THE DIFFERENT FACES OF COSMOGRAPHY

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Scuola Superiore Meridionale

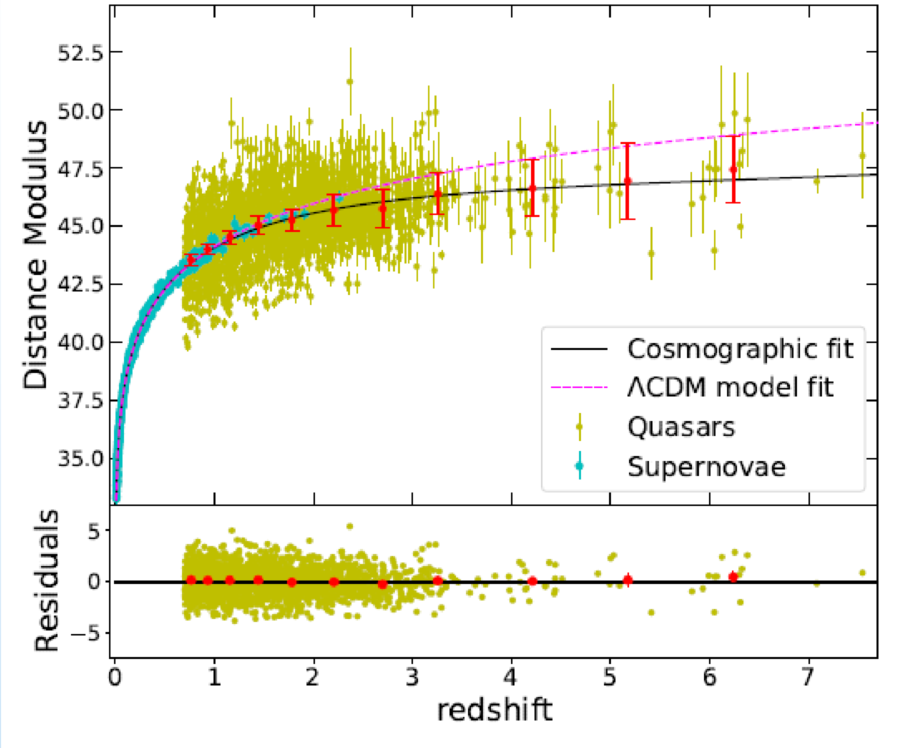
DΛrk Energy:
from Fundamental Theories to Observations (and back)
Frascati, September 11-15, 2023

OUTLINE

- ❑ what is cosmography?
- ❑ why it is useful?
- ❑ How is a cosmographic series constructed?
- ❑ what are pros and cons?
- ❑ Current research line in cosmography

Cosmography is a mathematical technique used to map the general characteristics of a given behavior.

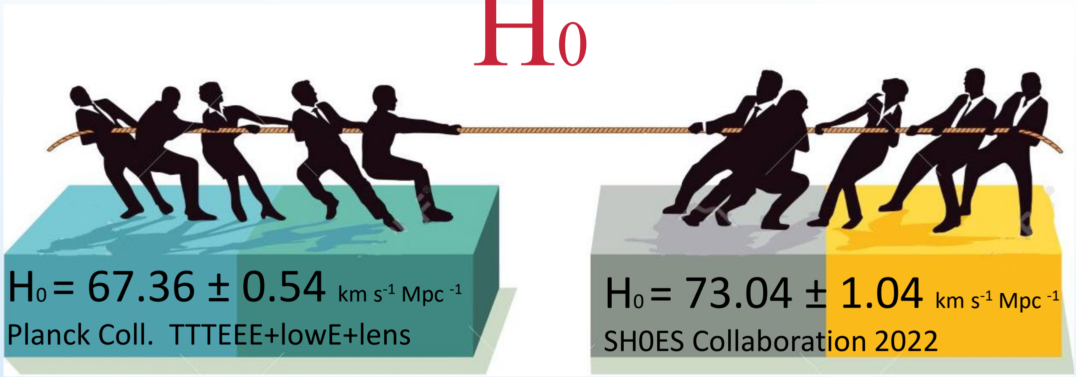
In **astrophysics**, it is used to determine the large-scale matter distribution and kinematics of the observable universe.



Credit: Bargiacchi *Astron.Astrophys.* 649 (2021) A65

In **cosmology**, it can be used in place of the DE concept into a cosmological model to parameterize the evolution of the recent universe.

H_0

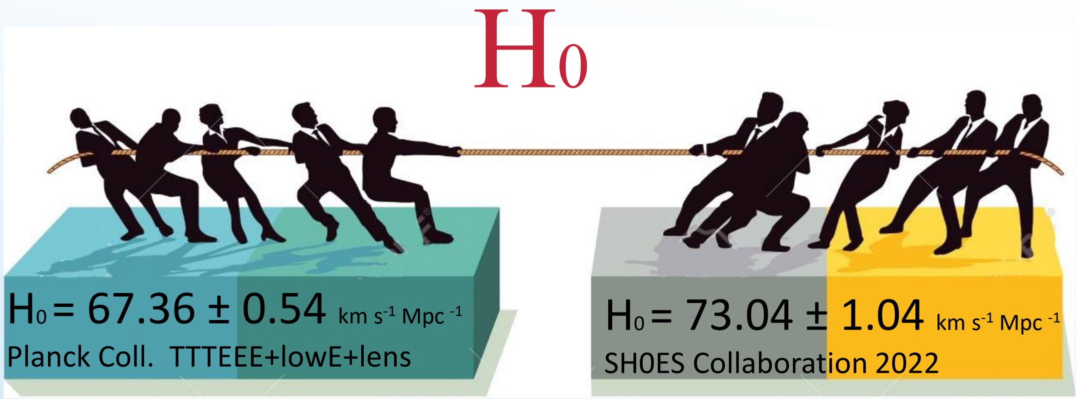


$5\ \sigma$

Models have been proposed to alleviate the tension, from simple modifications to DE physic (w, CPL, etc.) to Modified Gravity to modifications to Particle Physics but..

1- no final solving model

2- degeneracy problem between models: different models produce same predictions



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Cosmography as a parametric approach to describe the data

→ addresses the problem in a model-independent way with the aim of obtaining important clues to be considered in the theory

As the standard cosmological model, Cosmography assumes the **cosmological principle**: at large scale the universe is

homogeneous, with galaxies uniformly distributed in space (isomorphism under translations)

Isotropic, with galaxies uniformly distributed in different angular directions (isomorphism under rotations)

The cosmological principle demands the **scale factor** as degree of freedom governing the universe.

As the standard cosmological model, Cosmography assumes the **cosmological principle**: at large scale the universe is

Purely geometrical description of the Universe kinematic in which all the physics is hidden in the scale factor a(t)

$$a(t) = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^k a}{dt^k}\bigg|_{t=t_0} (t - t_0)^k$$

$$H(t) \equiv \frac{1}{a}\frac{da}{dt}$$

Hubble

$$q(t) \equiv -\frac{1}{aH^2}\frac{d^2 a}{dt^2}$$

Deceleration

$$j(t) \equiv \frac{1}{aH^3}\frac{d^3 a}{dt^3}$$

Jerk

$$s(t) \equiv \frac{1}{aH^4}\frac{d^4 a}{dt^4}$$

Snap

D_L = luminosity distance
D(z) = comoving angular diameter distance

$$D_L = (1+z)D(z)$$

Low redshift approximation

$$v(z) = H_0 D(z)$$

$$z \sim \frac{v(z)}{c}$$

$$D_L = (1+z)\frac{zc}{H_0}$$

High redshift

$$D_L = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{H(z')}$$

$$z = \frac{a(t_0)}{a(t_e)} - 1$$

$$a(t) = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^k a}{dt^k}\bigg|_{t=t_0} (t - t_0)^k$$

Low redshift approximation

$$v(z) = H_0 D(z)$$

$$z \sim \frac{v(z)}{c}$$

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$$H(t) \equiv \frac{1}{a}\frac{da}{dt} \qquad j(t) \equiv \frac{1}{aH^3}\frac{d^3a}{dt^3}$$

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Low redshift approximation

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$$D_L(z) = \frac{zc}{H_0}\Big[1 + \frac{z}{2}(1-q_0) - \frac{z^2}{6}\left(1 - q_0 - 3q_0^2 + j_0\right) + \\ + \frac{z^3}{24}\left(2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0 j_0 + s_0\right) + \mathcal{O}(z^4)\Big]$$

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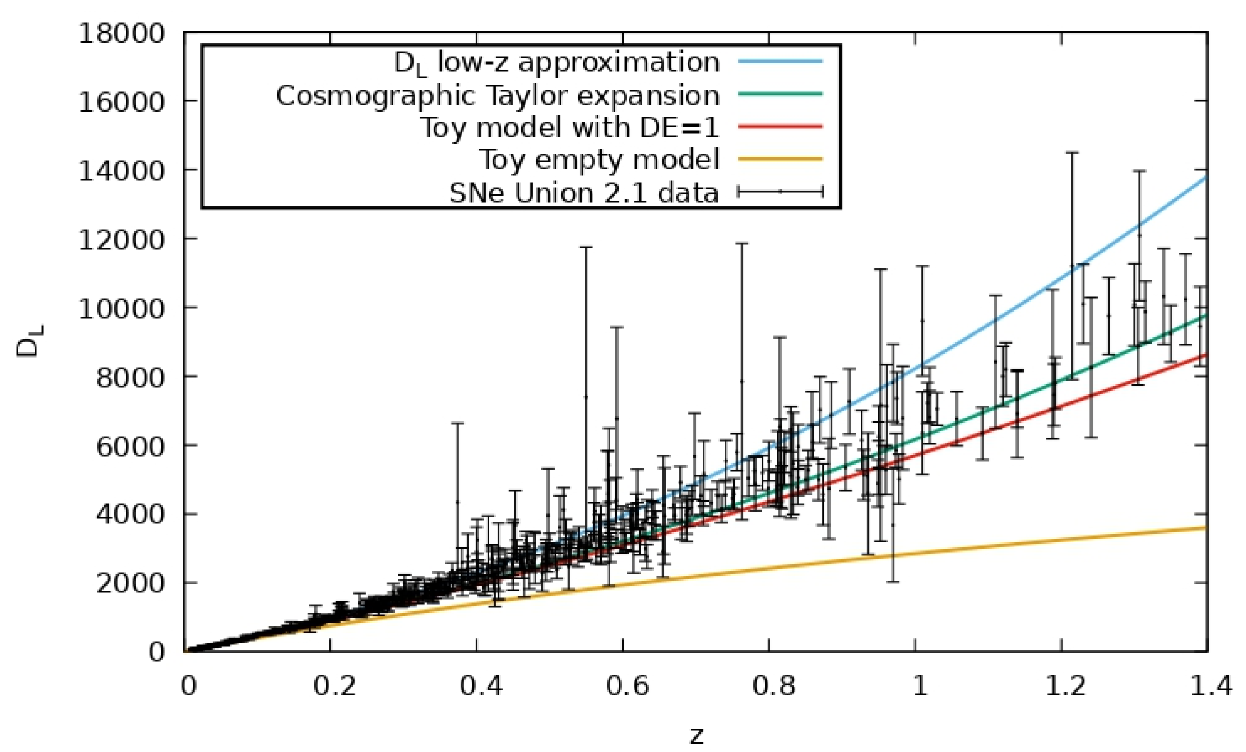

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$q_0 = -1/2$
$j_0 = s_0 = 0$

For a flat ΛCDM model:

$$D_L(z) = \frac{c}{H_0}(1+z)\int_0^z \frac{dz'}{\sqrt{\Omega_{M,0}(1+z')^3 + (1-\Omega_{M,0})}}$$

$$D_L(z) = \frac{c}{H_0}\left[z + \left(\frac{1-q_0}{2}\right)z^2 + \left(\frac{3q_0^2+q_0-1-j_0}{6}\right)z^3 - \left(\frac{15q_0^3+15q_0^2+2q_0-2-5j_0-s_0-10q_0j_0}{24}\right)z^4\right]$$

$$q_0 = \frac{3}{2}\Omega_{M,0} - 1, j_0 = 1, s_0 = 1 - \frac{9}{2}\Omega_{M,0}$$

From a 4th-order Taylor expansion at z=0 of both cosmographic and cosmological $D_L(z)$

Good news:

- If adopted expansion sufficiently flexible, it is able to fit observational data with high accuracy

- Possibility to reduce the degeneracy of cosmological models

- Cosmographic parameters used to test any cosmological model:
  - is it possible to get relations between the physical parameters of the model and cosmographic parameters
  - by cosmographic fit we can constrain the parameters of the model

Issues:

- Arbitrary truncation limits the predictive power and may result in possible misleading outcomes

- A large number of cosmographic parameters makes their estimation difficult and introduces degeneracy among them

- To detect deviations from flat ΛCDM we need to explore high redshift data → **Taylor show convergence issues for $z \geq 1$!!**


Possible solutions:

- orthogonal polynomials of logarithmic functions
- Rational polynomials
- Link cosmography with cosmology



NON-Orthogonal polynomials of logarithmic functions

$$D_L = k\left[a_1 \log(1+z) + a_2 \log^2(1+z) + a_3 \log^3(1+z) + ..\right]$$

Substitute the Taylor expansion with a new analytical function: **expand D_L as power series of $\log_{10}(1 + z)$**

NON-Orthogonal polynomials of logarithmic functions

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$$D_L = \frac{cz}{H_0}$$

$$D_L = \frac{c}{H_0}\ln(1+z) = \frac{c}{H_0}\ln(10)\log(1+z)$$

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Substitute the Taylor expansion with a new analytical function: **expand D_L as power series of $\log_{10}(1 + z)$**

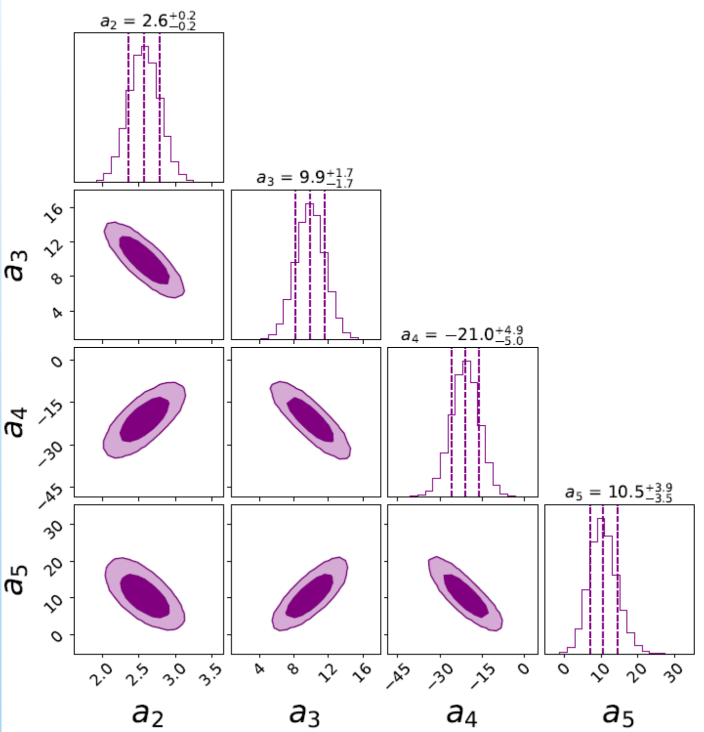
$$D_L = \frac{cz}{H_0}$$

k **$a_1 = 1$**

$$D_L = \frac{c}{H_0}\ln(1+z) = \frac{c}{H_0}\ln(10)\log(1+z)$$

$$D_L = \frac{c}{H_0}\ln(10)\left[\log(1+z) + a_2 \log^2(1+z) + a_3 \log^3(1+z) + ..\right]$$

NON-Orthogonal polynomials of logarithmic functions



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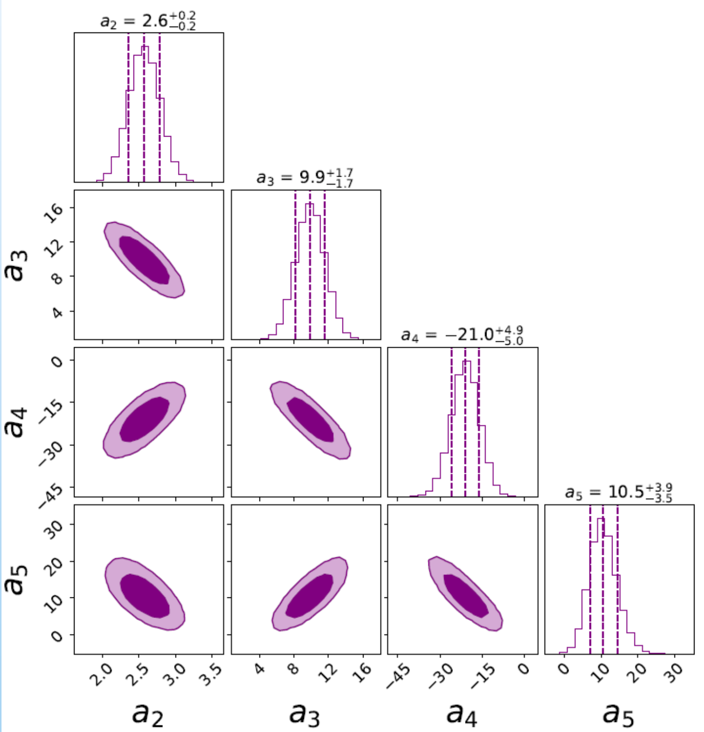

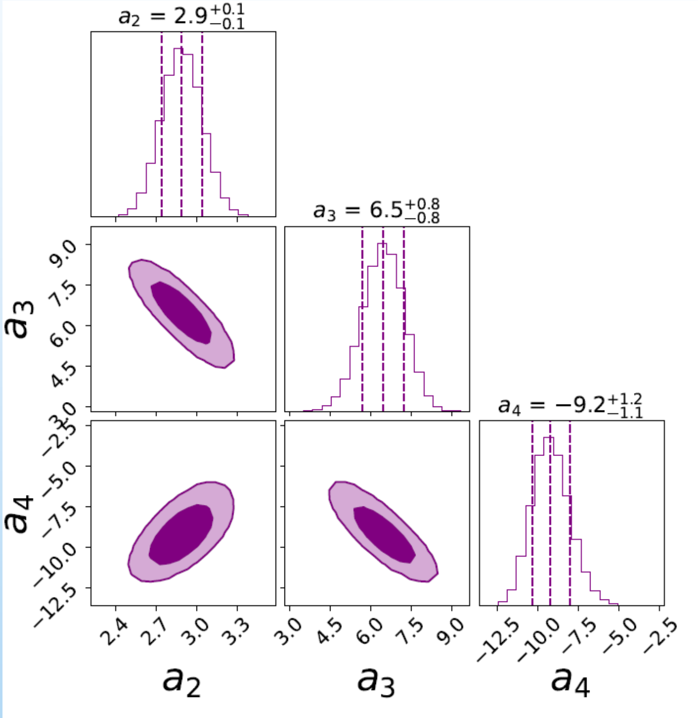

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Orthogonal polynomials of logarithmic functions

$$D_L(z) = \frac{\ln(10)}{H_0}\left\{\log(1+z) + a_2\log^2(1+z) + a_3\left[k_{32}\log^2(1+z) + \log^3(1+z)\right] + \right.$$
$$\left. + a_4\left[k_{42}\log^2(1+z) + k_{43}\log^3(1+z) + \log^4(1+z)\right] + a_5\left[k_{52}\log^2(1+z) + k_{53}\log^3(1+z) + k_{54}\log^4(1+z) + \log^5(1+z)\right]\right\}$$

Remove the correlation among coefficients

- A change in the truncation order of the series does not change the values of the cosmographic coefficients
- Allows to test the significance of a possible additional term in the expansion

$$D_L = \frac{c}{H_0}\ln(10)\left[\log(1+z) + a_2\log^2(1+z) + a_3\log^3(1+z) + ..\right]$$

$$D_L(z) = \frac{\ln(10)}{H_0}\left\{\log(1+z) + a_2\log^2(1+z) + a_3\left[k_{32}\log^2(1+z) + \log^3(1+z)\right] + \right.$$
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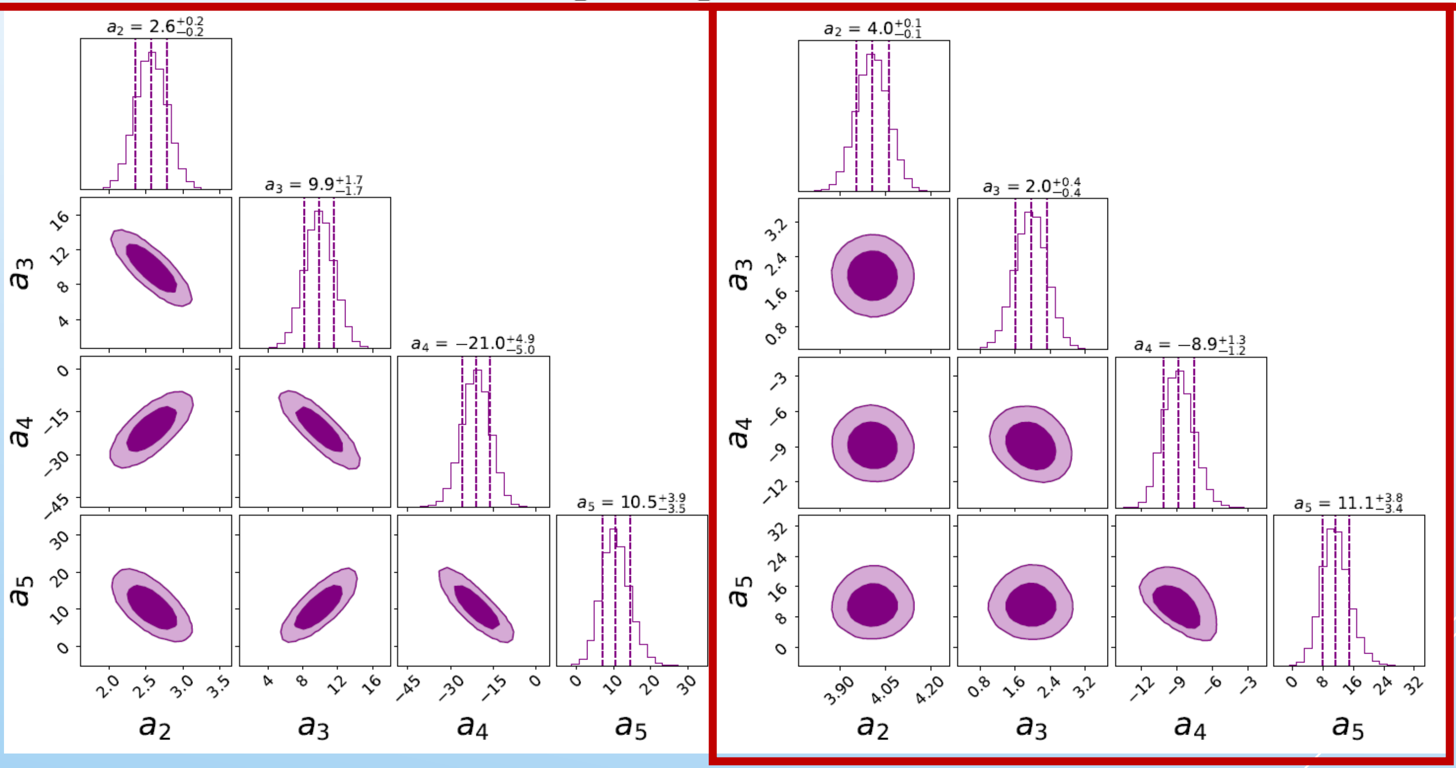

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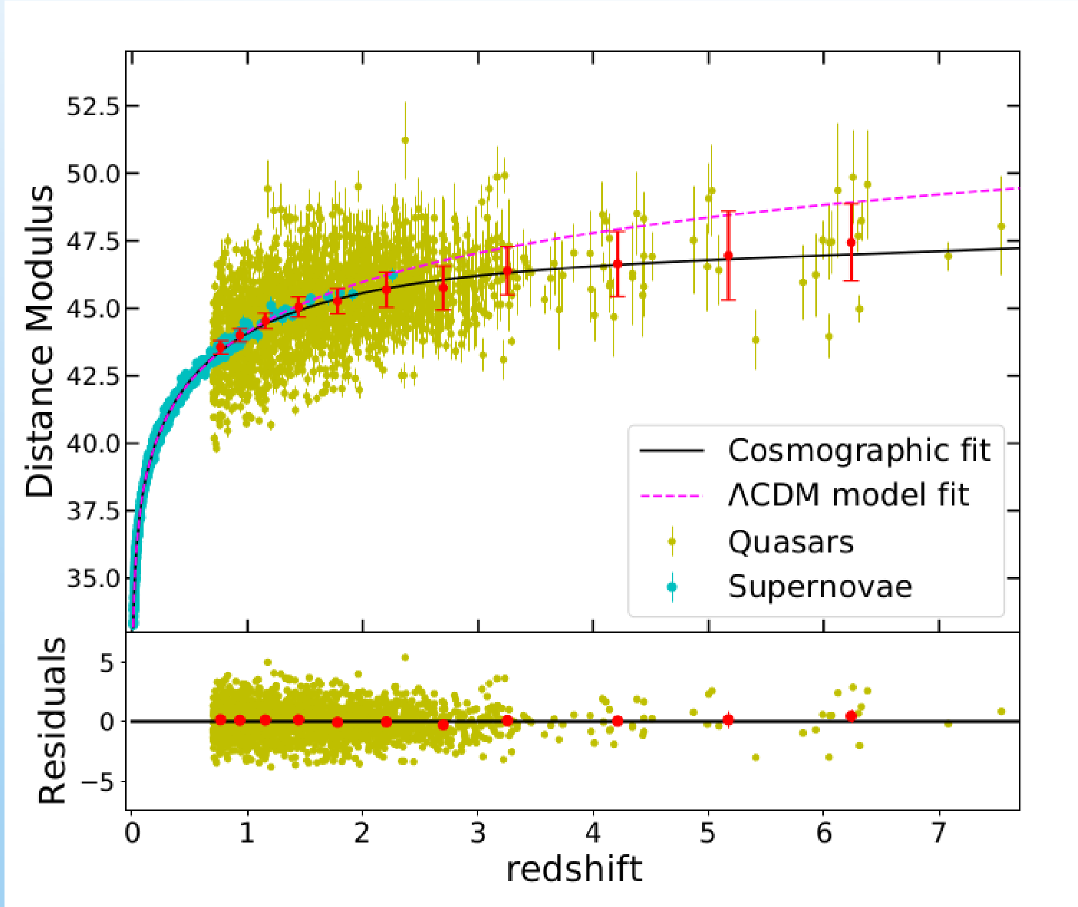

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Fifth-order in the logarithmic polynomial is needed to fit data up to the maximum redshifts of quasars

A sixth-order would not be significant.

Possible solutions:

- ✓ orthogonal polynomials of logarithmic functions
- ➢ **Rational polynomials**
- ➢ Link cosmography with cosmology

Padè rational polynomial

Padè approximation: $$P_{n,m}(z) = \frac{\sum_{i=0}^{n} a_i z^i}{1 + \sum_{j=1}^{m} b_j z^j}$$

Standard Taylor series: $$\mathrm{f(z)} = \sum_{i=0}^{\infty} c_i z^i$$

Padè rational polynomial

Padè approximation:

$$P_{n,m}(z) = \frac{\sum_{i=0}^{n} a_i z^i}{1 + \sum_{j=1}^{m} b_j z^j}$$

$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$P_{22}(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

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$D_L = f(P_{nm})$ →

$$\begin{cases} P_{nm}(0) = D_L(0) \\ P'_{nm}(0) = D'_L(0) \\ \dots \\ P_{nm}^{(n+m)}(0) = D_L^{(n+m)}(0) \end{cases}$$

↑ PADE ↑ Taylor expansion

S. Capozziello, R. D'Agostino, and O. Luongo, Mon. Not. Roy. Astron. Soc. **494**, 2576 (2020), arXiv:2003.09341 [astro-ph.CO] .

Padè rational polynomial

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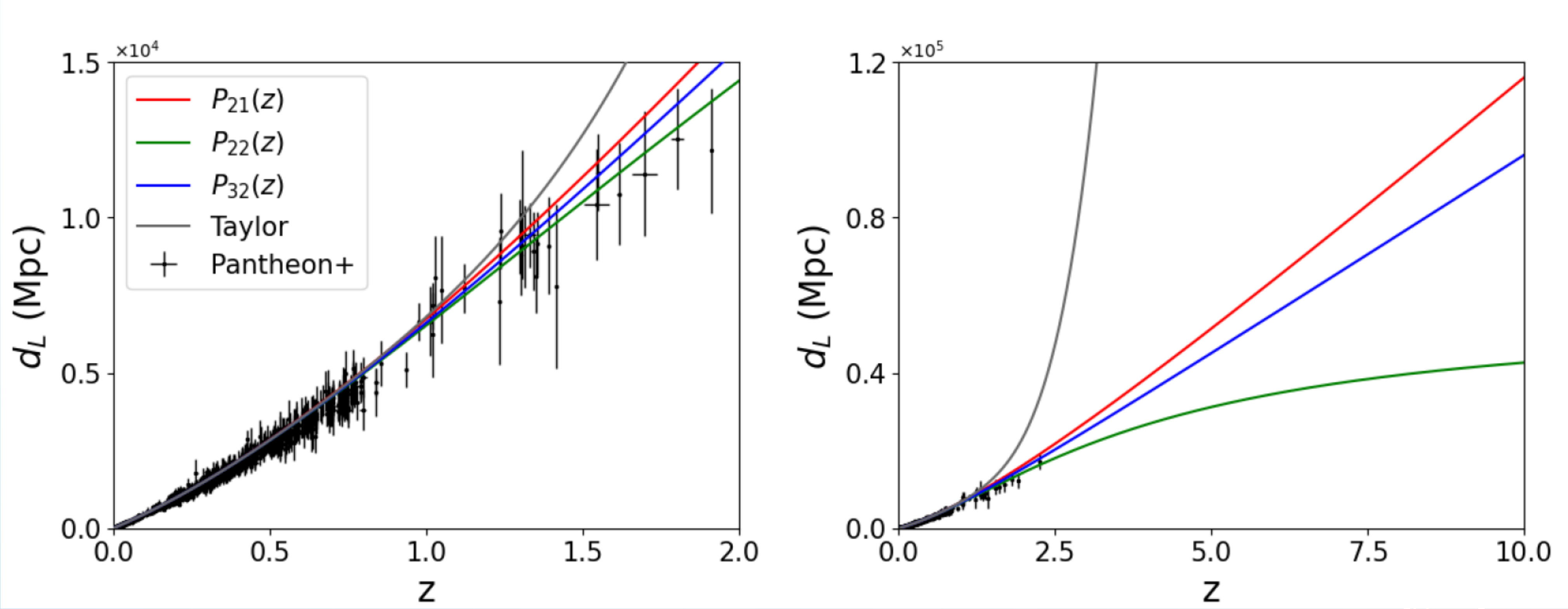
$$\begin{cases} P_{12}(0) = D_L(0) \\ P'_{12}(0) = D'_L(0) \\ P''_{12}(0) = D''_L(0) \\ P'''_{12}(0) = D'''_L(0) \end{cases}$$

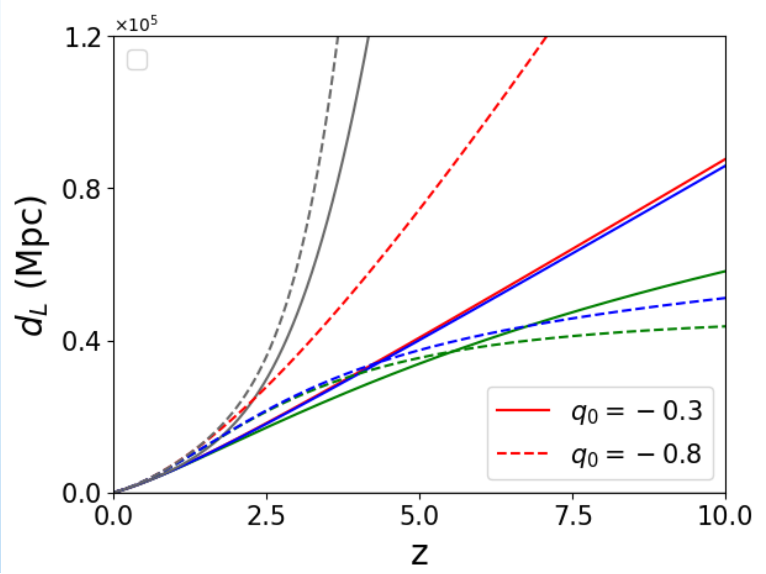
$D_L = f(P_{21})$

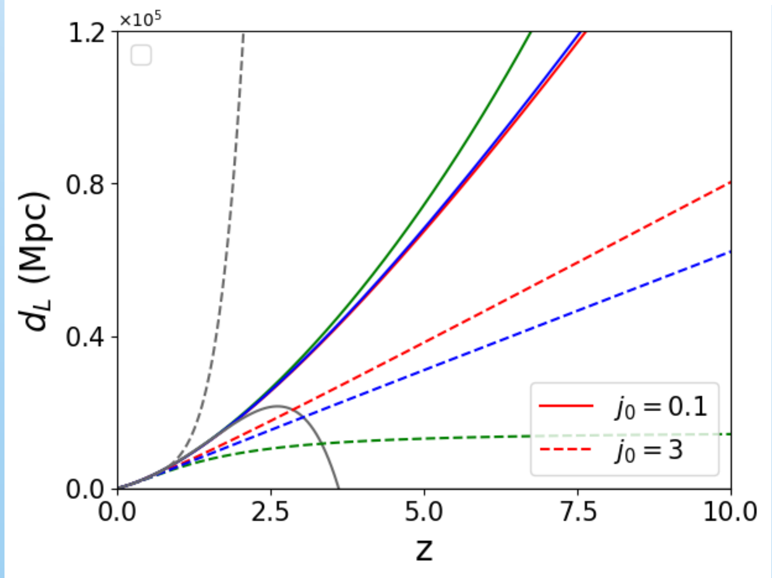
$$P_{21}(z) = \frac{cz}{H_0}\frac{6(q_0 - 1) + z[-5 - 2j_0 + q_0(8 + 3q_0)]}{-2(3 + z + j_0 z) + 2q_0(3 + z + 3zq_0)}$$

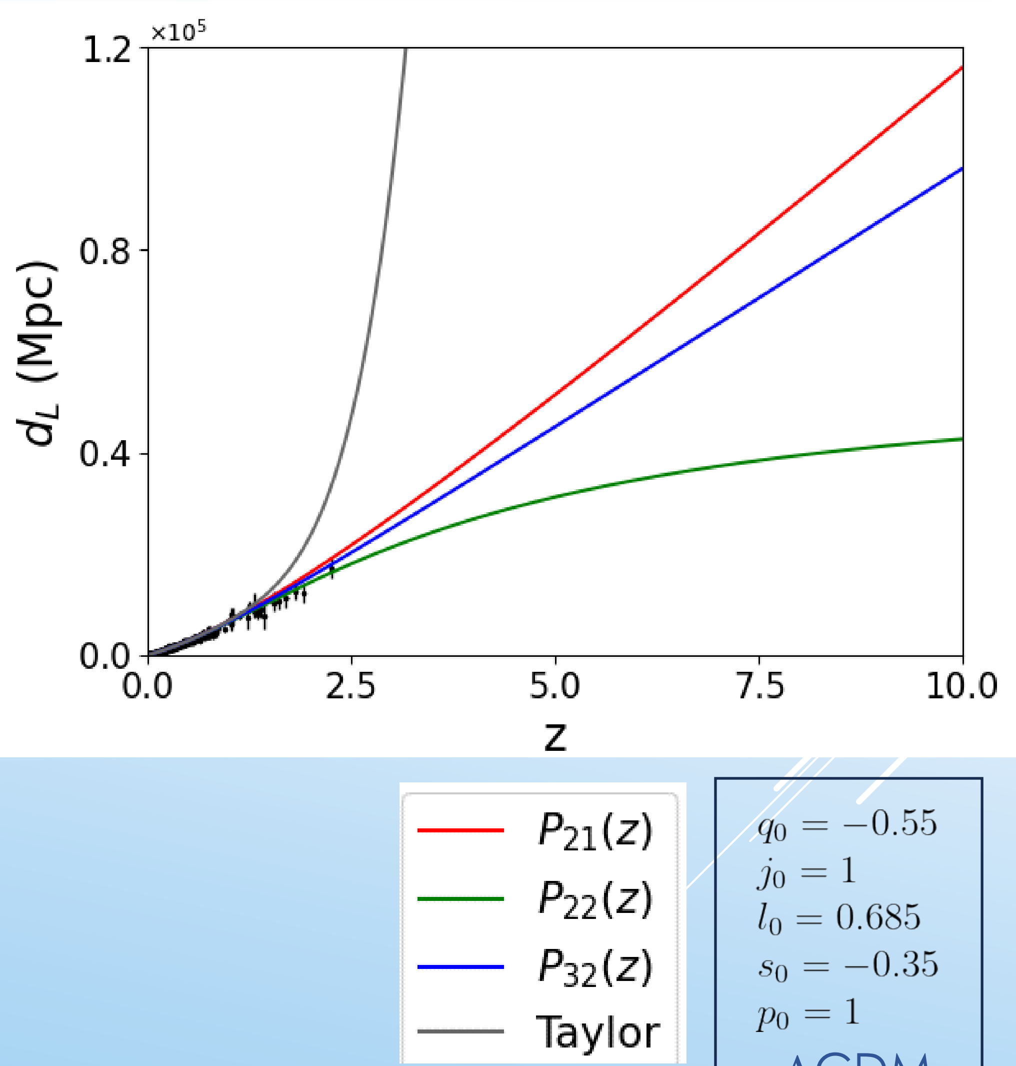


$q_0 = -0.55$
$j_0 = 1$
$l_0 = 0.685$
$s_0 = -0.35$
$p_0 = 1$

ΛCDM







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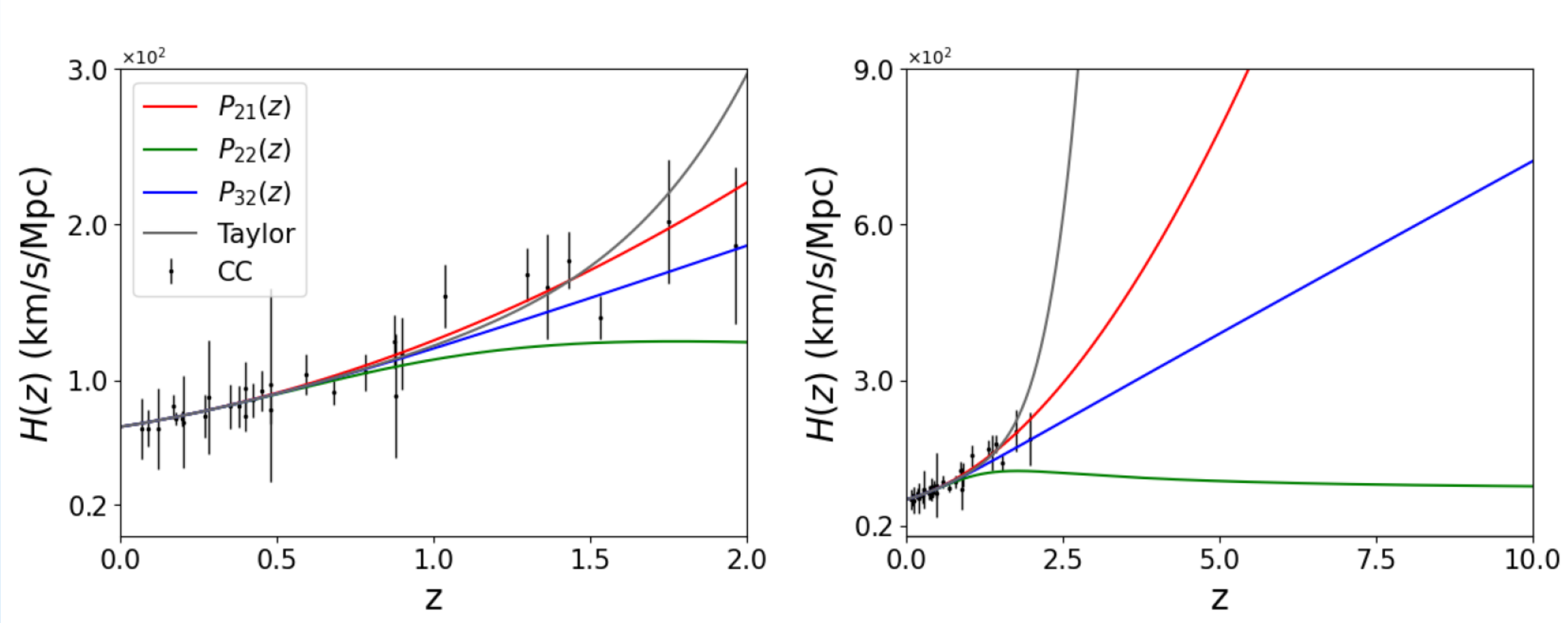
H(z) = f(P_{nm}) →

$$P_{nm}(0) = H(0)$$
$$P'_{nm}(0) = H'(0)$$
$$P_{nm}^{(n+m)}(0) = H^{(n+m)}(0)$$

$$H(z) = H_0\{1 + (1+q_0)z + \frac{1}{2}(j_0 - q_0^2)z^2 - \frac{1}{6}[-3q_0^2 - 3q_0^3 + j_0(3+4q_0) + s_0]z^3 + \frac{1}{24}[-4j_0^2 + l_0 - 12q_0^2 - 24q_0^3 - 15q_0^4 + j_0(12 + 32q_0 + 25q_0^2) + 8s_0 + 7q_0 s_0]z^4 + \frac{1}{120}(p_0 + 15l_0 + 60(s_0 + j_0 - j_0^2 + 4j_0 q_0 + s_0 q_0^2 - q_0^2 - 3q_0^3) - 15s_0 j_0 + 11l_0 q_0 + 105 s_0 q_0 - 70 j_0^2 q_0 + 375 j_0 q_0^2 + 210 j_0 q_0^3 - 225 q_0^4 - 105 q_0^5)z^5 + O(z^6)\}$$



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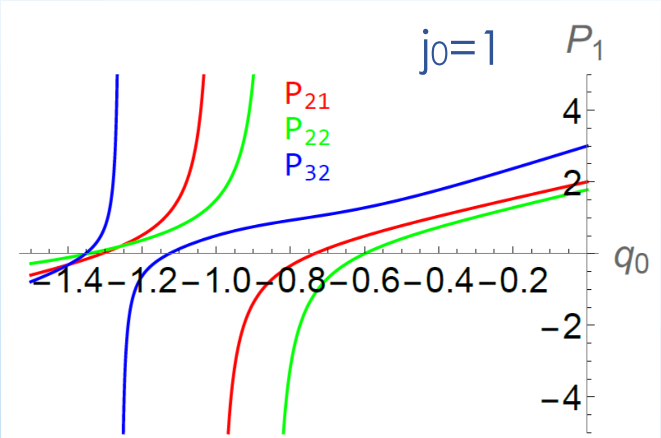
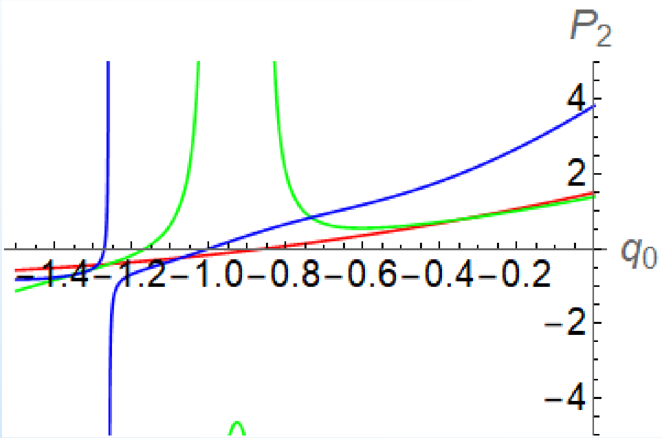
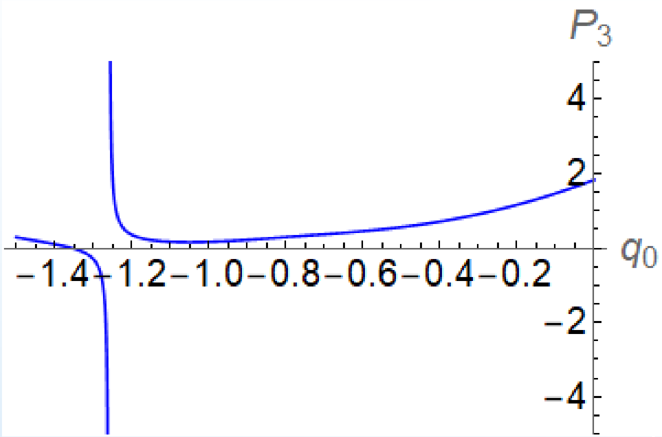
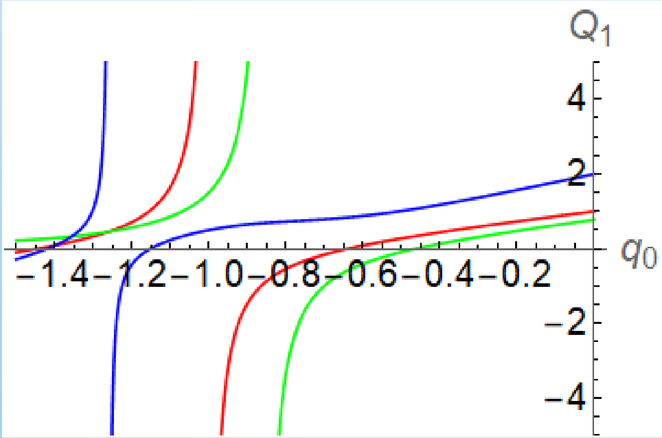
ΛCDM

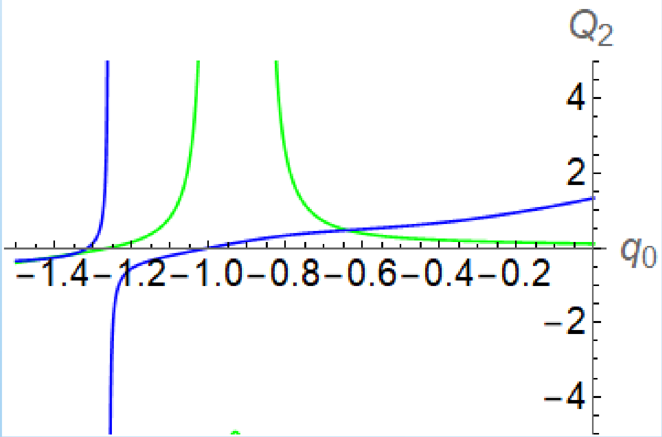
Which is the best choice (stability, best performance-complexity ratio, ...) among these polynomials?











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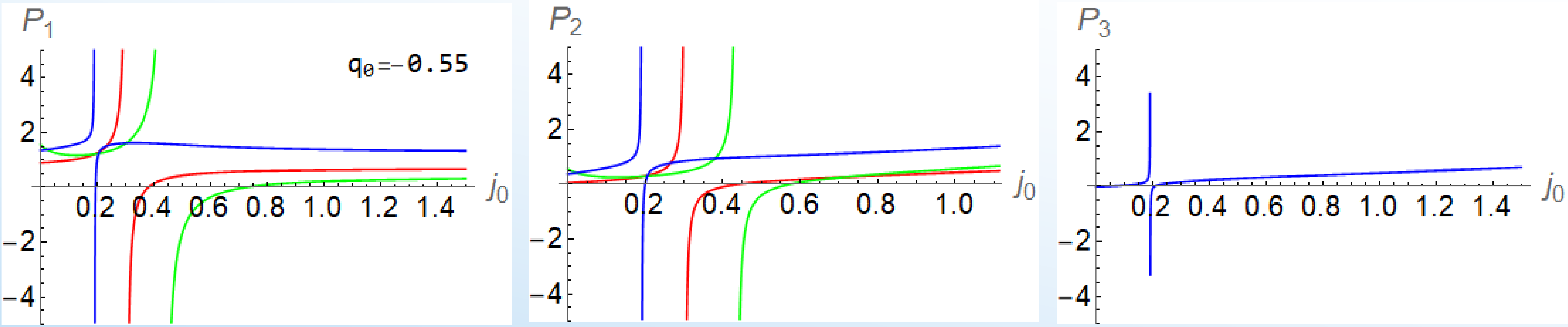

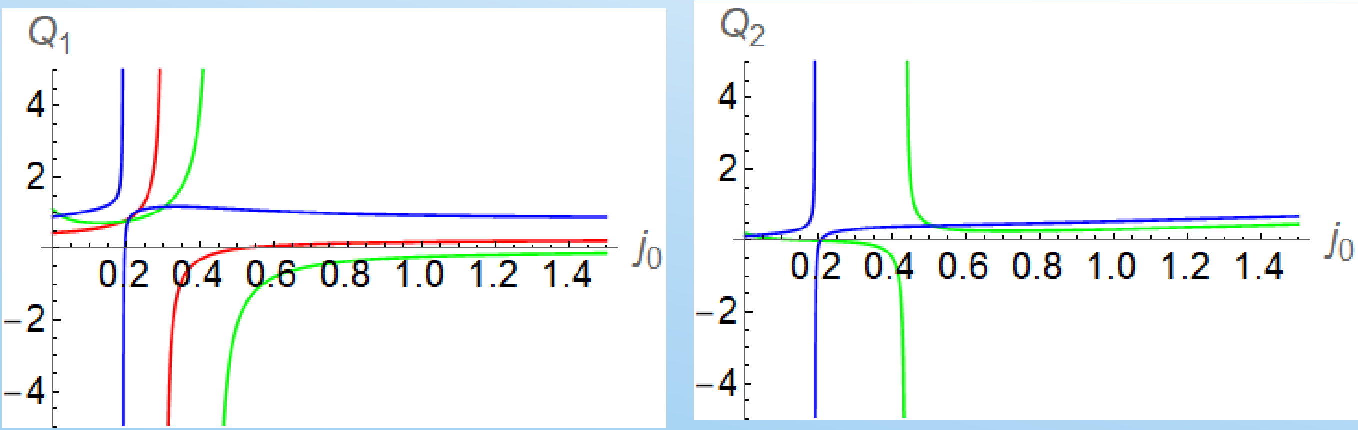

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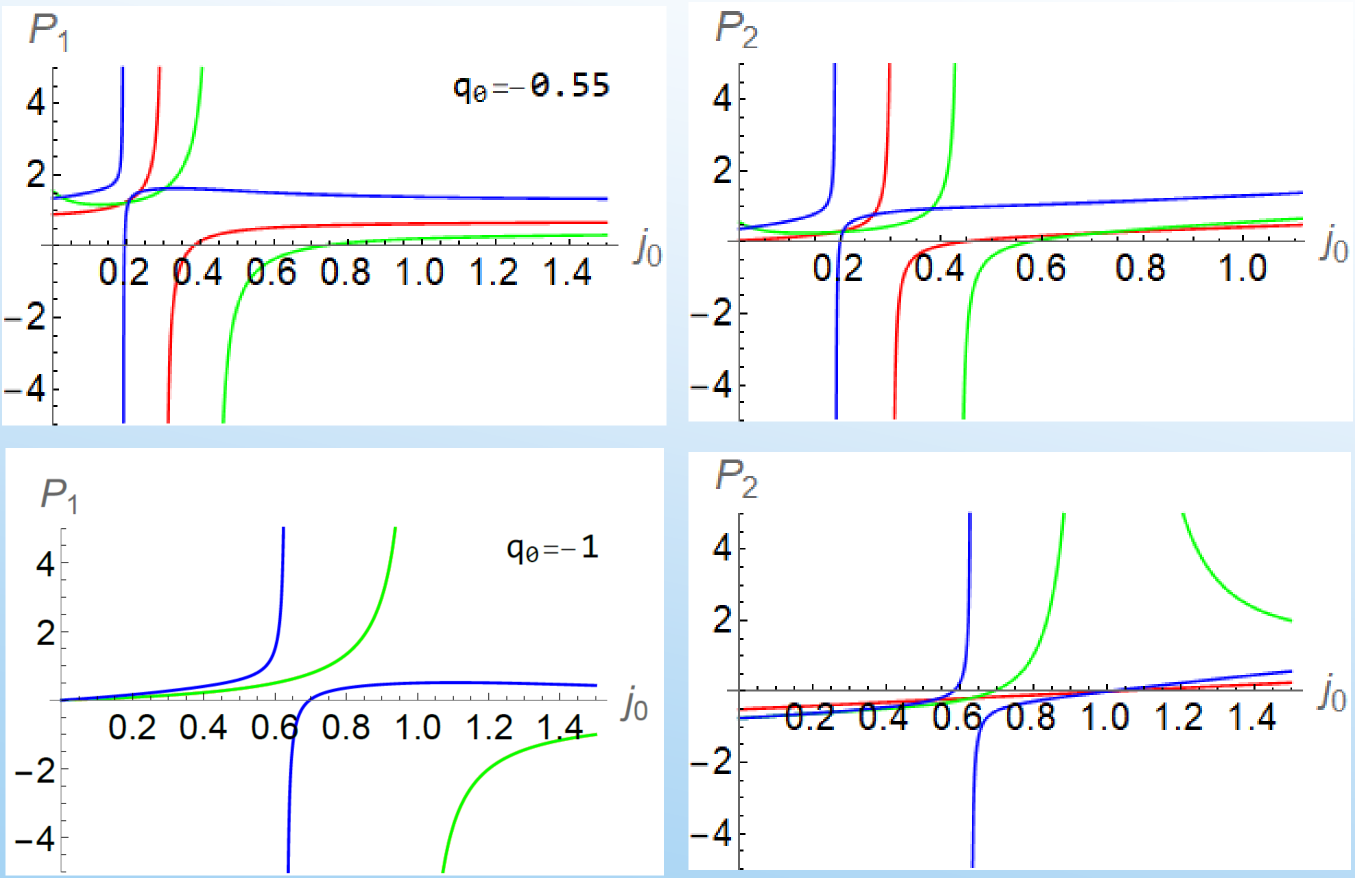

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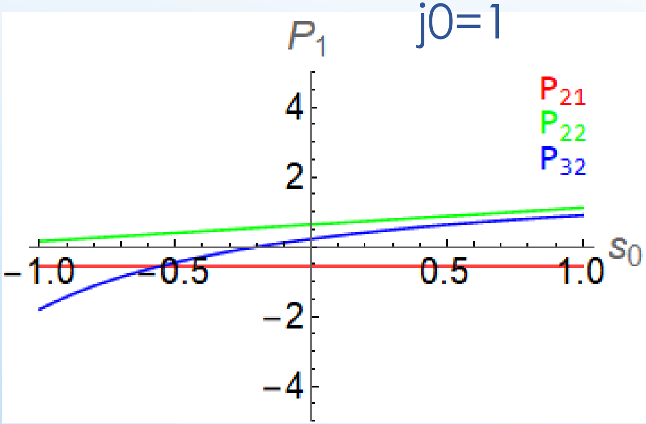
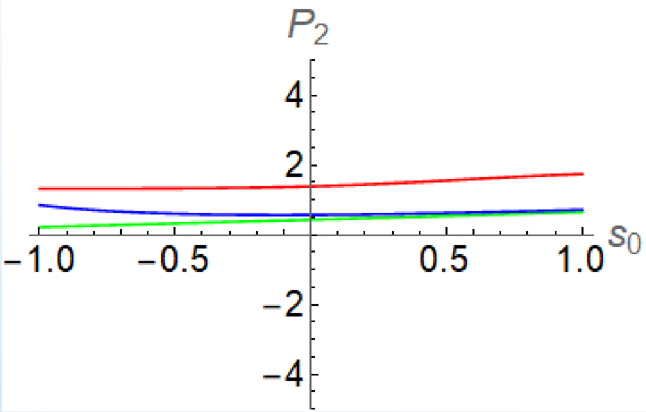
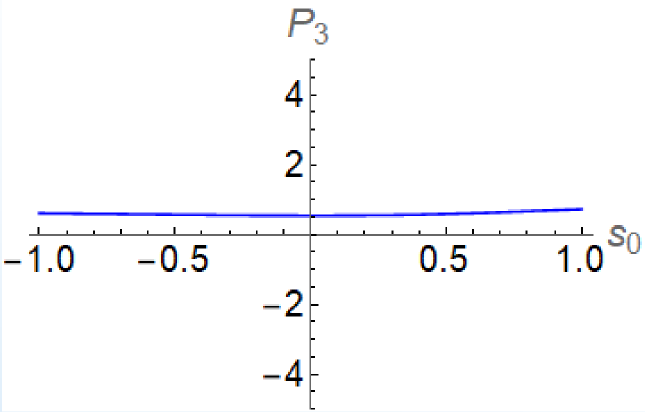
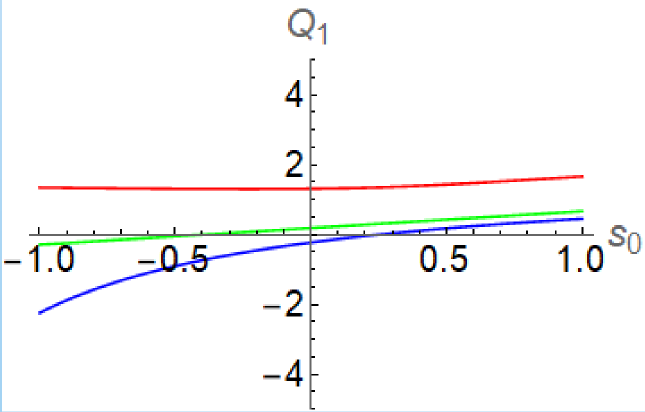
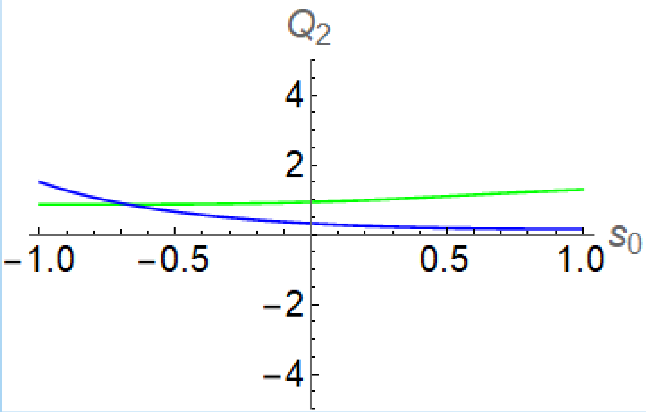








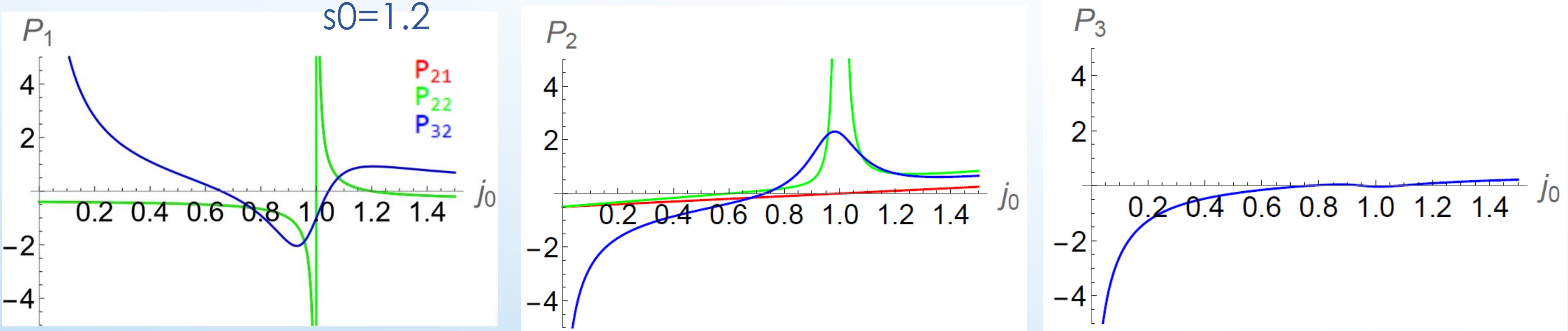


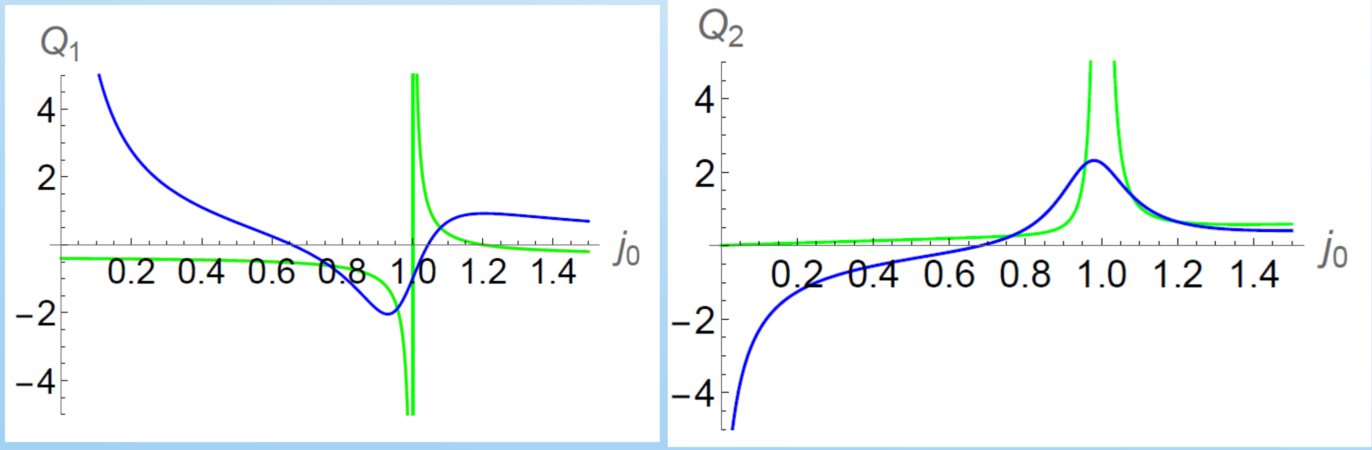
$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$P_{22}(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$P_{32}(z) = \frac{P_0 + P_1 z + P_2 z^2 + P_3 z^3}{1 + Q_1 z + Q_2 z^2}$$

q0= -0.55
s0=1.2





$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$P_{22}(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$P_{32}(z) = \frac{P_0 + P_1 z + P_2 z^2 + P_3 z^3}{1 + Q_1 z + Q_2 z^2}$$

- Significant correlation between cosmographic parameter
- All works well at low redshift
- At high redshift, P21 and P32 converge better than P22
- P21 is simpler than P32, but P32 is more stable at very-high redshift
- P22 is more studied in literature

S. Capozziello, R. D'Agostino, and O. Luongo, Mon. Not. Roy. Astron. Soc. **494**, 2576 (2020), arXiv:2003.09341 [astro-ph.CO] .

S. Capozziello, R. D'Agostino, and O. Luongo, Int. J. Mod. Phys. D **28**, 1930016 (2019), arXiv:1904.01427 [gr-qc] .

S. Capozziello, R. D'Agostino, and O. Luongo, JCAP **05**, 008 (2018), arXiv:1709.08407 [gr-qc] .

M. Benetti and S. Capozziello, Journal of Cosmology and Astroparticle Physics **2019**, 008 (2019).

K. Dutta, Ruchika, A. Roy, A. A. Sen, and M. M. Sheikh-Jabbari, Gen. Rel. Grav. **52**, 15 (2020), arXiv:1808.06623 [astro-ph.CO] .

K. Dutta, A. Roy, Ruchika, A. A. Sen, and M. M. Sheikh-Jabbari, Phys. Rev. D **100**, 103501 (2019), arXiv:1908.07267 [astro-ph.CO] .

S. Capozziello, Ruchika, and A. A. Sen, Mon. Not. Roy. Astron. Soc. **484**, 4484 (2019), arXiv:1806.03943 [astro-ph.CO] .

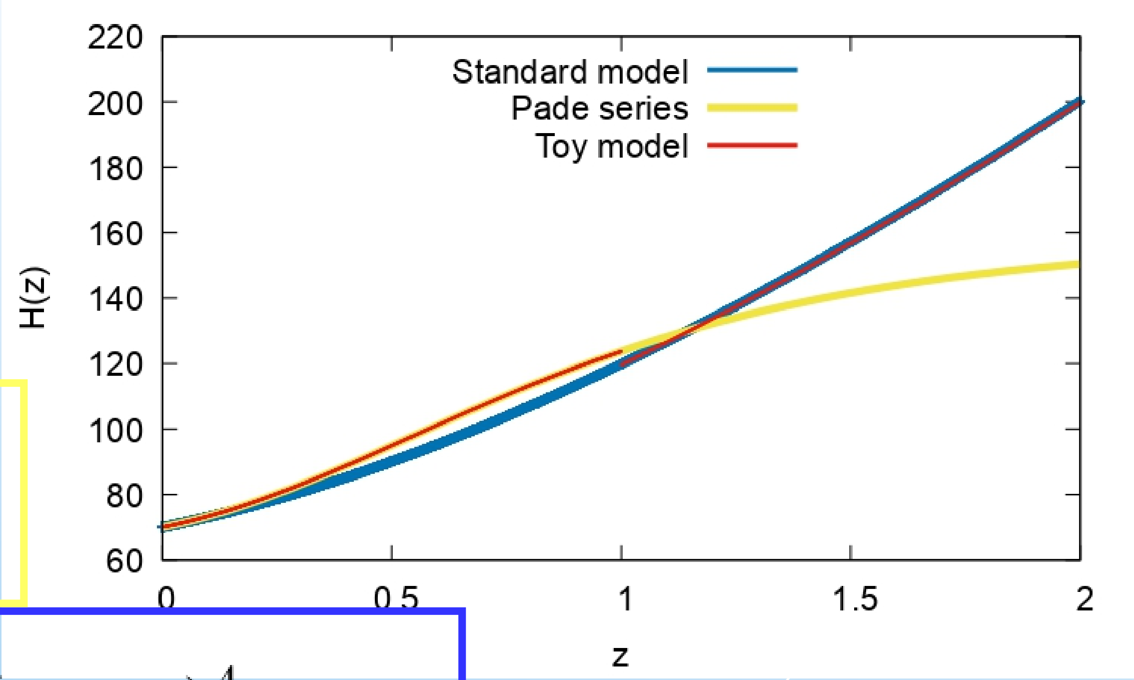
Possible solutions:

- ✓ orthogonal polynomials of logarithmic functions
- ✓ Rational polynomials
- ➢ **Link cosmography with cosmology**

f(z)CDM model

q_0=-0.6
j_0=2
s_0=0

Ω_m = 0.3
Ω_r = 0
Ω_Λ = 0.7



Padè P_{22}

$$\frac{H(z)}{H_0} = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

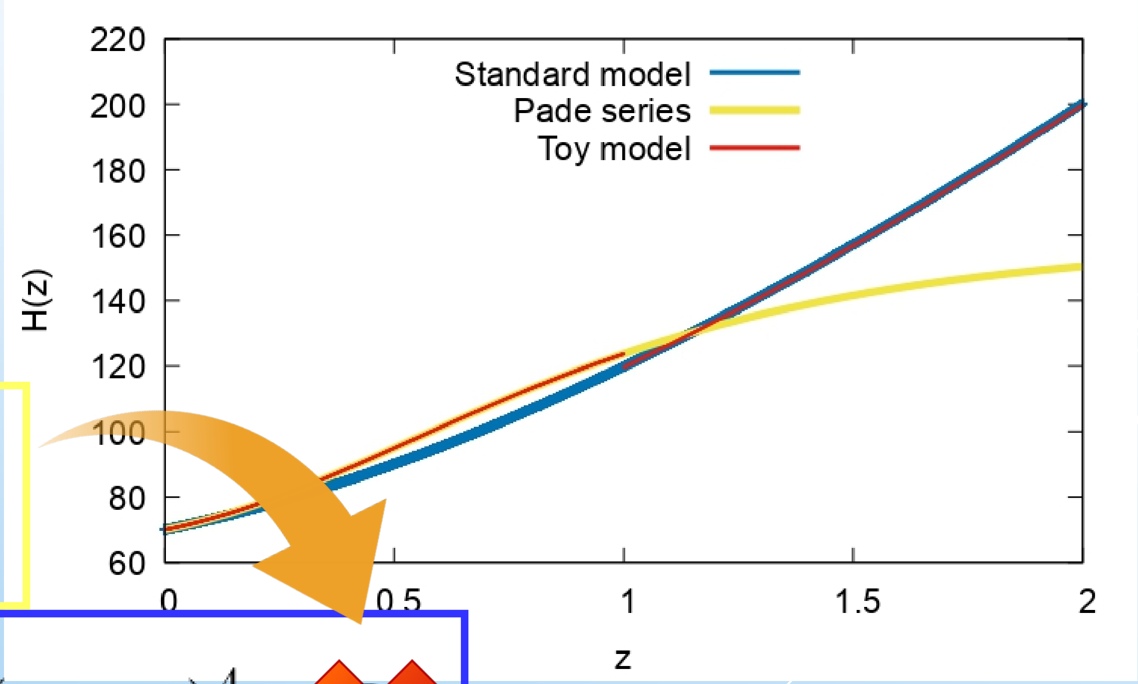
$$\frac{H(z)}{H_0} = \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda$$

ΛCDM

f(z)CDM model

q_0=-0.6
j_0=2
s_0=0

Ω_m = 0.3
Ω_r = 0
Ω_Λ = 0.7



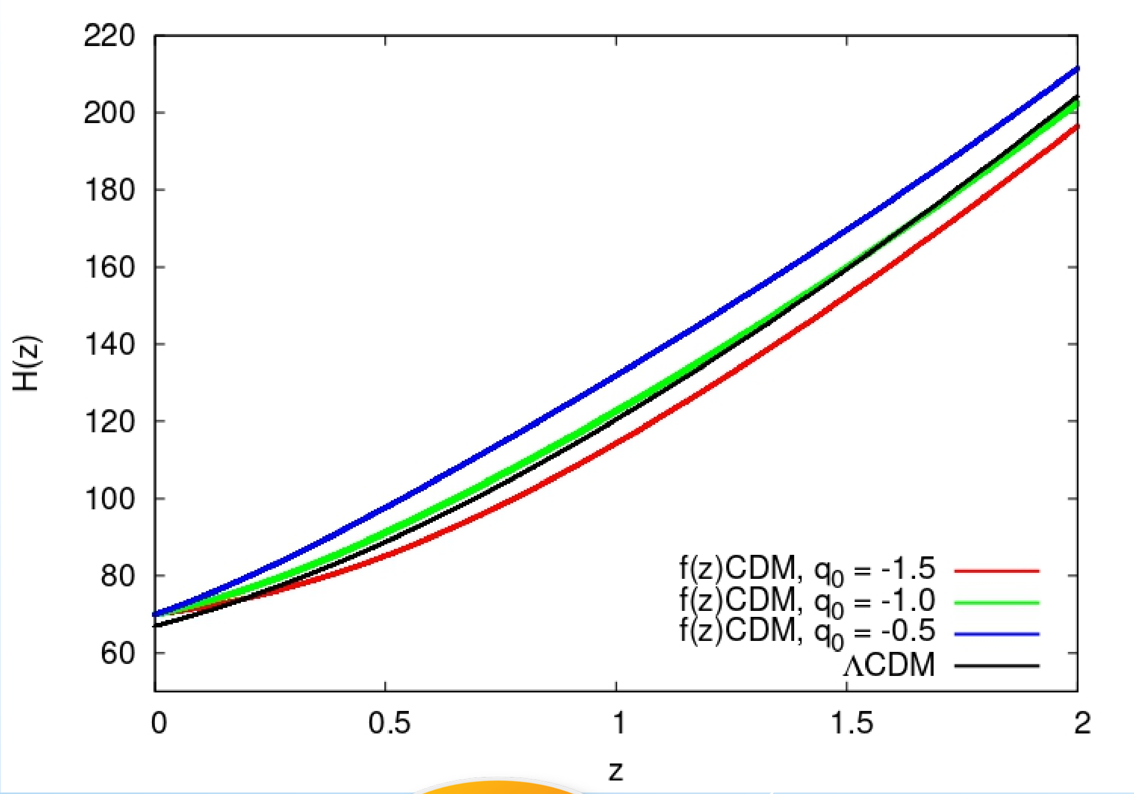
Padè P_{22}

$$\frac{H(z)}{H_0} = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$\frac{H(z)}{H_0} = \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda$$

f(z)CDM

f(z)CDM model



$\Omega_m = 0.3$
$\Omega_r = 0$
$\Omega_f = 0.7$

$j_0=2$
$s_0=0$

$$H(z)^2 = H_0^2(\Omega_m(1+z)^3+\Omega_r(1+z)^4+\Omega_f f(z))$$

$$f(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

f(z)CDM model

$$\frac{H(z)}{H_0} = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$\rightarrow$ $q_0, j_0, s_0, ..$

$\overline{q_0}, \overline{j_0}, \overline{s_0}, ..$ $\uparrow$

How can we find a relationship between the cosmographic coefficients?

$$H(z)^2 = H_0^2(\Omega_m(1+z)^3+\Omega_r(1+z)^4+\Omega_f f(z))$$

$$f(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

f(z)CDM model

$$\frac{H(z)}{H_0} = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

How can we find a relationship between the cosmographic coefficients?

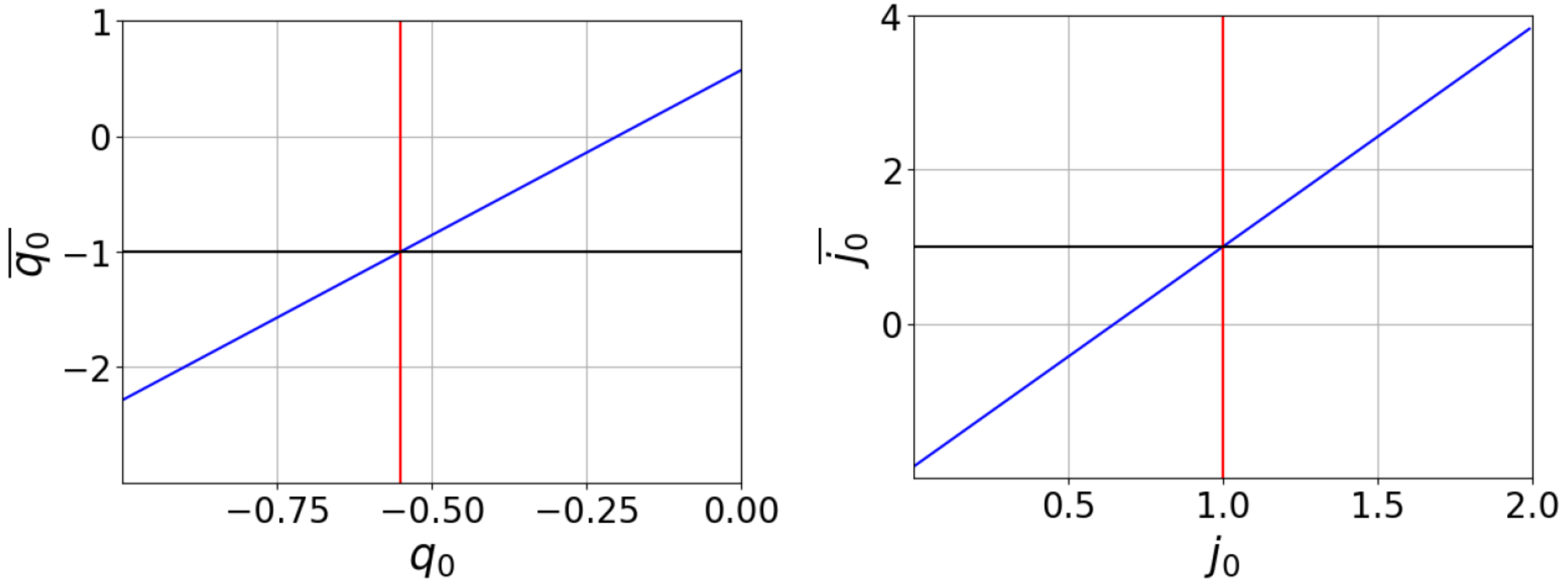
$$\frac{H'}{H_0}(0) = f_z'(0)$$

$$\frac{H''}{H_0}(0) = f_z''(0)$$

$$\frac{H^{(n+m)}}{H_0}(0) = f_z^{(n+m)}(0)$$

$$H(z)^2 = H_0^2(\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_f f(z))$$

$$f(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$



$$\overline{q_0} = \frac{-1 - 2q_0 + 2\Omega_m}{-1 + \Omega_m}$$

$$\overline{j_0} = \frac{3 + 4q_0^2 + q_0(8 - 12\Omega_m) - 2j_0(-1 + \Omega_m) - 12\Omega_m + 10\Omega_m^2}{(-1 + \Omega_m)^2}$$

PADÈ - P_{22}

Connecting early and late epochs by f(z)CDM cosmography
MB, S. Capozziello - JCAP 12 (2019) 008

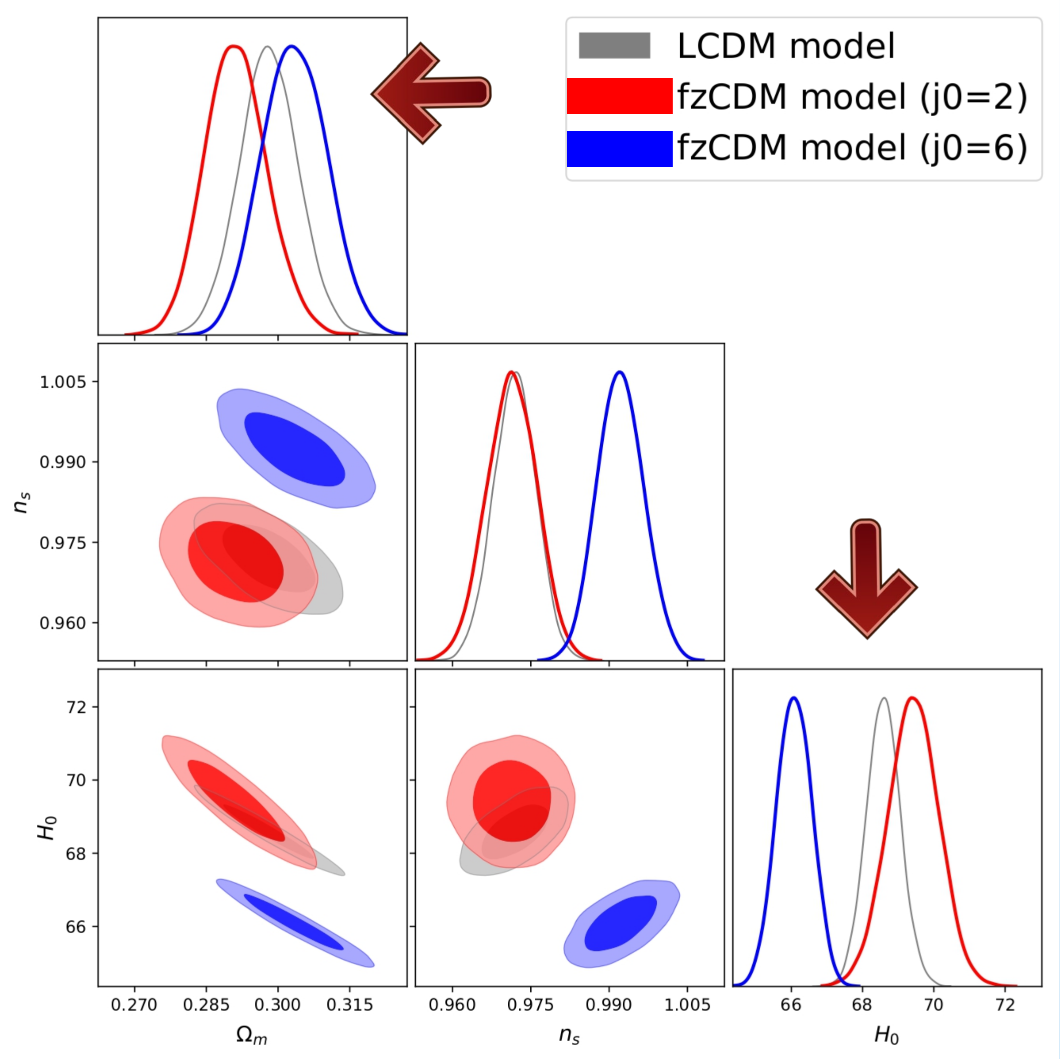
- **Models**
- f(z)CDM model truncated to 2° order → q0
- f(z)CDM model truncated to 3° → q0 and j0
- f(z)CDM model truncated to 4° order → q0, j0, s0

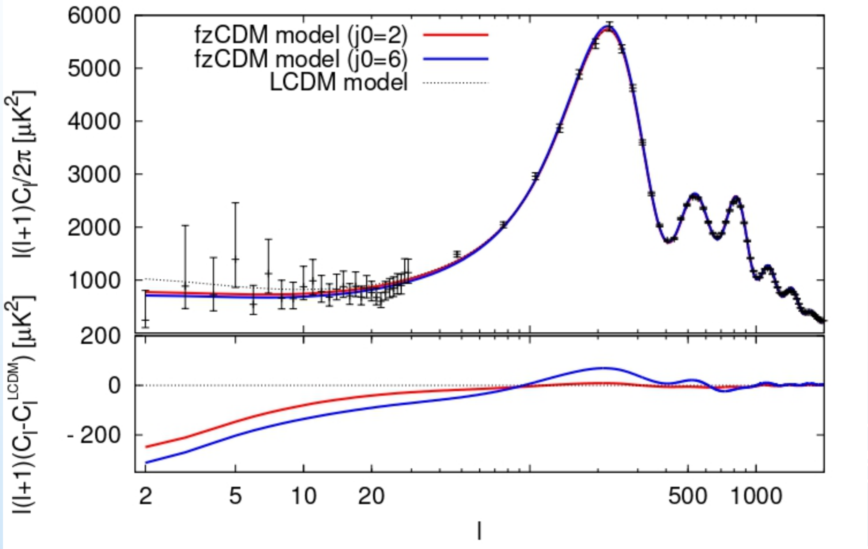
$$H(t) \equiv \frac{1}{a}\frac{da}{dt}$$
$$q(t) \equiv -\frac{1}{aH^2}\frac{d^2a}{dt^2}$$
$$j(t) \equiv \frac{1}{aH^3}\frac{d^3a}{dt^3}$$
$$s(t) \equiv \frac{1}{aH^4}\frac{d^4a}{dt^4}$$

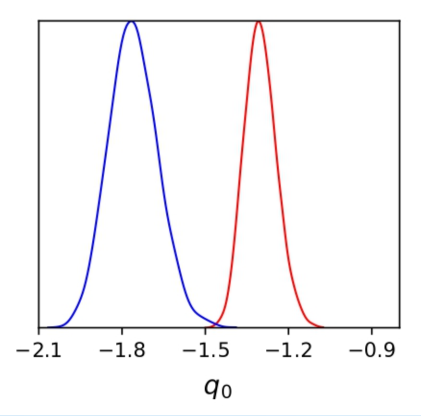
Base-dataset

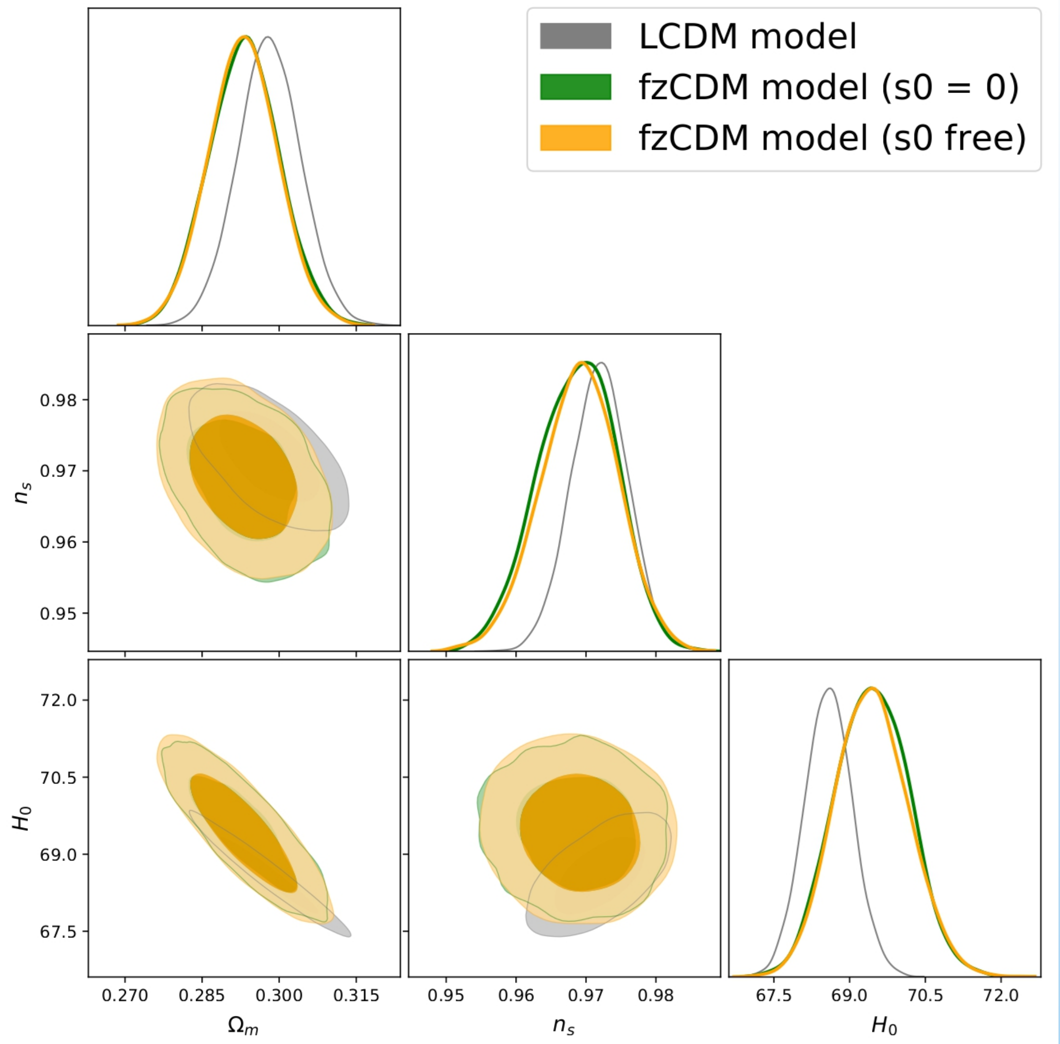
- Cosmic Microwave Background (CMB)
- Baryon Acoustic Oscillation (BAO)
- Supernovae Type Ia (Pantheon sample)
- Cosmic Clock (CC) data

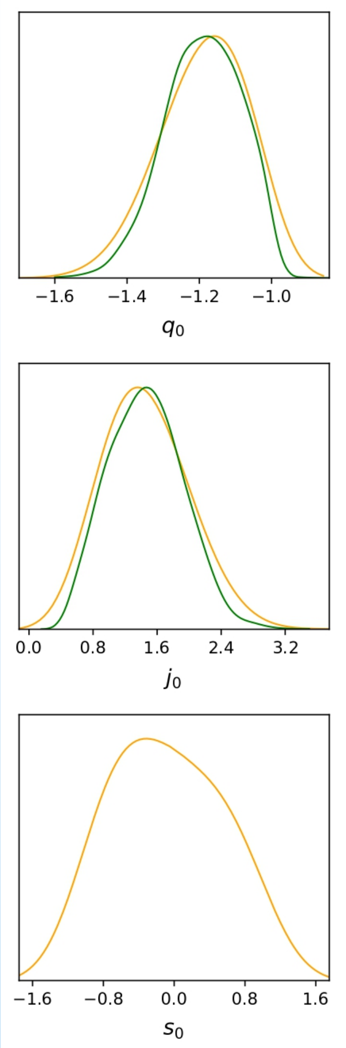
f(z)CDM model truncated to 2°











f(z)CDM model truncated t0 3°

$q_0 = -1.2 \pm 0.1$

$j_0 = 1.5 \pm 0.5$

$s_0 = 0$

f(z)CDM model truncated t0 4°

$q_0 = -1.2 \pm 0.1$

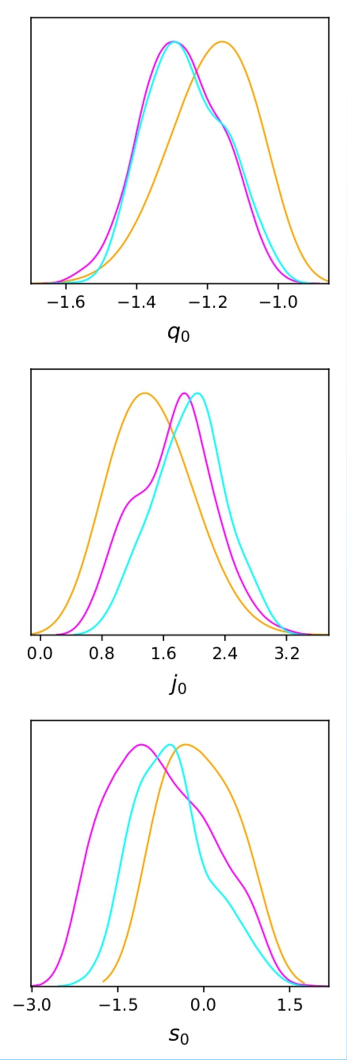
$j_0 = 1.5 \pm 0.5$

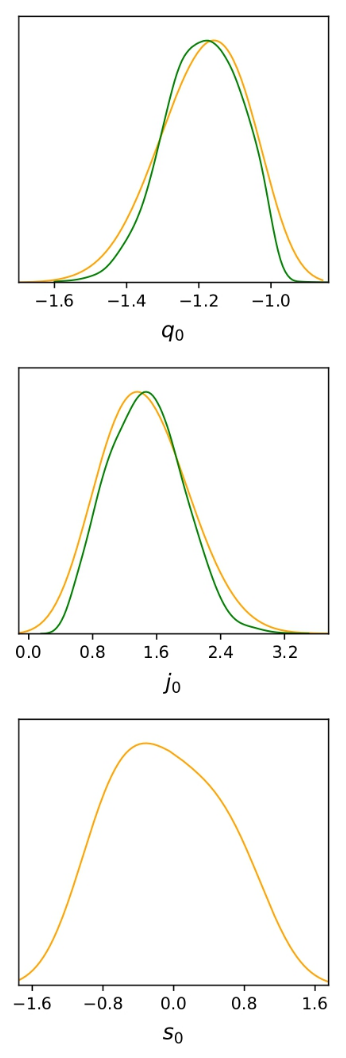
$s_0 = -0.1 \pm 0.6$

4° f(z)model base 2015
4° f(z)model base 2018
4° f(z)model base 2018 + DES

$q_0 = -1.27 \pm 0.11$
$j_0 = 1.7 \pm 0.5$
$s_0 = -1.1 \pm 0.9$

$q_0 = -1.26 \pm 0.11$
$j_0 = 1.9 \pm 0.5$
$s_0 = -0.6 \pm 0.7$





f(z)CDM model truncated to 3°

$q_0 = -1.2 \pm 0.1$
$j_0 = 1.5 \pm 0.5$
$s_0 = 0$

f(z)CDM model truncated to 4°

$q_0 = -1.2 \pm 0.1$
$j_0 = 1.5 \pm 0.5$
$s_0 = -0.1 \pm 0.6$

We found:

- Correlation between cosmographic parameter, significant impact on cosmological parameters (Ωm, H0)
- Sensitivity of cosmographic parameters with CMB data
- good parameter constraints on q0 and j0, while s0 needs further data accuracy
- f(z)CDM model truncated at third order show $\Delta\chi^2$~7 with respect to vanilla LCDM model → can this be interpreted as a requirement to consider higher orders, with respect to the General Relativity theory, to properly describe the data?

F(Z) CDM WITH PADÈ - P_{21} P_{22} P_{32}

Beyond ΛCDM with f (z)CDM - criticalities and solutions of Padè Cosmography. A. Turmina Petreca, *MB, S. Capozziello*
Paper coming soon

$$H(z)^2 = H_0^2(\Omega_m(1+z)^3+\Omega_r(1+z)^4+\Omega_f f(z))$$

Base-dataset

- ✓ Cosmic Microwave Background (CMB)
- ✓ Baryon Acoustic Oscillation (BAO)
- ✓ Supernovae Type Ia (**Pantheon +**)
- ✓ Cosmic Clock (CC) data

$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$P_{22}(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$P_{32}(z) = \frac{P_0 + P_1 z + P_2 z^2 + P_3 z^3}{1 + Q_1 z + Q_2 z^2}$$

$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$\begin{aligned} H^{P_{21}}(z) = {} & H_0(3(j_0^2 - q_0^4) + 2q_0 s_0)z^2 + (-12q_0^3 + 2s_0)z(1 + z) - 6q_0^2(1 + z)^2 + 2j_0(3 + (6 \\ & + 7q_0)z + (3 + 7q_0 + q_0^2)z^2)/(-6q_0^3 z + 2s_0 z - 6q_0^2(1 + z) + j_0(6 + (6 + 8q_0)z)) \end{aligned}$$

$$P_{22}(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$\begin{aligned} H^{P_{22}}(z) = {} & H_0(24s_0 + 6l_0 z + 72s_0 z + 30j_0^3 z^2 + 6l_0 z^2 - 45q_0^6 z^2 + 48s_0 z^2 + 4s_0^2 z^2 - 90q_0^5 z(1 + 2z) - 18q_0^4(2 \\ & + 21z + 21z^2) + j_0^2(36 + 12(4 + 5q_0)z + (48 + 120q_0 - 5q_0^2)z^2) + 3q_0^3(-48 - 144z + (-96 \\ & + 5s_0)z^2) + 9q_0^2(-8 + 2(-8 + 3s_0)z + (-8 + l_0 + 12s_0)z^2) + 6q_0(l_0 z(1 + 2z) + s_0(4 \\ & + 23z + 23z^2)) + j_0(60q_0^4 z^2 + 90q_0^3 z(1 + 2z) + 6q_0^2(4 + 77z + 77z^2) + 7q_0(24 + 72z + (48 \\ & + 5s_0)z^2) + 3(24 + 4(12 + s_0)z + (24l_0 + 8s_0)z^2))/(24s_0 + 6l_0 z - 54q_0^5 z + 48s_0 z + 12j_0^3 z^2 \\ & - 9q_0^6 z^2 + 4s_0^2 z^2 - 18q_0^4(2 + 11z) + 3q_0^2(-24 + 2(-12 + 5s_0)z + l_0 z^2) + j_0^2(36 + 12(1 + 2q_0)z \\ & - 23q_0^2 z^2) - 3q_0^3(48 + 72z + s_0 z^2) + 6q_0(l_0 z + s_0(4 + 15z)) + j_0(72 + 66q_0^3 z + 12(6 + s_0)z \\ & - 3l_0 z^2 + 24q_0^4 z^2 + 6q_0^2(4 + 45z) + q_0(168 + 264z + 11s_0 z^2)))) \end{aligned}$$

$$P_{32}(z) = \frac{P_0 + P_1 z + P_2 z^2 + P_3 z^3}{1 + Q_1 z + Q_2 z^2}$$

$$\begin{aligned}
H^{P_{32}}(z) = cH_0\{&-540q_0^6 - 180q_0^3 s_0 + 240 s_0^2 + 36 p_0 q_0^2 z - 1620 q_0^6 z - 1620 q_0^7 z - 540 q_0^3 s_0 z - 180 q_0^4 s_0 z + \\
&720 s_0^2 z + 660 q_0 s_0^2 z + 72 p_0 q_0^2 z^2 + 72 p_0 q_0^3 z^2 - 1620 q_0^6 z^2 - 3240 q_0^7 z^2 - 1215 q_0^8 z^2 - 12 p_0 s_0 z^2 - \\
&540 q_0^3 s_0 z^2 - 360 q_0^4 s_0 z^2 + 360 q_0^5 s_0 z^2 + 720 s_0^2 z^2 + 1320 q_0 s_0^2 z^2 + 315 q_0^2 s_0^2 z^2 + 36 p_0 q_0^2 z^3 + 72 p_0 q_0^3 z^3 \\
&+ 18 p_0 q_0^4 z^3 - 540 q_0^6 z^3 - 1620 q_0^7 z^3 - 1215 q_0^8 z^3 - 135 q_0^9 z^3 - 12 p_0 s_0 z^3 - 12 p_0 q_0 s_0 z^3 - 180 q_0^3 s_0 z^3 \\
&- 180 q_0^4 s_0 z^3 + 360 q_0^5 s_0 z^3 + 270 q_0^6 s_0 z^3 + 240 s_0^2 z^3 + 660 q_0 s_0^2 z^3 + 315 q_0^2 s_0^2 z^3 - 45 q_0^3 s_0^2 z^3 - 40 s_0^3 z^3 \\
&+ 15 l_0^2 z^2 (1 + z + q_0 z) + 60 j_0^4 z^2 (10 + (10 + 9q_0) z) + 10 j_0^3 (72 + 12(18 + 19 q_0) z + 3(72 + 152 q_0 \\
&+ 29 q_0^2) z^2 + (72 + 228 q_0 + 87 q_0^2 - 82 q_0^3 + 3 s_0) z^3) + 6 l_0 (-12 q_0^5 z^3 + 12 q_0^4 z^2 (1 + z) + 23 q_0 s_0 z^2 (1 \\
&+ z) + 66 q_0^3 z (1 + z)^2 + 10 s_0 z (1 + z)^2 + 3 q_0^2 (10 + 30 z + 30 z^2 + (10 + s_0) z^3)) + 6 j_0 (45 q_0^7 z^3 + \\
&600 q_0^6 z^2 (1 + z) - 105 q_0^3 s_0 z^2 (1 + z) + 780 q_0^5 z (1 + z)^2 - 120 q_0^4 (-2 - 6z - 6z^2 + (-2 + s_0) z^3) + \\
&l_0 (-30 - 2(45 + 28 q_0) z + (-90 - 112 q_0 + 41 q_0^2) z^2 + (-30 - 56 q_0 + 41 q_0^2 + 33 q_0^3 + 10 s_0) z^3) - \\
&2z(1 + z)(-25 s_0^2 z + 3 p_0 (1 + z)) + 2 q_0^2 z (-p_0 z^2 + 95 s_0 (1 + z)^2) + 2 q_0 (10 s_0^2 z^3 - 7 p_0 z^2 (1 + z) + \\
&55 s_0 (1 + z)^3)) + 3 j_0^2 (75 q_0^5 z^3 - 1125 q_0^4 z^2 (1 + z) - 1580 q_0^3 z (1 + z)^2 + 20 q_0^2 (-23 - 69 z - 69 z^2 + \\
&(-23 + 10 s_0) z^3) + 2 q_0 z^2 (-13 l_0 z + 225 s_0 (1 + z)) + 2z(50 s_0 (1 + z)^2 - z(3 p_0 z + 35 l_0 (1 + z)))))\} \times \\
&\times \{3(-180 q_0^6 - 60 q_0^3 s_0 + 80 s_0^2 + 12 p_0 q_0^2 z - 360 q_0^6 z - 360 q_0^7 z - 120 q_0^3 s_0 z + 160 s_0^2 z + 140 q_0 s_0^2 z + \\
&80 j_0^4 z^2 + 5 l_0^2 z^2 + 12 p_0 q_0^2 z^2 + 12 p_0 q_0^3 z^2 - 180 q_0^6 z^2 - 360 q_0^7 z^2 - 135 q_0^8 z^2 - 4 p_0 s_0 z^2 - 60 q_0^3 s_0 z^2 + \\
&90 q_0^5 s_0 z^2 + 80 s_0^2 z^2 + 140 q_0 s_0^2 z^2 + 5 q_0^2 s_0^2 z^2 + 40 j_0^3 (6 + (12 + 13 q_0) z + (6 + 13 q_0 + 3 q_0^2) z^2) + \\
&l_0 (-18 q_0^4 z^2 + 26 q_0 s_0 z^2 + 72 q_0^3 z (1 + z) + 20 s_0 z (1 + z) + 60 q_0^2 (1 + z)^2) - 5 j_0^2 (8 l_0 z^2 + 95 q_0^4 z^2 - \\
&48 q_0 s_0 z^2 + 224 q_0^3 z (1 + z) - 20 s_0 z (1 + z) + 92 q_0^2 (1 + z)^2) + 2 j_0 (225 q_0^6 z^2 - 115 q_0^3 s_0 z^2 + 540 q_0^5 z (1 + \\
&z) + 80 q_0^2 s_0 z (1 + z) + 240 q_0^4 (1 + z)^2 - 6z(p_0 + p_0 z - 5 s_0^2 z) + l_0 (-30 - 2(30 + 13 q_0) z + (-30 - \\
&26 q_0 + 37 q_0^2) z^2) + 2 q_0 (-4 p_0 z^2 + 55 s_0 (1 + z)^2)))\}^{-1}
\end{aligned}$$

	P_{21}	P_{22}	P_{32}
$100\Omega_b h^2$	2.240 ± 0.014	2.244 ± 0.016	2.240 ± 0.014
$\Omega_c h^2$	0.1195 ± 0.0010	0.1187 ± 0.0011	0.1196 ± 0.0012
Ω_m	0.3177 ± 0.0073	0.3207 ± 0.0086	0.3233 ± 0.0069
n_s	0.9661 ± 0.0038	0.9673 ± 0.0041	0.9656 ± 0.0045
H_0	67.00 ± 0.76	66.51 ± 0.84	66.45 ± 0.70
σ_8	0.8065 ± 0.0087	0.7980 ± 0.0084	0.8035 ± 0.0100
$\overline{q_0}$	-0.86 ± 0.06	-1.06 ± 0.12	-0.70 ± 0.09
$\overline{j_0}$	0.45 ± 0.17	1.71 ± 0.38	0.32 ± 0.24

	P_{21}	P_{22}	P_{32}
$\overline{q_0}$	-0.79	-1.03	-0.69
$\overline{j_0}$	0.22	1.59	0.28
Ω_m	0.3126	0.3288	0.3159
q_0	-0.46	-0.52	-0.42
j_0	0.73	1.19	0.75

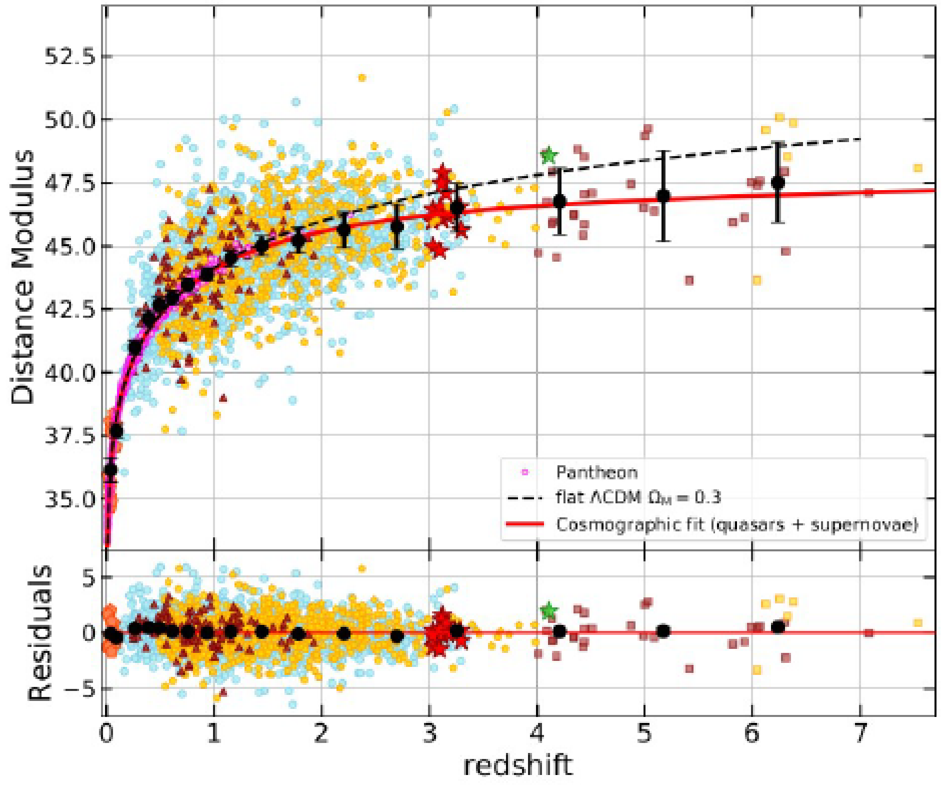
Beyond ΛCDM with f (z)CDM - criticalities and solutions of Padè Cosmography.
A. Turmina Petreca, *MB, S. Capozziello*
Paper coming soon

What next?

- Using P_{32}, test QSO data from Lusso-Risaliti gold sample (2036 sources covering up to $z = 7.54$)

- Orthogonalyse Padè (?!?)

- Any other ideas??



Lusso et al. 2020

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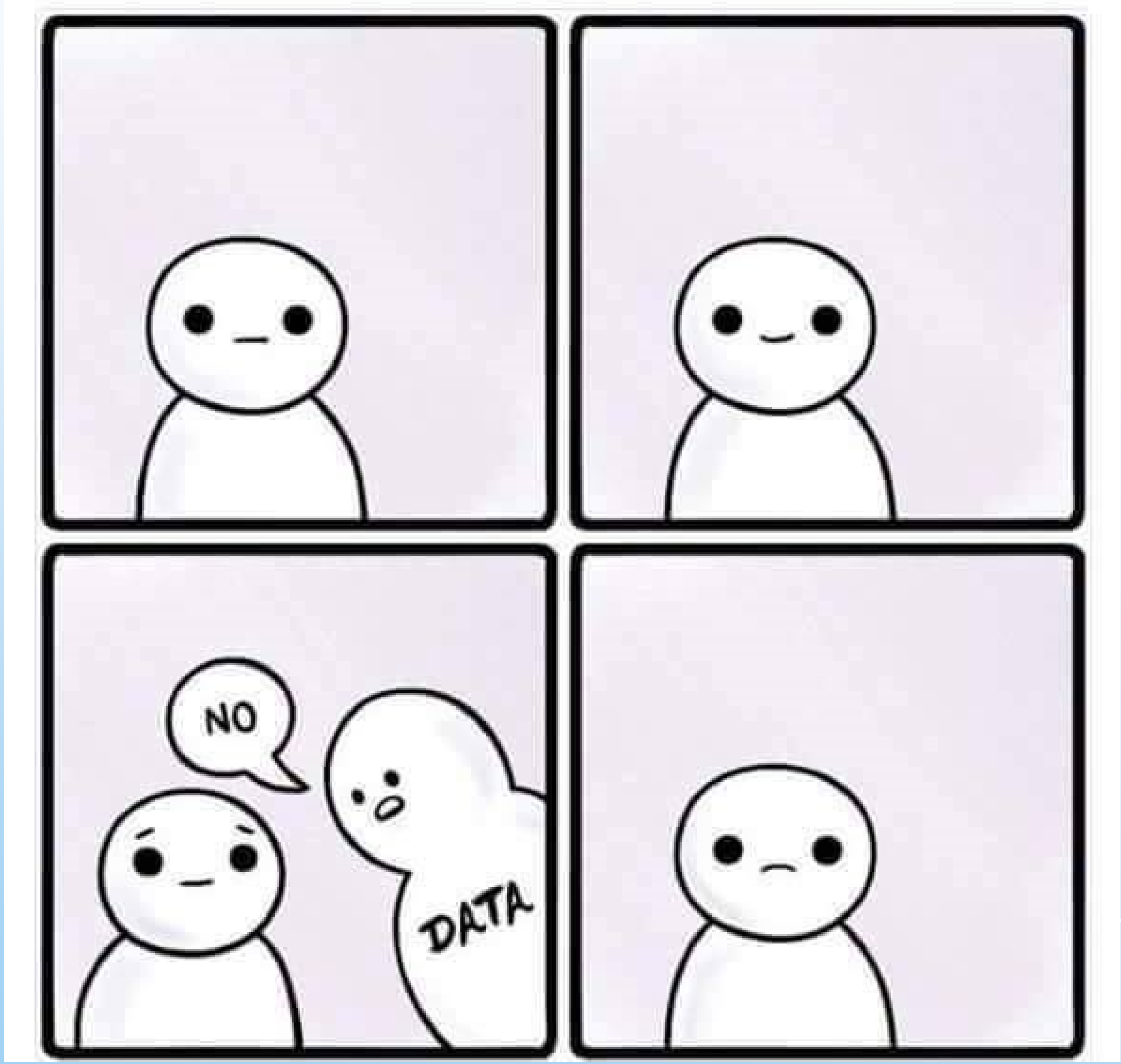
School for Advanced Studies – SSM
Naples, Italy


NO
DATA

QSO as standard candles

the method to determine QSO distances is based on the non-linear relation between their UV and X-ray luminosity

$$\log(\mathrm{L}_X) = \gamma \, log(L_{UV}) + \beta \qquad \text{X: 2 KeV, UV: 2500 Å}$$

The fitted distance moduli are obtained from

$$\mathbf{DM}(z) = \mathbf{5log}[D\mathbf{L}(z)\ (\mathbf{Mpc})] + \mathbf{25} + k$$

where

$$\log D_{\mathrm{L}}(z) = \frac{[\log F_X - \beta - \gamma\,(\log F_{UV} + 27.5)]}{2(\gamma - 1)} - \frac{1}{2}\log(4\pi) + 28.5.$$

The slope γ and the intercept β of the logarithmic X-UV luminosity relation are free parameters of the fit.

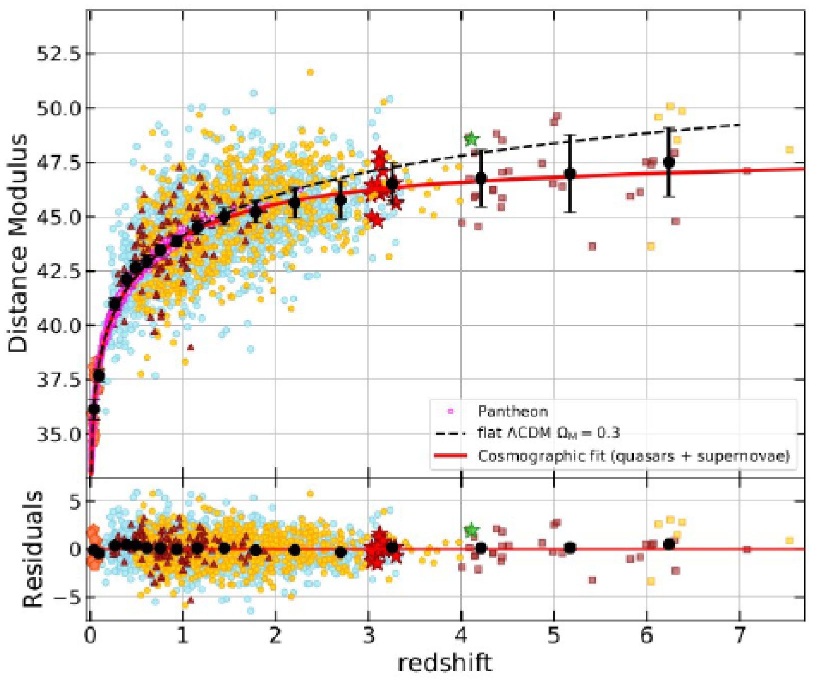
QSO as standard candles

the method to determine QSO distances is based on the non-linear relation between their UV and X-ray luminosity

$$\log(\mathrm{L}_X) = \gamma\, log(L_{UV}) + \beta$$ X: 2 KeV, UV: 2500 Å

The fitted distance moduli are obtained from **DM(z) = 5log[DL(z) (Mpc)] + 25 +** k

k is shared by both SNe and QSOs and is a rigid shift of the QSO Hubble diagram to match the one of SNe in the common redshift range.



Lusso et al. 2020