Modified Gravity in Two-Body Systems

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1809.09844; 1910.08831; 2011.01213; 2107.10841, 2212.03098, 2305.19977

Dark Energy: from Fundamental Theories to Observations

Frascati September 2023

Outline

Introduction — Motivation, The Action and the approximations

Constructing the Effective Action — Brax &ACD 1809.09844; Brax, ACD & Kuntz 1903.03842

a diagrammatic approach

Beyond the Ladder Approximation — ACD & Melville 1910.08831

resuming the ladder and a new screening mechanism

Spinning Binaries — Brax, ACD, Melville, Wong 2011.01213, 2107.10841

EFT, Gravity Probe B

Pulsar Constraints — Benisty, Brax, ACD 2212.03098

Lens Thirring Effect — Benisty, Brax, ACD 2305.19977

Conclusions

Why Modified Gravity?

Original Motivation was that it could potentially 'explain' dark energy. Despite our best efforts this is still, at best, an open question.

Modified Gravity theories allow one to test General Relativity against one of the simplest extensions over a variety of ranges. This could either constrain the model parameters or give a hint of new physics and where to look for it in detail.

We consider scalar-tensor gravity with the most general coupling to matter, containing both conformal and disformal couplings. This extends the programme of Damour and collaborators. For example Damour & Deruelle Annales de l'IHP Physique Theorique 43 (1985) 107; Damour 2010.01641, Juli and Deruelle 1703.0536

The Action

$$S = \int d^4x\sqrt{-g_E}\left(\frac{R_E}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_m(\psi_i, g_{\mu\nu})$$

This is the action of scalar-tensor gravity. The scalar field is coupled to matter with the most general coupling containing a conformal and disformal coupling

$$g_{\mu\nu} = A^2(\phi)g^E_{\mu\nu} + B^2(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

Bekenstein

with $A(\phi) = e^{\beta\phi/m_{\rm Pl}}$ and $B(\phi, X) = \dfrac{1}{M^4}$

The Approximations

We work consistently to leading order in

$$G_N \qquad v^2 << 1$$

and in the conformal and disformal couplings

$$\beta \qquad 1/M^4$$

our work generalises that of Damour and collaborators

To proceed one uses Einstein and Klein-Gordon equations, which contains the conformal coupling to matter. The disformal coupling is added perturbatively

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N (T_{\mu\nu} + T^{\phi}_{\mu\nu})$$

$$T^{\phi}_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{(\partial\phi)^2}{2} g^E_{\mu\nu}.$$

$$\Box\phi = -\beta \frac{T}{m_{\rm Pl}} + \frac{1}{M^4} D_\mu \partial_\nu \phi T^{\mu\nu}$$

$$\phi = \phi^{(0)} + \delta\phi$$

$$\Box\phi^{(0)} = -\beta \frac{T}{m_{\rm Pl}}$$

$$\Box\delta\phi - \frac{1}{M^4} D_\mu \partial_\nu \delta\phi T^{\mu\nu} = \frac{1}{M^4} D_\mu \partial_\nu \phi^{(0)} T^{\mu\nu}$$

$$\delta\phi = \sum_{n \geq 0} \delta\phi^{(n)}$$

This gives the ladder approximation

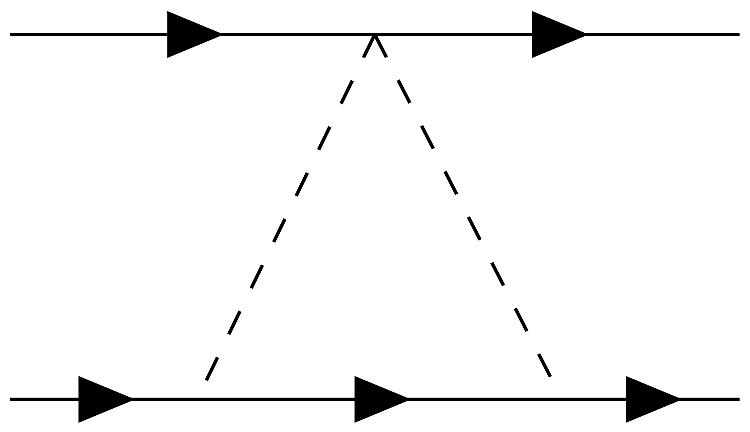
The Two Body System

For two bodies, A and B we have

$$\Box\phi^{(0)} = -\beta\frac{T^A + T^B}{m_{\mathrm{Pl}}}$$

$$\phi^{(0)} = \phi_A^{(0)} + \phi_B^{(0)}$$

diagrammatically



with the disformal vertex insertion.

this gives us the ladder approximation with each insertion bringing in the energy-momentum tensor and a suppression of

$$1/M^4$$

The Effective Action

In the centre of mass frame the effective Lagrangian is

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\mu\vec{v}^2 + \frac{G_N\mu\mathcal{M}(1+2\beta^2)}{x} + \frac{G_N\mu\mathcal{M}}{2x}((3-2\beta^2)\vec{v}^2 + 2(1+2\beta^2)\nu\vec{v}^2) + \frac{\beta^2 G_N(v_\perp^2 - (\vec{v}.\vec{n})^2)}{4\pi}\frac{\mu\mathcal{M}^2}{x^4M^4}$$

where we have introduced the reduced and total mass

$$\mu = \frac{m_A m_B}{m_A + m_B} \qquad \mathcal{M} = m_A + m_B \qquad \nu = \frac{m_A m_B}{(m_A+m_B)^2}$$

this is actually equivalent to having the effective metric as

$$g_{00}^{\rm eff} = -(1 - \frac{2G_N\mathcal{M}(1+2\beta^2)}{x})$$

$$g_{ij}^{\rm eff} = (1 + \frac{2G_N\mathcal{M}(1-2\beta^2)}{x})\delta_{ij} + \frac{\beta^2 G_N}{2\pi}\frac{\mathcal{M}^2}{x^4M^4}(\delta_{ij} - 2n_in_j)$$

the conformal part is familiar and provides the exact, post Minkowski limit, ie leading order in G. here we include the disformal term as well.

Let us now consider the effect of a light body moving in the background of a heavier body. For example this could be a planet moving in orbit around the sun.

The effective metric is not the same as the metric followed by photons. The disformal coupling involves both perpendicular and parallel velocities whilst the metric for photons only involves parallel velocities. Thus the equivalence principle is violated between photons and matter. This could have an effect on the Shapiro time delay and the perihelion advance. In fact to this order the disformal coupling doesn't affect the Shapiro time delay, but does effect the perihelion advance, which is.

$$\Delta\theta = 2\pi \frac{G_N m_A}{p}\left((3-2\beta^2) + 5\frac{\beta^2 m_A}{2\pi M^4 p^3}\right) \qquad p = a(1-e^2)$$

so the perihelion advance of Mercury gives

$$M \gtrsim 10^{-4}\ \mathrm{MeV}$$

Resumming the Ladder Expansion

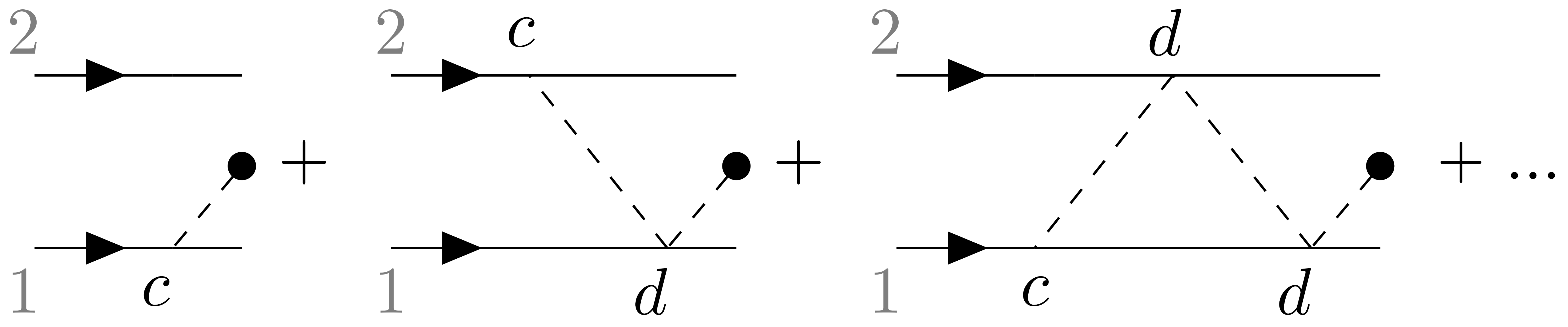
Melville and ACD - 1910.08831

So far we have treated the disformal coupling perturbatively

$$\Box\phi^0 = -\beta\frac{T}{m_{pl}}$$

$$\phi = \phi^0 + \delta\phi$$

which gives us $\phi^{(0)}, \phi^{(1)}, \phi^{(2)}$



for two bodies

The ladder approximation breaks down when corrections are O(1)

corrections from subsequent ladder diagrams become O(1) when

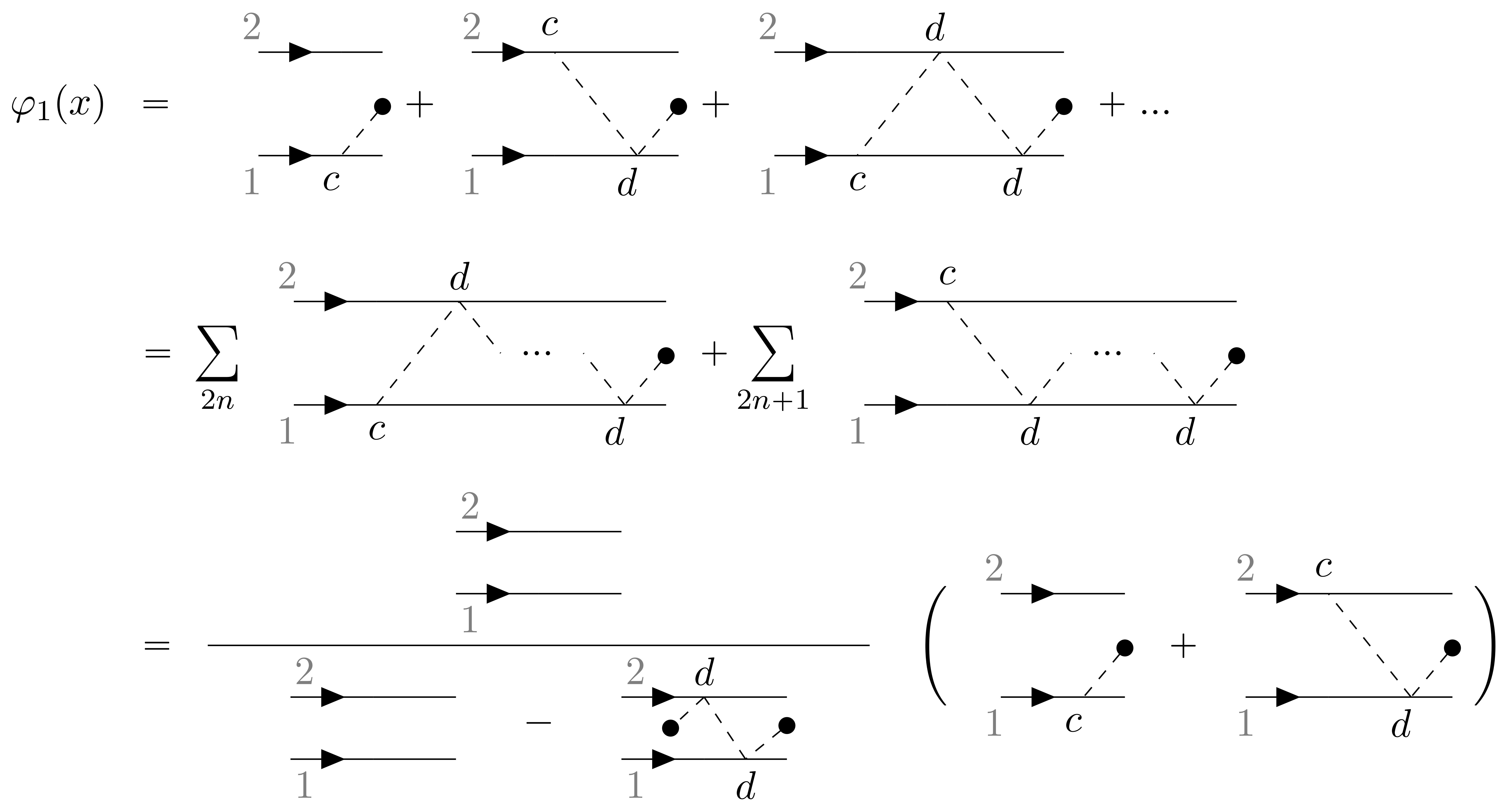
$$r^3 \sim \frac{m}{M^4} =: R_V^3$$

and we define the ladder parameter as

$$L \sim \frac{v^2 R_V^3}{r^3}$$

so the perturbative expansion is only valid for L <<1, ie low velocity and large distances. Otherwise we need to resum the ladder expansion

Note L is very important as it marks the change from the pertubative to resummed regimes. For L>>1 the disformal effects are summed and become screened



$\varphi_1(x)$ =
+ ...
$\sum_{2n}$
$\sum_{2n+1}$
c
d
1
2

In the ladder resummation regime we obtain the correction to the force

$$\delta F_{\varphi\,2}^{(c)r} = -\frac{\beta^2}{4da}\frac{R_{S_2}}{a}\frac{a^4}{R_{S_1}R_{V_1}^3}\left[\frac{c_1}{\rho}+\frac{1}{2}\rho+\frac{1}{3}\rho^2\right]$$

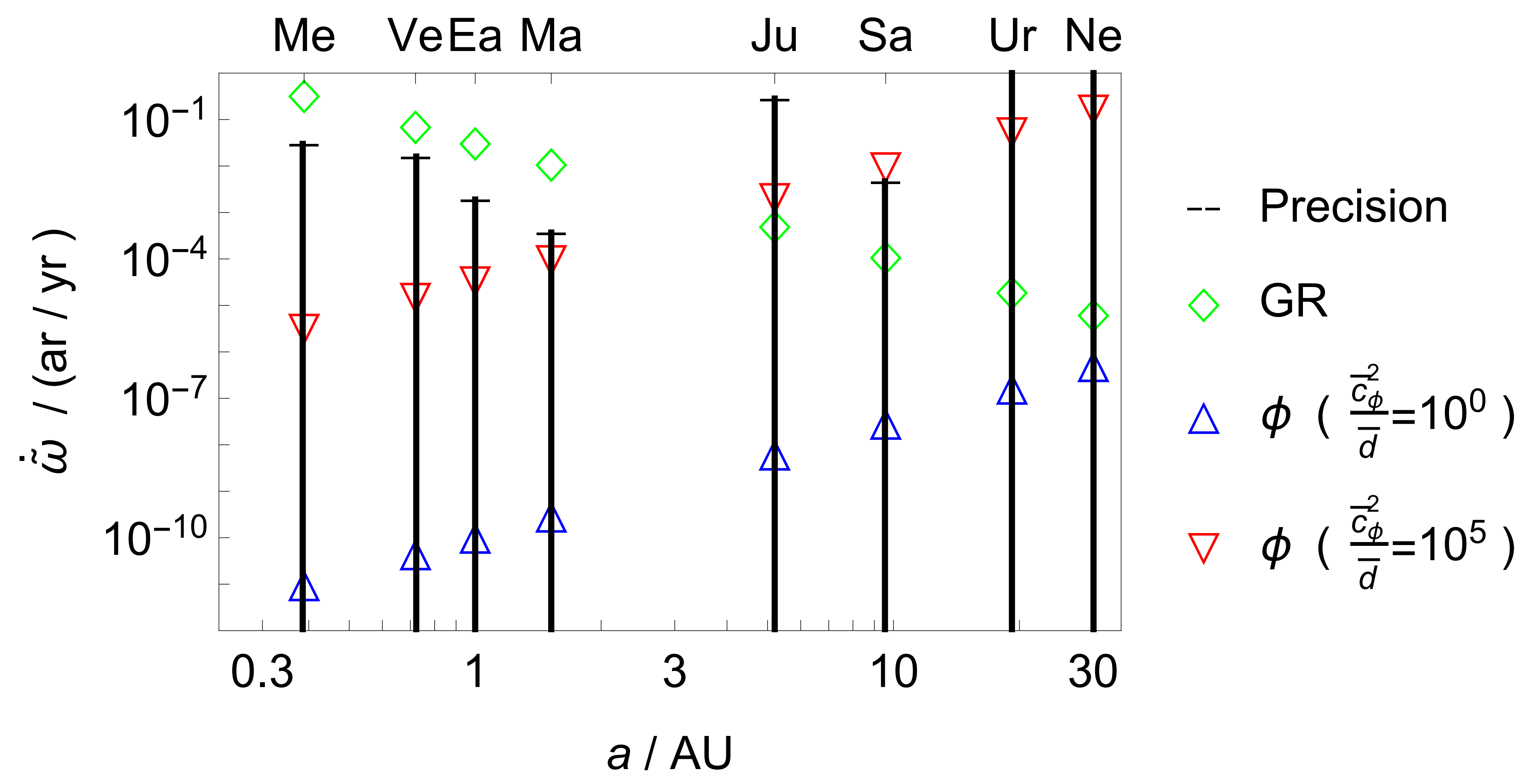
$$\delta F_{\varphi\,2}^{(d)r} = \frac{\beta^2}{48da}\frac{R_S}{a}\frac{a^4}{R_{S_1}R_{V_1}^3}\Big[\frac{36c_1^2}{\rho^6}-\frac{96c_1^2}{\rho^5}+\frac{12c_1\left(7c_1+5\right)}{\rho^4}-\frac{24c_1\left(c_1+5\right)}{\rho^3}+\frac{21-4c_1}{\rho^2} + \frac{4\left(26c_1-7\right)}{\rho}-5\left(8c_1+7\right)-\frac{32\rho^3}{3}+\frac{40\rho^2}{3}+\frac{118\rho}{3}$$

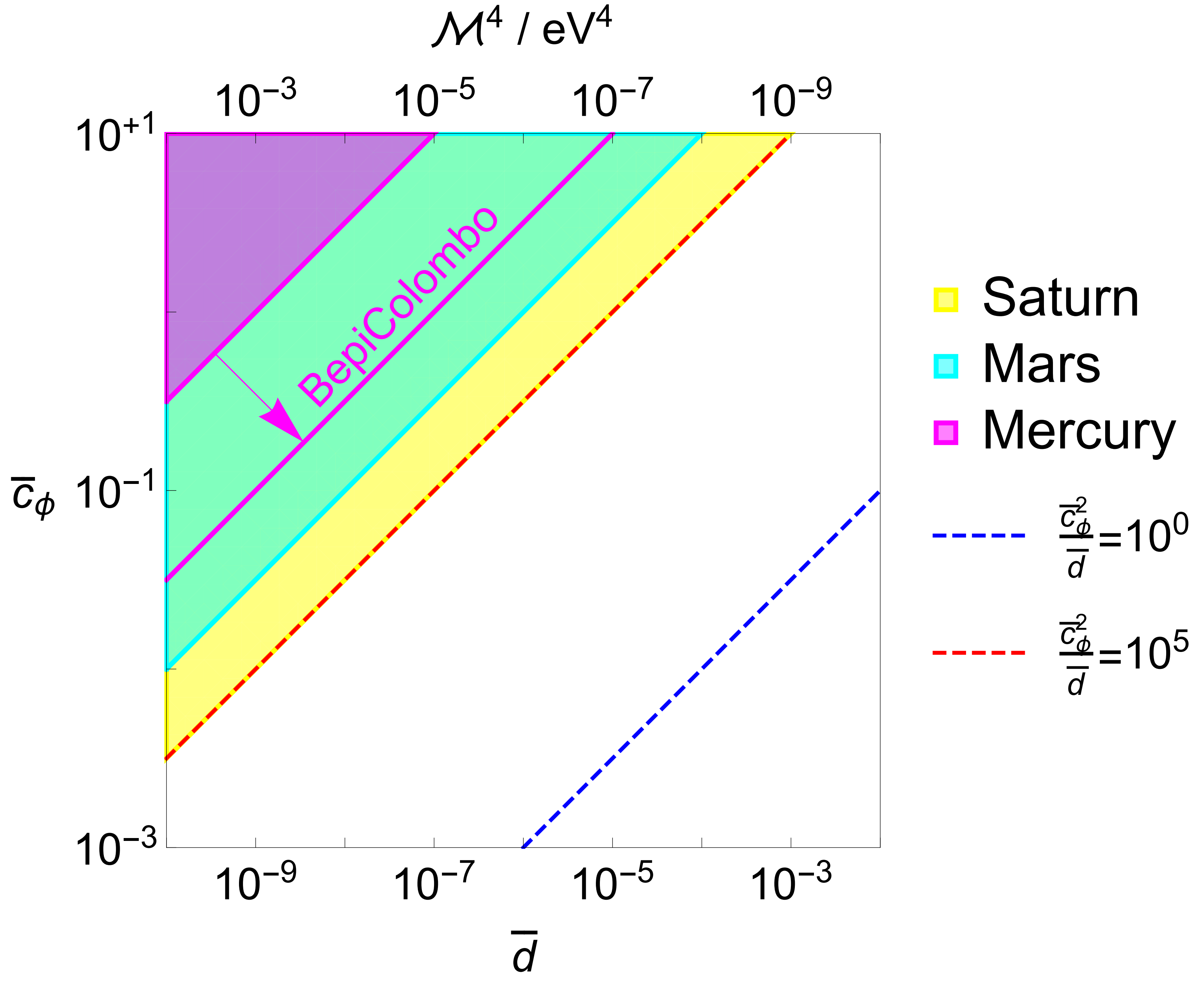
where $\rho = r(t)/a$ a the semi-major axis and e the eccentricity

The conformal and disformal forces are of the same order and suppressed like 1/L for L>>1

In this regime with L>>1 there is an efficient screening mechanism. This is for the two-body case and is distinct from Vainshtein screening

Precession of the planets due to ladder screening





Spin Effects

Jacobson, astro-ph/9905303, in a time dependent background a black hole can support scalar 'hair'. We, (Melville, Wong, Brax, ACD) 2107.10841, used the EFT approach of Goldberger and Rothstein, hep-th/0409156, to compute the effect of spinning bodies in scalar-tensor theories.

Model the inspiral phase by an EFT with point particles coupled to (g, ϕ)

$$S_{\text{eff}} = S_{\text{fields}}[g, \phi] + \sum_{\kappa=1}^{2} S_{\text{pp},\kappa}$$

As this regime is perturbative, we can write

$$S_{\text{fields}}[g, \phi] = \frac{1}{16\pi} \int \sqrt{-g}\, \mathrm{d}^4 x \left(R - 2(\partial\phi)^2 + (\text{interaction terms}) \right)$$

GR

Massless scalar

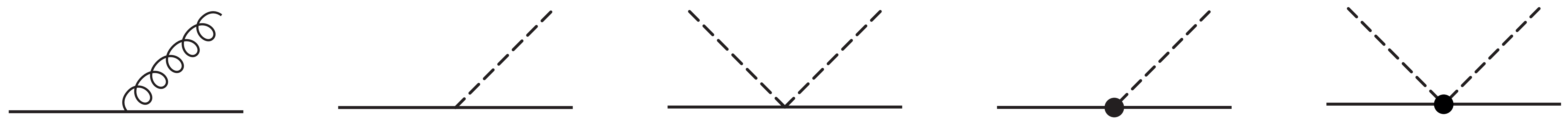
Not needed at leading PN order

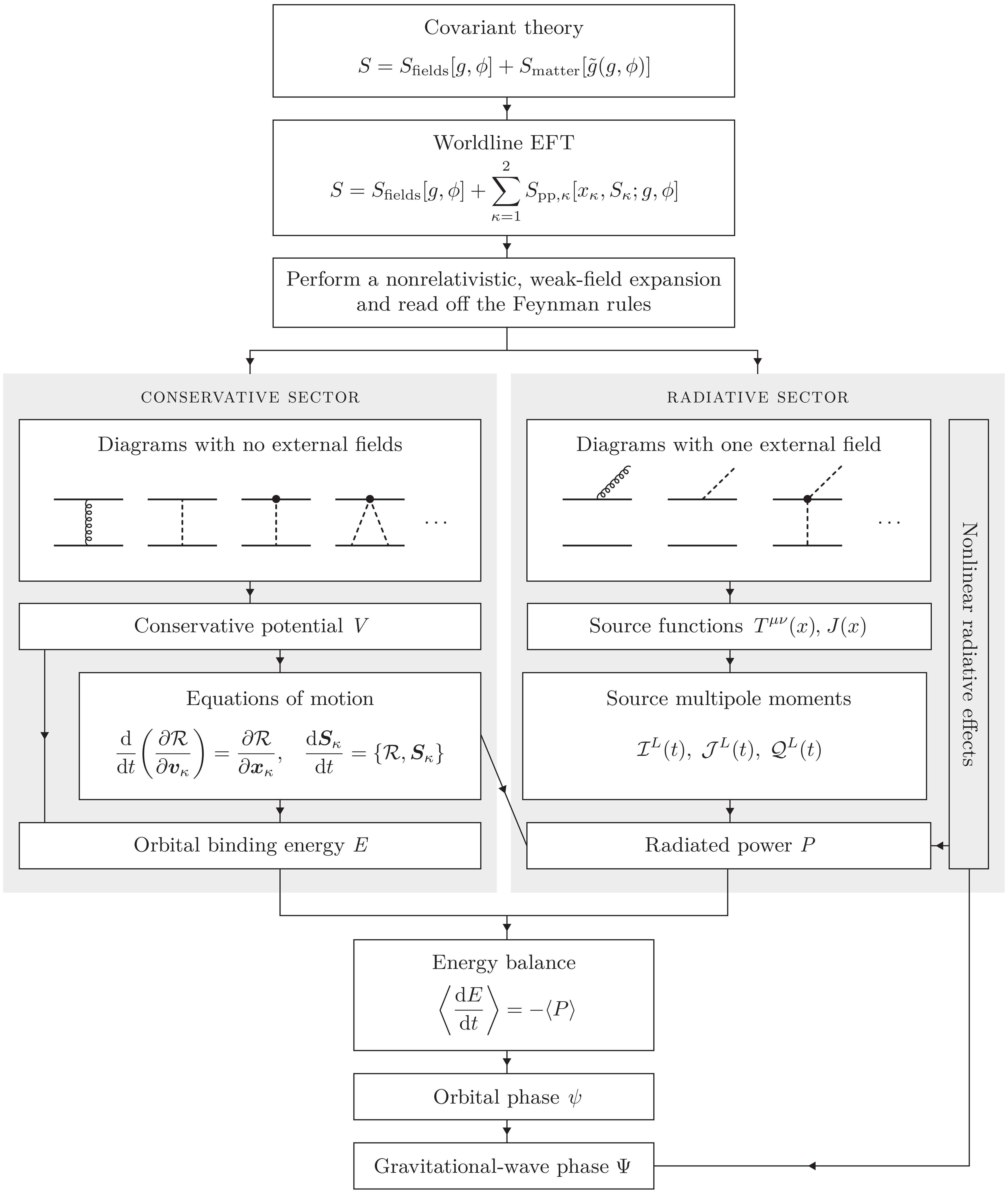
Effective theory for the binary

$$S_{\text{eff}} = S_{\text{fields}}[g, \phi] + \sum_{\kappa=1}^{2} S_{\text{pp},\kappa}$$

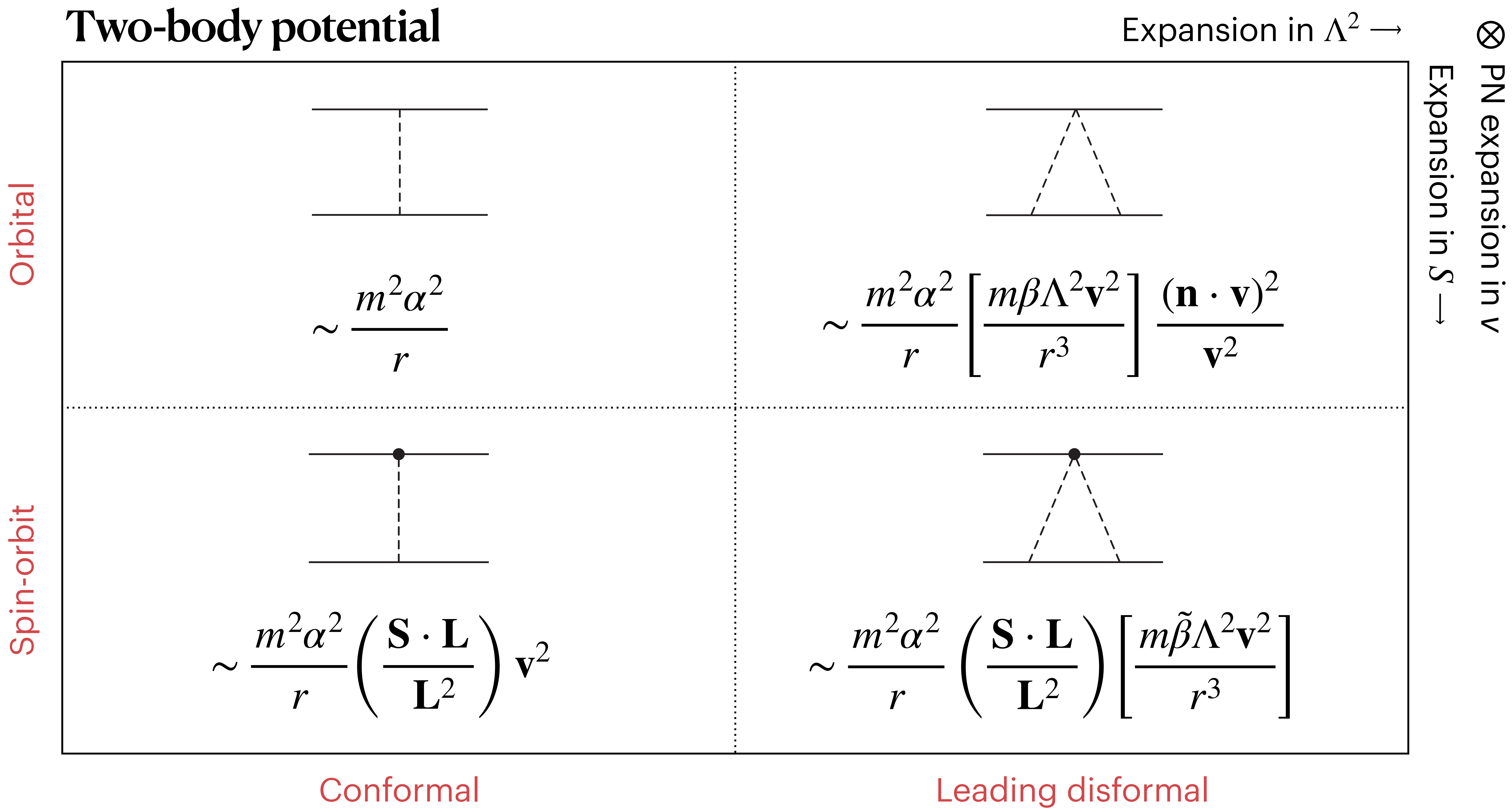
Nonrelativistic, weak-field expansion

Feynman rules

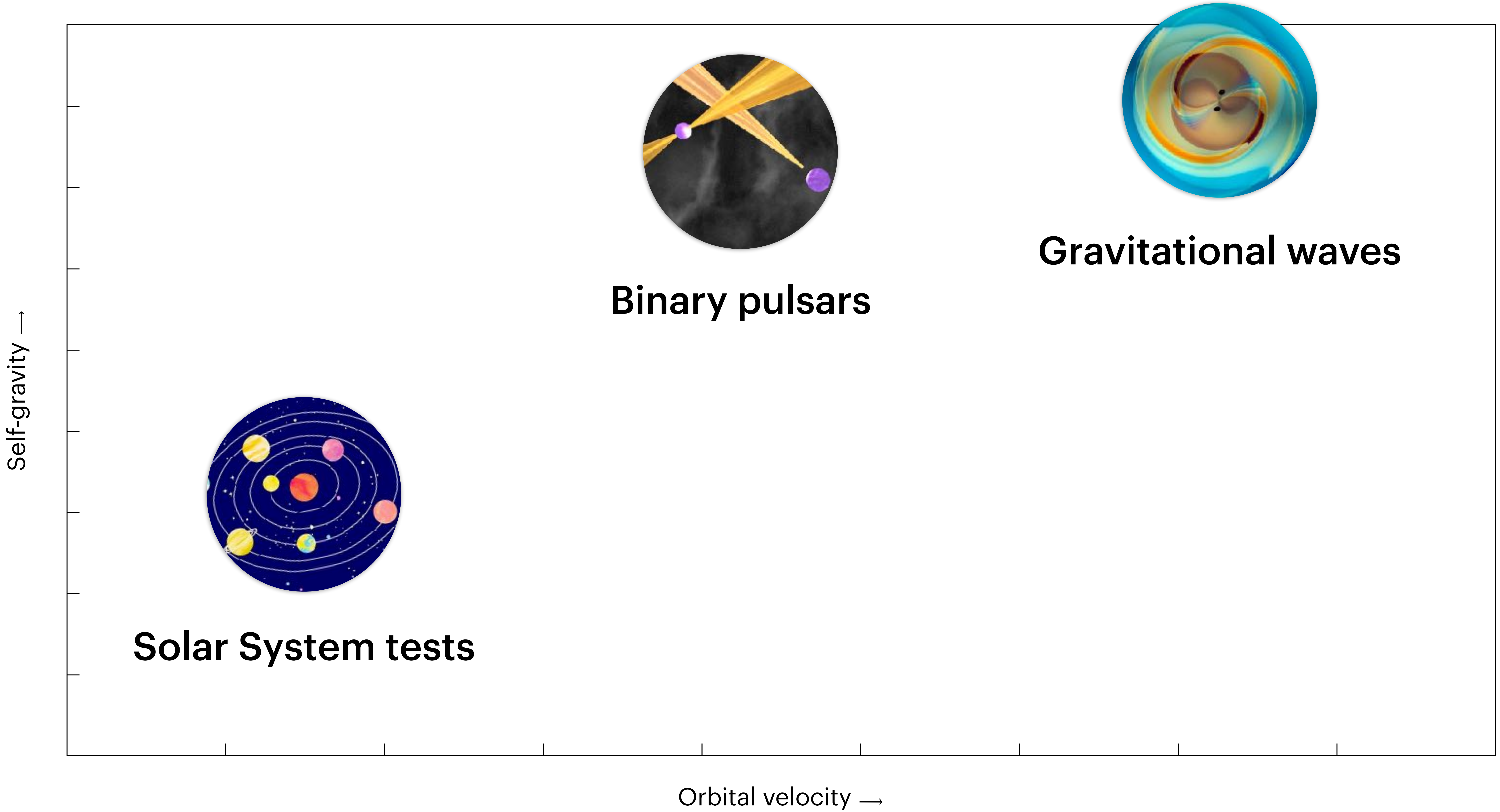


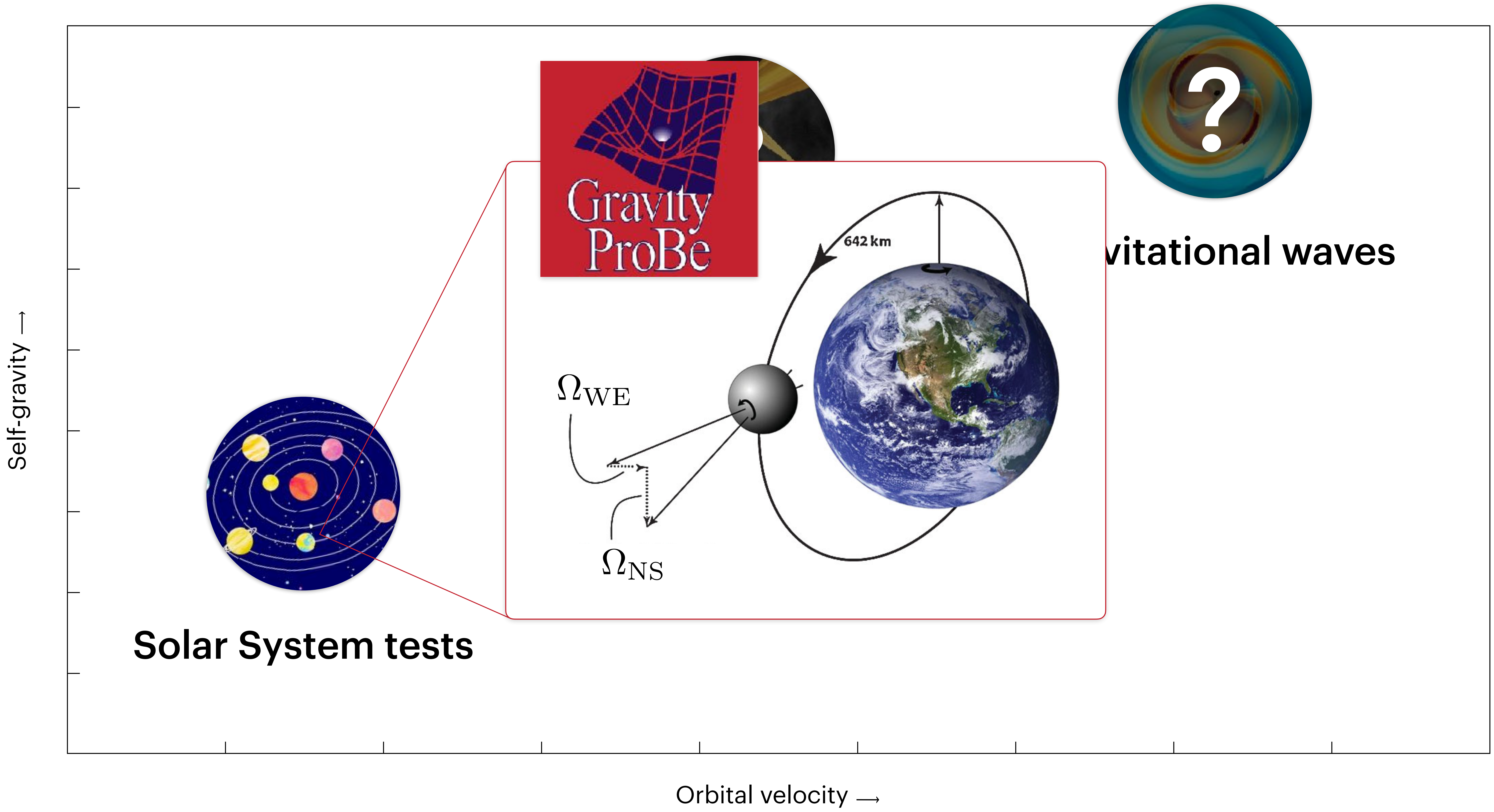


Covariant theory
$S = S_{\text{fields}}[g,\phi] + S_{\text{matter}}[\tilde{g}(g,\phi)]$
Worldline EFT
$S = S_{\text{fields}}[g,\phi] + \sum_{\kappa=1}^{2} S_{\text{pp},\kappa}[x_\kappa, S_\kappa; g, \phi]$
Perform a nonrelativistic, weak-field expansion and read off the Feynman rules
CONSERVATIVE SECTOR
Diagrams with no external fields
Conservative potential V
Equations of motion
$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{R}}{\partial \boldsymbol{v}_\kappa}\right) = \frac{\partial \mathcal{R}}{\partial \boldsymbol{x}_\kappa}, \quad \frac{\mathrm{d}\boldsymbol{S}_\kappa}{\mathrm{d}t} = \{\mathcal{R}, \boldsymbol{S}_\kappa\}$
Orbital binding energy E
RADIATIVE SECTOR
Diagrams with one external field
Source functions $T^{\mu\nu}(x)$, $J(x)$
Source multipole moments
$\mathcal{I}^L(t),\ \mathcal{J}^L(t),\ \mathcal{Q}^L(t)$
Radiated power P
Nonlinear radiative effects
Energy balance
$\left\langle \frac{\mathrm{d}E}{\mathrm{d}t} \right\rangle = -\langle P \rangle$
Orbital phase ψ
Gravitational-wave phase Ψ

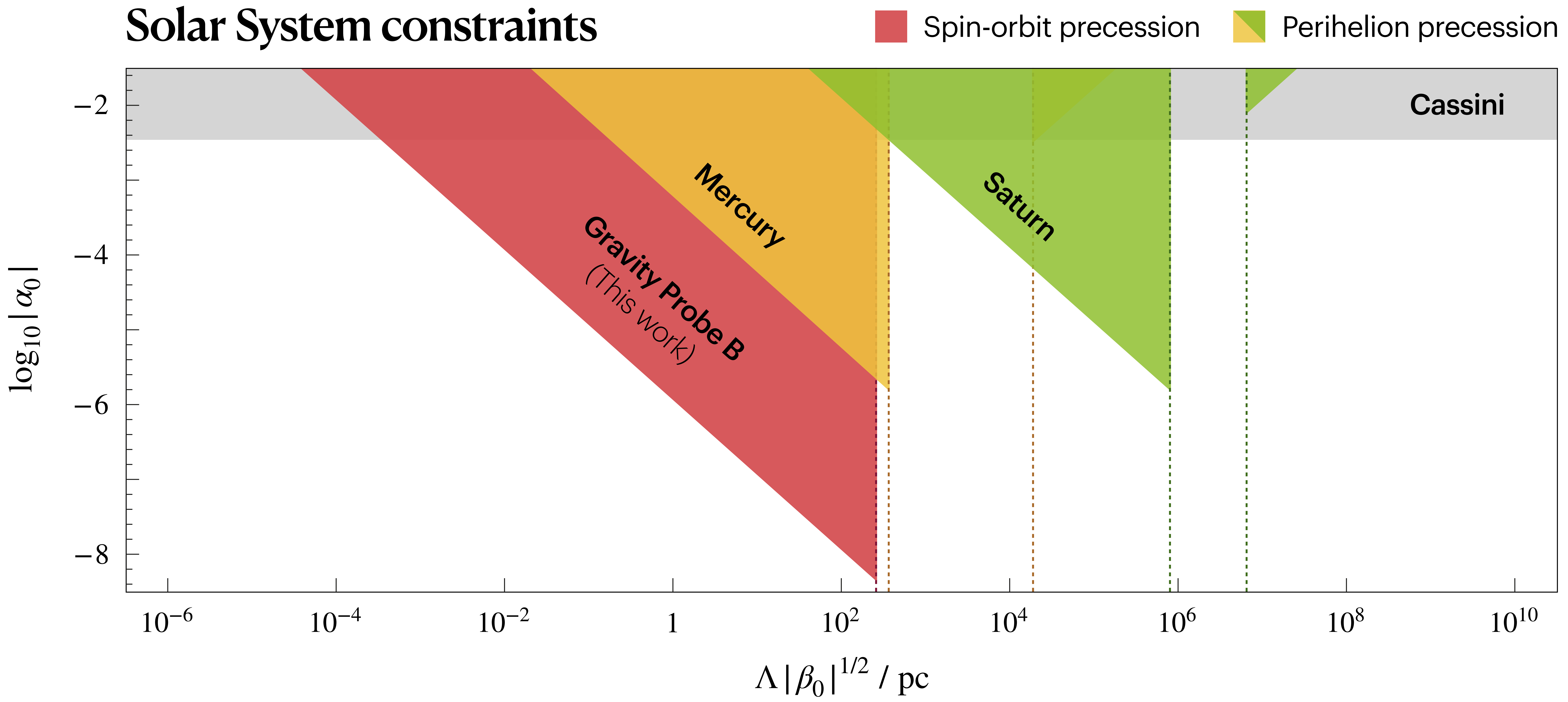

Two-body potential
Expansion in $\Lambda^2 \longrightarrow$
$\otimes$ PN expansion in v
Expansion in $S \longrightarrow$
Orbital
Spin-orbit
$\sim \dfrac{m^2\alpha^2}{r}$
$\sim \dfrac{m^2\alpha^2}{r}\left[\dfrac{m\beta\Lambda^2\mathbf{v}^2}{r^3}\right]\dfrac{(\mathbf{n}\cdot\mathbf{v})^2}{\mathbf{v}^2}$
$\sim \dfrac{m^2\alpha^2}{r}\left(\dfrac{\mathbf{S}\cdot\mathbf{L}}{\mathbf{L}^2}\right)\mathbf{v}^2$
$\sim \dfrac{m^2\alpha^2}{r}\left(\dfrac{\mathbf{S}\cdot\mathbf{L}}{\mathbf{L}^2}\right)\left[\dfrac{m\tilde{\beta}\Lambda^2\mathbf{v}^2}{r^3}\right]$
Conformal
Leading disformal

In nearly circular binaries, spin-orbit effects can be better at probing disformal interactions than purely orbital effects.


Gravitational waves
Binary pulsars
Solar System tests
Self-gravity ⟶
Orbital velocity ⟶



Gravity ProBe
642 km
Ω_{WE}
Ω_{NS}
vitational waves
Solar System tests
Self-gravity ⟶
Orbital velocity ⟶



Solar System constraints
Spin-orbit precession
Perihelion precession
Cassini
Mercury
Saturn
Gravity Probe B
(This work)
-2
-4
-6
-8
$\log_{10}|\alpha_0|$
10^{-6}
10^{-4}
10^{-2}
1
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}
$\Lambda\,|\beta_0|^{1/2}$ / pc

Two important pulsar events

- Hulse Taylor Pulsar PSR B1913+16.

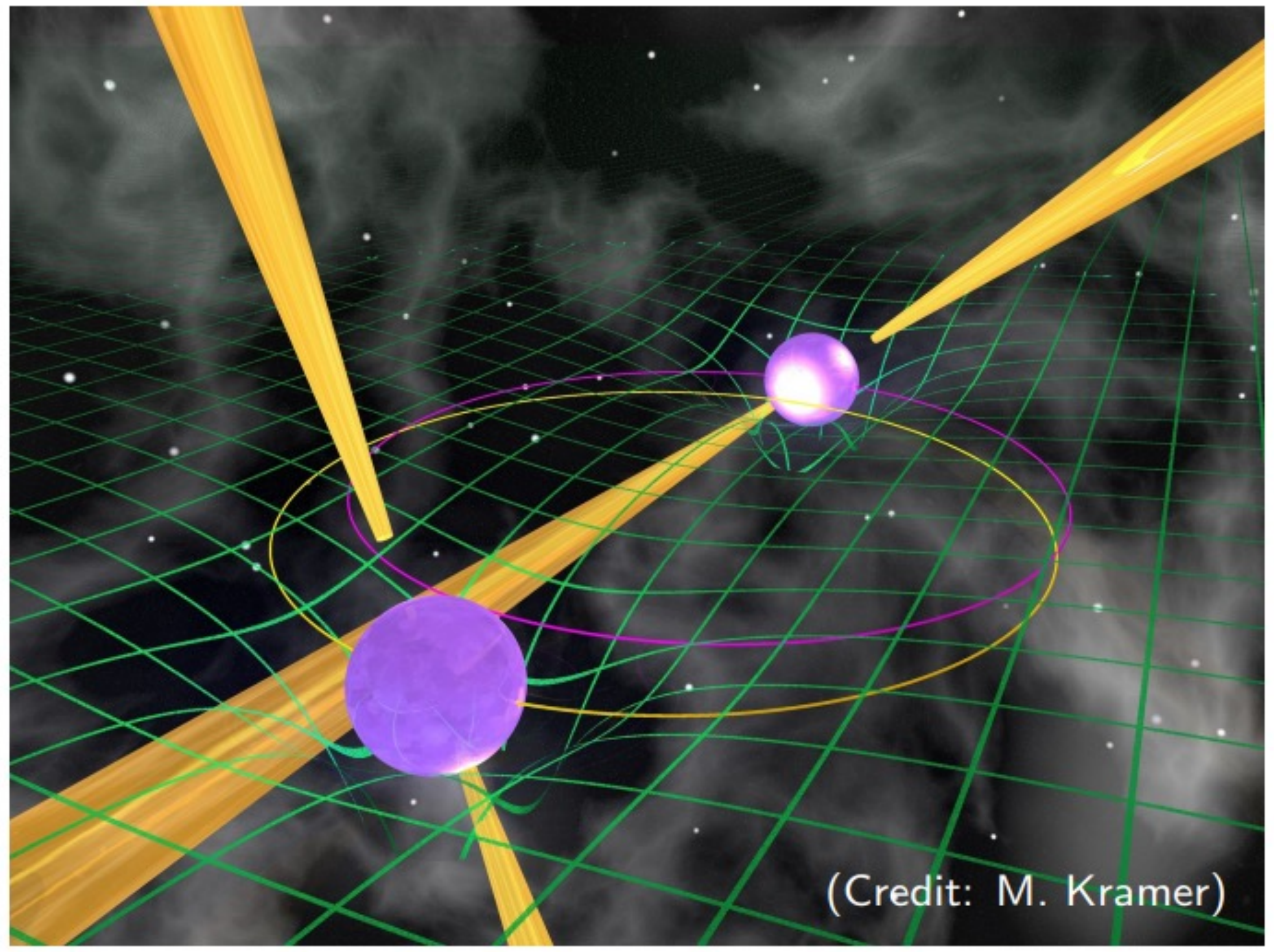
  Astrophysical Journal, 829:55 (10pp), 2016 September 20

T_0 (MJD)	52144.90097849(3)
$x \equiv a_1 \sin i$ (s)	2.341776(2)
e	0.6171340(4)
P_b (d)	0.322997448918(3)
ω_0 (deg)	292.54450(8)
$\langle\dot{\omega}\rangle$ (deg yr^{-1})	4.226585(4)
γ (ms)	0.004307(4)
$\dot{P}_b^{\mathrm{obs}}$	$-2.423(1) \times 10^{-12}$
$\delta_\theta^{\mathrm{obs}}$	$4.0(25) \times 10^{-6}$
$\dot{x}^{\mathrm{obs}}$	$-0.014(9) \times 10^{-12}$
$\dot{e}^{\mathrm{obs}}$ (s^{-1})	$0.0006(7) \times 10^{-12}$
Shapiro Gravitational Propagation Delay Parameters	
Damour & Deruelle (1986) Parametrization	
s	$0.68^{+0.10}_{-0.06}$
r (μs)	$9.6^{+2.7}_{-3.5}$

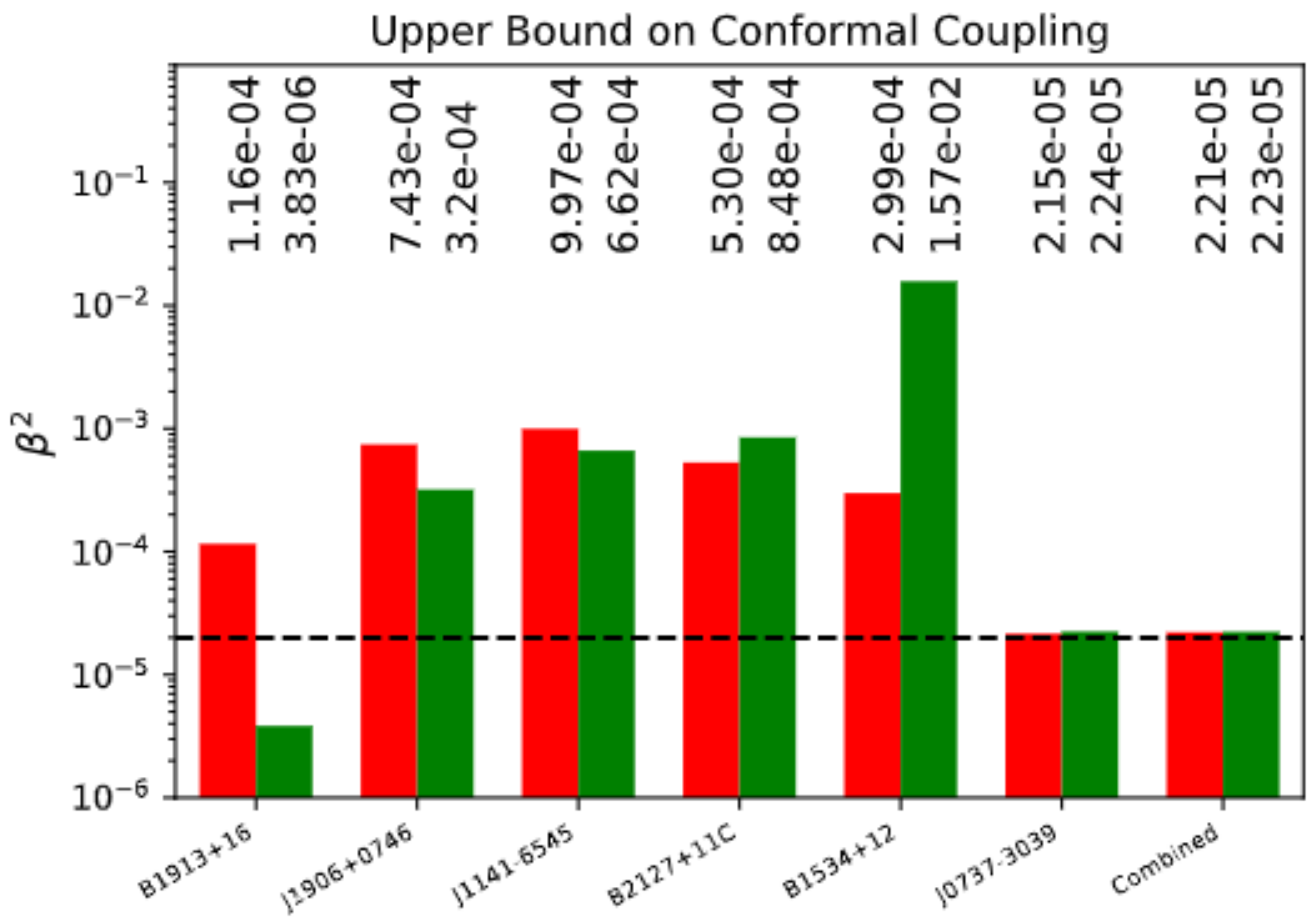
- The double pulsar PSR J0737-3039.

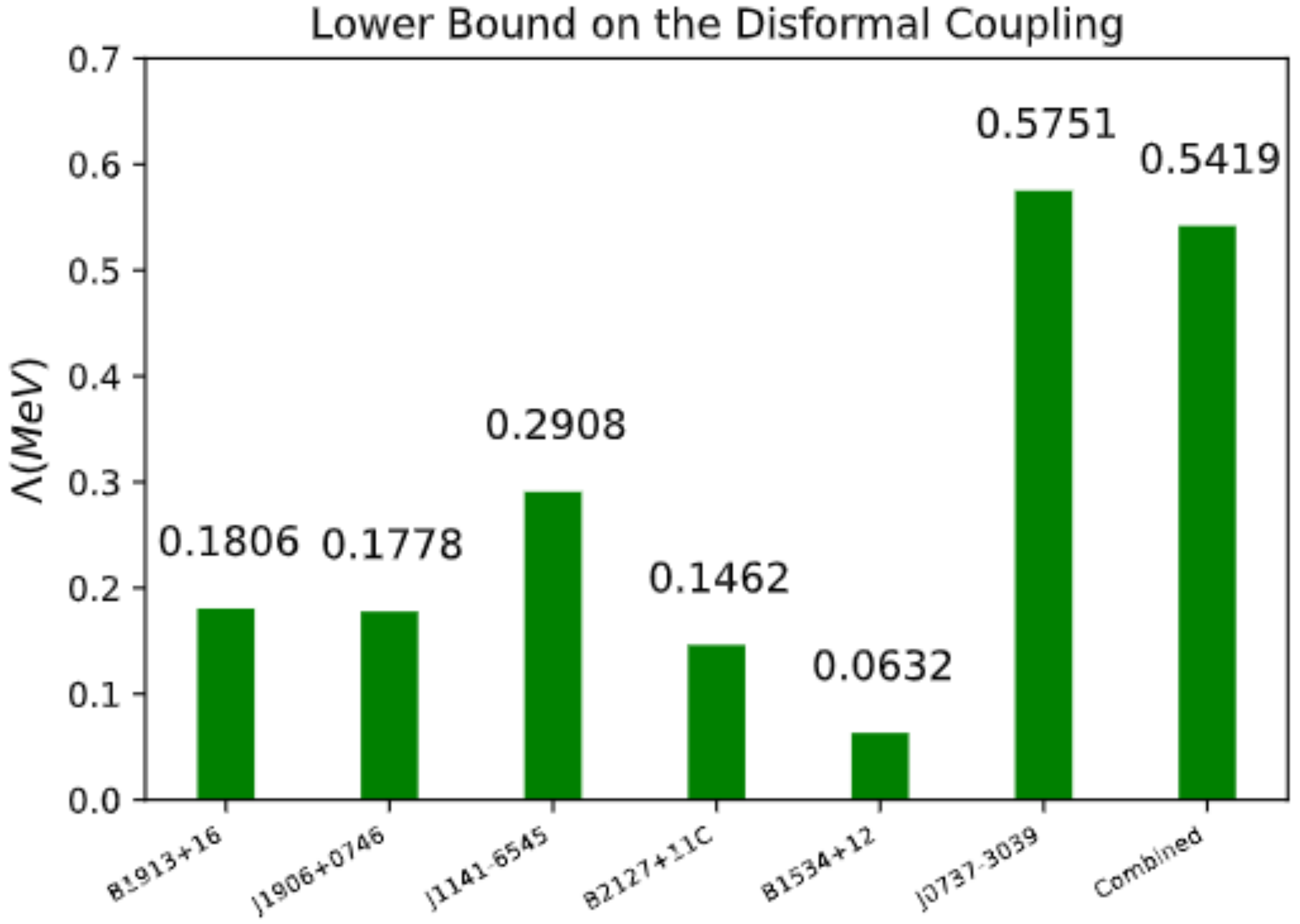
PHYSICAL REVIEW X 11, 041050 (2021)

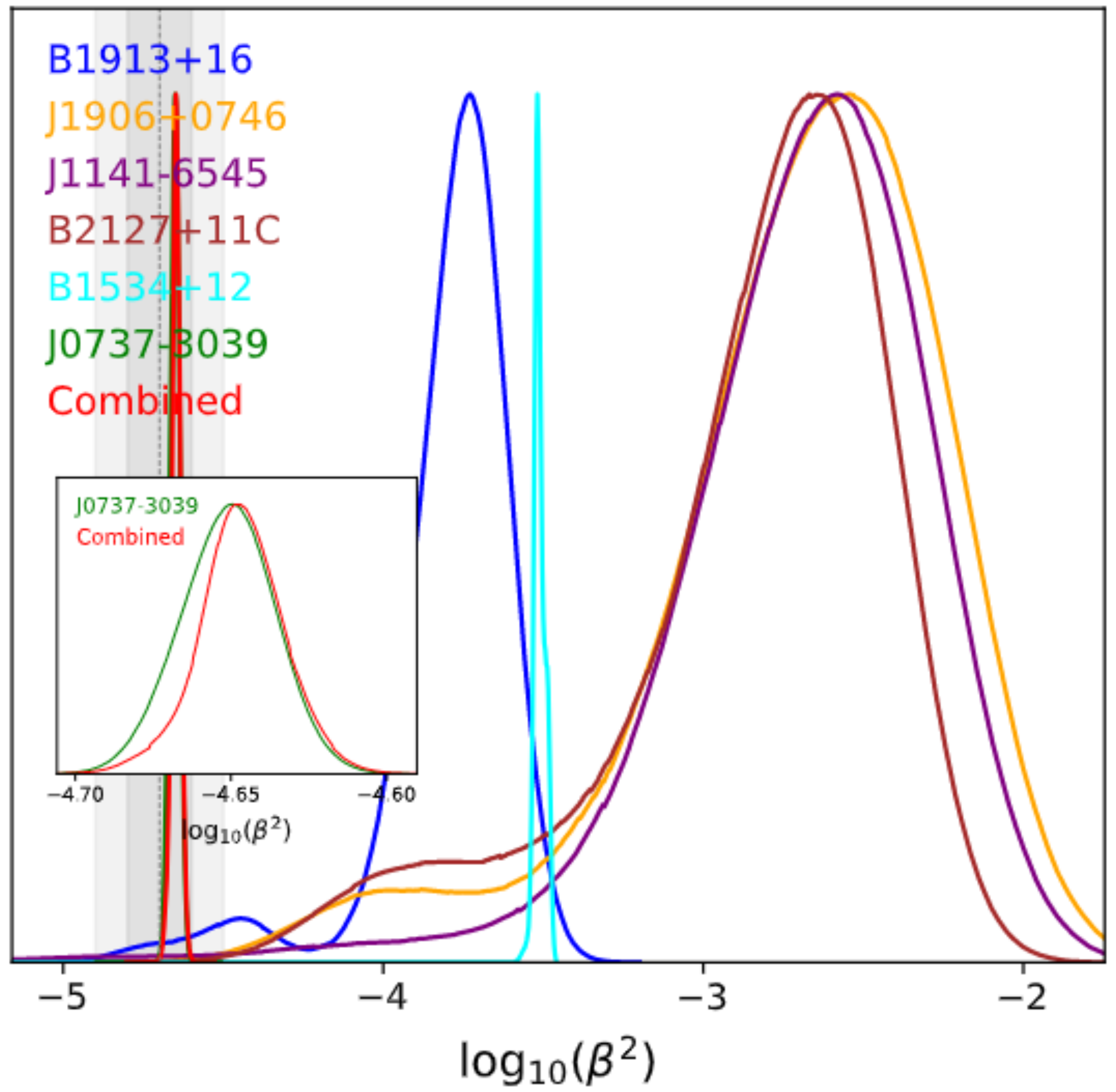
Orbital period, P_b (day)	0.102 251 559 297 3(10)
Projected semimajor axis, x (s)	1.415 028 603(92)
Eccentricity (Kepler equation), e_T	0.087 777 023(61)
Epoch of periastron, T_0 (MJD)	55 700.233 017 540(13)
Longitude of periastron, ω_0 (deg)	204.753 686(47)
Periastron advance, $\dot{\omega}$ (deg yr^{-1}) [c]	16.899 323(13)
Change of orbital period, $\dot{P}_b$	$-1.247920(78) \times 10^{-12}$
Einstein delay amplitude, γ_E (ms)	0.384 045(94)
Logarithmic Shapiro shape, z_s	9.65(15)
Range of Shapiro delay, r (μs)	6.162(21)
NLO factor for signal prop., q_{NLO}	1.15(13)
Relativistic deformation of orbit, δ_θ	$13(13) \times 10^{-6}$
Change of proj. semimajor axis, $\dot{x}$	$8(7) \times 10^{-16}$
Change of eccentricity, $\dot{e}_T$ (s^{-1})	$3(6) \times 10^{-16}$

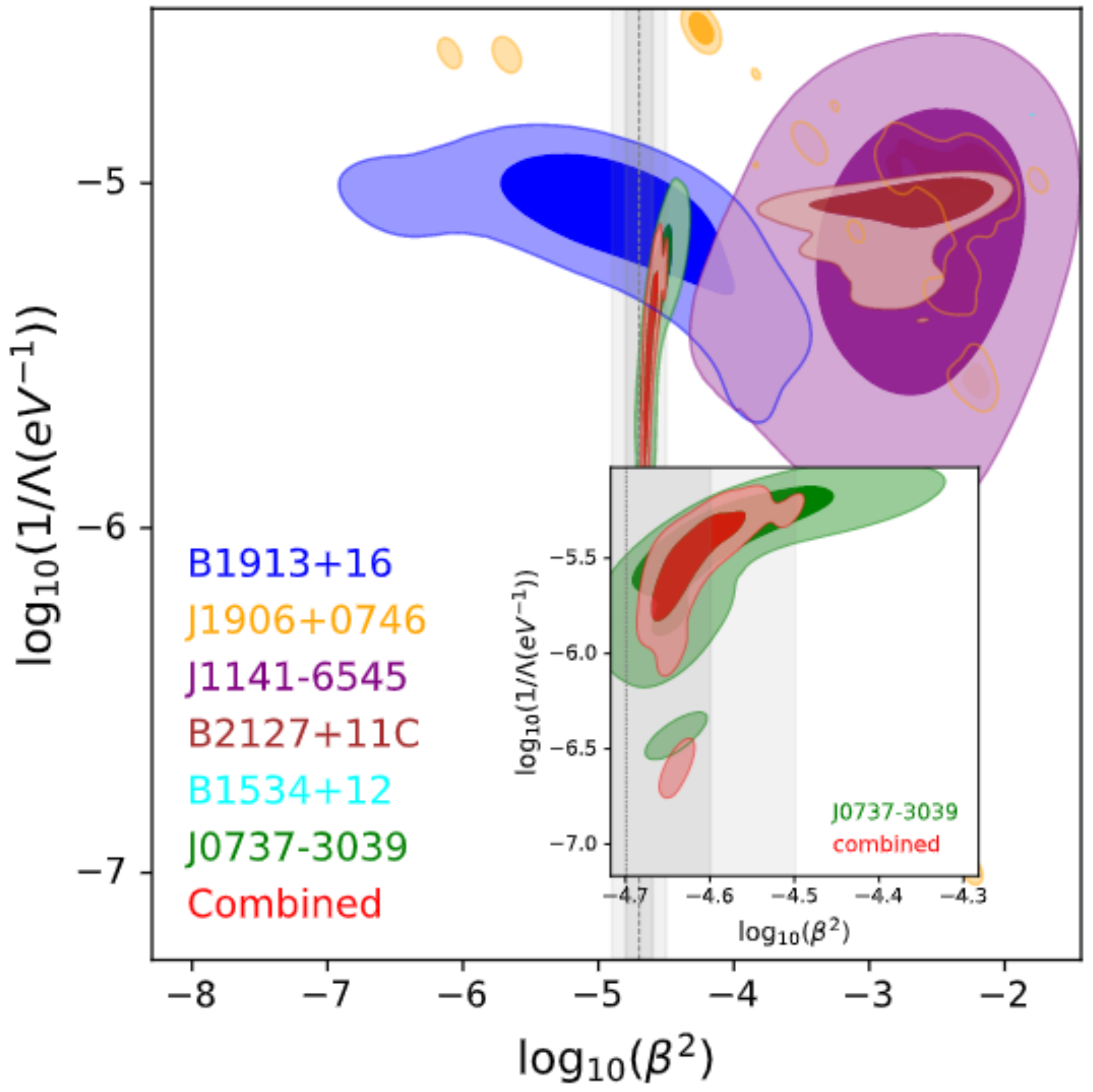


Dark Energy Interactions constraints









In general there are two effects to take into account for binaries - the geodesic effect and the Lens-Thirring effect

The geodesic effect arises from the curvature of space-time in general relativity whilst the Lens-Thirring Effect arises from the spin of the central body

$$\Omega_{dS} = \frac{3Gn_b}{2ac^2\left(1-e^2\right)}\frac{m_2\left(4m_1+3m_2\right)}{\left(m_1+m_2\right)^{4/3}},$$

where S is the spin, $n_b = 2\pi/P_b$ the orbital frequency

$$\Omega_{FD} = \frac{3GS}{2ac^2\left(1-e^2\right)^{3/2}},$$

To proceed take the spin evolution equation and from this the total precession, separate out the geodesic/de Sitter term and the Lens-Thirring/frame dragging term, including both the GR and modified gravity terms

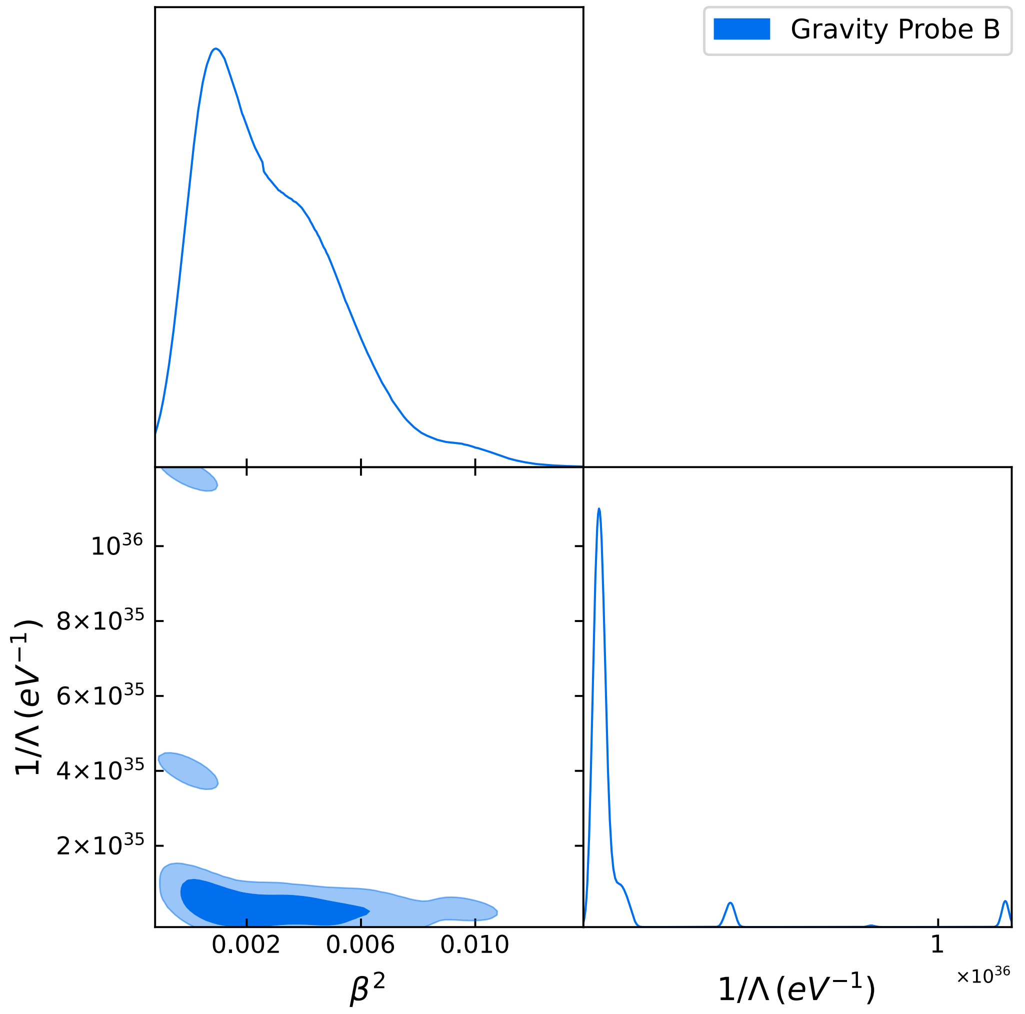
Defining $\lambda_{dS} := \Delta\Omega_{\rm dS}/\Omega_{\rm dS}^{(GR)}$

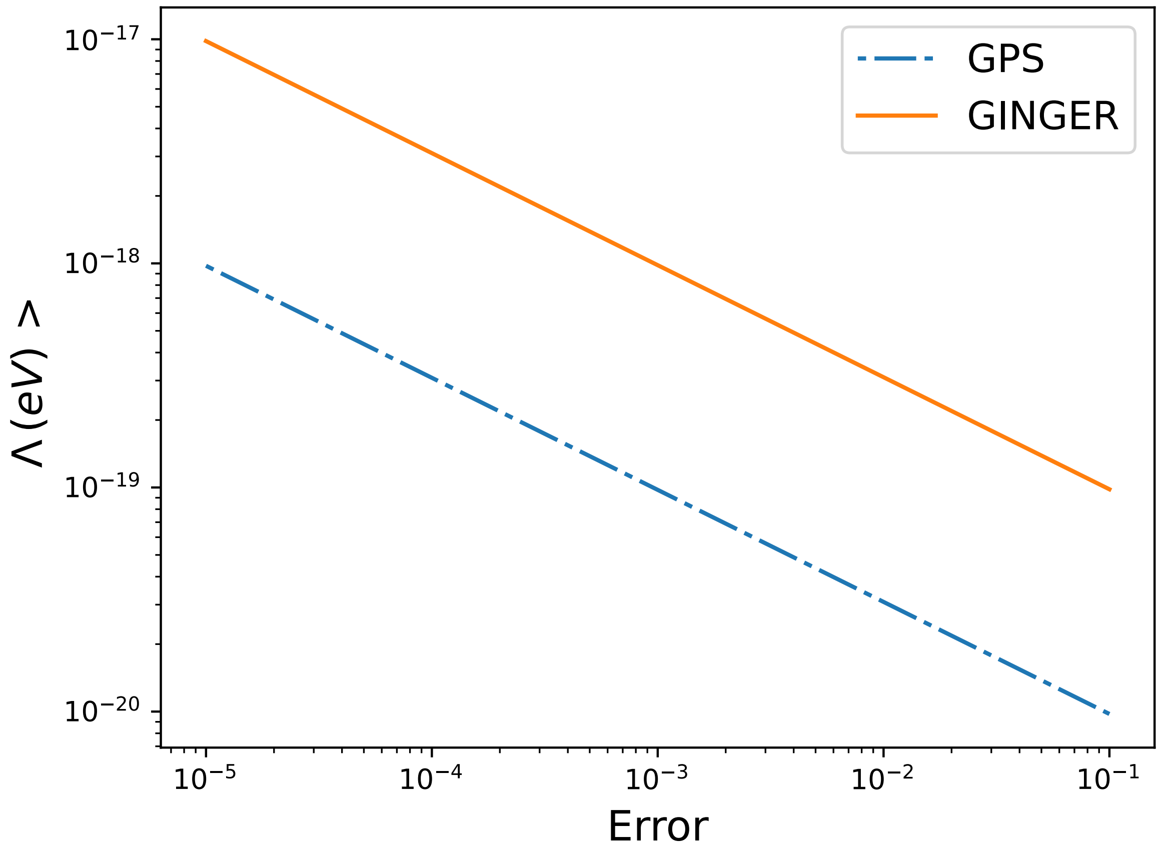
$$\lambda_{dS} = -\frac{2m_1}{3m_1 + 4m_2}\left[\beta^2 - \epsilon_\Lambda\left(1 + 3e^2 + \frac{3}{8}e^4\right)\right]$$

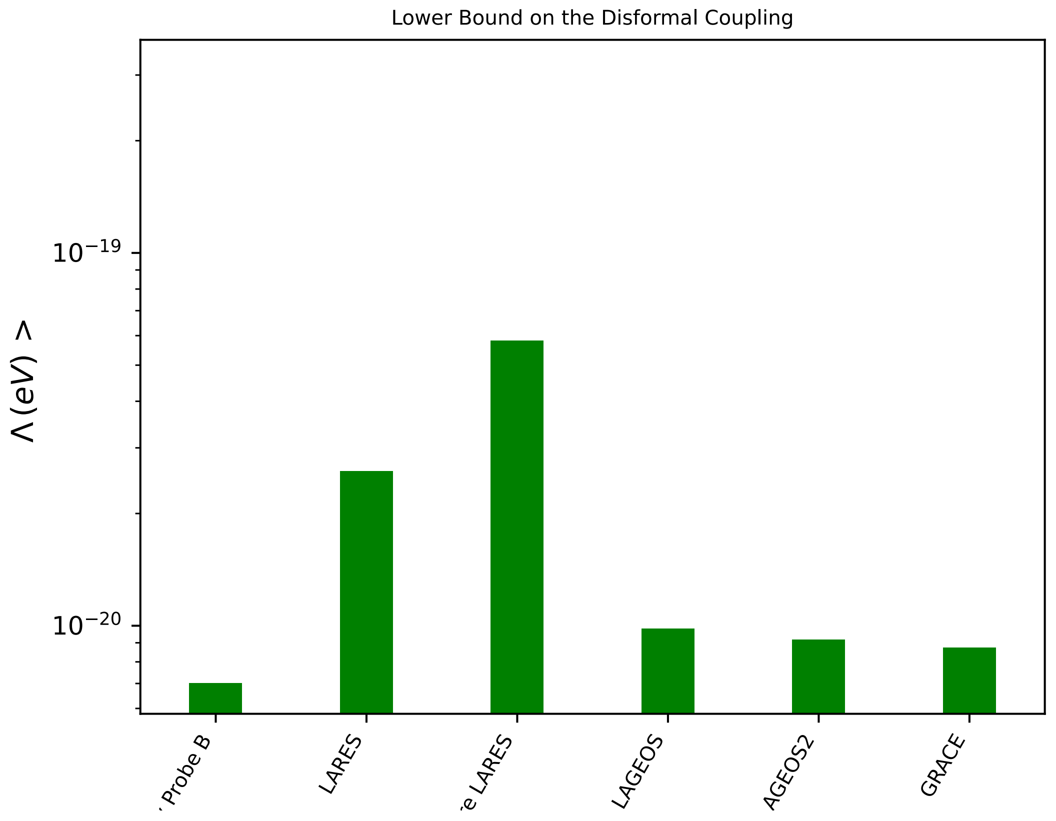
$$\lambda_{FD} = \epsilon_\Lambda \Phi_s \frac{1 + 8e^2 + \frac{51}{8}e^4 + \frac{3}{8}e^6}{1 - e^2}\sin\psi \qquad \epsilon_\Lambda = \frac{(\beta \cdot n_b/\Lambda)^2}{(1-e^2)^3}$$

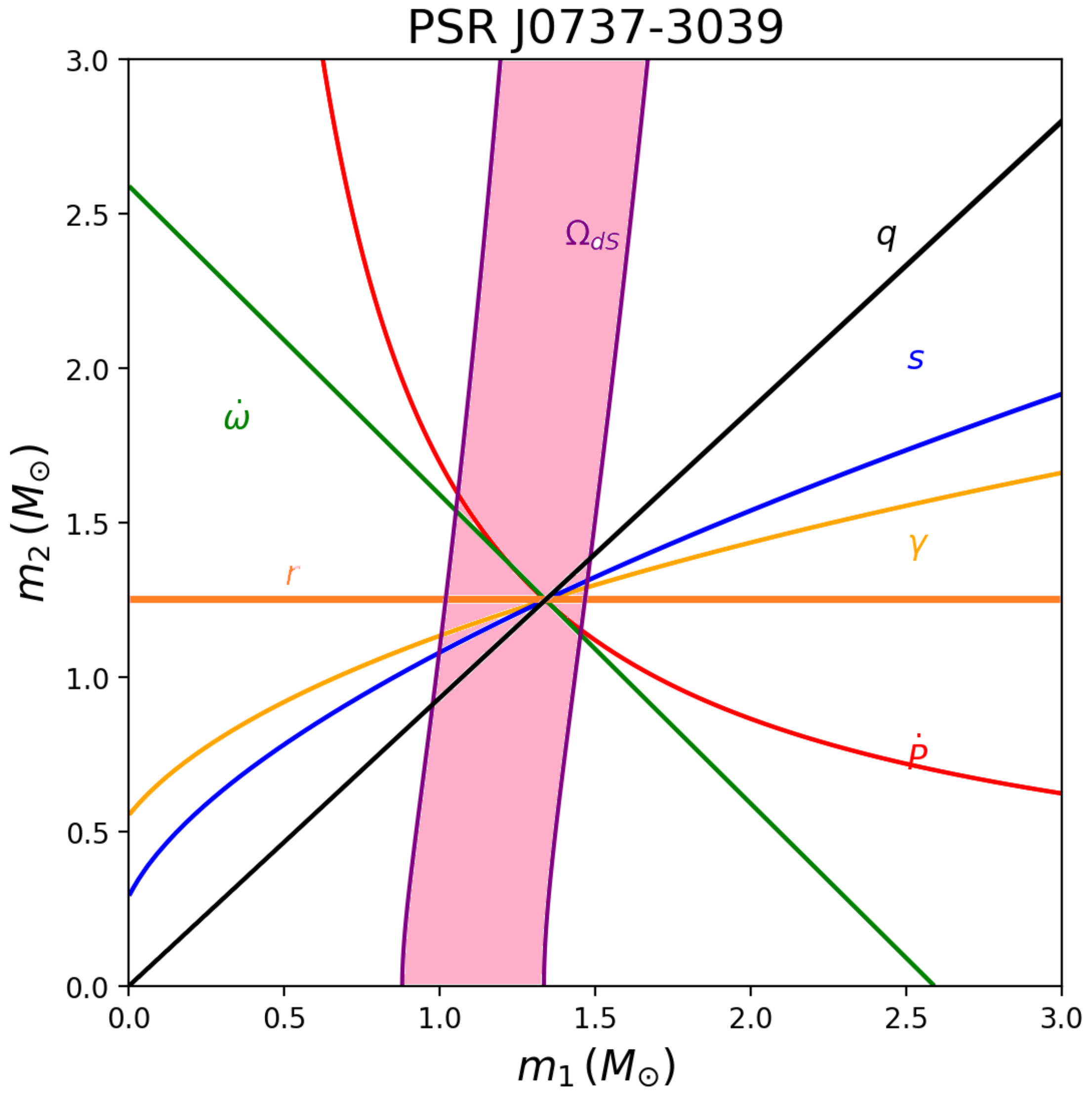
where $\Phi_s = Gm_s/\left(ac^2\right)$ is the potential of the companion body and ψ the angle between the spin vectors

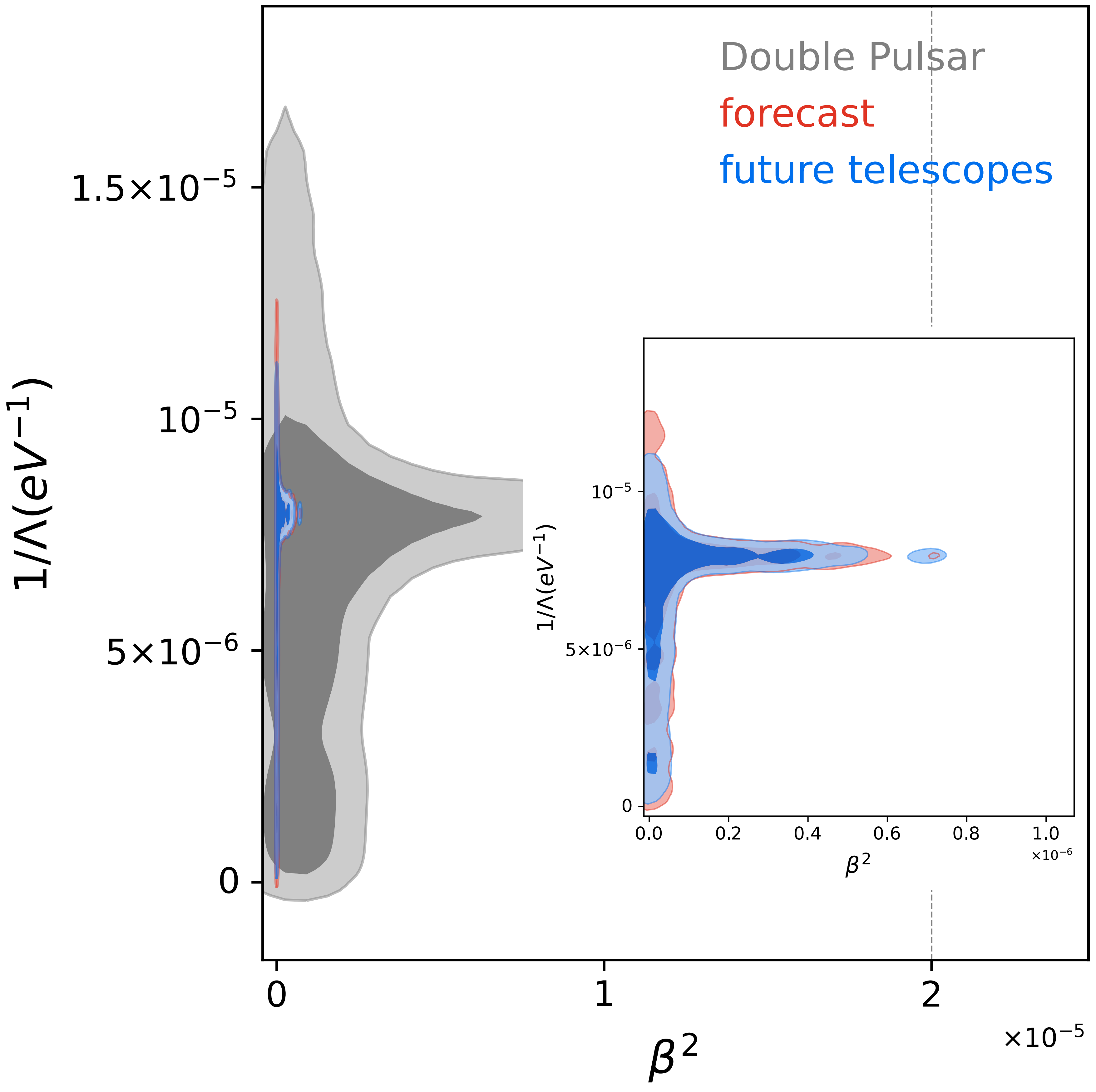
We can now test and constrain against current and future experiments: GPB, GPS, GINGER, Pulsars











	β^2	$\Lambda\,(MeV)$ >
Current	$(1.94 \pm 0.72)\cdot 10^{-5}$	1.62
Forecast	$(1.16 \pm 1.84)\cdot 10^{-7}$	2.1
future telescopes	$(0.99 \pm 1.53)\cdot 10^{-7}$	3.0

Summary

We have shown the effects of modified gravity on the dynamics of the two-body system, including both conformal and disformal couplings.

We have constrained the parameters using solar system constraints

We have shown how the ladder expansion can be resumed to uncover a new screening mechanism in the two-body case

We include spin producing constraints from Gravity Probe B and pulsars. We have made predictions for the Lens Thirring Effect which could be probed in future