

Ultra-light axions in string cosmology

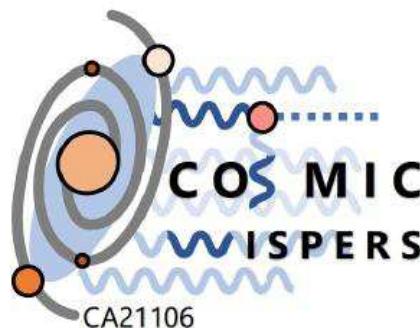


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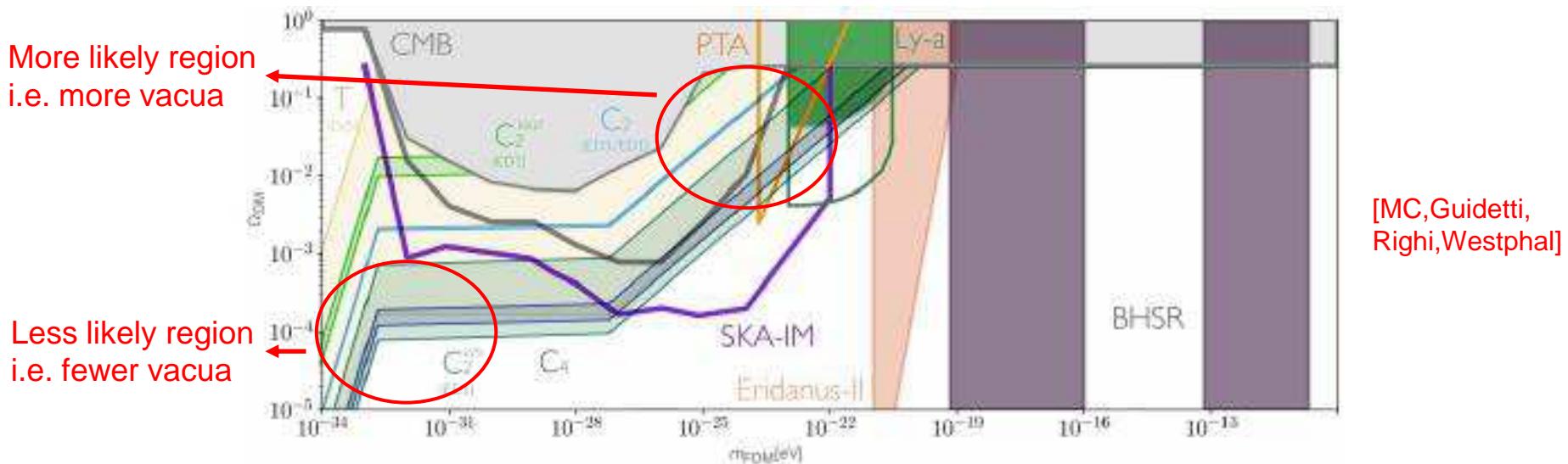


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Road to stringy predictions

- Dream: stringy predictions superimposed on exclusion plots
- Is it doable with a **landscape** of 4D solutions from string theory?
 - i) Yes but just for classes of motivated models (with EFT under control)

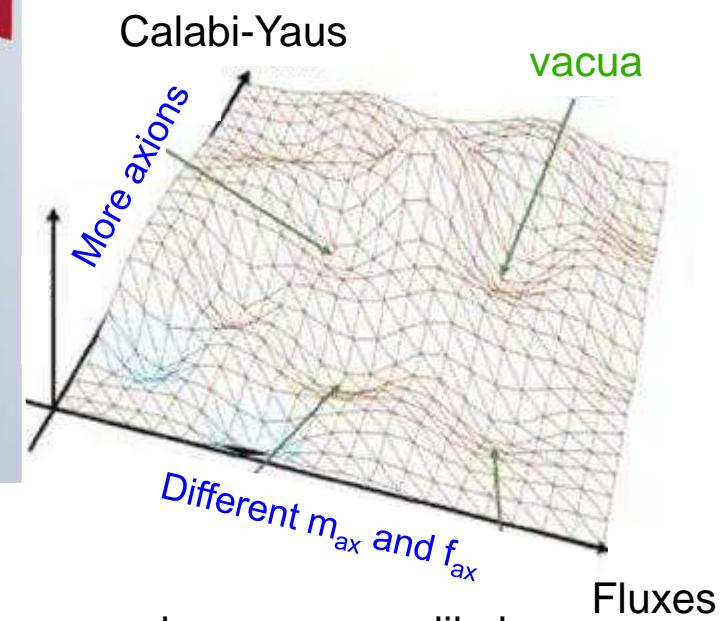
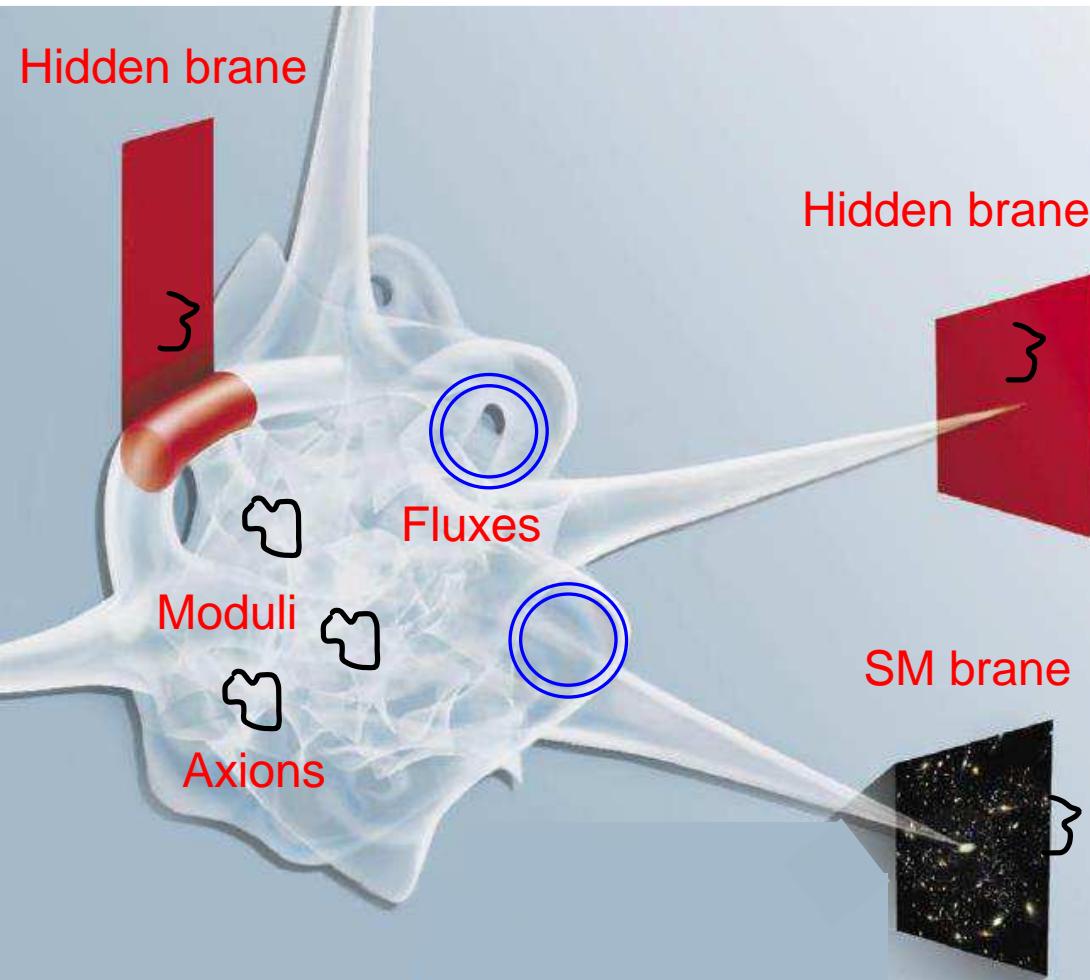


- i) Better to search for **scenarios** more than models
generic stringy signatures (especially if not motivated from **QFT** viewpoint):
 - a) $O(100)$ ultra-light **axions** with **gravitational** couplings
applications: fuzzy DM, dark radiation, stellar cooling, quintessence, early dark energy....
 - b) **Non-standard** cosmological histories with **early matter domination** or **kinetic domination**
applications: dilution effects on DM, baryogenesis, GWs, dark radiation, growth of pert...

String compactifications

- String theory lives in 10D and needs SUSY for consistency
- Compactification: $X_{10D} = M_{4D} \times Y_{6D}$
- 4D EFT for $E \ll M_{KK} = \text{Vol}(Y_{6D})^{-1/6}$
- Geometrical and topological properties of Y_{6D} determine 4D physics
- N=1 SUSY in 4D if Y_{6D} is a Calabi-Yau manifold \longrightarrow chiral theory \longrightarrow realistic!
- Y_{6D} can be deformed in size and shape
 - i) maths: deformations parametrised by moduli
 - ii) 4D physics: moduli ϕ_i are new scalar particles with gravitational couplings and axionic partners
- Only 1 free parameter: string length \longrightarrow all properties of EFT are ϕ -dependent
 $g_{YM}(\phi_i)$, $Y_{ijk}(\phi_i)$, $M_{SUSY}(\phi_i)$, $m_{ax}(\phi_i)$, $m_\phi(\phi_i)$, $H_{inf}(\phi_i)$, $\Lambda(\phi_i)$, ...
- Need to know ϕ_i to make predictions
 - \longrightarrow moduli stabilisation: develop $V(\phi_i)$ to fix $\langle \phi_i \rangle$ at minimum
- $V(\phi_i)$ sourced by background fluxes = non-zero VEVs of anti-symmetric forms
- 2 choices:
 - i) Calabi-Yau topology gives number of moduli and axions
 - ii) VEV of fluxes determines $\langle \phi_i \rangle$ \longrightarrow landscape of string vacua $\sim 10^{500}$

4D string models



Different fluxes can also yield same m_{ax} and f_{ax} → some values are more likely

Stringy axions

- 4D string axions:
 - i) closed string ϑ ($T = \tau + i\vartheta$): KK 0-modes of antisymmetric forms
ex: $U(N)$ theory on N D7-branes wrapping internal 4-cycle Σ_4

$$\tau \equiv \text{Vol}(\Sigma_4) = g_{YM}^{-2}$$

$$\vartheta = \int_{\Sigma_4} C_4$$

$$f_a \simeq M_{KK} \simeq \frac{M_p}{\tau} \simeq 10^{16-17} \text{ GeV}$$

→ “stringy” QCD axion, inflation, quintessence, fuzzy DM, dark radiation...
 - ii) open string ζ ($\phi = \rho e^{i\zeta}$): matter field ϕ charged under anomalous $U(1)$
ex: $SU(N) \times U(1)$ theory on D3-branes at singularities with blow-up $\tau_{loc} \ll 1$

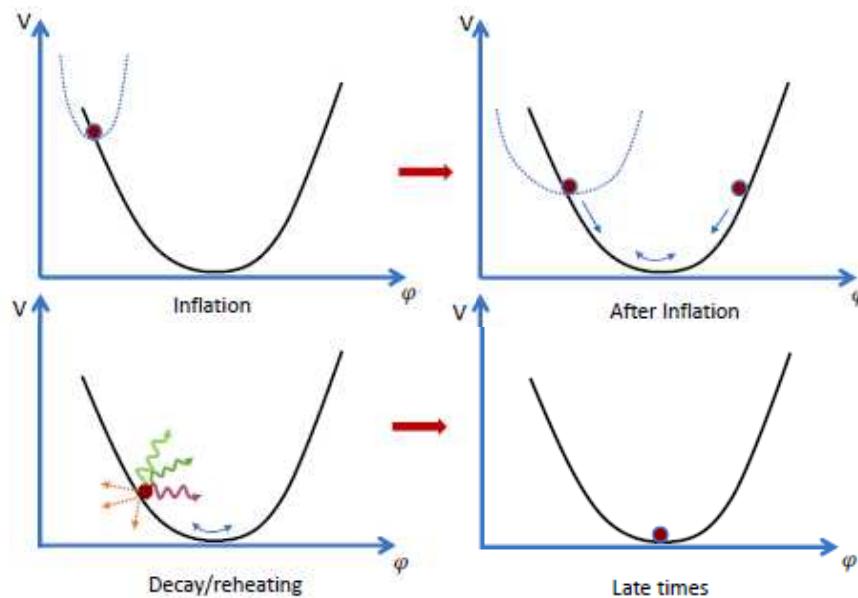
$$V_D \simeq g_{U(1)}^2 (\rho^2 - \xi)^2 \simeq 0$$

$$f_a = \langle \rho \rangle \simeq \sqrt{\xi} \simeq \tau_{loc} M_s \simeq 10^{10-11} \text{ GeV}$$

→ “field-theory” QCD axion, astrophysical hints,....
- Axion shift symmetry exact at perturbative level and broken by instantons
 - $m_{\vartheta,inst} \simeq m_\tau \simeq m_{3/2} > 50 \text{ TeV}$ if τ fixed non-perturbatively as in KKLT models
 - $m_{\vartheta,inst} \simeq m_\tau e^{-c\tau} \ll m_\tau \simeq m_{3/2}$ if τ fixed perturbatively as in LVS models → ultra-light axions
- Generic prediction: ultra-light axions unavoidable in controlled EFT with $\tau \gg 1$
- Generic implications:
 - i) early matter domination from moduli oscillations → dilution and non-standard DM
 - ii) relativistic axions from moduli decay → extra dark radiation $\Delta N_{\text{eff}} \neq 0$
 - iii) Ultra-light axions suitable for quintessence, EDE and fuzzy DM

Reheating and moduli oscillations

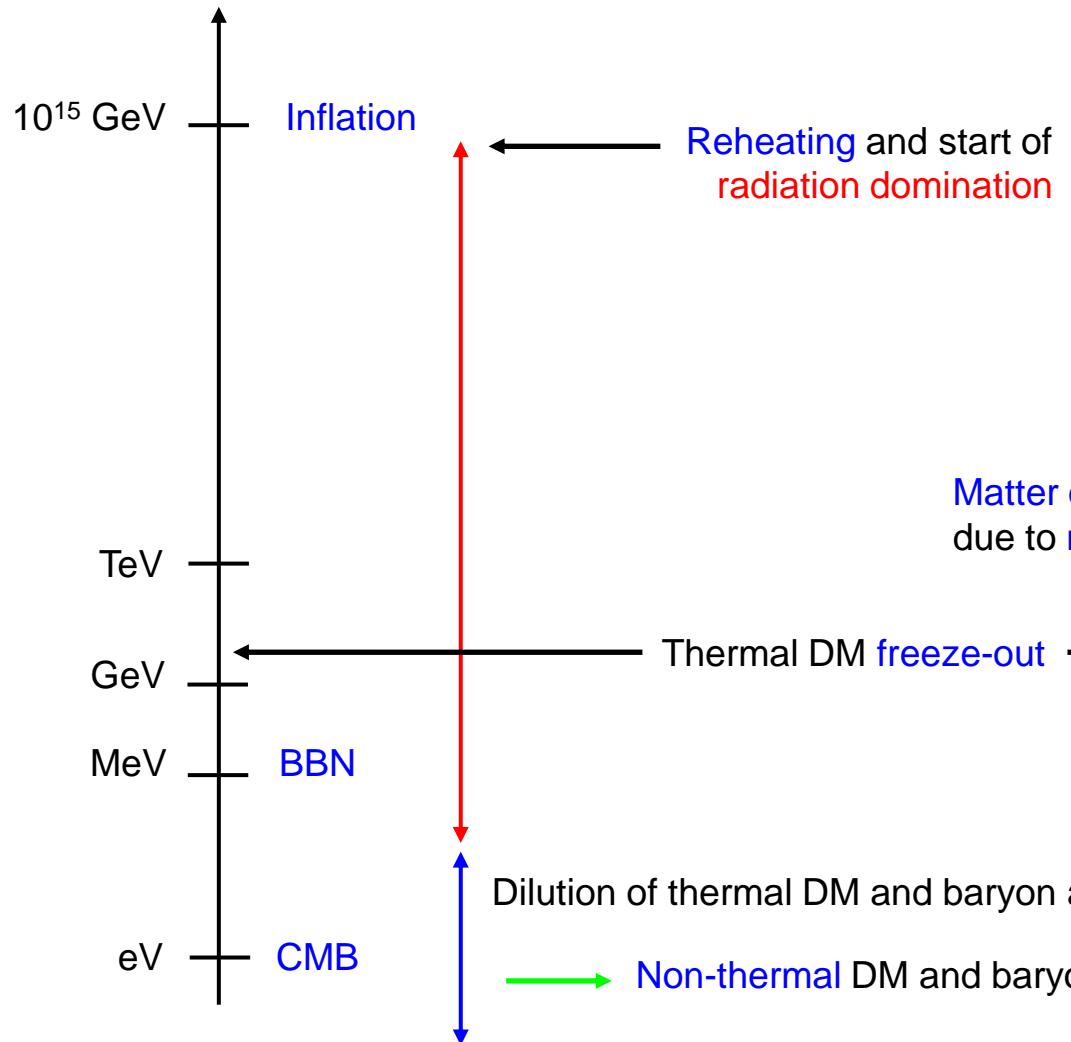
- **Reheating:** production of SM fields from inflaton decay after the end of inflation
→ radiation domination
- Early epoch of **matter domination** due to **moduli oscillations** prior to BBN



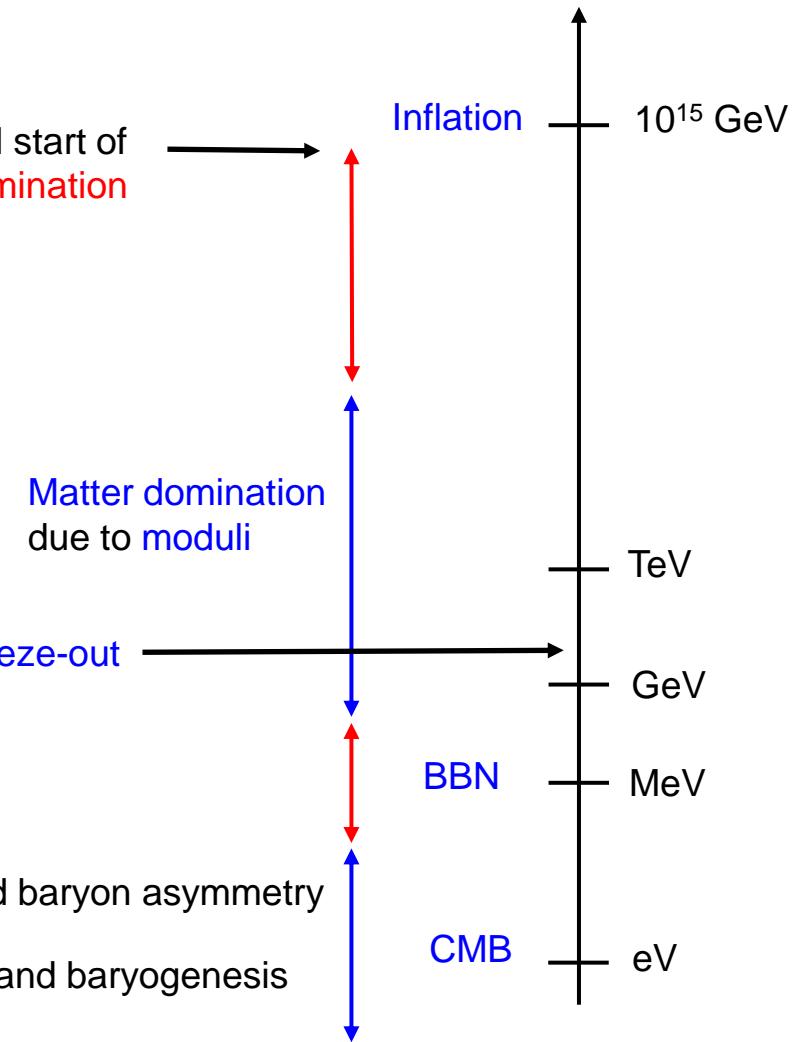
Non-standard cosmology from strings

[Erickcek,Kane,Sinha,Watson,...]

Standard history

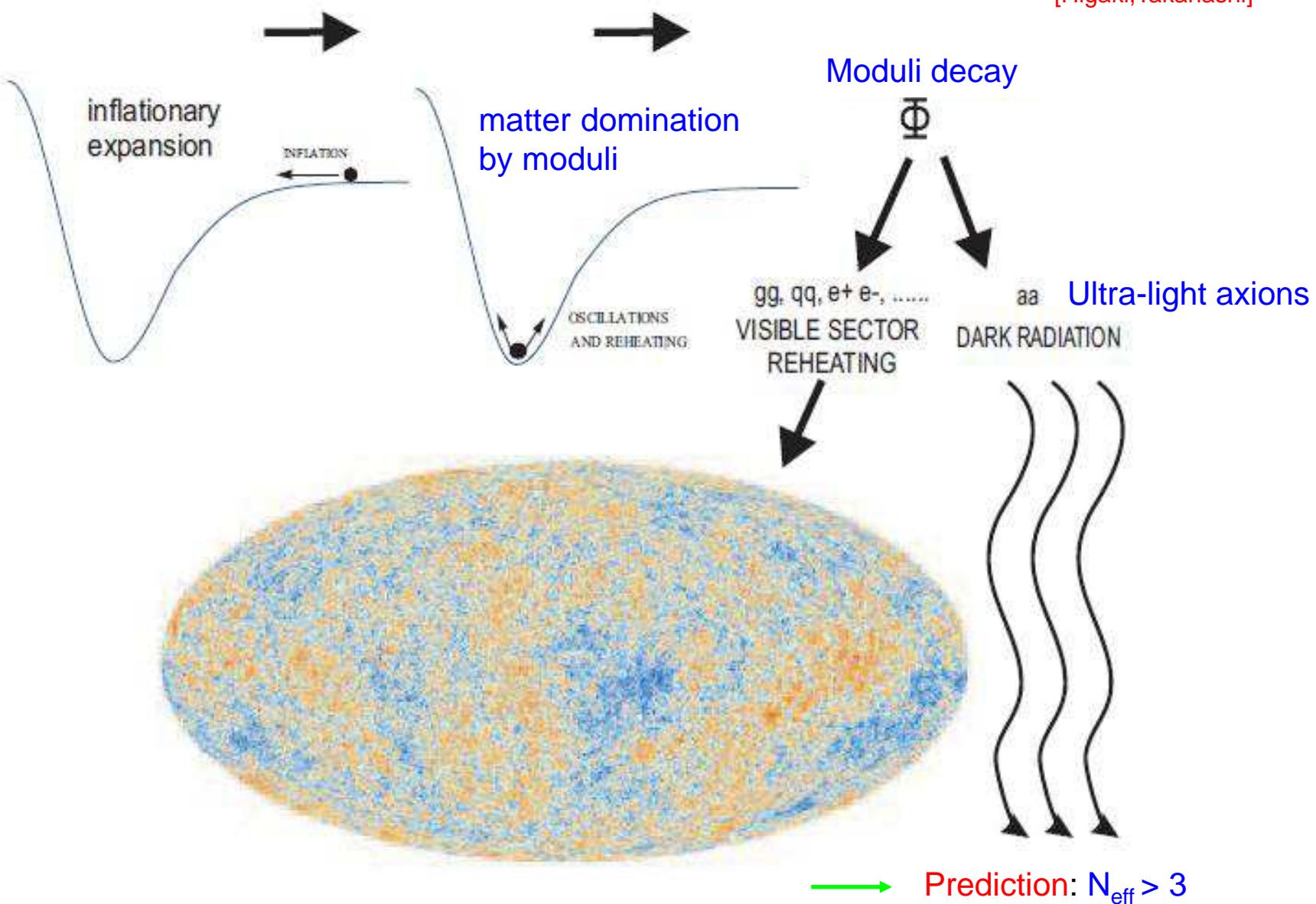


Non-standard history



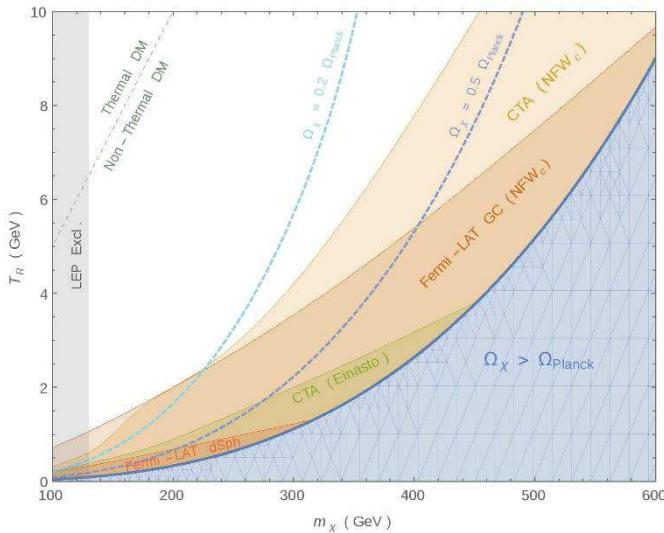
Axionic dark radiation

[MC,Conlon,Quevedo]
[Higaki,Takahashi]

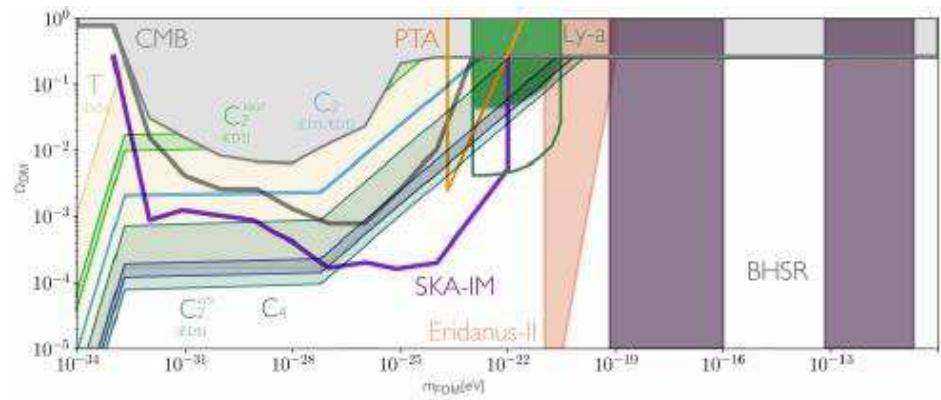


Non-standard dark matter

Non-thermal WIMPs



Fuzzy DM

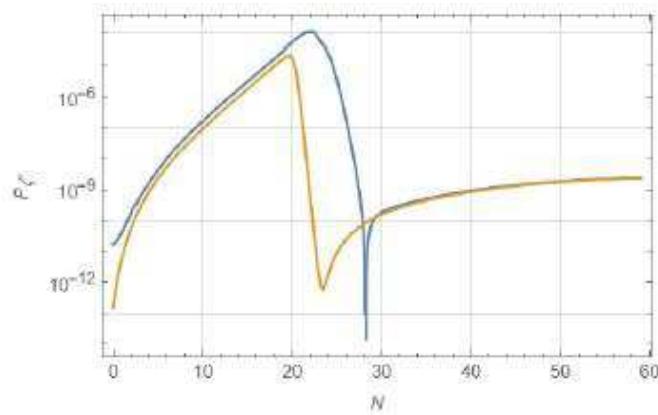


Fuzzy DM from ultra-light ALPs with $m \sim 10^{-22}$ eV

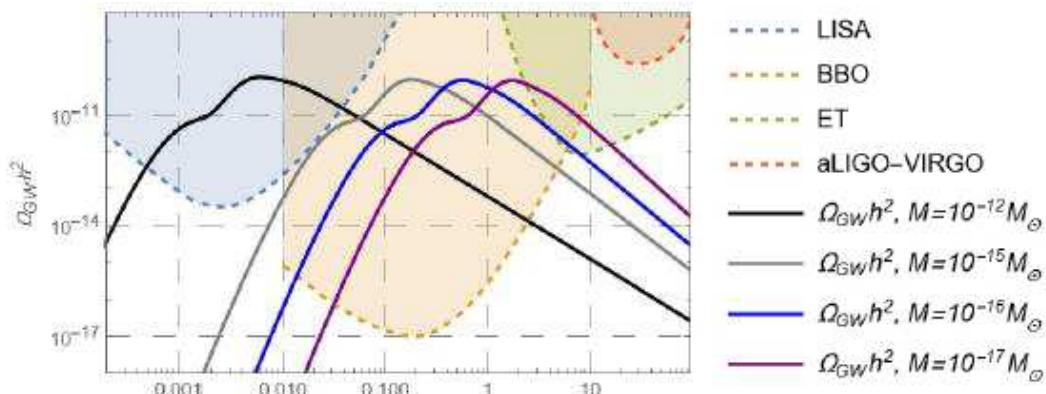
[MC, Guidetti, Righi, Westphal]

Higgsino DM with $m \sim 300$ GeV or WIMPs with $m \sim 10^{10}$ GeV
[Aparicio et al] [Allahverdi et al]

PBH DM



Detectable secondary GWs



[MC, Pedro, Pedron]

dS from STRING THEORY?

- NO:**
- DIFFICULTY to get dS with EFT under CONTROL
 - METASTABLE dS may exist BUT might be SHORT-LIVED
 - ⇒ dS might be INCOMPATIBLE with RG
 - ⇒ NO dS CONJECTURES
- [Obied,Ooguri,Spodyneiko,Vafa,Palti,Shiu,Andriot,...]

- YES:**
- NO dS with PARAMETRIC CONTROL BUT can have dS with NUMERICAL CONTROL due to SMALL parameters [MC,de Alwis,Maharana,Muia,Quevedo]

$$\begin{cases} W_0 \ll 1 \text{ in KKLT} \\ 1/V \simeq e^{-1/\gamma_s} \ll 1 \text{ for } \gamma_s \lesssim 0.1 \text{ in LVS} \end{cases}$$
 - SEVERAL UPLIFTING MECHANISMS:
 \overline{D}_3 , D-TERMS, T-BRANES, α' CORR., $F^U \neq 0$, NON-PERT. EFFECTS at SING.
 - PROGRESS in CLASSIFYING α' AND γ_s CORR. using 10D SYMMETRIES
 [Burgess,MC,Ciupke,Krippendorf,Quevedo]
 - GLOBAL CY MODELS with SM on D_3 s and dS from T-BRANES
 [MC,Garcia Etxebarria,Quevedo,Schachner,Shukla,Valandro]

QUINTESSENCE from STRING THEORY?

- TAKE NO dS POINT of VIEW

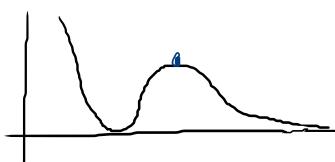
⇒ IMPLICATIONS for QUINTESSENCE?

- MODELS that would be RULED OUT:

- 1) SAXION QUINT. slow-roll down a SHALLOW POTENTIAL
- 2) AXION QUINT. with $f_a \gtrsim M_p$ (due to WGC)

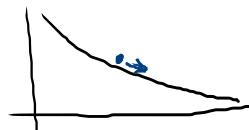
- MODELS that would be OK:

- 1) SAXION HILLTOP for MINKOWSKI/AdS VACUUM



- 2) AXION HILLTOP for MINK. VACUUM

- 3) SAXION RUNAWAY



NO QUINTESSENCE at BOUNDARY of MODULI SPACE

- SIMILAR to dS. FOCUS on TYPE IIB (VALID also for IIA and HETEROITC)

VOLUME MODE

[MC,Cunillera,Padilla,Pedro]

$$K = -3 \ln(\tau + \bar{\tau}) \quad \tau = \mathbb{T} + i\vartheta$$

$$\Rightarrow L_{kin} \supset \frac{3}{4\tau^2} \gamma_\mu \tau^\mu \tau = \frac{1}{2} \gamma_\nu \varphi^\nu \gamma^\mu \varphi \quad \text{for} \quad \mathbb{T} = e^{\frac{\sqrt{2}}{3}\varphi}$$

SCALAR POTENTIAL for $\nabla_\tau W = 0$ and $\tau \rightarrow \infty$ (\propto EXPANSION under CONTROL)

NO-SCALE CANCELLATION at TREE-LEVEL

$$V = e^K (|D_\mu W|^2 + |D_\nu W|^2) = \frac{V_0}{\tau^3}$$

QUANTUM CORRECTIONS give a LARGER τ -SUPPRESSION for $\tau \gg 1$

$$\Rightarrow V = \frac{V_0}{\tau^{3+\mu}} = V_0 e^{-\lambda \varphi} \quad \lambda = \sqrt{6}(1+\mu) \quad \mu > 0$$

$$\Rightarrow \varepsilon = \frac{1}{2} \left(\frac{V_0}{V} \right)^2 = \frac{\lambda^2}{2} = 3(1+\mu)^2 > 1 \quad \text{NO ACCELERATION}$$

SIMILAR RESULT for DILATON $\rightarrow \infty$ (g_s EXPANSION under CONTROL)

MULTIFIELD QUINTESSENCE ?

QUINT. could still work due to KINETIC COUPLING with AXION

\Rightarrow NON-GEODESIC MOTION in CURVED FIELD SPACE gives ACCELERATION

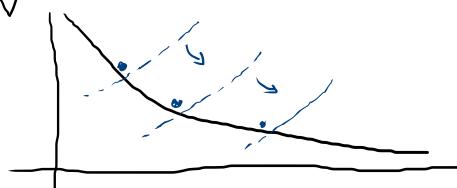
IDEA:

$$\frac{3}{4\zeta^2} \gamma_\mu \partial^\mu \gamma^\nu \partial_\nu = \frac{3}{4} e^{-2\sqrt{\frac{2}{3}}\phi} \dot{\phi}^2$$

gives EFFECTIVE TIME-DEPENDENT CONTRIBUTION to $V(\phi)$ if $\dot{\phi} \neq 0$

$$\Rightarrow V_{\text{eff}}(\phi) = V_0 e^{-\lambda\phi} - \frac{3}{4} e^{-2\sqrt{\frac{2}{3}}\phi} \dot{\phi}^2$$

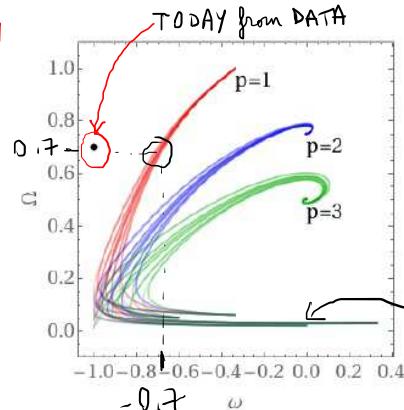
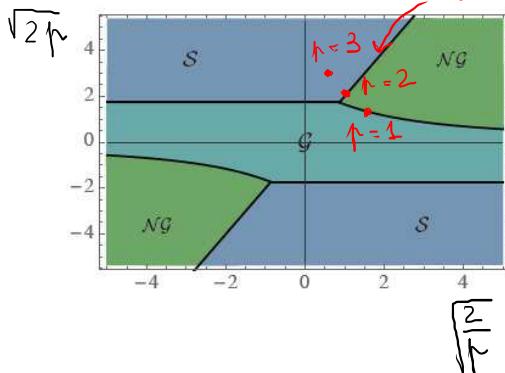
[MC,Dibitetto,Pedro]



$$\dot{\phi}^2 \propto \frac{1}{a^6} \quad \text{for } m_g \approx 0$$

HOWEVER it does NOT work in STRINGS: $K = -p \ln(x + \bar{x}) \Rightarrow V = V_0 e^{-\sqrt{2p}\phi} - \frac{1}{4} e^{-2\sqrt{\frac{2}{3}}\phi} \dot{\phi}^2$

FIXED POINTS of DYNAMICAL SYSTEM



CHALLENGES for QUINTESSENCE

\Rightarrow QUINTESSENCE, as dS , has to be in **BULK** of MODULI SPACE

\Rightarrow SAME CONTROL ISSUES of dS + EXTRA CHALLENGES:

1) ULTRA-LIGHT QUINTESSENCE FIELD

$$m_\varphi \lesssim H_0 \sim 10^{-60} M_p \quad \text{from} \quad \gamma \sim \frac{V_{\varphi\varphi}}{V} \lesssim 1 \quad \begin{matrix} \text{RADIATIVELY STABLE?} \\ \text{FIFTH-FORCES?} \end{matrix}$$

2) STRING SCALE ABOVE 1 TeV

$$M_s \simeq \frac{M_p}{\sqrt{V}} \gtrsim 1 \text{ TeV} \quad \Leftrightarrow \quad V \lesssim 10^{30}$$

3) HEAVY VOLUME MODE

$$m_\nu \gtrsim 1 \text{ meV} \simeq 10^{-30} M_p \quad \text{from FIFTH-FORCES} \Rightarrow m_\nu \gg m_\varphi$$

\Rightarrow LEADING ORDER: V is LIFTED while φ is FLAT

$$V = V_{\text{lead}}(v) + V_{\text{sub}}(\varphi, v)$$

$$\frac{V_{\text{sub}}}{V_{\text{lead}}} \sim \left(\frac{m_\varphi}{m_\nu} \right)^2 \lesssim 10^{-60} \quad \text{CANNOT be OBTAINED with PERT. CORR.}$$

SINCE $\frac{V_{\alpha'^4 g_s^2}}{V_{\alpha'^3}} \simeq \frac{1}{V^{1/3}} \lesssim 10^{-60} \Leftrightarrow V \gtrsim 10^{180} \Rightarrow M_s \ll 1 \text{ TeV}$

LIGHT VOLUME PROBLEM

\Rightarrow for SAXION QUINT with $m_\phi \sim 10^{-32} \text{ eV}$

$\Rightarrow m_\psi \ll 1 \text{ meV}$ + RADIATIVE INSTABILITY

WAY-OUT: Consider AXION QUINTESSENCE where

$$V_{\text{sub}} \sim e^{-a\tau} \sim e^{-a\sqrt{\nu}^{2/3}} \sim V_{\text{non-pert}}$$

$$\Rightarrow \frac{V_{\text{lead}}}{V_{\text{sub}}} \sim \frac{e^{a\sqrt{\nu}^{2/3}}}{\nu^{3/2}} \gtrsim 10^{60} \quad \text{for } \nu \lesssim 10^{30} \text{ and } M_s \gtrsim 1 \text{ TeV}$$

+ AXIONIC PERTURBATIVE SHIFT SYMM. gives RADIATIVE STABILITY

$\Rightarrow \psi$ has to be an AXIONIC FLAT DIRECTION
LIFTED BY NON-PERT. EFFECTS

QUINTESSENCE MODEL BUILDING

$V(\nu)$ has a SUSY MINK. VACUUM and φ is FLAT

$V(\varphi, \nu)$ generated by TINY NON-PERT. EFFECTS for $\varphi = \text{AXION}$:

- RIGHT HIERARCHY: $V(\varphi, \nu) \ll V(\nu)$
 \Rightarrow NO KL PROBLEM + NO ν DESTABILISATION by QUINT.
- NO RADIATIVE INSTABILITY due to PERT. SHIFT SYMM.
- NO 5-th FORCE PROBLEM

HOWEVER $V(\varphi, \nu) = \Lambda(\nu) \left(1 - \cos\left(\frac{\varphi}{f}\right)\right)$ gives ACCELERATION
ONLY for $f > M_p$
 \nwarrow NEVER OBTAINED in EFT + FORBIDDEN by WGC

\Rightarrow FOCUS on <
 ALIGNMENT \nwarrow requires TUNING + CONTROL ISSUES
 AXION HILLTOP \nwarrow requires TUNING of INIT. COND
 + LOW H_{inf} in TENSION with NO dS CONJ.

AXION HILLTOP

FOCUS on AXIONS in LVS

$$\mathcal{V} = \tau_B^{3/2} - \tau_S^{3/2} \quad \bar{\tau}_B = \tau_B + i\vartheta_B \quad \bar{\tau}_S = \tau_S + i\vartheta_S$$

$$K = -2 \ln \left(\mathcal{V} + \frac{\xi}{2 \tau_S^{3/2}} \right) \quad W = W_0 + A_S e^{-a_S \bar{\tau}_S} + A_B e^{-a_B \bar{\tau}_B}$$

LEADING ORDER \mathcal{V} DEPENDS on $\tau_B \simeq \nu^{2/3}$, τ_S and ϑ_S

$$\mathcal{V} \simeq \frac{C_0}{\nu^{8/3}} + \frac{C_1}{\nu} \sqrt{\tau_S} e^{-2a_S \bar{\tau}_S} + \frac{C_2}{\nu^2} \cos(a_S \vartheta_S) e^{-a_S \bar{\tau}_S} + \frac{C_3}{\nu^3}$$

T-BRANE UPLIFTING ($F_{\text{matter}} \neq 0$ due to $D=0$)

GENERIC in FLUX COMPACTIFICATIONS because of CONSISTENCY

- $D7'$, from TADPOLE CANCE. with $\Xi_2 \neq 0$ due to FREED-WITTEN ANOMALY CANCE.

$$\Rightarrow \xi_{F1} \sim \frac{1}{\nu} \int_D J_1 \Xi_2 \sim \frac{\lambda}{\nu^{2/3}} \Rightarrow V_D \sim g^2 \left(|\chi|^2 - \xi_{F1} \right)^2 = 0 \Leftrightarrow |\chi|^2 = \xi_{F1}$$

$$\bullet H_3, F_3 \Rightarrow V \sim m_{3/2}^2 |\chi|^2 \sim \frac{W_0^2}{\nu^2} \xi_{F1} \sim \frac{C_0 \nu}{\nu^{8/3}}$$

[MC,Quevedo,Valandro]

AXION HILLCLOUD

- LEADING ORDER STABILISATION:

SUSY MINK at $\vartheta_s = 0$, $\tau_s \sim \frac{1}{g_s}$, $V \sim e^{a_s \tau_s} \sim e^{\frac{a_s}{g_s}} \gg 1$

V
 ϑ_B is FLAT

for $g_s \lesssim 0.1$

$$V_{\text{lead}}(V_{\max}) \sim \frac{W_0^2}{V^3} \sim M_V^2$$

- SUBLADING ORDER

$$V_{\text{sub}}(\vartheta_B, V) \sim L(V) \left(1 - \cos(a_B \vartheta_B)\right) \ll V_{\text{lead}} \quad \text{since } L(V) \sim W_0 e^{-a_B T_B} \ll 1$$

KINETIC TERMS

$$\mathcal{L}_{\text{kin}} = \frac{3}{4 T_B^2} \partial_\mu \vartheta_B \partial^\mu \vartheta_B = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \quad \varphi = \sqrt{\frac{3}{2}} \frac{\vartheta_B}{T_B} \Rightarrow a_B \vartheta_B = \sqrt{\frac{2}{3}} a_B T_B \varphi = \frac{\varphi}{f}$$

$$\Leftrightarrow f = \sqrt{\frac{3}{2}} \frac{M_P}{a_B T_B} \Rightarrow V_{\text{sub}} = C \underbrace{e^{-\sqrt{\frac{3}{2}} \frac{M_P}{f} \varphi}}_{M_P^4} \left(1 - \cos\left(\frac{\varphi}{f}\right)\right)$$

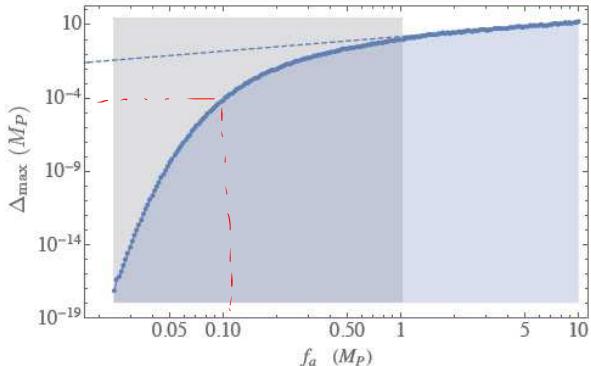
$$\Rightarrow M_V \sim 10^{12} \text{ GeV} \quad \text{OK!}$$

$$10^{120} \text{ for } \frac{M_P}{f} \sim 300 \Leftrightarrow V \sim \vartheta_B^{3/2} \sim 10^3 \text{ NATURAL!}$$

\Rightarrow EFT under CONTROL!

HILLTOP and INITIAL CONDITIONS

HOW CLOSE should φ be to the MAXIMUM to get ACCELERATION with $\omega_\varphi = -1$ and $\mathcal{R}_\varphi \approx 0.7$?



$$\left. \begin{array}{l} \text{EFT under control?} \\ f_a = 0.1 M_P \Rightarrow \Delta_{\max} \lesssim 10^{-4} M_P \\ f_a = 0.02 M_P \Rightarrow \Delta_{\max} \lesssim 10^{-18} M_P \\ f_a \sim \frac{M_P}{\sqrt[2/3]{V}} \lesssim 10^{-2} M_P \quad \text{for } V \gtrsim 10^3 \end{array} \right\}$$

↑ EFT under control

$$\text{as } \alpha' - \text{EXPANSION} \quad \frac{\alpha'}{\sqrt[2/3]{V}} = \frac{1}{V^{1/3}} \lesssim 0.1$$

QUANTUM DIFFUSION during INFLATION causes FLUCTUATIONS $\Delta\varphi \sim H_{\text{inf}}$

\Rightarrow need to REQUIRE $H_{\text{inf}} \lesssim \Delta_{\max}$

\Rightarrow for $f_a = 0.02 M_P$ where EFT is under control

$$H_{\text{inf}} \lesssim 10^{-18} M_P \sim 1 \text{ GeV} \quad \text{VERY STRONG BOUND!}$$

CONCLUSIONS on DARK ENERGY

BUT

$$\left(\frac{S\rho}{\rho}\right)^2 \sim \frac{H_{inf}^2}{\varepsilon} \sim 10^{-10} \Rightarrow \varepsilon \sim 10^{+10} H_{inf}^2 \lesssim 10^{-26} \text{ for } f = 0.02 M_p$$

\Rightarrow SUPER SHALLOW V in TENSION with NO dS CONJ.

NO dS CONJ. + THEORETICAL and PHENO CONSIDERATIONS

\Rightarrow QUINTESSENCE is in the SWAMPLAND

OR QUINTESSENCE is as CHALLENGING as dS + EXTRA CONSTRAINTS
(FIFTH FORCES, KL PROBLEM, RADIATIVE STABILITY)

BUT OBSERVATIONS TODAY REQUIRE ACCELERATED EXPANSION

\Rightarrow WORK HARD and SEARCH for dS in BULK of MODULI SPACE
with NUMERICAL, even if NOT PARAMETRIC, CONTROL

└ KNOWN EXAMPLES in PHYSICS:

- QED $d=1/137$

- COSMOLOGICAL PERT. THEORY $S\rho/\rho \sim 10^{-5}$

Quintessence from axion alignment

- Focus on fibred CYs:

$$Vol = \sqrt{\tau_f} \tau_b - \tau_s^{3/2}$$

[Angus, Choi, Shin]

[MC, Padilla, Pedro]

In progress

- Scalar potential:

$$V = V_{lead}(Vol, \tau_s, \vartheta_s) + V_{inf}(\tau_f) + V_{sub}(\vartheta_b, \vartheta_f)$$

$$V_{inf} \simeq V_0(1 - \frac{4}{3}e^{-\phi/\sqrt{3}}) \quad \text{and} \quad H_{inf} \simeq 10^{-5} M_p \quad [\text{MC, Burgess, Quevedo}]$$

- 3 ultra-light axions: ζ = QCD axion DM, ϑ_b and ϑ_f = DM (up to 0.1%) and DE via alignment
- Alignment mechanism [Kim, Nilles, Peloso]

$$W = W_{LVS} + A_1 e^{-\frac{2\pi}{N_1}(q_{1f}T_f + q_{1b}T_b)} + A_2 e^{-\frac{2\pi}{N_2}(q_{2f}T_f + q_{2b}T_b)}$$

- Canonical normalisation: $\phi_H \propto q_{1f}\vartheta_f + q_{1b}\vartheta_b$ and $\phi_L \propto (q_{1b}/\tau_f^2)\vartheta_f - (2q_{1f}/\tau_b^2)\vartheta_b$

$$\longrightarrow V_{sub} = \Lambda_1^4 \left[1 - \cos \left(\frac{\phi_H}{f_H} \right) \right] + \Lambda_2^4 \left[1 - \cos \left(\frac{\phi_H}{\tilde{f}_H} + \frac{\phi_L}{f_L} \right) \right]$$

$$f_H \sim \tilde{f}_H \sim O(\tau_f^{-1}, \tau_b^{-1}) \ll 1 \quad \text{while} \quad f_L \sim \frac{\sqrt{q_{1b}^2/\tau_f^2 + 2q_{1f}^2/\tau_b^2}}{|q_{1f}q_{2b} - q_{2f}q_{1b}|} \rightarrow \infty \quad \text{for} \quad q_{1f}q_{2b} \simeq q_{2f}q_{1b}$$

$$m_H^2 \simeq \frac{\Lambda_1^4}{f_H^2} \quad \text{and} \quad m_L^2 \simeq \frac{\Lambda_2^4}{f_L^2} \rightarrow 0$$

$$V_{DE} = \Lambda_2^4 \left[1 - \cos \left(\frac{\phi_L}{f_L} \right) \right] \quad \text{after fixing } \phi_H = 0$$

- Numerical results:

$$\tau_f \sim \tau_b \sim O(100) \quad N_1 \sim N_2 \sim O(20) \quad q_{ij} \sim O(10) \quad \longrightarrow \quad f_H \sim 10^{-3} \quad m_H \sim 10^{-25} \text{ eV} \\ f_L \sim 10^{-1} \quad m_L \sim 10^{-32} \text{ eV}$$

Early dark energy

- EDE proposed to solve H_0 tension: [Poulin et al]
 10% of energy density before recombination and then decays faster than radiation
 → late-time evolution is unchanged
 → expansion rate is increased shortly before CMB formation raising H_0 from CMB

$$V_{EDE} = V_0 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]^n \quad \text{with} \quad V_0 \sim eV^4 \quad n \simeq 3 \quad f \simeq 0.2 M_p$$

- Embedding in string theory: Swiss-cheese LVS with 1 orientifold odd axion

$$G = \int_{\Sigma_2} B_2 + \int_{\Sigma_2} C_2 = b + ic \quad [\text{MC,Licheri,Mahanta, McDonough,Pedro,Scalisi}]$$

- W from 3 gaugino condensates on D7s with gauge fluxes k , $2k$ and $3k$

$$W = W_{LVS} + A_1 e^{-a(T_b+kG)} + A_2 e^{-a(T_b+2kG)} + A_3 e^{-a(T_b+3kG)} \quad a = 2\pi/M$$

$$\rightarrow V_{EDE} = V_0 [A + A_1 \cos(akc) + A_2 \cos(2akc) + A_3 \cos(3akc)] = V_0 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]^3$$

$$\text{if } A = \frac{5}{2}, A_1 = -\frac{15}{4}, A_2 = \frac{3}{2}, A_3 = -\frac{1}{4} \quad f \simeq 0.2\sqrt{g_s} M Vol^{-1/3}$$

- Need to violate WGC to get right V_0 without tuning prefactors since

$$V_0 \simeq A e^{-S} M_p^4 = A e^{-\frac{\lambda M_p}{f}} M_p^4 = A e^{-5\lambda} M_p^4 \sim 10^{-110} M_p^4 \quad \text{for } A \sim O(1) \quad \text{only if } \lambda \gg 1$$

→ C_2 axions with fluxed D7s have $\lambda \simeq \sqrt{g_s} Vol^{1/3} \gg 1$

- Can get right EDE scale and decay constant for $g_s \sim 0.1$, $Vol \sim 10^5$ and $M \sim O(100)$