Cosmological tensions guiding the path beyond ΛCDM

Late time solutions to H_0 and σ_8

Jann Zosso

L. Heisenberg, H. Villarrubia-Rojo, JZ - **arXiv:2201.11623** and **arXiv:2202.01202**

DΛark Energy, Frascati 2023 **-** September 13th

ETH zürich

OUTLINE

1. Introduction
   - H_0 and σ_8 tensions
2. General model agnostic method
   - Generic late time modifications of expansion history
   - Analytic response functions
3. Results
   - Necessary conditions on modifications
   - Example:
     - Dark energy fluid model
     - Implications on beyond GR
   - Beyond background modifications

COSMOLOGICAL STANDARD MODEL - ΛCDM

Basis: General relativity

Background homogeneity and isotropy

Particle physics

Ingredients: Cosmological constant Λ

Cold dark matter (CDM)

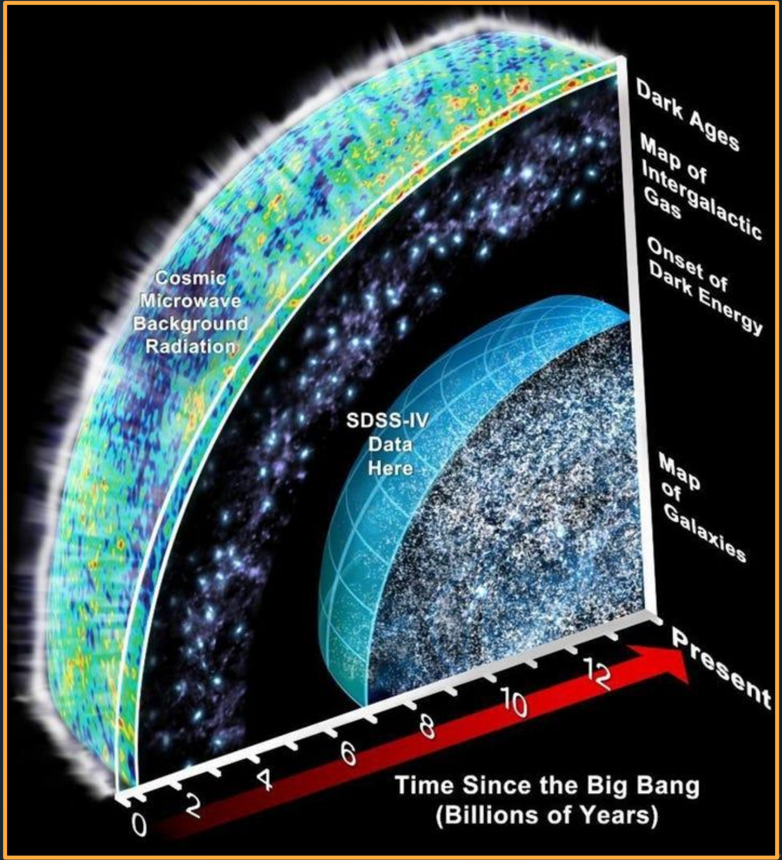
Inflation

Parameters: $\left\{H_0, \Omega_m, \Omega_b, \tau_{\text{reio}}, A_s, n_s\right\}$

only 6 free parameters

COSMOLOGICAL STANDARD MODEL - ΛCDM

Fundamental observations



[SLOAN DIGITAL SKY SURVEY (SDSS)]

COSMOLOGICAL STANDARD MODEL - ΛCDM

Fundamental observations

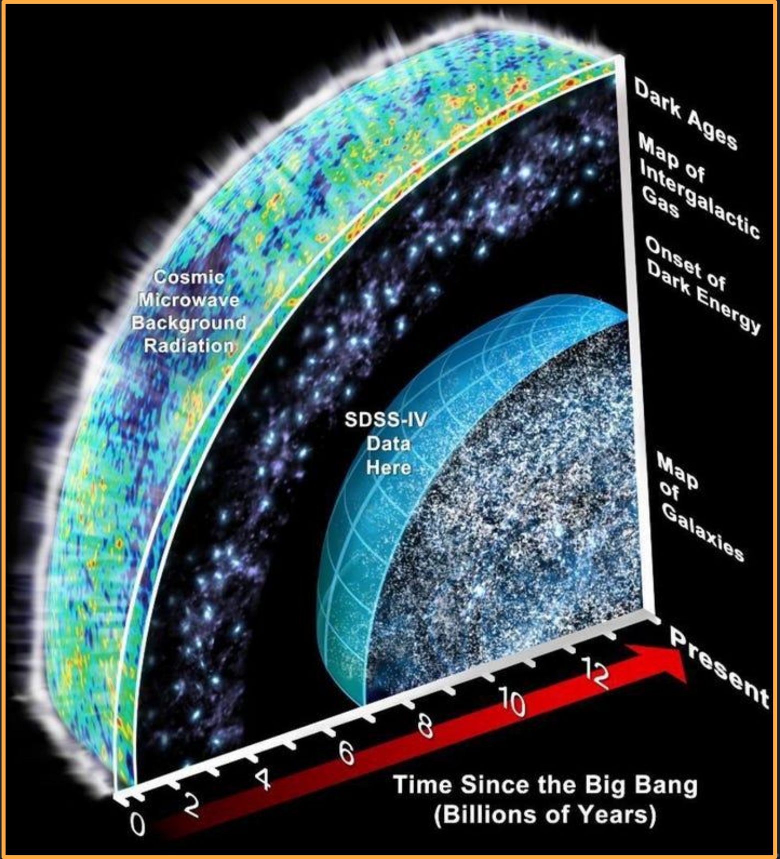
Expansion rate today

$$H_0 \equiv 100\ h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Direct/local

- Distance redshift relation $d = c \int \frac{\mathrm{d}z}{H}$ ⇨ $d(z) = \frac{z}{H_0} + \mathcal{O}(z^2)$

cf. talk by Stefano Maria



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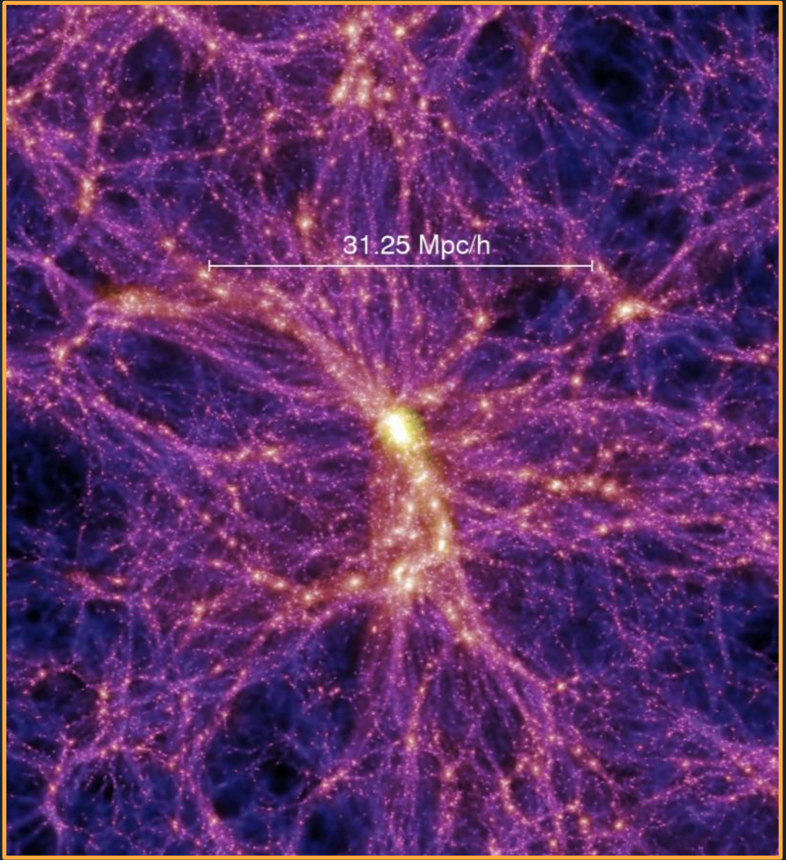
- Galaxy surveys σ_8^2

  large scale structure (LSS)



[Millenium Simulation]

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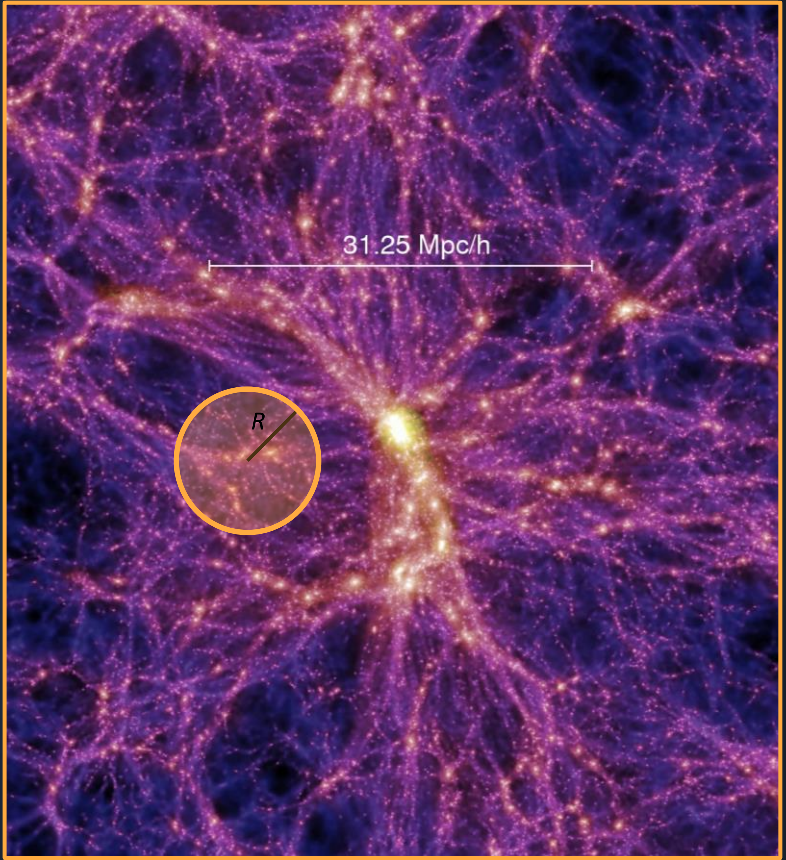
matter overdensities

$$\delta_m \equiv \frac{\rho_m(t,\boldsymbol{x}) - \langle \rho_m \rangle(t)}{\langle \rho_m \rangle(t)}$$

smoothing

$$\delta_R \equiv \int_{|\boldsymbol{x}-\boldsymbol{x}'|<R} \delta_m(\boldsymbol{x}') \frac{\mathrm{d}^3\boldsymbol{x}'}{V}$$



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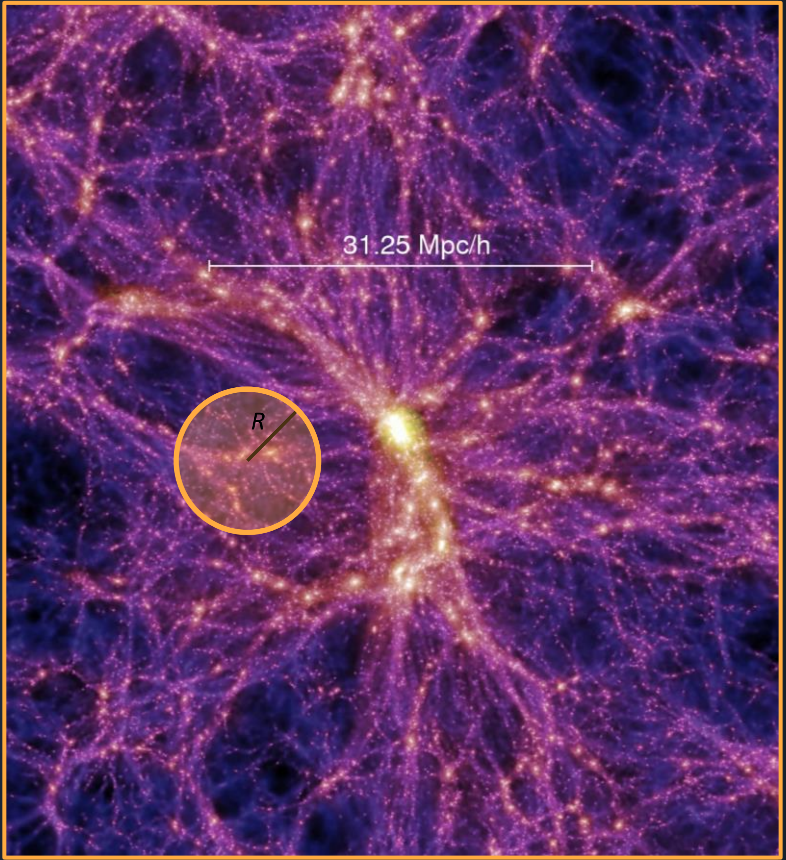
cf. talk by Vincenzo Carmelita



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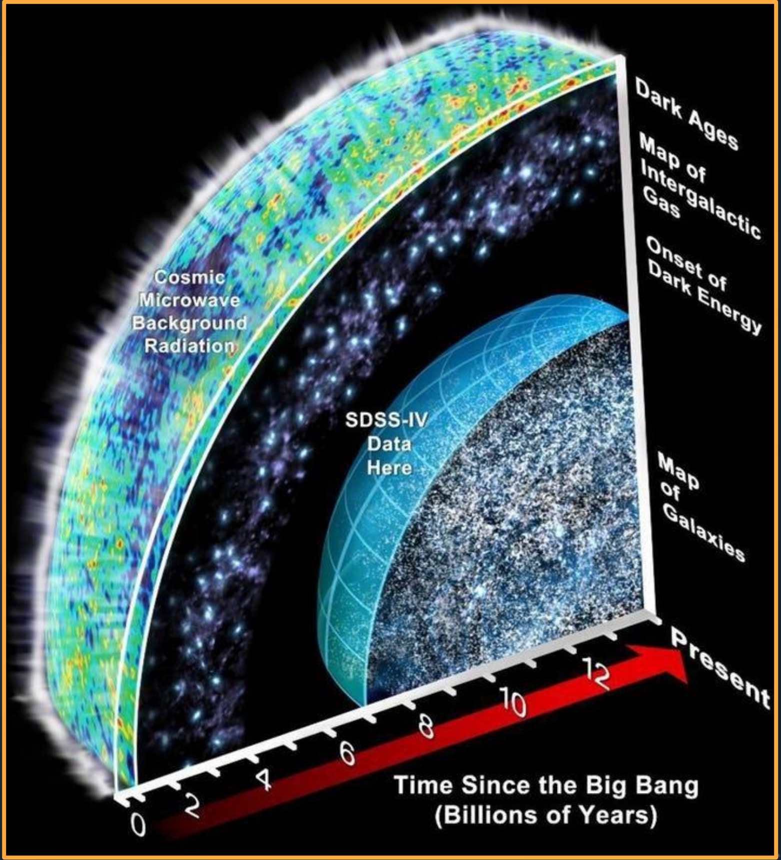

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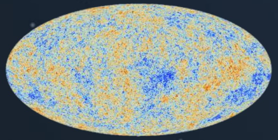

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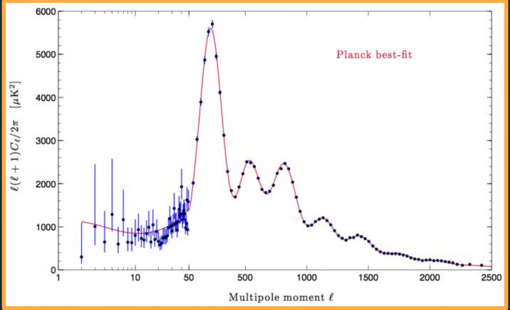
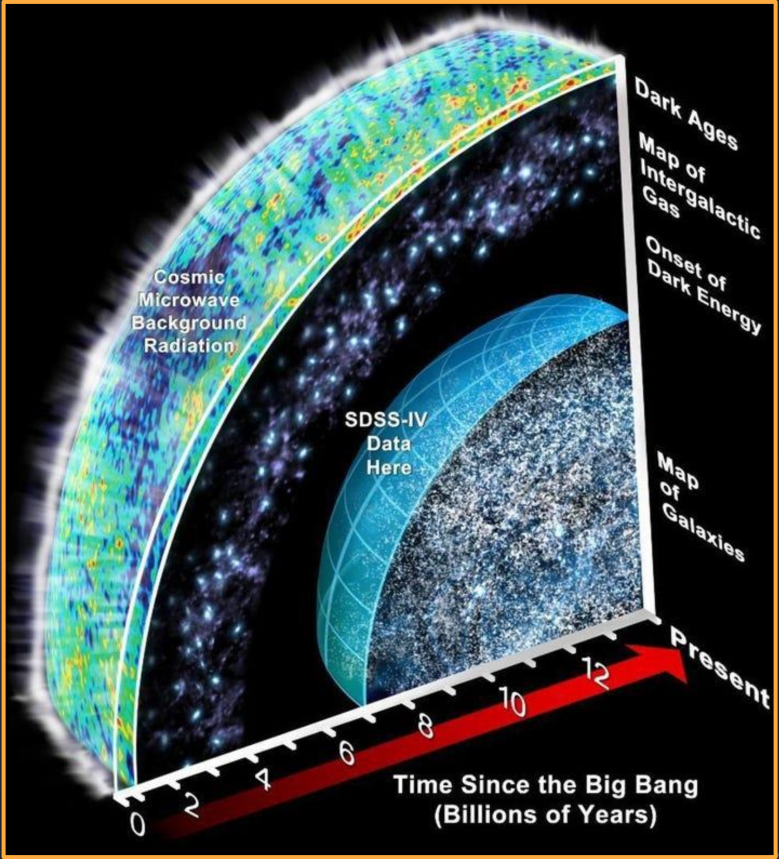


cf. talk by Martina

[Planck]



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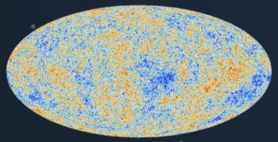

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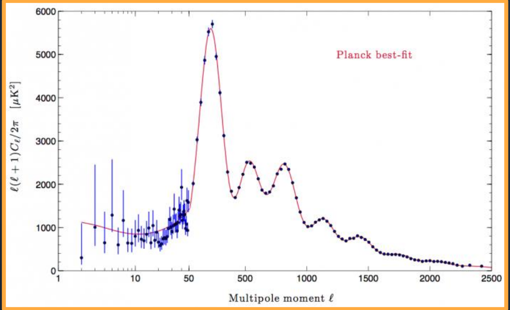

[Planck]

cf. talk by Martina

CMB distance priors

angular scale $\theta_* = \frac{r_s(z_*)}{d(z_*)}$

shift parameter $R_* = d(z_*)\sqrt{\Omega_m H_0^2}$

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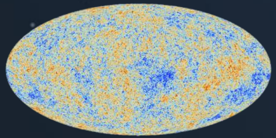

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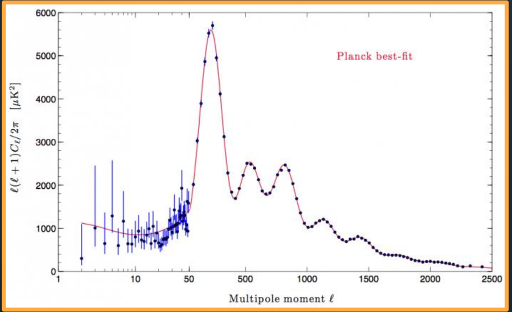

[Planck]

$\left\{ H_0, \Omega_m, \Omega_b, \tau_{\mathrm{reio}}, A_s, n_s \right\}$ ⇦

ΛCDM – best fit

CMB distance priors

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  large scale structure (LSS)

$$\sigma_8^2 = \int \frac{\mathrm{d}k}{2\pi^2}\, k^2 \underbrace{P_m(k)}_{D^2(a)}\, W(kR)$$

growth factor

$$\delta_m \propto D(a)$$

growth rate

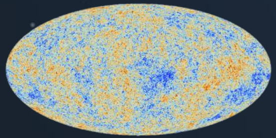
$$f = \frac{d \log D}{d \log a}$$

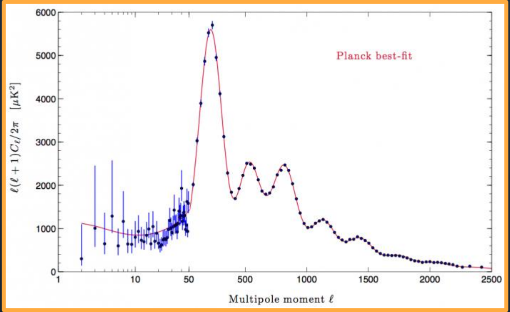
Einstein equations

$$\frac{\mathrm{d}^2 D}{\mathrm{d}a^2} + \frac{\mathrm{d}\log a^3 H}{\mathrm{d}a}\frac{\mathrm{d}D}{\mathrm{d}a} - \frac{4\pi G \rho_m}{a^5 H^2} D = 0$$

Indirect/distant

- CMB





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$$\left\{ H_0, \Omega_m, \Omega_b, \tau_{\mathrm{reio}}, A_s, n_s \right\}$$

ΛCDM – best fit

COSMOLOGICAL STANDARD MODEL - ΛCDM

Fundamental observations

Direct/local

- Distance redshift relation

measured

- Galaxy surveys

large scale structure (LSS)

H_0 and σ_8

Indirect/distant

- CMB + ΛCDM

H_0 and σ_8

inferred

H_0 TENSION

Fundamental observations

Direct/local

- Distance redshift relation

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large scale structure (LSS) H_0 and σ_8

Indirect/distant v

- CMB + ΛCDM H_0 and σ_8

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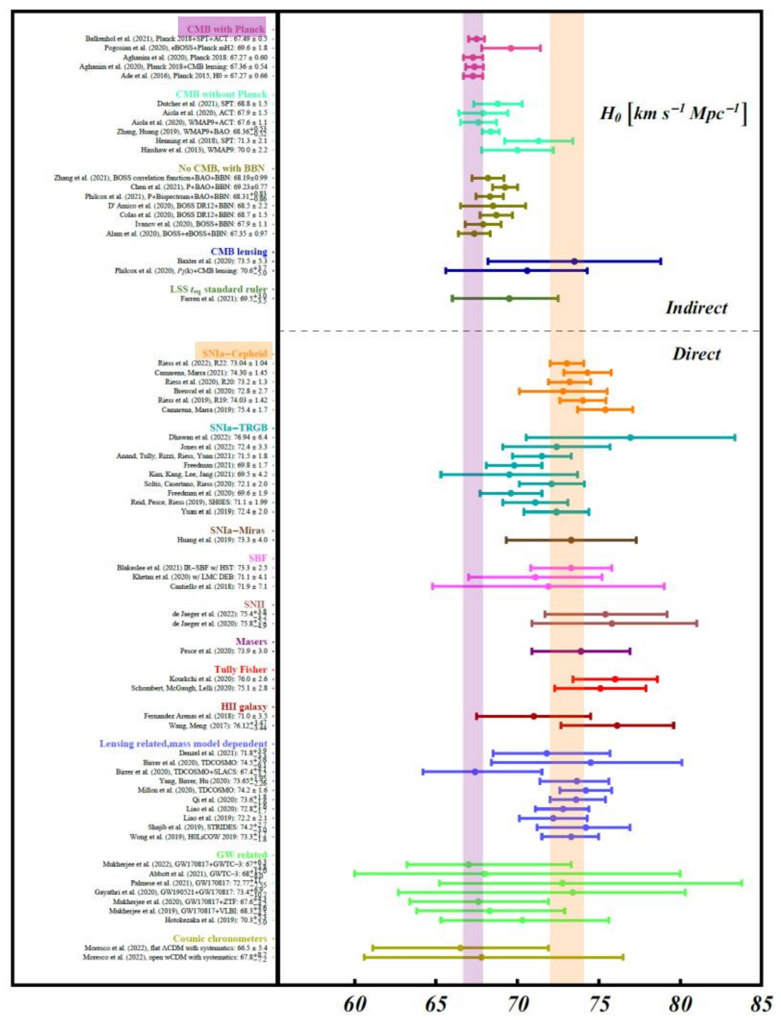

[Abdalla et al. 2203.06142]

σ_8 TENSION

Fundamental observations

Direct/local

- Distance redshift relation
- Galaxy surveys

  large scale structure (LSS)

measured

H_0 and σ_8

$\vee$ $\wedge$

H_0 and σ_8

inferred

Indirect/distant

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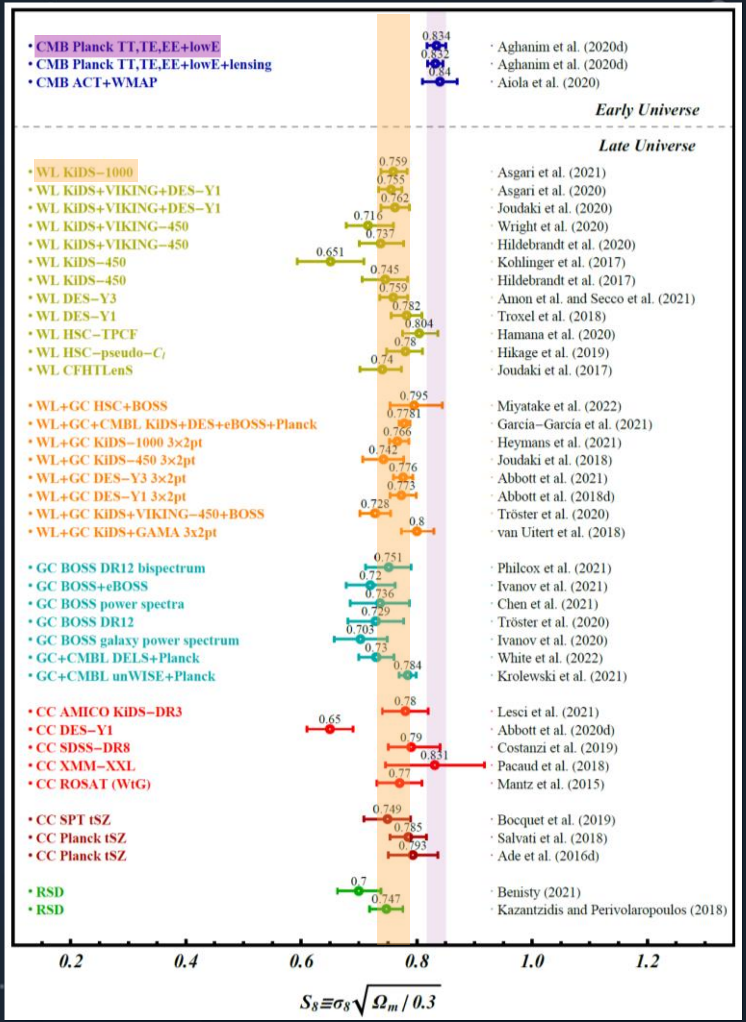

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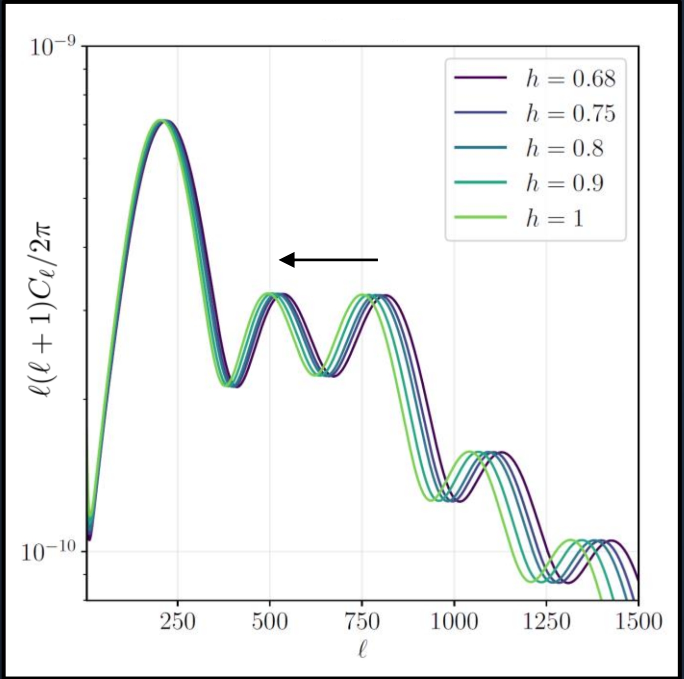
H_0 TENSION

$$H_0 \equiv 100\ h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$



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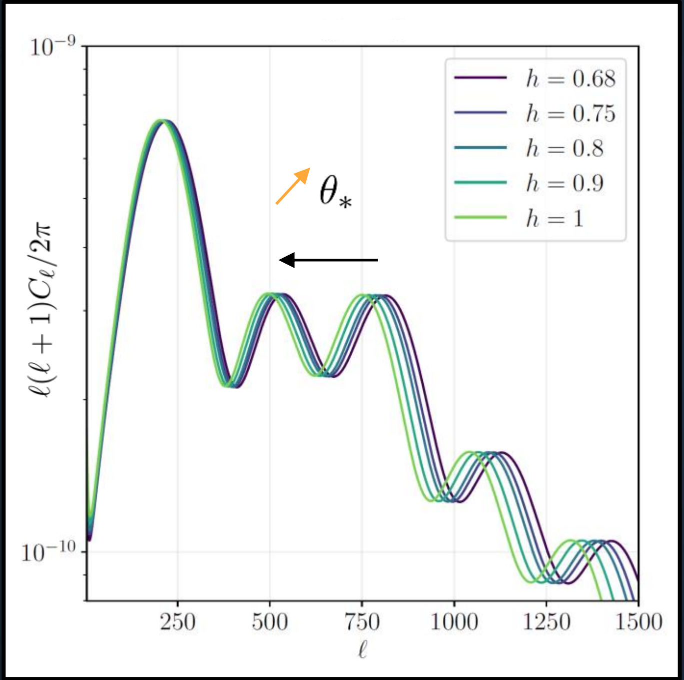

[H. Villarrubia-Rojo]

angular scale

$$\theta_* = \frac{r_s(z_*)}{d(z_*)}$$

sound horizon

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{\mathrm{d}z}{H} c_s$$

distance to us

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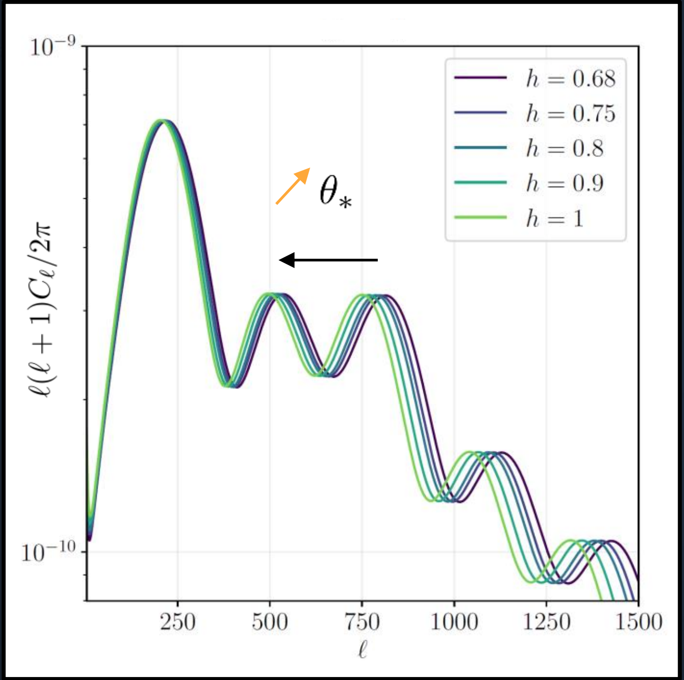

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early time solutions

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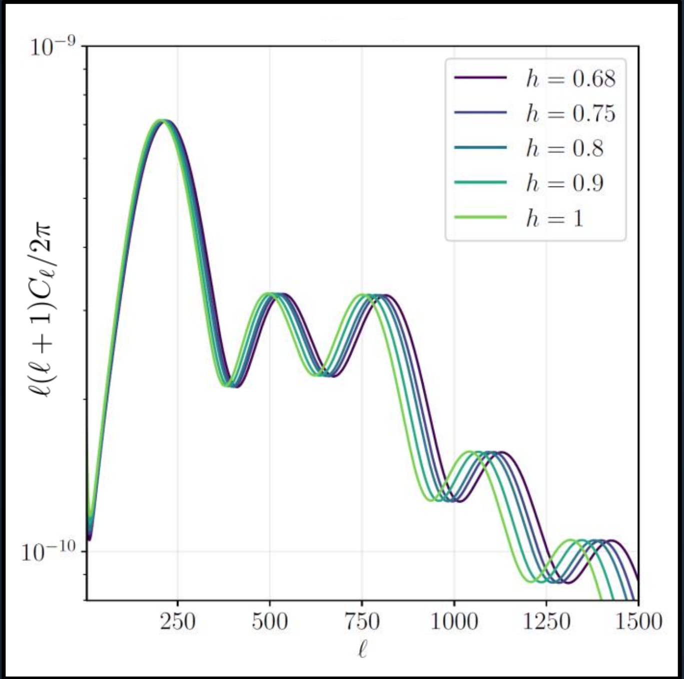

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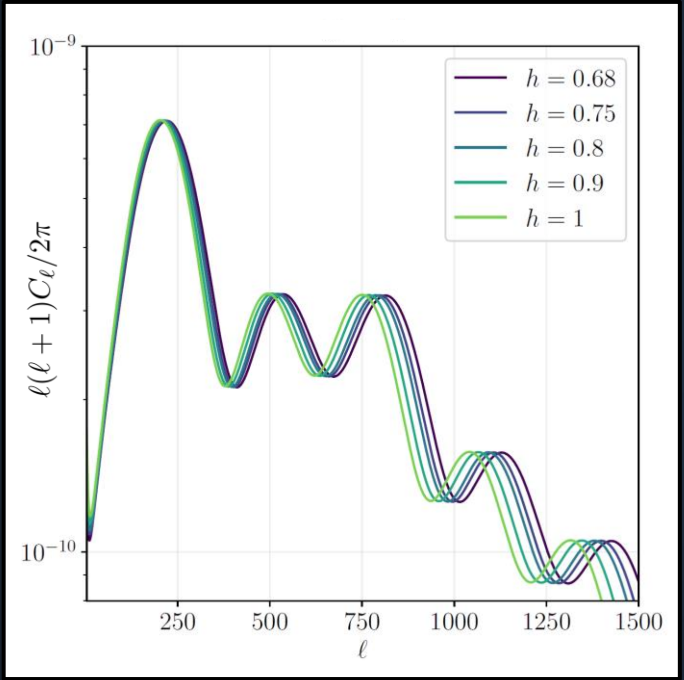

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late time solutions

There exist many proposed modifications of ΛCDM

Example: covariant dark enegy models (Horndeski, GenProca)

- Aleviate H_0 tension
- At the cost of worsening σ_8 tension!

perturbation level: expensive Boltzman simulations

[arXiv:1603.05806]

[arXiv:1901.07183]

[arXiv:2002.06782]

[arXiv:2010.00513]

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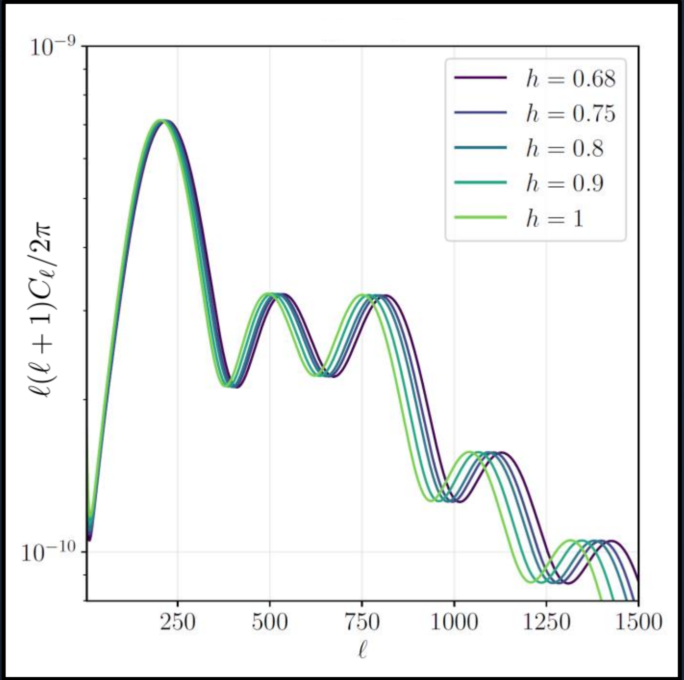

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[arXiv:1603.05806]
[arXiv:1901.07183]
[arXiv:2002.06782]
[arXiv:2010.00513]

Goal: Formulate analytic, model independent conditions to solve both tensions simultaneously

METHOD

Late time ΛCDM: 2 parameters H_0 and ω_m

Assume: generic ΛCDM modification produces small deviations in *H(z)* at late times

$$H(H_0, \omega_m) = H_{\Lambda\mathrm{CDM}}(H_0, \omega_m) + \delta H(z)$$

model independent

δH(z) an arbitrary function, no parametrization!

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⇨

$$H_0 + \delta H_0$$

$$\omega_m + \delta\omega_m$$

where

$$\omega_\lambda \equiv \Omega_\lambda h^2$$

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On background $\Rightarrow$

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2(z)} \frac{\delta H_0}{H_0} + \frac{\delta H(z)}{H(z)}$$

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Calculate total variation of any observable $\Rightarrow$

$$\frac{\Delta \mathcal{O}(z)}{\mathcal{O}(z)} = I_{\mathcal{O}}(z) \frac{\delta H_0}{H_0} + \int_0^\infty \frac{\mathrm{d}x_z}{1+x_z} R_{\mathcal{O}}(x_z, z) \frac{\delta H(x_z)}{H(x_z)}$$

METHOD

?

relate

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METHOD

Choose a single observable and impose a vanishing variation

angular scale

$$\boxed{\Delta\theta_* \overset{!}{=} 0}$$

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METHOD

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$$\Rightarrow \boxed{\frac{\delta H_0}{H_0} = \int \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{H_0}(x_z) \frac{\delta H(x_z)}{H(x_z)}}$$

Response functions

relate

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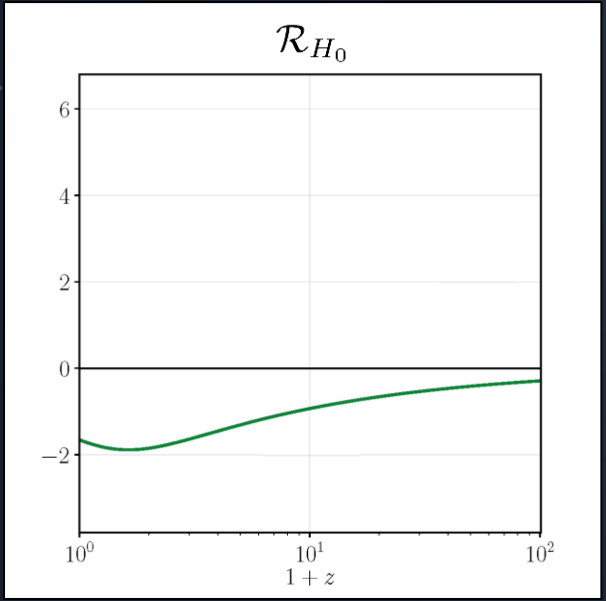

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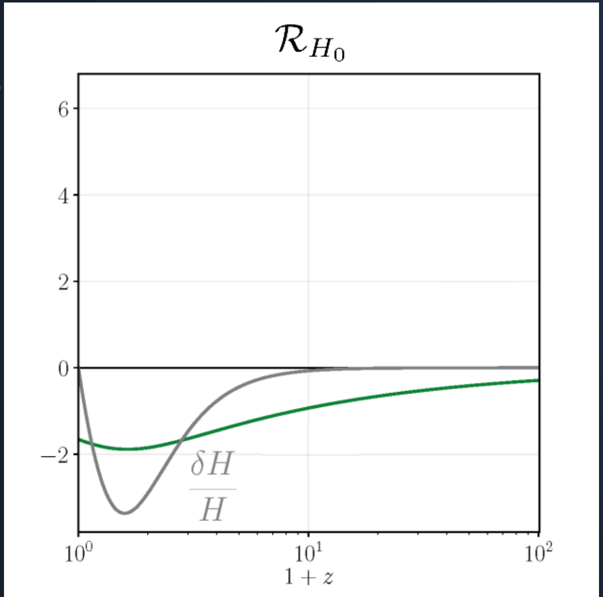

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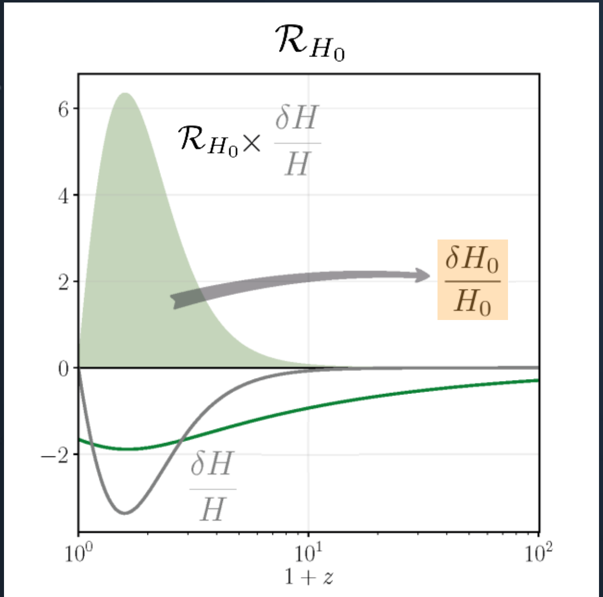
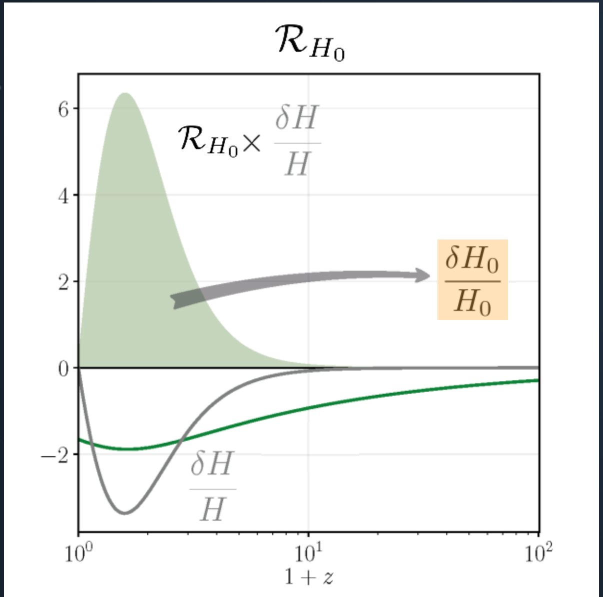
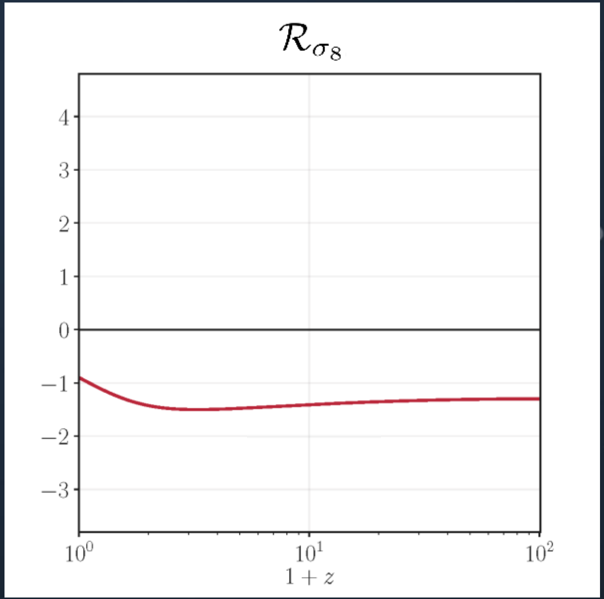


RESULTS

$$\frac{\delta H_0}{H_0} = \int \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{H_0}(x_z) \frac{\delta H(x_z)}{H(x_z)} > 0$$

$$\frac{\Delta\sigma_8}{\sigma_8} = \int_0^\infty \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{\sigma_8} \frac{\delta H}{H}$$





RESULTS

$$\frac{\delta H_0}{H_0} = \int \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{H_0}(x_z) \frac{\delta H(x_z)}{H(x_z)}$$ > 0

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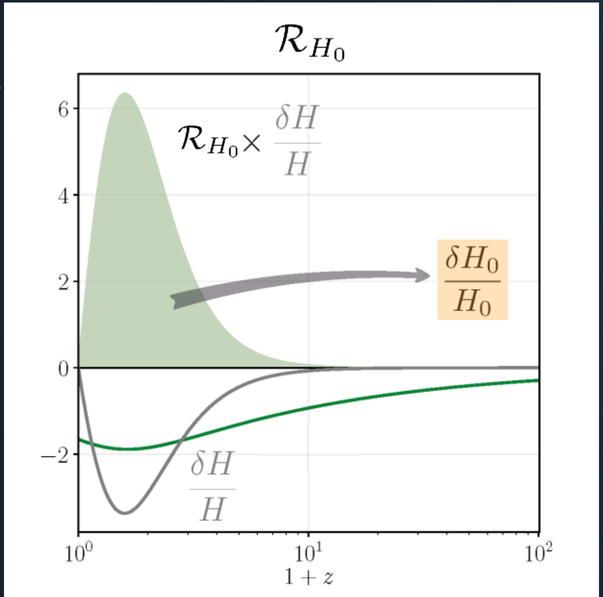

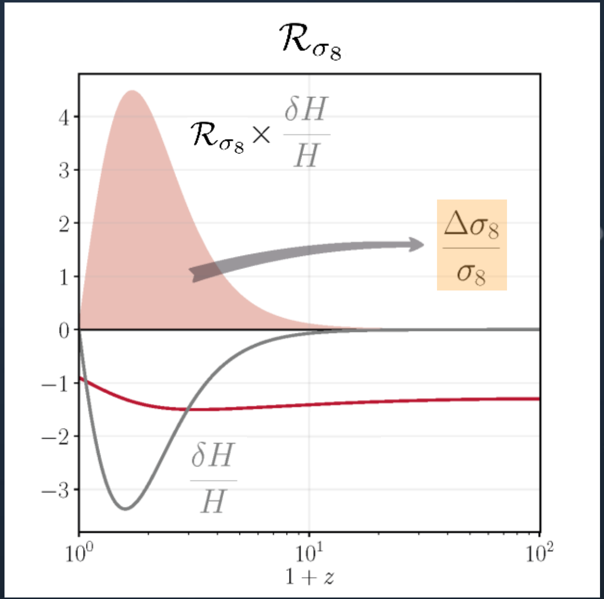

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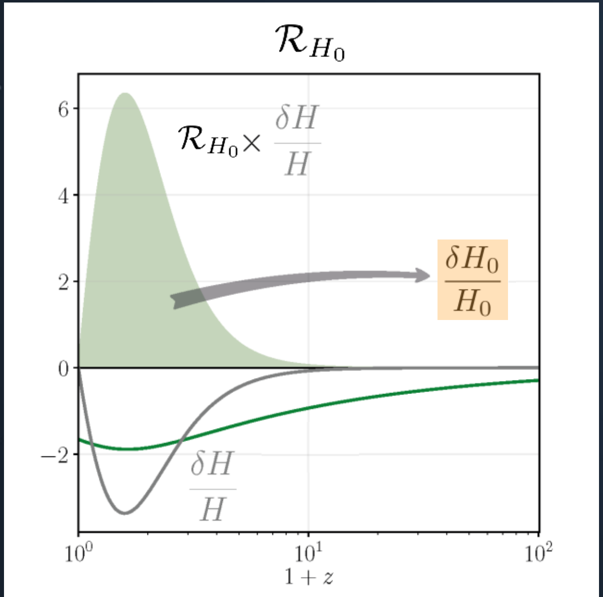

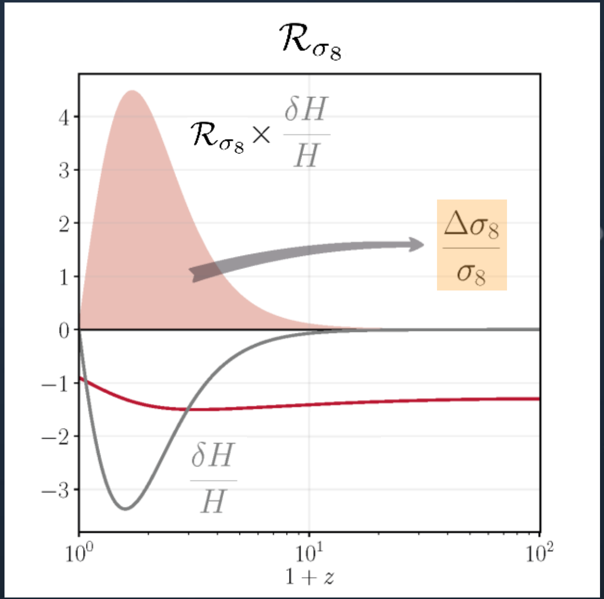

At the background level:

⇨ Increase H0 ↗

⇨ Inevitably increase σ8! ↗

δH(z) needs to change sign

EXAMPLE

$$\boxed{\frac{\delta H_0}{H_0} = \int \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{H_0}(x_z) \frac{\delta H(x_z)}{H(x_z)}}$$

Dark energy fluid model

Equation of state

$$\boxed{\frac{p}{\rho} = \omega(z)} \quad , \quad \boxed{\frac{\delta H(z)}{H(z)} = \frac{H_0^2 \Omega_\Lambda}{2H^2(z)} \left\{ \exp\left(3 \int_0^z \left(1 + w(z)\right) \frac{dz}{1+z}\right) - 1 \right\}}$$

cf. talk by Martin

EXAMPLE

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phantom dark energy ω<-1

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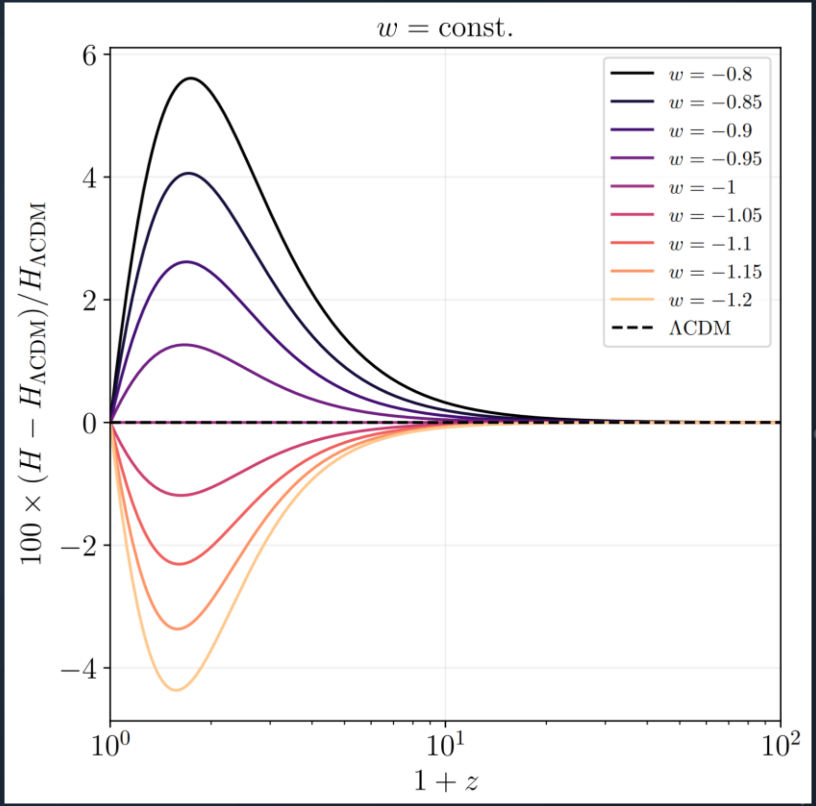

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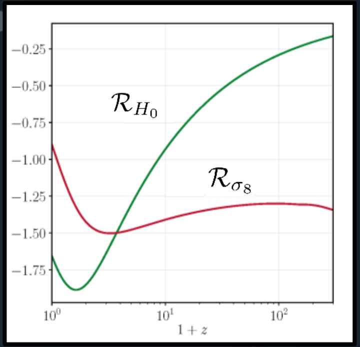

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δH(z) needs to change sign

At the background level

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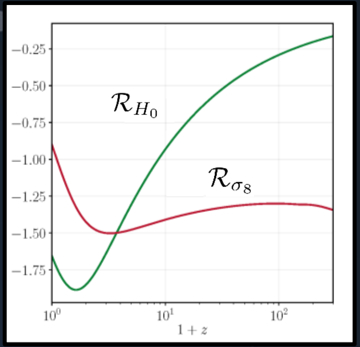

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Equation of state $\boxed{\frac{p}{\rho} = \omega(z)}$

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phantom dark energy ω<-1

need to cross the phantom divide!

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At the background level

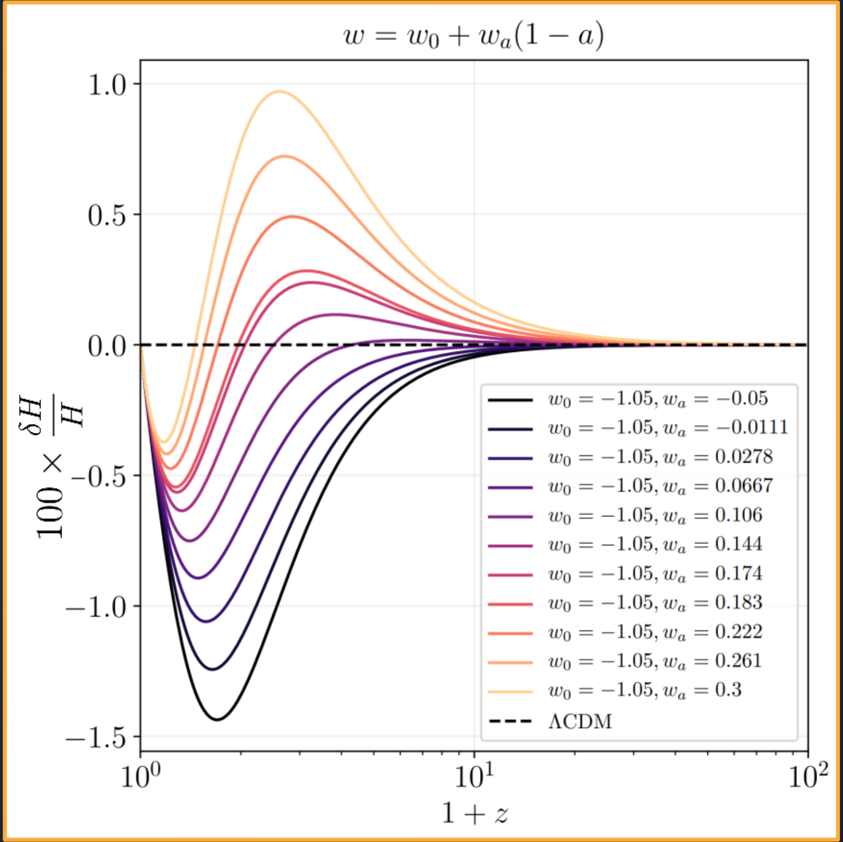
Dark energy fluid model

EXAMPLE

Solve H_0 tension: phantom dark energy $\omega<-1$

Solve both tensions: need to cross the phantom divide!

CLP parametrization



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Numerical Benchmarks

w_a	$100 \times \delta h/h$ class	$100 \times \delta h/h$ Analytical	$100 \times \Delta\sigma_8/\sigma_8$ class	$100 \times \Delta\sigma_8/\sigma_8$ Analytical
-0.05	2.81	2.61	2.24	2.08
-0.01	2.35	2.22	1.86	1.75
0.03	1.89	1.80	1.47	1.39
0.07	1.42	1.37	1.07	1.03
0.11	0.946	0.93	0.66	0.64
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0.174	0.095	0.093	-0.089	-0.092
0.18	-0.022	-0.025	-0.19	-0.20
0.22	-0.52	-0.53	-0.64	-0.65
0.26	-1.02	-1.07	-1.09	-1.13
0.3	-1.54	-1.62	-1.56	-1.64

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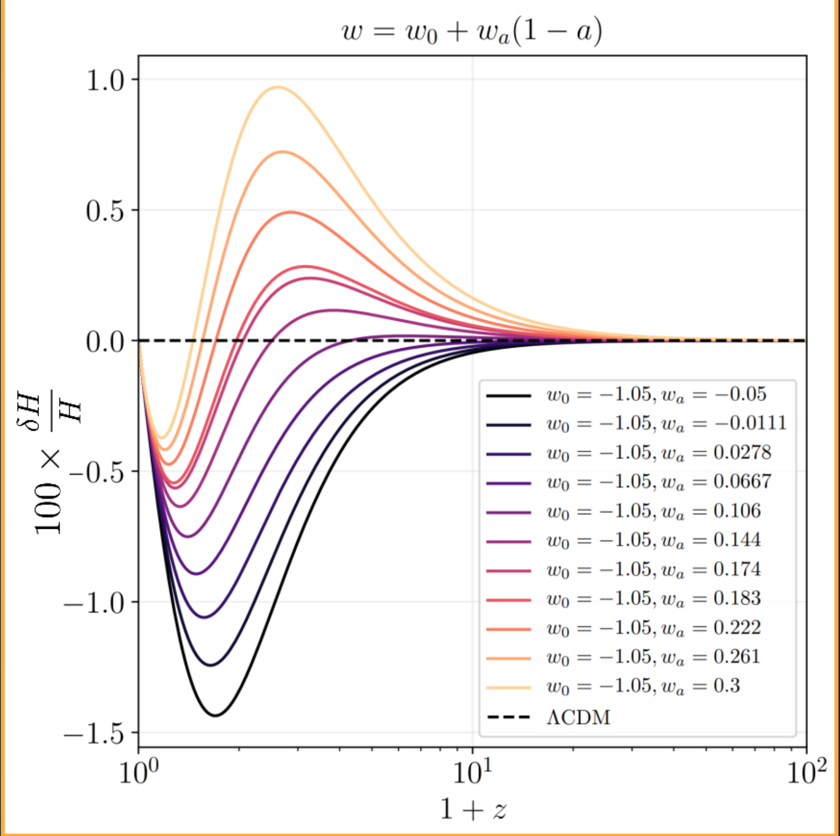

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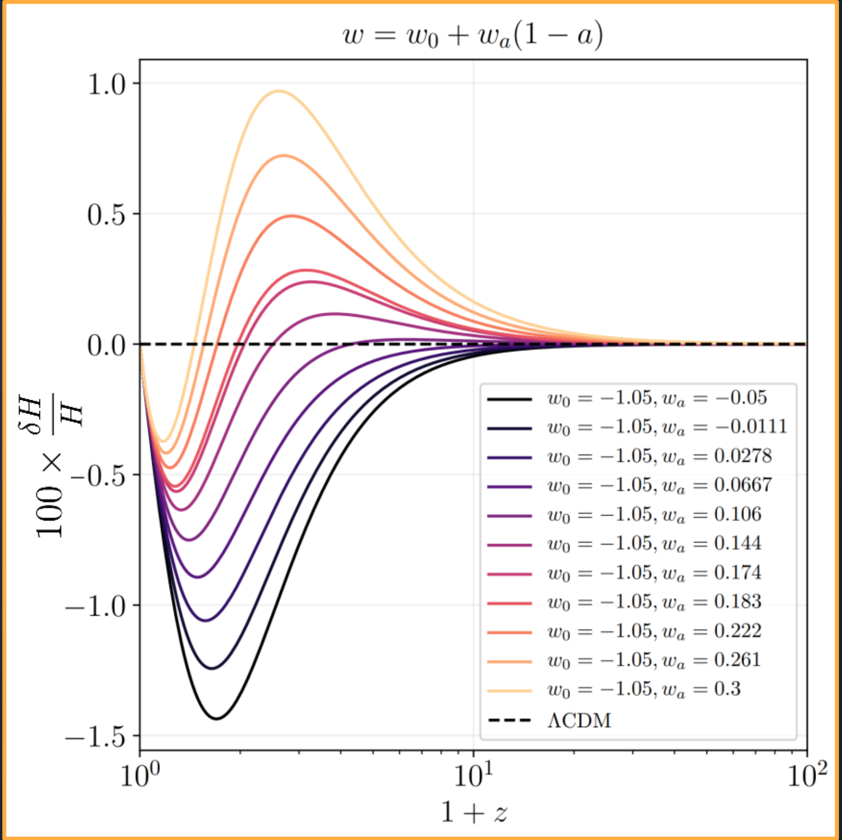

IMPLICATIONS FOR BEYOND GR THEORIES

Naively, $\omega<-1$ ⇨ Kinetic instability / Ostrogradski ghost

(in connection to violation of energy conditions)

[arXiv:astro-ph/0301273]

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Higher derivative interactions allow for consistent phantom equations of state

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Crossing $\omega=-1$: difficult without running into divergences [arXiv:1008.1684]

BEYOND BACKGROUND MODIFICATIONS

$$\frac{\delta H_0}{H_0} = \int \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{H_0}(x_z) \frac{\delta H(x_z)}{H(x_z)}$$

$$\frac{\Delta\sigma_8}{\sigma_8} = \int_0^\infty \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{\sigma_8} \frac{\delta H}{H} + \sum_i \int_0^\infty \frac{\mathrm{d}x_z}{1+x_z} \mathcal{Q}_i \frac{\delta Q_i}{Q_i}$$

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If solve the H_0-tension at the background level but not σ_8

$\delta H(z) < 0$

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If solve the H_0-tension at the background level but not σ_8

$\delta H(z) < 0$

New effects need to reduce σ_8 even more!

EXAMPLE

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Include perturbations $\boxed{\delta_m \propto D(a)}$

Change in Newtons const. $\boxed{G_{\text{eff}} = G + \delta G}$

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RESULTS

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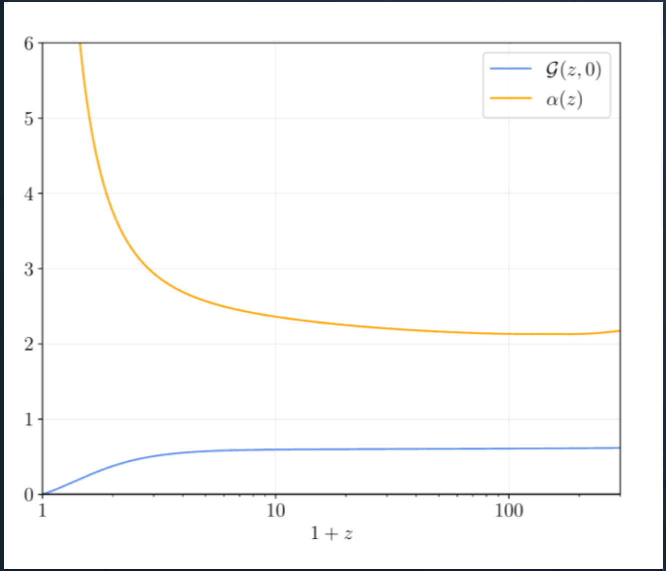

If $\delta H(z) < 0$

$$\frac{\delta G}{G} < \alpha(z) \frac{\delta H}{H} < 0$$

$$\alpha(x_z) \equiv -\mathcal{R}_{\sigma_8}(x_z,0)/\mathcal{G}_{\sigma_8}(x_z,0)$$

RESULTS

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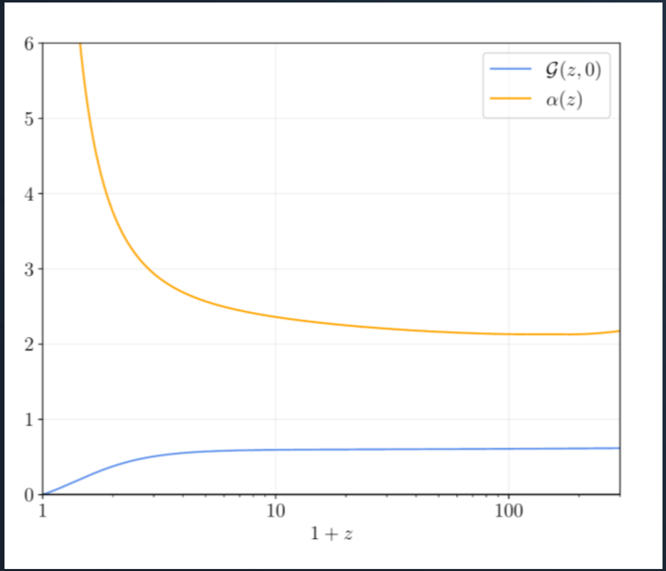

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But usually clustering of DE results in $G_{eff} > G$!

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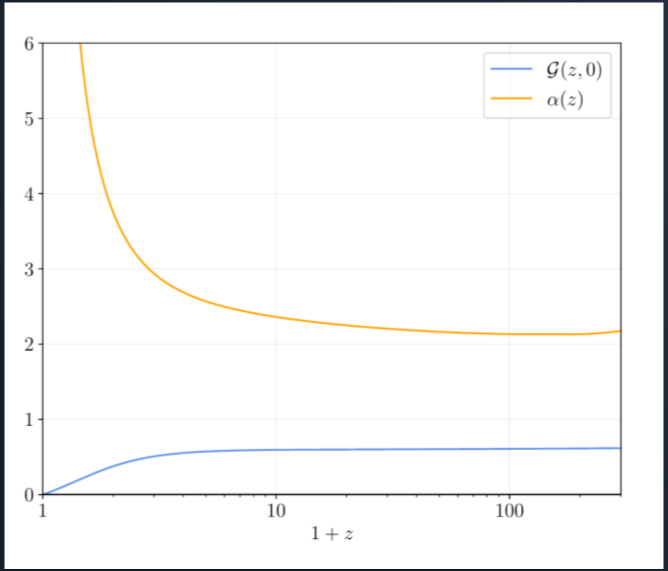

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But usually clustering of DE results in $G_{eff} > G$!

Conclusion: Solving both tensions is not easy

⇨ **Hints towards ΛCDM extension**

SUMMARY AND OUTLOOK

- Analytic, model independent method, based on single observational constraint

i) Solving the H_0 tension $\Rightarrow$ $\delta H(z) < 0$ for some z $\Rightarrow$ $w(z) < -1$ for some z.

ii) If the perturbations are not modified ($G_{\rm eff} = G$) then:
Solving the H_0 and σ_8 tensions $\Rightarrow$ $\delta H(z)$ changes sign $\Rightarrow$ $w(z)$ crosses the phantom divide.

iii) If $G_{\rm eff} = G + \delta G(z)$ and $\delta H(z)$ does not change sign then:

$$\text{Solving the } H_0 \text{ and } \sigma_8 \text{ tensions} \Rightarrow \frac{\delta G(z)}{G} < \alpha(z)\frac{\delta H(z)}{H(z)} < 0 \quad \text{for some } z.$$

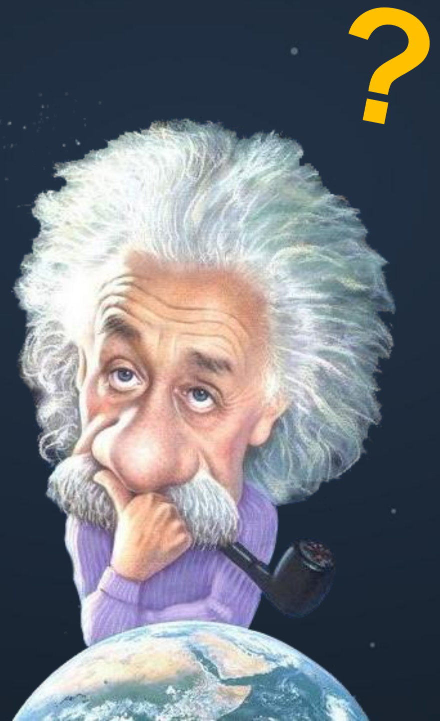
where $\delta H(z) < 0$ and $\alpha(z) > 0$.

- Combine with other constraints eg. GW speed , BAO …

  Example: luminal Horndeski/GenProca theoreis are possibly ruled out

- Use method for other observables: late ISW
- Early time solutions

THANK YOU FOR THE ATTENTION



BONUS

COSMOLOGICAL STANDARD MODEL - ΛCDM

Basis:

General relativity

Background homogeneity and isotropy

Particle physics

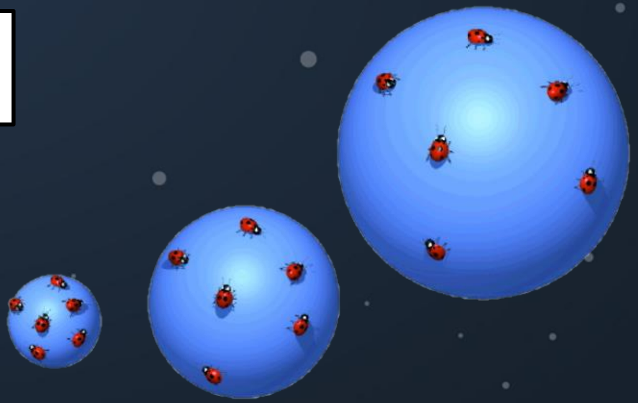
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\boldsymbol{x}^2$$

$$H = \frac{\dot{a}}{a}$$

$$a \equiv \frac{1}{1+z}$$

Comoving distance

$$d = \int_0^d \mathrm{d}r = c\int_{t_e}^{t_0} \frac{\mathrm{d}t}{a(t)} = c\int_0^z \frac{\mathrm{d}z}{H(z)}$$



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$$H = \frac{\dot{a}}{a}$$

Expansion rate today

$$H_0 \equiv 100\ h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Ingredients: Cosmological constant Λ

Cold dark matter (CDM)

Inflation

Parameters: $\left\{H_0, \Omega_m, \Omega_b, \tau_{\mathrm{reio}}, A_s, n_s\right\}$

only 6 free parameters

$$H^2_{\Lambda\mathrm{CDM}} = H_0^2\left(\Omega_m(1+z)^3 + \Omega_\Lambda\right)$$

Late time: 2 parameters $\Omega_m + \Omega_\Lambda = 1$

COSMOLOGICAL STANDARD MODEL - ΛCDM

Fundamental observations

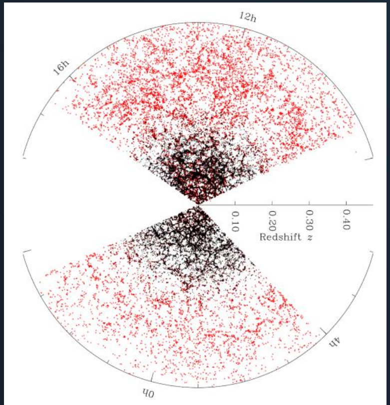
Direct/local

- Distance redshift relation $d = c\int \frac{\mathrm{d}z}{H}$ ⇨ $d(z) = \frac{z}{H_0} + \mathcal{O}(z^2)$

- Galaxy surveys / tracers $\sigma_8^2 \equiv \langle \delta_R^2(\boldsymbol{x}) \rangle \,, \quad R = 8\ \mathrm{Mpc}/h$

  large scale structure (LSS)

Redshift space distortions (RSD) and RS Galaxy Clustering (GC)



[SDSS Collaboration]

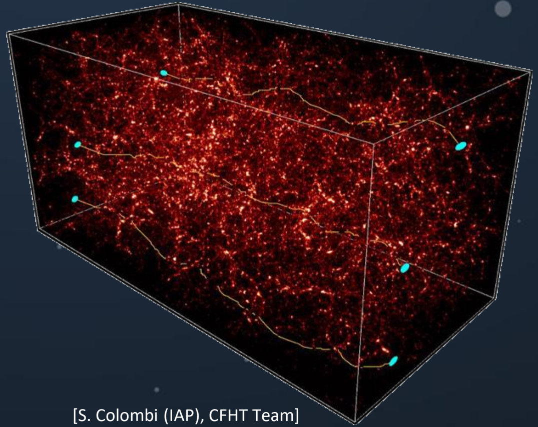
measure

$f\sigma_8$

growth rate

$f = \frac{d\log D}{d\log a}$

Weak lensing (WL)



[S. Colombi (IAP), CFHT Team]

measure

$S_8 \equiv \sigma_8\sqrt{\frac{\Omega_m}{0.3}}$

cosmic shear

galaxy shape distortion correlations

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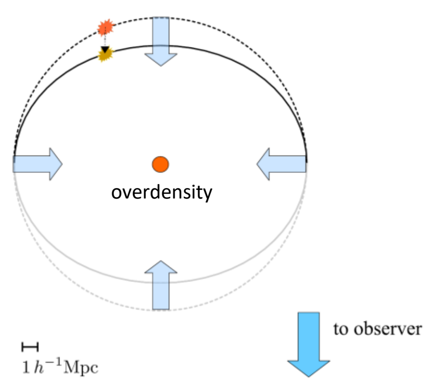

[Modern Cosmology, S. Dodelson & F. Schmidt (2021)]

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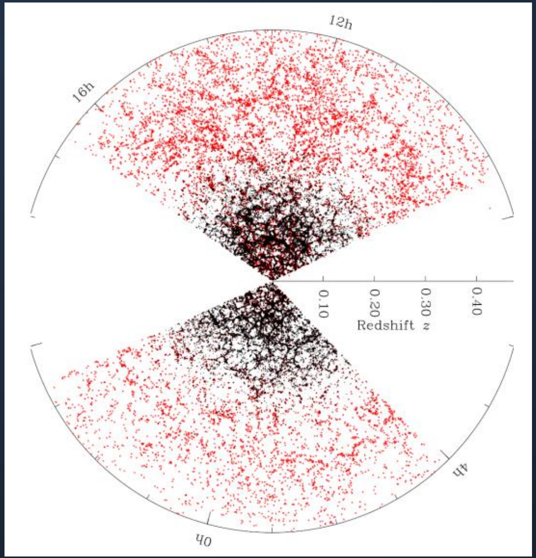

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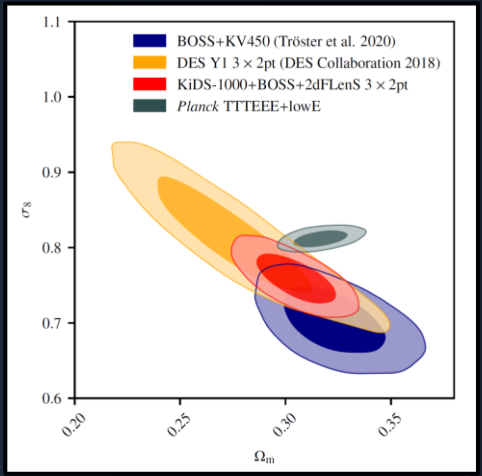

[KiDS-1000 arXiv:2007.15632]

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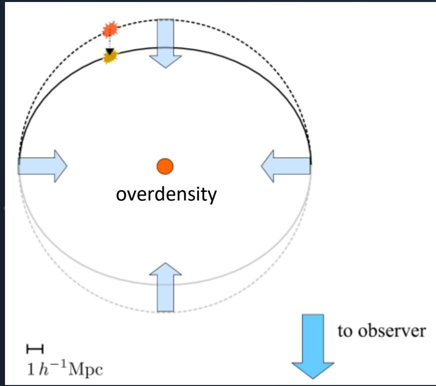

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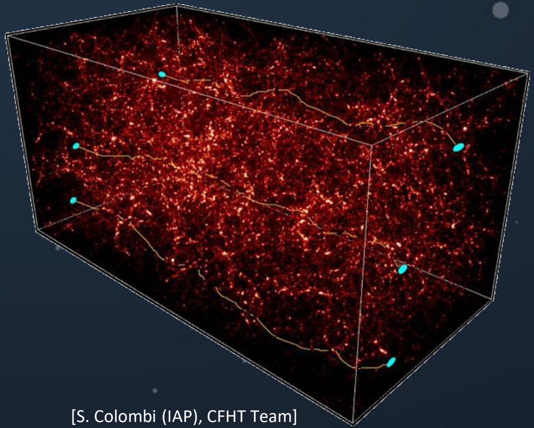
$$\boxed{f = \frac{d \log D}{d \log a}}$$

Weak lensing (WL)



[S. Colombi (IAP), CFHT Team]

measure

$$\boxed{S_8 \equiv \sigma_8 \sqrt{\frac{\Omega_m}{0.3}}}$$

cosmic shear

galaxy shape distortion correlations

COSMOLOGICAL STANDARD MODEL - ΛCDM

Fundamental observations

Direct/local

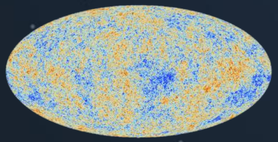
- Distance redshift relation $d = c\int \frac{\mathrm{d}z}{H}$ ⇨ $d(z) = \frac{z}{H_0} + \mathcal{O}(z^2)$

- Galaxy surveys $\sigma_8^2 \equiv \langle \delta_R^2(\boldsymbol{x}) \rangle \;, \quad R = 8\ \mathrm{Mpc}/h$

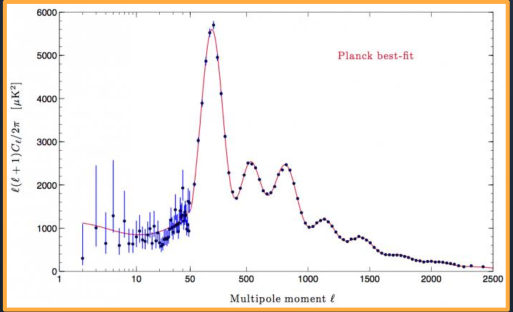
  large scale structure (LSS)

Indirect/distant

- CMB



⇨



[Planck]

⇦ $\left\{ H_0, \Omega_m, \Omega_b, \tau_{\mathrm{reio}}, A_s, n_s \right\}$

⇨ $\sigma_8^2 \equiv \int \frac{\mathrm{d}k}{k} \mathcal{P}_m(k)\, W(kR)$

CMB distance priors

angular scale $\theta_* = \frac{r_s(z_*)}{d(z_*)}$

shift parameter $R_* = d(z_*)\sqrt{\Omega_m H_0^2}$

METHOD

angular scale

Choose a single observable and impose a vanishing variation $\boxed{\Delta\theta_* \stackrel{!}{=} 0}$

Sidenote

With $\boxed{\omega_m + \delta\omega_m}$ would need to impose second constraint $\boxed{\Delta R_* \stackrel{!}{=} 0}$

shift parameter $\boxed{R_* = d(z_*)\sqrt{\Omega_m H_0^2}}$

Controls the overall amplitude and relative heights of the peaks

METHOD

angular scale

Choose a single observable and impose a vanishing variation $\boxed{\Delta\theta_* \overset{!}{=} 0}$

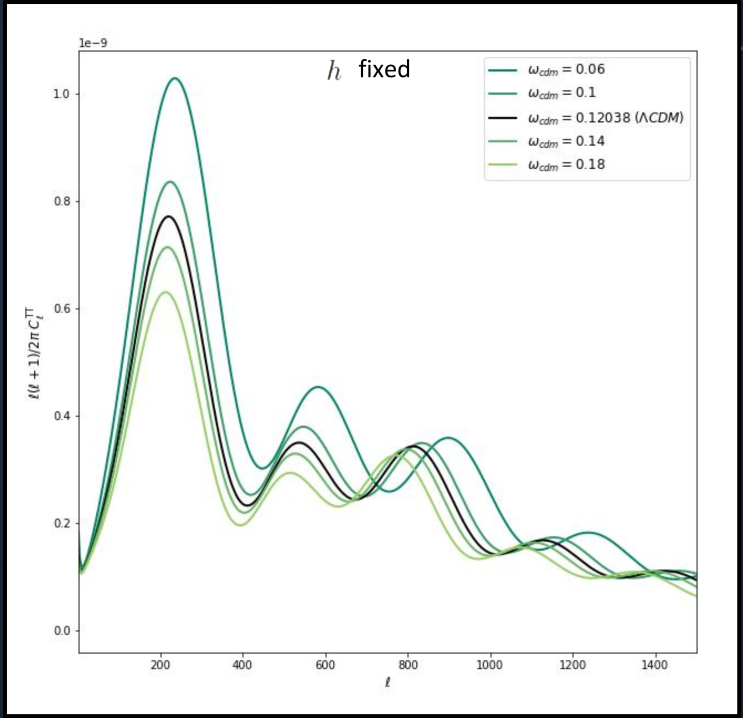
Sidenote

With $\boxed{\omega_m + \delta\omega_m}$ would need to impose second constraint $\boxed{\Delta R_* \overset{!}{=} 0}$

shift parameter $\boxed{R_* = d(z_*)\sqrt{\Omega_m H_0^2}}$

Controls the overall amplitude and relative heights of the peaks

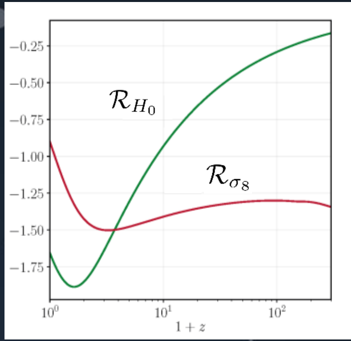
Effects similar to varying ω_m



EXAMPLE

$$\frac{\delta H_0}{H_0} = \int \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{H_0}(x_z) \frac{\delta H(x_z)}{H(x_z)}$$

$$\frac{\Delta\sigma_8}{\sigma_8} = \int_0^\infty \frac{\mathrm{d}x_z}{1+x_z} \mathcal{R}_{\sigma_8} \frac{\delta H}{H}$$



Dark energy fluid model

Equation of state $\frac{p}{\rho} = \omega(z)$

$$\delta H \propto \exp\left[3\int_0^z (1+\omega(z))\frac{dz}{1+z}\right] - 1 \Rightarrow \boxed{\delta H(z) < 0 \Rightarrow \omega(z) < -1}$$

At the background:

⇨ Increase H0

⇨ Inevitably increase σ8!

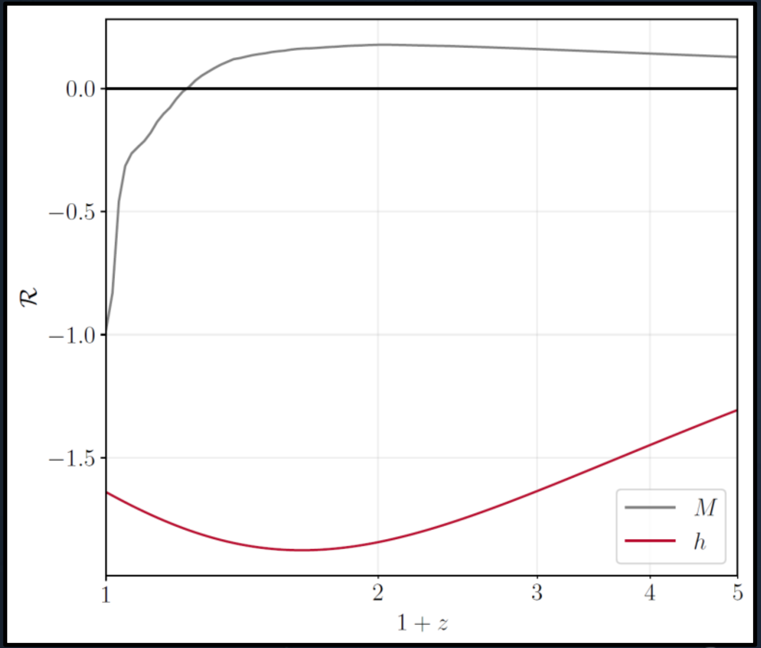
phantom dark energy ω<-1

need to cross the phantom divide

δH(z) needs to change sign

RESULTS

What about truly measured quantities?



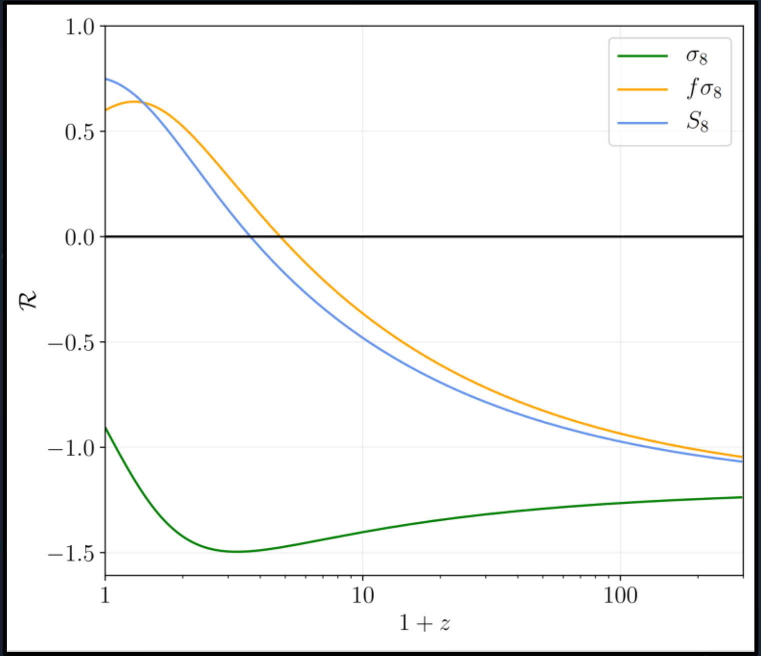
Supernova observations

Measure the absolute magnitude *M*
(used to calibrate the observed apparent magnitude)

at very low redshifts could *increase* H_0
while *decreasing* *M* !

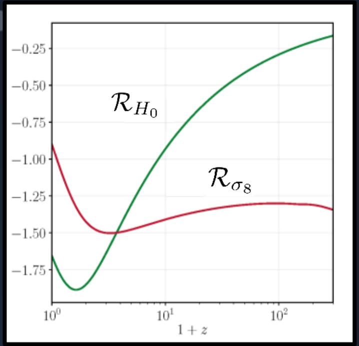
RESULTS

What about truly measured quantities?



RSD $f\sigma_8$

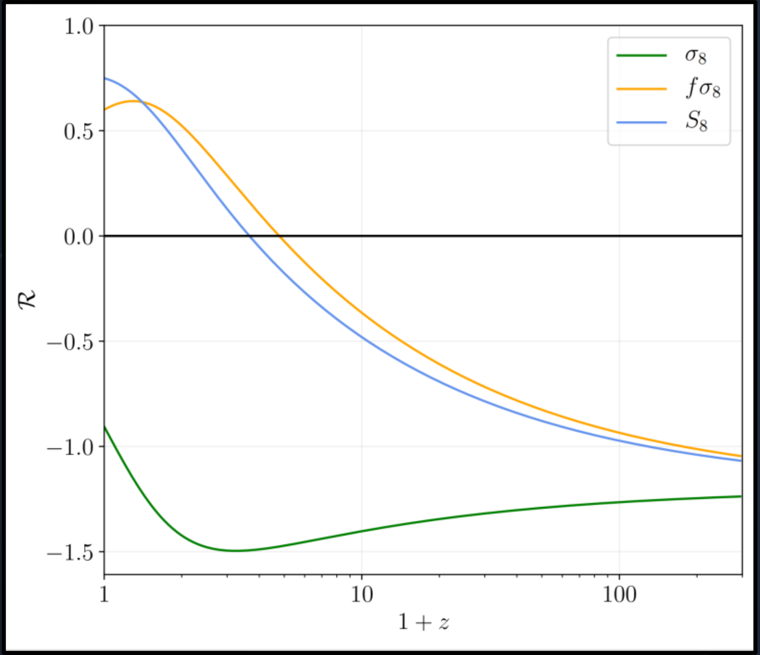
WL $S_8 \equiv \sigma_8 \sqrt{\frac{\Omega_m}{0.3}}$



δH(z) needs to change sign

RESULTS

What about truly measured quantities?

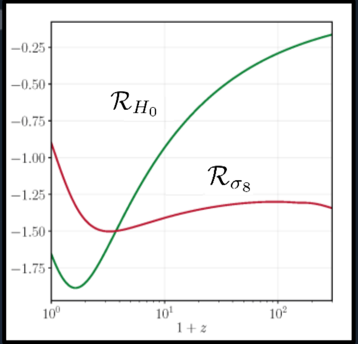


RSD $\boxed{f\sigma_8}$

WL $\boxed{S_8 \equiv \sigma_8\sqrt{\frac{\Omega_m}{0.3}}}$

at very low redshifts could *increase* σ_8 while *decreasing* $f\,\sigma_8$ and S_8

This could be interesting if in the future a discrepancy between σ_8 and measured quantities at very late times arises



$\delta H(z)$ needs to change sign

EXAMPLE: SCALAR-TENSOR THEORIES

$$\mathcal{L}_T^{(2)} = \sum_\lambda q_T \left(\dot{h}_\lambda^2 - c_T^2 (\partial_z h_\lambda)^2\right) \propto \sum_\lambda h_\lambda \mathcal{G}_{\mu\nu}\partial^\mu\partial^\nu h_\lambda \quad \Rightarrow \quad \mathcal{G}_{\mu\nu}k^\mu k^\nu = 0$$

Horndeski theory

$$\begin{aligned}
\mathcal{L}_2 &= G_2(\pi, X) \\
\mathcal{L}_3 &= -G_3(\pi, X)[\Pi] \\
\mathcal{L}_4 &= G_4(\pi, X)R + G_{4,X}\left([\Pi]^2 - [\Pi^2]\right) \\
\mathcal{L}_5 &= G_5(\pi, X)G_{\mu\nu}\Pi^{\mu\nu} - \frac{1}{6}G_{5,X}\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right)
\end{aligned}$$

$$\Pi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \pi$$

$$X \equiv -\frac{1}{2}(\partial\pi)^2$$

Choose lorentz breaking cosmological BG

$$\pi(t)$$

$$\begin{aligned}
q_T &= \frac{1}{4}\left[2(G_4 - 2XG_{4,X}) + 2XG_{5,\pi}\right] \\
c_T^2 &= \frac{2G_4 - 2XG_{5,\pi} - 2XG_{5,X}\ddot{\pi}}{4q_T}
\end{aligned}$$

$$G_4 \qquad G_5$$

$$\mathcal{G}_{\mu\nu} = C\,\eta_{\mu\nu} + D\,\Pi_\mu\Pi_\nu + E\,\Pi_{\mu\nu}$$

No robust cancellation between the two contributions!

⇨ Luminality $2G_{4,X} - 2G_{5,\pi} - \ddot{\pi}G_{5,X} = 0$ ⇨ $G_{4,X} = 0 \quad \text{and} \quad G_5 = \text{const.}$

EXAMPLE: SCALAR-TENSOR THEORIES

Luminal Horndeski theory

$$\mathcal{L}_2 = G_2(\pi, X)$$
$$\mathcal{L}_3 = -G_3(\pi, X)[\Pi]$$
$$\mathcal{L}_4 = G_4(\pi, \cancel{X})R + \cancel{G_{4,X}\left([\Pi]^2 - [\Pi^2]\right)}$$
$$\mathcal{L}_5 = \cancel{G_5(\pi, X)G_{\mu\nu}\Pi^{\mu\nu}} - \cancel{\tfrac{1}{6}G_{5,X}\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right)}$$

$$\boxed{G_{4,X} = 0 \qquad \text{and} \qquad G_5 = \text{const.}}$$

EXAMPLE: SCALAR-TENSOR THEORIES

Luminal Horndeski theory

$$\mathcal{L}_2 = G_2(\pi, X)$$
$$\mathcal{L}_3 = -G_3(\pi, X)[\Pi]$$
$$\mathcal{L}_4 = G_4(\pi)R$$

Implications

- Radiatively stable even in presence of Galileon breaking interaction
- Luminal, but scalar still mediates a propagating scalar GW breathing mode h_b (monopole and dipole radiation)
- In conflict with cosmological observations, e.g. $G_{eff} > G_N$ ⚡ H_0 and σ_8 tensions