Collective behaviours: from the Ising model to bird flocks

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Definition of collective behaviours

Why do biological systems behave collectively?

Mainly to **efficiently respond to external stimuli,** propagating information in a very efficient way.
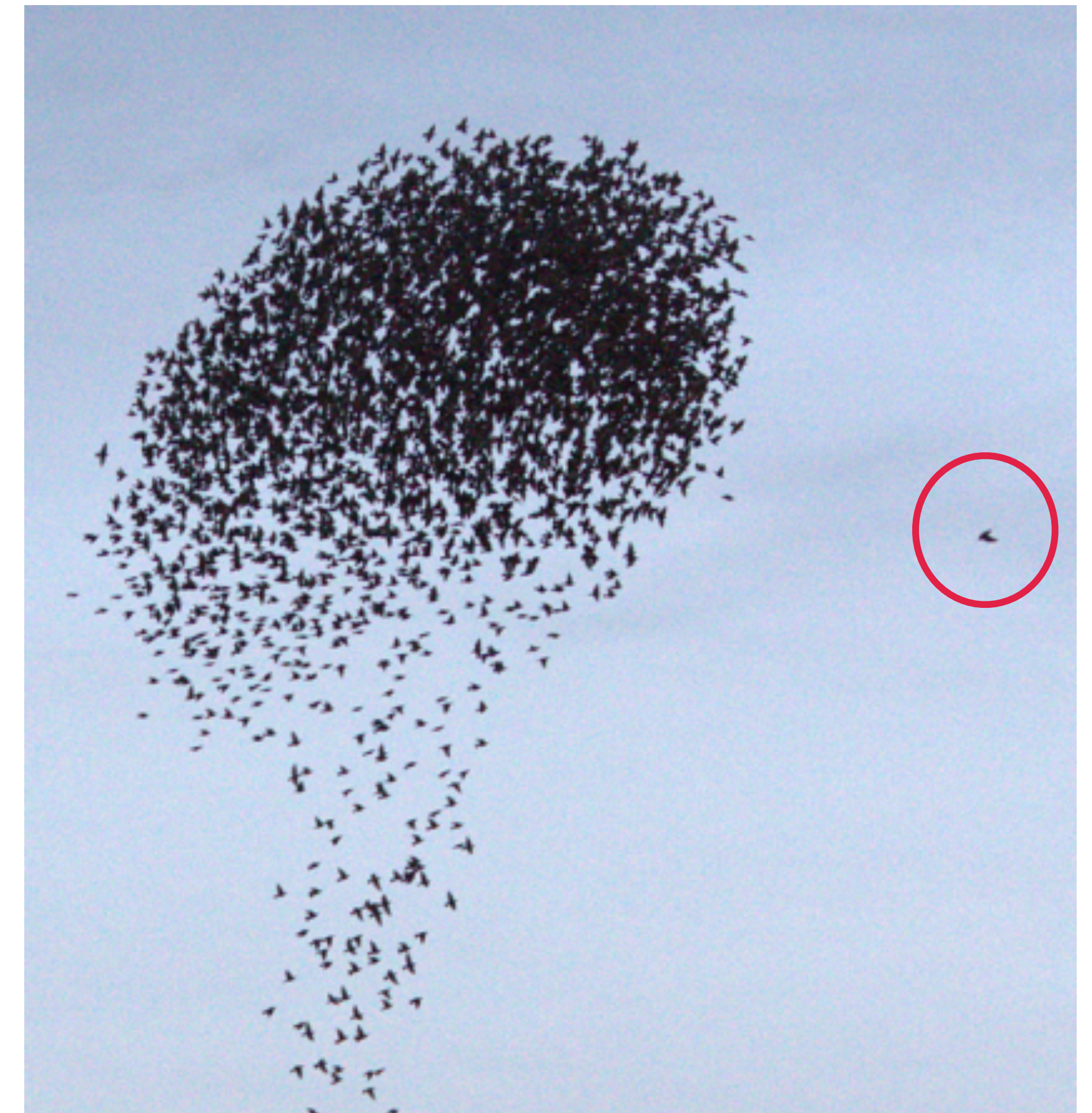
For example:
- Search for food;
- Resist the attack of a predator.

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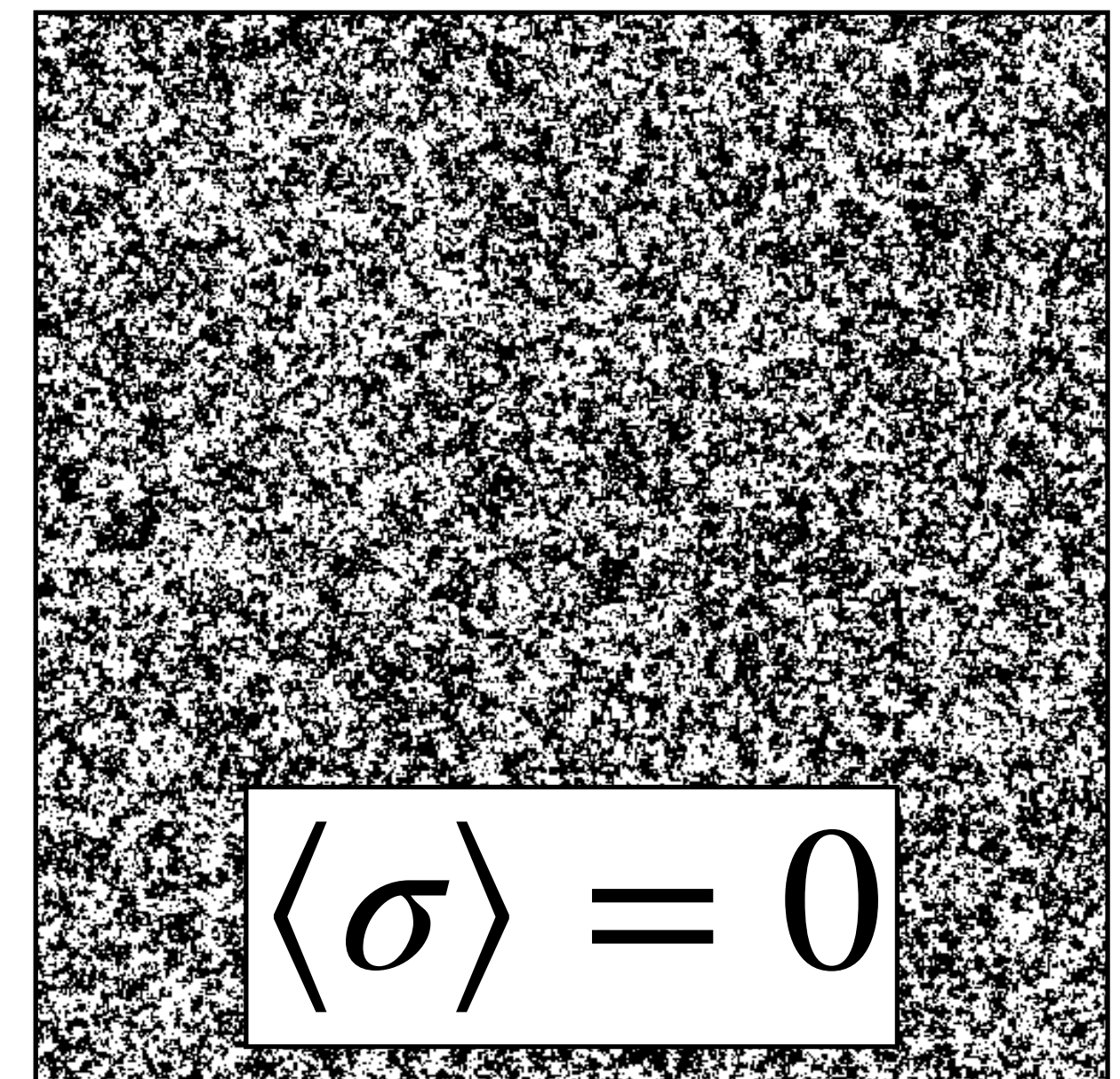- when the system is **strongly correlated**.
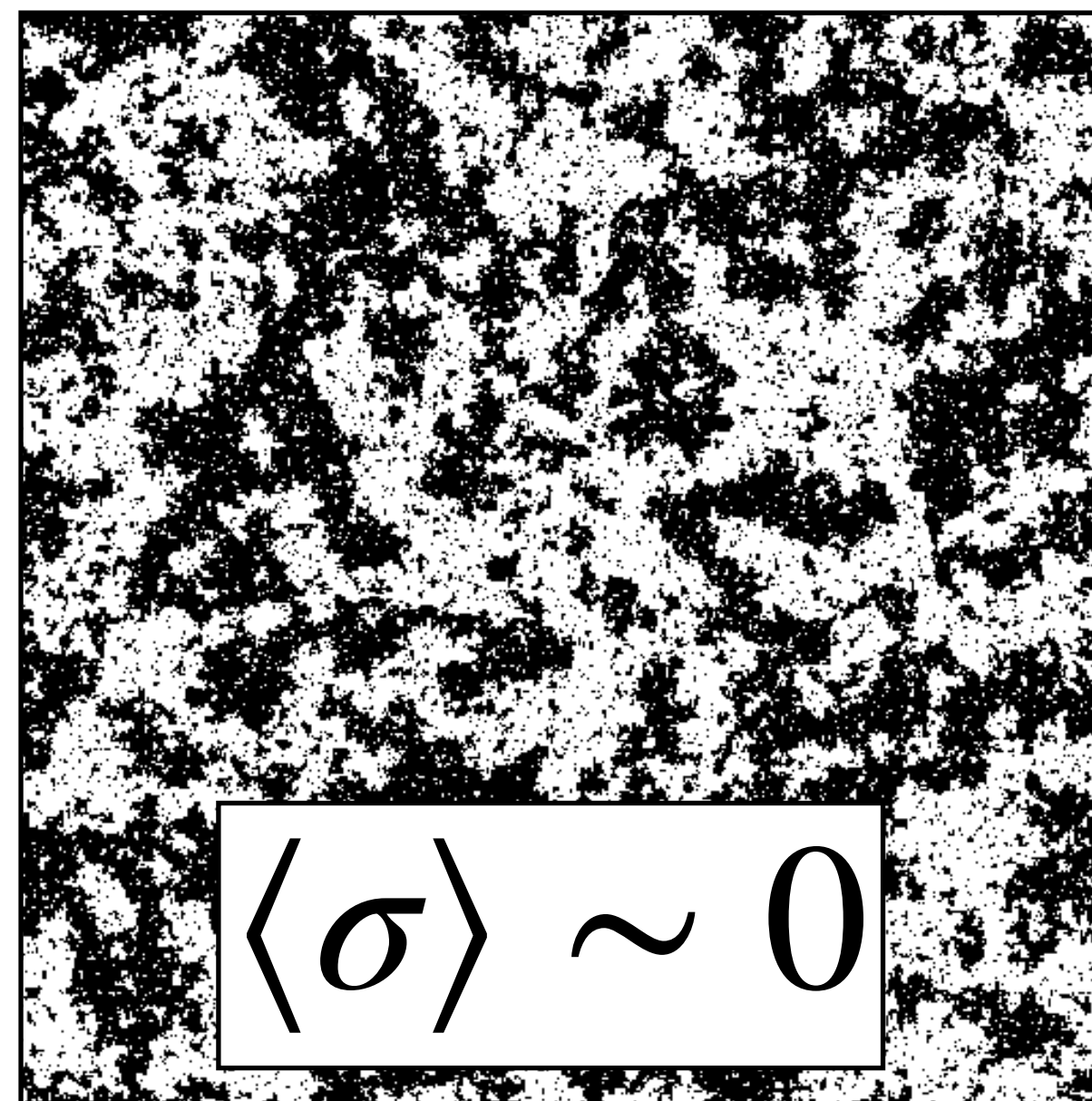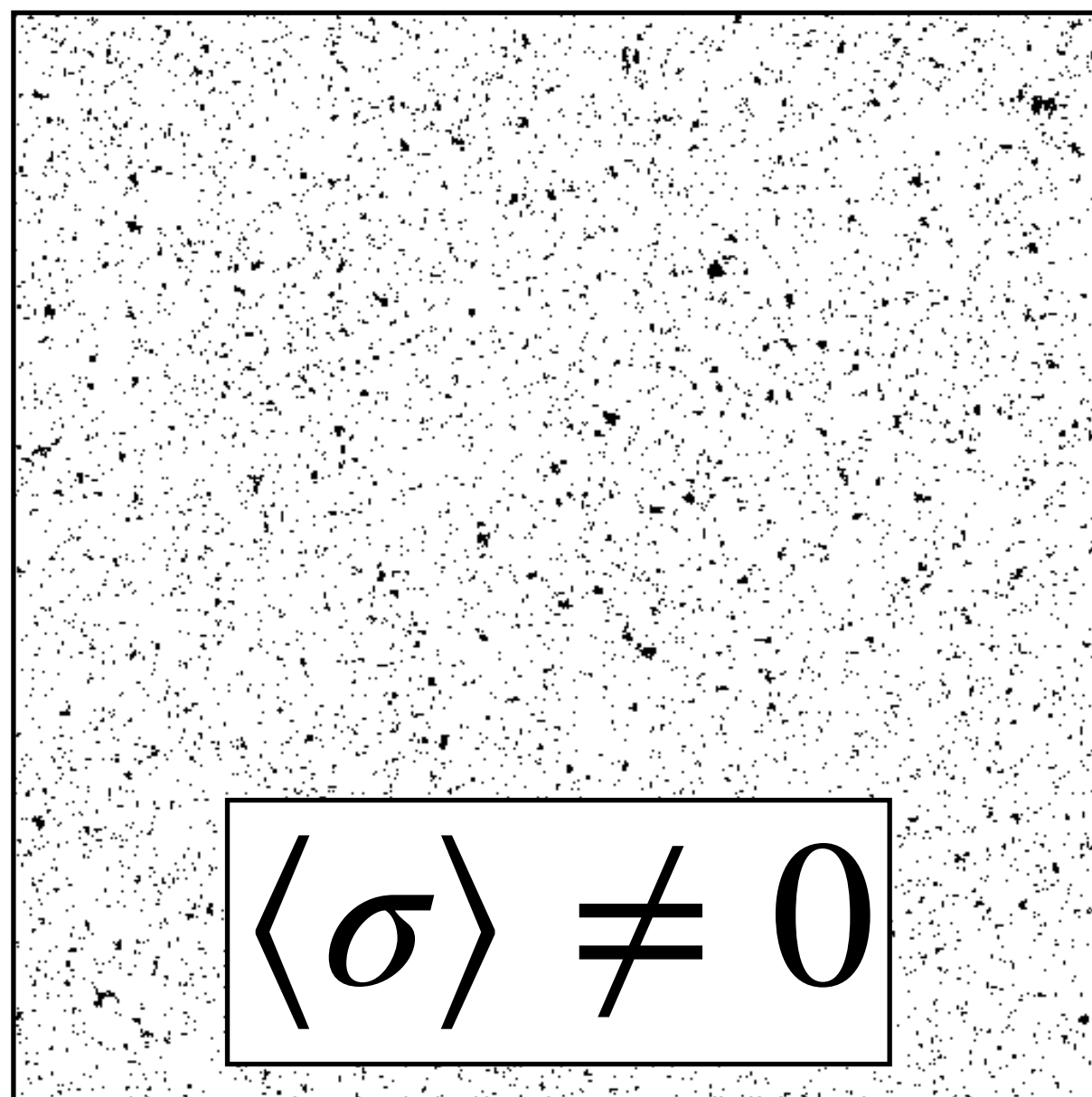
Collective behaviours in the Ising Model

The Ising Model:

$$H\left(\vec{\sigma}\right) = -\sum J_{ij}\sigma_i\sigma_j \quad \rightarrow \quad P\left(\vec{\sigma}\right) \propto e^{-\beta H(\vec{\sigma})}, \quad \beta = T^{-1}$$

$$T \ll T_c \qquad\qquad T \sim T_c \qquad\qquad T \gg T_c$$

$$\langle\sigma\rangle \neq 0$$

$$\langle\sigma\rangle \sim 0$$

$$\langle\sigma\rangle = 0$$
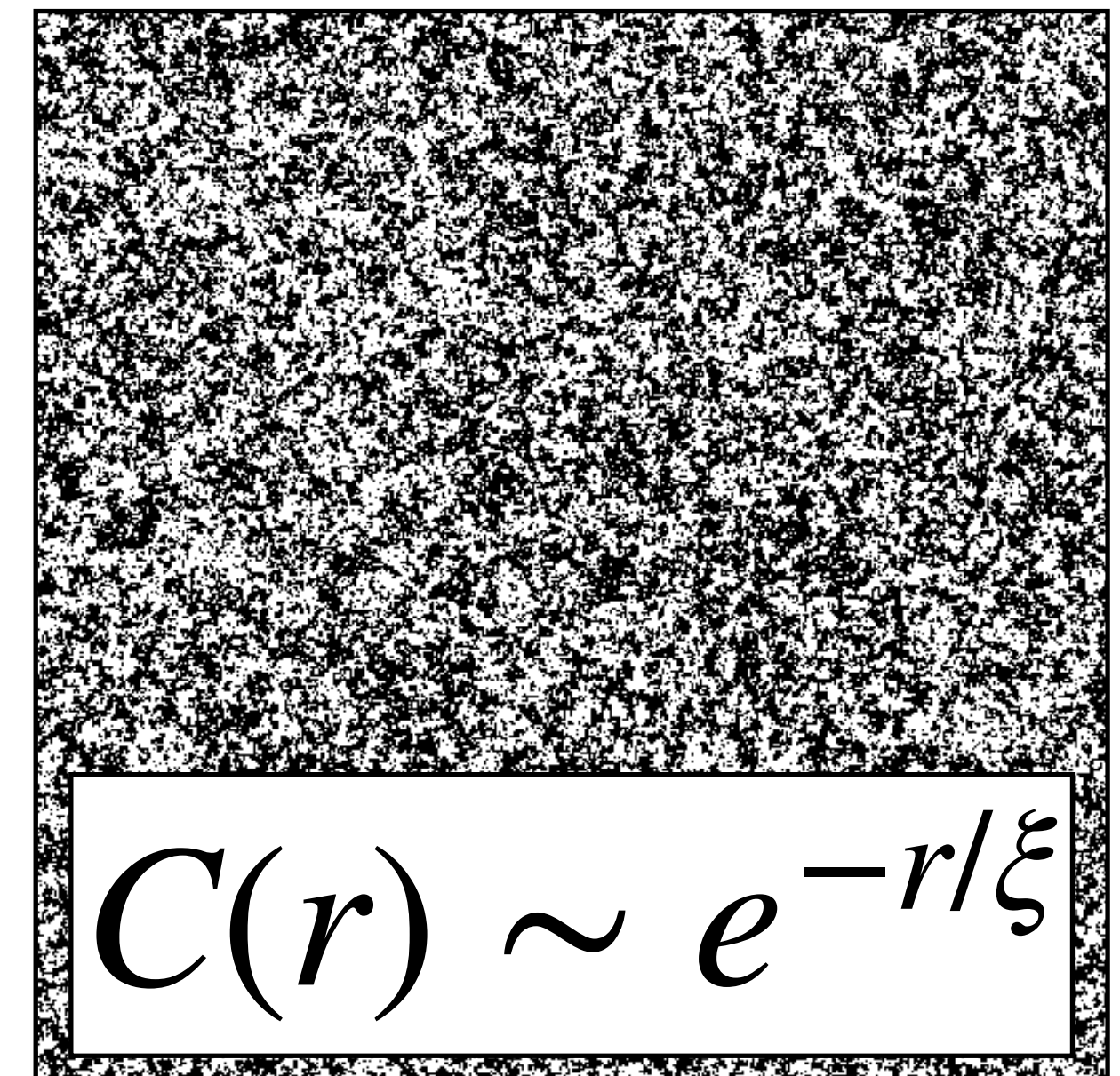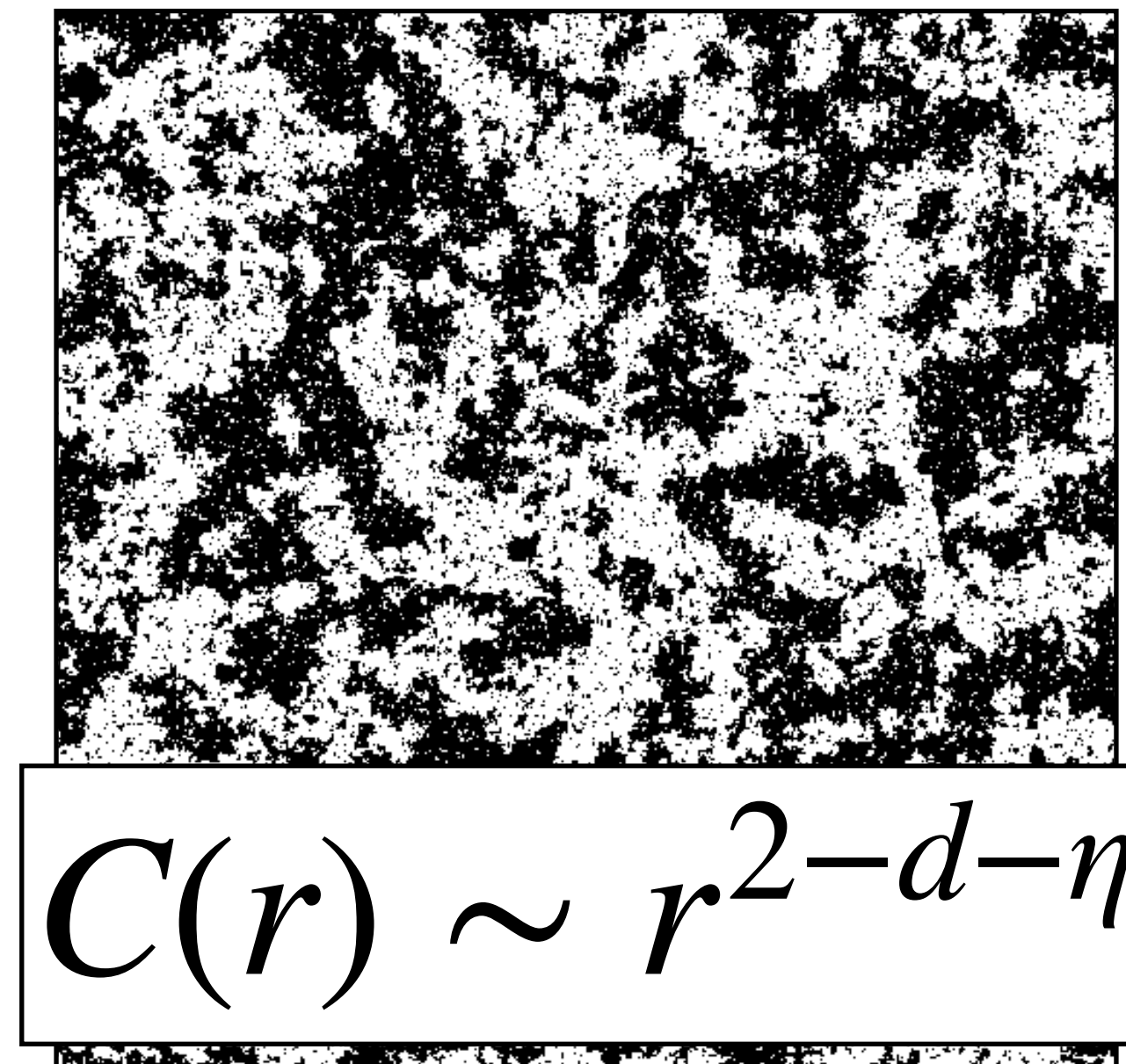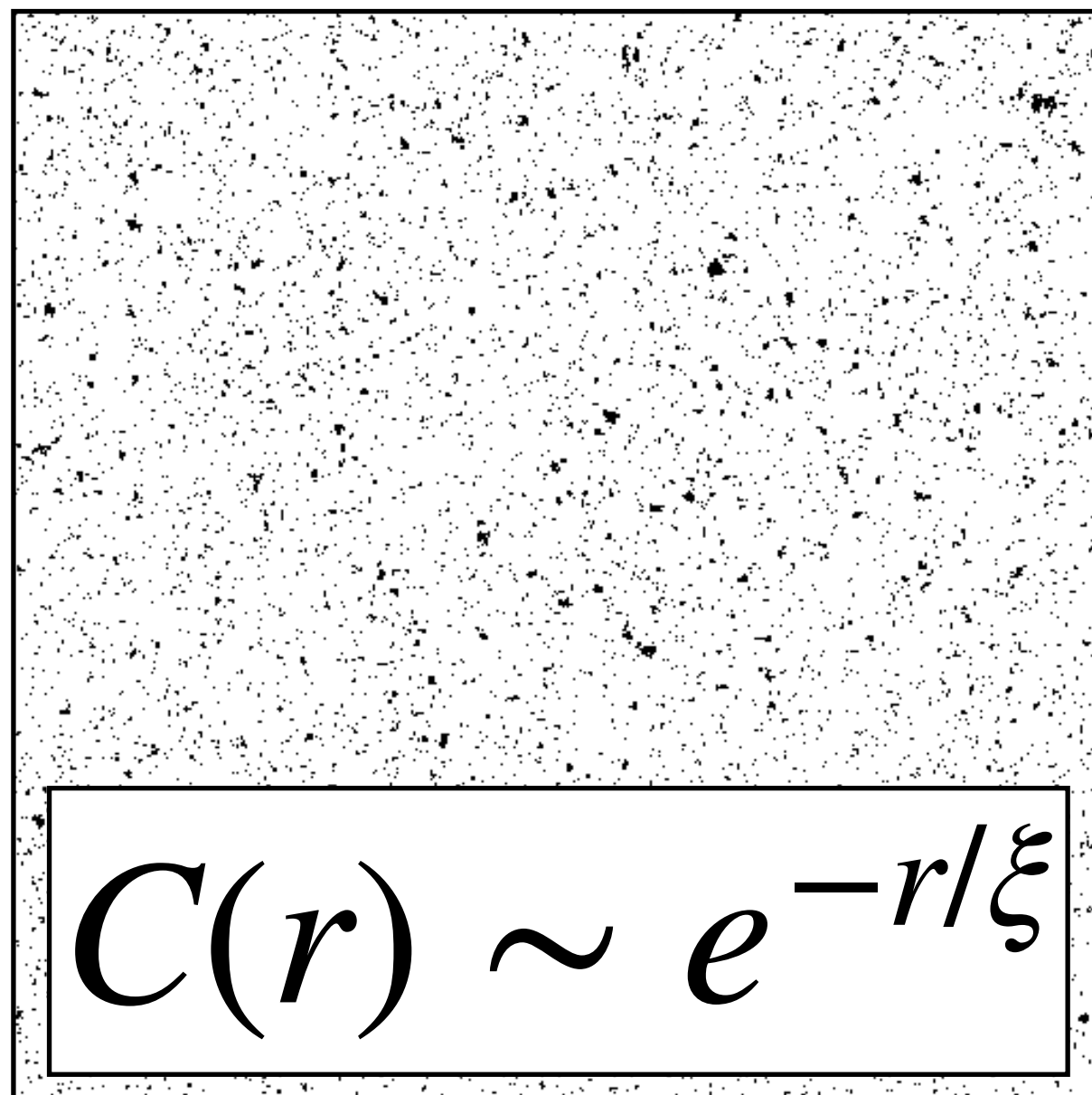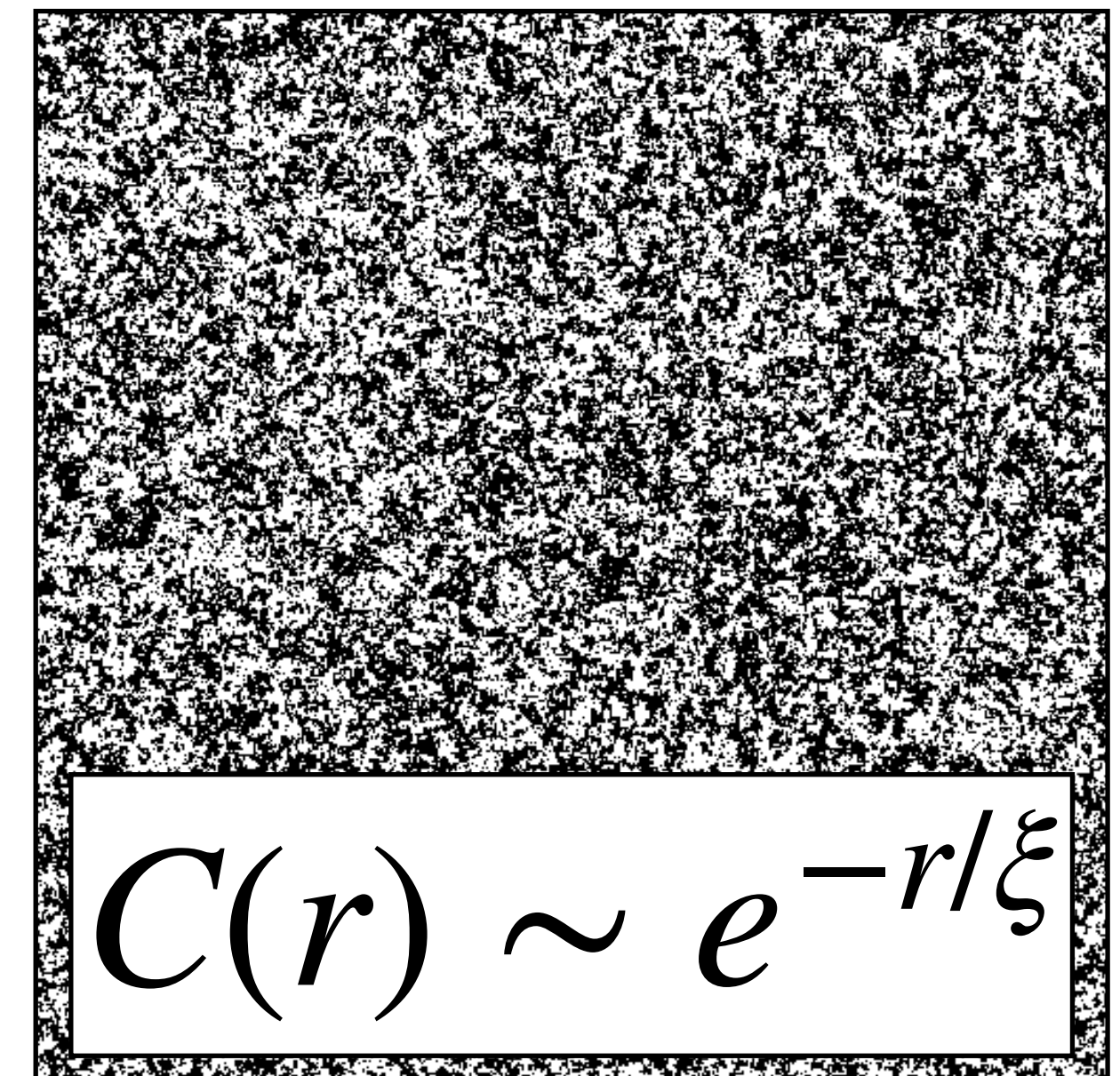
Collective behaviours in the Ising Model

Correlation functions as a way to probe collective behaviours:

$$C(r) = \sum_{i,j} \delta\sigma_i \, \delta\sigma_j \, \delta\left(r - \left| R_i - R_j \right| \right)$$

$$\delta\sigma_i = \sigma_i - \langle \sigma \rangle$$

$T \ll T_c$

$T \sim T_c$

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$$C(r) \sim e^{-r/\xi}$$

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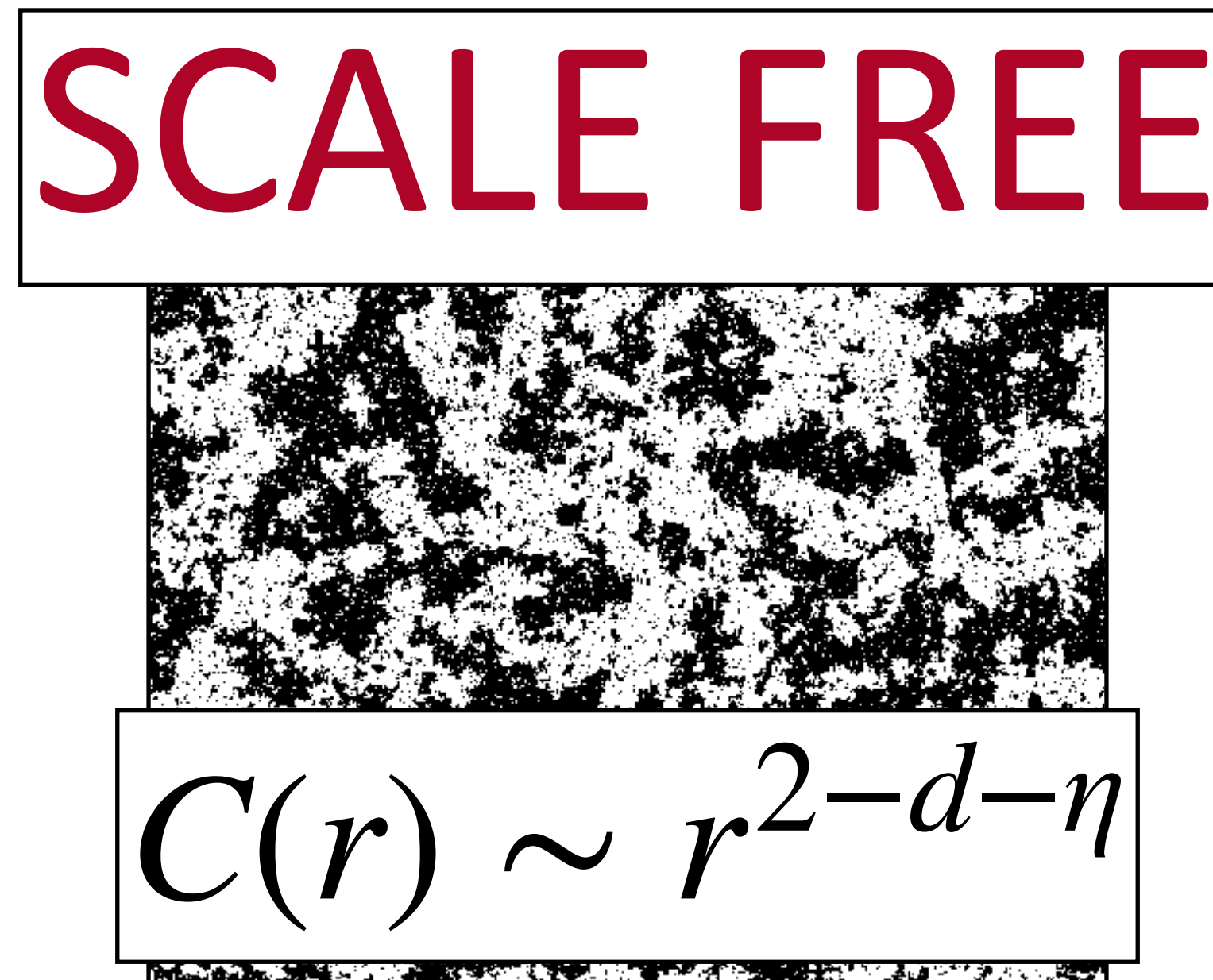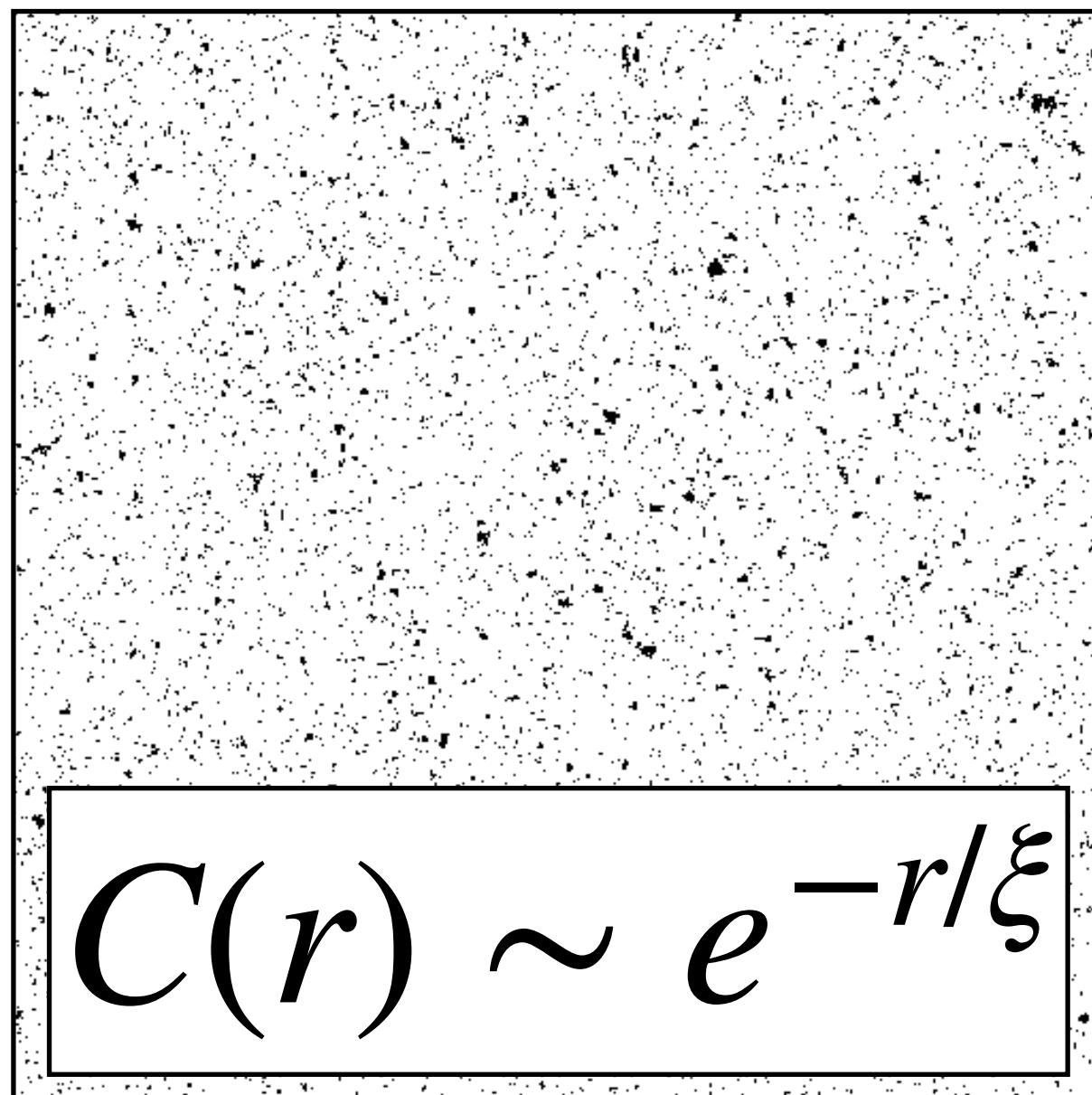
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SCALE FREE

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Collective behaviours in the Ising Model
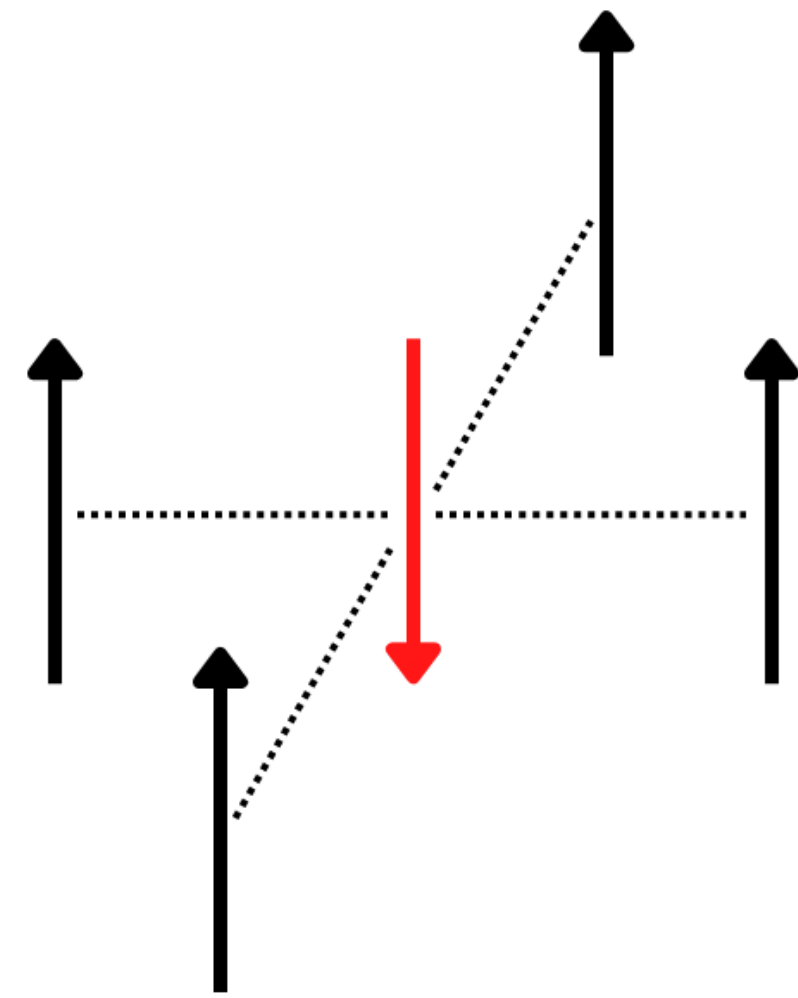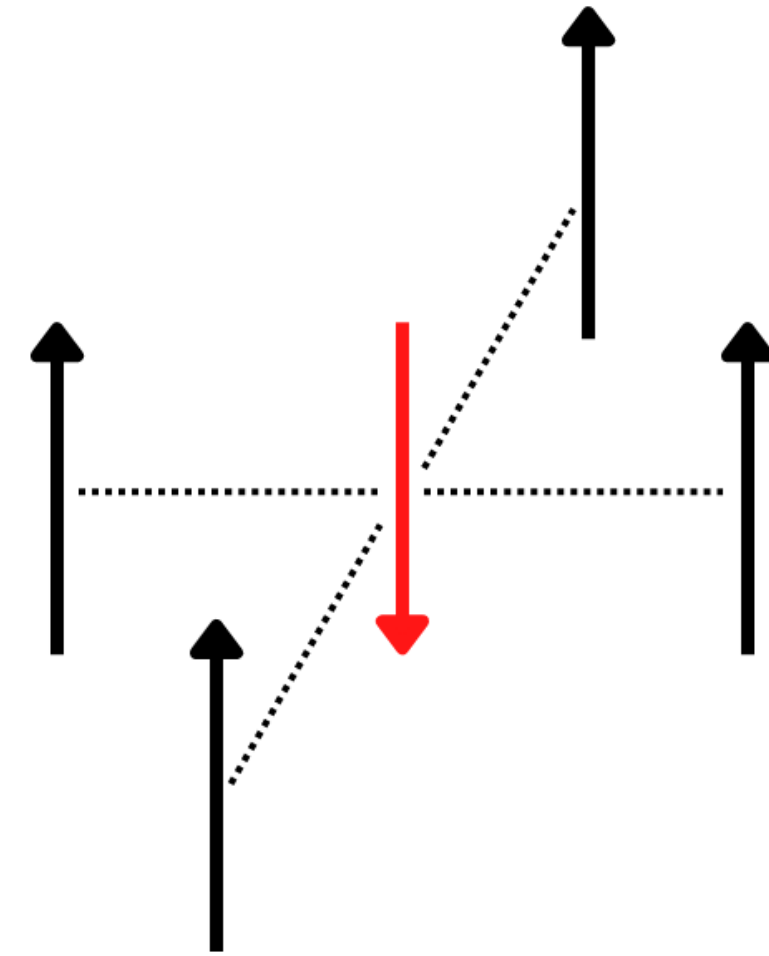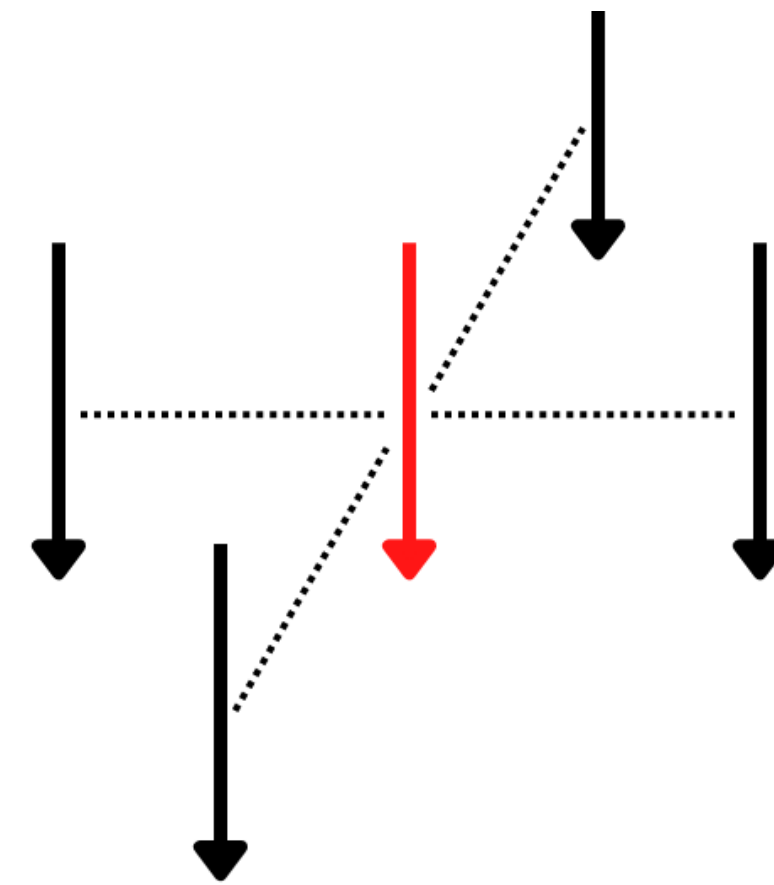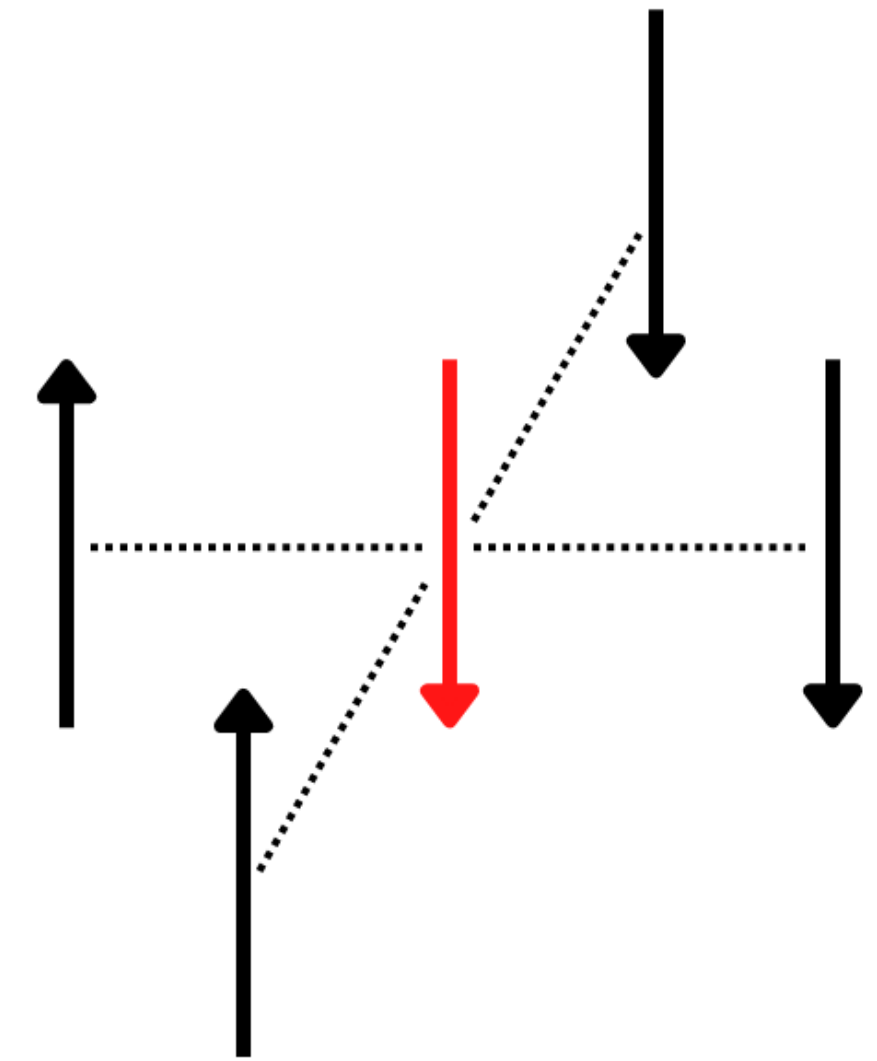
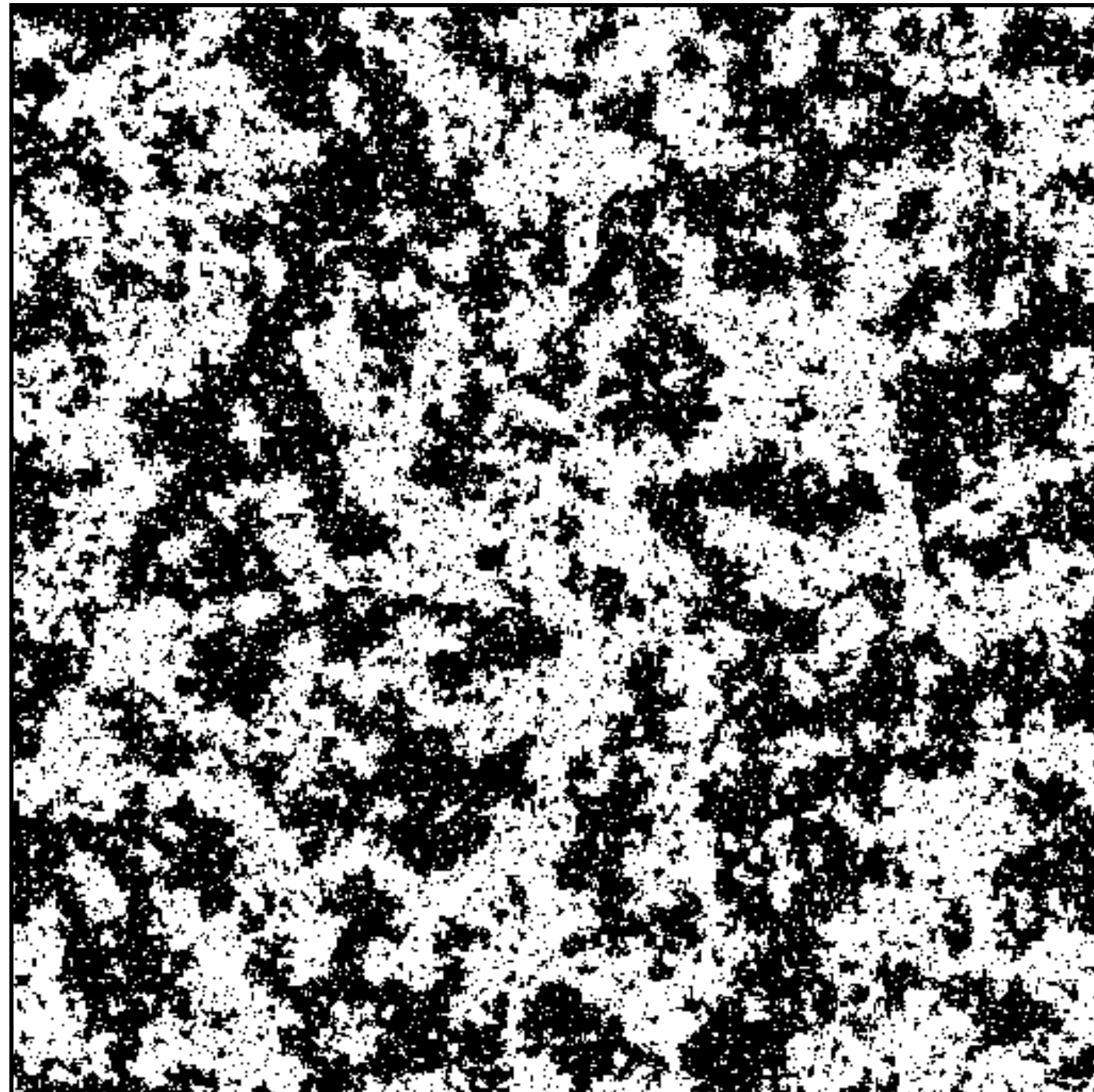Start from the configuration:

$$T \ll T_c$$

$$T \sim T_c$$

$$T \gg T_c$$

Collective behaviours in the Ising Model

$$T \sim T_c$$

Diverging correlation length:

$$\xi \sim \tau^{-\nu} \qquad \tau \propto \left| T - T_c \right|$$

Diverging respons to external fields:

$$\chi = \frac{\partial M}{\partial H} \sim \tau^{-\gamma}$$

Can we model flocks through an Ising model?

Analogies:
- Birds play the role of spins, with the direction of the spin pointing in the direction of motion;
- Birds tend to align their direction to that of neighbours;

Differences:
- birds' direction of motion can point in any direction of space;
- birds in flocks are all aligned, while collective behaviours in Ising model occur only near the transition

Collective behaviours in flocks

Simple model for flocks (XY or Heisenberg models):

$$H\left(\vec{\sigma}\right) = -\sum J_{ij}\vec{v}_i \cdot \vec{v}_j \qquad |\vec{v}_i| = 1$$

In $d = 2$ the velocity of each bird is $\vec{v}_i = \left(\cos\varphi_i, \sin\varphi_i\right)$

and hence: $H\left(\vec{\sigma}\right) = -\sum J_{ij}\cos\left(\varphi_i - \varphi_j\right)$
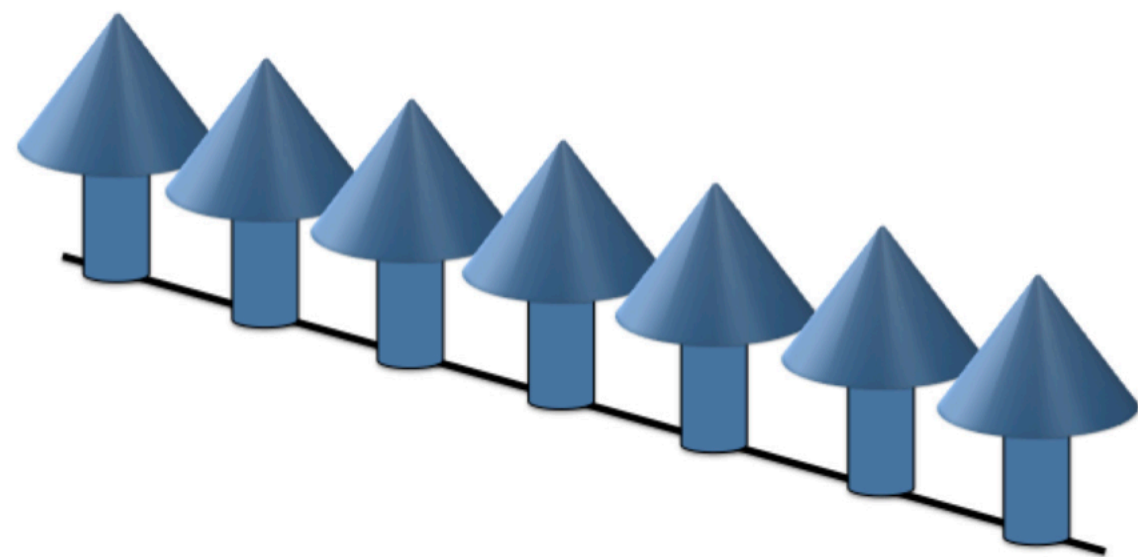
Collective behaviours in flocks

Also this model has a transition between a disordered high temperature phase and an ordered low temperature phase.
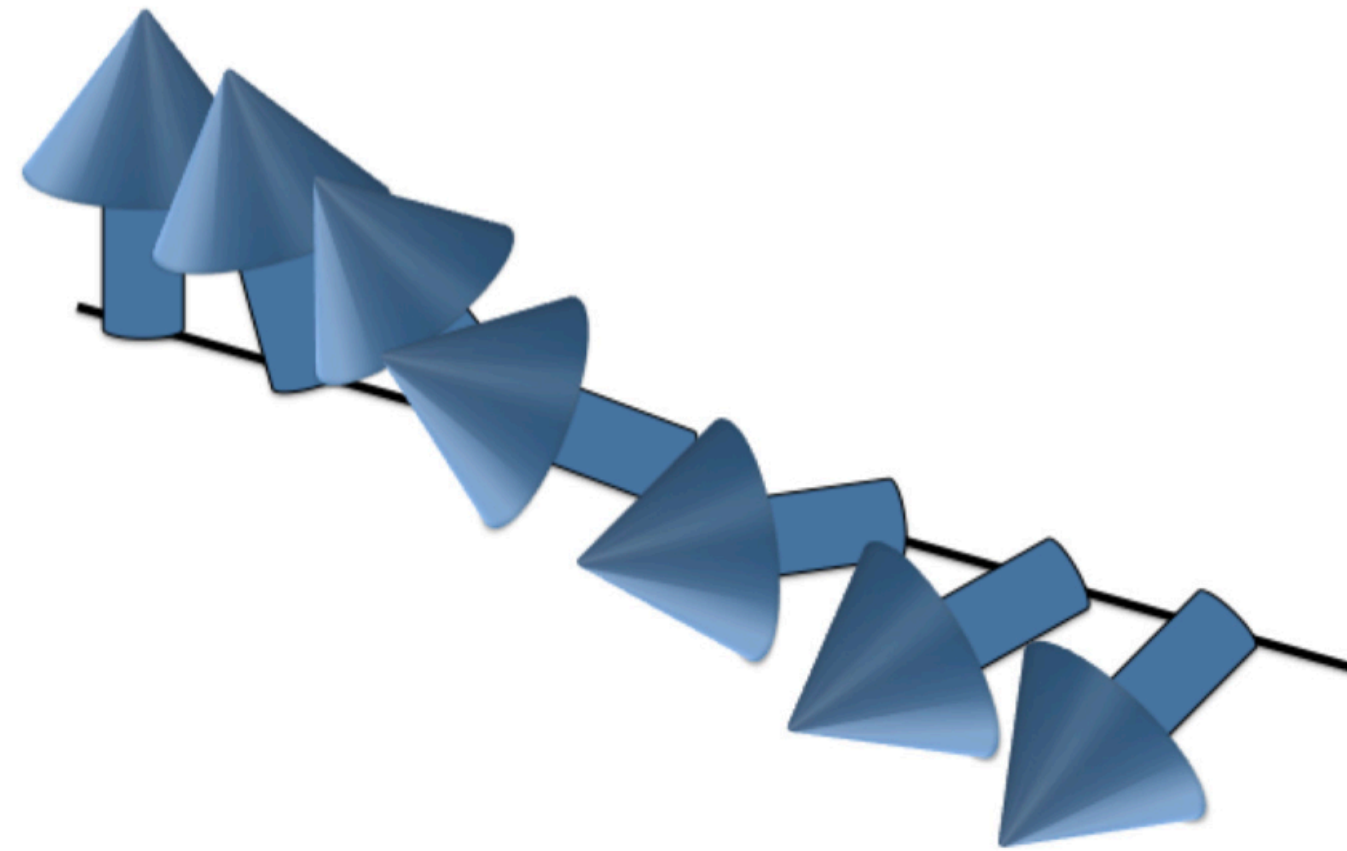
Collective behaviours in flocks

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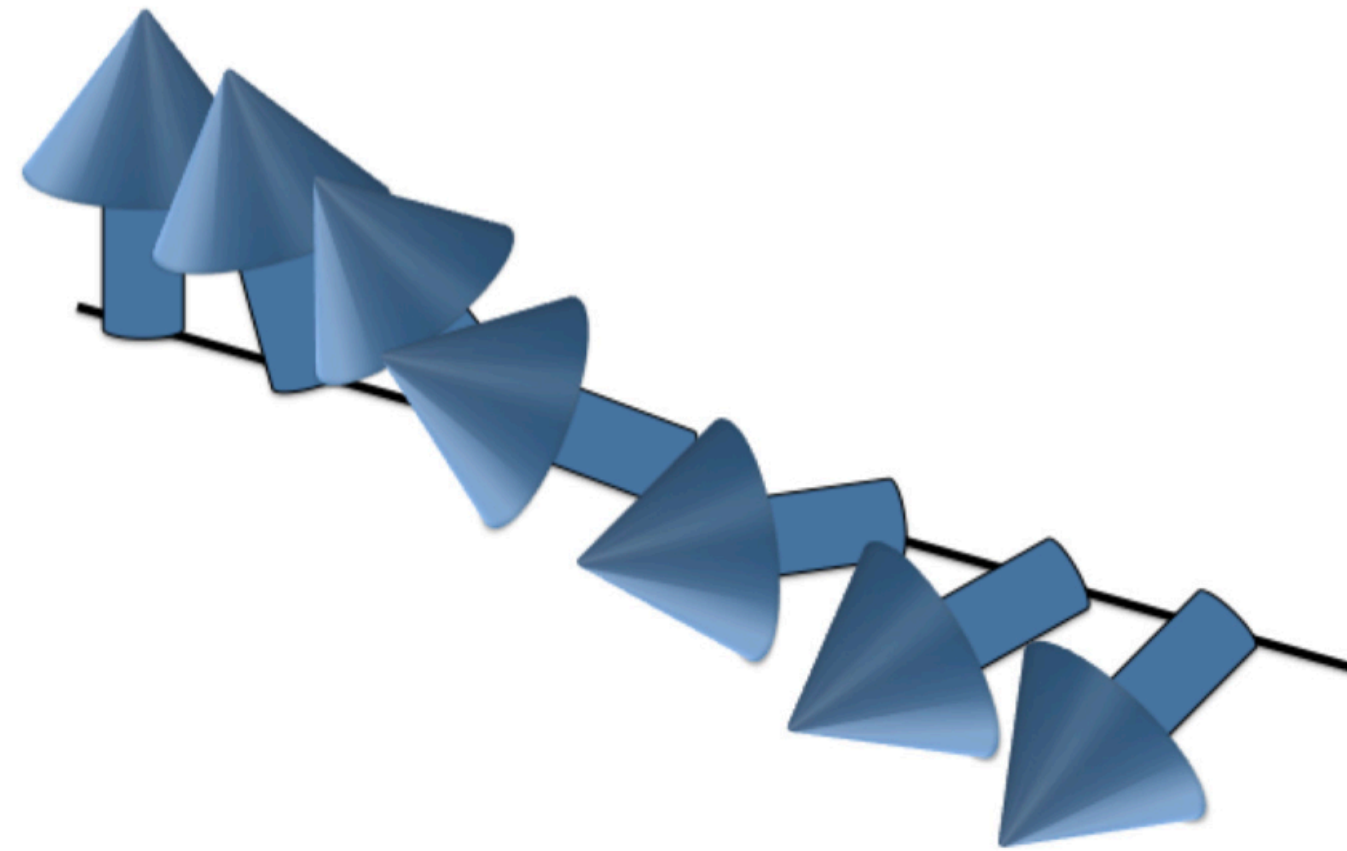At low "temperatures", all birds are aligned but may point in any direction
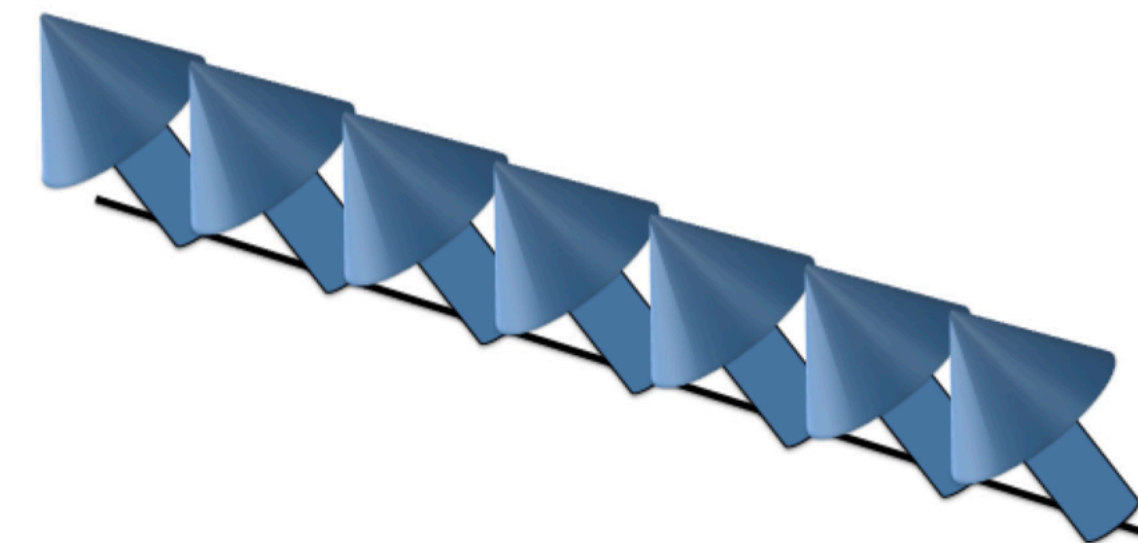
Collective behaviours in flocks

The following exitation has a cost which is smaller the longer the wavelength

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This exitation, known as Goldstone mode, has a "gapless" spectrum: it is a collective behaviour!
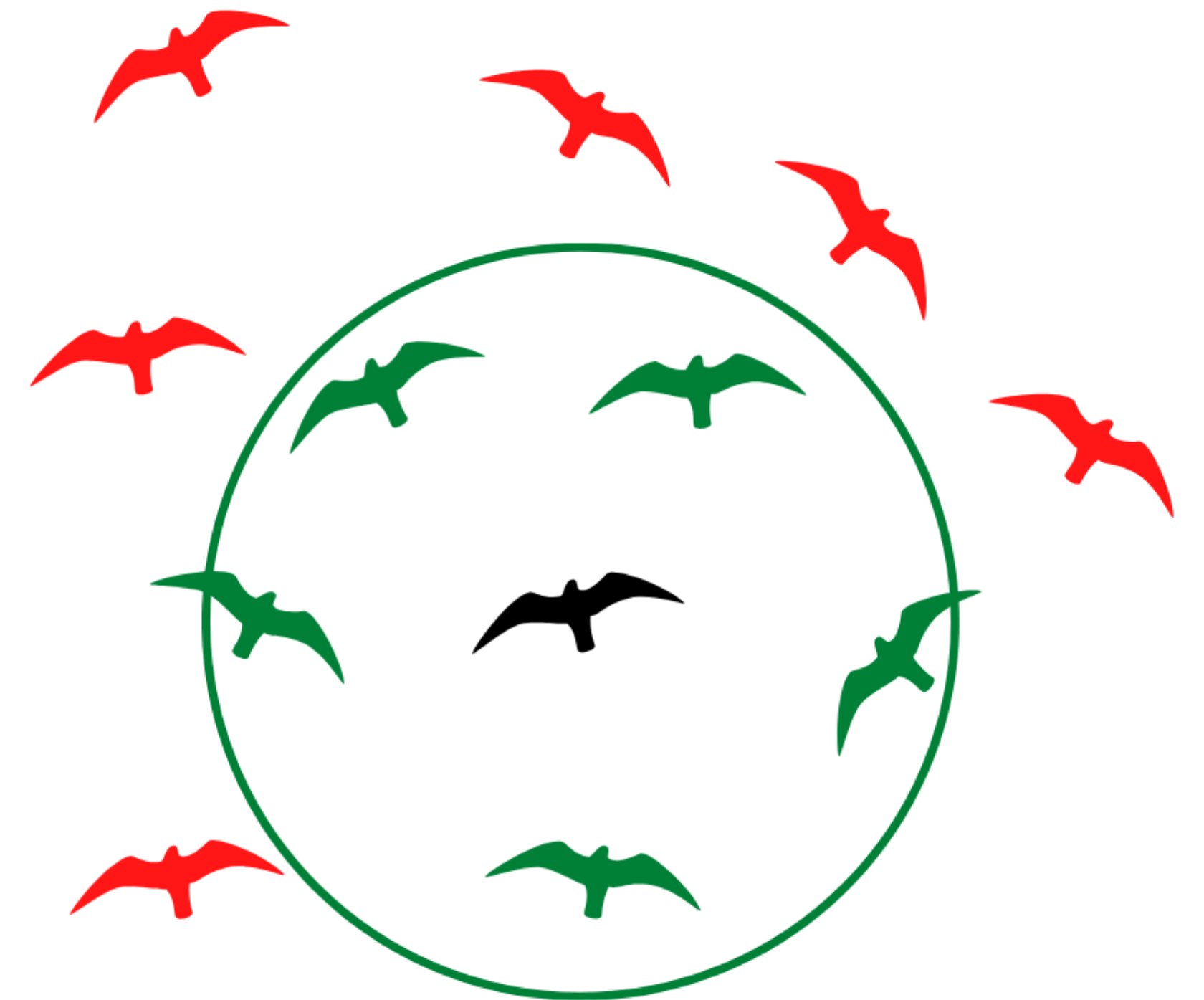
Vicsek Model for flocking

One missing ingredient so far: activity.

$$\frac{d\vec{v}_i}{dt} = -\frac{dH}{d\vec{v}_i} + \eta \qquad |\vec{v}_i| = 1$$

$$\frac{d\vec{r}_i}{dt} = \vec{v}_i$$

$$H\left(\vec{\sigma}\right) = -\sum J(|\vec{r}_i - \vec{r}_j|)\, \vec{v}_i \cdot \vec{v}_j$$

Vicsek Model for flocking

My research focuses on:

- Dynamic collective behaviours occuring in the presence of activity;

- How to "upgrade" the Vicsek model to describe collective behaviours in real biological systems.

Conclusions

Collective behaviours are symthoms of the presence of long wavelength modes that are easy to "activate"

Collective behaviours occure in a wide variety of biological systems for surviving reasons

Collective behaviours can be detected by looking at correlations