# Model independent reconstruction of deviations from GR with cosmological data

Matteo Martinelli

INAF
Osservatorio Astronomico di Roma

Frascati, September 12th, 2023

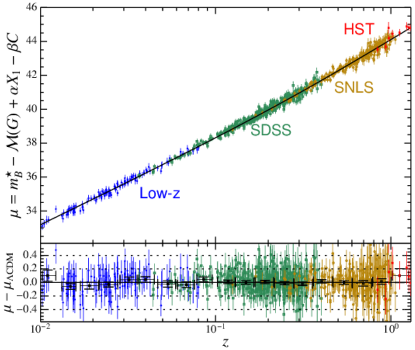
based on :
L. Perenon, MM, S. Ilic, R. Maartens, M. Lochner Phys.Dark Univ. 34 (2021)
L. Perenon, MM, R. Maartens, S. Camera, C. Clarkson Phys.Dark Univ. 37 (2022)

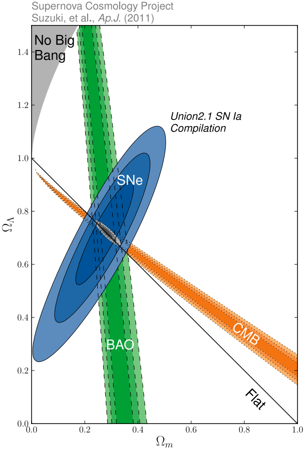
# ΛCDM: a good match for observations

Predictions obtained within the ΛCDM (GR+FLRW+CDM+Λ) can be compared with observations. The ΛCDM model has great success in explaining the observed Universe!

# ΛCDM: a good match for observations

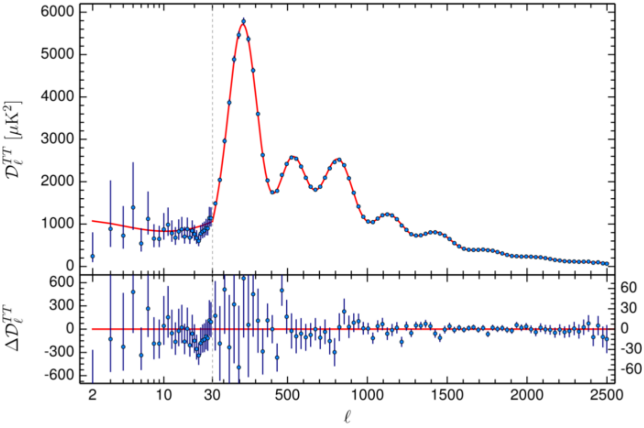
Predictions obtained within the ΛCDM (GR+FLRW+CDM+Λ) can be compared with observations. The ΛCDM model has great success in explaining the observed Universe!



# ΛCDM: a good match for observations

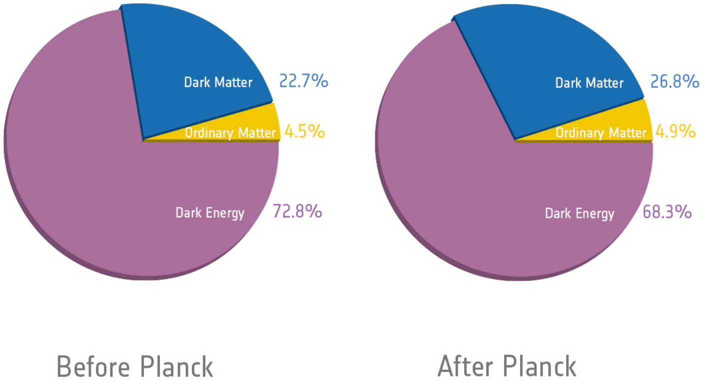
Predictions obtained within the ΛCDM (GR+FLRW+CDM+Λ) can be compared with observations. The ΛCDM model has great success in explaining the observed Universe!




Planck 2015 results I. Overview of products and scientific results

# ΛCDM: a good match for observations

Predictions obtained within the ΛCDM (GR+FLRW+CDM+Λ) can be compared with observations. The ΛCDM model has great success in explaining the observed Universe!



# Cracks in the cosmological constant model

Despite its success, ΛCDM still has some open theoretical questions. From the Λ point of view

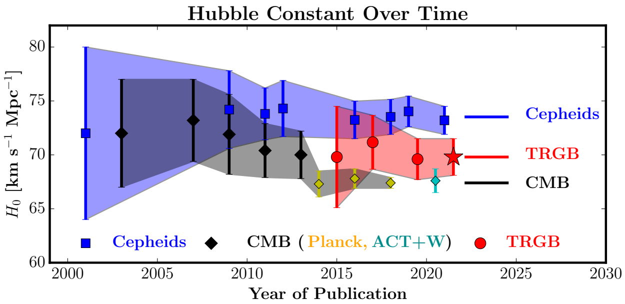
- fine-tuning: why does Λ get the value that we observe?
- coincidence: why does DE and DM have comparable relevance right now?

# Cracks in the cosmological constant model

Despite its success, ΛCDM still has some open theoretical questions. From the Λ point of view

- fine-tuning: why does Λ get the value that we observe?
- coincidence: why does DE and DM have comparable relevance right now?

Moreover, the improved sensitivity of observations also made some tensions emerge, with the most striking being the one on $H_0$ ($\approx 5\sigma$)



# Cracks in the cosmological constant model

Despite its success, ΛCDM still has some open theoretical questions. From the Λ point of view

- fine-tuning: why does Λ get the value that we observe?
- coincidence: why does DE and DM have comparable relevance right now?

Moreover, the improved sensitivity of observations also made some tensions emerge, with the most striking being the one on $H_0$ ($\approx 5\sigma$)

How can we solve these tensions?

- Blame systematics!
- New physics!

# Deviations from the cosmological constant

Ideally, one would like to compare available alternative models with data and their performances against $\Lambda$CDM.
However we have too many available models!

# Deviations from the cosmological constant

Ideally, one would like to compare available alternative models with data and their performances against ΛCDM.
However we have too many available models!

We need a model independent approach!
We can identify the key features of ΛCDM and parameterize deviations from them

$$w_{\rm de} = -1$$
$$k^2\Psi = -4\pi G a^2 \rho\Delta$$
$$k^2\left[\Phi + \Psi\right] = -8\pi G a^2 \rho\Delta$$
$$\frac{\Phi}{\Psi} = 1$$

# Deviations from the cosmological constant

Ideally, one would like to compare available alternative models with data and their performances against ΛCDM.
However we have too many available models!

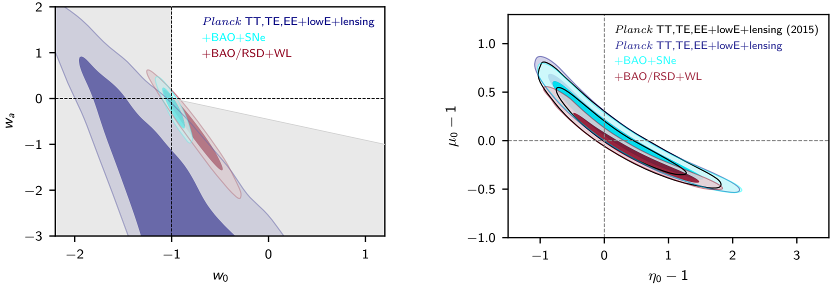
We need a model independent approach!
We can identify the key features of ΛCDM and parameterize deviations from them

$$w_{\rm de} = -1$$
$$k^2\Psi = -4\pi G a^2 \rho\Delta$$
$$k^2\left[\Phi + \Psi\right] = -8\pi G a^2 \rho\Delta$$
$$\frac{\Phi}{\Psi} = 1$$

$$w_{\rm de} = w(a)$$
$$k^2\Psi = -4\pi G \mu(a,k) a^2 \rho\Delta$$
$$k^2\left[\Phi + \Psi\right] = -8\pi G \Sigma(a,k) a^2 \rho\Delta$$
$$\frac{\Phi}{\Psi} = \eta(a,k)$$

# Deviations from GR

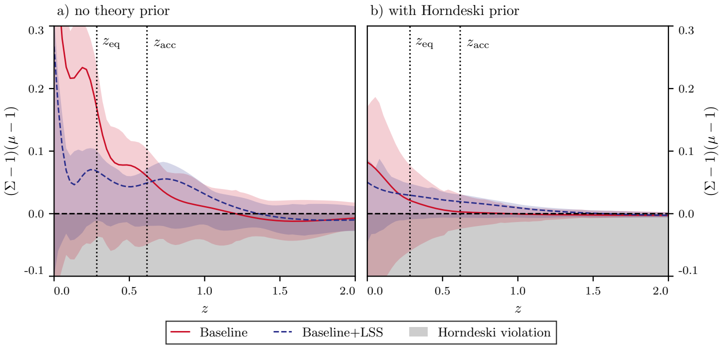
This is an approach that is extensively used in literature, e.g. for the results obtained by the Planck collaboration




Planck 2018 results. VI A&A 641 (2020)

# Deviations from GR

More general reconstructions have been also used, e.g. binning the functions in redshift




Pogosian, Raveri, Koyama, MM, Silvestri, Zhao, Li, Peirone, Zucca Nature Astron. 6 (2022)

# DE evolution as a consistency test

Rather than focusing on parameterized deviations from the ΛCDM model, what we want to do is create a consistency tests that could point to failures of our model.

We are testing the model without proposing any alternative. We decide to do this focusing on $w_{\rm DE}$.

$$w_{\rm de} = w(a)$$

$$k^2\Psi = -4\pi G\mu(a,k)a^2\rho\Delta$$

$$k^2\left[\Phi+\Psi\right] = -8\pi G\Sigma(a,k)a^2\rho\Delta$$

$$\frac{\Phi}{\Psi} = \eta(a,k)$$

# Observables for cosmological evolution

In order to test the ΛCDM model, we want now to focus on the function $w_{\rm de}(z)$.

This is not directly observable, but enters theoretical predictions for quantities that are, e.g. $H(z)$ and $f(z)$. We want to obtain this function from background or perturbation observables.

# Observables for cosmological evolution

In order to test the ΛCDM model, we want now to focus on the function $w_{\rm de}(z)$.

This is not directly observable, but enters theoretical predictions for quantities that are, e.g. $H(z)$ and $f(z)$. We want to obtain this function from background or perturbation observables.

At the background level, one can obtain measurements of $H(z)$ through **Cosmic Chronometers** (CC), i.e. measurements of the rate of expansion through redshift and age measurements of galaxies (based on star formation models).

**Galaxy surveys** can instead provide information on the growth rate $f(z)$ by observing the clustering of cosmological structures

# Observables for cosmological evolution

In order to test the ΛCDM model, we want now to focus on the function $w_{\rm de}(z)$.

This is not directly observable, but enters theoretical predictions for quantities that are, e.g. $H(z)$ and $f(z)$. We want to obtain this function from background or perturbation observables.

At the background level, one can obtain measurements of $H(z)$ through **Cosmic Chronometers** (CC), i.e. measurements of the rate of expansion through redshift and age measurements of galaxies (based on star formation models).

**Galaxy surveys** can instead provide information on the growth rate $f(z)$ by observing the clustering of cosmological structures

Here we use a compilation of $H(z)$ and $f(z)$ data points that is extracted from current observations

R. Bernardo, D. Grandón, J. Levi Said, V. Cardenas Phys. Dark Univ. 36 (2022)
F. Avila, A. Bernui, A. Bonilla, R. Nunes arXiv:2201.07829

# Connecting $w_{\rm de}$ to observables: $w_{\rm bg}$

While $w_{\rm de}(z)$ is not directly observable, this function can be connected to observations.

# Connecting $w_{\rm de}$ to observables: $w_{\rm bg}$

While $w_{\rm de}(z)$ is not directly observable, this function can be connected to observations.
Exploiting the continuity equation for DE in a flat ($\Omega_k = 0$) GR+FLRW Universe

$$\frac{{\rm d}\ln H(z)}{{\rm d}\ln(1+z)} \equiv -\frac{\dot{H}(z)}{H^2(z)} = \frac{3}{2} + \frac{3}{2}\left[1-\Omega_{\rm m}(z)\right] w_{\rm de}(z)\,,$$

we can express the EoS parameter of dark energy as a function of only background quantities

$$w_{\rm bg}(z) = \frac{1}{1-\Omega_{\rm m}(z)}\left[\frac{2}{3}\frac{{\rm d}\ln H(z)}{{\rm d}\ln(1+z)} - 1\right]$$

We saw that $H(z)$ can be obtained from CC observations, while

$$\Omega_{\rm m}(z) = \Omega_{\rm m,0}(1+z)^3\frac{H_0^2}{H^2(z)}\,.$$

Thus, $w_{\rm bg}$ can be completely specified by observations.

# Connecting $w_{\rm de}$ to observables: $w_{\rm gr}$

The same approach can be used starting for the linear evolution equations of matter perturbations, again assuming GR+FLRW (perturbed)

$$\ddot{\delta}_{\rm m}(z) + 2\,H(z)\,\dot{\delta}_{\rm m}(z) - \frac{3}{2}\,\Omega_{\rm m}(z)\,H^2(z)\,\delta_{\rm m}(z) = 0\ ,$$

which can be rewritten as an evolution equation for the growth rate

$$\frac{{\rm d}\ln f(z)}{{\rm d}\ln(1+z)} = f(z) + \frac{1}{2} + \frac{3}{2}\,\left[\Omega_{\rm m}(z) - 1\right]\,w_{\rm de}(z) - 3\,\frac{\Omega_{\rm m}(z)}{2\,f(z)}\ ,$$

from which we can express $w_{\rm de}$ as a function of observable background and perturbation quantities

$$w_{\rm gr}(z) = \frac{1}{1-\Omega_{\rm m}(z)}\,\left[\frac{2}{3}\,\left(f(z) - \frac{{\rm d}\ln f(z)}{{\rm d}\ln(1+z)}\right) - \frac{\Omega_{\rm m}(z)}{f(z)} + \frac{1}{3}\right]\ .$$

# Consistency test

We have now two equations for the same quantity, obtained within the assumptions of the standard cosmological model:

$$w_{\rm bg}(z) = \frac{1}{1-\Omega_{\rm m}(z)}\left[\frac{2}{3}\frac{{\rm d}\ln H(z)}{{\rm d}\ln(1+z)} - 1\right]$$

$$w_{\rm gr}(z) = \frac{1}{1-\Omega_{\rm m}(z)}\left[\frac{2}{3}\left(f(z) - \frac{{\rm d}\ln f(z)}{{\rm d}\ln(1+z)}\right) - \frac{\Omega_{\rm m}(z)}{f(z)} + \frac{1}{3}\right] .$$

We can use the comparison of these two as a consistency test of the standard model, as they need to match if this holds

# Consistency checklist

Our aim is to constrain both $w_{\rm bg}$ and $w_{\rm gr}$ from observational data and compare the results we find.
Depending on this comparison we can obtain information on possible departures from ΛCDM:

- $w_{\rm bg} \neq -1$ departure from flat-ΛCDM caused by either curvature, DE, or MG, or even a combination of them.

# Consistency checklist

Our aim is to constrain both $w_{\rm bg}$ and $w_{\rm gr}$ from observational data and compare the results we find.
Depending on this comparison we can obtain information on possible departures from ΛCDM:

- $w_{\rm bg} \neq -1$ departure from flat-ΛCDM caused by either curvature, DE, or MG, or even a combination of them.

- $w_{\rm gr} \neq -1$ is instead an indication of different possibilities:

# Consistency checklist

Our aim is to constrain both $w_{\rm bg}$ and $w_{\rm gr}$ from observational data and compare the results we find.
Depending on this comparison we can obtain information on possible departures from ΛCDM:

- $w_{\rm bg} \neq -1$ departure from flat-ΛCDM caused by either curvature, DE, or MG, or even a combination of them.

- $w_{\rm gr} \neq -1$ is instead an indication of different possibilities:
  - $w_{\rm bg} = w_{\rm gr} \neq -1$, the same mechanism breaks ΛCDM and we can use the reconstructions to trace its behaviour;

# Consistency checklist

Our aim is to constrain both $w_{\rm bg}$ and $w_{\rm gr}$ from observational data and compare the results we find.
Depending on this comparison we can obtain information on possible departures from ΛCDM:

- $w_{\rm bg} \neq -1$ departure from flat-ΛCDM caused by either curvature, DE, or MG, or even a combination of them.
- $w_{\rm gr} \neq -1$ is instead an indication of different possibilities:
  - $w_{\rm bg} = w_{\rm gr} \neq -1$, the same mechanism breaks ΛCDM and we can use the reconstructions to trace its behaviour;
  - $w_{\rm bg} \neq -1 \neq w_{\rm gr}$, competing effects at play in the two sectors (e.g. massive neutrinos vs MG);

# Consistency checklist

Our aim is to constrain both $w_{\rm bg}$ and $w_{\rm gr}$ from observational data and compare the results we find.
Depending on this comparison we can obtain information on possible departures from ΛCDM:

- $w_{\rm bg} \neq -1$ departure from flat-ΛCDM caused by either curvature, DE, or MG, or even a combination of them.
- $w_{\rm gr} \neq -1$ is instead an indication of different possibilities:
  - $w_{\rm bg} = w_{\rm gr} \neq -1$, the same mechanism breaks ΛCDM and we can use the reconstructions to trace its behaviour;
  - $w_{\rm bg} \neq -1 \neq w_{\rm gr}$, competing effects at play in the two sectors (e.g. massive neutrinos vs MG);
  - If $w_{\rm bg} = -1$, only perturbations affected (MG?), it can also hint for problems in how we obtain the data on $f(z)$.

# Reconstruction instead of fitting

In order to perform our consistency test, we need to obtain the $w_{\rm de}$ functions from observables.

$$w_{\rm bg}(z) = \frac{1}{1-\Omega_{\rm m}(z)} \left[\frac{2}{3}\,\frac{{\rm d}\ln H(z)}{{\rm d}\ln(1+z)} - 1\right]$$

$$w_{\rm gr}(z) = \frac{1}{1-\Omega_{\rm m}(z)} \left[\frac{2}{3}\left(f(z) - \frac{{\rm d}\ln f(z)}{{\rm d}\ln(1+z)}\right) - \frac{\Omega_{\rm m}(z)}{f(z)} + \frac{1}{3}\right] .$$

How do we obtain these functions?

# Reconstruction instead of fitting

In order to perform our consistency test, we need to obtain the $w_{\rm de}$ functions from observables.

$$w_{\rm bg}(z) = \frac{1}{1-\Omega_{\rm m}(z)}\left[\frac{2}{3}\frac{{\rm d}\ln H(z)}{{\rm d}\ln(1+z)}-1\right]$$

$$w_{\rm gr}(z) = \frac{1}{1-\Omega_{\rm m}(z)}\left[\frac{2}{3}\left(f(z)-\frac{{\rm d}\ln f(z)}{{\rm d}\ln(1+z)}\right)-\frac{\Omega_{\rm m}(z)}{f(z)}+\frac{1}{3}\right].$$

How do we obtain these functions?

- parameterize $w_{\rm de}$ and constrain parameters (limited freedom)

# Reconstruction instead of fitting

In order to perform our consistency test, we need to obtain the $w_{\rm de}$ functions from observables.

$$w_{\rm bg}(z) = \frac{1}{1-\Omega_{\rm m}(z)} \left[\frac{2}{3}\frac{{\rm d}\ln H(z)}{{\rm d}\ln(1+z)} - 1\right]$$

$$w_{\rm gr}(z) = \frac{1}{1-\Omega_{\rm m}(z)} \left[\frac{2}{3}\left(f(z) - \frac{{\rm d}\ln f(z)}{{\rm d}\ln(1+z)}\right) - \frac{\Omega_{\rm m}(z)}{f(z)} + \frac{1}{3}\right] .$$

How do we obtain these functions?

- parameterize $w_{\rm de}$ and constrain parameters (limited freedom)
- model independent (?) reconstruction $\Rightarrow$ Gaussian Process

# Gaussian Process

Reconstructing a function with GP corresponds to assuming that the the values $f^*$ at points $X^*$ are Gaussian random variables with mean $\mu^*$ and variance $K$.

Observational data ($y$) are instead Gaussian variables, with noise entering the covariance matrix ($C$).

# Gaussian Process

Reconstructing a function with GP corresponds to assuming that the the values $f^*$ at points $X^*$ are Gaussian random variables with mean $\mu^*$ and variance $K$.

Observational data ($y$) are instead Gaussian variables, with noise entering the covariance matrix ($C$).

Assuming $f^*$ and $y$ come from the same distribution

$$\begin{bmatrix} y \\ f^* \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} \mu \\ \mu^* \end{bmatrix}, \begin{bmatrix} K(X,X)+C & K(X,X^*) \\ K(X^*,X) & K(X^*,X^*) \end{bmatrix} \right).$$

$K$ is the so-called kernel, which is an "arbitrary" function: this will contain hyper-parameters that need to be optimized by minimizing the likelihood

$$\ln \mathcal{L} = -\frac{1}{2}(y-\mu)^T \left[K(X,X)+C\right]^{-1}(y-\mu) - \frac{1}{2}\ln\left|K(X,X)+C\right| - \frac{n}{2}\ln 2\pi$$

# Reconstruction a function with GP

Once the kernel is chosen, and we obtain the optimal hyper parameters, we can reconstruct the mean and covariance of the function we are interested in

$$\mathrm{mean}(f^*) = \mu^* + K(X^*, X)[K(X, X) + C]^{-1}(y - \mu),$$
$$\mathrm{cov}(f^*) = K(X^*, X^*) - K(X^*, X)[K(X, X) + C]^{-1} K(X, X^*)$$

A big advantage is that we are able to obtain derivatives of the reconstructed functions, without taking derivatives of data.

# Reconstruction a function with GP

Once the kernel is chosen, and we obtain the optimal hyper parameters, we can reconstruct the mean and covariance of the function we are interested in

$$\mathrm{mean}(f^*) = \mu^* + K(X^*, X)[K(X, X) + C]^{-1}(y - \mu),$$
$$\mathrm{cov}(f^*) = K(X^*, X^*) - K(X^*, X)[K(X, X) + C]^{-1} K(X, X^*)$$

A big advantage is that we are able to obtain derivatives of the reconstructed functions, without taking derivatives of data.

The use of GP does not free us from the need of making choices. In particular the details of the reconstruction have to be defined.

# Reconstruction a function with GP

Once the kernel is chosen, and we obtain the optimal hyper parameters, we can reconstruct the mean and covariance of the function we are interested in

$$\mathrm{mean}(f^*) = \mu^* + K(X^*, X)[K(X, X) + C]^{-1}(y - \mu),$$
$$\mathrm{cov}(f^*) = K(X^*, X^*) - K(X^*, X)[K(X, X) + C]^{-1} K(X, X^*)$$

A big advantage is that we are able to obtain derivatives of the reconstructed functions, without taking derivatives of data.

The use of GP does not free us from the need of making choices. In particular the details of the reconstruction have to be defined.

- mean priors $\mu$ have to be chosen and we need to check that the reconstruction is not dominated by this choice;

# Reconstruction a function with GP

Once the kernel is chosen, and we obtain the optimal hyper parameters, we can reconstruct the mean and covariance of the function we are interested in

$$\mathrm{mean}(f^*) = \mu^* + K(X^*, X)[K(X, X) + C]^{-1}(y - \mu),$$
$$\mathrm{cov}(f^*) = K(X^*, X^*) - K(X^*, X)[K(X, X) + C]^{-1} K(X, X^*)$$

A big advantage is that we are able to obtain derivatives of the reconstructed functions, without taking derivatives of data.

The use of GP does not free us from the need of making choices. In particular the details of the reconstruction have to be defined.

- mean priors $\mu$ have to be chosen and we need to check that the reconstruction is not dominated by this choice;
- the hyperparameters contained in the kernel need to be optimized and special attention is needed for their prior range as it needs to be wider than the constraints obtained from data;

# Reconstruction a function with GP

Once the kernel is chosen, and we obtain the optimal hyper parameters, we can reconstruct the mean and covariance of the function we are interested in

$$\mathrm{mean}(f^*) = \mu^* + K(X^*, X)[K(X, X) + C]^{-1}(y - \mu),$$
$$\mathrm{cov}(f^*) = K(X^*, X^*) - K(X^*, X)[K(X, X) + C]^{-1} K(X, X^*)$$

A big advantage is that we are able to obtain derivatives of the reconstructed functions, without taking derivatives of data.

The use of GP does not free us from the need of making choices. In particular the details of the reconstruction have to be defined.

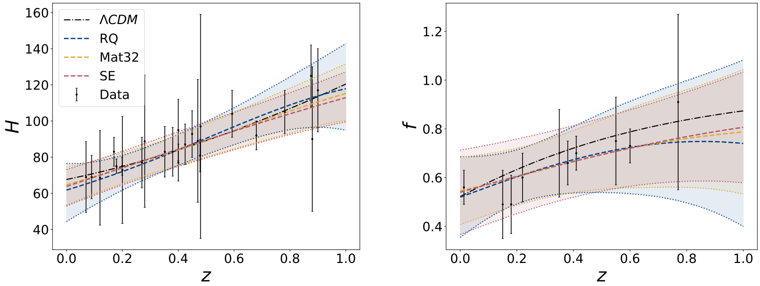
- mean priors $\mu$ have to be chosen and we need to check that the reconstruction is not dominated by this choice;
- the hyperparameters contained in the kernel need to be optimized and special attention is needed for their prior range as it needs to be wider than the constraints obtained from data;
- results should be reasonably stable changing the kernel choice.

# Reconstruction a function with GP

Once the kernel is chosen, and we obtain the optimal hyper parameters, we can reconstruct the mean and covariance of the function we are interested in

$$\mathrm{mean}(f^*) = \mu^* + K(X^*, X)[K(X, X) + C]^{-1}(y - \mu),$$
$$\mathrm{cov}(f^*) = K(X^*, X^*) - K(X^*, X)[K(X, X) + C]^{-1}\, K(X, X^*)$$

A big advantage is that we are able to obtain derivatives of the reconstructed functions, without taking derivatives of data.



# Pipeline for ΛCDM consistency test

In order to apply our consistency test, we follow the procedure below:

1. we choose kernel, hyper parameters range and mean prior for the GP;
2. we apply the GP reconstruction to our CC and galaxy data and obtain the reconstructed $H(z)$ and $f(z)$ functions;
3. from the former, we derive the reconstruction of $\Omega_{\rm m}(z) = \Omega_{\rm m,0}(1+z)^3 H_0^2/H^2(z)$, where $\Omega_{\rm m,0}$ is given by an external prior (independent measurements);
4. using these reconstructed functions and their derivatives, we obtain $w_{\rm bg}(z) = w_{\rm bg}(H, H', \Omega_{\rm m})$ and $w_{\rm gr}(z) = w_{\rm gr}(f, f', \Omega_{\rm m})$ via MCMC sampling;
5. we compare the two functions and draw conclusions on the validity of ΛCDM.

# Are we really model independent?

While GP allows to reconstruct functions without assumptions on their trend, we are still affected by assumptions done in obtaining the equations for $w_{\rm de}$ and $w_{\rm gr}$:

- scale independent growth: this is an assumption that can break down in MG (or with massive neutrinos). Our approach needs to be generalized to include this possibility;

# Are we really model independent?

While GP allows to reconstruct functions without assumptions on their trend, we are still affected by assumptions done in obtaining the equations for $w_{\rm de}$ and $w_{\rm gr}$:

- scale independent growth: this is an assumption that can break down in MG (or with massive neutrinos). Our approach needs to be generalized to include this possibility;
- flat Universe: we assume $\Omega_k = 0$. While the effects of this possible extra component would show up as deviations from $w_i = -1$, this assumption leads to reconstruction issue as this requires $\Omega_{\rm m}(z) < 1$ and

$$w_i \propto \frac{1}{1 - \Omega_{\rm m}(z)}$$

as we don't have data on $\Omega_{\rm m}(z)$ we use external priors on $\Omega_{\rm m,0}$ and obtain

$$\Omega_{\rm m}(z) = \Omega_{\rm m,0}(1+z)^3(H_0/H(z))^2$$

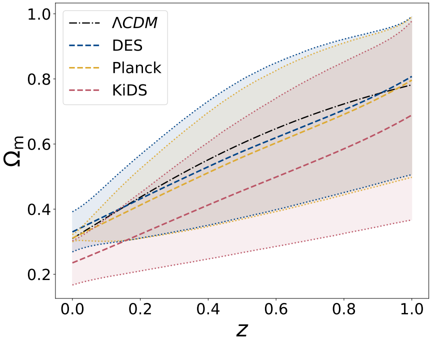
# The $\Omega_{\rm m}(z)$ issue

The last point is quite relevant as for realizations crossing $\Omega_{\rm m} = 1$, the $w_i$ functions will diverge!

To account for this we introduce a hard cut, rejecting all realizations that cross this boundary.

The amount of rejected changes depending on the choice of prior for $\Omega_{\rm m,0}$, with KiDS being the least affected.

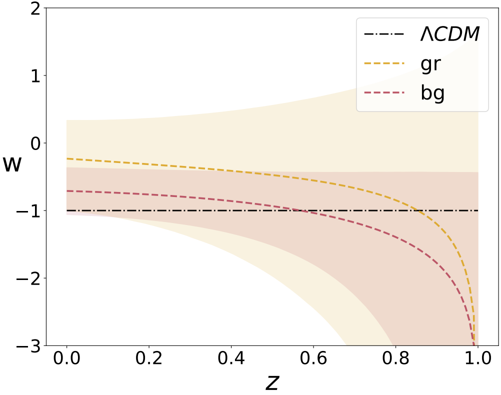
Having data on $\Omega_{\rm m}(z)$ could help with this issue.



# Current data results

When we reconstruct our $w(z)$ function using the current CC and galaxy data we find that the two functions are compatible with each other.

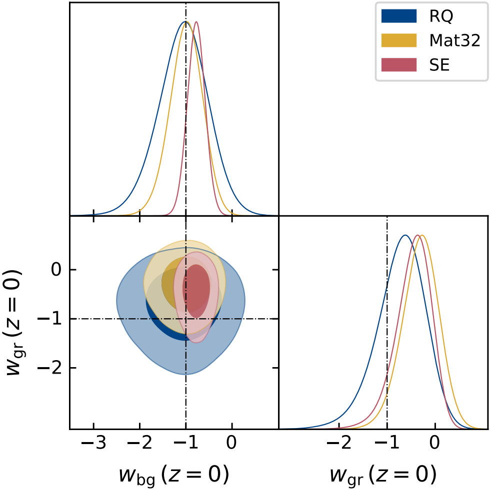
At "high" redshifts we see the effect of $\Omega_{\rm m}(z)$ getting close to one, when matter dominates over dark energy.



# Current data results ($z = 0$)

If we project at current time ($z = 0$) we see an interesting hint of $w_{\rm gr} > -1$, although not statistically significant ($< 2\sigma$).

This trend is stable changing the specifics of GP, and it is most likely due to the preference of galaxy data for a suppressed growth (the $S_8$ tension).



# Expectation for future experiments

We also attempted to understand how much future data can improve this test, and if this can detect a breakdown of ΛCDM should the Universe not follow this model.

In order to do so, we produced synthetic data for $H(z)$ and $f(z)$ in 10 redshift bins with a 1% error, so that we mimic what is expected from Stage IV experiments.

# Expectation for future experiments

We also attempted to understand how much future data can improve this test, and if this can detect a breakdown of ΛCDM should the Universe not follow this model.

In order to do so, we produced synthetic data for $H(z)$ and $f(z)$ in 10 redshift bins with a 1% error, so that we mimic what is expected from Stage IV experiments.

We produce the data assuming two fiducial cosmologies:

- a simple DE model that affects both the reconstruction of $H(z)$ and $f(z)$

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

- a modified gravity model where $H(z)$ is unchanged but perturbations growth is modified

$$f(z) = \Omega_{\rm m}^{\gamma}(z)$$

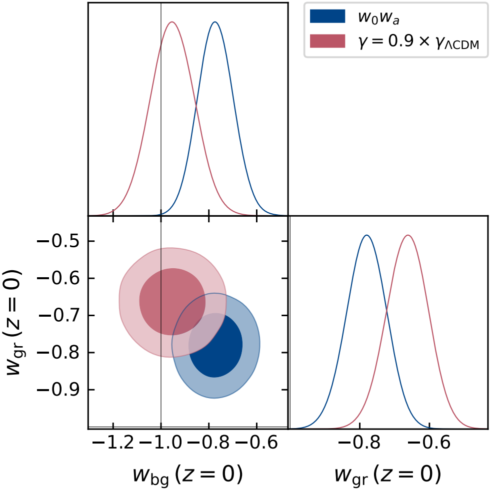
with $\gamma = 0.545$ being the GR limit, while we take a 10% variation from it.

E. Linder Phys.Rev.D 72 (2005)

# Forecast results

The method is able to detect both departures from ΛCDM that we assume and to distinguish between them.

The more precise data and prior on $\Omega_{\mathrm{m},0}$ reduce the dependency on the choice of kernel and priors, while the realization rejected by the $\Omega_{\mathrm{m}}(z) < 1$ are reduced.



# Conclusions

- Starting from the predictions of the ΛCDM model, we can derive equations for the DE EoS starting from background and perturbations;
- we compare $w_{\rm bg}$ and $w_{\rm gr}$ to detect failures of ΛCDM and the reason for it (DE, MG, ...);
- we use GP, rather than parameterizations, to reconstruct the quantities on which $w_i$ depend;
- applying this method to current data (CC and galaxy surveys) shows agreement with ΛCDM... as usual :(
- A hint for $w_{\rm gr} > -1$ is present in the results ($< 2\sigma$) most likely connected to lower growth preferred by galaxy data.

# Take home messages

- Exploring deviations from standard model with consistency checks allows to test the current model without assuming another;
- using GP or other ML reconstruction methods (e.g. GA) allows to avoid parameterizing these functions to fit the data, thus ensuring that we can catch all features;
- assumptions are still present in how the $w_i$ functions are obtained, and they affect our reconstruction. A more general derivation is needed;
- future data will allow to lower the dependency of the results on the fine details of the GP (and on assumptions like the $\Omega_{\rm m}(z)$ cut). Model independent approaches require very good data to work!

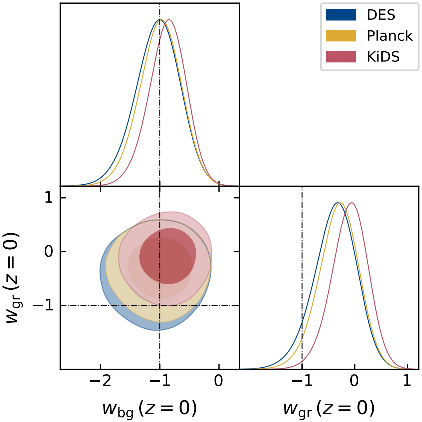
*EXTRA SLIDES*

# Impact of $\Omega_{\mathrm{m},0}$ prior

We cannot obtain estimates for $w_{\mathrm{bg}}$ and $w_{\mathrm{gr}}$ by only reconstructing $H(z)$ and $f(z)$; we need an external prior on $\Omega_{\mathrm{m},0}$ to obtain $\Omega_{\mathrm{m}}(z) = \Omega_{\mathrm{m},0}(1+z)^3(H_0/H(z))^2$.

This introduces an arbitrary choice of the prior, but we tested different possibility finding results compatible with each other.

The KiDS prior gives the most significant deviation from ΛCDM, as this survey is the one with the highest $S_8$ tension.



# Derivatives of GPs

In the expressions for $w_{\rm bg}$ and $w_{\rm gr}$ we do not have only $H(z)$ and $f(z)$ but also their derivatives.

The use of GP allows to obtain such derivatives without the need of directly obtaining data on these, which would imply to "take derivatives of the data" (very noisy procedure)

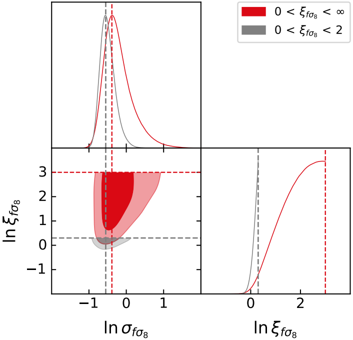
Indeed, once a function $g(x)$ is reconstructed via GP, it is possible to obtain its derivative as the derivative of a GP is still a GP

$$y' \sim \mathcal{N}(\frac{\mathrm{d}}{\mathrm{d}x}\mu, \frac{\mathrm{d}}{\mathrm{d}x}\Sigma)$$

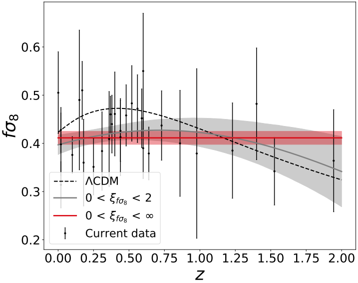
with the derivative of the covariance matrix defined by the derivative of the original kernel

$$k'(x, x^*) = \frac{\partial^2}{\partial x \partial x^*} k(x, x^*)$$

# Issues with hyperparameters




If data are scattered, sparse, with large errors, it can be extremely difficult to find a suitable prior range for hyper-parameters.



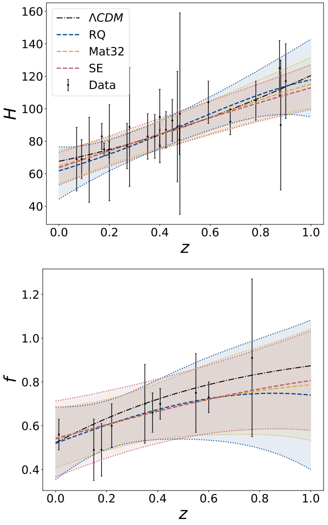

Different ranges can produce completely different results. Important to have data that can produce constraints.

L. Perenon, MM, S. Ilic, R. Maartens, M. Lochner Phys.Dark Univ. 34 (2021)

# Stability checks

We perform our reconstruction with 3 different kernels to test its stability:

- Squared Exponential (SE), simple and smooth reconstruction;
- Matern 3/2 (Mat32), captures sharp variations and performs well with noisy data;
- Rational Quadratic (RQ), combination of many SE kernels, one extra parameter.



# Stability checks

We perform our reconstruction with 3 different kernels to test its stability:

- Squared Exponential (SE), simple and smooth reconstruction;
- Matern 3/2 (Mat32), captures sharp variations and performs well with noisy data;
- Rational Quadratic (RQ), combination of many SE kernels, one extra parameter.

We also check that

- prior ranges for hyper parameters are wide enough, i.e. they are completely constrained by data and not limited by the prior;
- changing the mean prior (we use $\mu = 0$ and the $\Lambda$CDM prediction) does not change the results.

# Multi-task GP

Current and upcoming galaxy data do not only provide information on $f(z)$, but also on $\sigma_8(z)$ and $f\sigma_8(z)$. We might want to exploit all information we have available, but these functions are obviously correlated, we should not reconstruct them separately

The solution for this a "Multi-task" GP, where we consider the correlation both in data and kernels

$$C = \begin{bmatrix} \mathrm{cov}(f,f) & \mathrm{cov}(f,\sigma_8) & \mathrm{cov}(f,f\sigma_8) \\ \mathrm{cov}(f,\sigma_8) & \mathrm{cov}(\sigma_8,\sigma_8) & \mathrm{cov}(\sigma_8,f\sigma_8) \\ \mathrm{cov}(f,f\sigma_8) & \mathrm{cov}(\sigma_8,f\sigma_8) & \mathrm{cov}(f\sigma_8,f\sigma_8) \end{bmatrix},$$

$$\tilde{K} = \begin{bmatrix} K_{f,f} & K_{f,\sigma_8} & K_{f,f\sigma_8} \\ K_{f,\sigma_8} & K_{\sigma_8,\sigma_8} & K_{\sigma_8,f\sigma_8} \\ K_{f,f\sigma_8} & K_{\sigma_8,f\sigma_8} & K_{f\sigma_8,f\sigma_8} \end{bmatrix}$$

with the off-diagonal terms in $\tilde{K}$ being the convolution of the two single kernels.

B. Haridasu, V. Lukovic, M. Moresco, N. Vittorio JCAP 10 (2018)