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{\cos^2 \theta/2}{1 + (2E/M) \sin^2 \theta/2}$$
$$\times \left\{ (F_1{}^p(t))^2 - \frac{t}{4M^2}\left( 4M^2 (F_2{}^p(t))^2 + 2(F_1{}^p(t) + 2M\, F_2{}^p(t))^2 \tan^2 \frac{\theta}{2} \right) \right\}$$

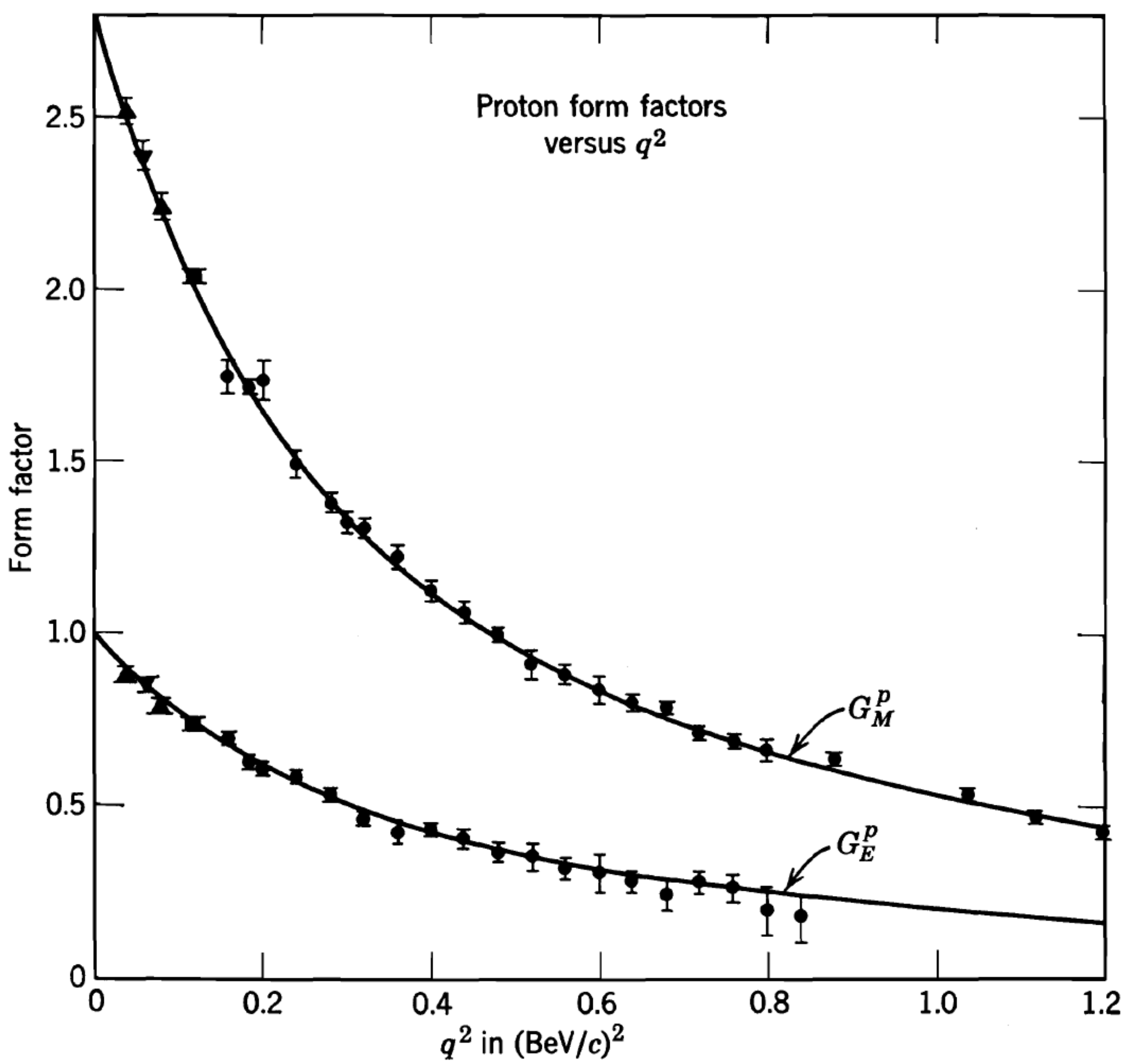


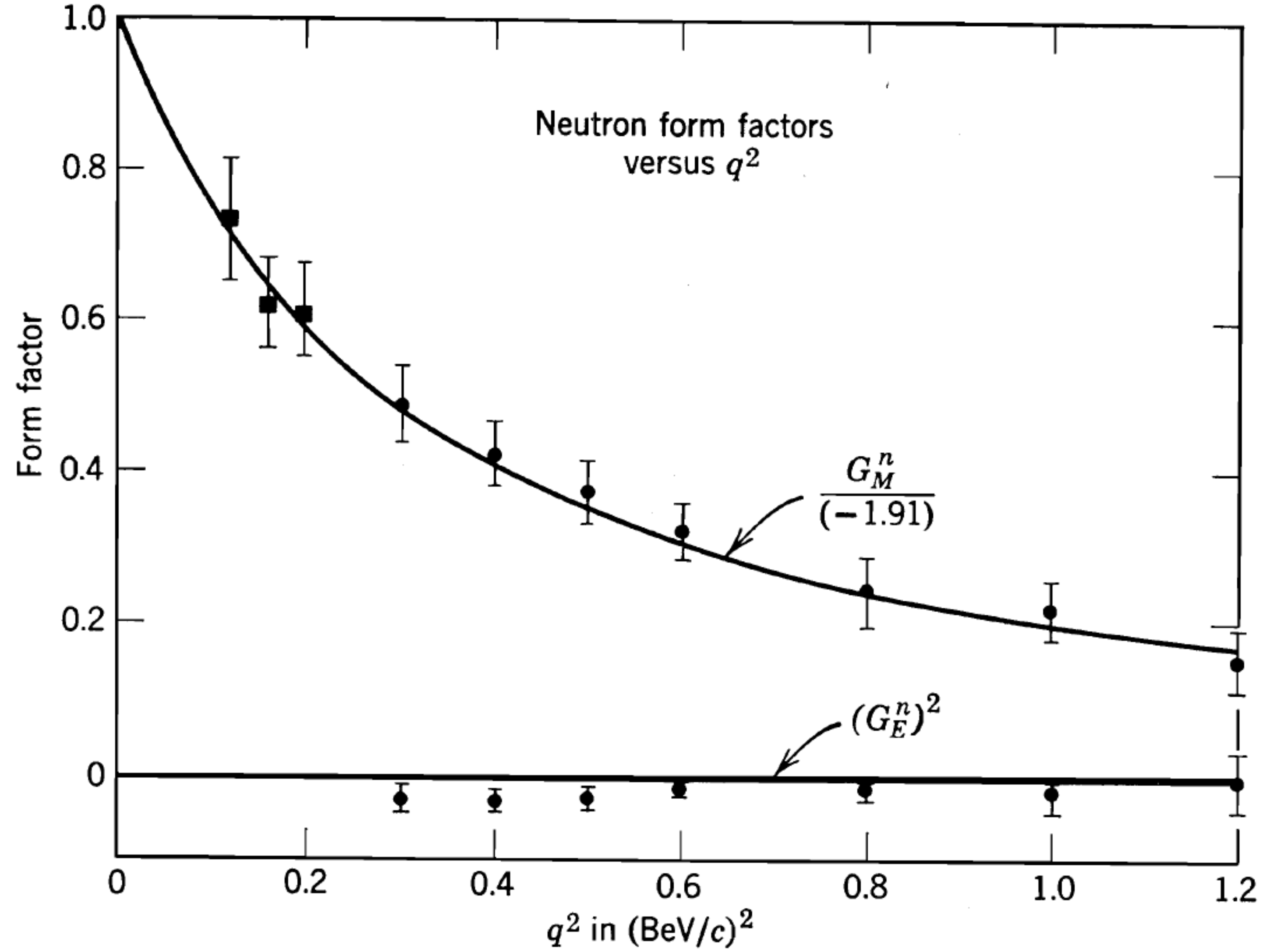

$$G_E{}^p(t) = F_1{}^p(t) + \frac{t}{2M} F_2{}^p(t)$$
$$G_M{}^p(t) = F_1{}^p(t) + 2M\, F_2{}^p(t)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left\{ \frac{(G_E{}^p(t))^2 - \frac{t}{4M^2}(G_M{}^p(t))^2}{1 - t/4M^2} - \frac{t}{2M^2}(G_M{}^p(t))^2 \tan^2 \frac{\theta}{2} \right\}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{\cos^2 \theta/2}{1 + (2E/M) \sin^2 \theta/2}$$

# Nucleon Form Factors



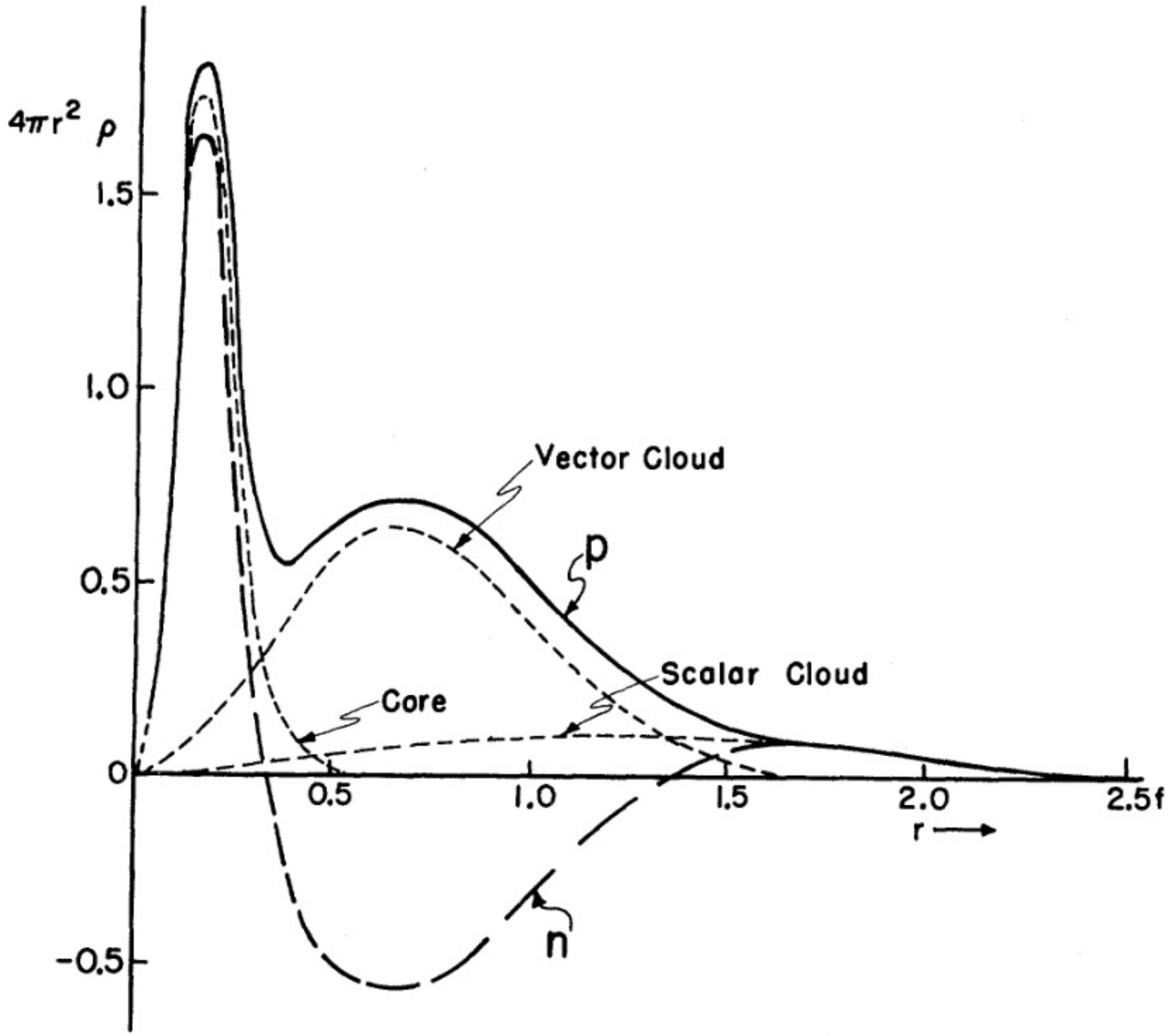

$$G_E{}^p(0) = 1$$

$$G_M{}^p(0) = 1 + \mu_p \cong 2.79$$

$$G_E{}^n(0) = 0$$

$$G_M{}^n(0) = \mu_n \cong -1.91$$

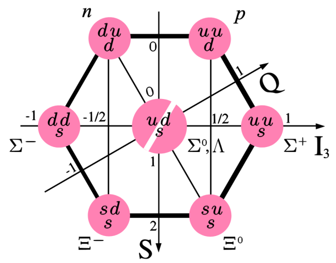
# Nucleon Form Factors

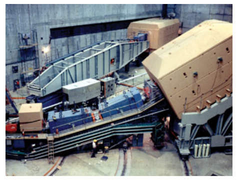


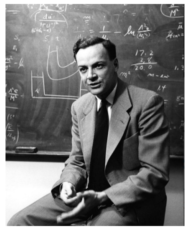

D. N. Olson, H. F. Schopper, and R. R. Wilson
Phys. Rev. Lett. 6, 286 (1961)

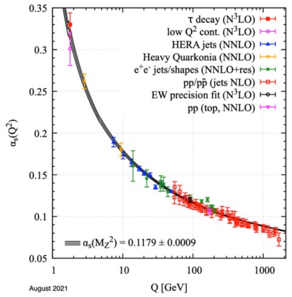
# Part I

- Quark model
- SLAC-MIT
- Parton Model
- Gargamelle
- pQCD

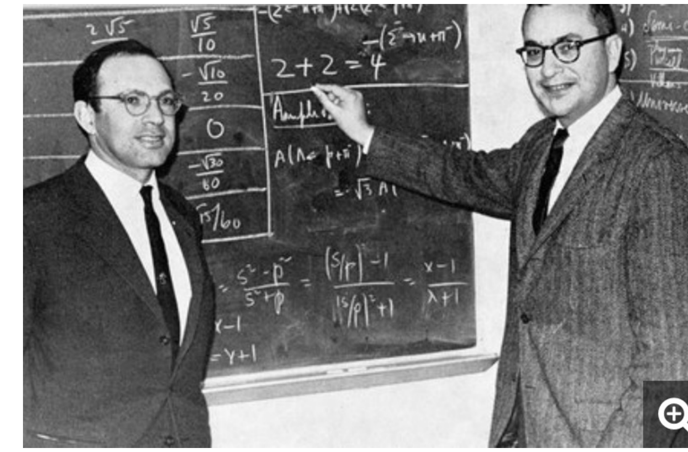






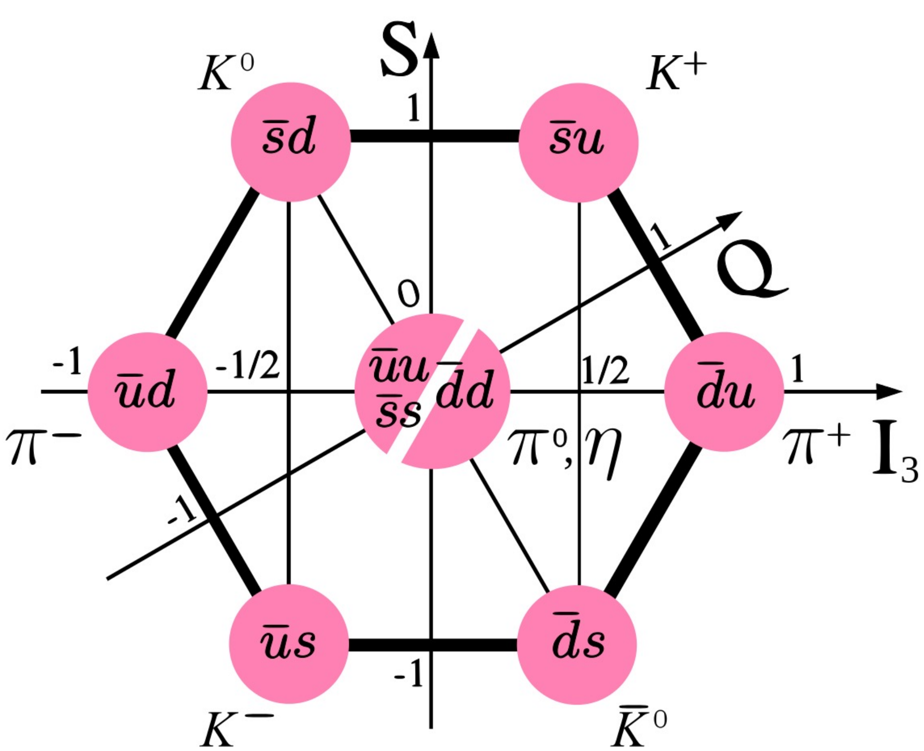
# The Eightfold Way

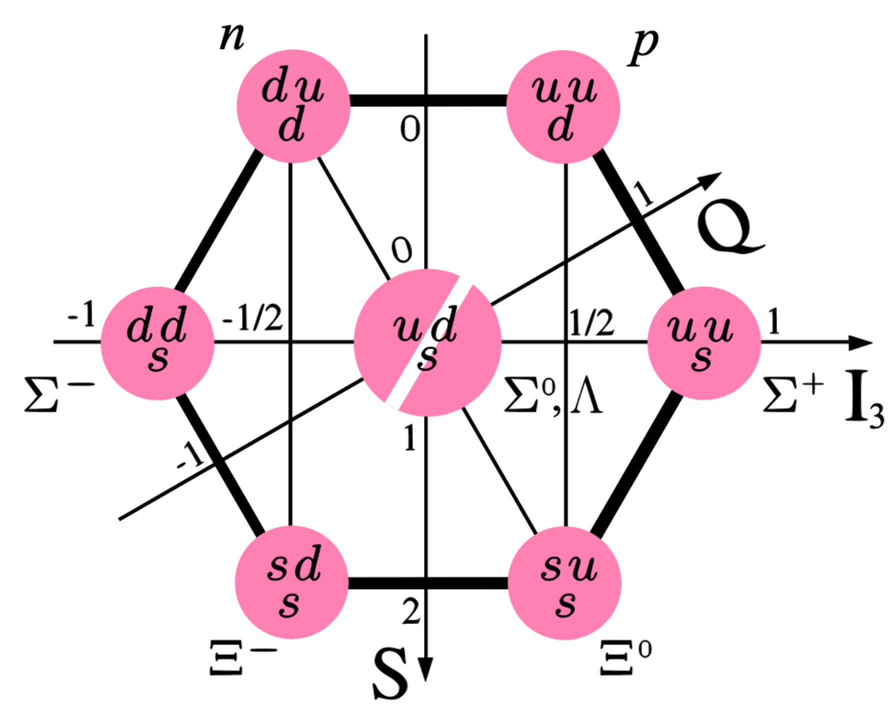


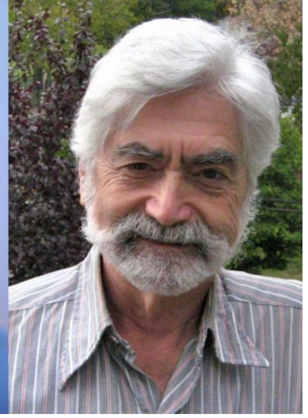
By the early 1960s, many particles (mesons and baryons) had been discovered thanks to the use of accelerator machines and new detectors.

In 1961 M. Gell-mann and Y. Ne'eman introduced a scheme for the classification of mesons and baryons into families/multiplets.

Mathematical Framework: Irr. representations of the SU(3) group



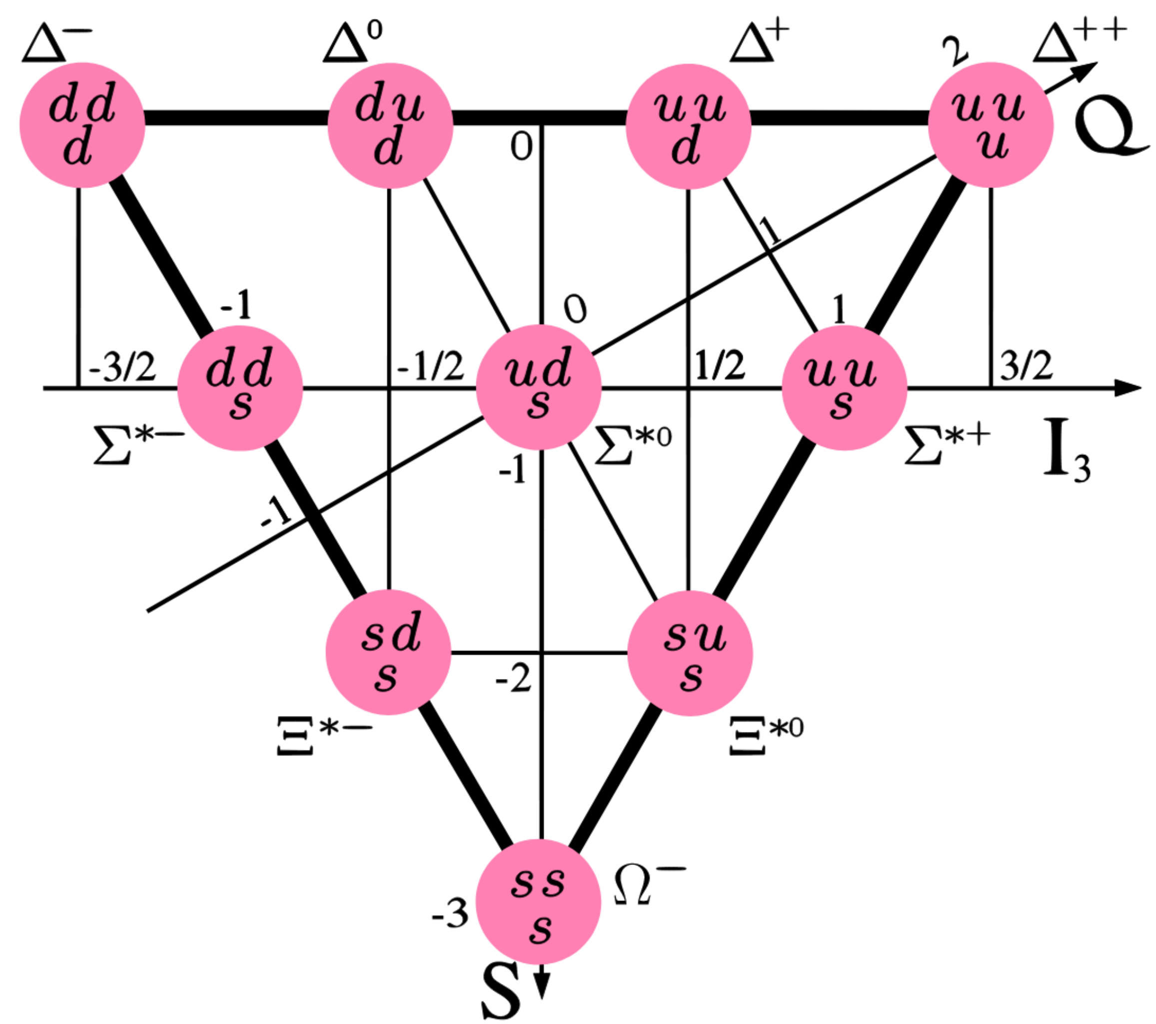





# The quark model

1964: Gell-Mann and Zweig independently proposed the existence of elementary particles of spin ½ and fractional charge (the quarks). Three types of quarks (up, down and strange) and the corresponding quark-anti-quarks explained the regularities observed in hadrons.

| | $d$ | $u$ | $s$ | $c$ | $b$ | $t$ |
|---|---|---|---|---|---|---|
| Q – electric charge | $-\frac{1}{3}$ | $+\frac{2}{3}$ | $-\frac{1}{3}$ | $+\frac{2}{3}$ | $-\frac{1}{3}$ | $+\frac{2}{3}$ |
| I – isospin | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | 0 |
| $I_z$ – isospin $z$-component | $-\frac{1}{2}$ | $+\frac{1}{2}$ | 0 | 0 | 0 | 0 |
| S – strangeness | 0 | 0 | $-1$ | 0 | 0 | 0 |
| C – charm | 0 | 0 | 0 | $+1$ | 0 | 0 |
| B – bottomness | 0 | 0 | 0 | 0 | $-1$ | 0 |
| T – topness | 0 | 0 | 0 | 0 | 0 | $+1$ |




Descriptive and partially predictive model (e.g. masses, magnetic moments)

# Are the quarks real?

Prevailing interpretation (~1965s) :

- The quark model is only a useful organisational scheme for hadron spectroscopy
- Particles have a "diffuse substructure" but no "elementary constituents"
  (Nuclear democracy - Bootstrap model)

"…the idea that mesons and baryons are made primarily of quarks and gluons is hard to believe…"

M. Gell-mann 1966

"Additional data are necessary and very welcome to destroy the picture of elementary constituents."

J. Bjorken 1967

"I think Professor Bjorken and I constructed the sum rules in the hope of destroying the quark model."

K. Gottfried 1967

"Of course the whole quark idea is ill founded."

J.J. Kokkedee 1967

# SLAC and the "M(onster)-Project"

On 10 April 1956, the Stanford staff met at W. Panofsky's house to discuss R. Hofstadter's proposal to build a linear accelerator 10 times more powerful than Mark III. The project was given the provisional name 'M(onster) Project' because it was it was estimated that the accelerator should be 2 miles long and reach an energy of 20 GeV!!!

- 1957 a detailed project is presented
- 1959 Eisenhower says yes
- 1961 Congress approves the project ($114 Million)

1962, construction began

During excavation work, the skeleton of a mammal that livedin the Miocene, (Paleoparadoxia), which populated the region 14 million years ago, was found ...

# The (CIT)-SLAC-MIT Collaboration

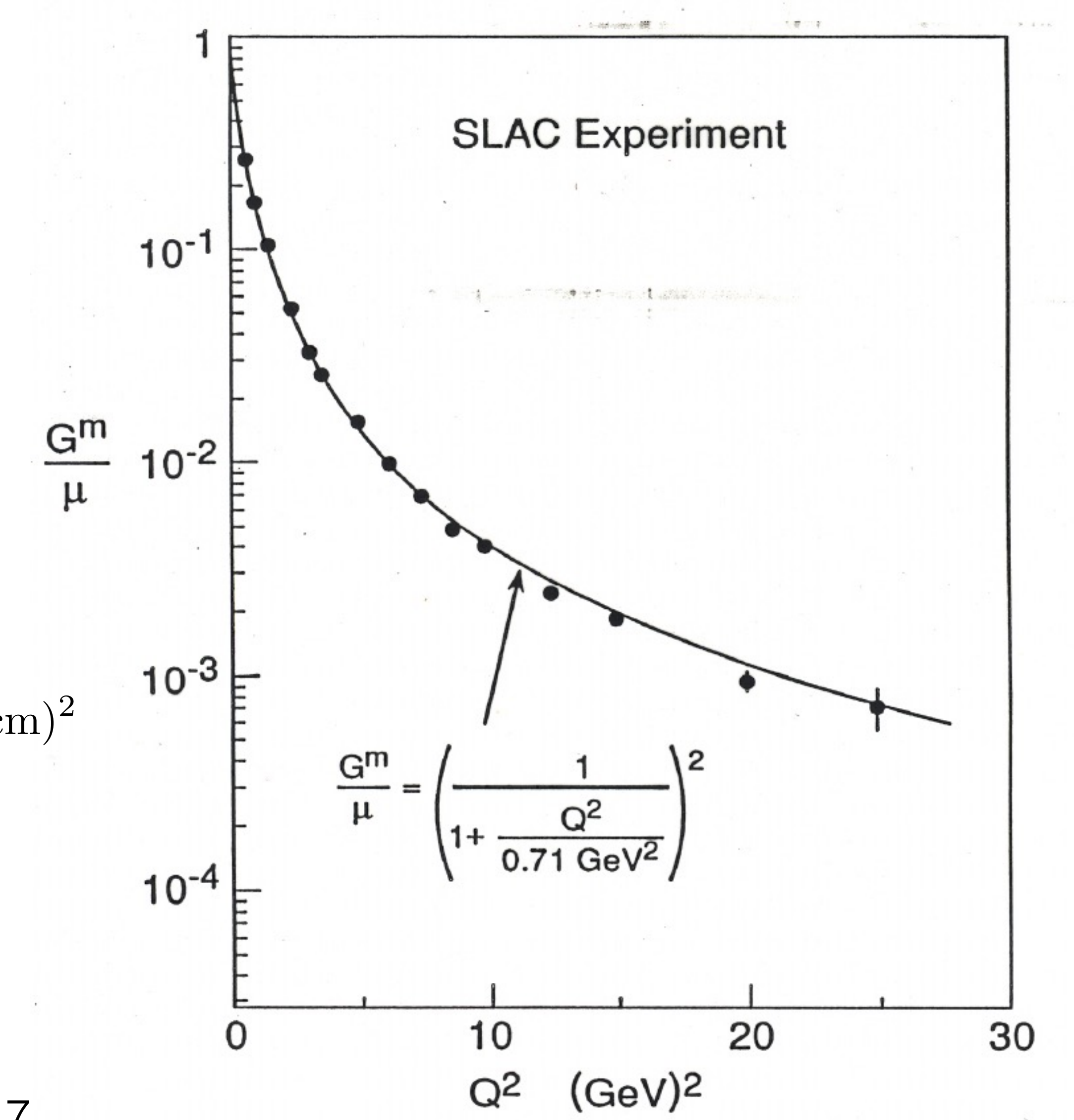
SLAC began its operational phase in 1966.

The CIT-SLAC-MIT collaboration built the electron spectrometer to study the structure of the proton through the elastic diffusion process e-p

The electron is an ideal 'probe':
- elementary particle
- interacts electromagnetically (QED)

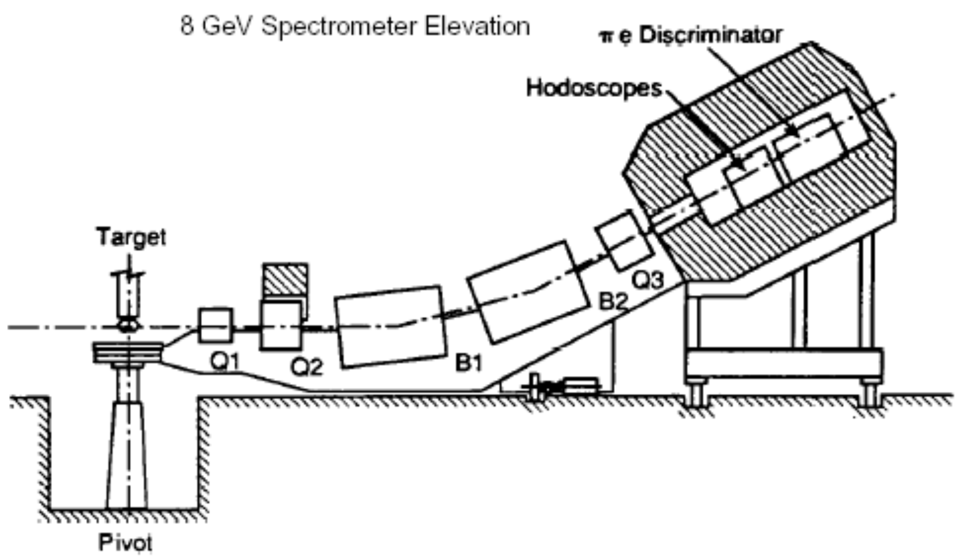
Magnetic Form Factor $\Longrightarrow$

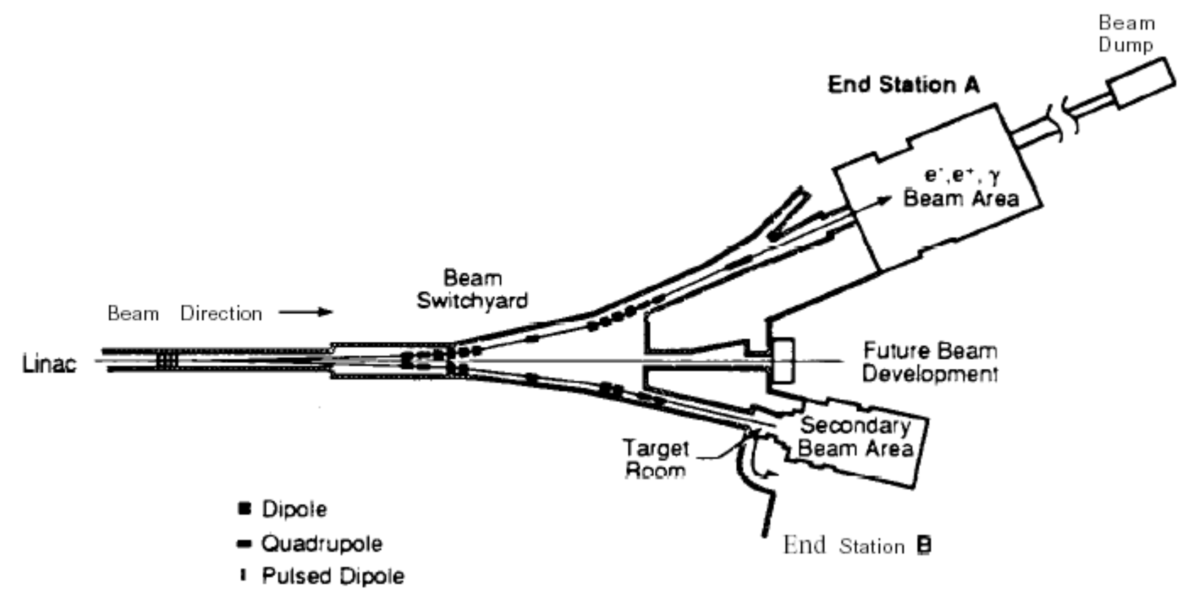
$$< r^2 >= \int d^3 r r^2 \rho(\mathbf{r}) = -6 \frac{dG_E(q^2)}{dq^2}\bigg|_{q^2=0} = ((0.81 \pm 0.04) \cdot 10^{-13}\ \mathrm{cm})^2$$

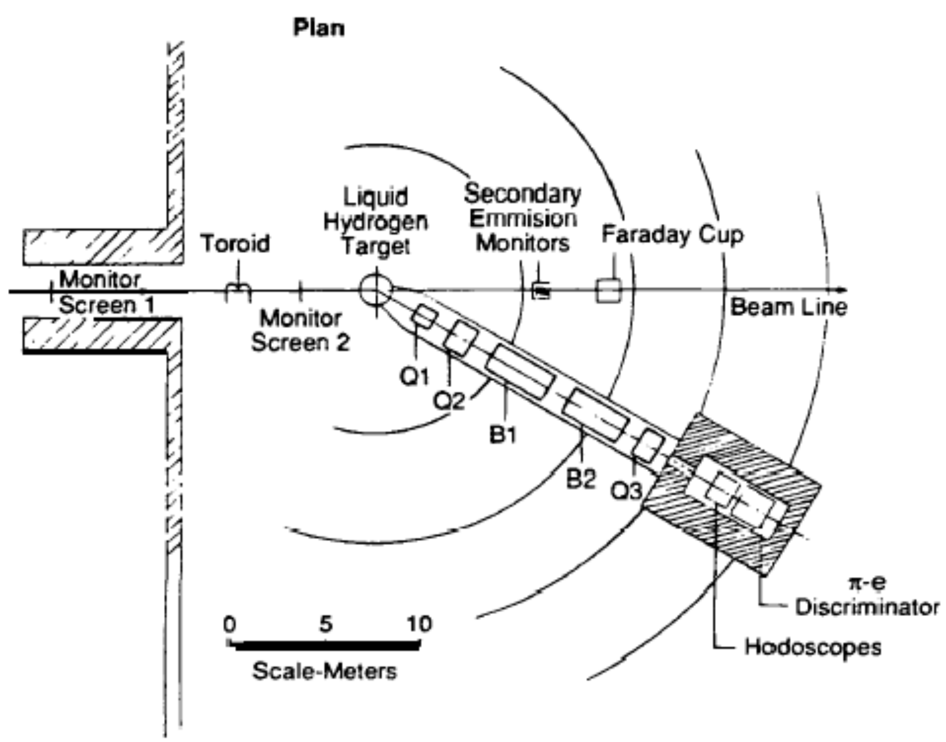


# SLAC-MIT

....under the direction of Taylor, Friedman and Kendall

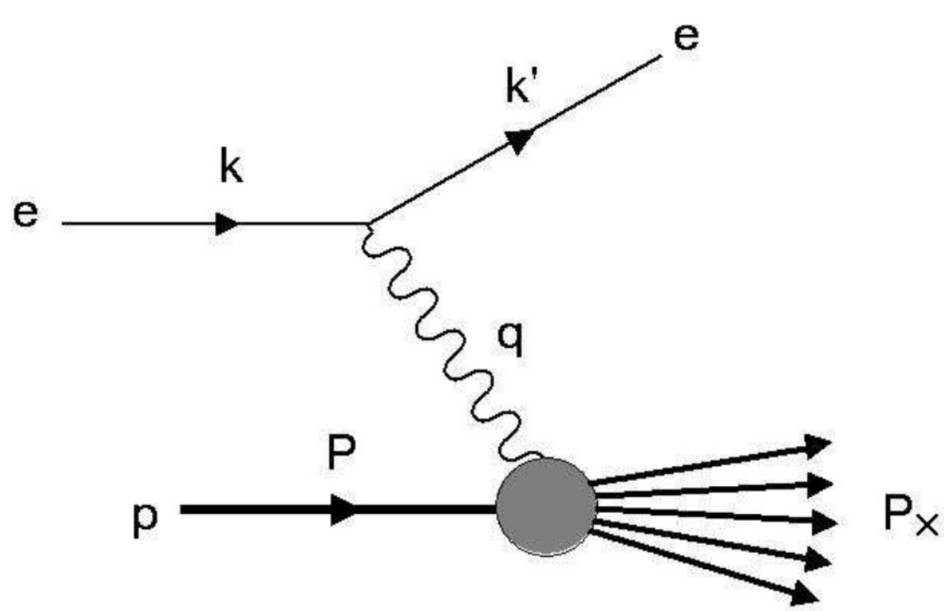



# SLAC-MIT

In 1967, the SLAC-MIT collaboration began the systematic study of inelastic scattering:




$$Q^2 = -(k-k')^2 = 4EE'\sin^2(\theta/2)$$

$$\omega = \frac{1}{x} = \frac{2m_p\nu}{Q^2}$$

$$\nu = \frac{P\cdot q}{m_p} = E - E'$$

$$W^2 = P_X^2 = 2m_p\nu + m_p^2 - Q^2$$

Elastic vs inelastic scattering:

Elastic scattering provides information on 'time-averaged' electric charge and magnetic moment distributions

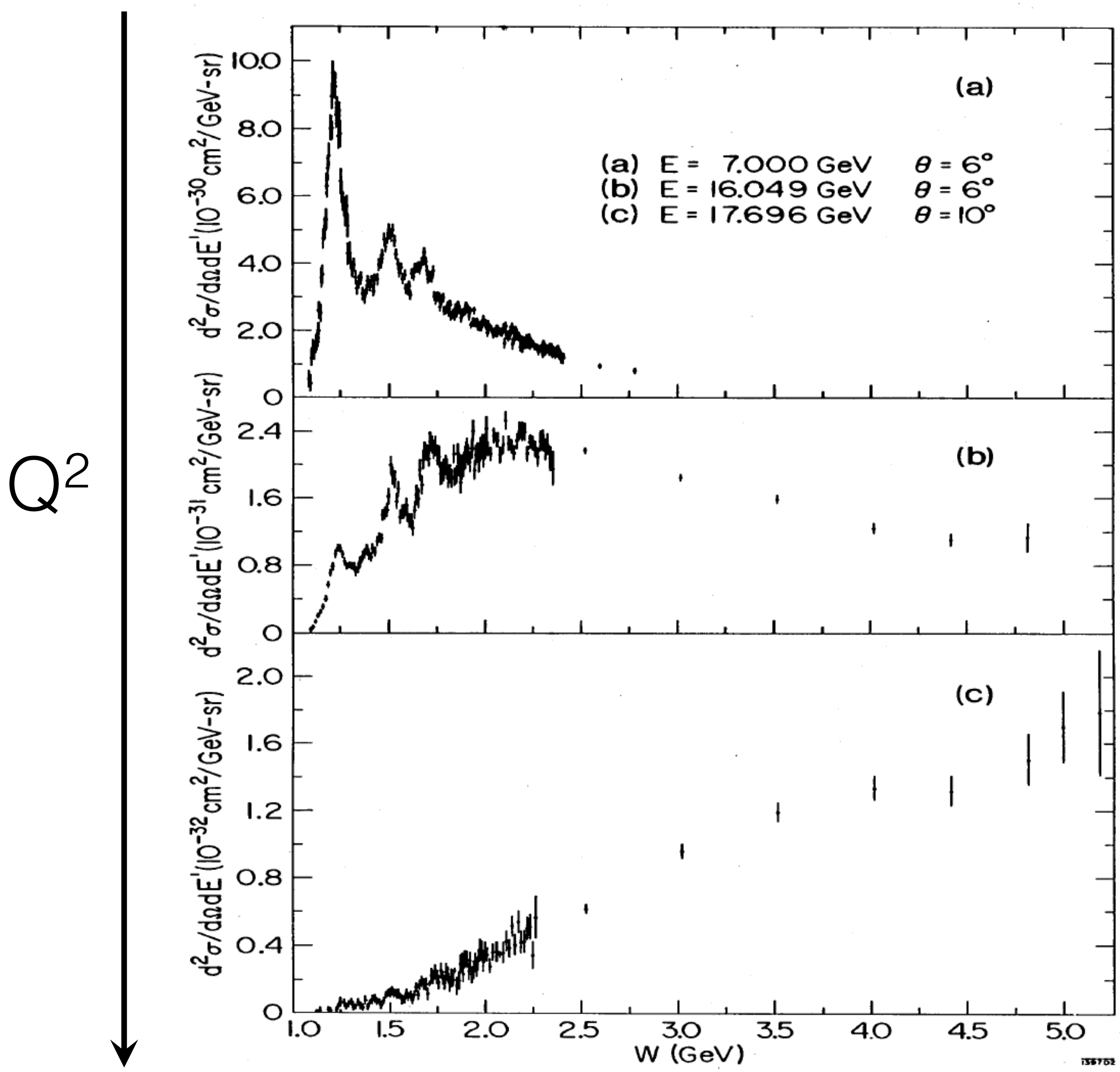
Inelastic scattering gives us a 'snapshot' of the proton's structure

$$\Delta t = \frac{h}{\Delta E}$$

$$\Delta E = 2\ \mathrm{GeV} \rightarrow \Delta t = 3\cdot 10^{-25}\ \mathrm{s}$$

$$\rightarrow \Delta x \simeq 10^{-16}\ \mathrm{m} \quad (v=c)$$

# SLAC MIT




It was decided to study the region of the continuum.

Two unexpected results :

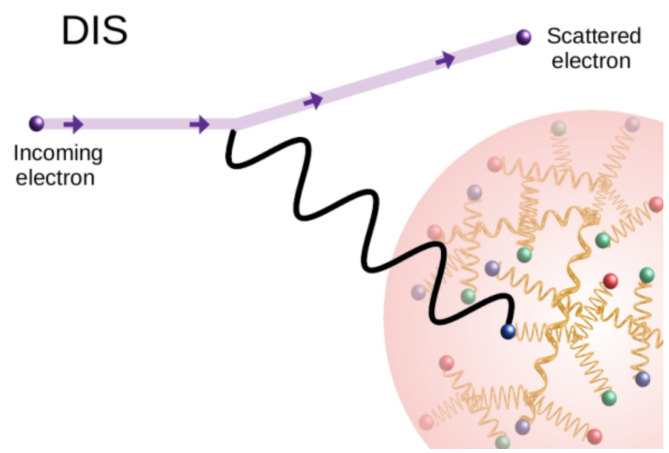
- Bjorken scaling
- Slight dependence on $Q^2$

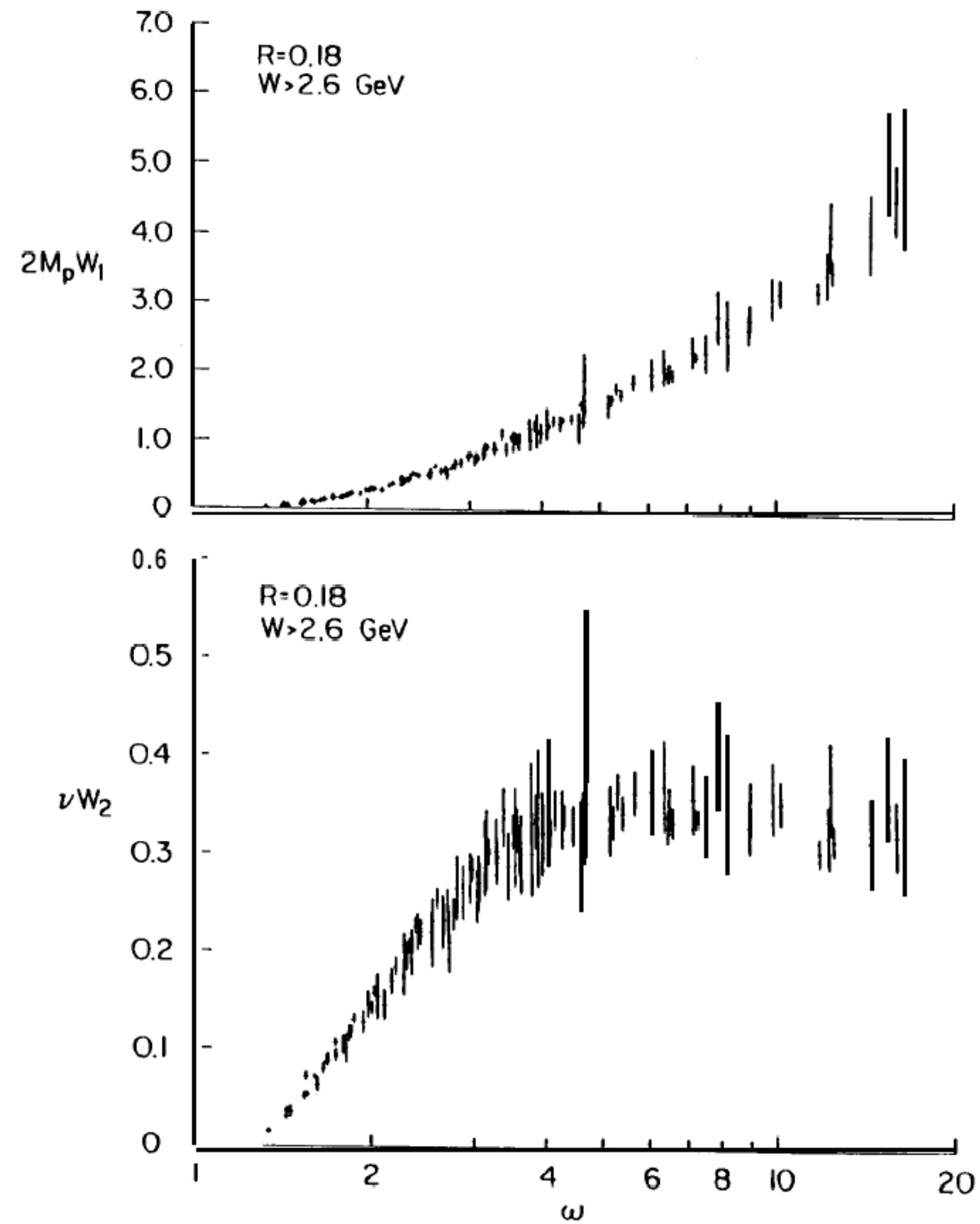
Reminder:

$$Q^2_{el} = 2E_e E'_e(1 - cos\theta'_e)$$

# Bjorken scaling

$$W > 2.6 \text{ GeV}$$
$$2 < Q^2 < 20 \text{ GeV}^2$$




Differential Cross section:

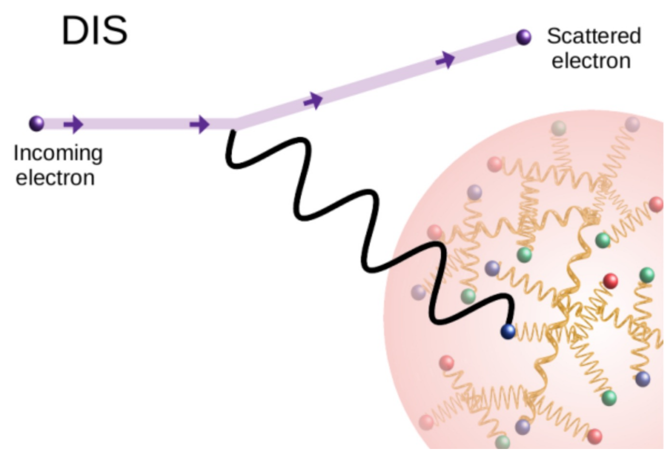
$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_M \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\tan^2(\theta/2)\right]$$

Bjorken scaling:

$$\text{for } \nu \to \infty \;,\; Q^2 \to \infty \;\; (\text{with } \omega = \frac{2m_p\nu}{Q^2} \text{ fixed})$$

$$\nu W_2(\nu, Q^2) \to F_2(\omega)$$
$$2m_p W_1(\nu, Q^2) \to F_1(\omega)$$

# Bjorken scaling




Differential Cross section:

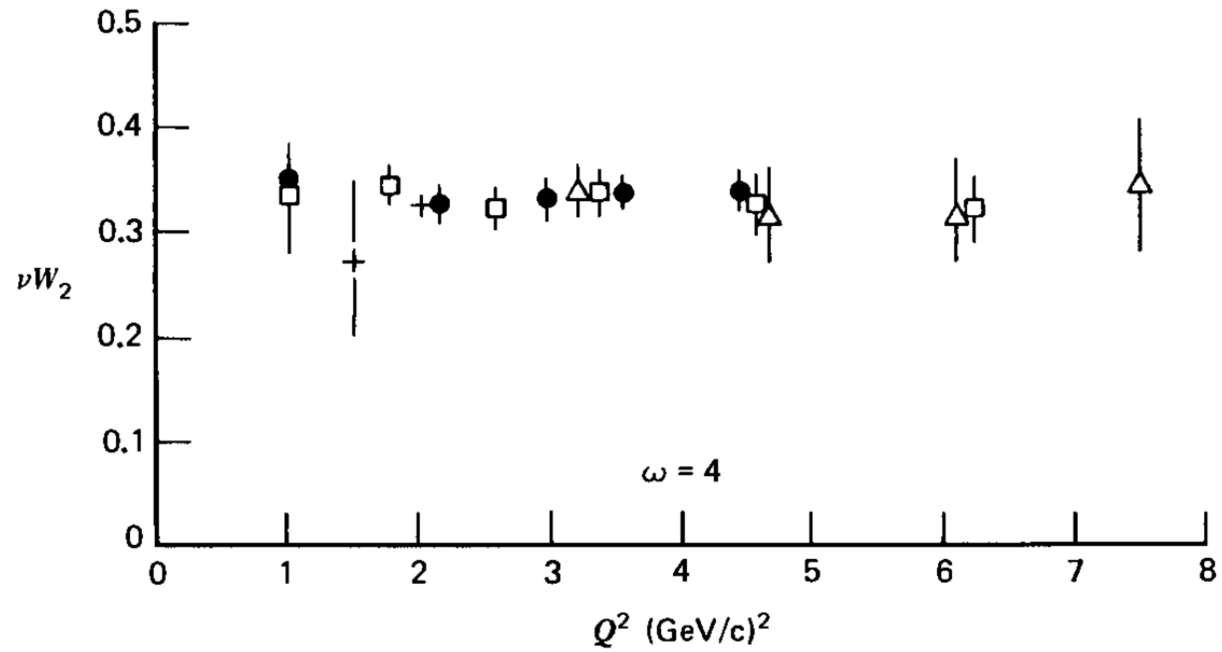
$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_M \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\tan^2(\theta/2)\right]$$

Bjorken scaling:

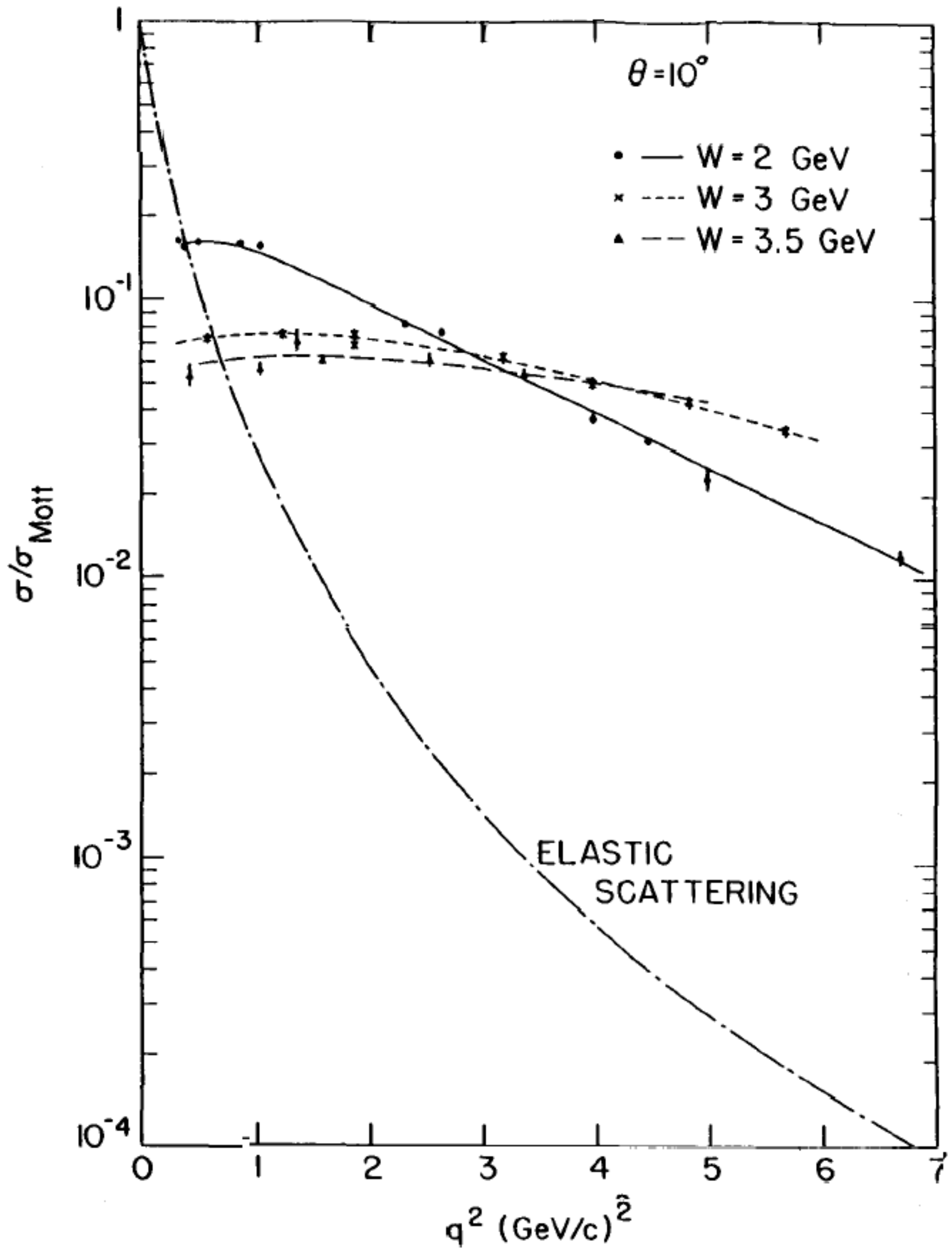
$$\text{for } \nu \to \infty \ , \ Q^2 \to \infty \ \ (\text{with } \omega = \frac{2m_p\nu}{Q^2} \text{ fixed})$$

$$\nu W_2(\nu, Q^2) \to F_2(\omega)$$

$$2m_p W_1(\nu, Q^2) \to F_1(\omega)$$



# $Q^2$ (in)dependence




Slight dependence on $Q^2$
(compared to elastic scattering)

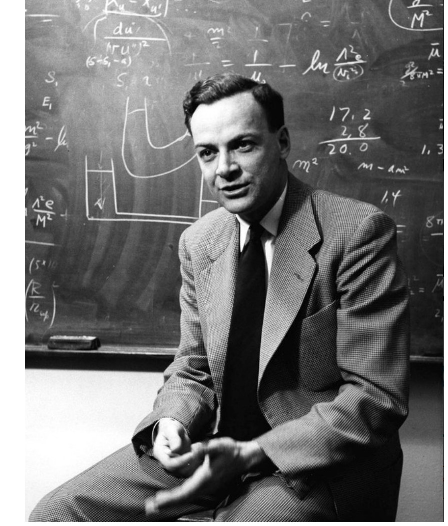
The measurement suggests the possible existence of "point-like" proton constituents

$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3 r$$

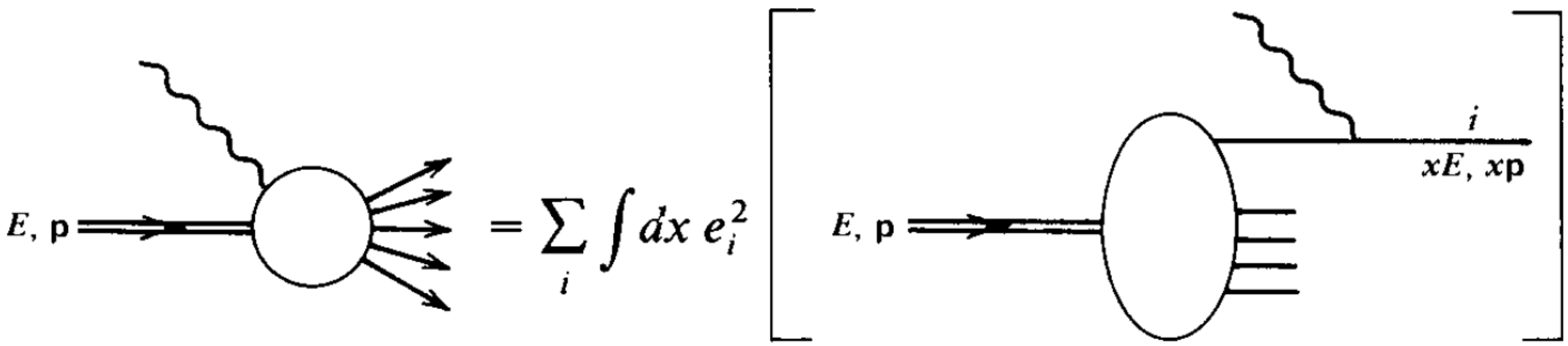
$$\text{if } \rho(\mathbf{r}) = \delta(\mathbf{r}) \to F(\mathbf{q}) = 1$$

“Pointlike” $\Longrightarrow$ weak $Q^2$ dependence

# The Parton Model



Feynman (1969): the proton is a collection of particles ("partons") and the virtual photon interacts with the single parton. The cross section is the incoherent sum of the photon-particle cross sections.



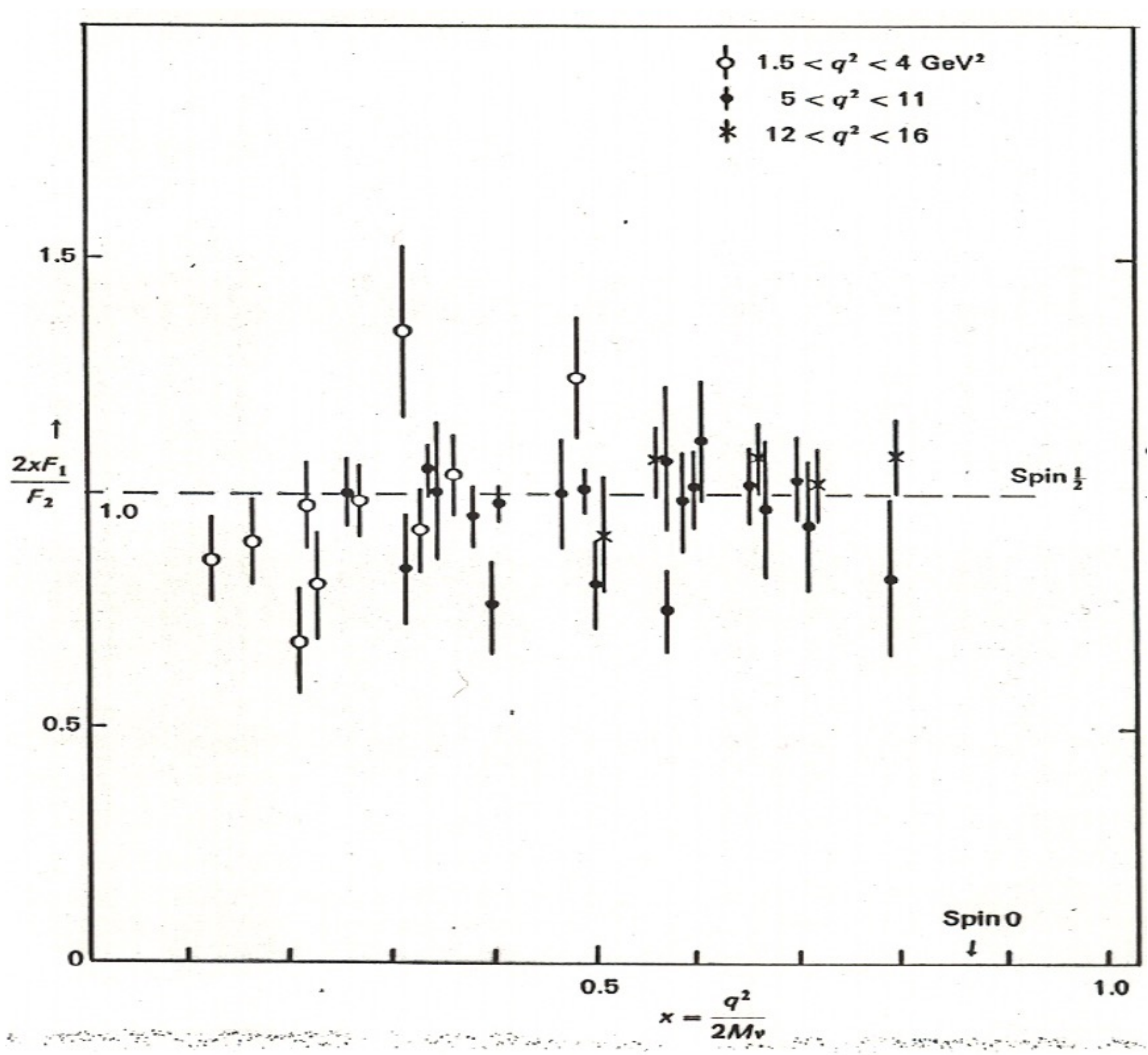

$$\nu W_2(\nu, Q^2) \rightarrow F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$m_p W_1(\nu, Q^2) \rightarrow F_1(x) = \frac{1}{2x} F_2(x)$$

# Partons and quarks

Can we identify partons with quarks?
- the spin must be 1/2
- the electric charge must be fractional (+2/3 , -1/3)

For spin, a first answer came from the experimental verification of the Callan-Gross relation:




$$R = \frac{\sigma_L}{\sigma_T} = \frac{W_2}{W_1}\left(1 + \frac{\nu^2}{q^2}\right) - 1$$

For spin 1/2 and in the "DIS Region":

$$R \rightarrow 0 \quad ; \quad F_2(x) = 2xF_1(x)$$

# Partons and quarks

For the fractional electric charge (+2/3 , -1/3) the first indications came from the "$F_2$ sum rule":

$$\frac{1}{2}\int \frac{[\nu W_2^p(\omega) + \nu W_2^n(\omega)]}{\omega^2} d\omega =$$

$$\frac{1}{2}\int [F_2^p(x) + F_2^n(x)]dx = \frac{Q_u^2 + Q_d^2}{2} \int x[u_p(x) + \bar{u}_p(x) + d_p(x) + \bar{d}_p(x)]dx$$

$$\frac{1}{2}\int [F_2^p(x) + F_2^n(x)]dx = \left(\frac{Q_u^2 + Q_d^2}{2}\right)(?) = \frac{5}{18} \cdot (?) \simeq 0.28 \cdot (?)$$

Experimental result (SLAC-MIT):

$$\frac{1}{2}\int [F_2^p(x) + F_2^n(x)]dx = 0.14 \pm 0.005$$

Conclusion: consistent with the quark model if quarks/antiquarks carry 50% of the proton's momentum. What about the remaining 50%?

# Gargamelle

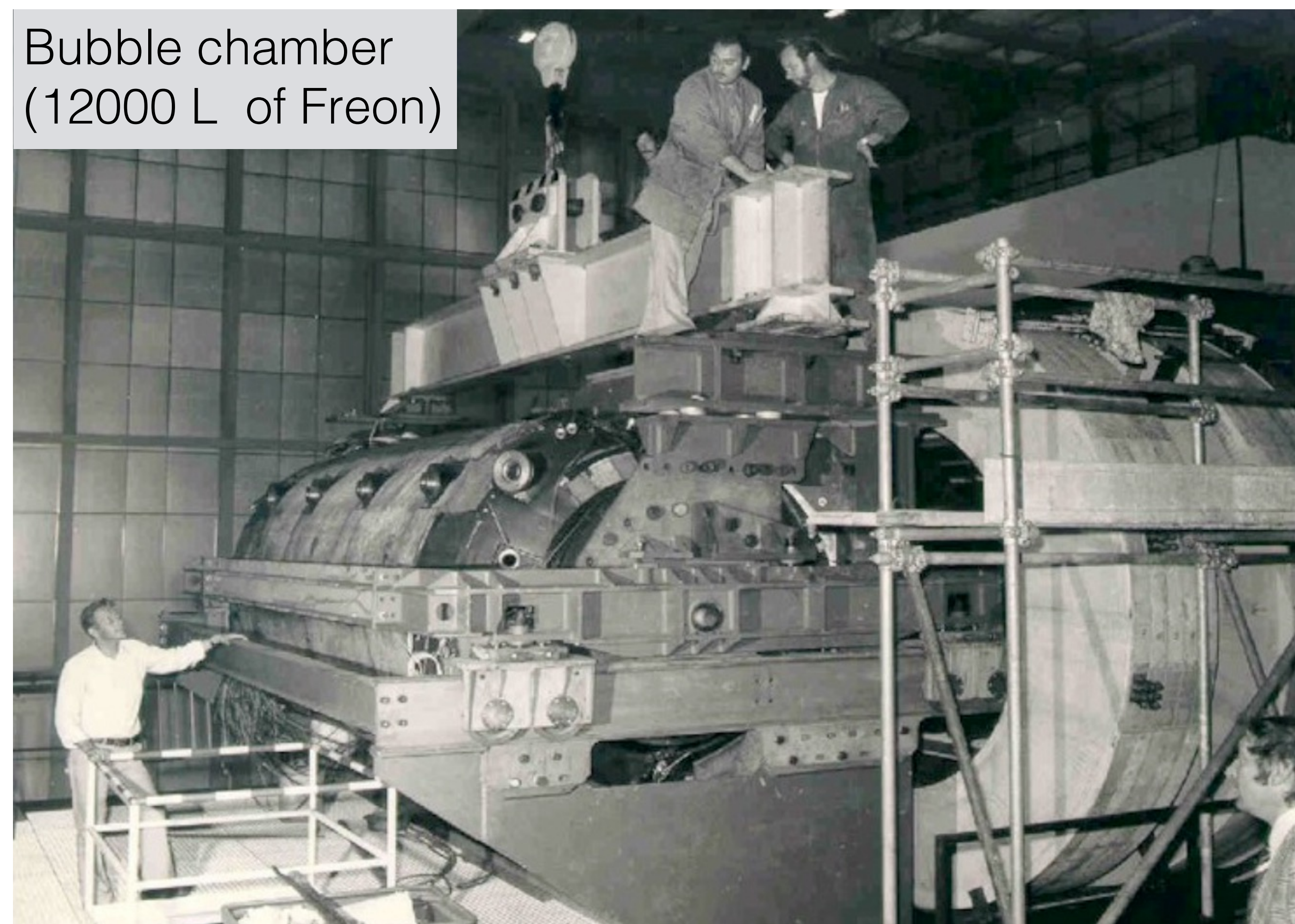
Can we identify partons with quarks?
- the spin must be 1/2
- the electric charge must be fractional (+2/3 , -1/3)

Further confirmation came from initial comparisons of electron and neutrino scattering data :

Gargamelle Experiment
(CERN 24 GeV PS Synchroton)

Processes:

$$\nu_\mu + N \to \mu^- + X$$
$$\bar{\nu}_\mu + N \to \mu^+ + X$$



# Gargamelle

Can we identify partons with quarks?
- the spin must be 1/2
- the electric charge must be fractional (+2/3 , -1/3)

Further confirmation came from initial comparisons of electron and neutrino scattering data :

Gargamelle Experiment
(CERN 24 GeV PS Synchroton)

Processes:

$$\nu_\mu + N \to \mu^- + X$$
$$\bar{\nu}_\mu + N \to \mu^+ + X$$

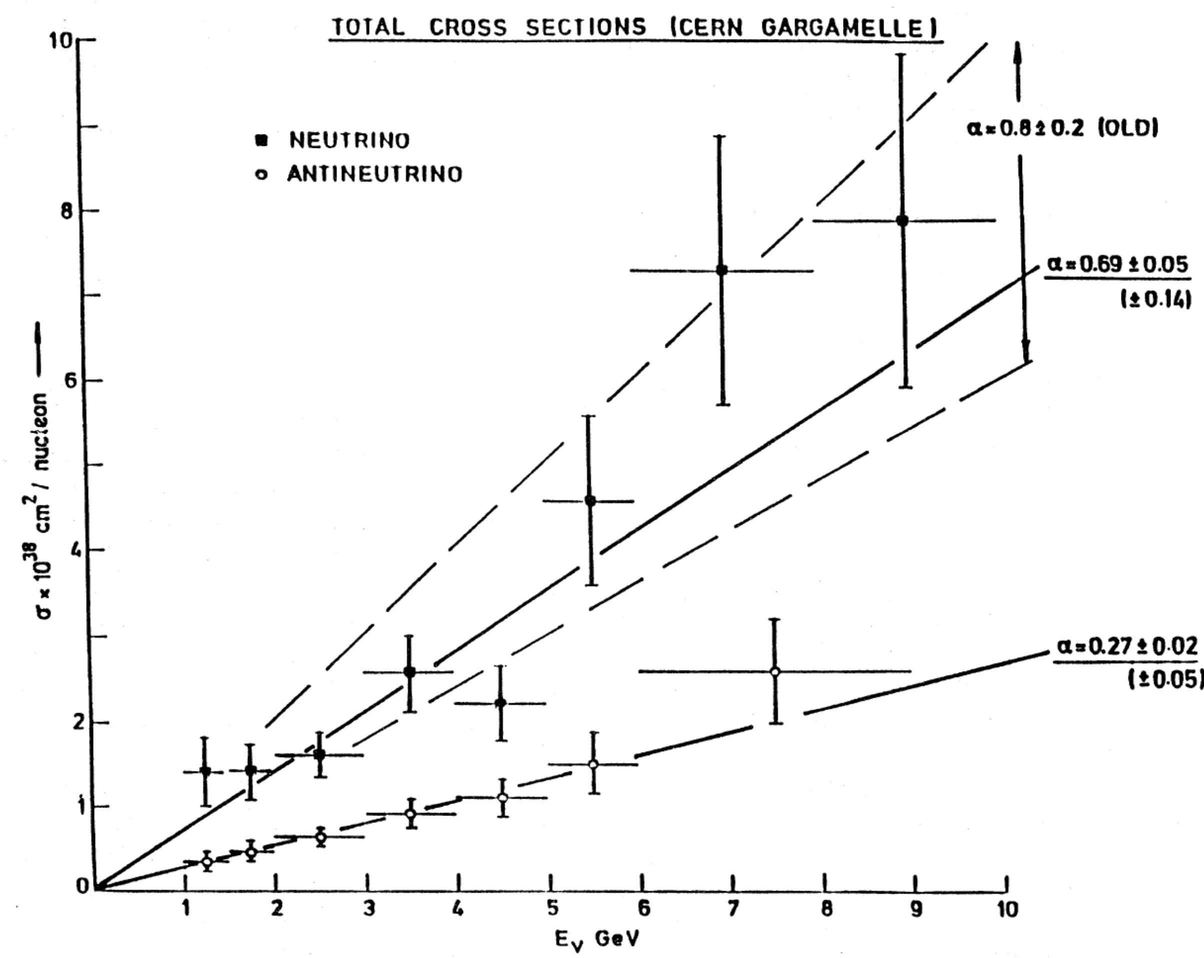
# Neutrinos

Linear Energy dependence of $\sigma_{tot}(\nu_\mu)$ , $(\sigma_{tot}(\bar{\nu}_\mu))$ provides further confirmation of the existence of point constituents in the proton and neutron.

Cross sections ("scaling region"):

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dx\,dy} = \frac{G^2ME}{\pi}\left[(1-y)F_2(x) + \frac{y^2}{2}\left[2xF_1(x)\right] \mp y\left(1-\frac{y}{2}\right)xF_3(x)\right],$$

$$\sigma_{tot}(\nu_\mu) = \alpha_\nu E$$
$$\sigma_{tot}(\bar{\nu}_\mu) = \alpha_{\bar{\nu}} E$$




ICHEP 1972

Perkins "...the preliminary data on the cross sections provide an astonishing verification for the Gell-Mann/Zweig quark model of hadrons."

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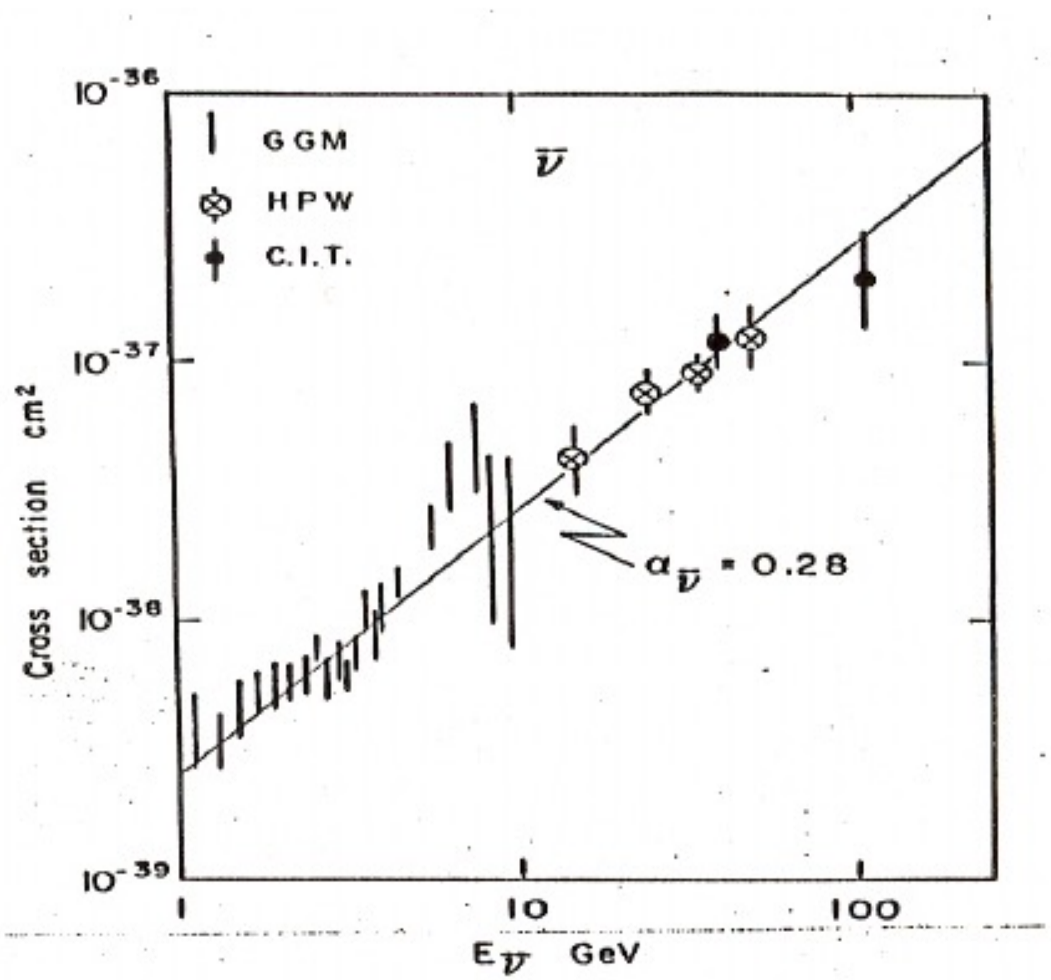


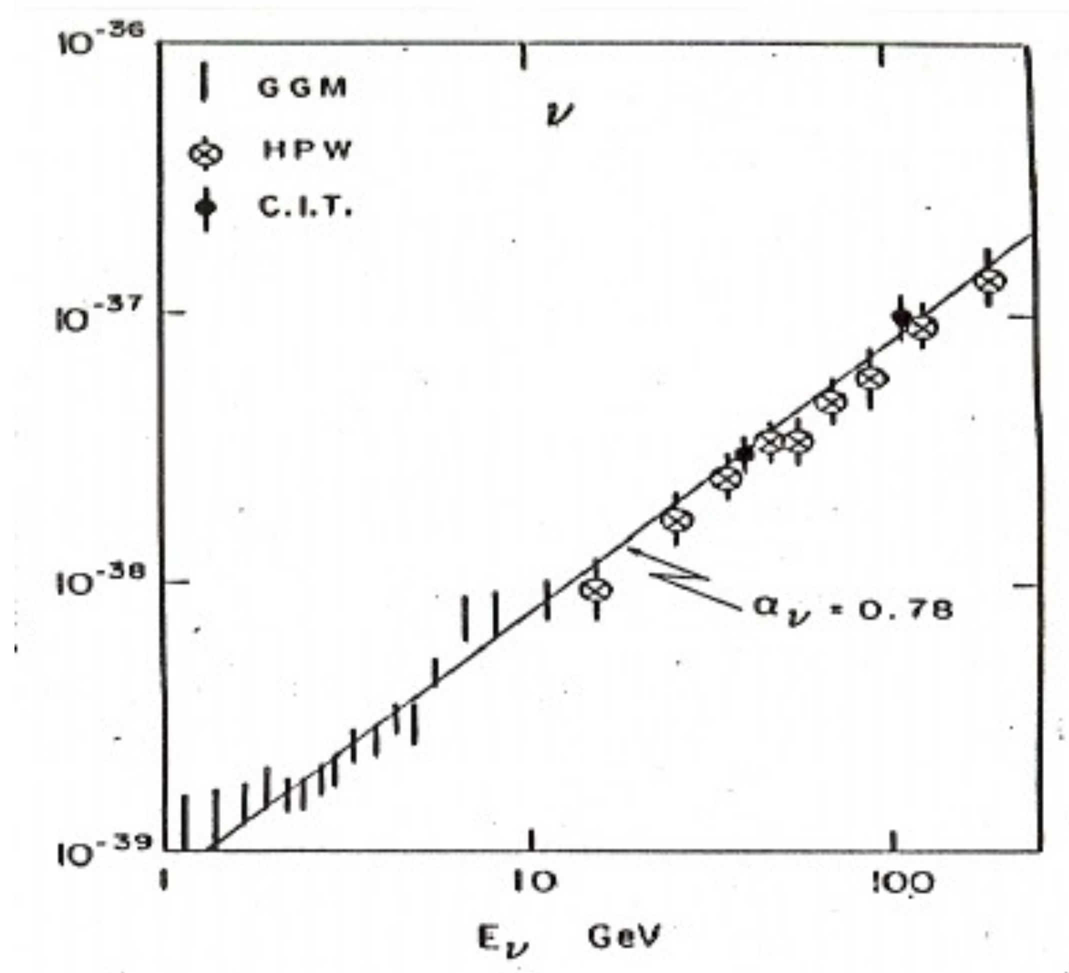


ICHEP 1972

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# Partons electric charge

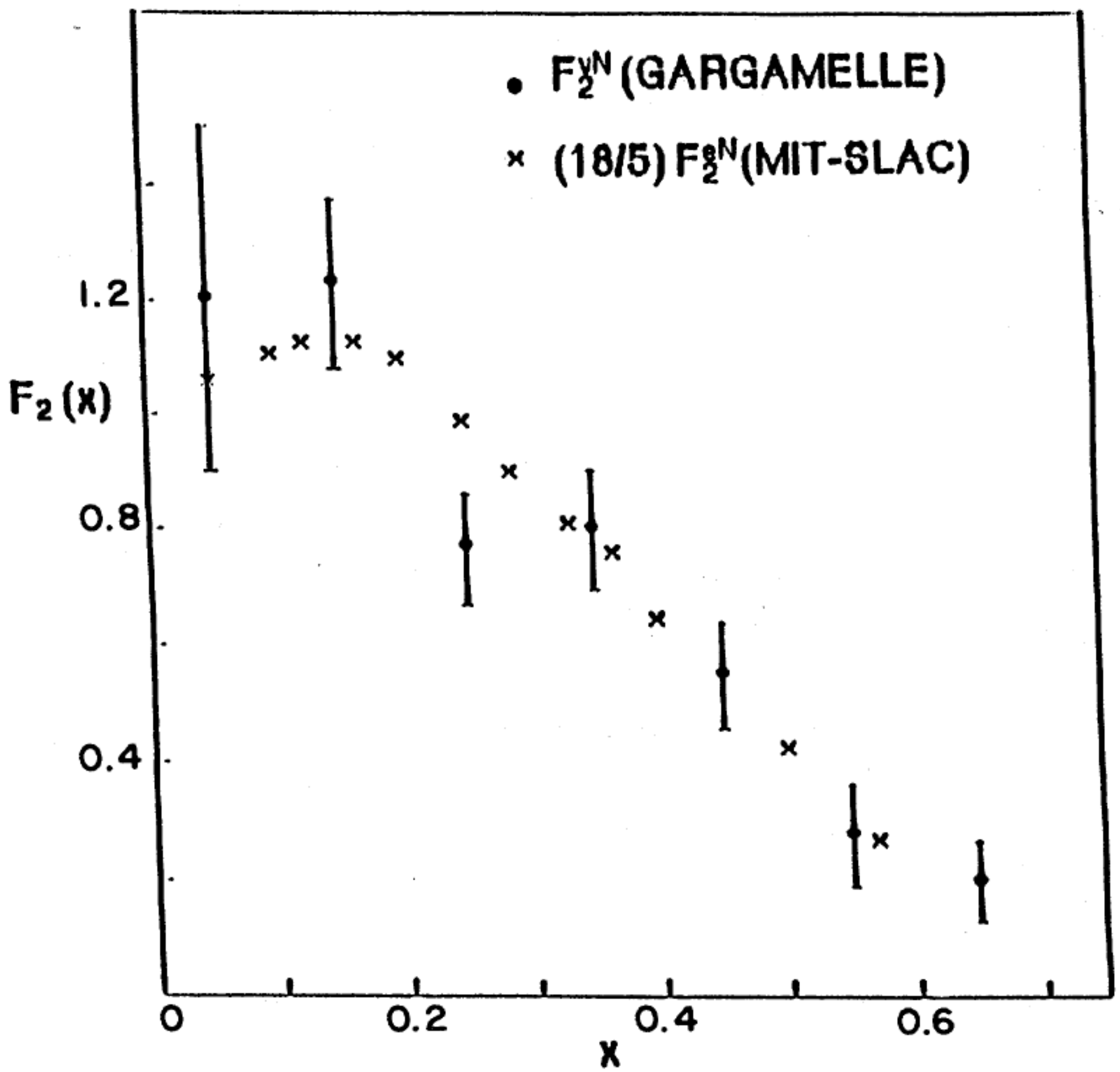
Partons have fractional charge (+2/3 , -1/3) ?

The answer came from the first comparison of electron and neutrino scattering data:

$$\frac{\frac{1}{2}\int\left[F_2^{\nu n}(x)+F_2^{\nu p}(x)\right]dx}{\frac{1}{2}\int\left[F_2^{en}(x)+F_2^{ep}(x)\right]dx}=\frac{2}{(Q_u^2+Q_d^2)}=\frac{2}{(2/3)^2+(1/3)^2}=\frac{18}{5}=3.6$$

Experimental result (SLAC-MIT, Gargamelle): $\mathbf{3.4 \pm 0.7}$

# Other early results with neutrinos




First comparison of $F_2^{\nu N}$ with $F_2^{eN}$

$$F_2^{\nu N} \sim (\frac{18}{5}) F_2^{eN}$$

# Other early results with neutrinos

$$\frac{1}{2}\int [F_2^{\nu p}(x) + F_2^{\nu n}(x)]\,dx = \int x[u_p(x) + \bar{u}_p(x) + d_p(x) + \bar{d}_p(x)]dx$$

Experimental result (Gargamelle): $0.49 \pm 0.7$

50% of the proton momentum is carried by quarks, in agreement with the SLAC-MIT results

$$\frac{1}{2}\int [F_3^{\nu p}(x) + F_3^{\nu n}(x)]dx = \text{Nr. of valence quarks}$$

Experimental result (Gargamelle): $3.2 \pm 0.6$

Consistent with the Quark Model

# Quantum ChromoDynamics (QCD)

$$\mathcal{L} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavours}} \bar{q}_a (i\not{D} - m)_{ab} q_b + \mathcal{L}_{\text{gauge-fixing}}$$

$$F^A_{\alpha\beta} = \partial_\alpha \mathcal{A}^A_\beta - \partial_\beta \mathcal{A}^A_\alpha - g f^{ABC} \mathcal{A}^B_\alpha \mathcal{A}^C_\beta$$

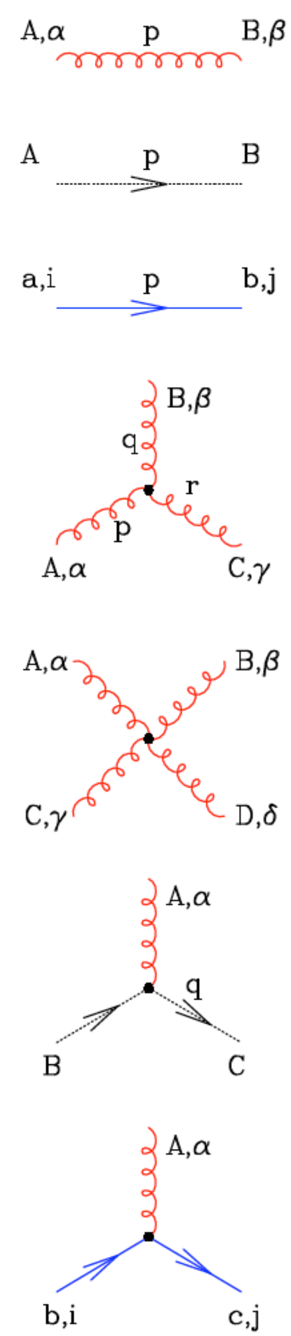
$$(D_\alpha)_{ab} \quad = \quad \partial_\alpha \delta_{ab} + ig \left( t^C \mathcal{A}^C_\alpha \right)_{ab}$$

$$\frac{\partial \alpha_S(Q)}{\partial \tau} = \beta(\alpha_S(Q)) \qquad \tau = \ln\left(\frac{Q^2}{\mu^2}\right)$$

$$\beta(\alpha_S) = -b\alpha_S^2 (1 + b'\alpha_S) + \mathcal{O}(\alpha_S^4)$$

$$b = \frac{(11C_A - 2N_f)}{12\pi}, \quad b' = \frac{(17C_A^2 - 5C_A N_f - 3C_F N_f)}{2\pi(11C_A - 2N_f)}$$

$$\alpha_S(Q) = \frac{1}{b \ln(Q^2/\Lambda^2)} \left[ 1 - \frac{b'}{b} \frac{\ln\ln(Q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)} \right] \qquad \text{(NLO)}$$




$$\delta^{AB} \left[ -g^{\alpha\beta} + (1-\lambda) \frac{p^\alpha p^\beta}{p^2 + i\epsilon} \right] \frac{i}{p^2 + i\epsilon}$$

$$\delta^{AB} \frac{i}{(p^2 + i\epsilon)}$$

$$\delta^{ab} \frac{i}{(\not{p} - m + i\epsilon)_{ji}}$$

$$-g\ f^{ABC}\left[(p-q)^\gamma g^{\alpha\beta} + (q-r)^\alpha g^{\beta\gamma} + (r-p)^\beta g^{\gamma\alpha}\right]$$

(all momenta incoming)

$$\begin{aligned} &-ig^2\ f^{XAC} f^{XBD}\ [g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\delta} g^{\beta\gamma}] \\ &-ig^2\ f^{XAD} f^{XBC}\ [g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta}] \\ &-ig^2\ f^{XAB} f^{XCD}\ [g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma}] \end{aligned}$$

$$g\ f^{ABC} q^\alpha$$

$$-ig\ (t^A)_{cb}\ (\gamma^\alpha)_{ji}$$

# DGLAP Equations

Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations

$$\frac{dq_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q_i(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{qg}\left(\frac{x}{y}\right)\right]$$

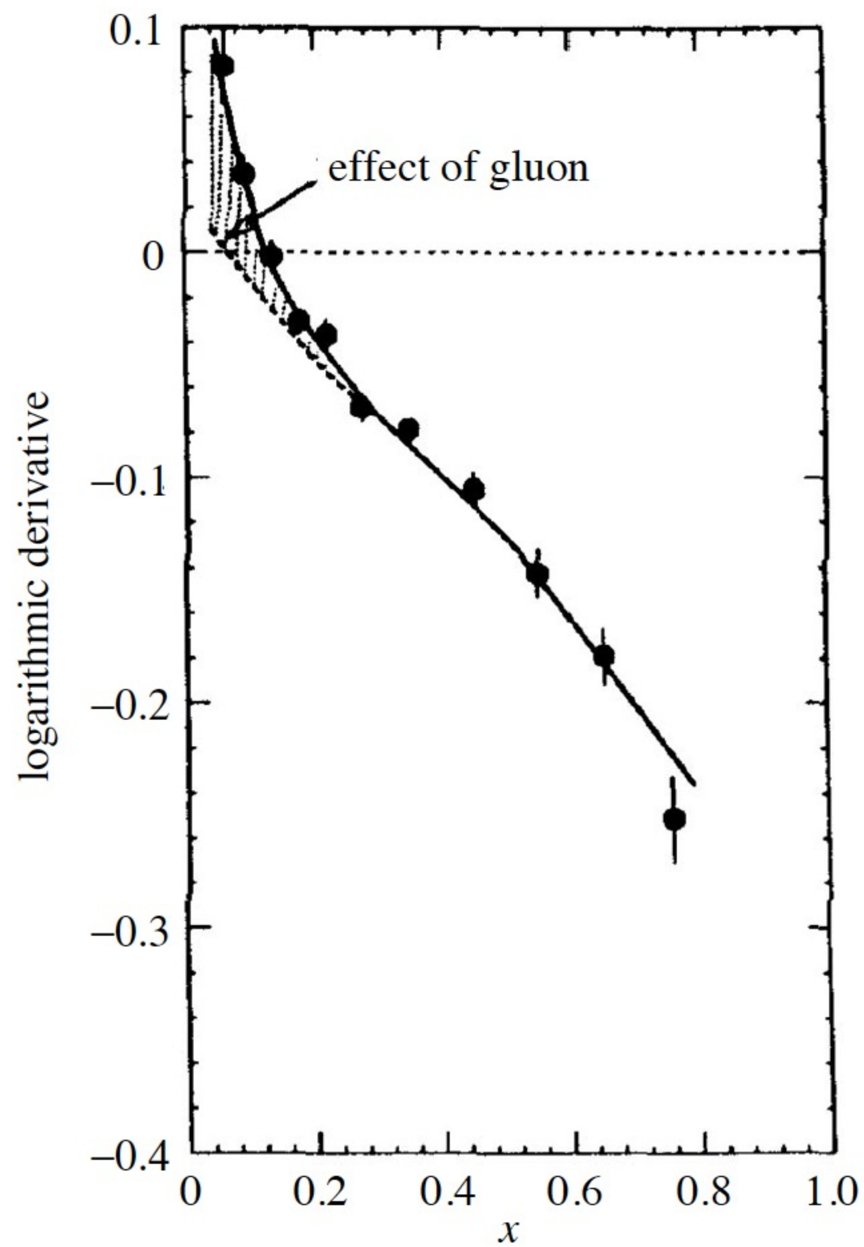
$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[\sum_i q_i(y,Q^2)P_{gq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{gg}\left(\frac{x}{y}\right)\right]$$

Splitting functions:

$$\begin{aligned}
P^{(0)}_{\mathrm{qq}}(z) &= \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z)\right] \\
P^{(0)}_{\mathrm{qg}}(z) &= \frac{1}{2}\left[z^2 + (1-z)^2\right] \\
P^{(0)}_{\mathrm{gq}}(z) &= \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right] \\
P^{(0)}_{\mathrm{gg}}(z) &= 6\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-z)\right]
\end{aligned}$$

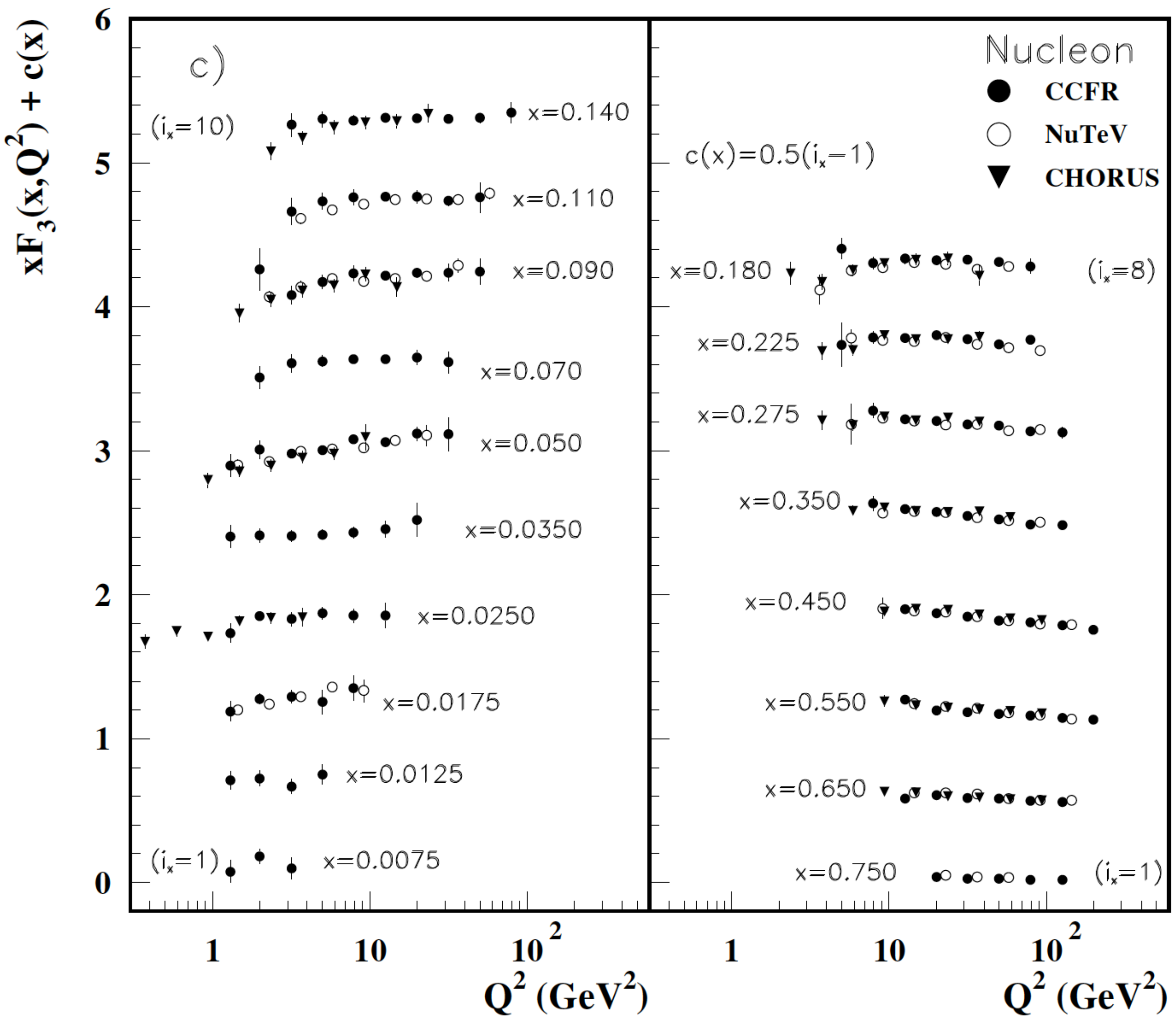
# Scaling violation

$$\frac{\partial F_2}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{F_2(\xi,Q^2)P_{qq}\left(\frac{x}{\xi},\alpha_s\right) + 2fG(\xi,Q^2)P_{qg}\left(\frac{x}{\xi},\alpha_s\right)\right\}$$



# Scaling violation

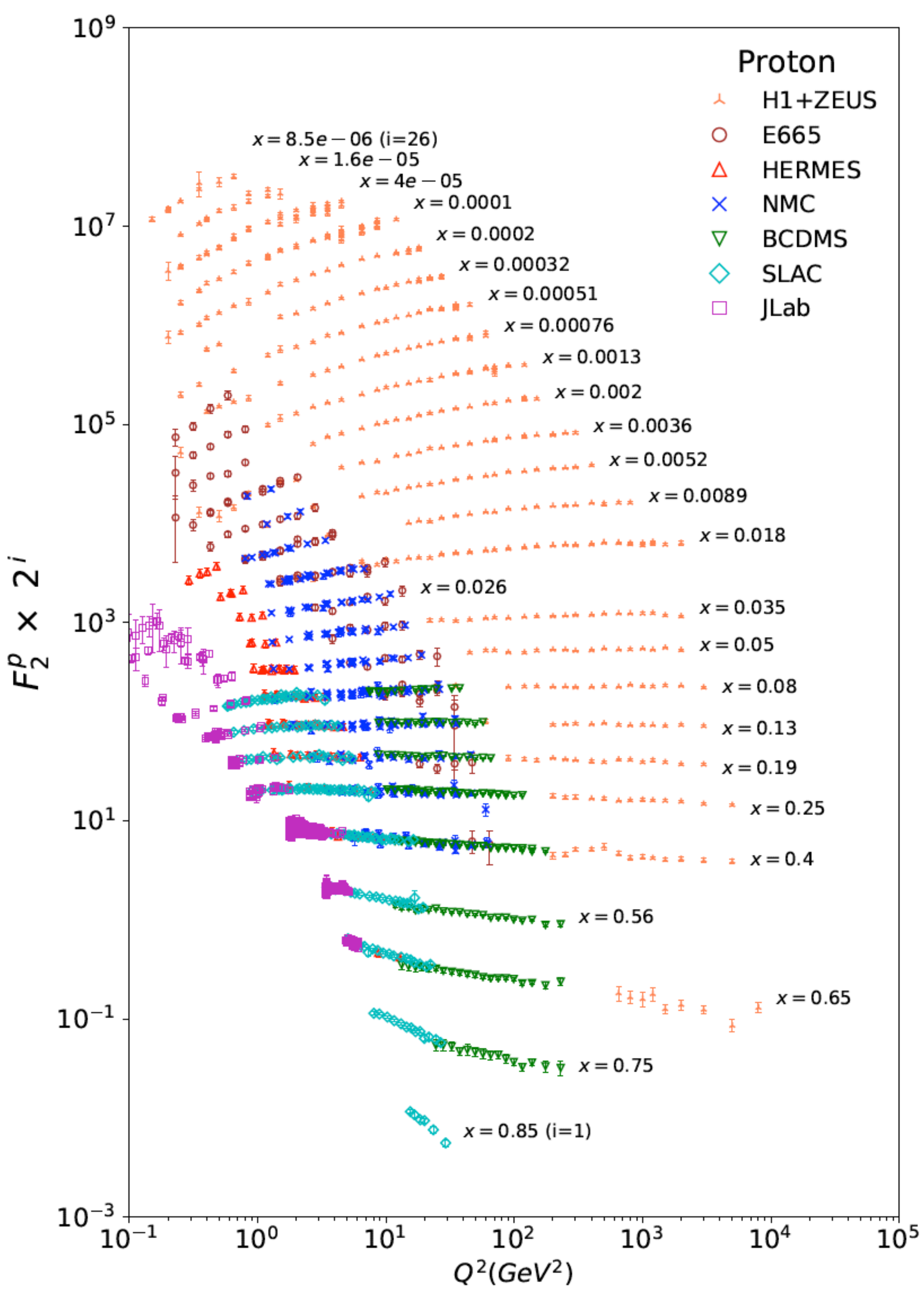
$$\frac{\partial x F_3}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left\{ \xi F_3(\xi, Q^2) P_{qq}\left(\frac{x}{\xi}, \alpha_s\right) \right\}$$

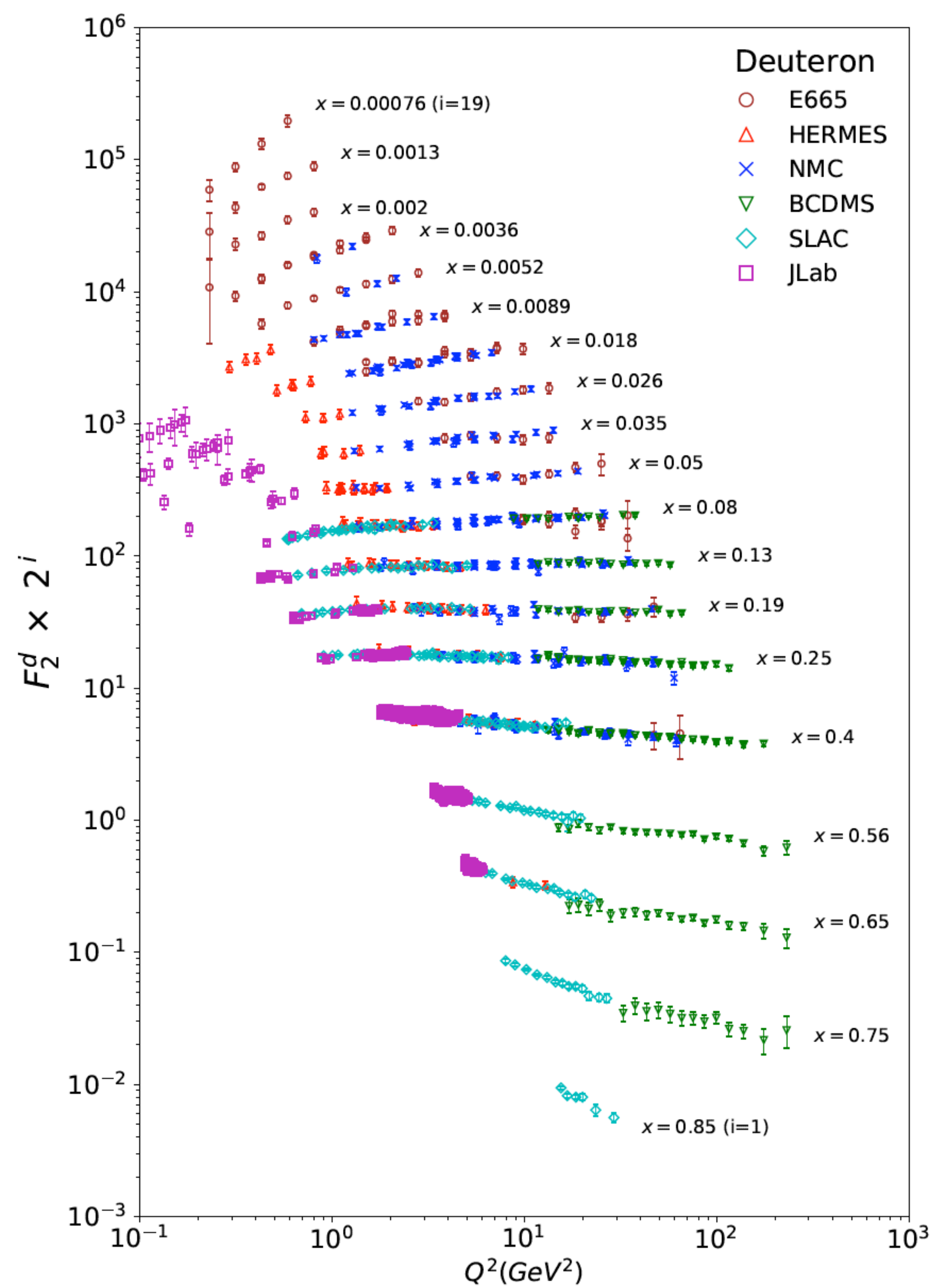


# Other Fixed Target Experiments

Experiments with muon beams: EMC, BFP, NMC, BCDMS, etc.

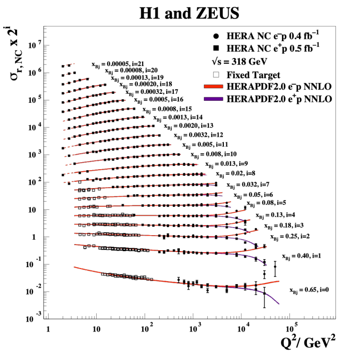
Experiments with neutrino beams: CCFR, HPWF, CDHSW, CHARM, WA24, WA21, etc.

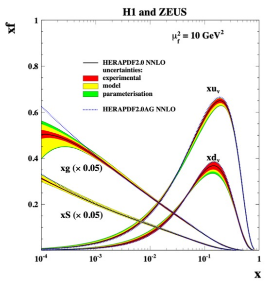


# II. HERA

- H1-ZEUS
- High-lights
- LHC & PDFs



# HERA

The world's first and only e-p collider


$e^{\pm}$ → ← p

27.5 GeV 920 GeV

$\sqrt{s}$ = 318 GeV

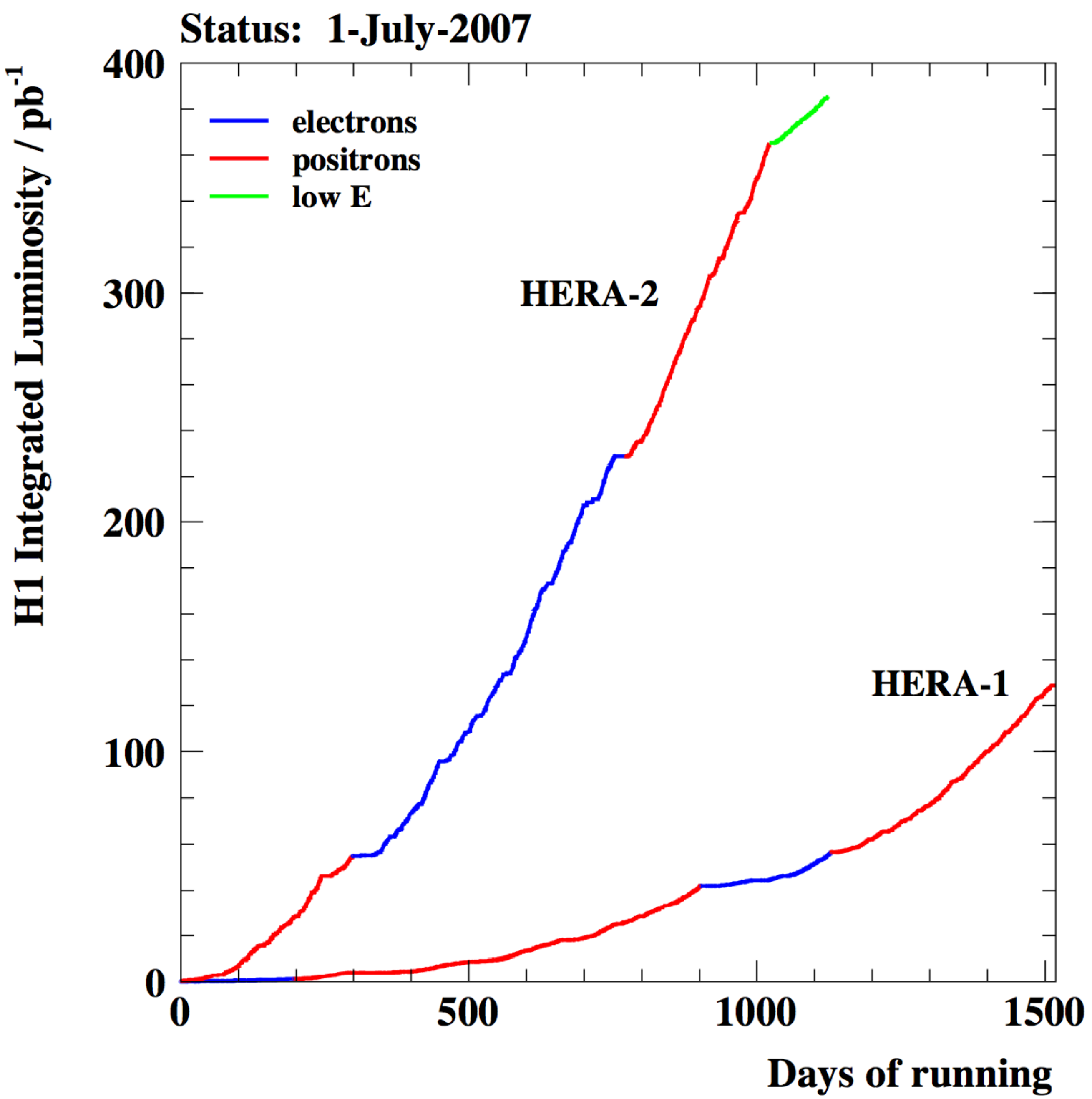
'Equivalent' to a fixed target experiment with $e^{\pm}$ of 50 TeV

# HERA Operation

HERA-I (1992-2000)
Ee=27.6 GeV
Ep=820 & 920 GeV
$L_{int}$ ~ 130 $pb^{-1}$ per experiment
Mostly $e^{+}p$

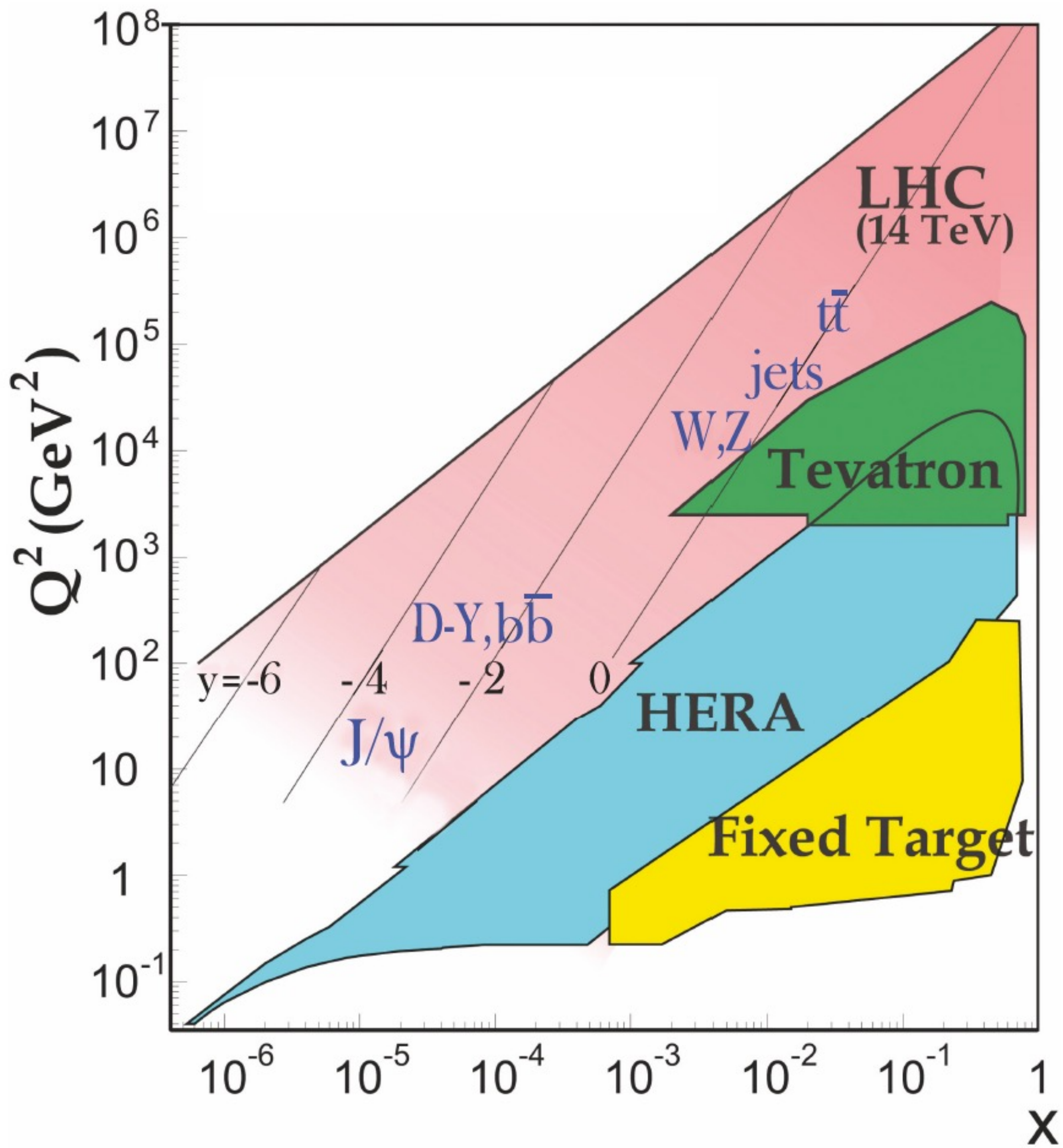
HERA-II (2003-2007)
Ee=27.6 GeV
Ep=920 GeV
$L_{int}$ ~ 360 $pb^{-1}$ per experiment
Longitudinally polarized lepton beams
Similar amounts of $e^{+}p$ and $e^{-}p$

Low Energy Run 2007
Ee=27.6 GeV
Ep=460 & 575 GeV
Runs at reduced $\sqrt{s}$ :
225 GeV (LER), 252 (MER) GeV
Dedicated $F_L$ measurements

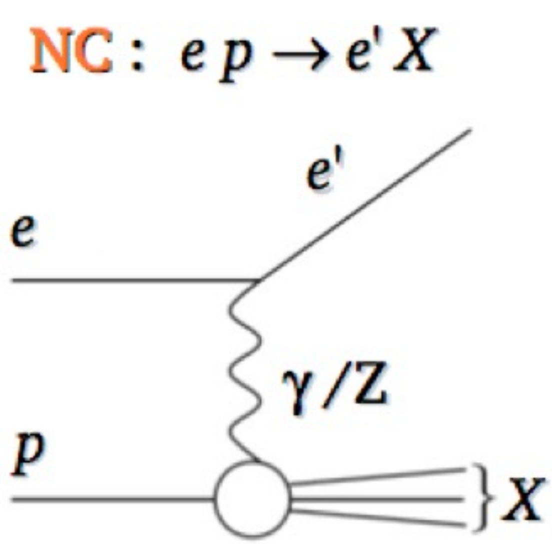


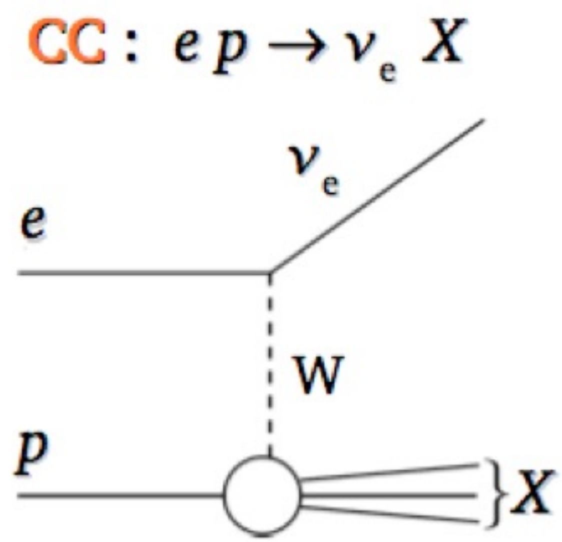

1 $fb^{-1}$ Integrated lumi, H1+ZEUS

# HERA Kinematic domain



# DIS processes and cross sections




Kinematic variables:

- Virtuality exchanged boson

$$Q^2 = -q^2 = -(k - k')^2$$

- Bjorken scaling variable

$$x = \frac{Q^2}{2p \cdot q}$$

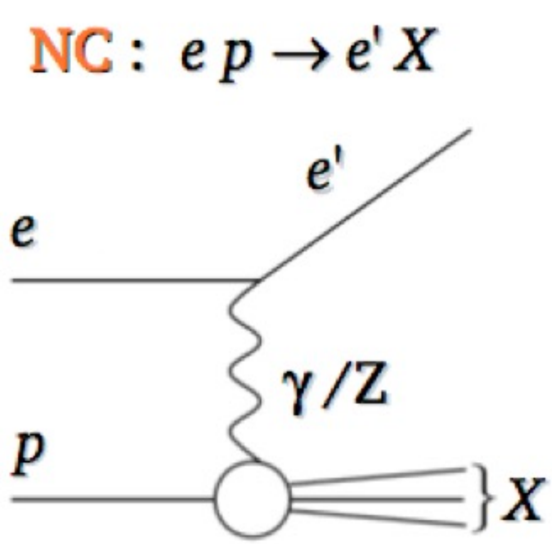
## "Reduced" Cross sections

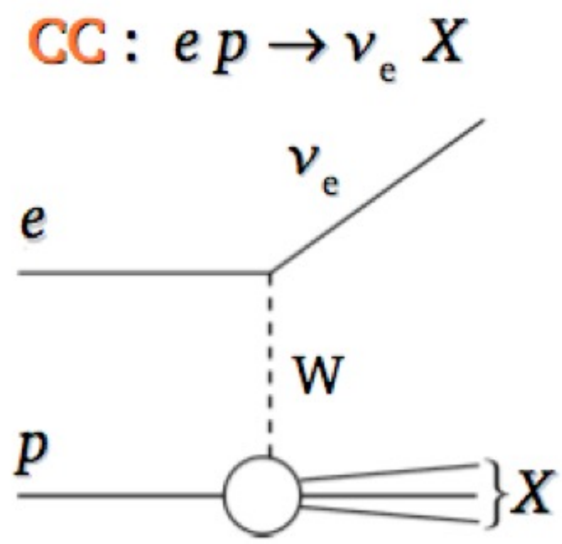
NC:
$$\sigma^{\pm}_{r,\mathrm{NC}} = \frac{d^2\sigma^{e^{\pm}p}_{\mathrm{NC}}}{dx dQ^2} \cdot \frac{Q^4 x}{2\pi\alpha^2 Y_+} = F_2 \mp \frac{Y_-}{Y_+} x F_3 - \frac{y^2}{Y_+} F_L$$

CC:
$$\sigma^{\pm}_{r,\mathrm{CC}} = \frac{d^2\sigma^{e^{\pm}p}_{\mathrm{CC}}}{dx dQ^2} \cdot \frac{2\pi x}{G_F^2} \left[ \frac{M_W^2 + Q^2}{M_W^2} \right]^2 = \frac{1}{2} \left( Y_+ W_2^{\pm} \mp Y_- x W_3^{\pm} - y^2 W_L^{\pm} \right)$$

with $\quad Y_{\pm} = 1 \pm (1-y)^2$

# DIS processes and cross sections




Kinematic variables:

- Virtuality exchanged boson

$$Q^2 = -q^2 = -(k - k')^2$$

- Bjorken scaling variable

$$x = \frac{Q^2}{2p \cdot q}$$

Structure Functions, PDFs and DGLAP evolution equations (LO, NLO and NNLO):

$$x^{-1} F_2(x, Q^2) = \sum_{i=q,g} \int_x^1 \frac{d\xi}{\xi} C_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i(\xi, \mu^2)$$

$$\frac{d}{d\ln\mu^2} f_i(\xi, \mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2) \otimes f_k(\mu^2))\right](\xi)$$

# Combination: Data sets

## The HERA Legacy

| Data Set | | $x_{Bj}$ Grid from | to | $Q^2$[GeV$^2$] Grid from | to | $\mathcal{L}$ pb$^{-1}$ | $e^+/e^-$ | $\sqrt{s}$ GeV | $x_{Bj}$,$Q^2$ from equations | Ref. |
|---|---|---|---|---|---|---|---|---|---|---|
| HERA I $E_p$ = 820 GeV and $E_p$ = 920 GeV data sets | | | | | | | | | | |
| H1 svx-mb [2] | 95-00 | 0.000005 | 0.02 | 0.2 | 12 | 2.1 | $e^+p$ | 301, 319 | 13,17,18 | [3] |
| H1 low $Q^2$ [2] | 96-00 | 0.0002 | 0.1 | 12 | 150 | 22 | $e^+p$ | 301, 319 | 13,17,18 | [4] |
| H1 NC | 94-97 | 0.0032 | 0.65 | 150 | 30000 | 35.6 | $e^+p$ | 301 | 19 | [5] |
| H1 CC | 94-97 | 0.013 | 0.40 | 300 | 15000 | 35.6 | $e^+p$ | 301 | 14 | [5] |
| H1 NC | 98-99 | 0.0032 | 0.65 | 150 | 30000 | 16.4 | $e^-p$ | 319 | 19 | [6] |
| H1 CC | 98-99 | 0.013 | 0.40 | 300 | 15000 | 16.4 | $e^-p$ | 319 | 14 | [6] |
| H1 NC HY | 98-99 | 0.0013 | 0.01 | 100 | 800 | 16.4 | $e^-p$ | 319 | 13 | [7] |
| H1 NC | 99-00 | 0.0013 | 0.65 | 100 | 30000 | 65.2 | $e^+p$ | 319 | 19 | [7] |
| H1 CC | 99-00 | 0.013 | 0.40 | 300 | 15000 | 65.2 | $e^+p$ | 319 | 14 | [7] |
| ZEUS BPC | 95 | 0.000002 | 0.00006 | 0.11 | 0.65 | 1.65 | $e^+p$ | 300 | 13 | [11] |
| ZEUS BPT | 97 | 0.0000006 | 0.001 | 0.045 | 0.65 | 3.9 | $e^+p$ | 300 | 13, 19 | [12] |
| ZEUS SVX | 95 | 0.000012 | 0.0019 | 0.6 | 17 | 0.2 | $e^+p$ | 300 | 13 | [13] |
| ZEUS NC [2] high/low $Q^2$ | 96-97 | 0.00006 | 0.65 | 2.7 | 30000 | 30.0 | $e^+p$ | 300 | 21 | [14] |
| ZEUS CC | 94-97 | 0.015 | 0.42 | 280 | 17000 | 47.7 | $e^+p$ | 300 | 14 | [15] |
| ZEUS NC | 98-99 | 0.005 | 0.65 | 200 | 30000 | 15.9 | $e^-p$ | 318 | 20 | [16] |
| ZEUS CC | 98-99 | 0.015 | 0.42 | 280 | 30000 | 16.4 | $e^-p$ | 318 | 14 | [17] |
| ZEUS NC | 99-00 | 0.005 | 0.65 | 200 | 30000 | 63.2 | $e^+p$ | 318 | 20 | [18] |
| ZEUS CC | 99-00 | 0.008 | 0.42 | 280 | 17000 | 60.9 | $e^+p$ | 318 | 14 | [19] |
| HERA II $E_p$ = 920 GeV data sets | | | | | | | | | | |
| H1 NC $^{1.5p}$ | 03-07 | 0.0008 | 0.65 | 60 | 30000 | 182 | $e^+p$ | 319 | 13, 19 | [8]$^1$ |
| H1 CC $^{1.5p}$ | 03-07 | 0.008 | 0.40 | 300 | 15000 | 182 | $e^+p$ | 319 | 14 | [8]$^1$ |
| H1 NC $^{1.5p}$ | 03-07 | 0.0008 | 0.65 | 60 | 50000 | 151.7 | $e^-p$ | 319 | 13, 19 | [8]$^1$ |
| H1 CC $^{1.5p}$ | 03-07 | 0.008 | 0.40 | 300 | 30000 | 151.7 | $e^-p$ | 319 | 14 | [8]$^1$ |
| H1 NC med $Q^2$ $^{*y.5}$ | 03-07 | 0.0000986 | 0.005 | 8.5 | 90 | 97.6 | $e^+p$ | 319 | 13 | [10] |
| H1 NC low $Q^2$ $^{*y.5}$ | 03-07 | 0.000029 | 0.00032 | 2.5 | 12 | 5.9 | $e^+p$ | 319 | 13 | [10] |
| ZEUS NC | 06-07 | 0.005 | 0.65 | 200 | 30000 | 135.5 | $e^+p$ | 318 | 13,14,20 | [22] |
| ZEUS CC $^{1.5p}$ | 06-07 | 0.0078 | 0.42 | 280 | 30000 | 132 | $e^+p$ | 318 | 14 | [23] |
| ZEUS NC $^{1.5}$ | 05-06 | 0.005 | 0.65 | 200 | 30000 | 169.9 | $e^-p$ | 318 | 20 | [20] |
| ZEUS CC $^{1.5}$ | 04-06 | 0.015 | 0.65 | 280 | 30000 | 175 | $e^-p$ | 318 | 14 | [21] |
| ZEUS NC nominal $^{*y}$ | 06-07 | 0.000092 | 0.008343 | 7 | 110 | 44.5 | $e^+p$ | 318 | 13 | [24] |
| ZEUS NC satellite $^{*y}$ | 06-07 | 0.000071 | 0.008343 | 5 | 110 | 44.5 | $e^+p$ | 318 | 13 | [24] |
| HERA II $E_p$ = 575 GeV data sets | | | | | | | | | | |
| H1 NC high $Q^2$ | 07 | 0.00065 | 0.65 | 35 | 800 | 5.4 | $e^+p$ | 252 | 13, 19 | [9] |
| H1 NC low $Q^2$ | 07 | 0.0000279 | 0.0148 | 1.5 | 90 | 5.9 | $e^+p$ | 252 | 13 | [10] |
| ZEUS NC nominal | 07 | 0.000147 | 0.013349 | 7 | 110 | 7.1 | $e^+p$ | 251 | 13 | [24] |
| ZEUS NC satellite | 07 | 0.000125 | 0.013349 | 5 | 110 | 7.1 | $e^+p$ | 251 | 13 | [24] |
| HERA II $E_p$ = 460 GeV data sets | | | | | | | | | | |
| H1 NC high $Q^2$ | 07 | 0.00081 | 0.65 | 35 | 800 | 11.8 | $e^+p$ | 225 | 13, 19 | [9] |
| H1 NC low $Q^2$ | 07 | 0.0000348 | 0.0148 | 1.5 | 90 | 12.2 | $e^+p$ | 225 | 13 | [10] |
| ZEUS NC nominal | 07 | 0.000184 | 0.016686 | 7 | 110 | 13.9 | $e^+p$ | 225 | 13 | [24] |
| ZEUS NC satellite | 07 | 0.000143 | 0.016686 | 5 | 110 | 13.9 | $e^+p$ | 225 | 13 | [24] |

H1 & ZEUS have now published all their inclusive measurements (1992-2007)

- HERA-I
- HERA-II measurements at high-$Q^2$
- HERA-II measurements at reduced $\sqrt{s}$

$0.6 \times 10^{-6} < x_{Bj} < 0.65$ , $0.045 < Q^2 < 50000$

41 data sets are combined:

- NC & CC cross sections
- $e^+p$ and $e^-p$ scattering
- 4 different $\sqrt{s}$ (318, 301, 252 and 225 GeV)

2927 data points

1307 combined points

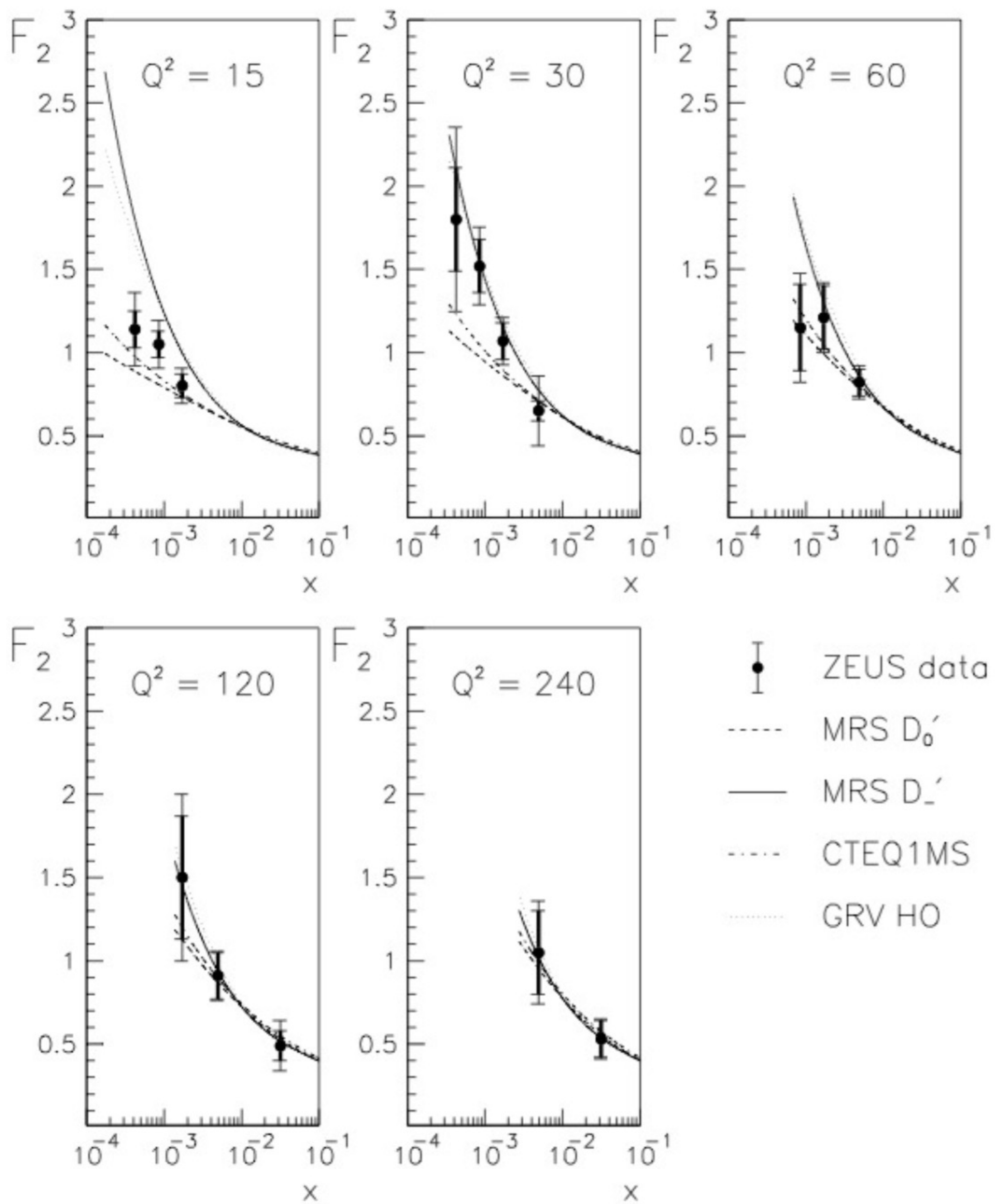
In typical cases 3 to 6 measurements contribute to a combined result

NC $e^+p$ accuracy reaches ~1%

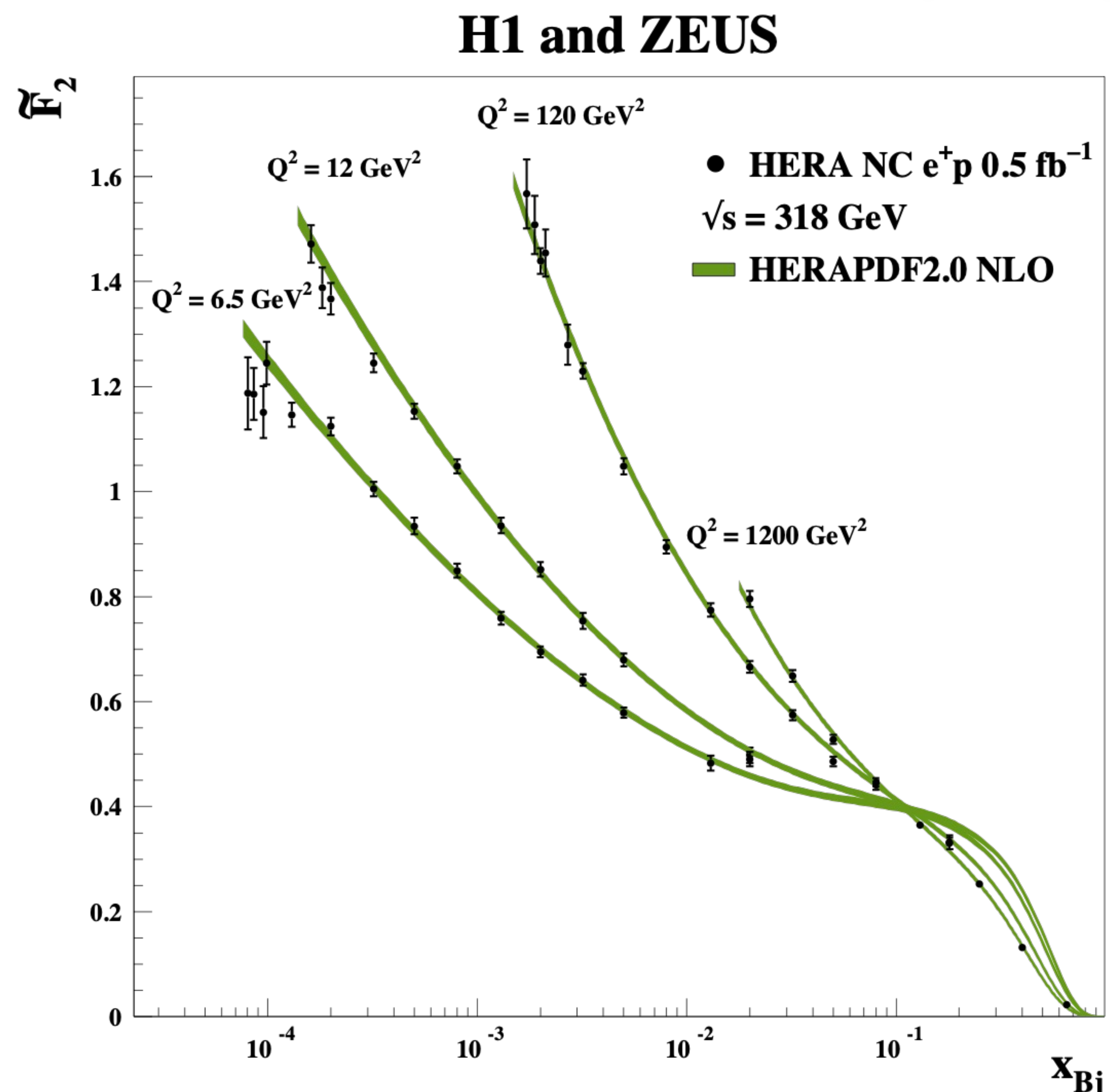
The usage of different reconstruction techniques and the differences in the strengths of the detector components of the two experiments lead to a substantial reduction of the systematic uncertainties of the combined cross sections.

# Rise of $F_2$

HERA $F_2$ (1993)




HERA $F_2$ (2015)



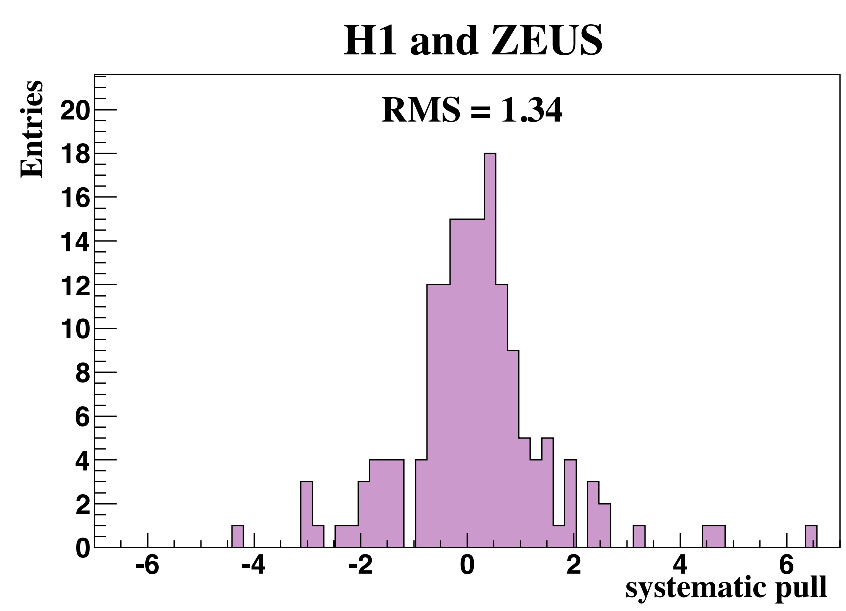

$$\tilde{F}_2 = \sigma^{\pm}_{r,\mathrm{NC}} \cdot \frac{\tilde{F}_2^{\text{predicted}}}{\sigma^{\pm\ \text{predicted}}_{r,\mathrm{NC}}} = \sigma^{\pm}_{r,\mathrm{NC}} \cdot (1 + C_F)$$

# Combination: Averaging Method

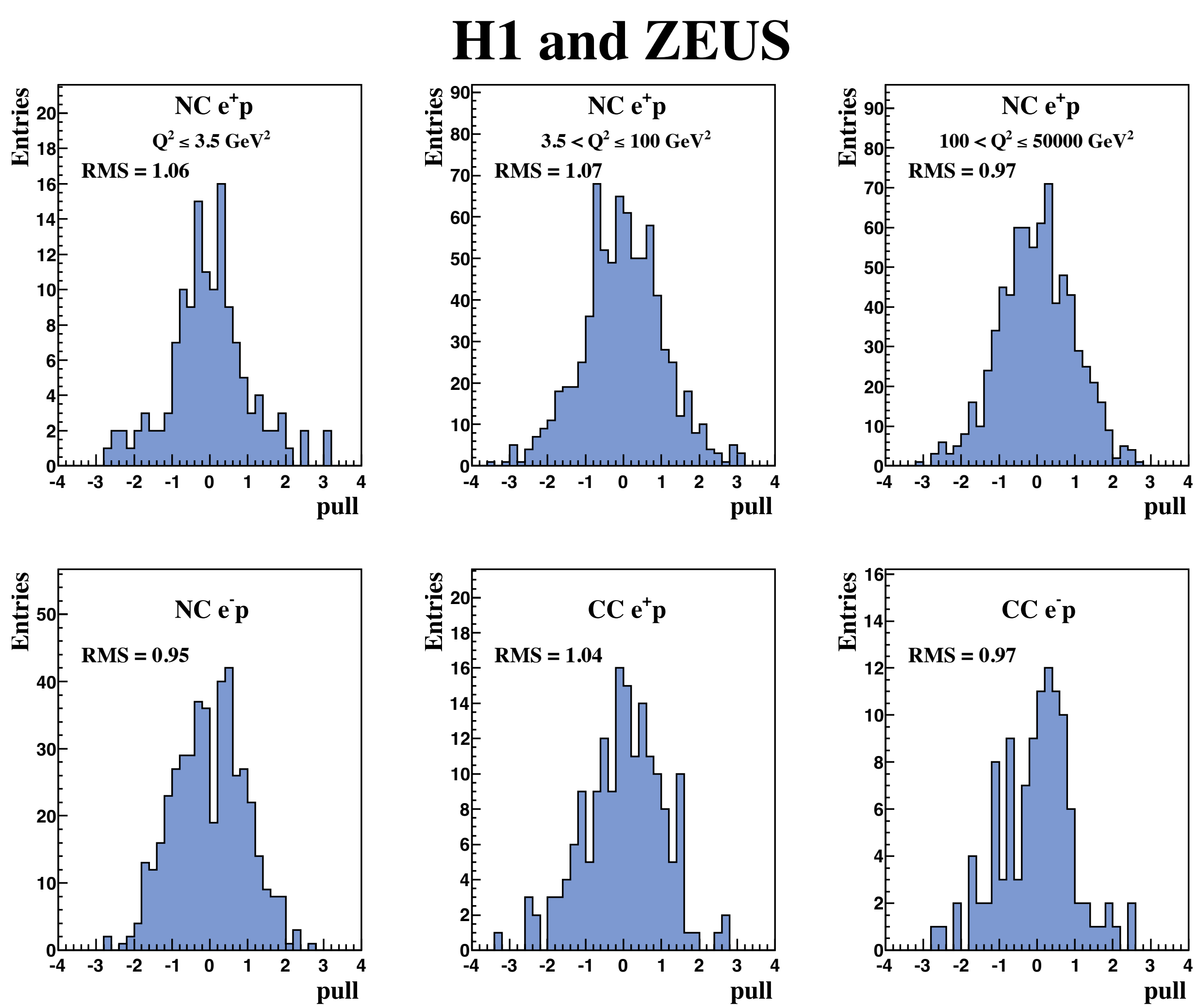
- Combination performed using the HERAverager package
- Averaging procedure take correlations of systematic unc. fully into account
- Multiplicative treatment of the systematic uncertainties (as a default choice)
- Minimisation procedure based on the following $\chi^2$ definition:

$$\chi^2_{\exp,ds}(\boldsymbol{m},\boldsymbol{b}) = \sum_i \frac{\left[m^i - \sum_j \gamma^i_j m^i b_j - \mu^i\right]^2}{\delta^2_{i,\mathrm{stat}}\,\mu^i\left(m^i - \sum_j \gamma^i_j m^i b_j\right) + \left(\delta_{i,\mathrm{uncor}}\, m^i\right)^2} + \sum_j b_j^2$$

- Procedural uncertainties:
  - Multiplicative vs additive nature of the systematic error sources
  - Correlations in photo-production background and hadronic energy scale across H1 and ZEUS measurements
  - Large pulls in correlated syst. uncert.



# Combination: Pulls




$\chi^2$ of the combination:

$$\frac{\chi^2}{d.o.f} = \frac{1687}{1620}$$

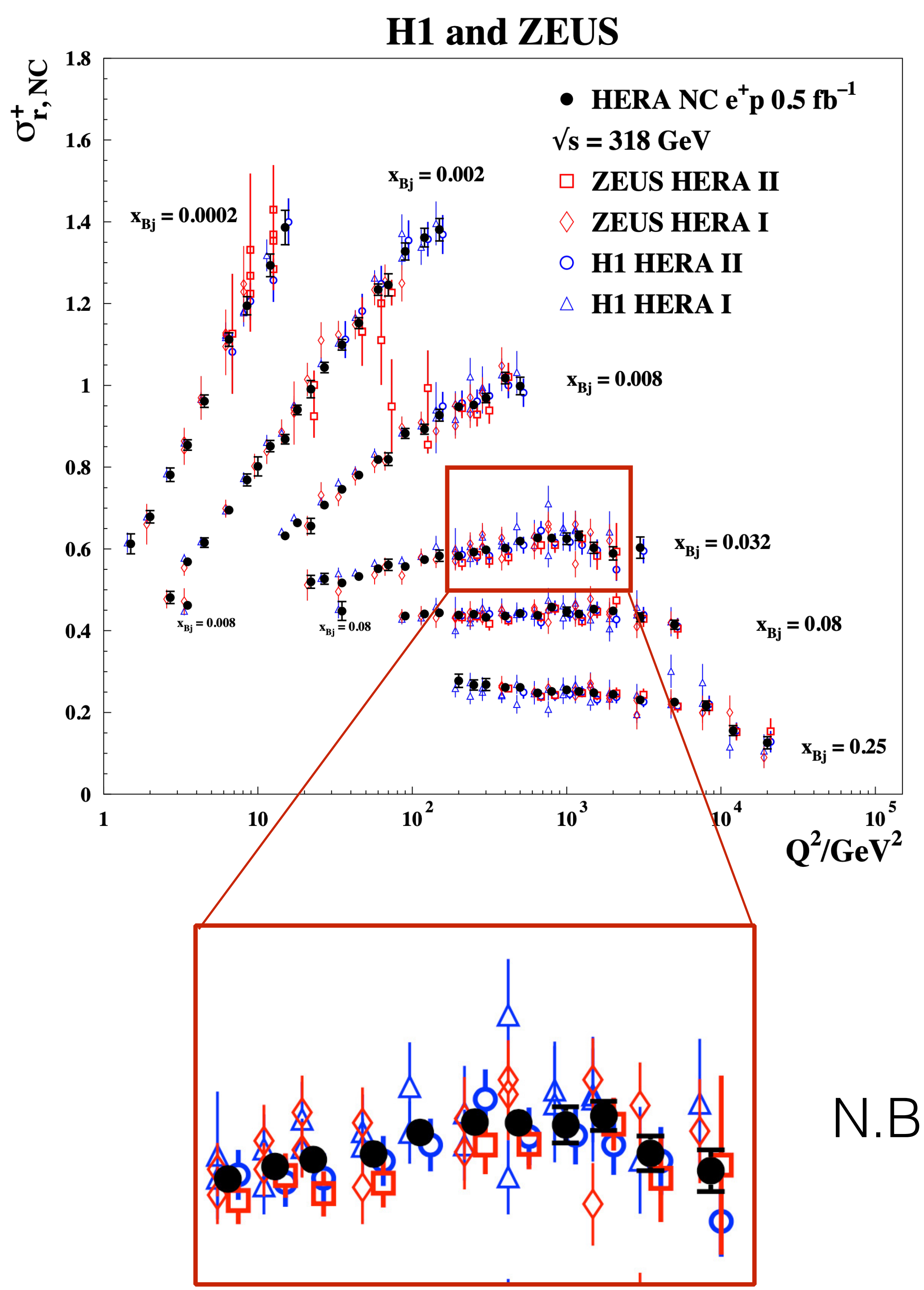
Pull definition:

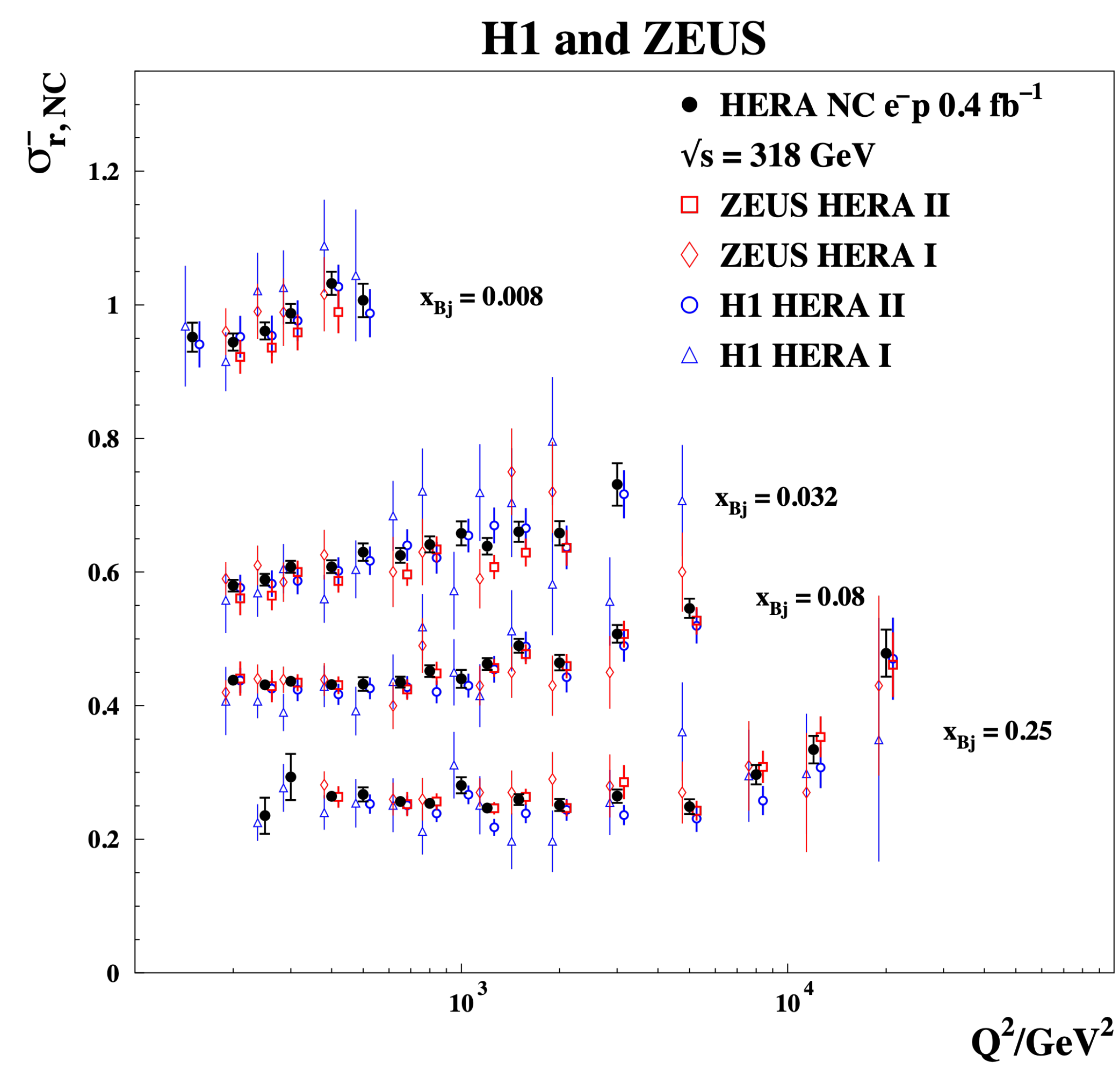
$$p^{i,k} = \frac{\mu^{i,k} - \mu^{i,ave}\left(1 - \sum_j \gamma_j^{i,k} b_{j,ave}\right)}{\sqrt{\Delta_{i,k}^2 - \Delta_{i,ave}^2}}$$

For each process pulls centred at zero with ~ unit width

# Combination: Results

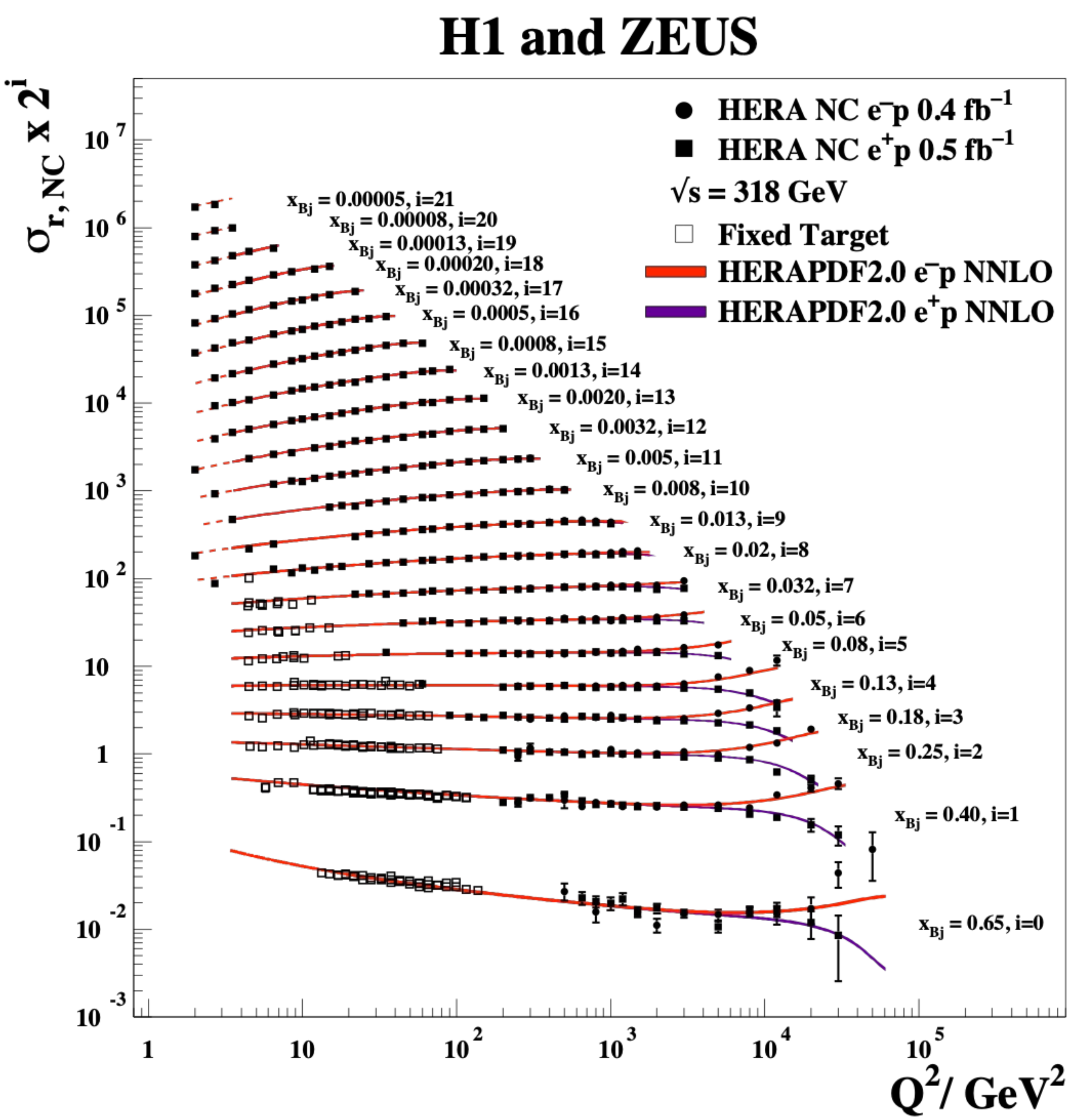
## NC $e^{\pm}p$ , $\sqrt{s}$ =318 GeV




N.B. only a few representative $x_{Bj}$ bins are shown

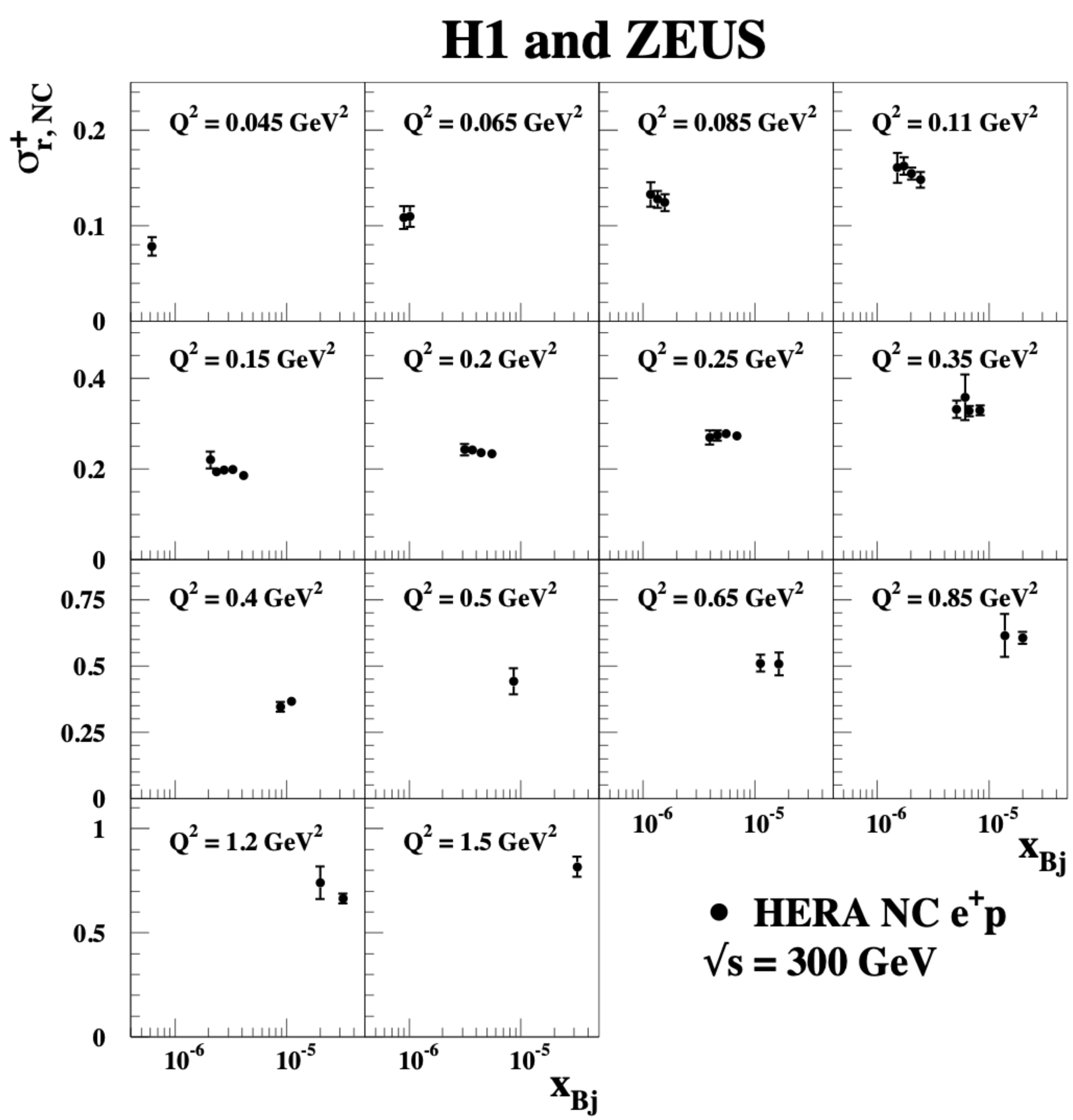
# Scaling violations

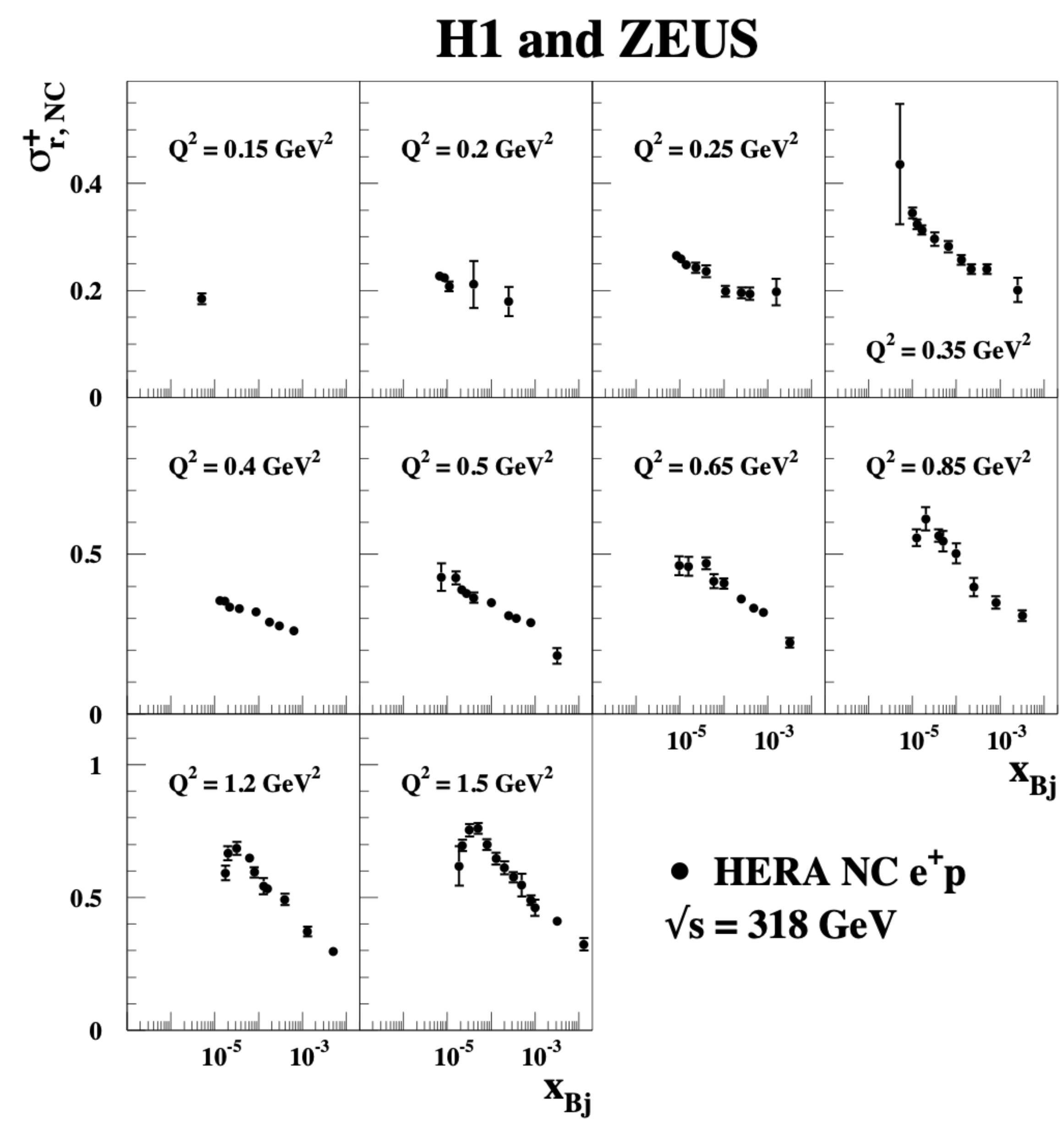



Textbook plots showing with great precision scaling violations patterns (and EW effects at high-$Q^2$ and high-x)

# Combination: Results

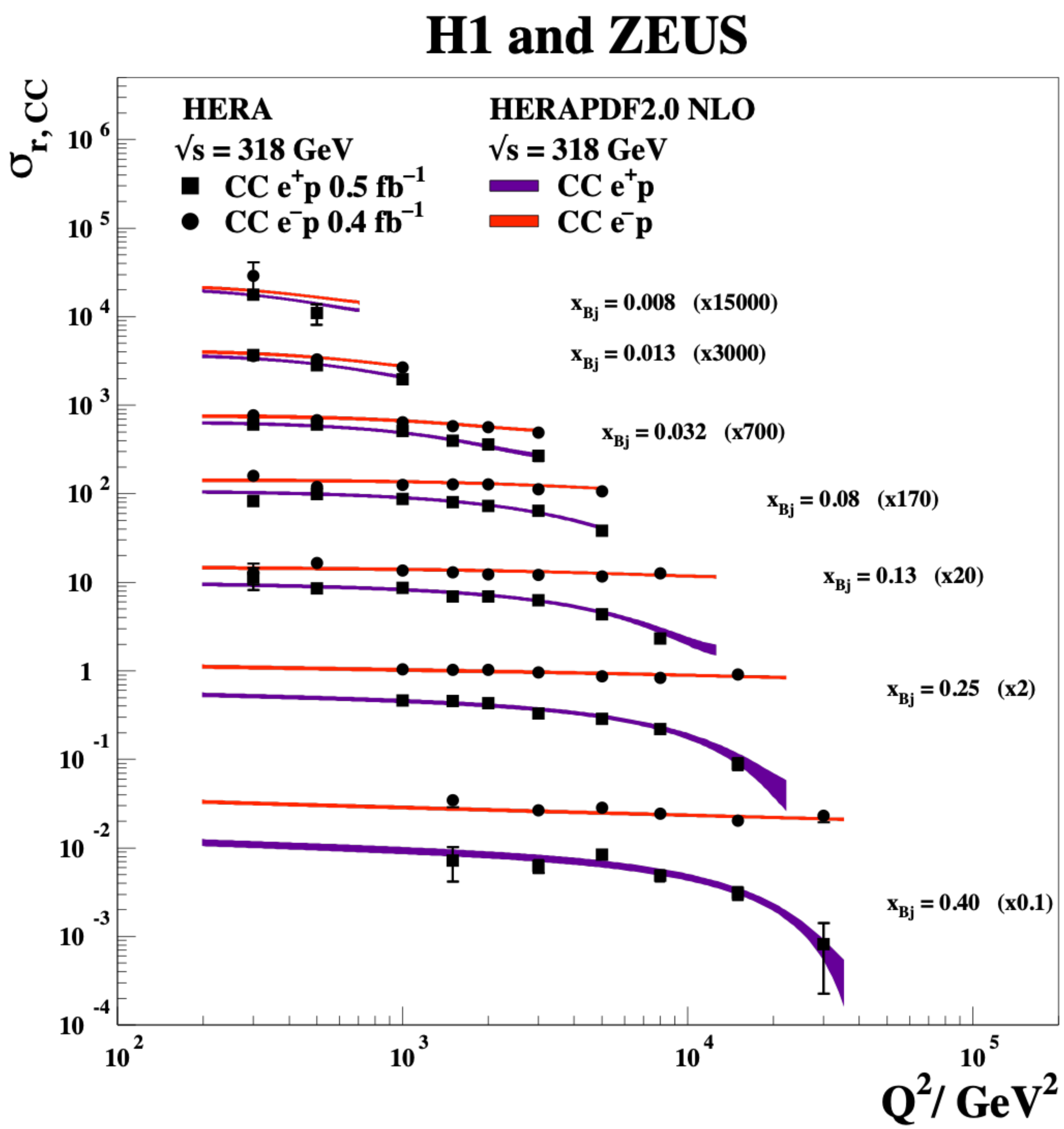
## Very low $Q^2$ and low $x_{Bj}$ data $\sqrt{s}$ =300, 318 GeV




- A very important data sample for QCD studies at low-$x_{Bj}$
- Interesting also for dipole/saturation models and higher-twist studies

# Helicity effects in CC interactions




Reminder:

$$\sigma^{+}_{r,\mathrm{CC}} \approx (x\bar{U} + (1-y)^2 xD)$$

$$\sigma^{-}_{r,\mathrm{CC}} \approx (xU + (1-y)^2 x\bar{D})$$

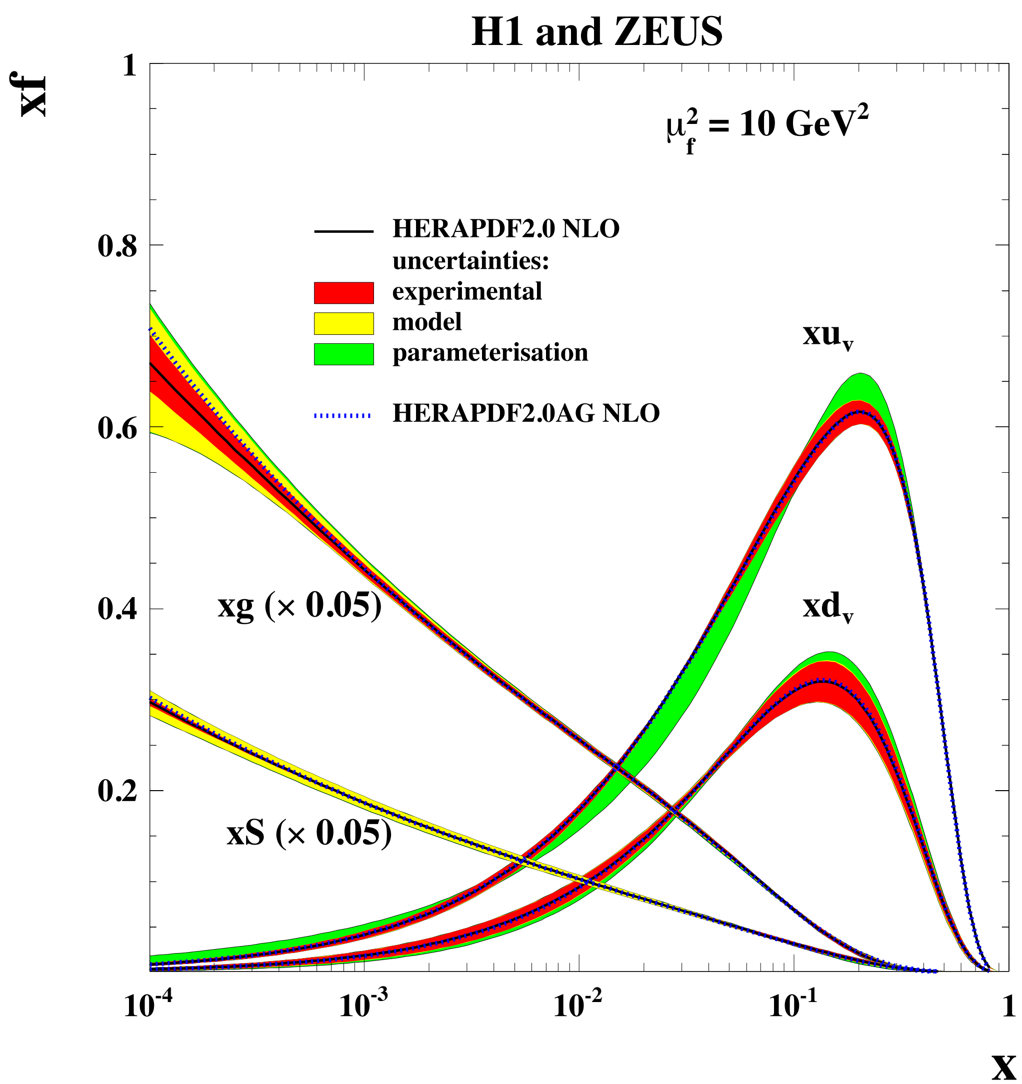
The helicity factor $(1\text{-}y)^2$ affects differently the $e^{\pm}p$ CC cross sections:

- The $e^{+}p$ cross section is suppressed at high-y (high-$Q^2$)
- The $e^{-}p$ cross section is almost unaffected

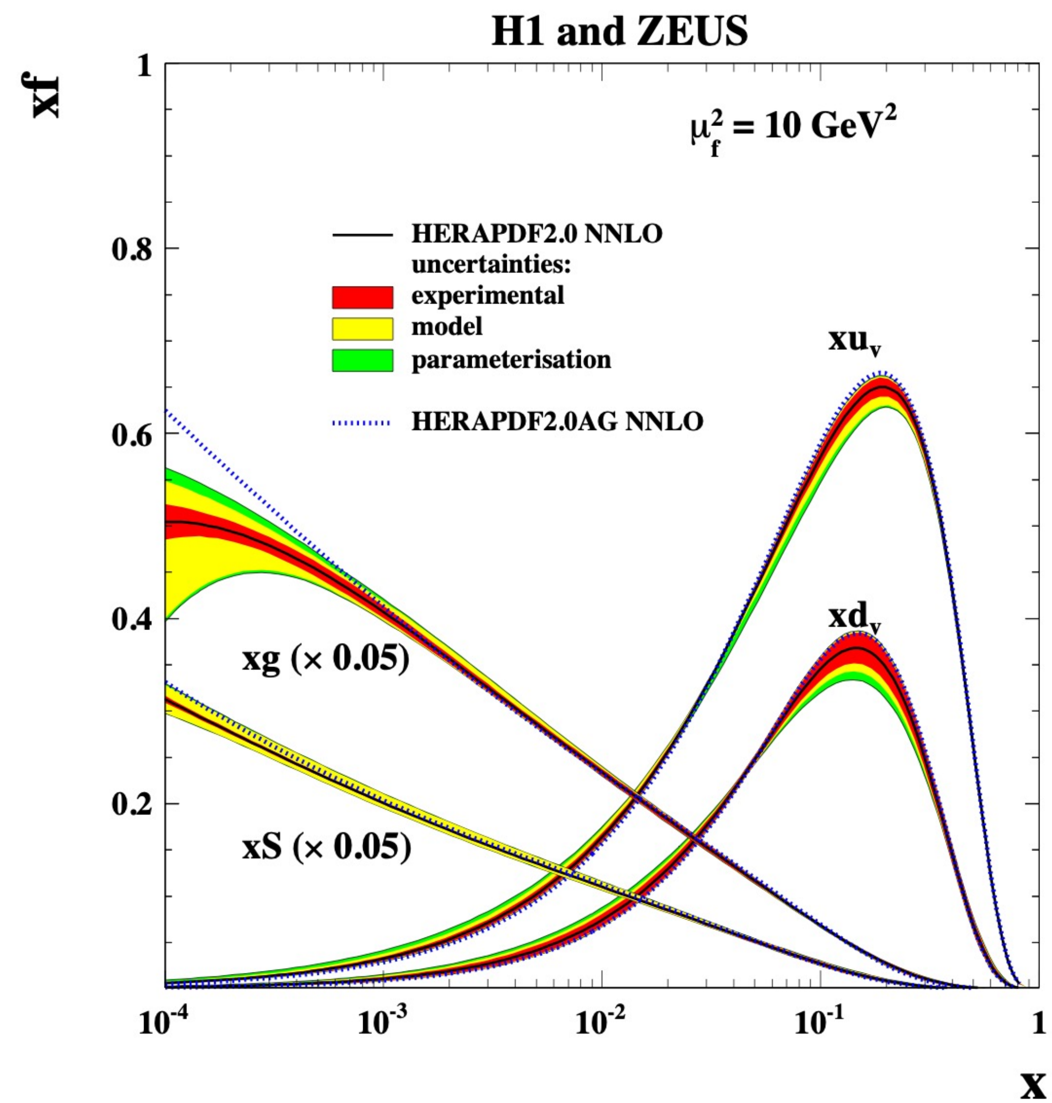
The precision of the CC cross sections at high-$Q^2$ allow the study of these helicity effects.

# HERAPDF2.0: NLO and NNLO PDFs

NLO

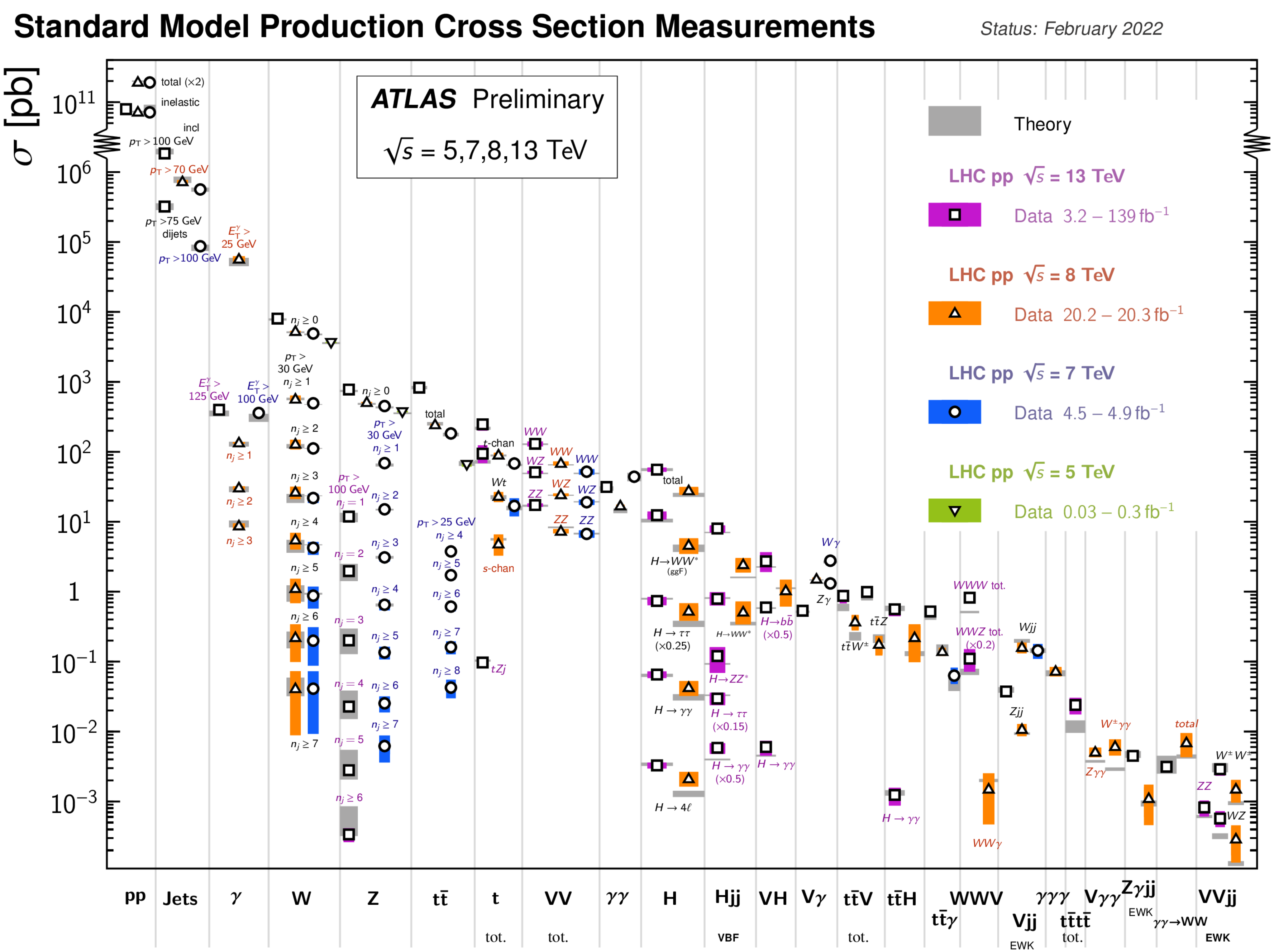



NNLO

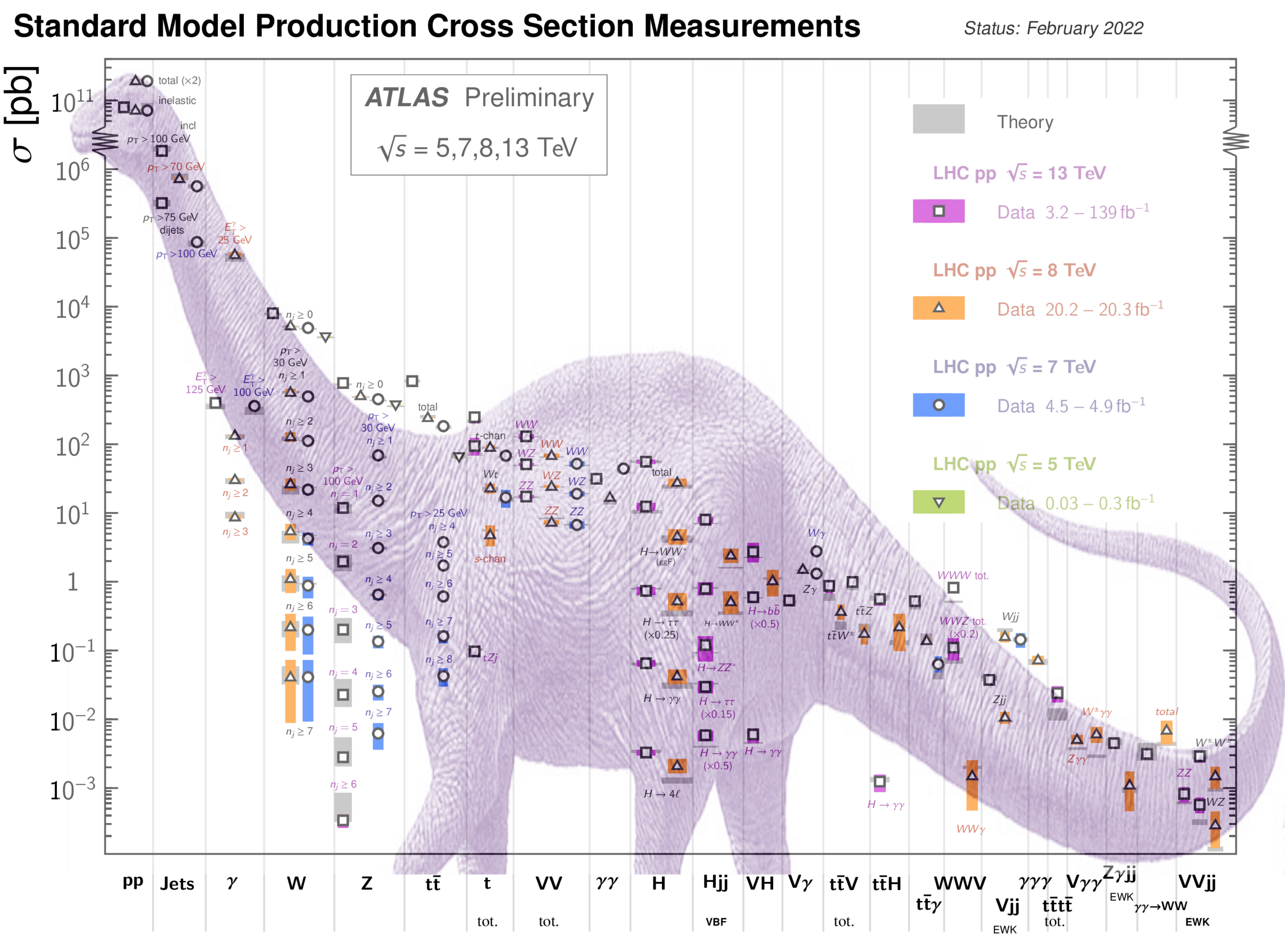



NNLO vs NLO:
- gluon ceases to rise at low-x
- sea at low-x somewhat steeper w.r.t. NLO

# Measurements at LHC and PDFs

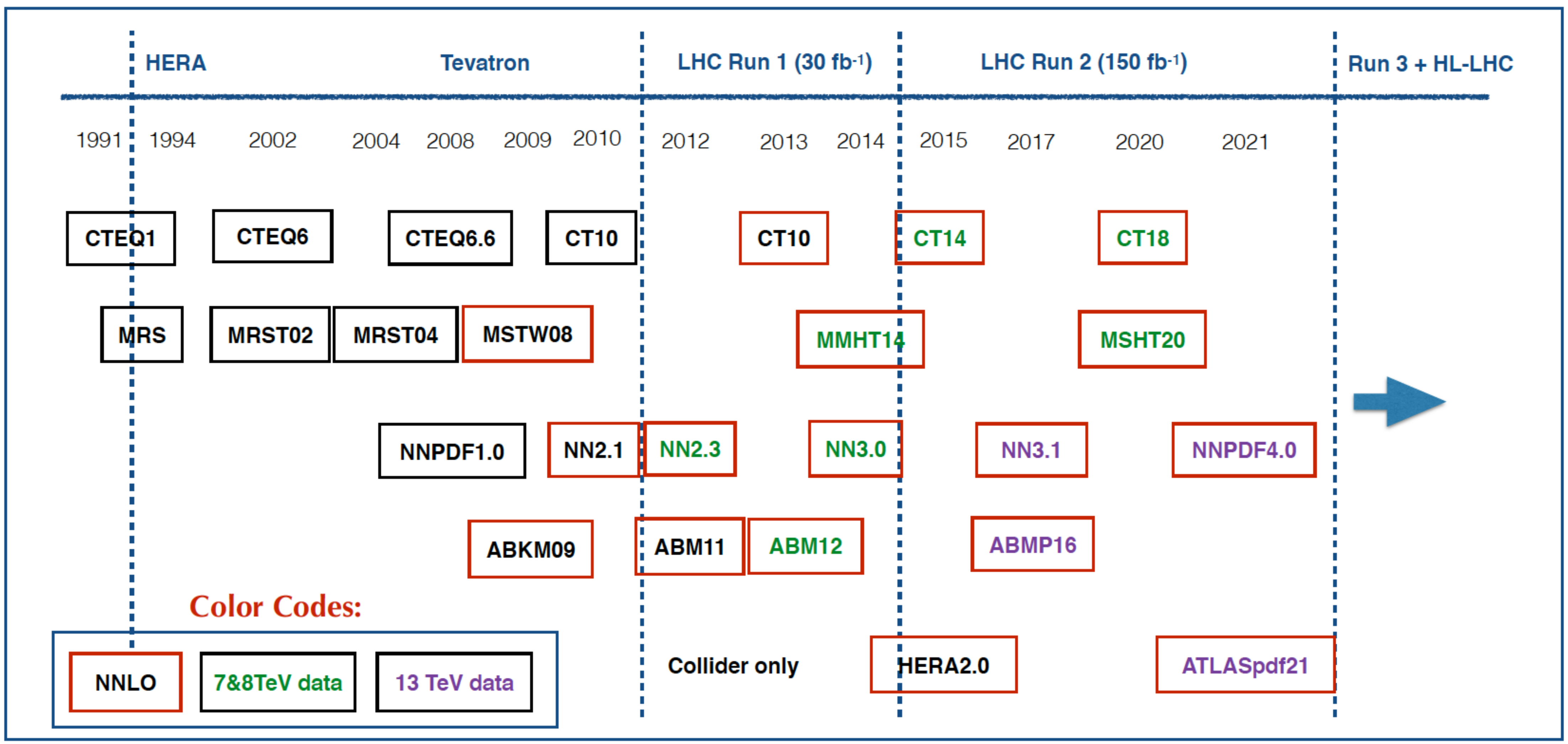


# Measurements at LHC and PDFs



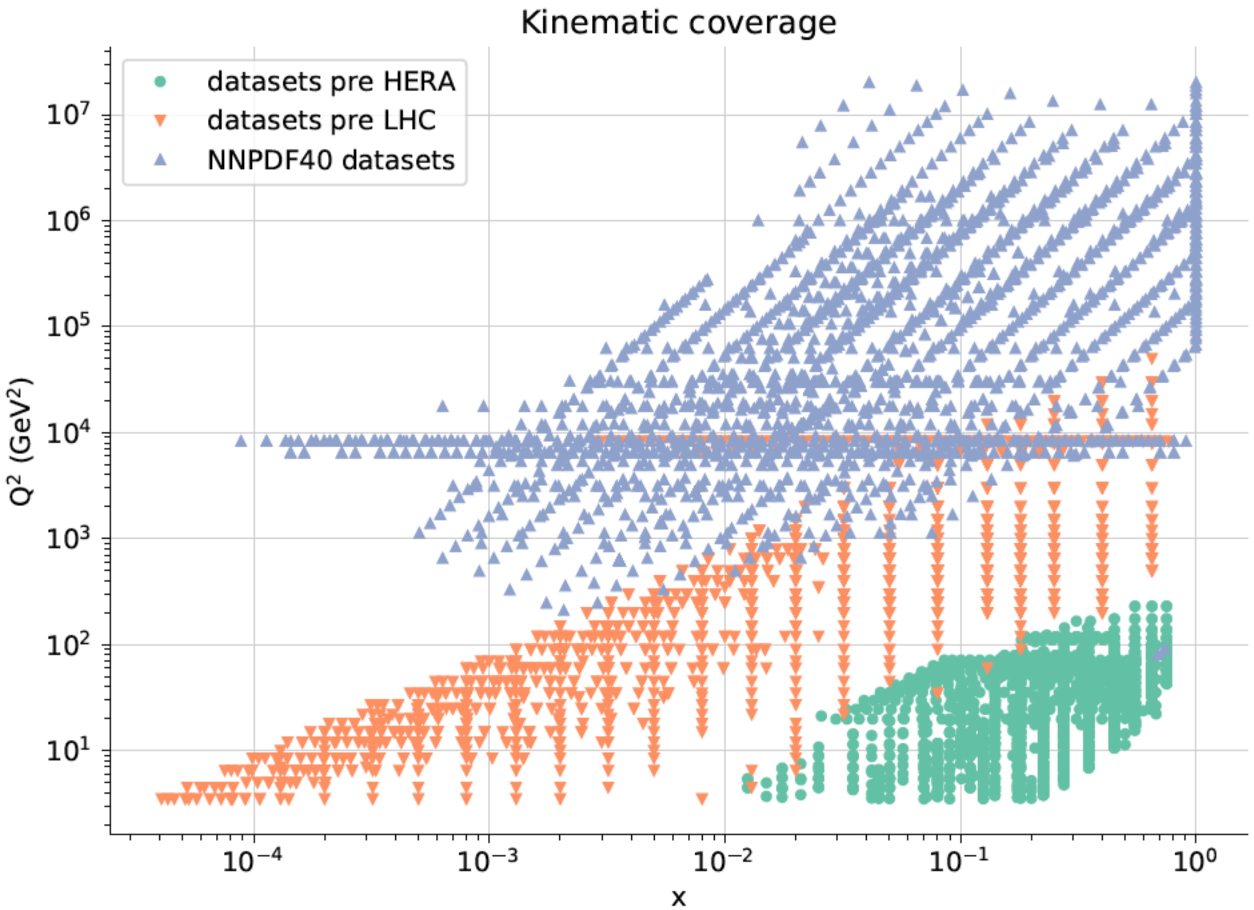
# PDF Groups

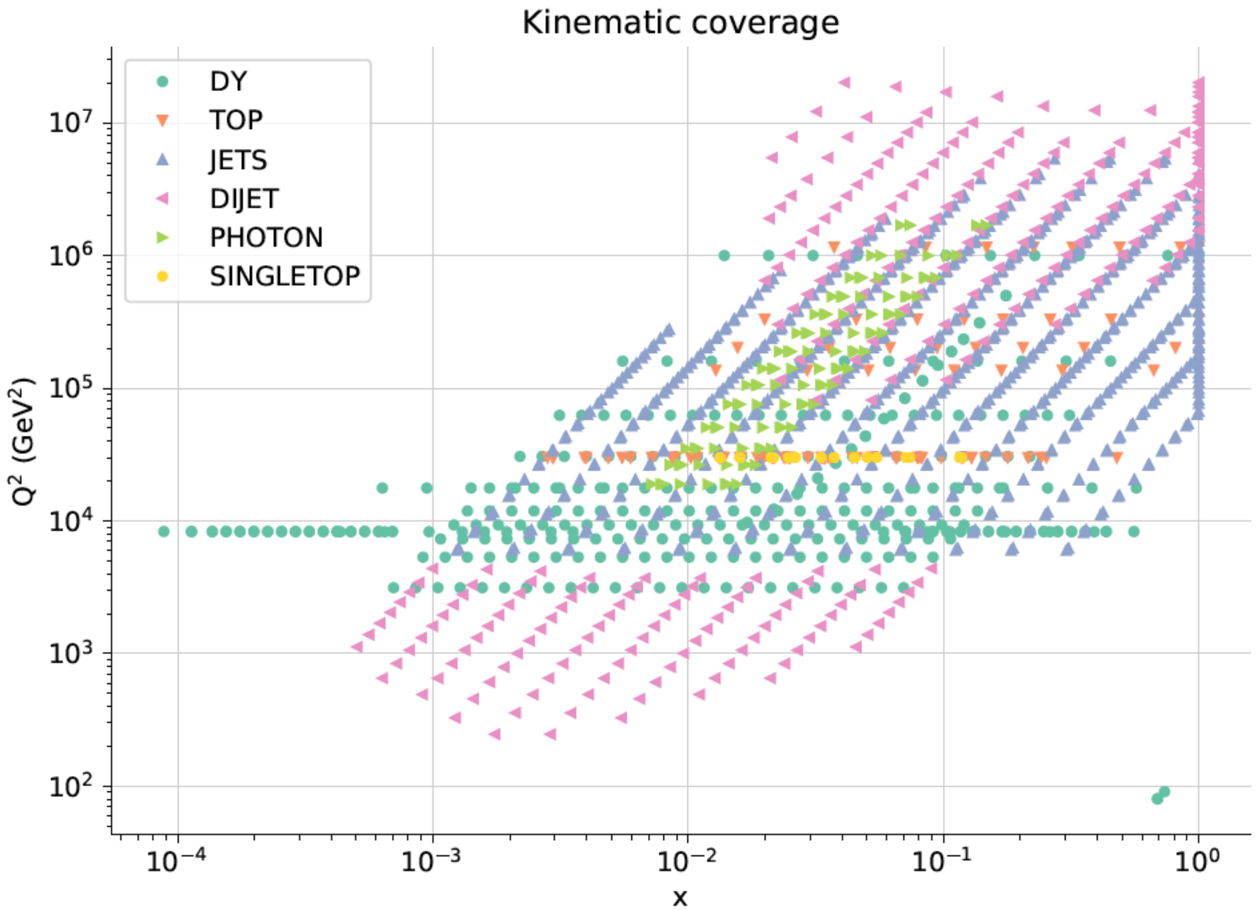
Collaborations/Groups active in DGLAP analyses and PDFs determinations:



# NNPDF4.0

Kinematic regions and observables:

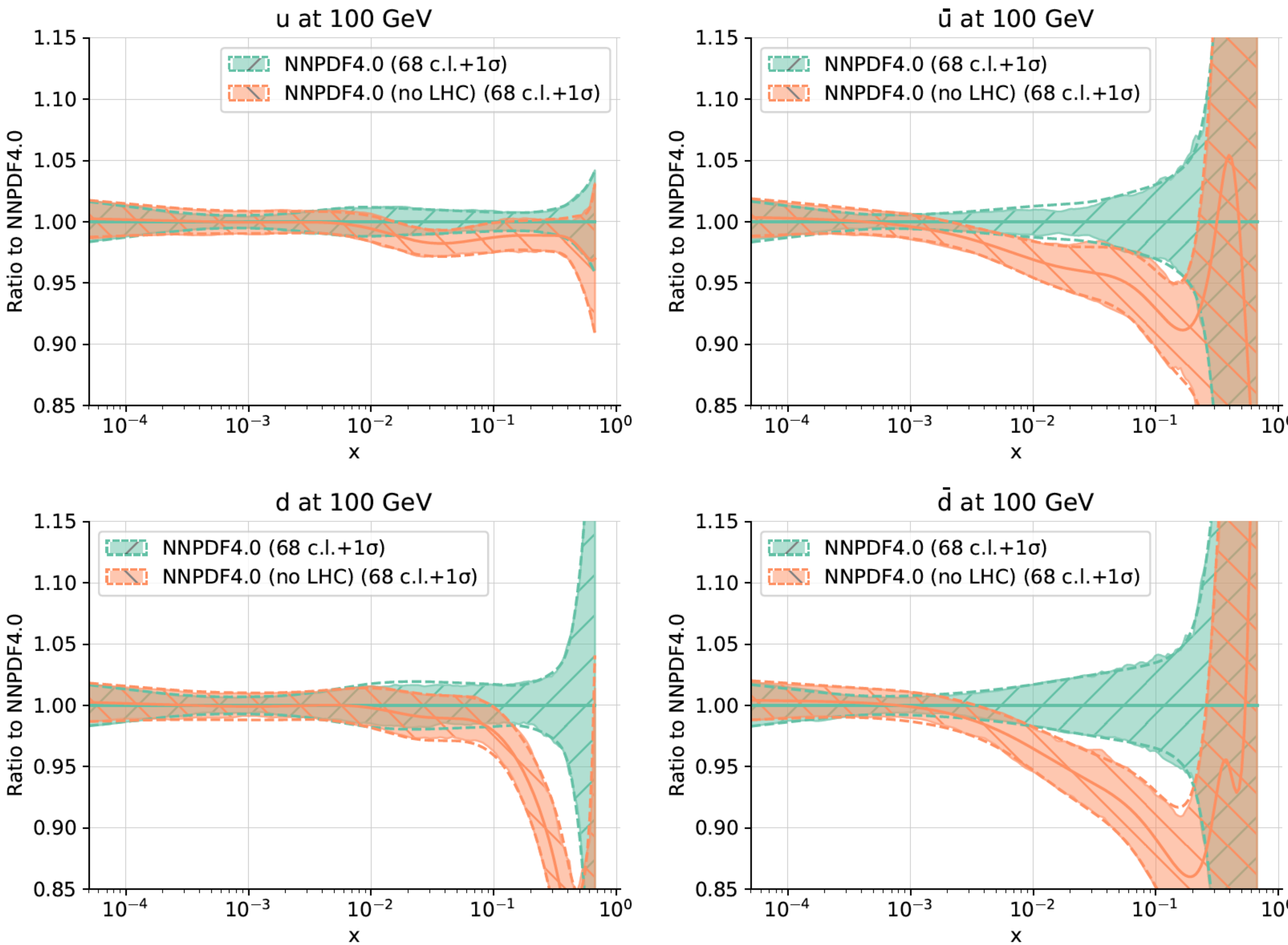



https://arxiv.org/abs/2109.02653

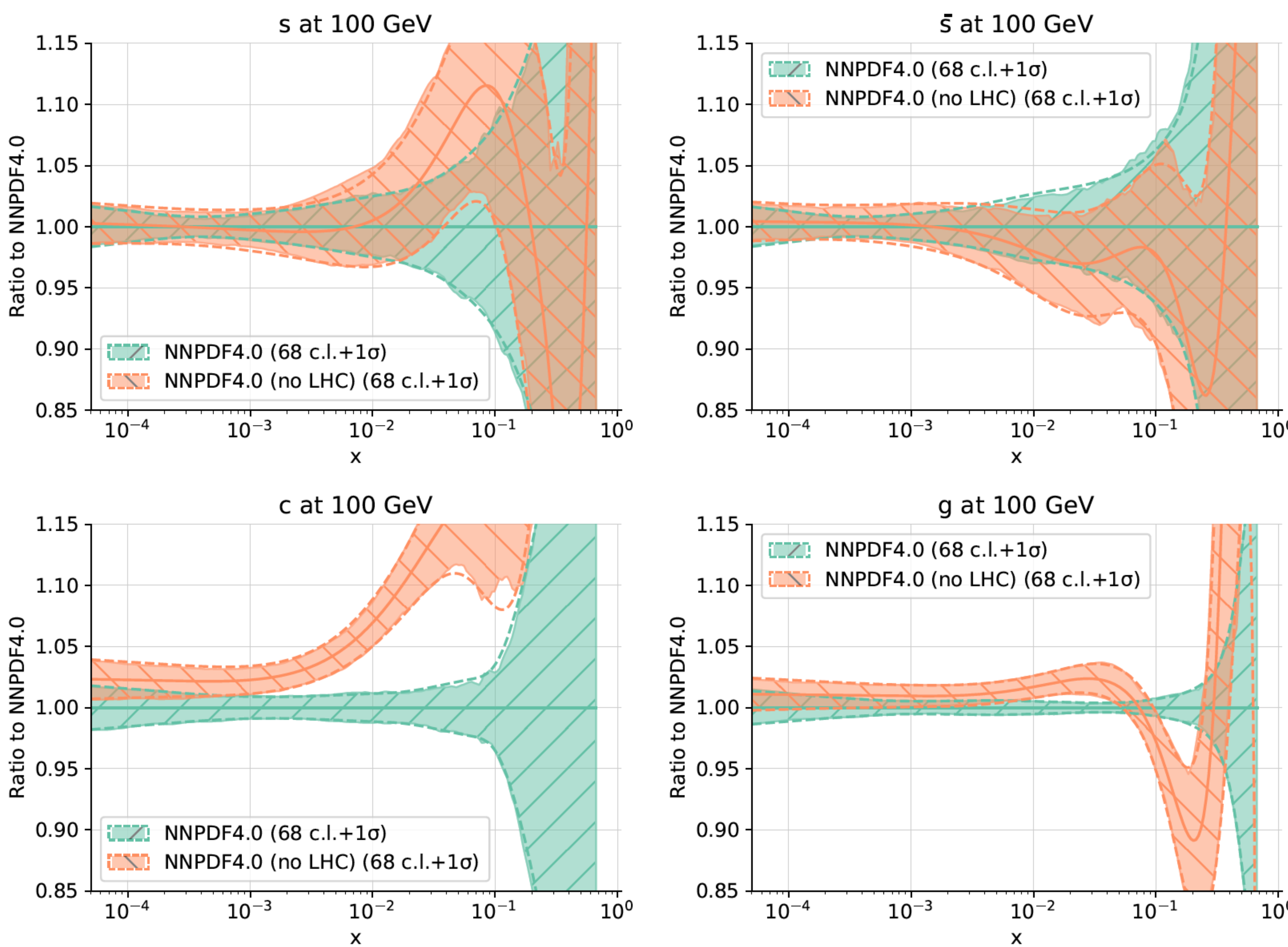
# NNPDF4.0

Impatto dati LHC:

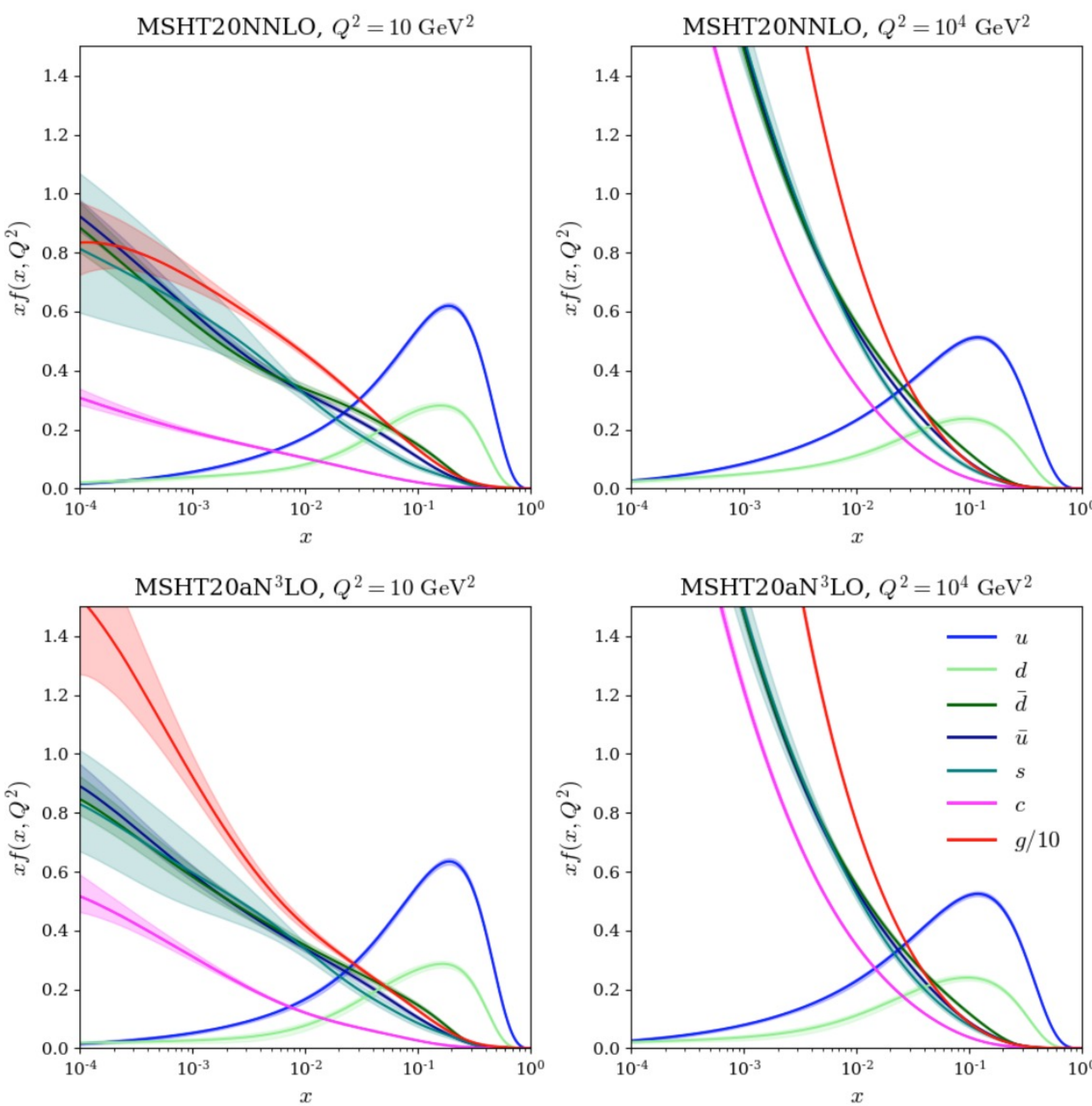


# NNPDF4.0

Impatto dati LHC:



# MSHT: Approximate N$^3$LO PDFs




First approximate N3LO PDFs

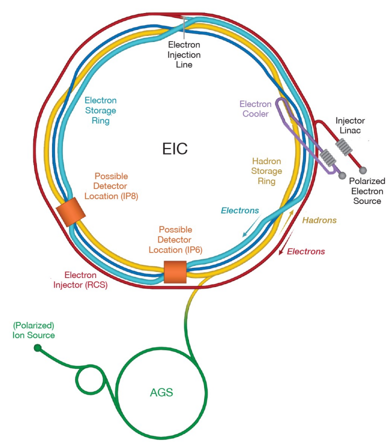
Gluon rises significantly at low

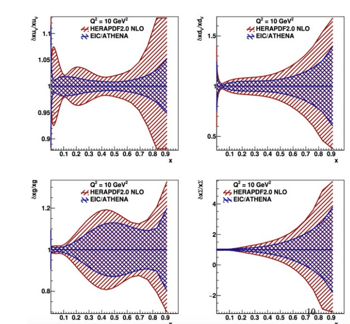
First attempt to quantify theoretical uncertainties

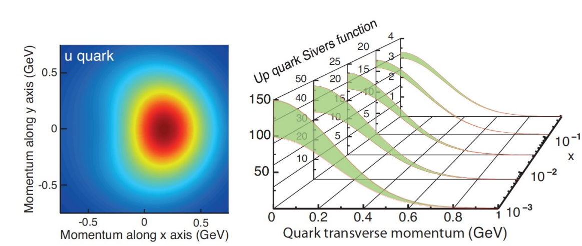
arXiv:2207.04739

# III. EIC

- Accelerator
- Scientific goals
- PDFs



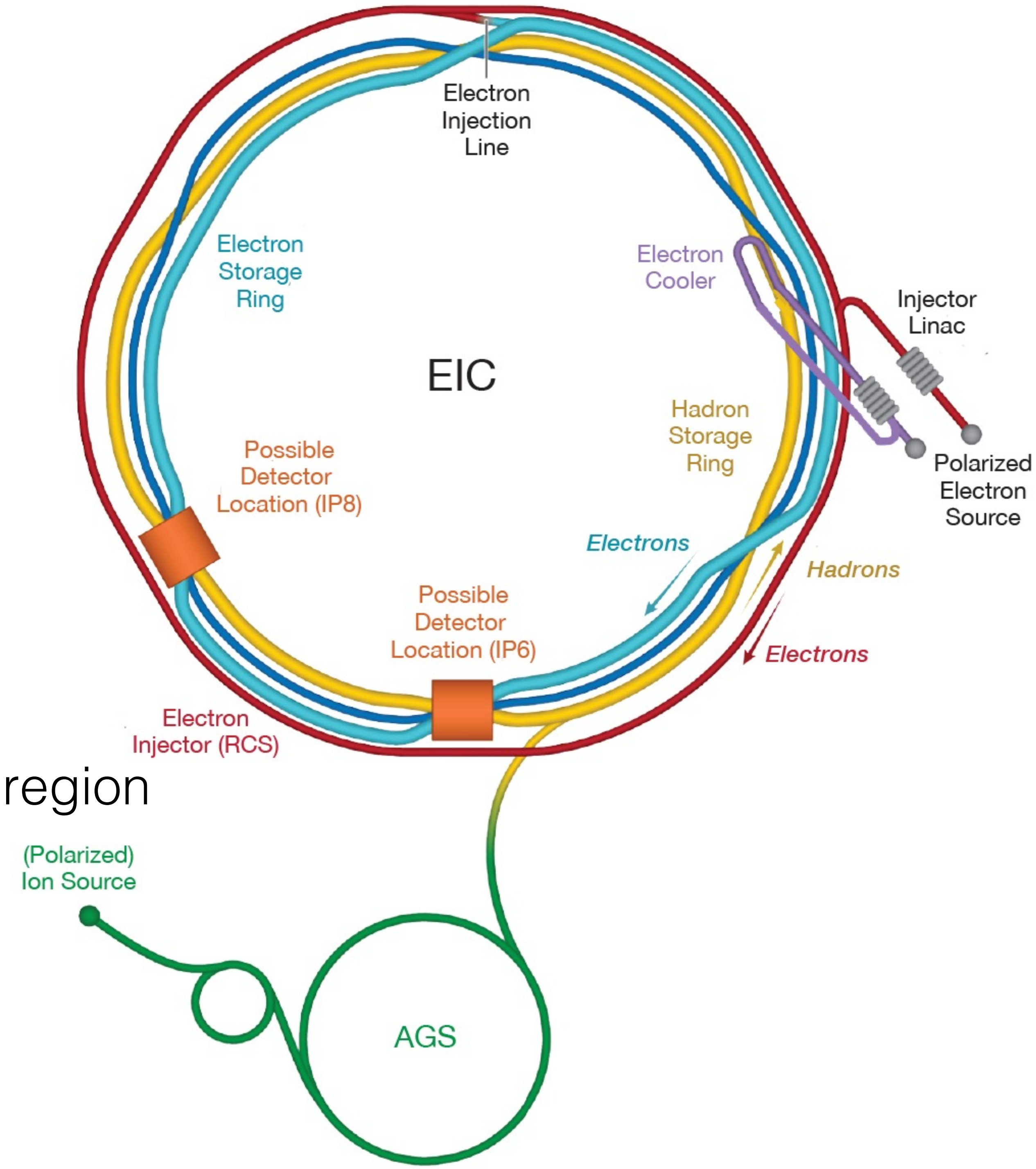



# EIC

The main design requirements of the EIC:

- Highly polarized electron ( 70%) and proton ( 70%) beams
- Ion beams from deuterons to heavy nuclei
- Variable e+p center-of-mass energies from 20-100 GeV, upgradable to 140 GeV
- High collision electron-nucleon luminosity $10^{33}$-$10^{34}$ $cm^{-2}$ $s^{-1}$
- Possibility to have more than one interaction region



# EIC Scientific Goals

Give answers to the 'key scientific questions' of the Yellow Report :

- How do nucleonic properties such as mass and spin emerge from the partons and their interactions?

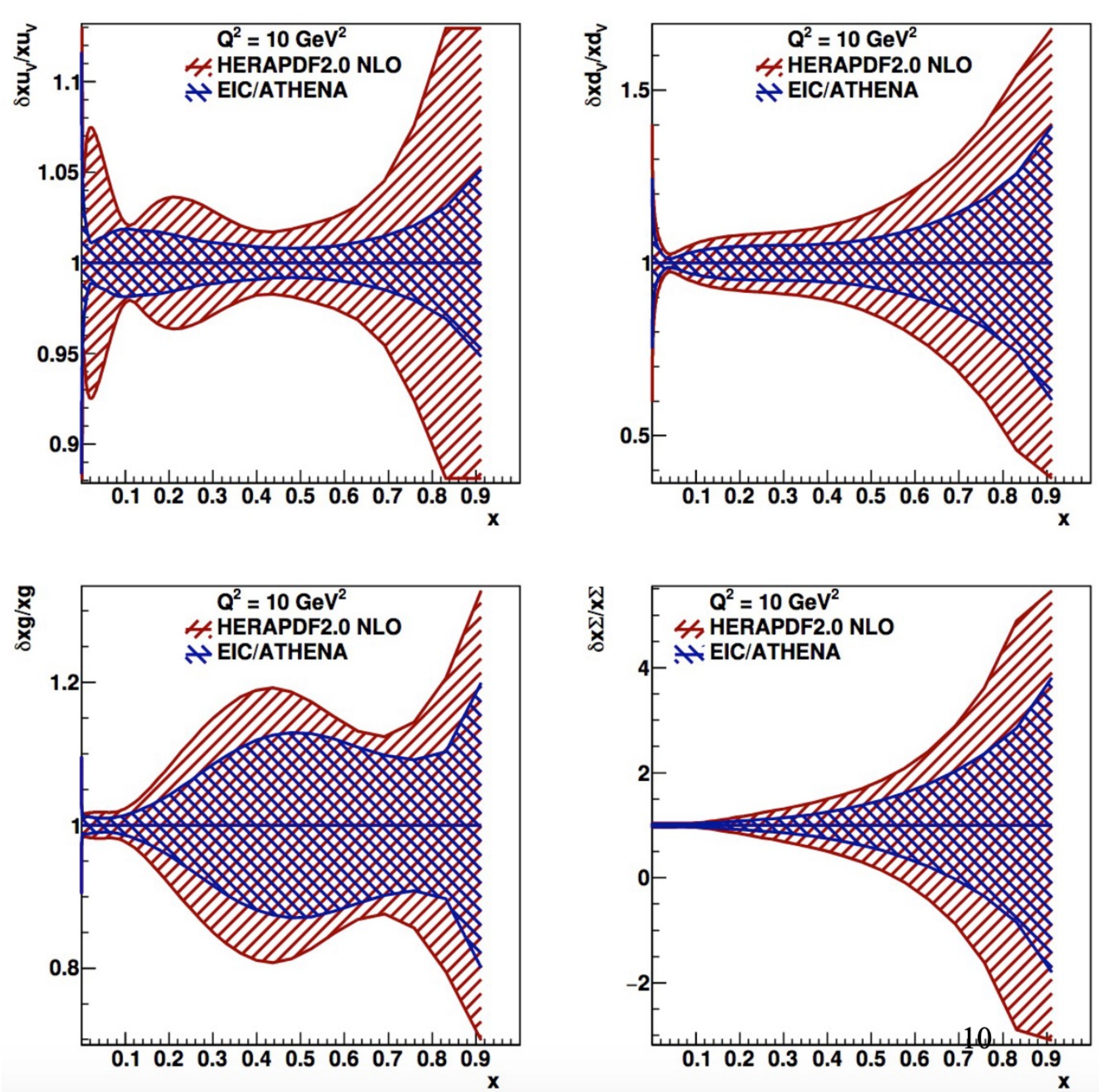
- How are the partons distributed within the nucleon both in momentum and in spatial position?

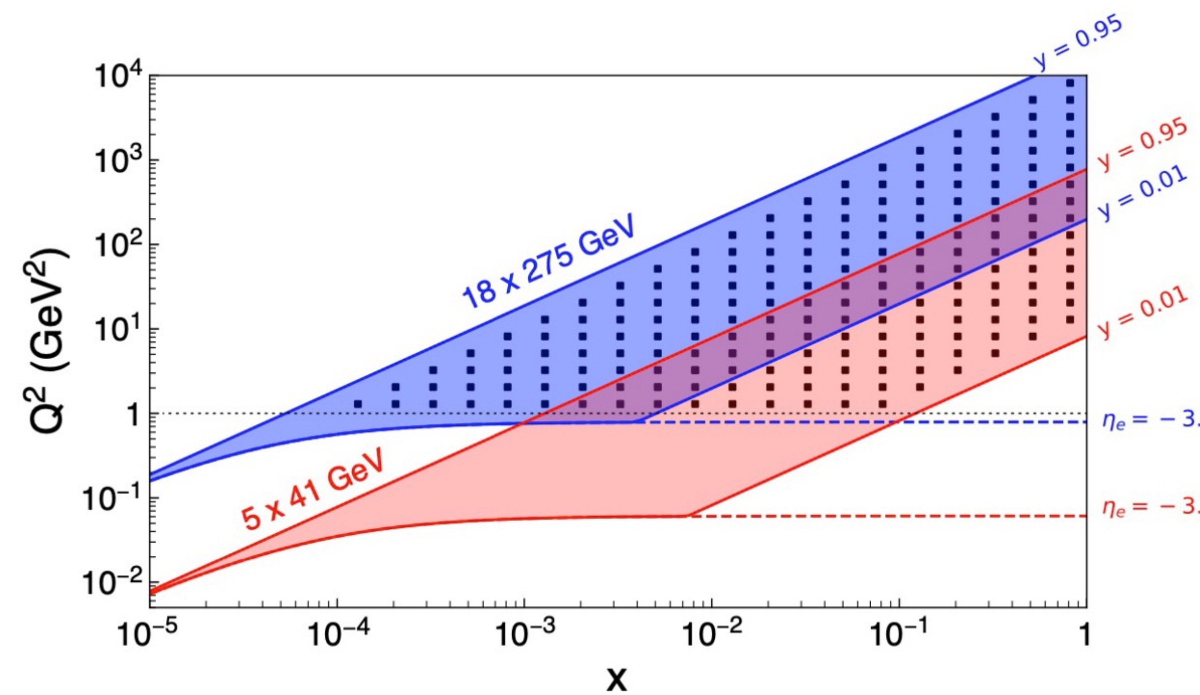
- How are the partons distributed within the nucleon both in momentum and in spatial position?
  How do confined hadronic states emerge from these quarks and gluons?
  How do quark-gluon interactions create nuclear binding?

- How does a dense nuclear medium affect the dynamics of quarks and gluons and their correlations and interactions? What happens to the gluon density in the nuclei?

# EIC: Collinear PDFs

Improved PDFs (simulated data (YR)):




| e-beam E | p-beam E | $\sqrt{s}$ (GeV) | inte. Lumi. (fb$^{-1}$) |
|---|---|---|---|
| 18 | 275 | 140 | 15.4 |
| 10 | 275 | 105 | 100.0 |
| 10 | 100 | 63 | 79.0 |
| 5 | 100 | 45 | 61.0 |
| 5 | 41 | 29 | 4.4 |

P. Newman et al.
DIS workshop 2022

# 3D Nucleon Imaging: Wigner functions, TMDs & GPDs