A minimal SM/LCDM cosmology

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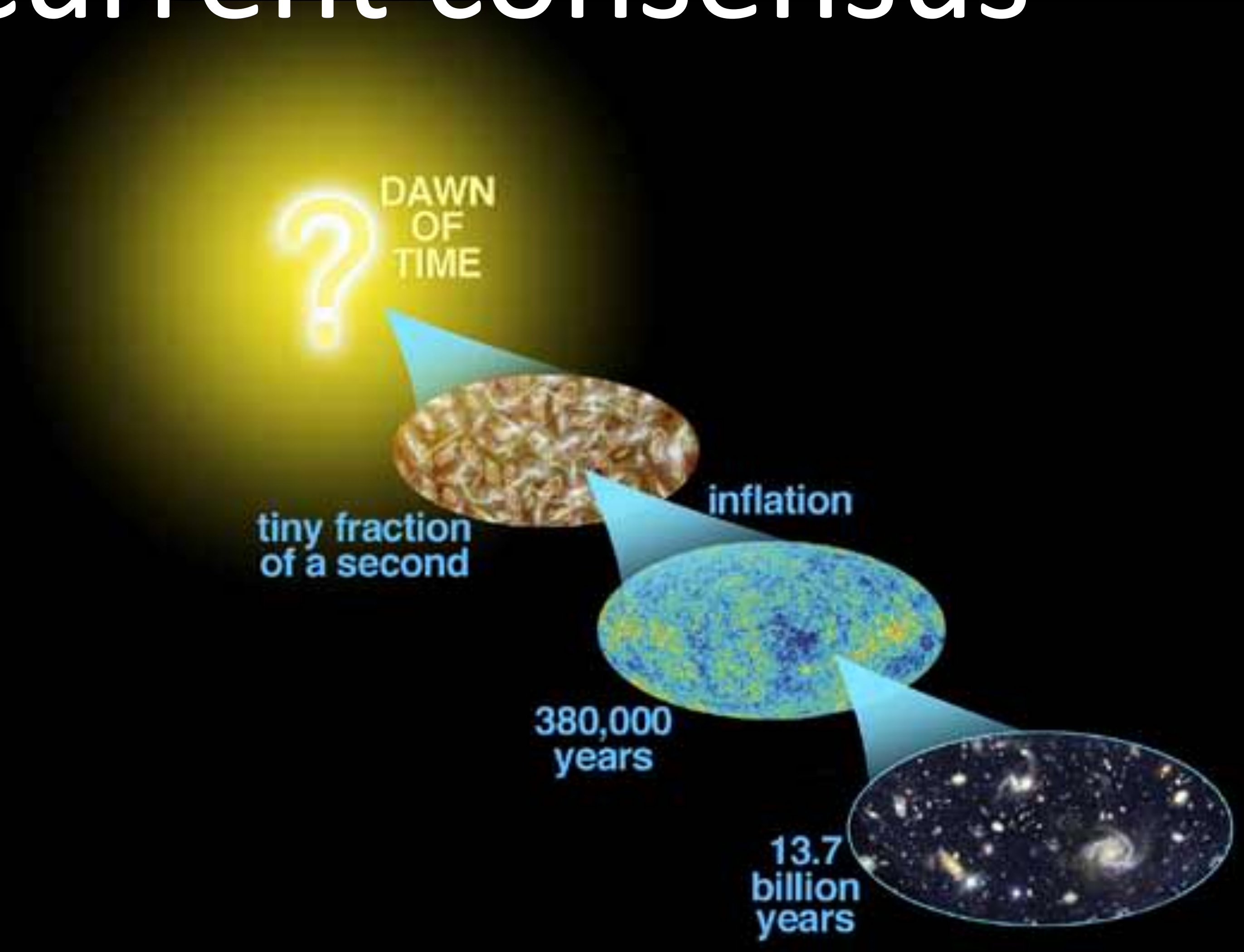
and
Perimeter Institute for Theoretical Physics

with Latham Boyle

current consensus



Inflation was a great idea and motivated many observational programs but

1. As it turned out, the observations are well fit by vanilla LCDM
   cf. superabundance of inflationary models

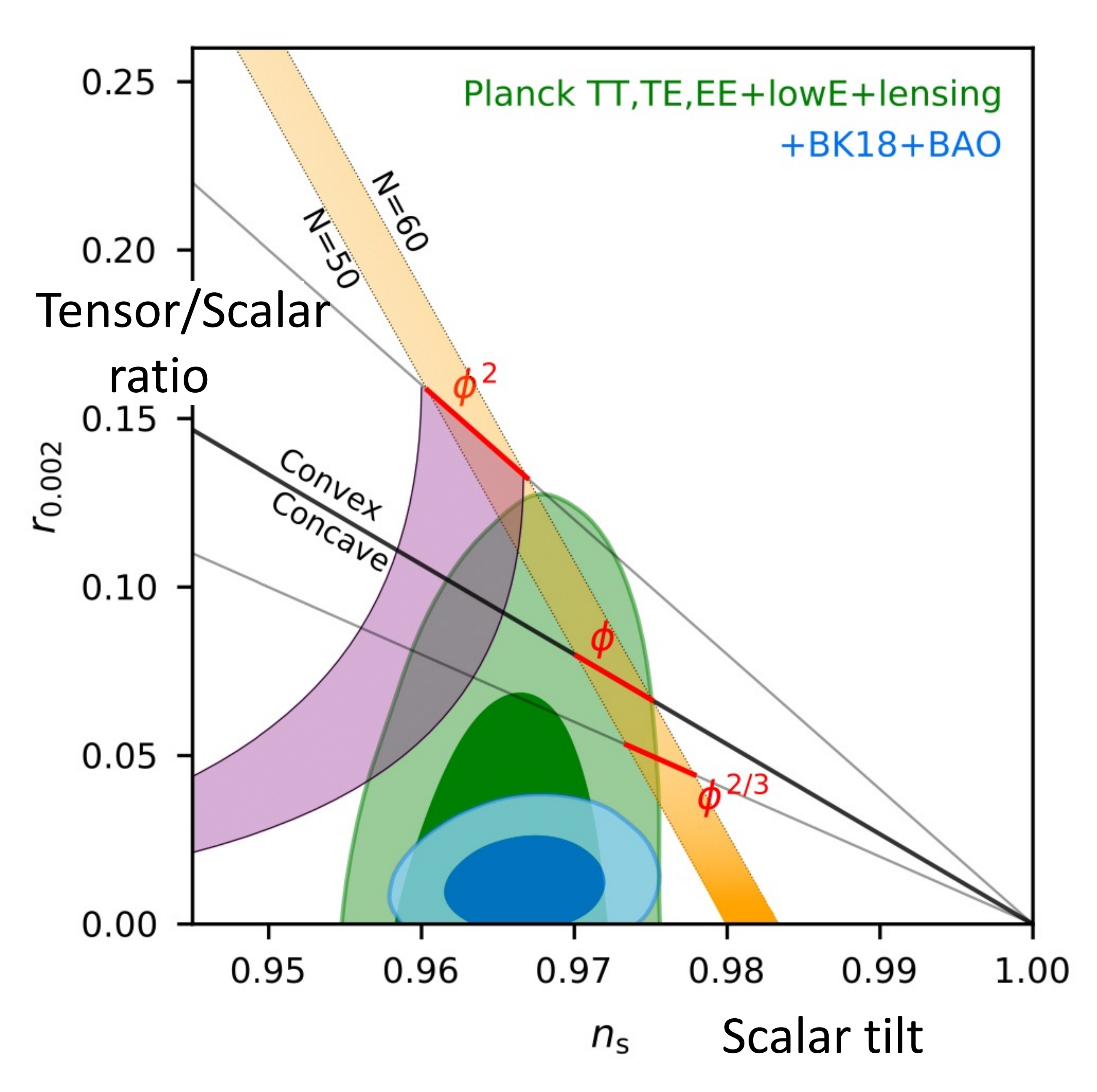
2. No sign of “smoking gun” signal: long wavelength gravitational waves
   (Current bound is $r < 0.04$; CMB experimenters project $r < 0.003$ by 2027)

3. There is no satisfactory measure on inflationary universes (or multiverses)

no sign of
inflationary
tensors

BICEP/Keck
Collaboration
2203.16556 [astro-ph]
PRL **127**, 151301 (2021)
r< 0.036 at 95% confidence

anticipated limit
r<.003
using SPT for
“delensing”
(2027)



vanilla LCDM:

just 5 fundamental physics parameters

3 for the matter/energy content

1. ρ_Λ cosmological constant
2. ρ_{DM}/ρ_B DM/baryon density
3. n_B/n_γ baryons per photon

2 for the fluctuations

Newtonian potential

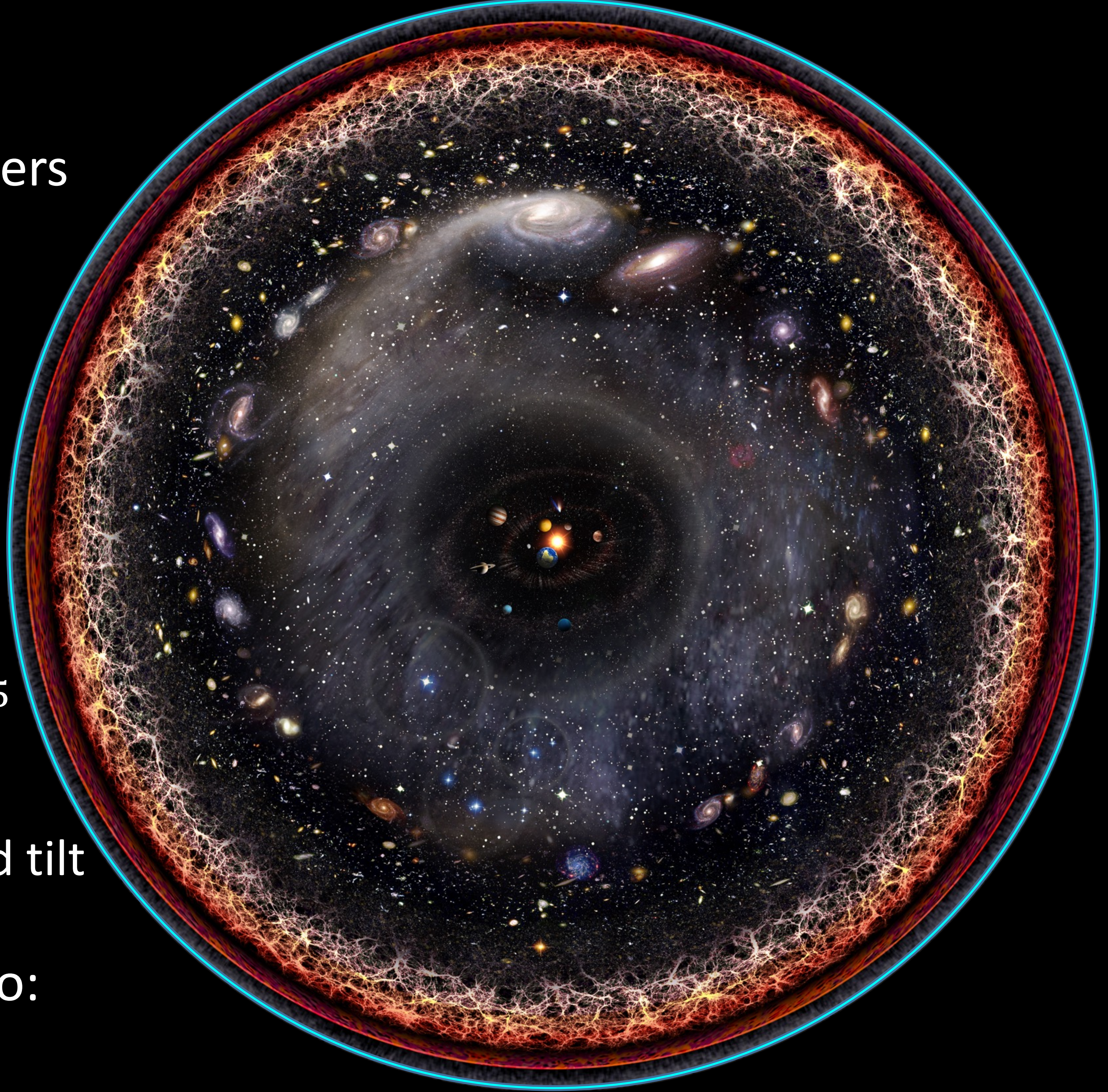
$$\langle\Phi^2\rangle = \int \frac{dk}{k} A \left(\frac{k}{k_*}\right)^{n_s-1} \quad (k_* \equiv 0.05\mathrm{Mpc}^{-1})$$

4. large scale $\Phi_{rms} \approx \sqrt{A} \approx 3\times10^{-5}$

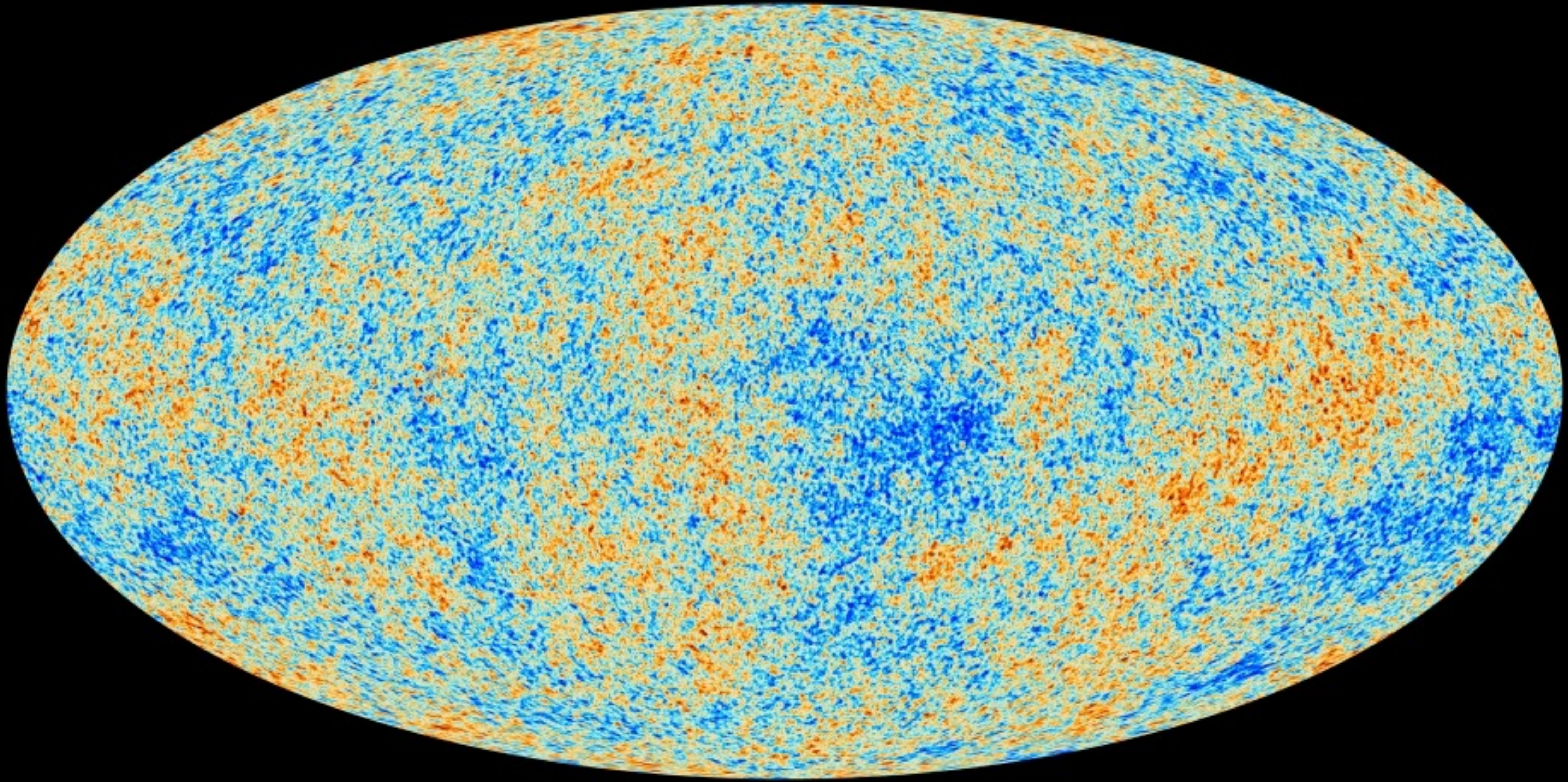
Sachs-Wolfe $\delta T/T \approx \frac{1}{3}\Phi \approx 10^{-5}$

5. $n_s - 1 \approx -0.04 \pm .006$ modest red tilt

many quantities consistent with zero:
suggests we apply Occam's razor

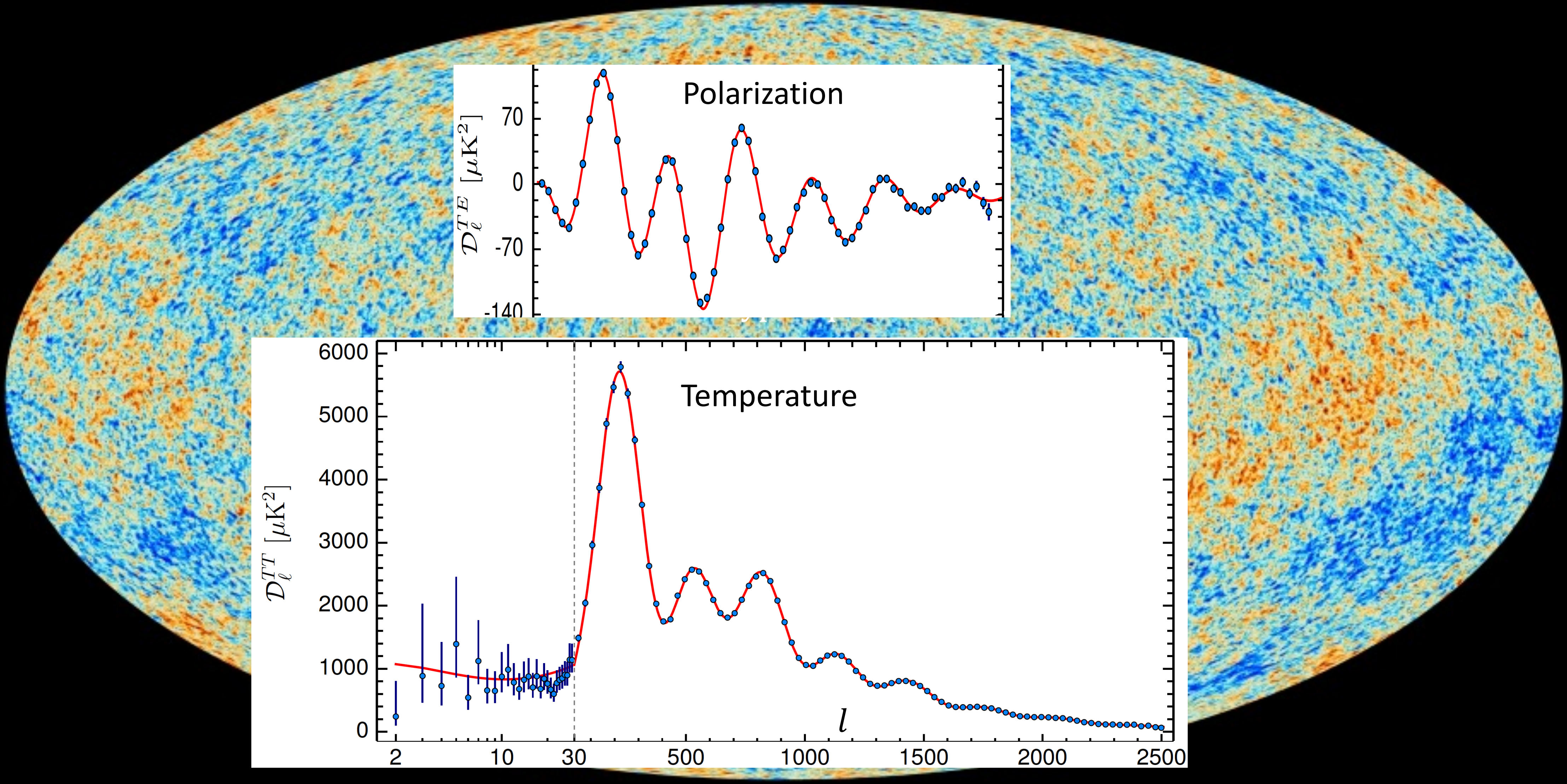


Looking back to the bang



ESA Planck satellite

Large scale perturbations

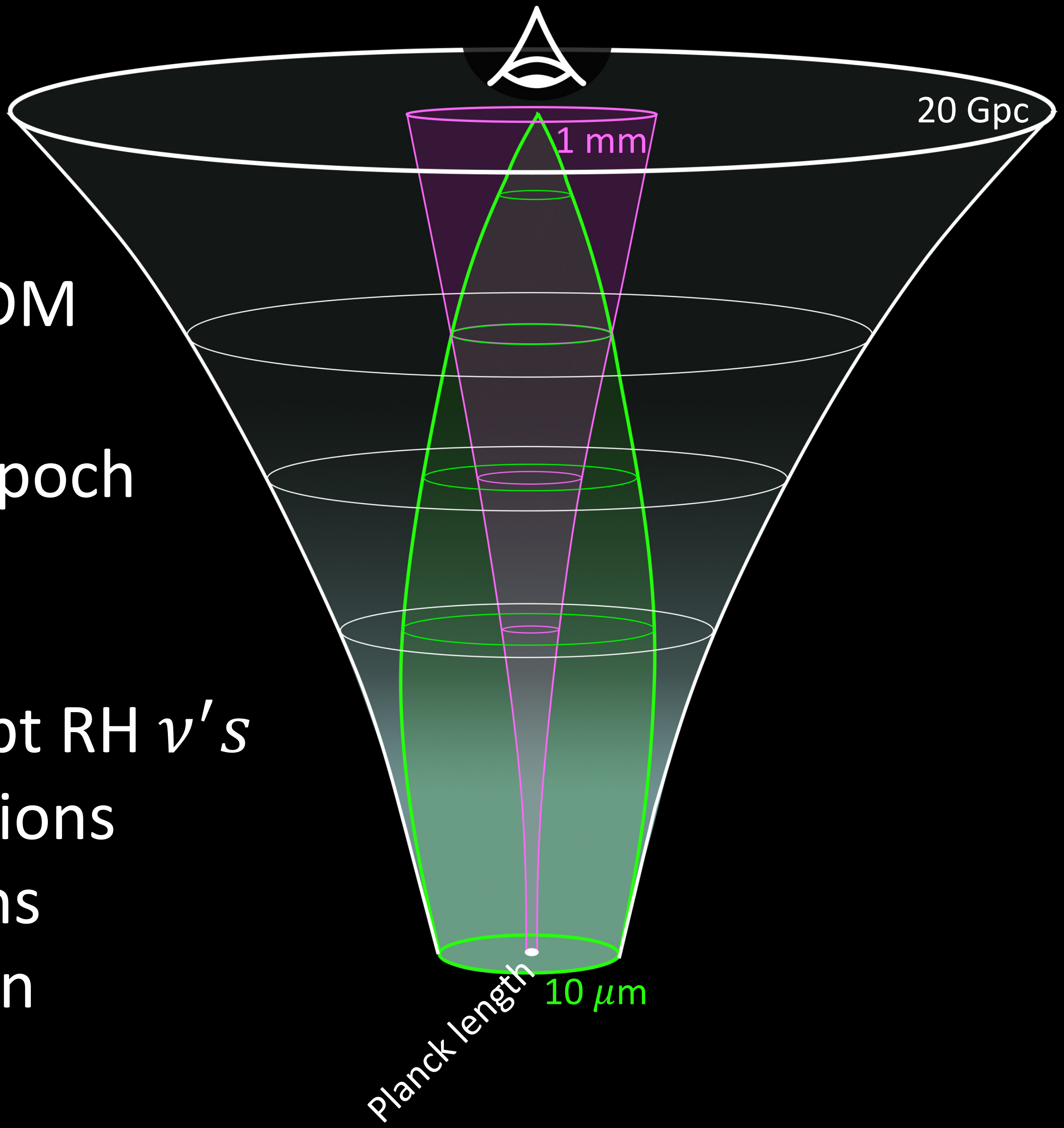


ESA Planck satellite

This talk:

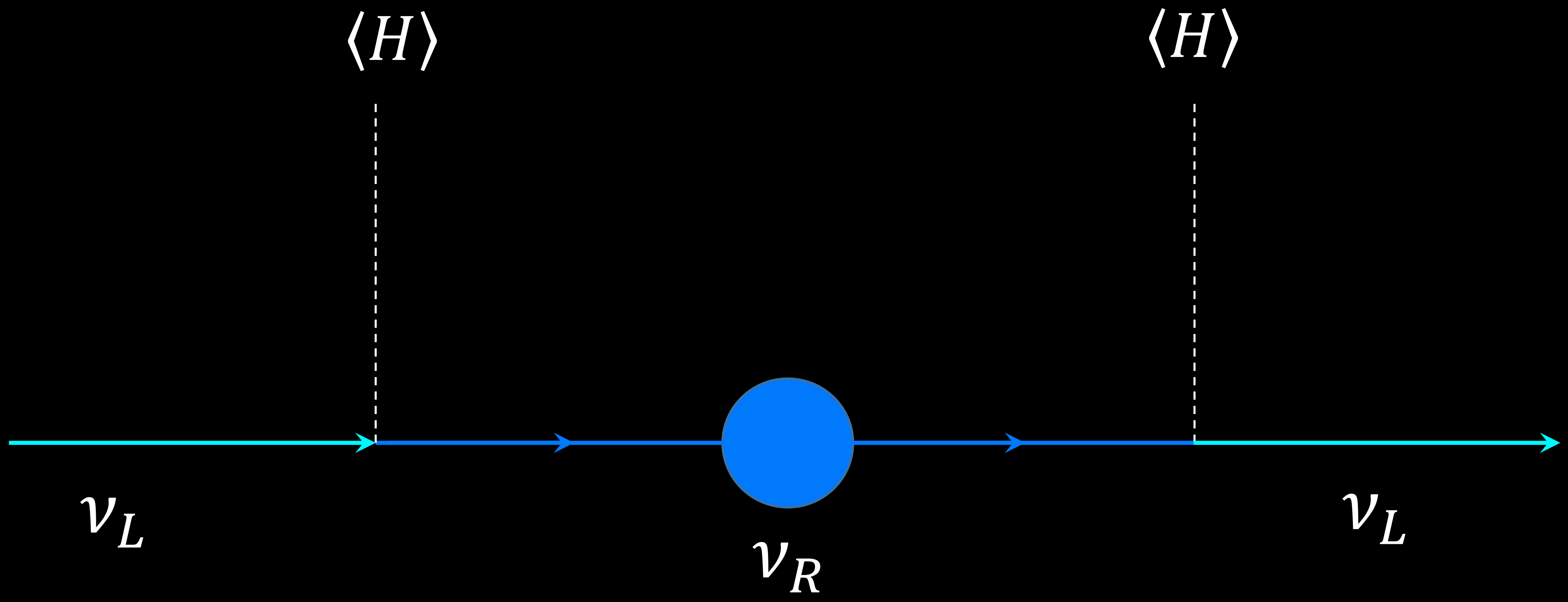
a unified framework
connecting SM and LCDM
(all 5 parameters)
extrapolate radiation epoch
back to the singularity

no extra particles except RH $\nu's$
explains why 3 generations
predicts the fluctuations
doesn't require inflation
no multiverse



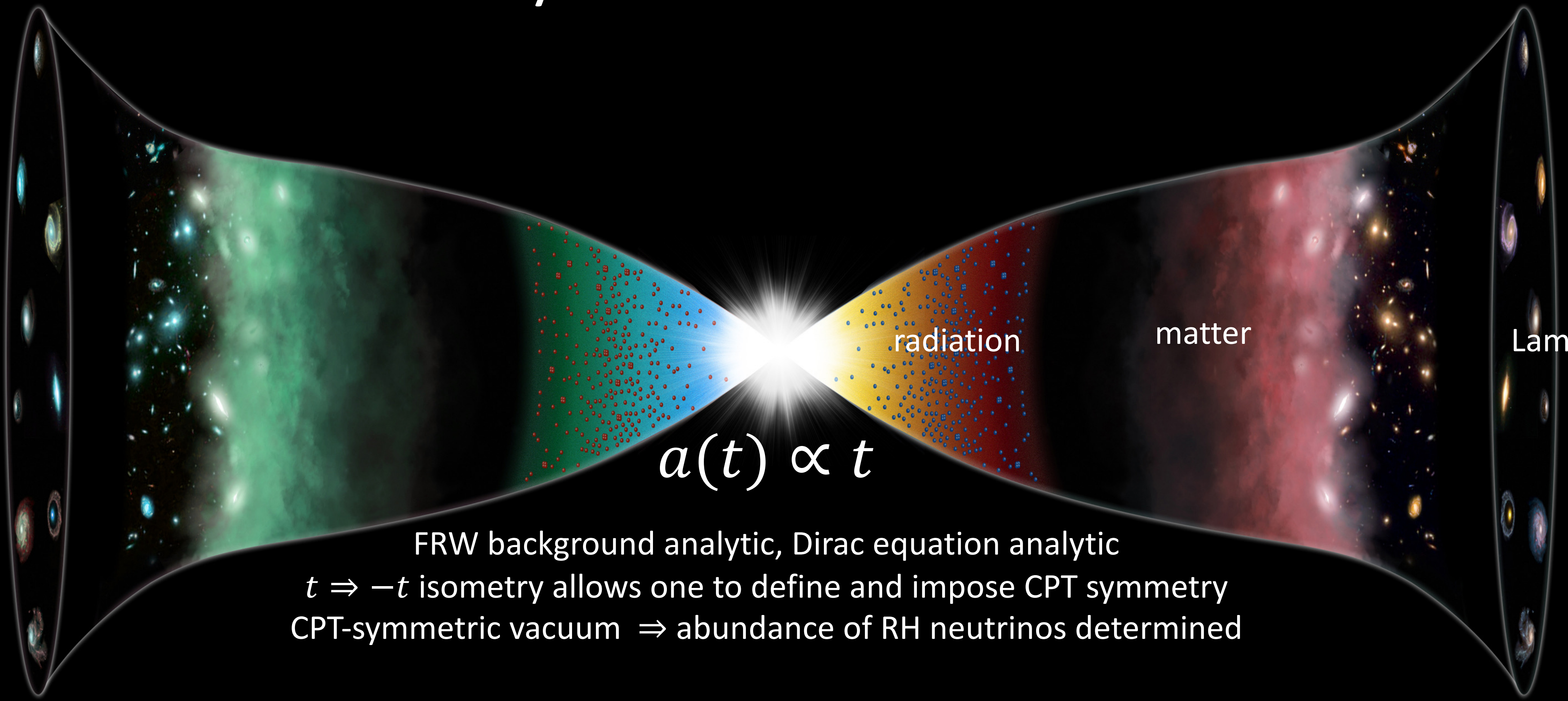
the simplest dark matter candidate

Right-handed neutrinos:



explain observed light neutrino masses (70's)

CPT symmetric universe



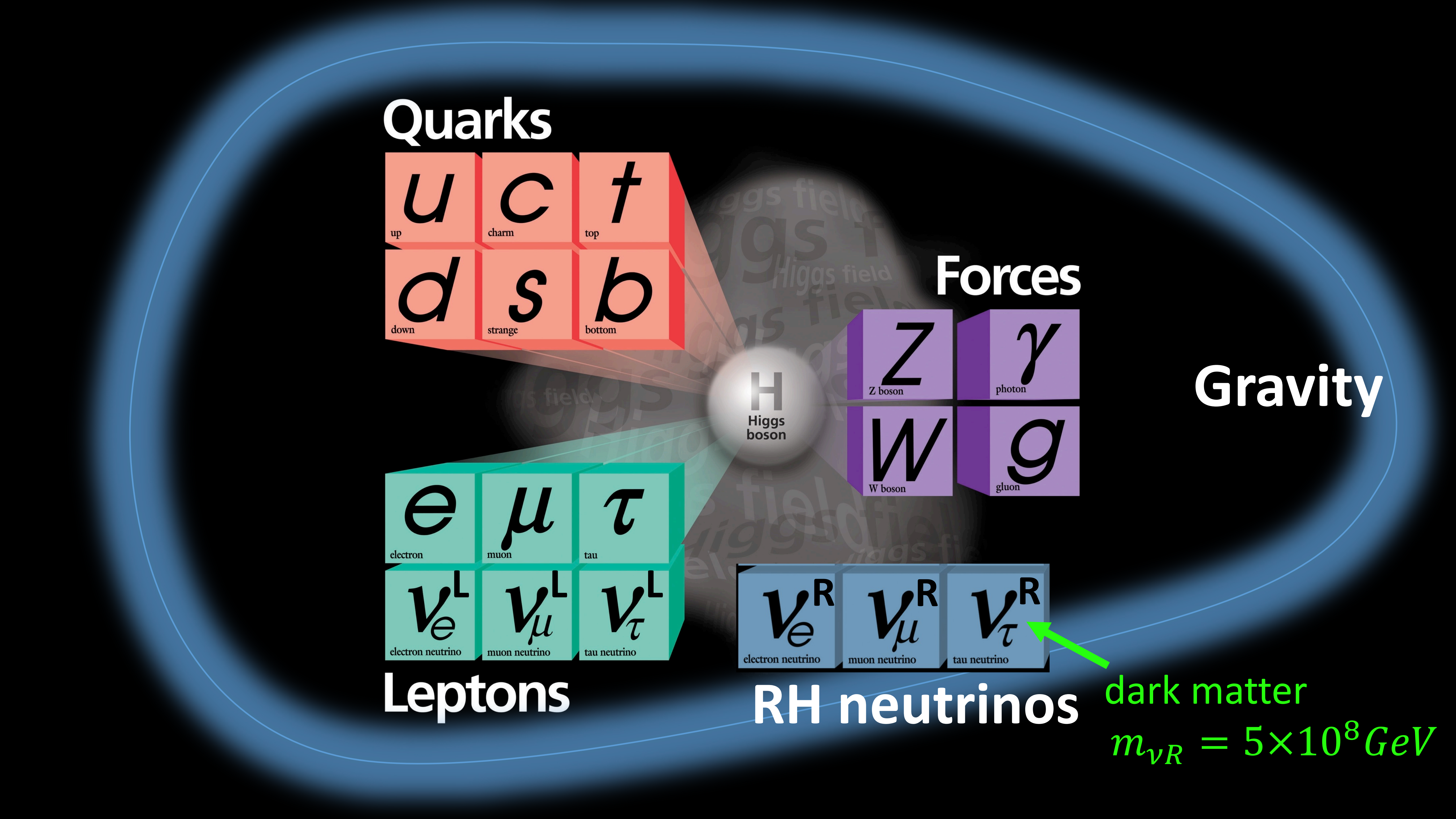
$$a(t) \propto t$$

FRW background analytic, Dirac equation analytic

$t \Rightarrow -t$ isometry allows one to define and impose CPT symmetry

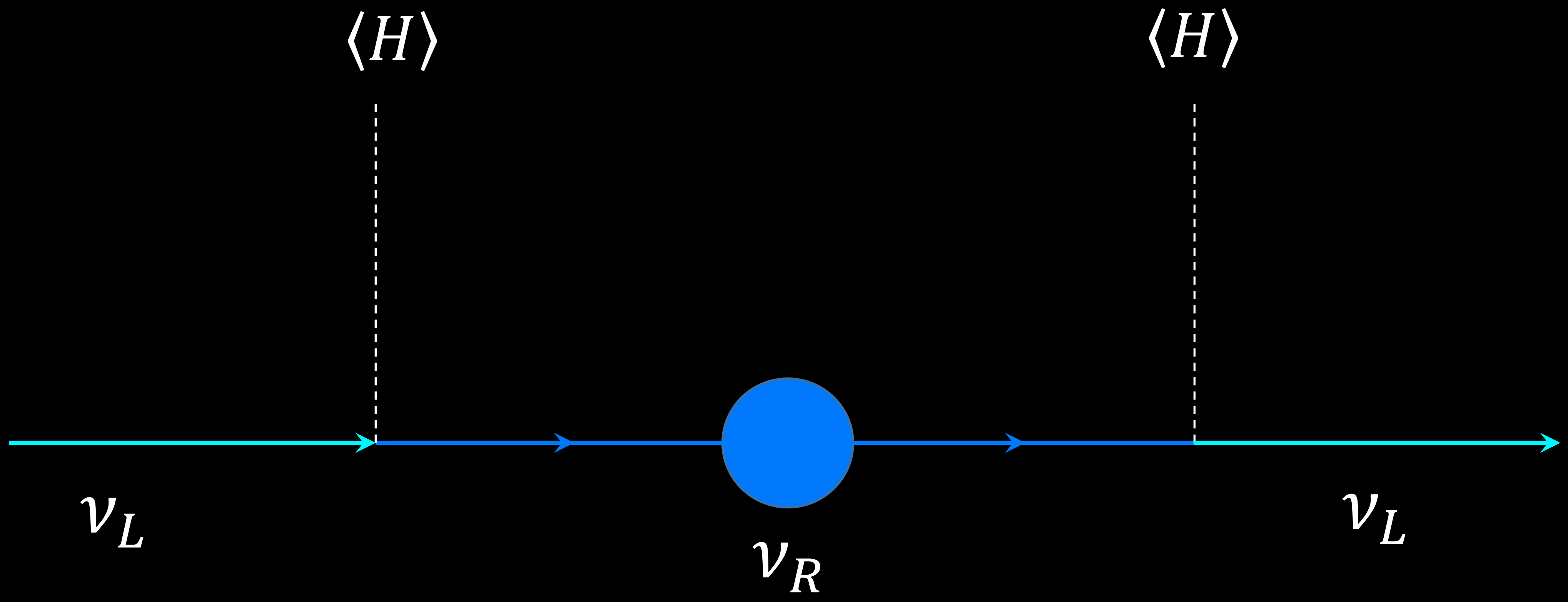
CPT-symmetric vacuum $\Rightarrow$ abundance of RH neutrinos determined

RH neutrinos produced as Hawking radiation



Quarks
u c t
up charm top
d s b
down strange bottom
H
Higgs boson
Forces
Z γ
Z boson photon
W g
W boson gluon
Gravity
e μ τ
electron muon tau
ν_e^L ν_μ^L ν_τ^L
electron neutrino muon neutrino tau neutrino
Leptons
ν_e^R ν_μ^R ν_τ^R
electron neutrino muon neutrino tau neutrino
RH neutrinos
dark matter
$m_{\nu R} = 5\times10^{8} GeV$

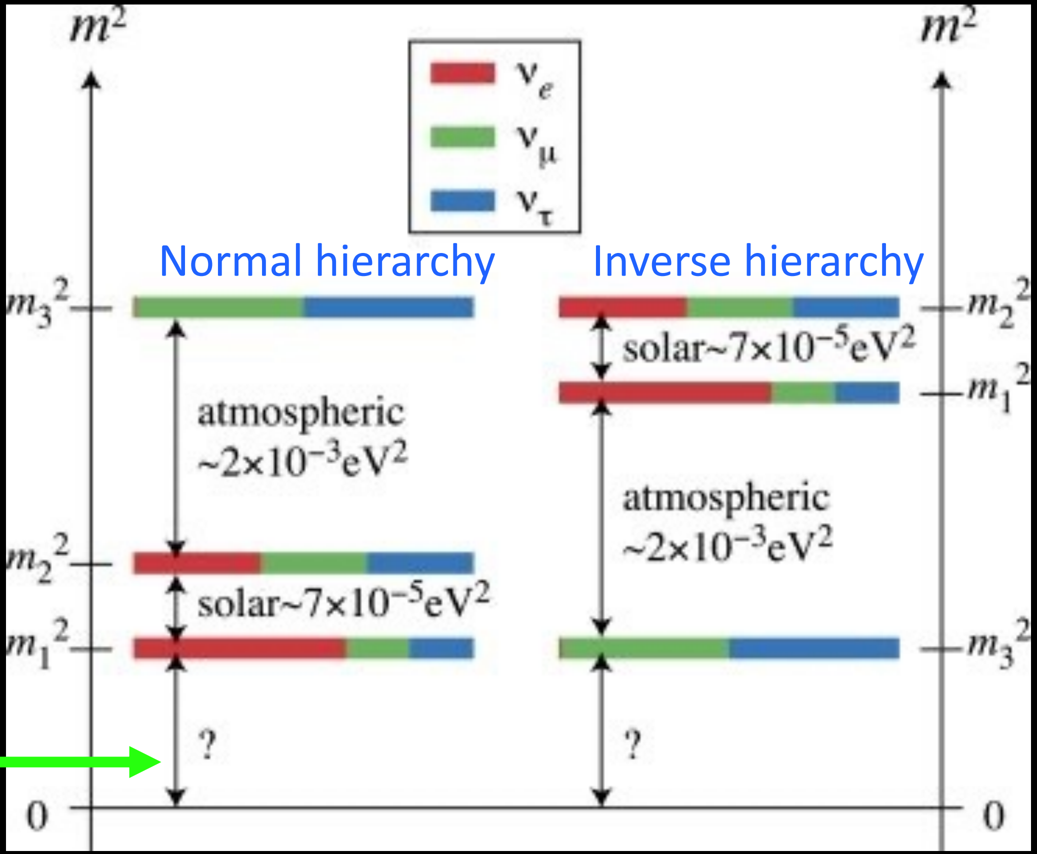
Stability of one RH neutrino $\Rightarrow$ $\mathbb{Z}_2$ symm $\Rightarrow$ lightest ν massless



Light neutrinos: observations

current data

eBOSS 2007.08991



we predict zero

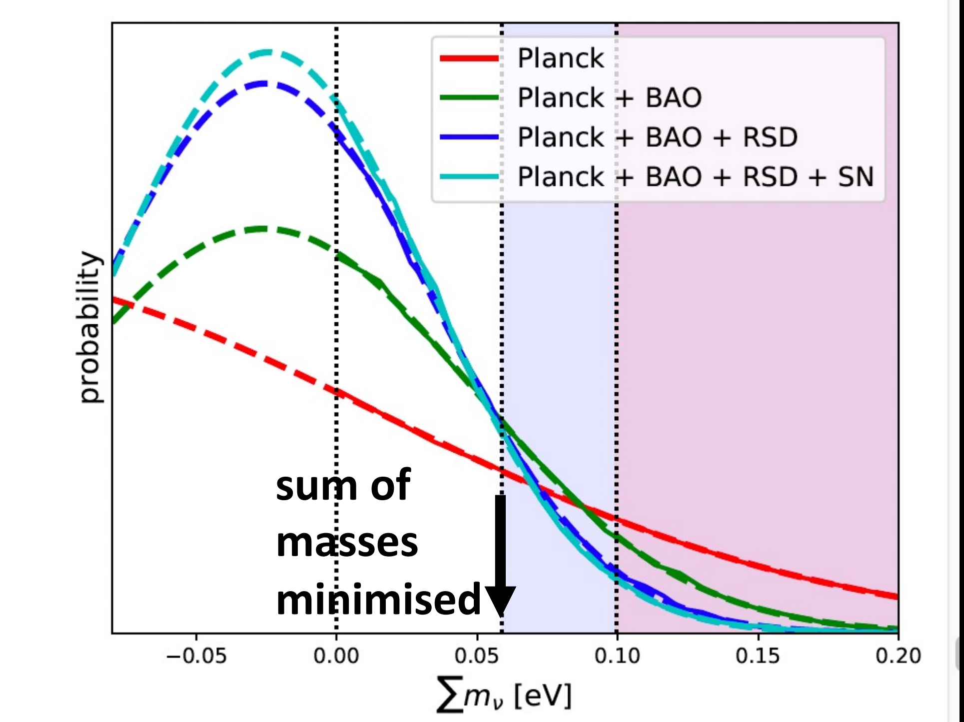
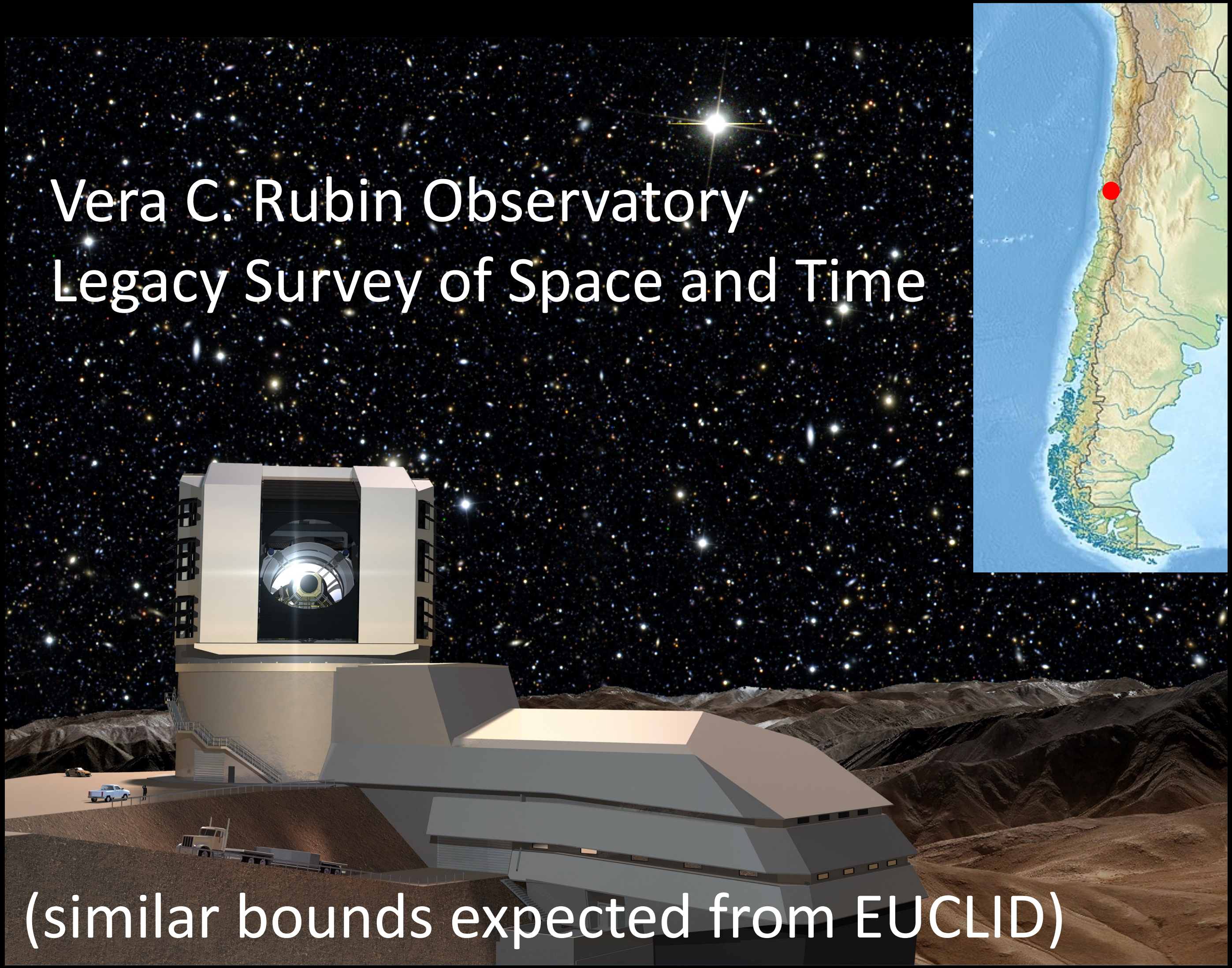


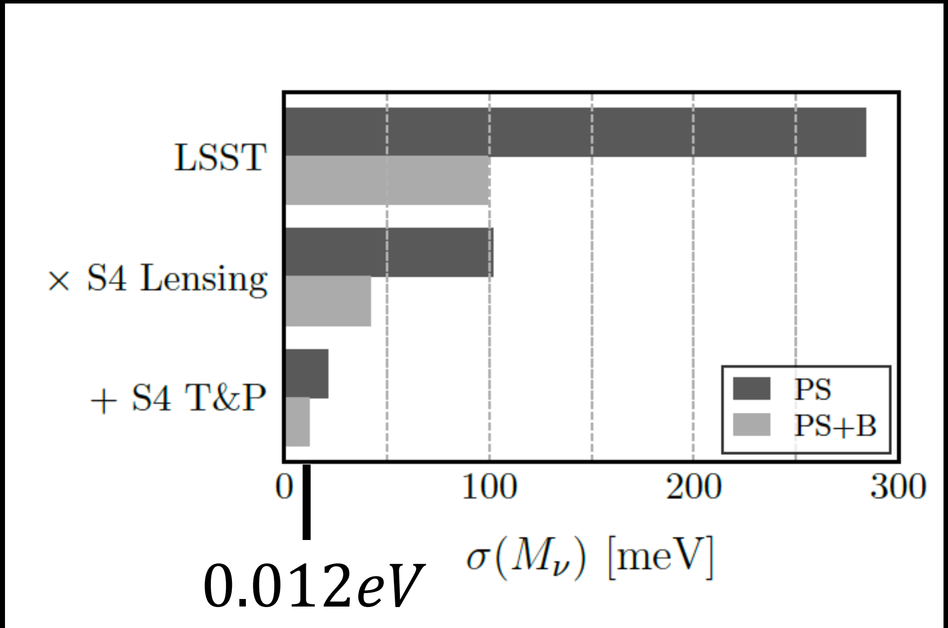
FIG. 13.— Posterior for sum of neutrino masses for selected con binations of data with a $\nu\Lambda$CDM cosmology. Dashed curves sho the implied Gaussian fits. Shaded regions correspond to lower lin its on normal and inverted hiearchies. Likelihood curves are no malized to have the same area under the curve for $\sum m_\nu > 0$.

Normal hierarchy: $M_\nu \equiv \sum m_\nu \approx 0.06\ eV$

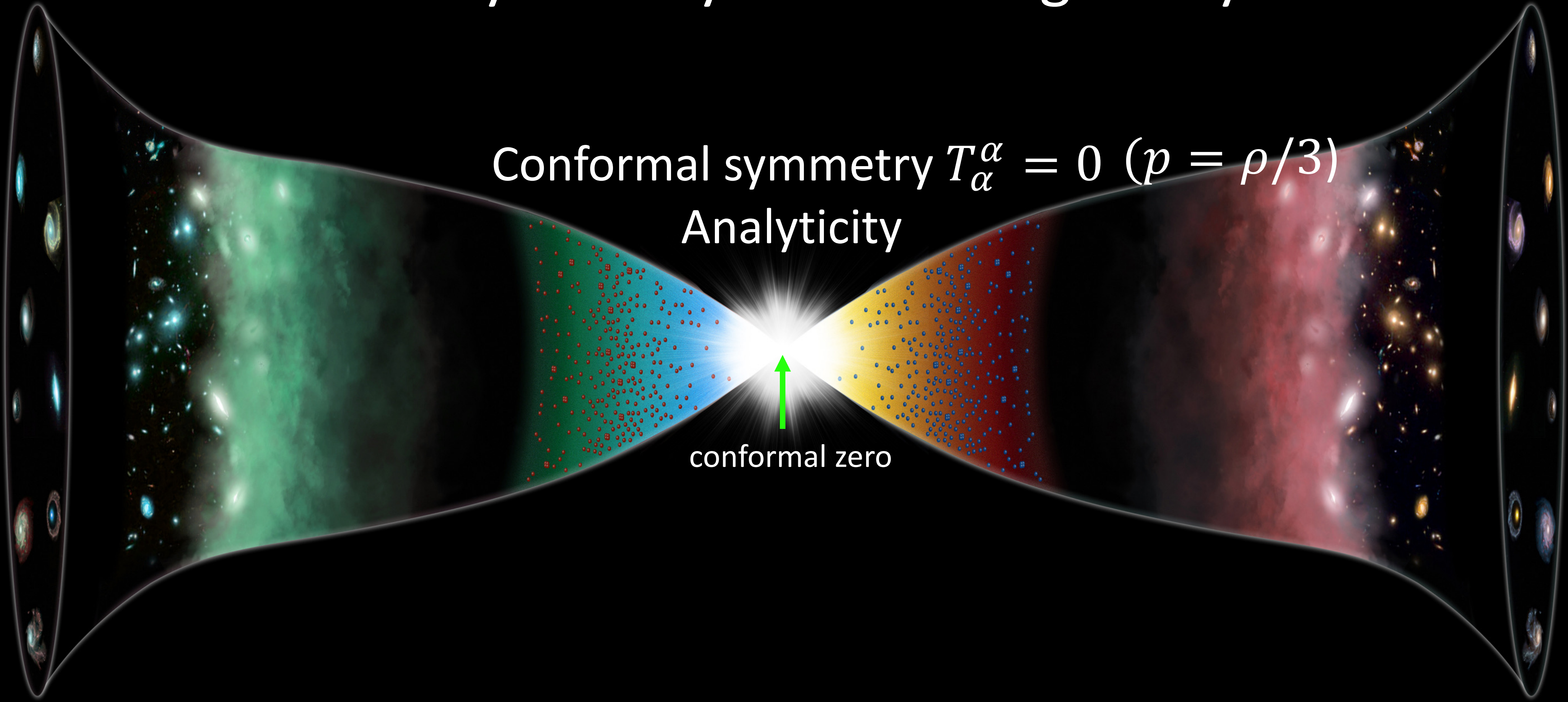
Inverted hierarchy: $M_\nu \approx 0.1\ eV$

Chen et al. 2103.01229





CPT symmetry and the singularity



CPT-symmetry imposed via the “method of images”

Striking fact: for a perfect fluid with ${T^\mu}_\mu = 0$, *i.e.*, local conformal symmetry,
$\exists\ \infty^3$ solutions to the Einstein equations which are analytic at $t = 0$:

$$ds^2 = t^2(-dt^2 + h_{ij}(t, \boldsymbol{x})dx^i\, dx^j);\ h_{ij}\,(t, \boldsymbol{x}) = h^0_{ij}(\boldsymbol{x}) + t^2\ h^2_{ij}(\boldsymbol{x}) + ..$$

regular 4-metric | regular 3-metric | determined by Einstein eqns

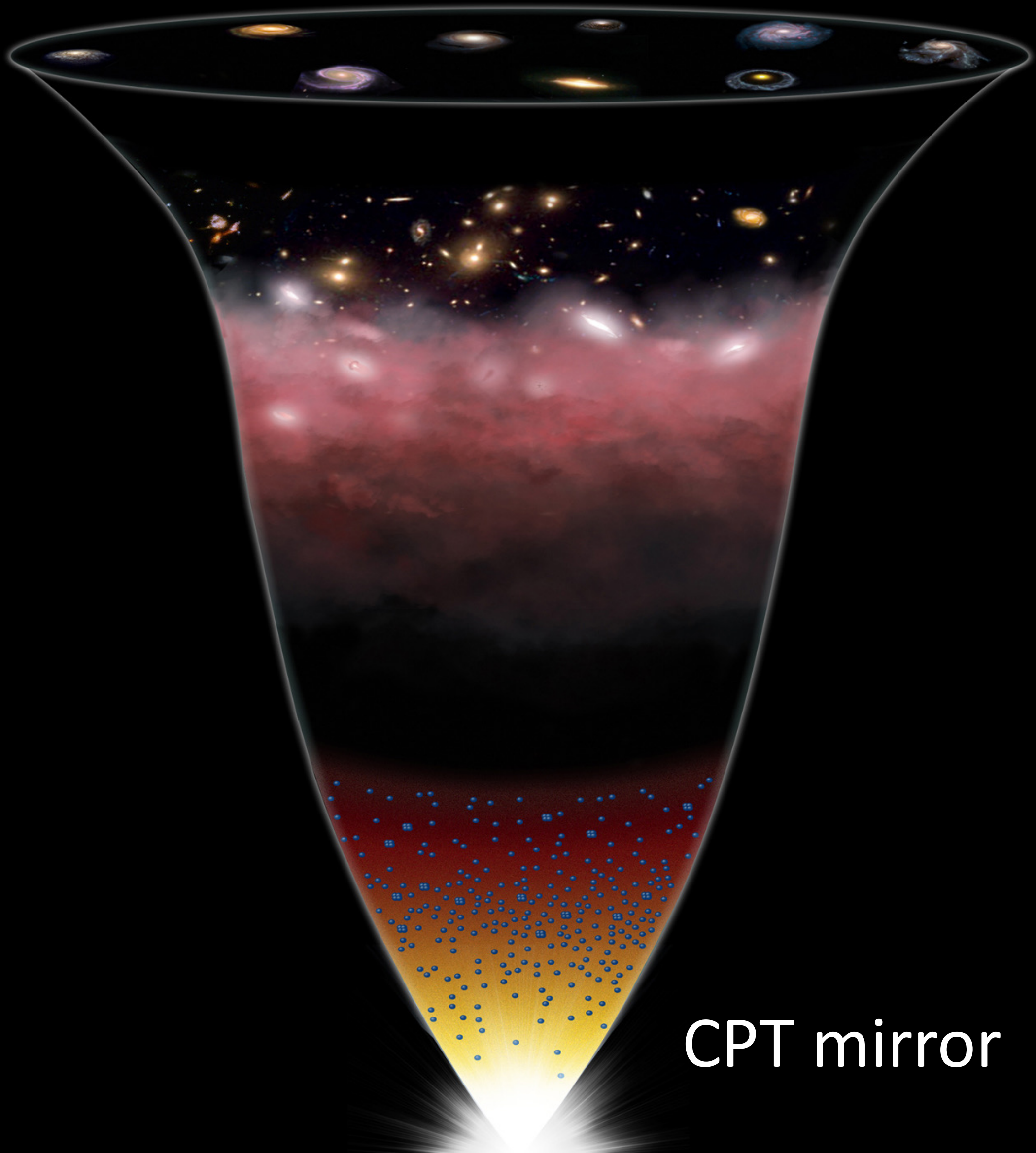
The extended spacetime is symmetric under $t \Rightarrow -t$; provides a saddle to the real-time path integral for gravity with CPT-symmetric boundary conditions

The big bang singularity is purely conformal: the Weyl tensor ${C^\lambda}_{\mu\nu\rho} = O(t^2)$ vanishes there

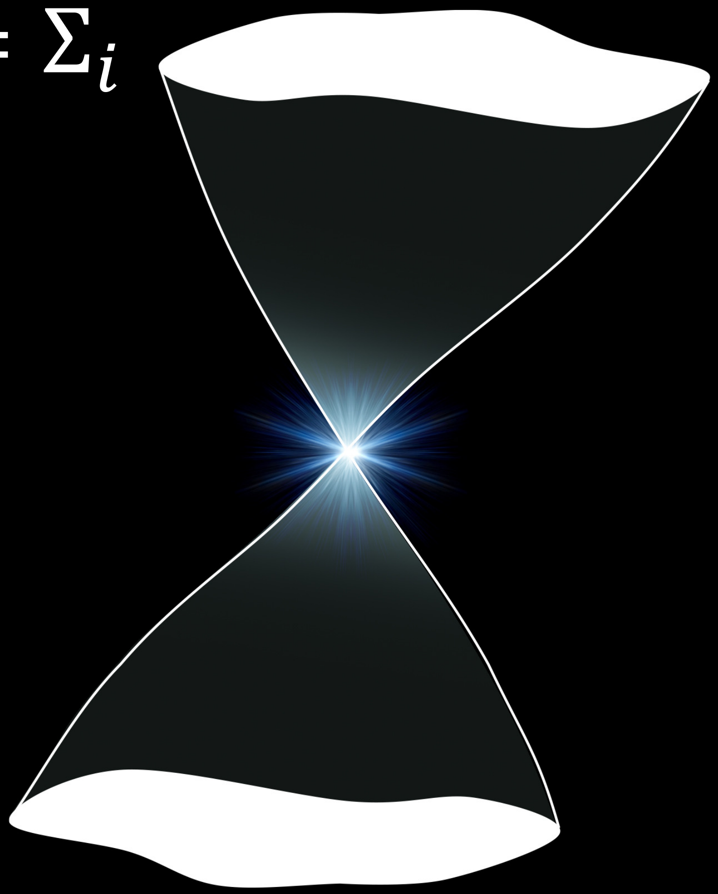
Penrose's conjecture follows

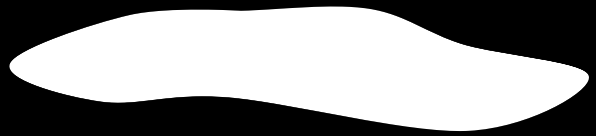
BKL or Mixmaster metrics excluded because they are singular hence not genuine saddles

classically, the big bang is an “analytic mirror”



Lorentzian path integral for gravity with CPT-symmetric boundary conditions





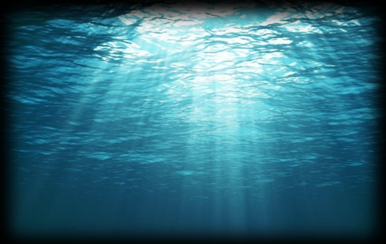
the puzzling large-scale geometry of the cosmos

Penrose

Path integrals and gravity

gauge fields

Higgs



$$\int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 + \overline{\psi} i \not{D}\psi - \lambda H\overline{\psi}\psi + |DH|^2 - V(H)\right)}$$

particles

gravity

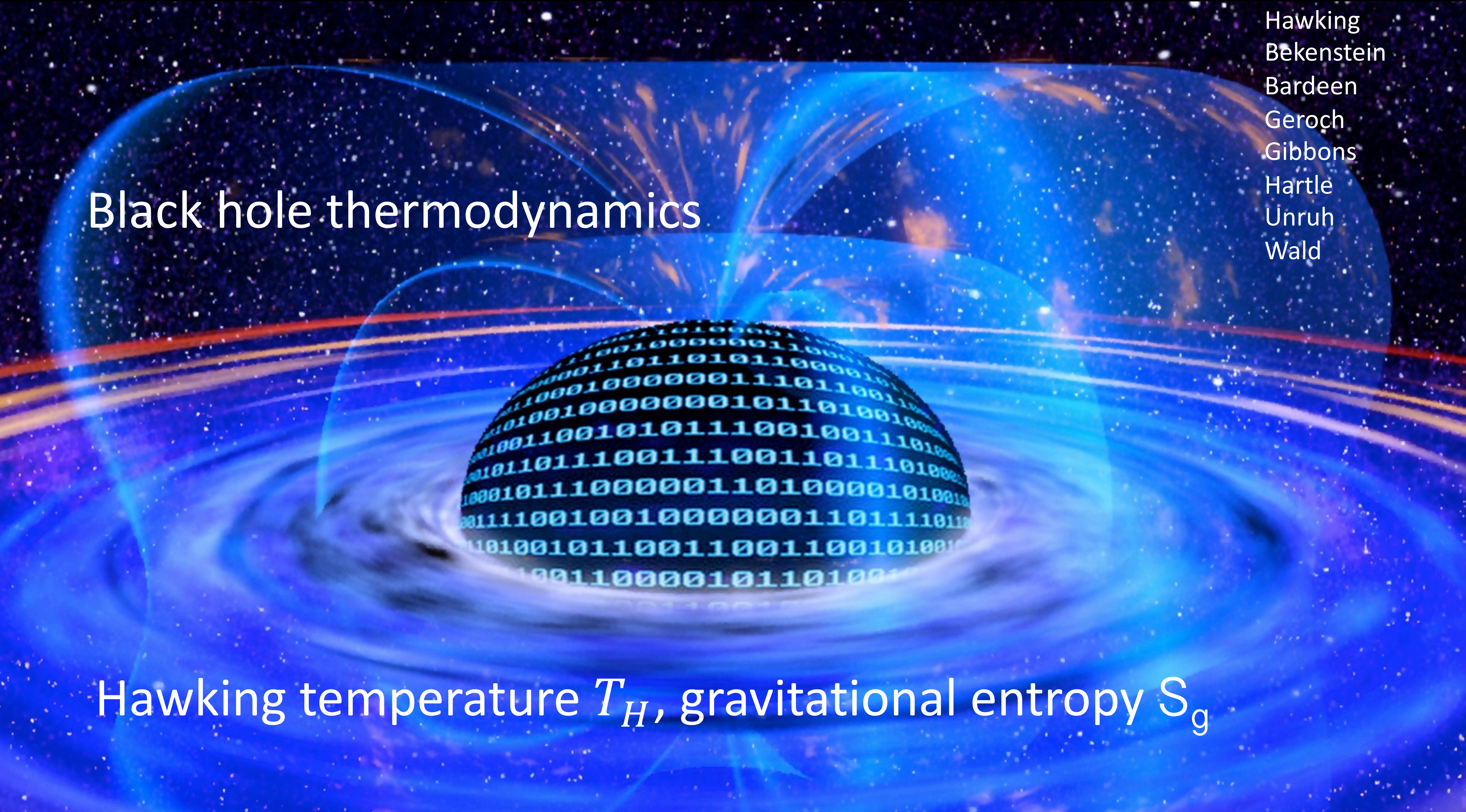
gravitational entropy

With pbcs in imaginary time, $Z = e^{S_g}$

partition function

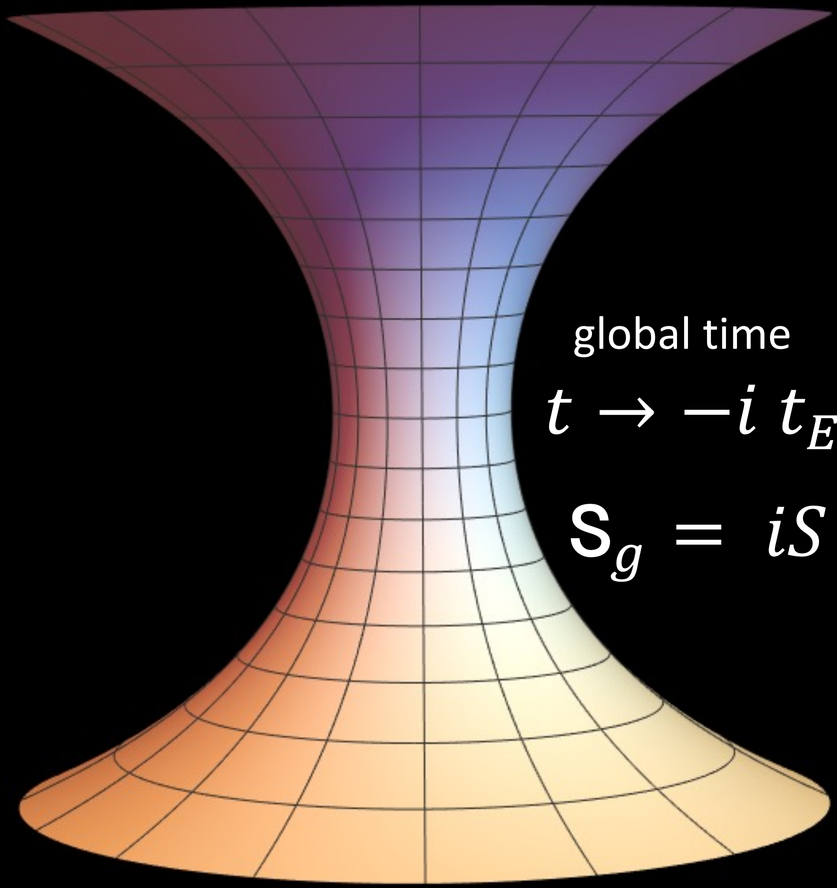
Black hole thermodynamics

Hawking
Bekenstein
Bardeen
Geroch
Gibbons
Hartle
Unruh
Wald



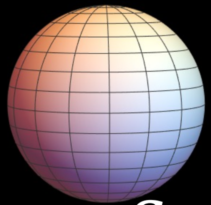
Hawking temperature T_H, gravitational entropy S_g

de Sitter



gravitational entropy from the Euclidean path integral

S^4



global time

$$t \to -i\, t_E$$

trace of Einstein

$$R = \frac{4\rho_\Lambda}{M_P^2}$$

$$\mathbf{S}_g = iS = -S_E = \int \left(\tfrac{1}{2} M_P^2 R - \rho_\Lambda\right) = \rho_\Lambda \,\mathrm{Vol} = \frac{24\pi^2 M_P^4}{\rho_\Lambda}$$

$$\equiv \mathbf{S}_\lambda \approx 3.26\times 10^{122} \text{ for measured } \rho_\Lambda$$

de Sitter Entropy

cf. entropy of radiation in our Hubble volume $\sim 10^{90}$

realistic cosmology:

scale factor

symmetric space $R^{(3)} = 6\kappa$

$$ds^2 = a(t)^2\left(-dt^2 + \gamma_{ij}dx^i dx^j\right)$$

conformal time

comoving space (assume compact)

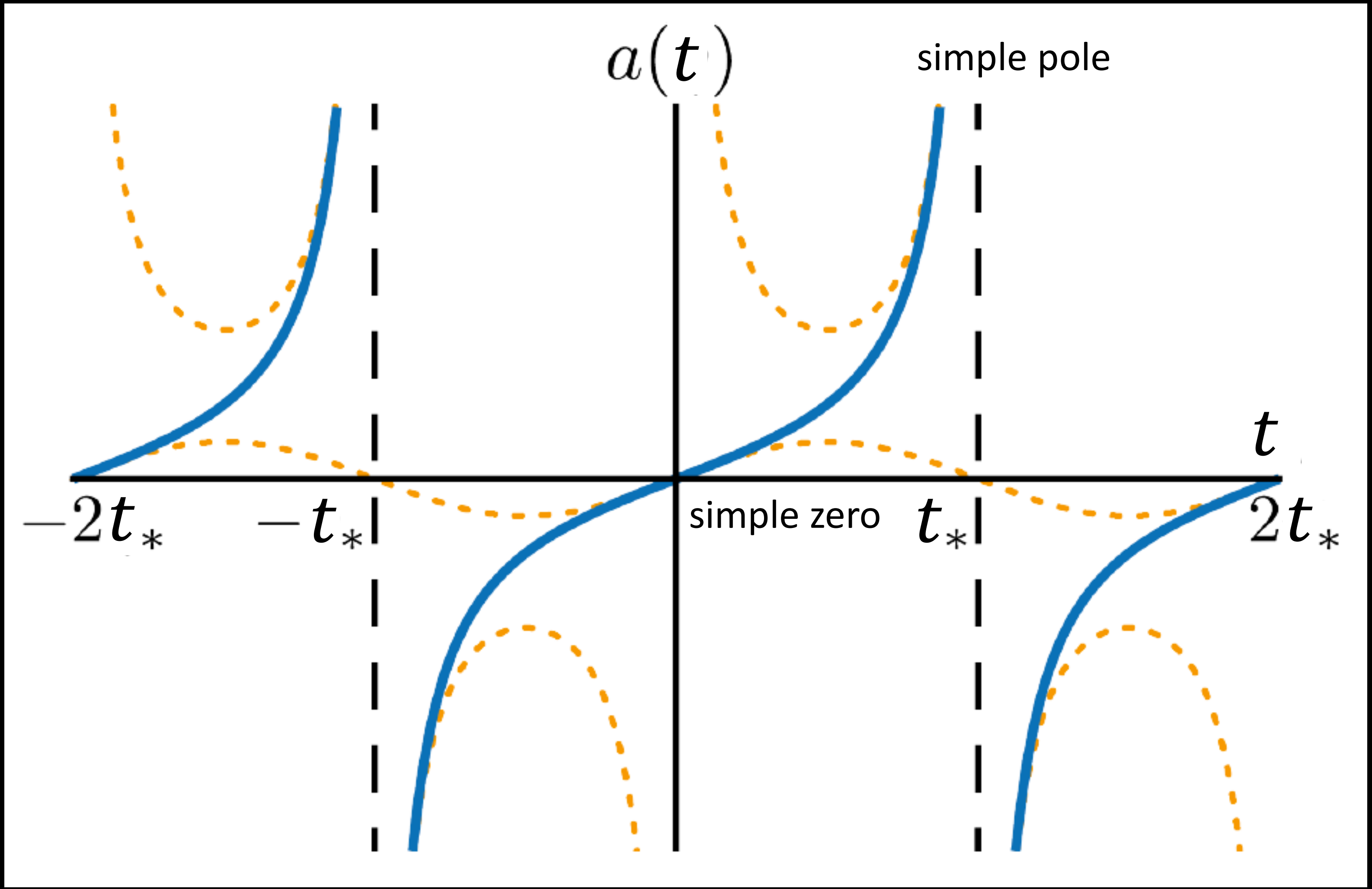
In suitable units

radiation matter space curvature Lambda

Friedmann $$3\dot{a}^2 = r + \mu a - 3\kappa a^2 + \lambda a^4$$

$$T^\mu{}_\mu = 0 \overset{\text{Einstein}}{\Longrightarrow} R = 0 \Longrightarrow a(t) \text{ analytic at } t = 0$$

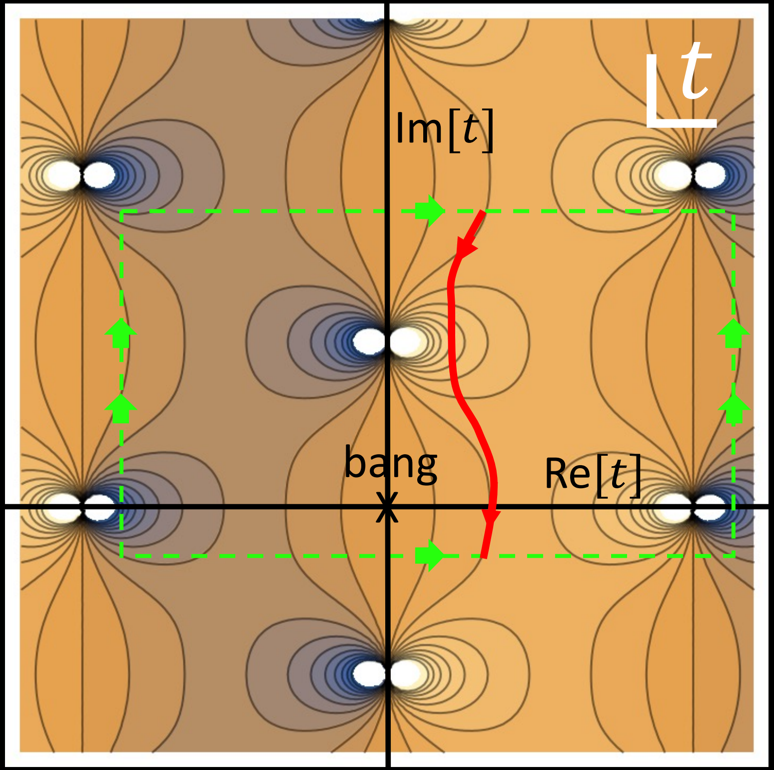
general solution has remarkable analytical properties real time



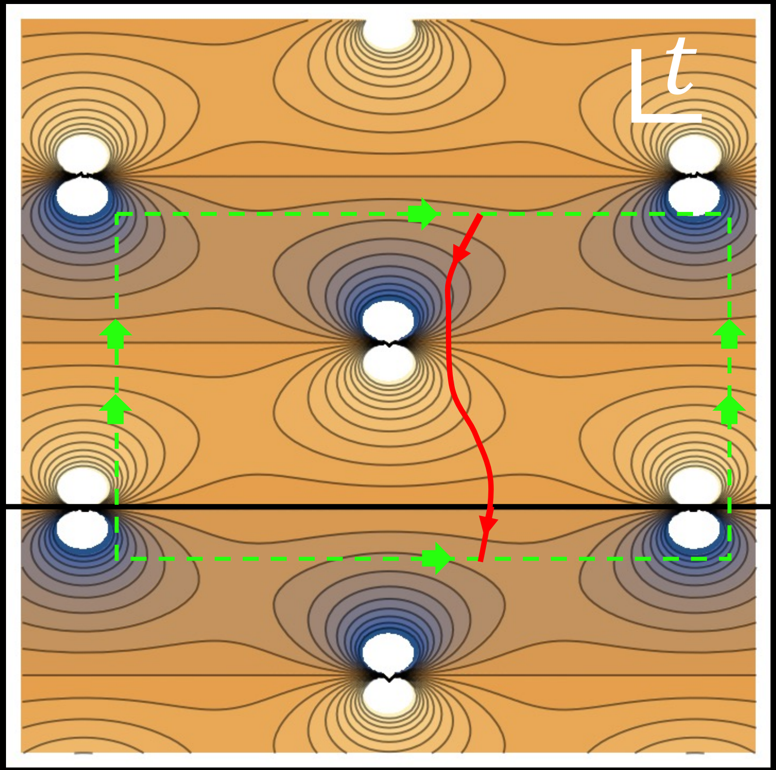
Jacobi

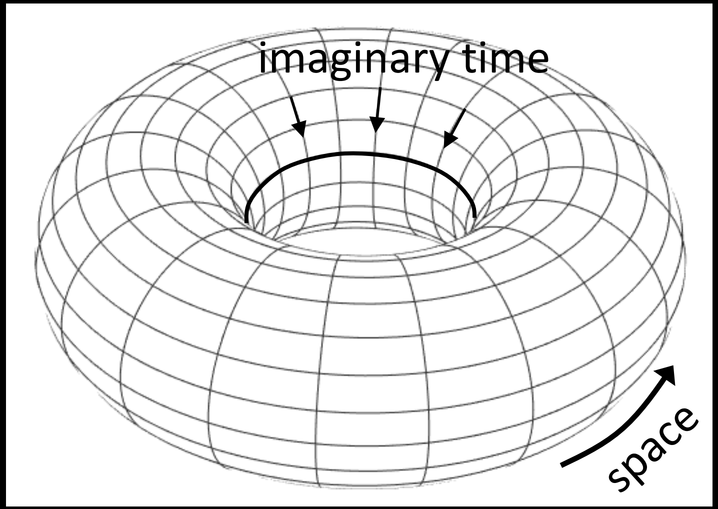
$a(t)$ is single-valued and doubly periodic in the complex t-plane: its only singularities are simple poles. The imaginary time period and the action computed over a period determine T_H and the gravitational entropy S_g

$\mathrm{Re}[a(t)]$



$\mathrm{Im}[a(t)]$





Euclidean instanton for a universe w/radiation, matter, curvature, Lambda

We recently computed S_g analytically for a general cosmology with radiation, matter, space curvature and a cosmological constant (*i.e.*, all conserved quantities).

We also treated inhomogeneities and anisotropies in cosmological perturbation theory.

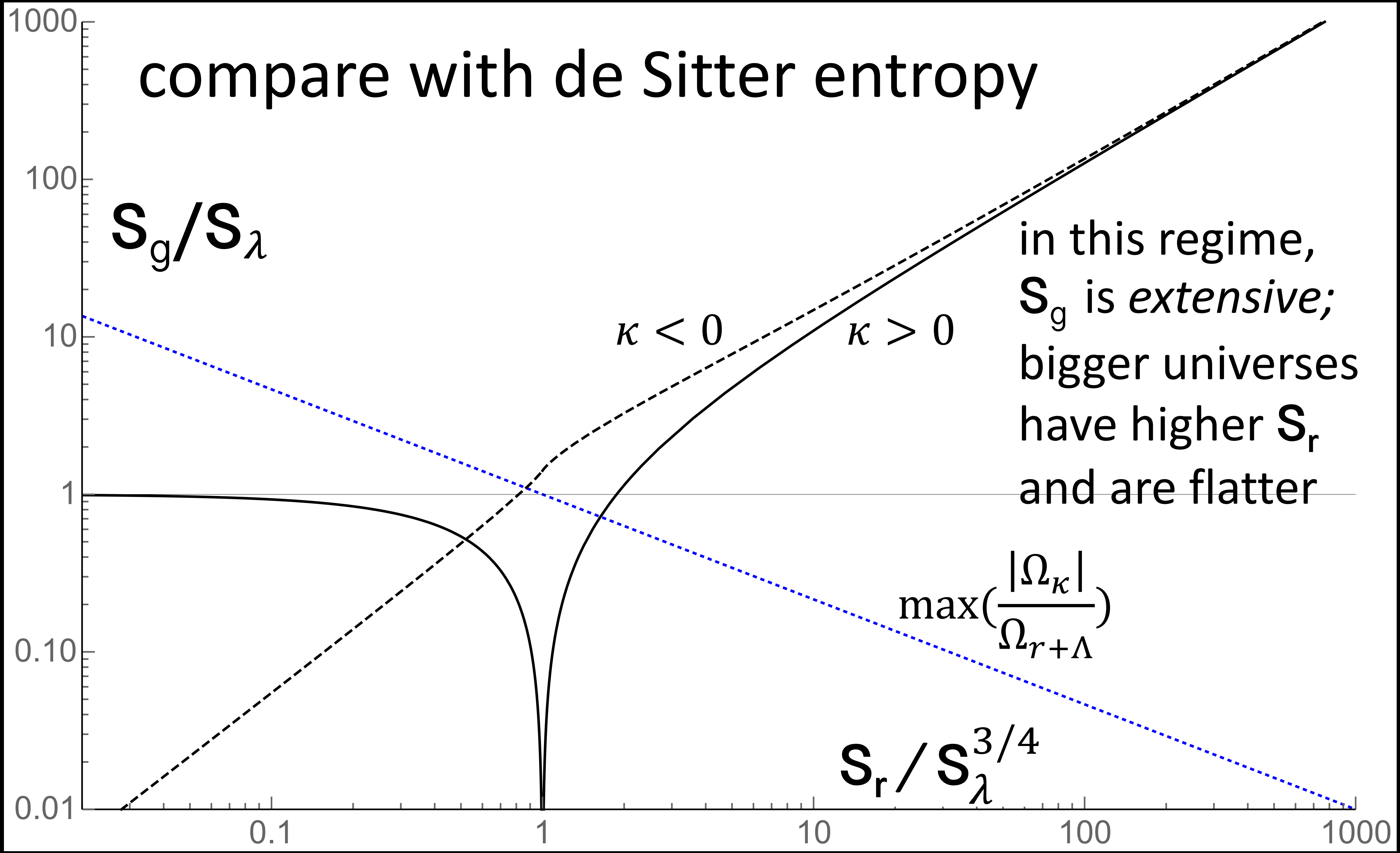
We found that S_g is greatest for:

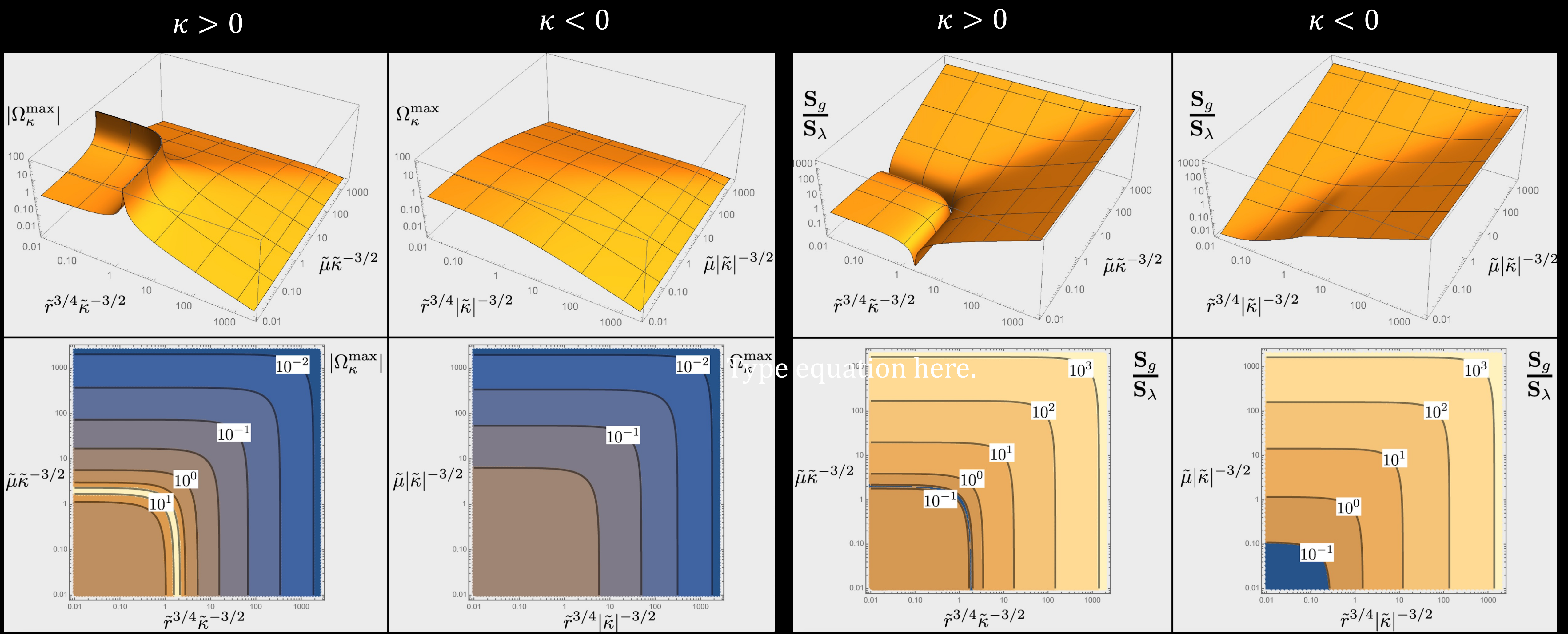
1. a spatially flat, homogeneous, isotropic universe
2. a small, positive cosmological constant

(echoing earlier arguments of Baum, Hawking, Coleman…)

Note:

S_g is the *global* entropy for the entire spacetime. It is a fixed number, independent of Lorentzian time (via Cauchy theorem), depending only on the cosmological parameters.


compare with de Sitter entropy
S_g/S_λ
in this regime,
S_g is *extensive;*
bigger universes
have higher S_r
and are flatter
$\kappa < 0$
$\kappa > 0$
$\max(\frac{|\Omega_\kappa|}{\Omega_{r+\Lambda}})$
$S_r/S_\lambda^{3/4}$
1000
100
10
1
0.10
0.01
0.1
1
10
100
1000

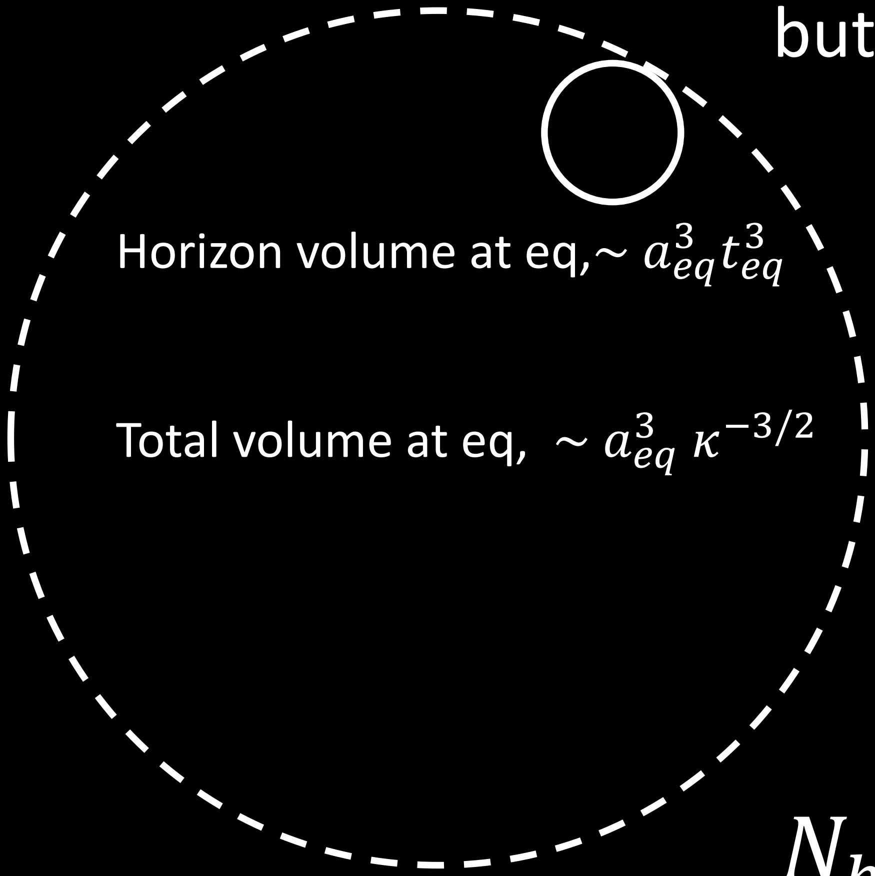


$$\tilde{r} \equiv \frac{r}{\lambda};\ \tilde{\mu} \equiv \frac{\mu}{\lambda};\ \tilde{\kappa} \equiv \frac{\kappa}{\lambda};\ \mathrm{S}_\lambda = \frac{24\pi^2}{L_{Pl}^2\lambda}$$

understanding in terms of horizons

$3\dot{a}^2 = -3\kappa a^2 + r + \lambda a^4$; equal Λ, radiation density at $a_{eq} = (r/\lambda)^{1/4}$

but for $a < a_{eq}$, $a \sim r^{1/2} t$ so $t_{eq} \sim 1/(r\lambda)^{1/4}$



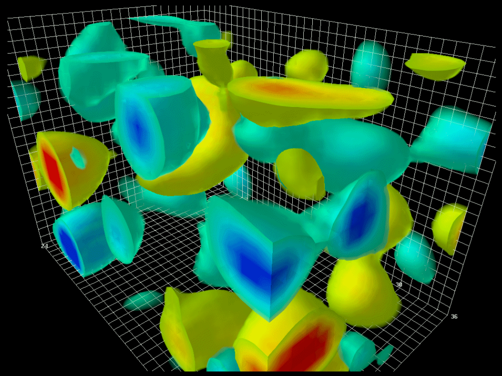
number of horizon volumes at equality

$N_{hor} \sim (\lambda r/\kappa^2)^{3/4}$

multiply by de Sitter entropy $\sim \lambda^{-1}$

$$N_{hor}\lambda^{-1} \sim (r/\kappa^2)^{3/4}\ \lambda^{-1/4} \sim \mathrm{S}_r \mathrm{S}_\lambda^{1/4}$$

Quantum fields and gravity



vacuum energy and pressure are divergent,
simple physical regularizations such as point splitting give (for, *e.g.*, Maxwell):

B.S.DeWitt, Phys. Rep. 19 (1975) 295

$$\Longrightarrow \langle T^{\mu\nu}\rangle_{vac} \sim \frac{3}{\pi^2 \Delta t^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix},$$

where $\Delta t^2 =$ invart time-like separation

Breaks Lorentz invariance! Can be renormalized away but leaves us with little physical understanding of the QFT vacuum.

Worse still are Weyl anomalies where quantum divergences spoil the local scale invariance of Maxwell and Dirac fields: violations cannot be renormalized away

Dimension zero scalars

A four-derivative, Weyl-invariant (*i.e.,* locally scale-invariant) action

$$S_4 = -\frac{1}{2}\int d^4x\sqrt{-g}\varphi\Delta_4\varphi; \quad \Delta_4 = \Box^2 + \ldots$$

Heisenberg (1957), Pauli, Thirring, Nakanishi, …
Flato, Fronsdal (‘70s , ‘80s) forerunner of AdS/CFT
Moschella+Strocchi (1989) Salvio+Strumia (2015)

φ is Heisenberg’s “dipole ghost” or Dirac’s “singleton”; a very interesting theory

Bogoliubov *et al.* recognized this as the simplest gauge theory (last, 1987 QFT text)

It has an infinite dimensional symmetry: $\varphi(x) \longrightarrow \varphi(x) + \alpha(x)$ with $\Box\alpha = 0$

which can be used to remove negative norm states (cf Gupta-Bleuler):

The only remaining physical state is the vacuum

The vacuum fluctuations are scale-invariant

$$\langle\varphi(0,\boldsymbol{x})\varphi(0,\boldsymbol{y})\rangle = \int \frac{d^3\boldsymbol{k}}{(2\,\pi)^3}\frac{e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}}{4\,k^3}$$

cf. observed Newtonian potential
in cosmology

SM + dim-zero scalar numerology:
the vacuum energy and trace anomalies (*at lowest, free-field order*)

$$E_{\boldsymbol{k}} = \frac{1}{2}\hbar k\left(\overset{\text{dim-one scalars}}{n_{s,1}} - \overset{\text{chiral fermions}}{2n_F} + \overset{\text{gauge fields}}{2n_A} + \overset{\text{dim-zero scalars}}{2n_{s,0}}\right) \quad \text{per mode } \boldsymbol{k}$$

$$\langle T^{\mu}{}_{\mu}\rangle = -a\,E + c\,C^2 \quad E = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2; \quad C^2 = C^{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

$$a = \frac{1}{360(4\,\pi)^2}\left[n_{s,1} + \frac{11}{2}\,n_F + 62\,n_A - 28\,n_{s,0}\right]$$

$$c = \frac{1}{120(4\,\pi)^2}\left[n_{s,1} + 3\,n_F + 12\,n_A - 8\,n_{s,0}\right]$$

Cancellation of all three implies $n_F = 4n_A$; $n_{s,0} = 3n_A$; $n_{s,1} = 0$.

Given SU3×SU2×U1, n_A=12, cancellation demands 3 generations of fermions (each with a ν_R)

Also requires no fundamental dim-1 scalar so the Higgs must be composite (exponentiating a dim 0 scalar gives an operator with nontrivial scaling dimension)

Promising independent (but related) development: twistor formulation of the exact path integral for self-dual gravity has anomalies which are removed *to all orders* by using dimension zero scalars

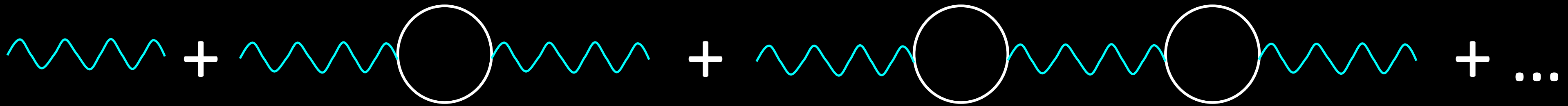
K. Costello 2111.08879, R. Bittleston *in prep*. 2023

Graviton propagator with 1 loop SM corrections

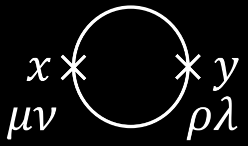
Tomboulis 70's

Han Willenbrook

Donoghue Menezes



Loop is given by the Fourier transform of the stress-energy correlator: for a CFT,



$$= \langle T^{\mu\nu}(x) T^{\rho\lambda}(y) \rangle = C^T \frac{1}{4\,\pi^4 x^8} I^{\mu\nu,\rho\lambda}(x-y)$$

where $I^{\mu\nu,\rho\lambda}(x) = \frac{1}{2}\big(I^{\mu\rho}(x) I^{\nu\lambda}(x) + I^{\mu\lambda}(x) I^{\rho\nu}(x)\big) - \frac{1}{4}\eta^{\mu\nu}\eta^{\rho\lambda}$ and $I^{\mu\nu}(x) = \eta^{\mu\nu} - 2\frac{x^\mu x^\nu}{x^2}$

$C^T = \frac{4}{3}[n_{s,1} + 3\,n_F + 12\,n_A - 8\,n_{s,0}] \equiv \frac{4}{3} n_{eff}$ ($\propto$ coefft c of Weyl squared in the trace anomaly)

Projector onto spin 2 component

- gauge invariant

Dim reg and min sub $\Rightarrow$ $$D^{\alpha\beta,\mu\nu}(k) = \frac{P^{\alpha\beta,\mu\nu}(k)}{k^2\left((1 - \frac{n_{eff}}{240\pi} G k^2 \ln(-\frac{k^2}{\mu^2})\right)}$$

SM corrections to the graviton propagator are problematic:

1. Inconsistent with Källén-Lehmann repn. $D(k) = \int_0^\infty dm^2\, \rho(m^2) \frac{1}{k^2 - m^2 + i\varepsilon}$

(follows from Poincare invariance and positivity of the physical Hilbert space)

2. Specifically, resummed $D(k)$ (i) falls off as $|k|^{-4}$ at large $|k|$

(ii) has complex (acausal) poles on physical sheet

Similarly, dim-0 scalar loops alone violate K-L: (i) $|k|^{-4}$ fall off; (ii) a tachyonic pole

BUT:

SM + dim-0 combination is consistent with Poincaré, positivity and microcausality
(at one loop in SM gauge+fermion fields: we are now examining higher orders)

A Minimal Explanation of the Primordial Cosmological Perturbations

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[2] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada, N2L 2Y5*

We outline a new explanation for the primordial density perturbations in cosmology. Dimension zero fields are a minimal addition to the Standard Model of particle physics: if the Higgs doublet is emergent, they cancel the vacuum energy and both Weyl anomalies without introducing any new particles. Furthermore, the cancellation explains why there are three generations of elementary particles, including RH neutrinos. We show how quantum zero point fluctuations of dimension zero fields seed nearly scale-invariant, Gaussian, adiabatic density perturbations. We determine their amplitude in terms of Standard Model couplings and find it is consistent with observation. Subject to two simple theoretical assumptions, both the amplitude and the tilt we compute *ab initio* agree with the measured values inferred from large scale structure observations, with no free parameters.

arXiv:2302.00344[hep-ph]

primordial perturbations from dim-0 fields and the SM

Running couplings violate scale symmetry: at high temperature,

$$T_\beta^{SM} \equiv \langle T^{SM\mu}{}_{\mu} \rangle_\beta = 3P - \rho \approx \sum c_i \alpha_i^2 T^4 \equiv c_\beta^{SM} T^4; \text{ in SM, } c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$$

This anomalous trace can be cancelled by introducing a linear coupling in the effective action,

$$\Gamma^\varphi = \sum_{j=1}^{n_{s,0}} \frac{1}{2} \int -a\varphi_j \Delta_4 \varphi_j + \left[a\left(E - \frac{2}{3}\Box R\right) + cC^2 - n_{s,0}^{-1} T_\beta^{SM}\right]\varphi_j$$

non-Weyl invariant term used to cancel anomalies

(generalizing sigma models in string theory)

The final linear term is chosen to cancel the trace anomaly due to running couplings at high T
It corrects the Einstein-fluid equations, converting quantum correlations in the dim-0 fields into large scale curvature fluctuations: Friedmann equation becomes

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_r a^4 (1 + c_\varphi \bar{\varphi}(x)) \text{ with } \bar{\varphi}(x) = n_{s,0}^{-1} \sum \varphi_j(x), \; c_\varphi = c_\beta^{SM} / \left(\frac{\pi^2}{30}\mathcal{N}_{eff}\right), \mathcal{N}_{eff} \approx 106\tfrac{1}{4}$$

Conformal factor translates directly into "comoving curvature perturbation" $\mathcal{R}(x) = \frac{1}{4} c_\varphi \bar{\varphi}(x)$
(adiabatic, Gaussian, scalar: no primordial long-wavelength gravitational waves)

Spectral tilt

Dominated by QCD: asymptotic freedom $\Longrightarrow$ red tilt!

To understand quantitatively, consider the trace anomaly (for QCD)

$S = -\int \frac{1}{4} F^2 \Rightarrow - \int \frac{1}{4g^2} F^2.\quad \alpha \equiv \frac{g^2}{4\pi};\ \mu\partial_\mu \alpha \equiv \beta_\alpha; \Longrightarrow \mu\partial_\mu S = \int \frac{\beta\alpha}{\alpha}\frac{1}{4g^2} F^2 \Rightarrow \int \frac{\beta\alpha}{4\,\alpha} F^2$

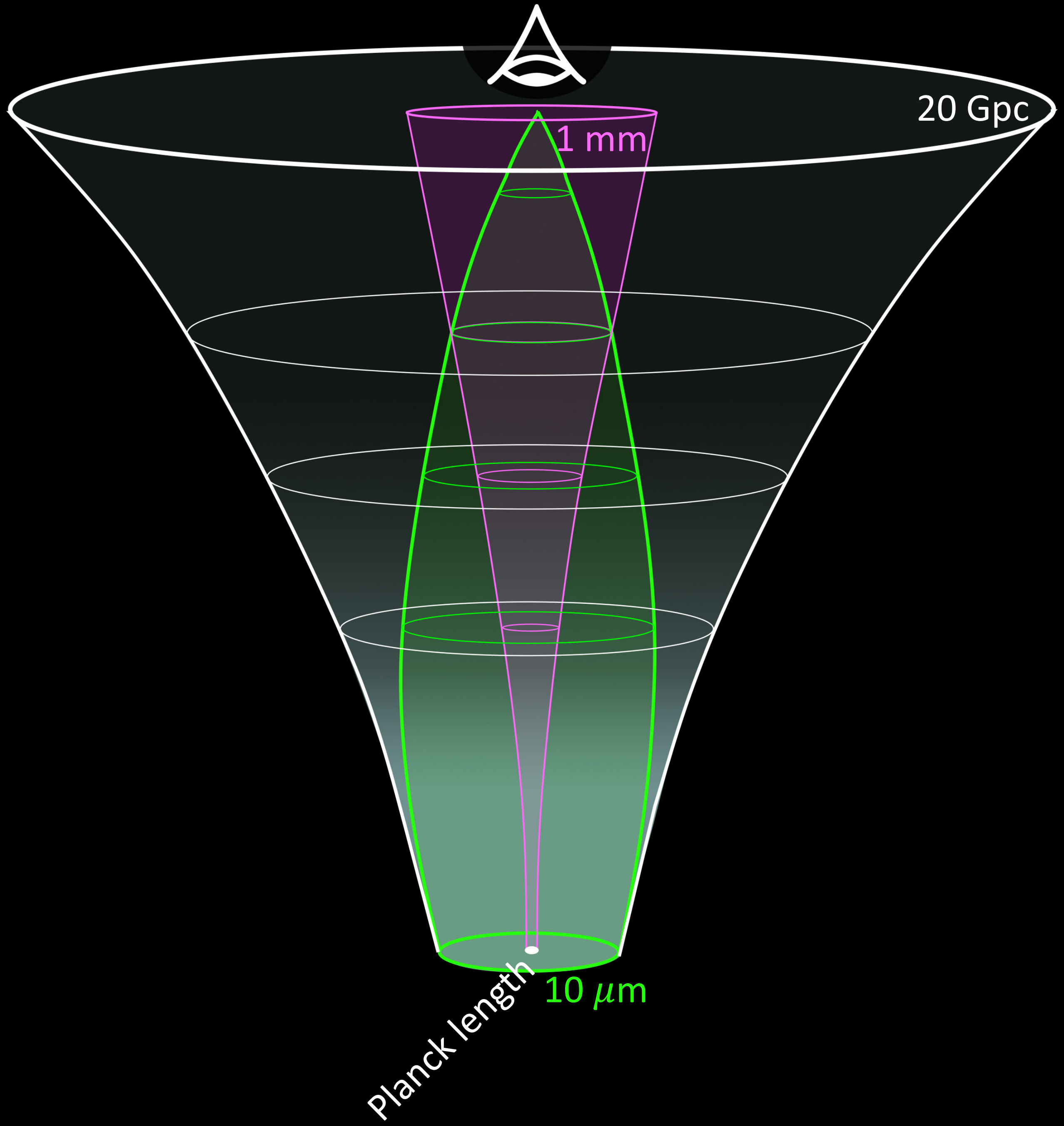
$\Longrightarrow T_\lambda^\lambda = \frac{\beta\alpha}{4\,\alpha} F^2; \quad \frac{\beta\alpha}{\alpha} = -(11 - \frac{2}{3} n_f)\frac{\alpha}{2\pi} T^4; \quad \langle F^2 \rangle_\beta = \frac{2\pi\alpha}{9}(12+5n_f)T^4$

running coupling: energy scale of φ plasma interactions: energy scale of T

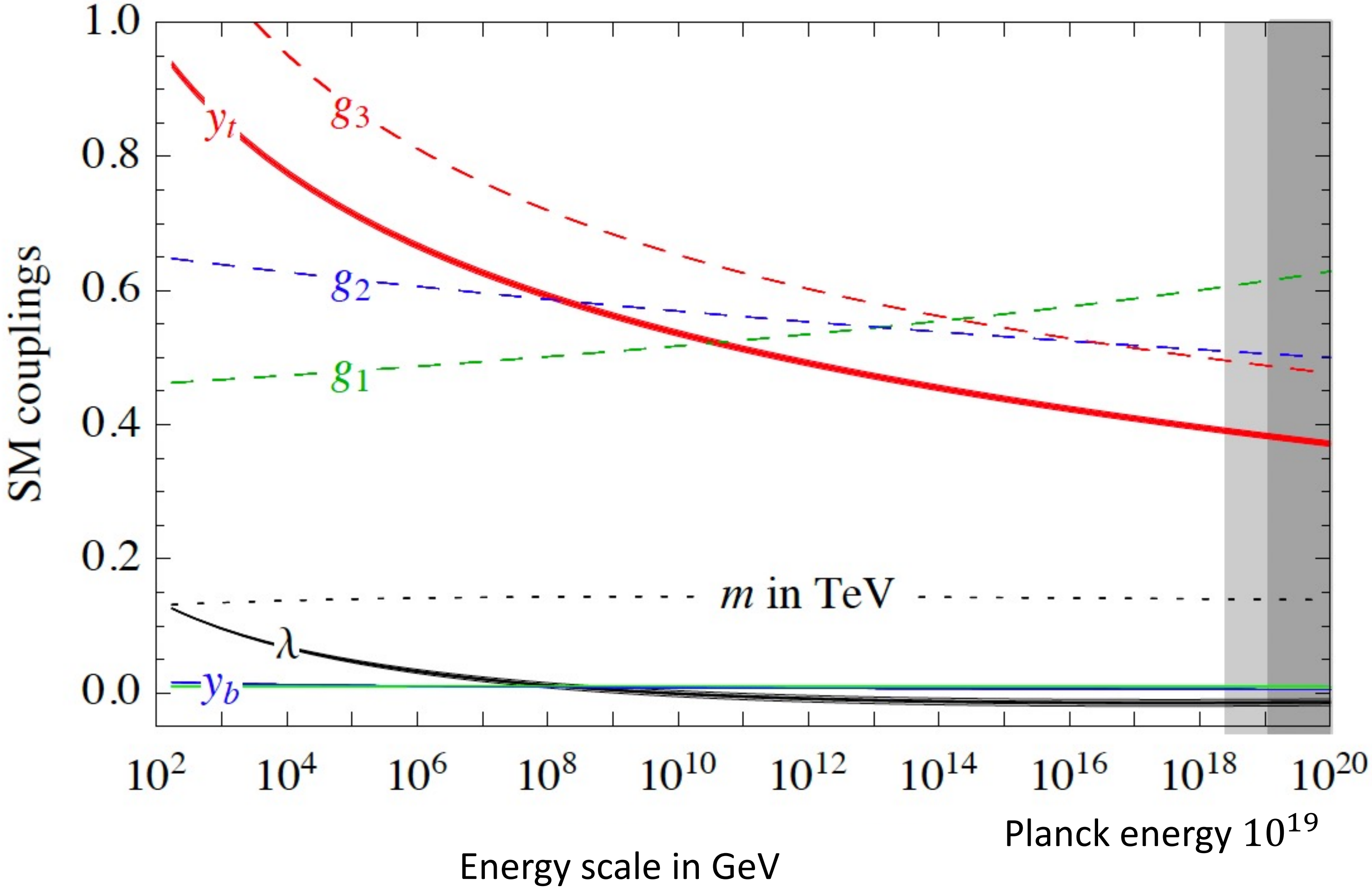
Thus, $\mathcal{P}_\mathcal{R}(k)$ scales with k as $\alpha^2(k)$; $n_s - 1 \equiv \frac{d \ln \mathcal{P}_\mathcal{R}(k)}{\mathrm{d} \ln k} = 2\frac{\beta\alpha}{\alpha} = -\frac{7}{\pi}\alpha_{QCD}(M_P)$

The red tilt is a critical exponent which can be computed perturbatively

If so, we can extrapolate over 30 orders of magnitude in length scale…


20 Gpc
1 mm
Planck length
10 μm

Buttazzo et al
1307.3536
[hep-ph]



Comparison with observation

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{3^2 5^2}{7(2\,\pi)^4}\left(\frac{c_\beta^{SM}}{\mathcal{N}_{eff}}\right)^2 \left(\frac{k}{k_P}\right)^{-\frac{7\alpha_3}{\pi}};$$ k_P= comoving Planck wavenumber

with $c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$ and $\mathcal{N}_{eff} = 106\frac{1}{4}$ (to lowest order, neglect Higgs)

Now use $(k_P/k_*)^{1-n_s} = 14.8 \pm 5.1,$ $k_* \equiv$0.05 Mpc^{-1}

Thus, we find $\mathcal{P}_{\mathcal{R}} = A\left(\frac{k}{k_*}\right)^{n_s-1}$, $A = (13 \pm 5)\times 10^{-10}$; $n_s = 0.958$

cf. Planck satellite: $A = (21 \pm 0.3)\times 10^{-10}$; $n_s = 0.959 \pm 0.006$

Comparison with observation

$11 - \frac{2}{3} n_f$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{3^2 5^2}{7(2\,\pi)^4}\left(\frac{c_\beta^{SM}}{\mathcal{N}_{eff}}\right)^2 \left(\frac{k}{k_P}\right)^{-\frac{7\alpha_3}{\pi}};$$ k_P= comoving Planck wavenumber

with $c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$ and $\mathcal{N}_{eff} = 106\frac{1}{4}$ (to lowest order, neglect Higgs)

Now use $(k_*/k_P)^{n_s-1} = 14.8 \pm 5.1,$ $k_* \equiv 0.05$ Mpc^{-1}

Thus, we predict $\mathcal{P}_{\mathcal{R}} = A\left(\frac{k}{k_*}\right)^{n_s-1},$ $A = (13 \pm 5)\times 10^{-10};\ n_s = 0.958$

cf. Planck satellite: $A = (21 \pm 0.3)\times 10^{-10};\ n_s = 0.959 \pm 0.006$

prediction will be tested further as observations and theory improve

summary

analytic extension of cosmological solutions of the Einstein equations lead to

- a new picture of the big bang singularity as a CPT “mirror”
- a calculation of the gravitational entropy for cosmologies

providing new explanations and predictions for

- the large-scale homogeneity, isotropy and flatness of the cosmos (and a hint about Lambda)
- the dark matter
- the arrow of time and the strong CP problem

including dimension zero scalars

- cancels the vacuum energy and both Weyl anomalies at leading (free field) order
- explains why there are 3 generations of SM fermions, each including a RH neutrino
- explains the amplitude, tilt and character of the primordial perturbations
- requires the Higgs to be emergent/composite, a new approach to the gauge-gravity hierarchy?

All without adding any new propagating degrees of freedom to the SM and Einstein gravity

These are very encouraging signs but much remains to be understood

Thank You!

Boyle, Finn, NT Phys. Rev. Lett. 121 (2018) 251301; Annals of Physics 438 (2022) 168767
arXiv: 2109.06204, 2110.06258, 2201.07279, 2208.10396, 2210.01142, 2302.00344