Nonlinearities in the dark sector in the early and the late Universe

Marco Bruni

Dark Energy : LNF-INFN, 09/2023

UNIVERSITY OF PORTSMOUTH

InDark INFN QUAGRAP

Institute of Cosmology and Gravitation

Outline

- Introduction: history of ΛCDM
- nonlinearity in the late Universe
  - Licia's option I: believe in ΛCDM and use Numerical Relativity simulations
- nonlinearity in the early Universe
  - Licia's option II: bounce and cycle cosmologies from nonlinear Dark Energy
- Conclusions

thanks to my students and collaborators: Kishore Ananda, Molly Burkarm, Rob Crittenden, Chandrima Ganguly, Natalie Hogg, Matteo Martinelli, Rodrigo Mayer, Robyn Munoz, David Wands

University of Durham

The ΛCDM model of cosmogony

Cosmological constant Cold dark matter

slide curtesy of
Carlos Frenk (Durham)

- Proposed in 1980s, it is an *ab initio,* fully specified model of cosmic evolution and the formation of cosmic structure
- Has strong predictive power and can, in principle, be ruled out
- Has made a number of predictions that were subsequently verified empirically (e.g. CMB, LSS, galaxy formation)

Three Nobel Prizes in Physics since 2006

Institute for Computational Cosmology

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CMB fluctuations (for COBE): George Smooth 2006
Λ inferred from SN-Ia: Nobel 2011
Peebles 2019: "for theoretical discoveries in physical cosmology", e.g. flat Universe with Ω_Λ=0.7

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History of Λ

- Myth #1: Λ proposed by Einstein to obtain a static Universe model, then rejected by Einstein after 1929 Hubble "discovery" of the expansion of the Universe
  - lesson #1: fake news are always a mix of facts and fiction
  - fact #1: Λ proposed by Einstein to obtain a static Universe model in 1916-7
  - fact #2: Einstein rejected Λ in a letter to Weyl in 1923, where he clearly say that, after the work of De Sitter that galaxies in his model move apart because of Λ, "then get rid of the cosmological term" https://einsteinpapers.press.princeton.edu/vol14-trans/71

History of Λ

- Myth #2: Λ was forgotten, until rediscovered in Cosmology after the SNae observations that inferred it from the acceleration of the Universe expansion
  - lesson #2: don’t trust astronomers narrative! 😂😂😂
  - fact #3: inflationary scenario (Guth 1981 but also earlier works by Starobinsky and Grishchuk) predicts a very flat Universe
  - fact #2: based on this and on data available at the time Peebles (1984) inferred a value for Ω_m=0.2±0.1

History of Λ

THE ASTROPHYSICAL JOURNAL, **284**:439–444, 1984 September 15

TESTS OF COSMOLOGICAL MODELS CONSTRAINED BY INFLATION

P. J. E. PEEBLES
Joseph Henry Laboratories, Princeton University
Received 1984 February 6; accepted 1984 March 23

ABSTRACT

The inflationary scenario requires that the universe have negligible curvature along constant-density surfaces. In the Friedmann-Lemaître cosmology that leaves us with two free parameters, Hubble's constant H_0 and the density parameter Ω_0 (or, equivalently, the cosmological constant Λ). I discuss here tests of this set of models from local and high-redshift observations. The data agree reasonably well with $\Omega_0 \sim 0.2$.

Subject heading: cosmology

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personal note: as a student of Occhionero in Rome in the '80s, I can testify that general-relativistic cosmological models with Λ where part of the undergraduates lectures, and the topic of research papers (one cited by Peebles) by Occhionero, Vittorio and collaborators

Λ
anticipated from theory
(flatness +
CMB fluctuations
+ simulations),
vs data
(CMB +
galaxy
distribution)
in
Nature 348 (1990)
705–707.

The cosmological constant and cold dark matter

G. Efstathiou, W. J. Sutherland & S. J. Maddox

Department of Physics, University of Oxford, Oxford OX1 3RH, UK

THE cold dark matter (CDM) model[1–4] for the formation and distribution of galaxies in a universe with exactly the critical density is theoretically appealing and has proved to be durable, but recent work[5–8] suggests that there is more cosmological structure on very large scales ($l > 10\,h^{-1}$ Mpc, where h is the Hubble constant H_0 in units of $100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$) than simple versions of the CDM theory predict. We argue here that the successes of the CDM theory can be retained and the new observations accommodated in a spatially flat cosmology in which as much as 80% of the critical density is provided by a positive cosmological constant, which is dynamically equivalent to endowing the vacuum with a non-zero energy density. In such a universe, expansion was dominated by CDM until a recent epoch, but is now governed by the cosmological constant. As well as explaining large-scale structure, a cosmological constant can account for the lack of fluctuations in the microwave background and the large number of certain kinds of object found at high redshift.

Cosmological Parameters

- expansion rate, Hubble parameter $H \equiv \frac{\dot{a}}{a}, \;\; H_0 = \dot{a}_0$
- Friedmann equation today

$$H_0^2 + K = \frac{8\pi G}{3}(\rho_{M0} + \rho_{R0}) + \frac{\Lambda}{3}$$

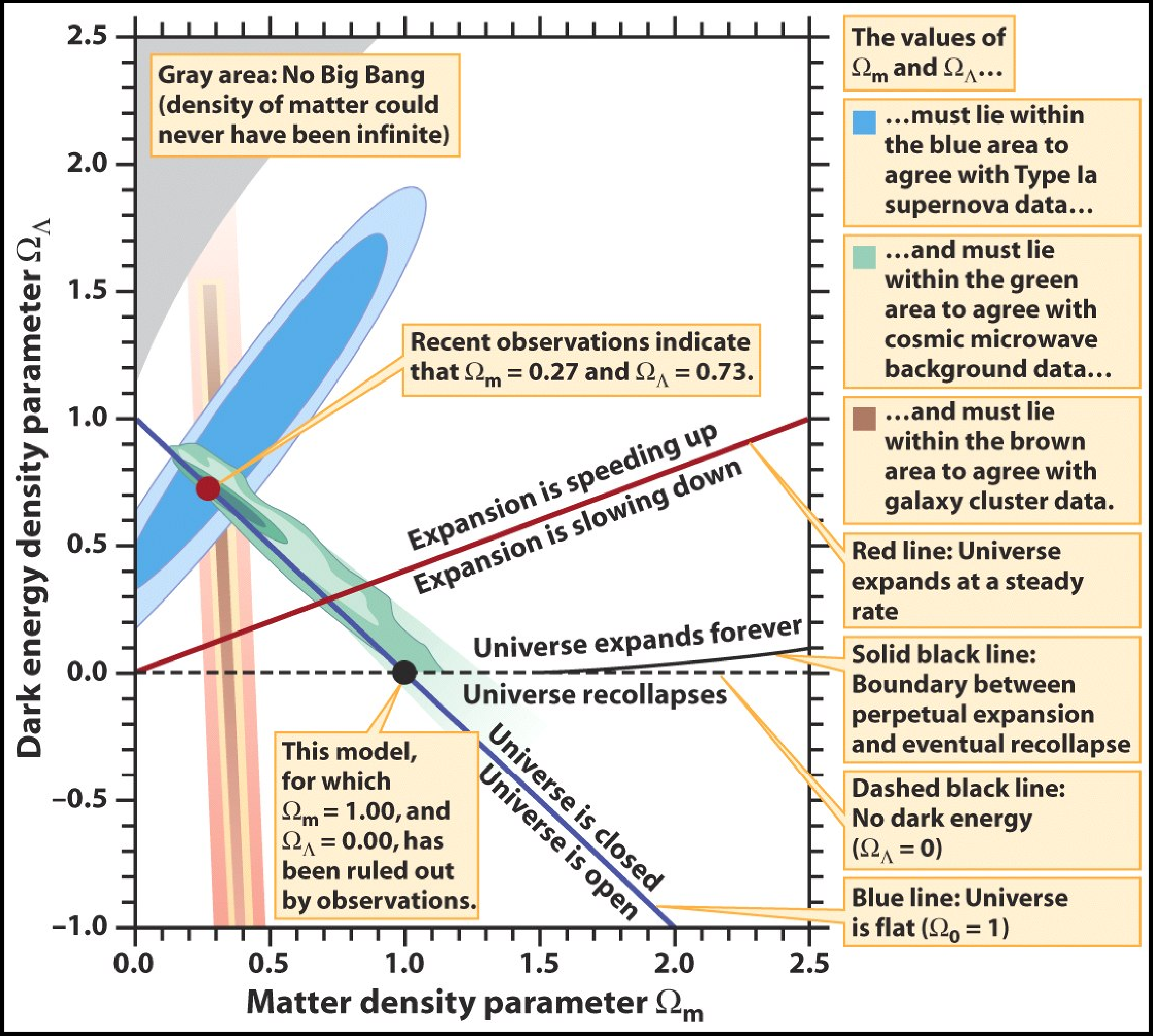
- dimensionless density parameters

$$\rho_c \equiv \frac{3H_0^2}{8\pi G}, \;\; \Omega_{M0} \equiv \frac{\rho_{M0}}{\rho_c}, \;\; \Omega_{R0} \equiv \frac{\rho_{R0}}{\rho_c}, \;\; \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}, \;\; \Omega_K \equiv -\frac{K}{H_0^2}$$

- today (neglecting radiation):

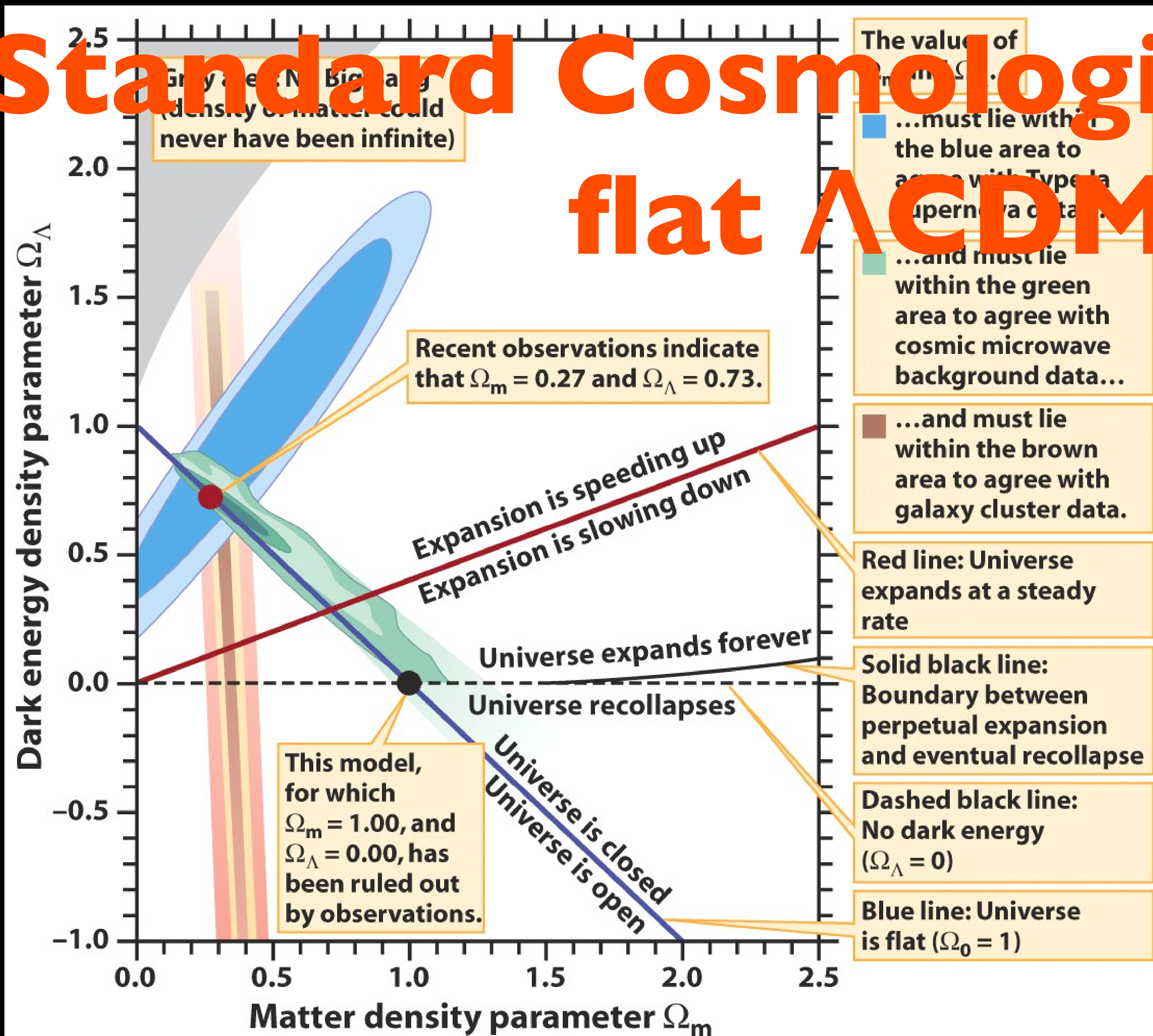
$$\Omega_{M0} + \Omega_\Lambda = 1 - \Omega_K$$

Standard Cosmology



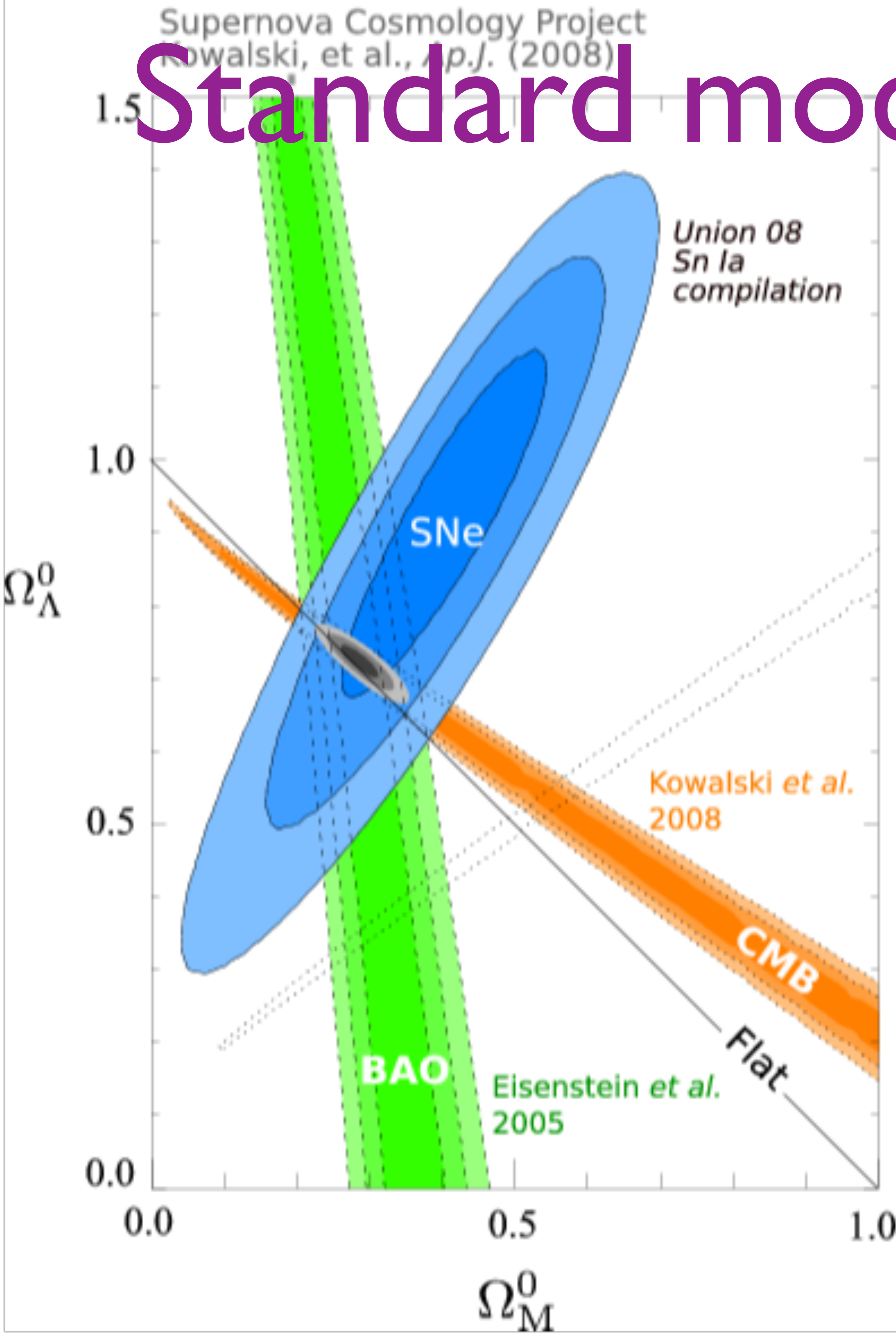
Standard Cosmology

Standard Cosmological Model: flat ΛCDM



Boomerang!!

Standard model: flat ΛCDM

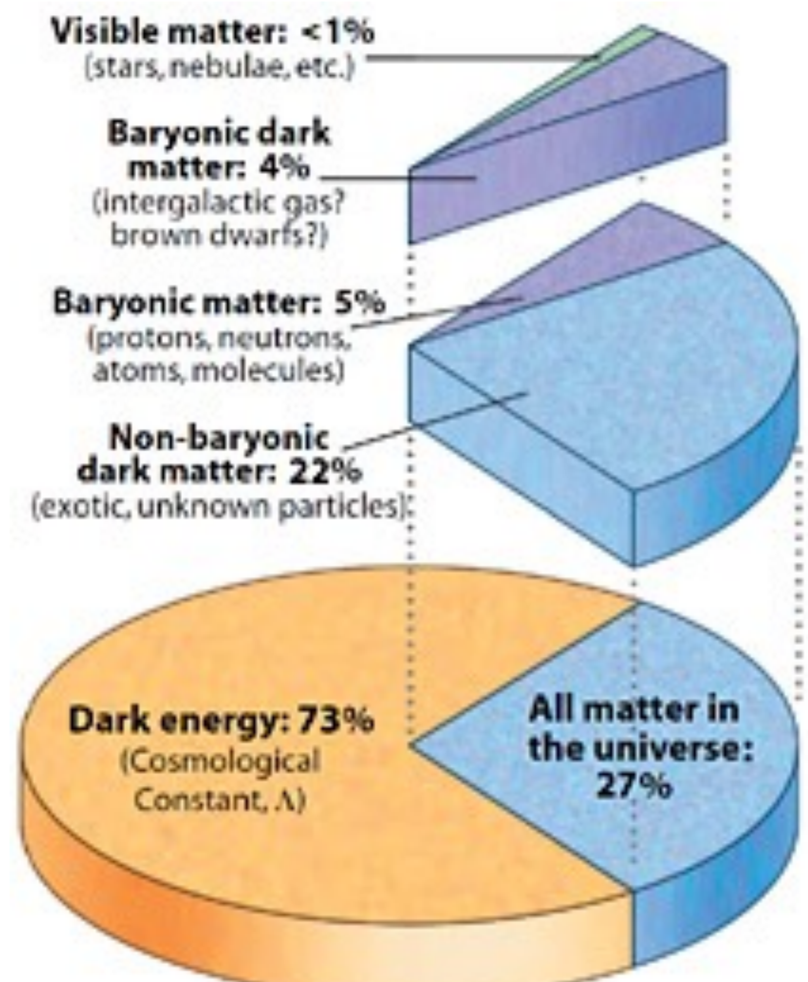


Plank 2018: $-0.095<\Omega_K<-0.007$

(Aghanim et al. [Planck], 2020, Astron. Astrophys. 641, A6)

cf. Di Valentino, Melchiorri & Silk, Nature Astron. 4, 2, 196

$$\Omega_\Lambda = 1 - \Omega_K - \Omega_{M0}$$



Standard ΛCDM Cosmology

Standard ΛCDM Cosmology

- Recipe for modelling based on 3 main ingredients:
  1. Homogeneous isotropic background, FLRW models
  2. Relativistic Perturbations, good for early times and/or for large scales, e.g. CMB and LSS; I-order, II order, "gradient expansion" (aka long-wavelength approximation)
  3. Newtonian study of non-linear structure formation (N-body simulations or approx. techniques, e.g. 2LPT) at small scales

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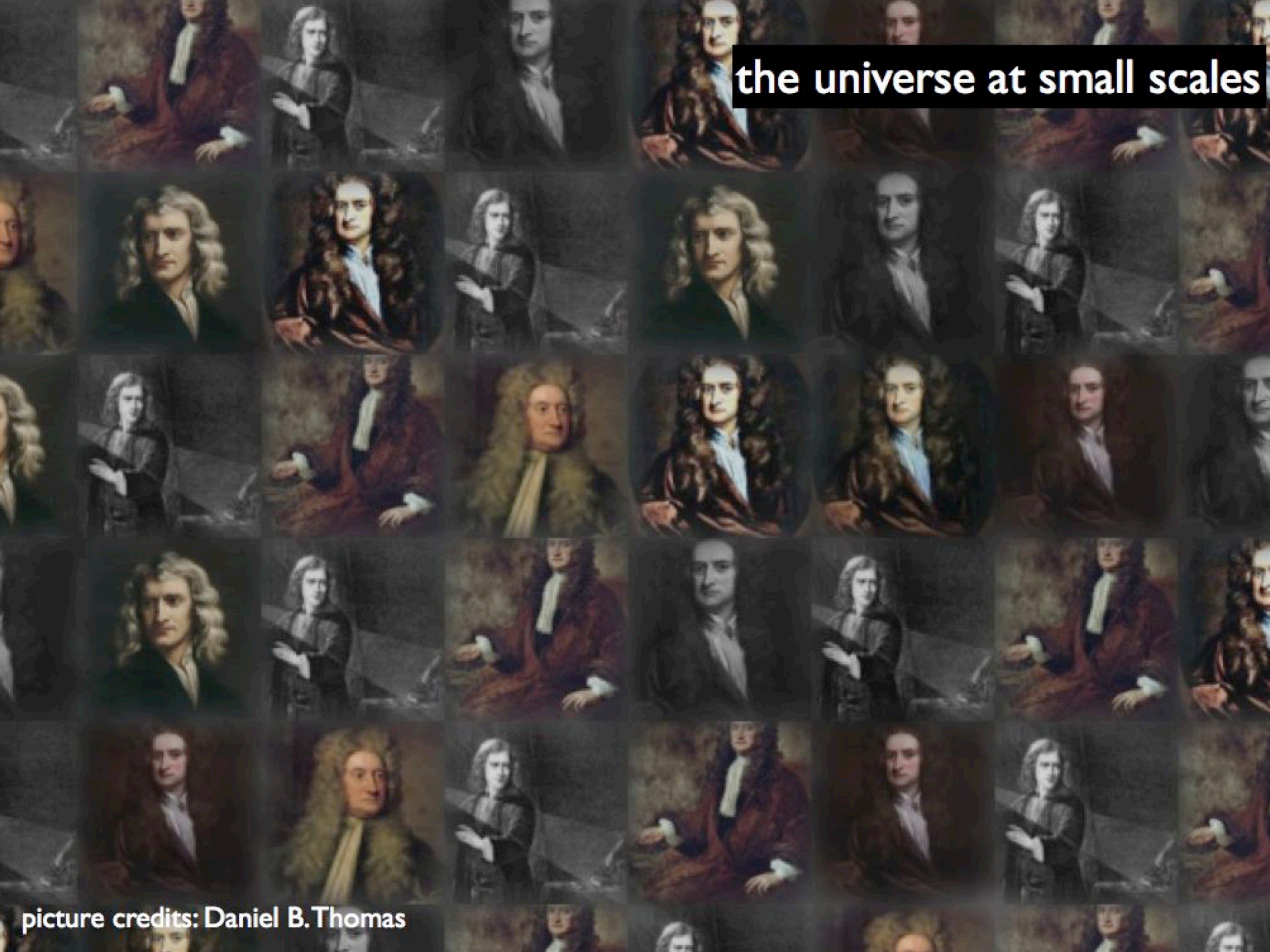
- on this basis, well supported by observations, the flat ΛCDM model has emerged as the Standard "Concordance" Model of cosmology.

the universe at very large scales: GR

picture credits: Daniel B. Thomas


the universe at small scales
picture credits: Daniel B. Thomas

Questions on ΛCDM

- Recipe for modelling based on 3 main ingredients:
  1. Homogeneous isotropic background, FLRW models
  2. Relativistic Perturbations (e.g. CMB; linear, nonlinear)
  3. Newtonian study of non-linear structure formation (numerical simulations or approx. techniques)
- Is 3 enough? (more data, precision cosmology, observations and simulations covering large fraction of H^{-1}, etc...)
  - It is timely to bridge the gap between 2 and 3

Structure Formation Simulations with Numerical Relativity

work with Robyn Munoz

Munoz and Bruni CQG **40**, 135010 (2023), 2211.08133

Munoz and Bruni PRD **107**, 123536 (2023), 2302.09033

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Robyn just passed her VIVA!

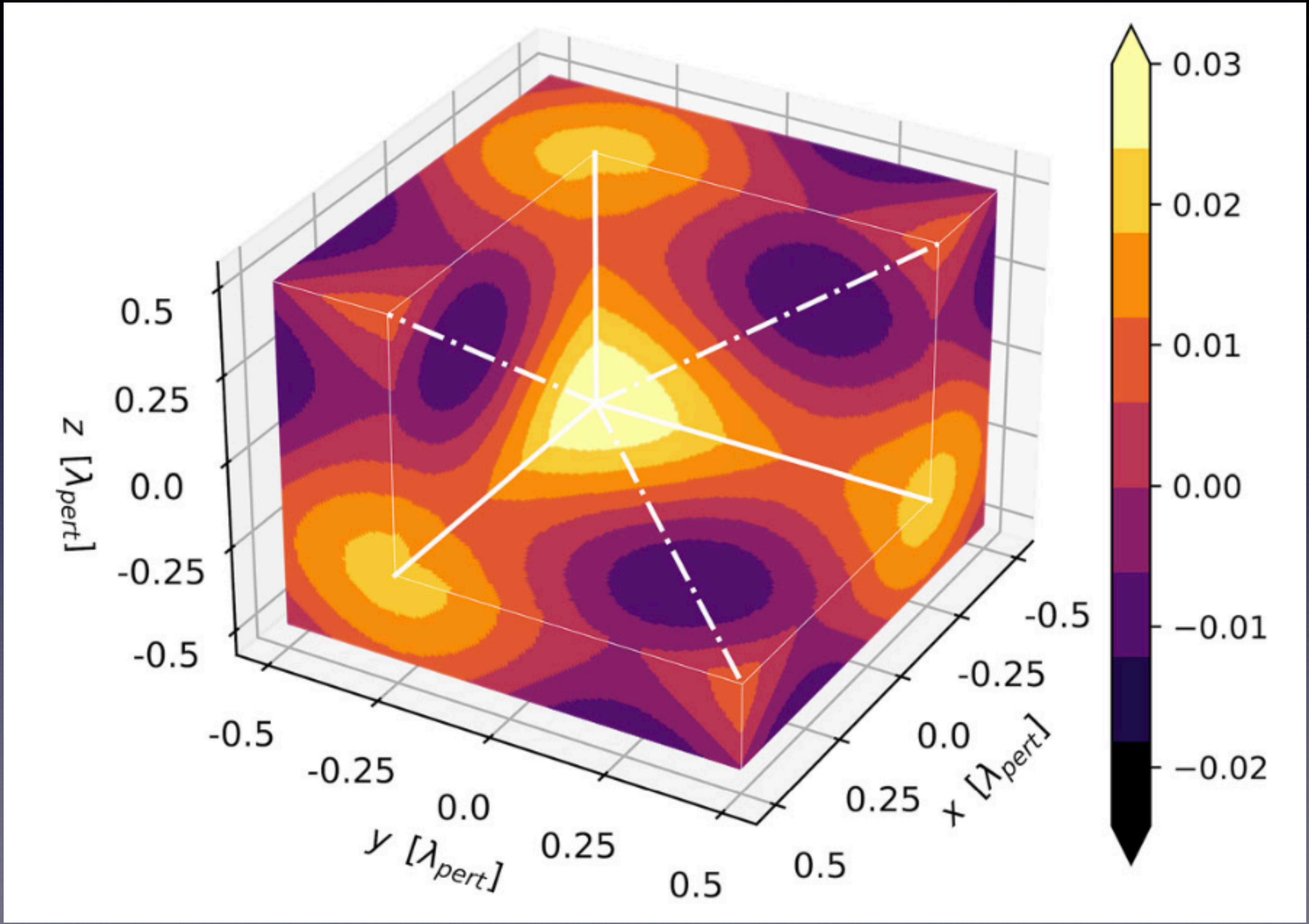
full GR with ET: Reductionism

- Cosmic web in LSS: a network of peaks connected by filaments and separated by voids, see J. R. Bond, L. Kofman, and D. Pogosyan, Nature (London) 380, 603 (1996).
- reductionistic approach:
  - use Einstein Toolkit to study how good the spherical TOP-HAT is in non-linear structure formation, and possible role of Λ
  - Fluid simulations valid up to first shall crossing, starting at z~300, lattice (periodic boundary conditions) of over-densities (OD)
  - initial conditions: inflationary curvature perturbation variable with simple spatial distribution (growing mode in flat ΛCDM)

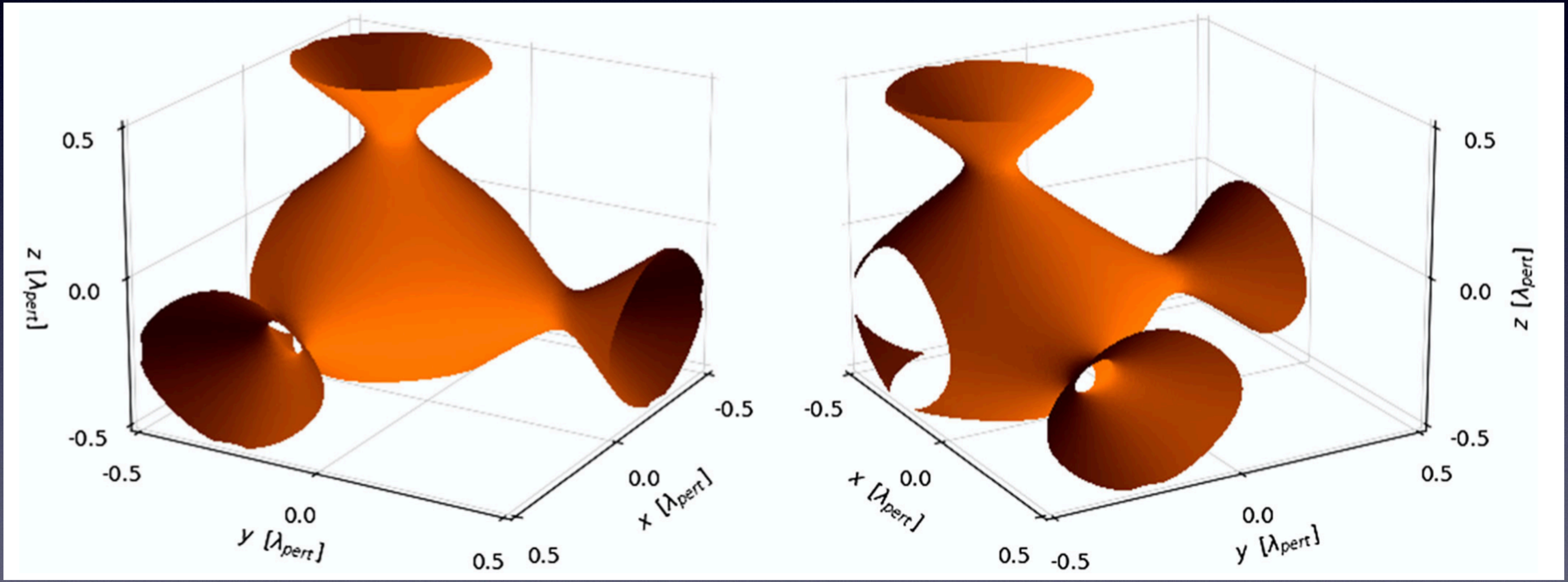
$$\mathcal{R}_c = A_{\rm pert}(\sin{(xk_{\rm pert})} + \sin{(yk_{\rm pert})} + \sin{(zk_{\rm pert})}),$$

- quasi-spherical around peak, but with filaments and voids

δ: density distribution

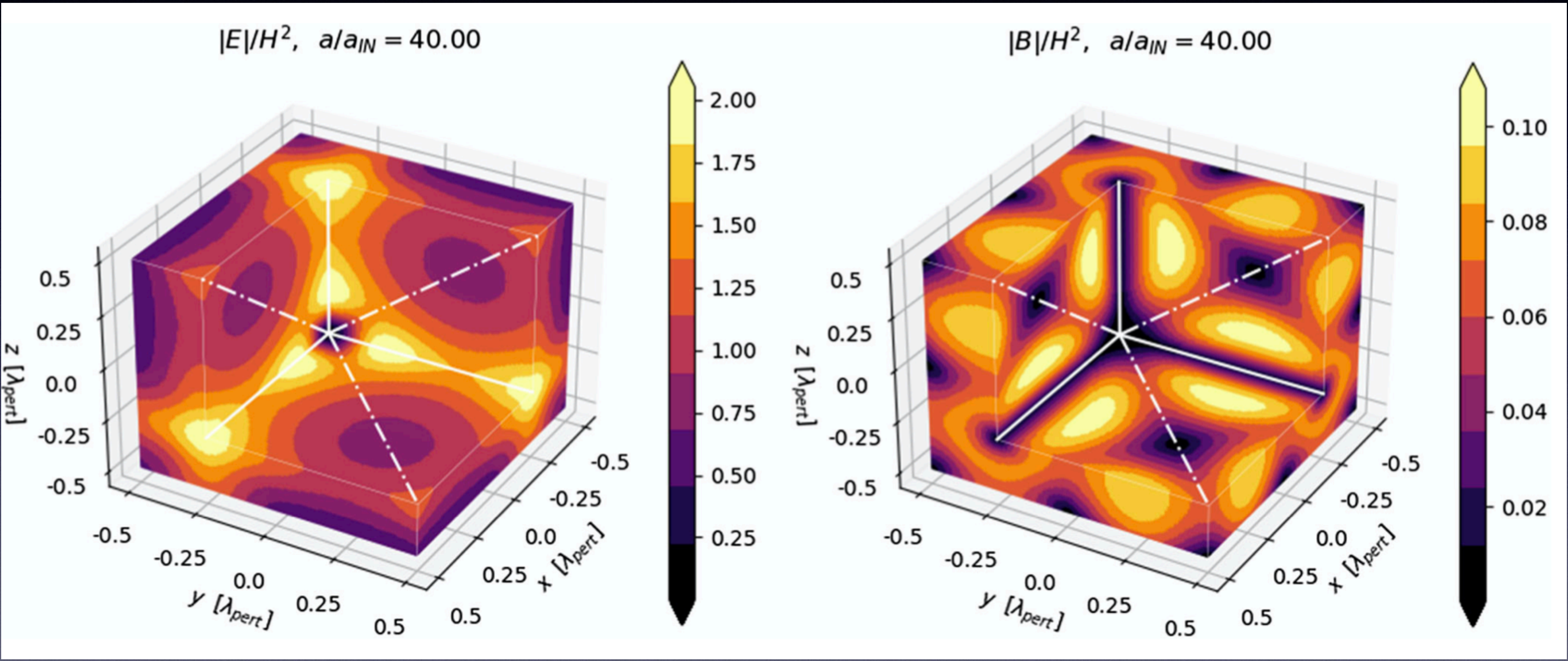


iso-density surface δ=0.01 at z_{in}=302



two different points of view

E and B parts of the Weyl curvature



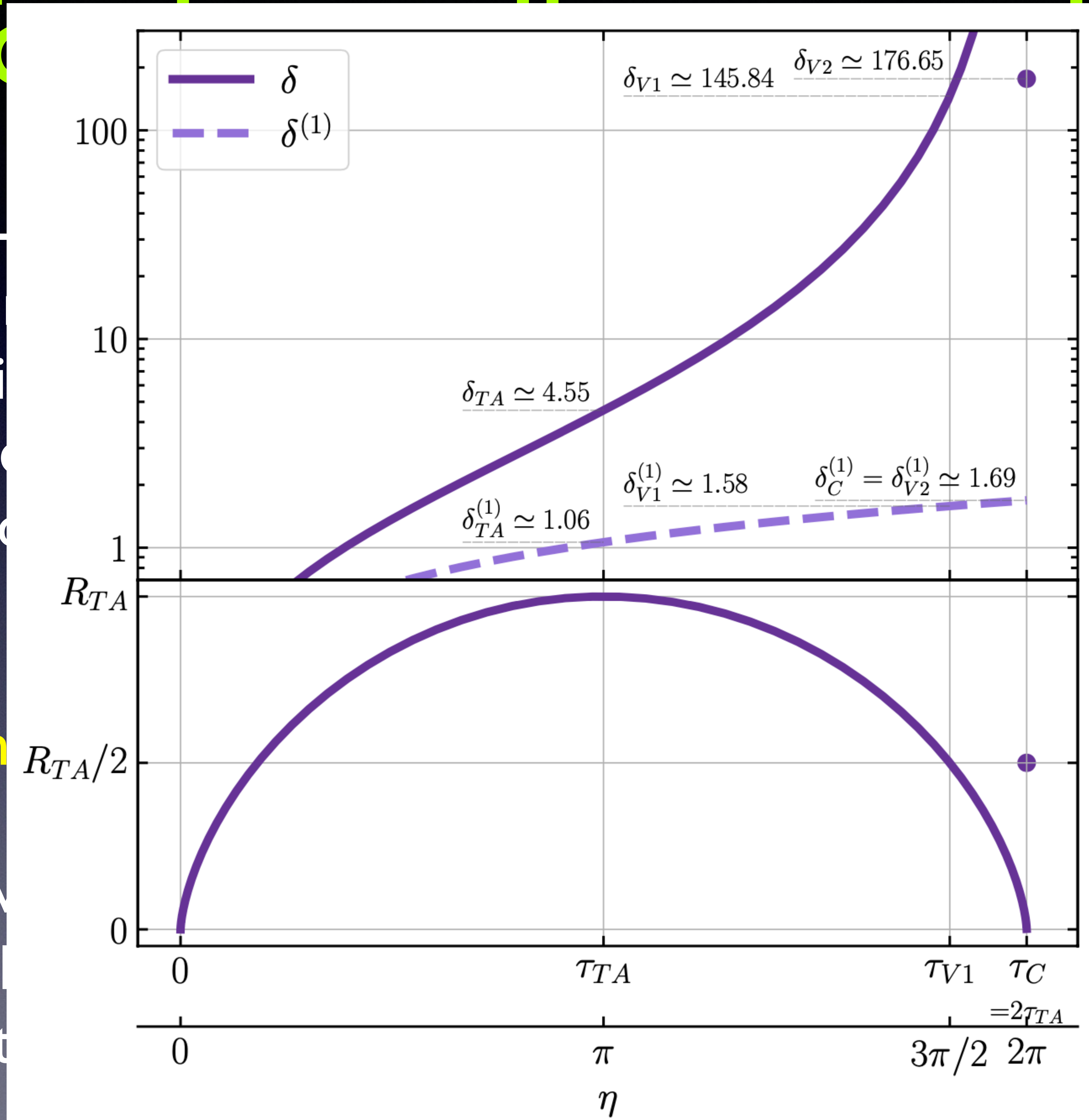
simple structure makes evident that E is stronger around the peaks and along filaments, while B is stronger around filaments

Top-hat collapse model

- the so-called Top-hat collapse model described the detachment of an overdensity from the Hubble expansion, and it is based on a closed model (negative energy, or positive curvature in GR) that expands, reaches a Turn Around, then recollapses.
- It is at the base of the mass function theory of Press-Schechter mass function and the Sheth-Tormen extension
- linear value of δ used as benchmark, conventional values of nonlinear δ used to flag virialization, collapse time predict first shell-crossing

To ... el

- the so- ... e
  detach ... xpansion,
  and it i ... y, or
  positiv ... Turn
  Aroun
- It is at ... ess-
  Schech ... xtension
- linear v ... l values
  of nonl ... ime
  predict



validity of Top-hat collapse

		Top-Hat, $\Lambda = 0$	Here, $\Lambda = 0$	Here, $\Lambda \neq 0$
Initially	z_{IN}		205.4	302.5
	a/a_{IN}	35.4137	$35.24467 \pm$ 7e-5	$35.195 \pm$ 3e-3
	z		$4.85620 \pm$ 1e-5	$7.6234 \pm$ 7e-4
Turn Around (TA)	$\gamma_{\mathrm{OD}}^{1/6}/\gamma_{\mathrm{IN,OD}}^{1/6}$		$20.10169 \pm$ 3e-5	$20.0600 \pm$ 1e-4
$K_{\mathrm{OD}} = 0$	$\langle\gamma^{1/6}\rangle_{\mathcal{D}}/\langle\gamma^{1/6}\rangle_{\mathcal{D},\mathrm{IN}}$		$35.2064 \pm$ 1e-4	$35.154 \pm$ 3e-3
	$\delta_{\mathrm{OD}}^{(1)}$	1.06241	$1.05734 \pm$ 2e-6	$1.05584 \pm$ 8e-5
	δ_{OD}	4.55165	$4.55164 \pm$ 1e-5	$4.5626 \pm$ 5e-4
	a/a_{IN}	56.22	$55.9 \pm$ 1e-1	$55.87 \pm$ 8e-2
	z		$2.692 \pm$ 7e-3	$4.432 \pm$ 8e-3
Collapse	$\gamma_{\mathrm{OD}}^{1/6}/\gamma_{\mathrm{IN,OD}}^{1/6}$		$0.4 \pm$ 6e-1	$0.8 \pm$ 2e-1
/Crash	$\langle\gamma^{1/6}\rangle_{\mathcal{D}}/\langle\gamma^{1/6}\rangle_{\mathcal{D},\mathrm{IN}}$		$55.8 \pm$ 1e-1	$55.77 \pm$ 2e-2
	$\delta_{\mathrm{OD}}^{(1)}$	1.686	$1.678 \pm$ 3e-3	$1.676 \pm$ 2e-3
	δ_{OD}	$+\infty$	$2e+6 \pm 2e+6$	$4e+5 \pm 4e+5$
Virialization	a/a_{IN}	52.64	$52.5055 \pm$ 9e-4	$52.469 \pm$ 2e-3
$R = R_{\mathrm{TA}}/2$	δ_{OD}	145.84	145.84	145.84
Virialization	a/a_{IN}	56.22	$52.83625 \pm$ 7e-5	$52.801 \pm$ 2e-3
$R = R_{\mathrm{TA}}/2$ & $\tau = \tau_C$	δ_{OD}	176.65	176.65	176.65

why the top-hat model works so well?

- It is all down to the Raychaudhuri equation! (local evolution)

$$\dot{\Theta} = -\frac{\Theta^2}{3} - 4\pi G \rho_M - 2\sigma^2 + 2\omega^2 + \Lambda$$

- Hamiltonian constraint

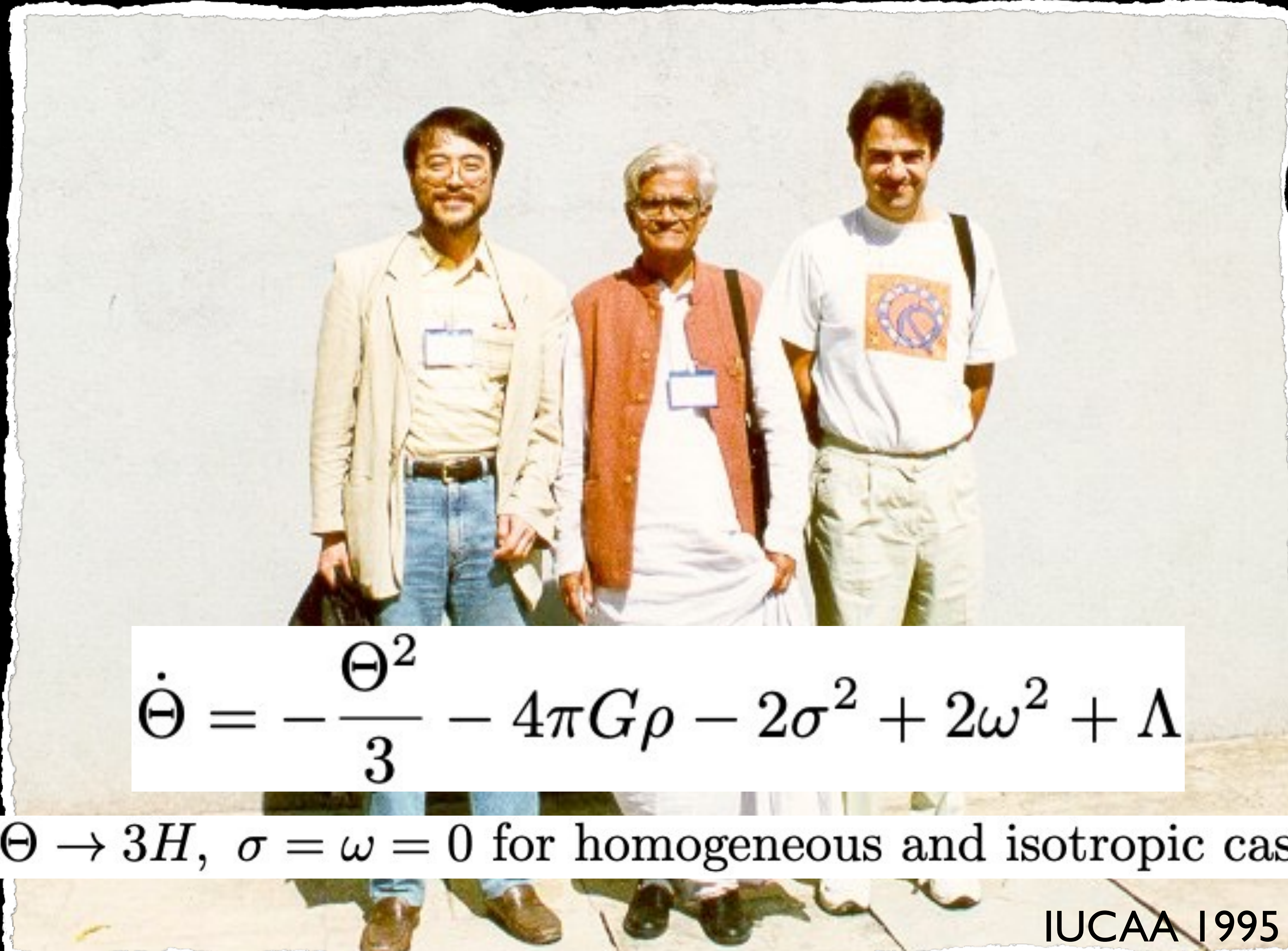
$$^{(3)}R = -\frac{2}{3}\Theta^2 + 16\pi G\rho + 2\sigma^2$$

$\Theta \to 3H = 3\frac{\dot{a}}{a}$, $\sigma = \omega = 0$ for homogeneous and isotropic case

reduces to Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_M + \frac{\Lambda}{3}$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho_M + \frac{\Lambda}{3}$$



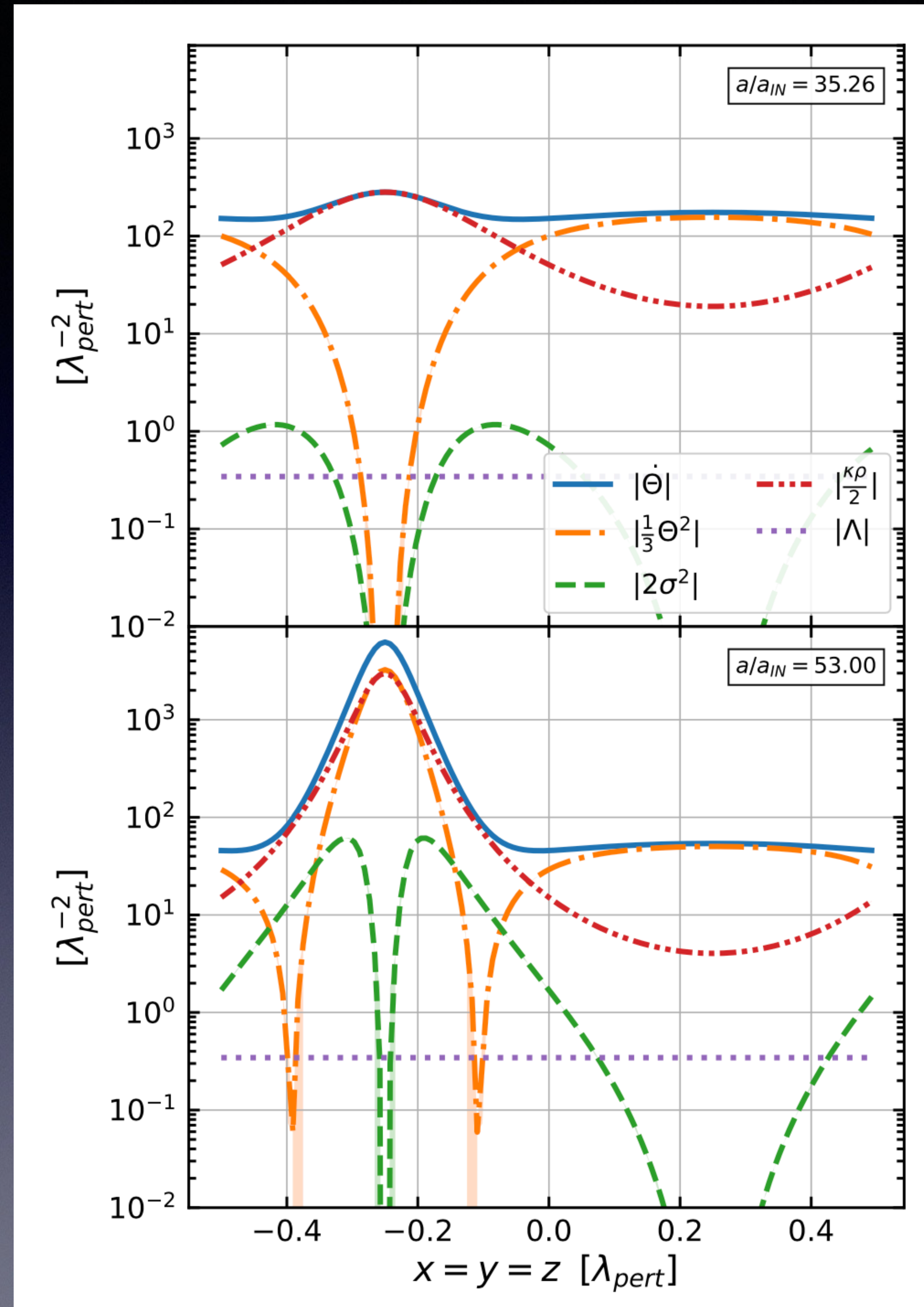
$$\dot{\Theta} = -\frac{\Theta^2}{3} - 4\pi G\rho - 2\sigma^2 + 2\omega^2 + \Lambda$$

$\Theta \to 3H,\ \sigma = \omega = 0$ for homogeneous and isotropic case

IUCAA 1995

contributions to Raychaudhuri

- in our quasi-spherical collapse the shear σ is negligible at the peak initially
- at Turn-Around (Top panel) Θ=0 and the shear remains negligible at the peak, and subdominant in general
- at collapse (Bottom panel) the shear remains negligible at the peak
- 3-Ricci curvature (not shown) becomes important at the peak
- Raychaudhuri is very well approximated by the Friedmann equation for closed models
- Top-Hat works very well to predict first shell-crossing



Early Universe: Cyclic and Bouncing with DE

- The idea of non singular models is not new
- Friedmann, Einstein, Lemaitre and Tolman, all wrote papers in the ‘30s addressing this subject
- many current attempts, e.g. Ijjas & Stenhardt 1904.08022
- there is also work outside GR, e.g. in LQC, but we stay in classical GR
- see e.g. Novello & Bergliaffa arXiv.0802.1634 for a review on bouncing comsmologies

Motivations

- classical ‘60s and ‘70s works on singularities are the result of assumptions on energy conditions
- once we introduce Dark Energy (DE) in cosmology, we give up on those assumptions
- it is then natural to investigate DE effects in the early universe
- “*David Hilbert famously argued that infinity cannot exist in physical reality. The consequence of this statement — still under debate today — has far-reaching implications*”, see Ellis, Meissner & Nicolai, Nature Physics, 14, 2018
- focus on closed FLRW (positive curvature) and Bianchi IX models
- observations: DE, with or without interaction with CDM, at least compatible (vast area, no time!)

basic GR equations

- definitions: for a fluid with 4-velocity u^μ, proper time derivative and expansion scalar are

$$\dot{\rho} \equiv u^\mu \nabla_\mu \rho, \quad \Theta \equiv \nabla_\mu u^\mu$$

- energy conservation equation

$$\dot{\rho} = -\Theta(\rho + P)$$

- Raychaudhuri

DE: a component that gives a positive contribution, P<-1/3ρ

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 + A - 2(\sigma^2 - \omega^2) - 4\pi G(\rho + 3P)$$

- Hamiltonian constraint

$${}^{(3)}R = -\frac{2}{3}\Theta^2 + 16\pi G\rho + 2\sigma^2$$

$$\Theta \to 3H, \ \sigma = 0 \text{ for homogeneous and isotropic case}$$

Singularities in GR

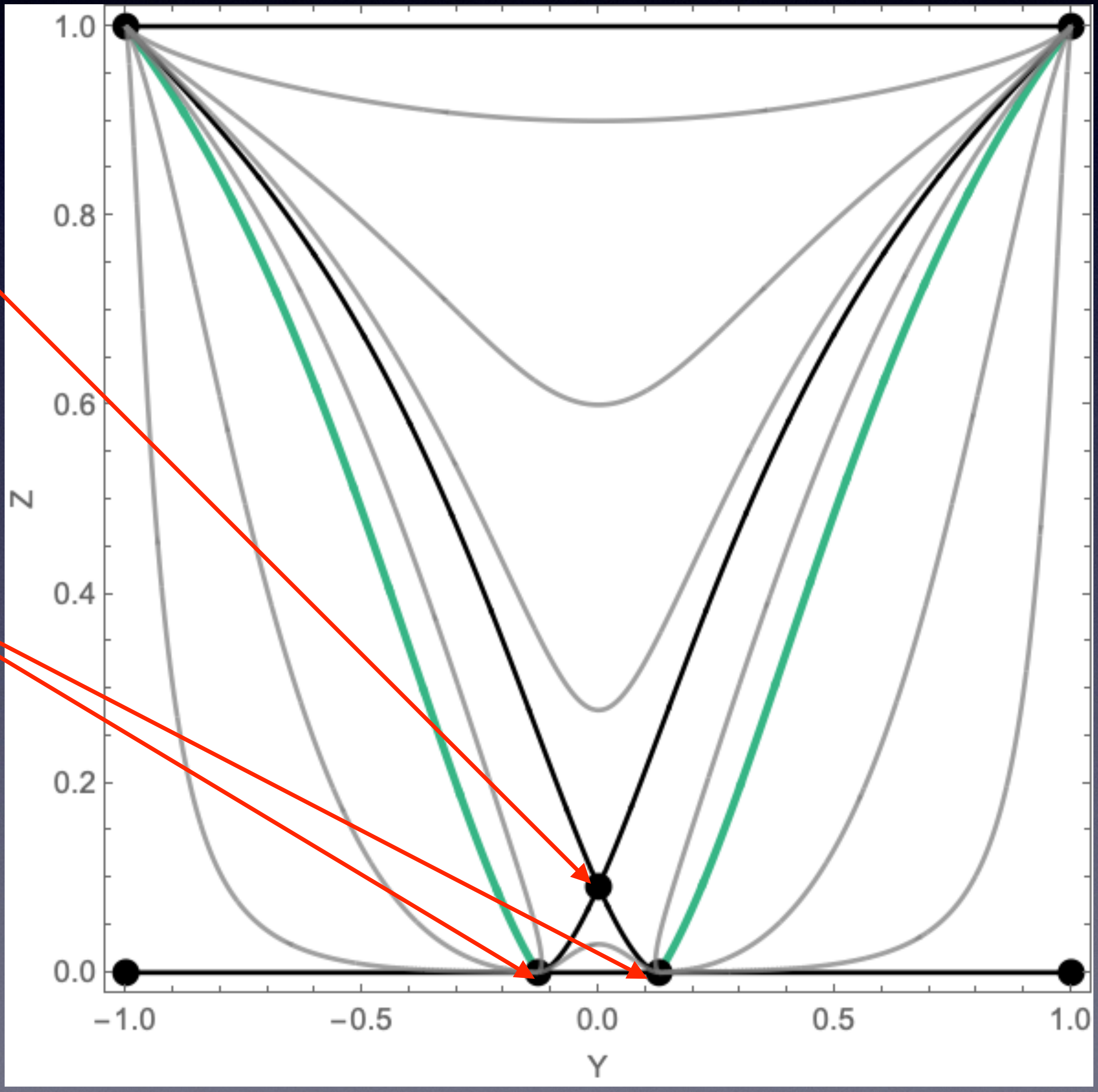
- standard FLRW models have a matter dominated singularity (spatial curvature negligible)
- This is non-generic: for $p = w\rho$ ($w < 1$) matter is negligible, singularities are "velocity dominated" (the shear σ wins), Weyl curvature dominates over Ricci curvature
- Kasner (vacuum Bianchi I) is paradigmatic
- BLK conjecture: Bianchi IX "mixmaster" (a chaotic sequence of Kasner phases) is generic
- No mixmaster for $w = 1$ and massless scalar field
- Isotropic singularity for $w > 1$

See Rendall and Nicolai *in* "100 years of Relativity", *ed.* Ashtekar (World Scientific, 2005)

qualitative analysis: ΛCDM

Y and Z: dimensionless compactified variables for *H*, and matter density ρ

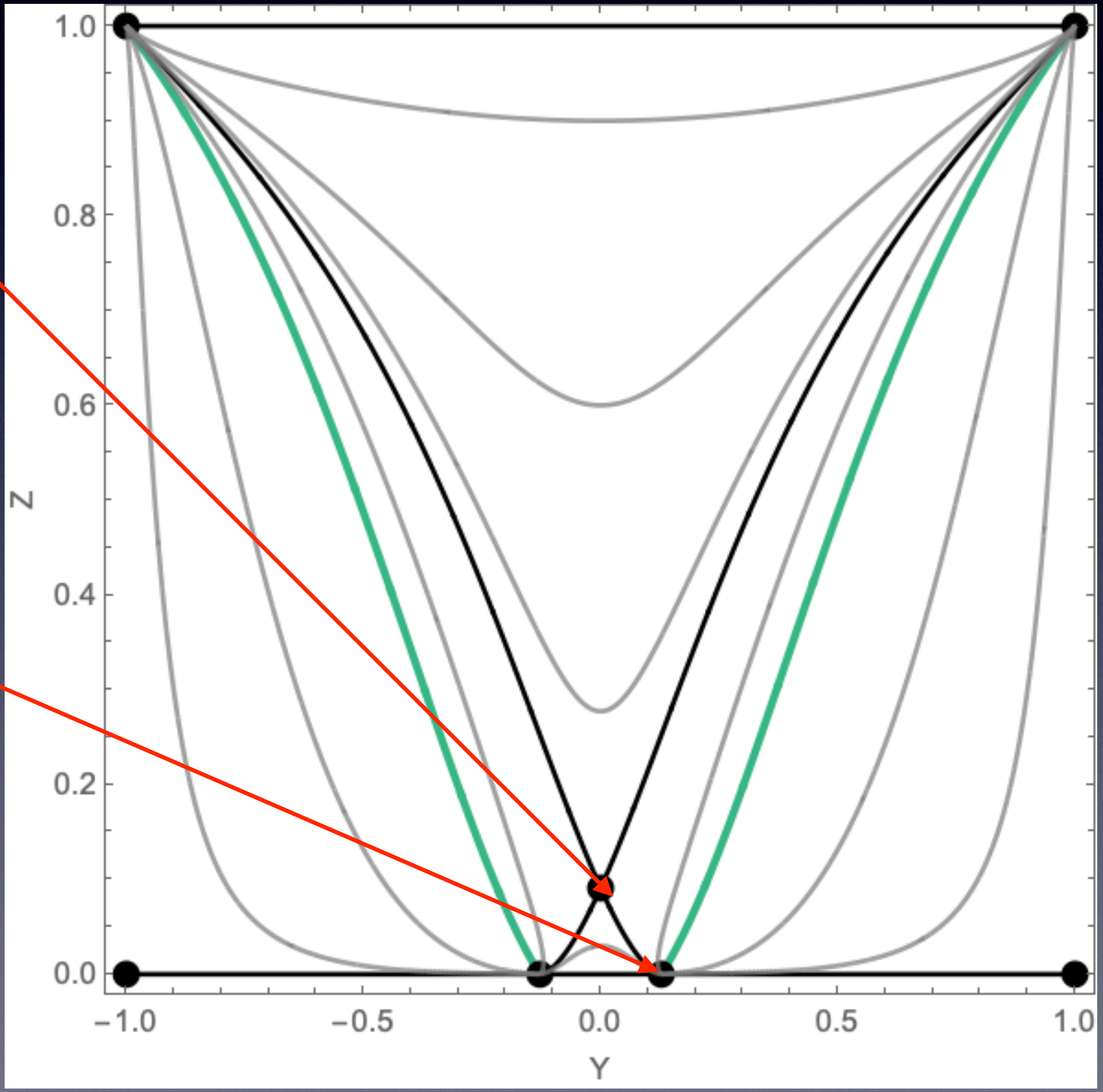
- Einstein fixed point: a fixed point at H=0 exists, representing a static Einstein static model
- de Sitter fixed points: two fixed points at ρ=0 exists:
$$H = \pm\sqrt{\Lambda/3}$$
- closed models may expand and recollapse, expand toward the asymptotically stable de Sitter point, some may evolve with a bounce between the two de Sitter points



qualitative analysis: ΛCDM

Y and Z: dimensionless compactified variables for H, and matter density ρ

- Einstein fixed point: asymptotic past for Eddington "Emergent Universe"
- Eddington "Emergent Universe" model is the separatrix between Einstein and de Sitter fixed points
- Einstein point is a saddle: Eddington Emergent Universe is a set of measure zero



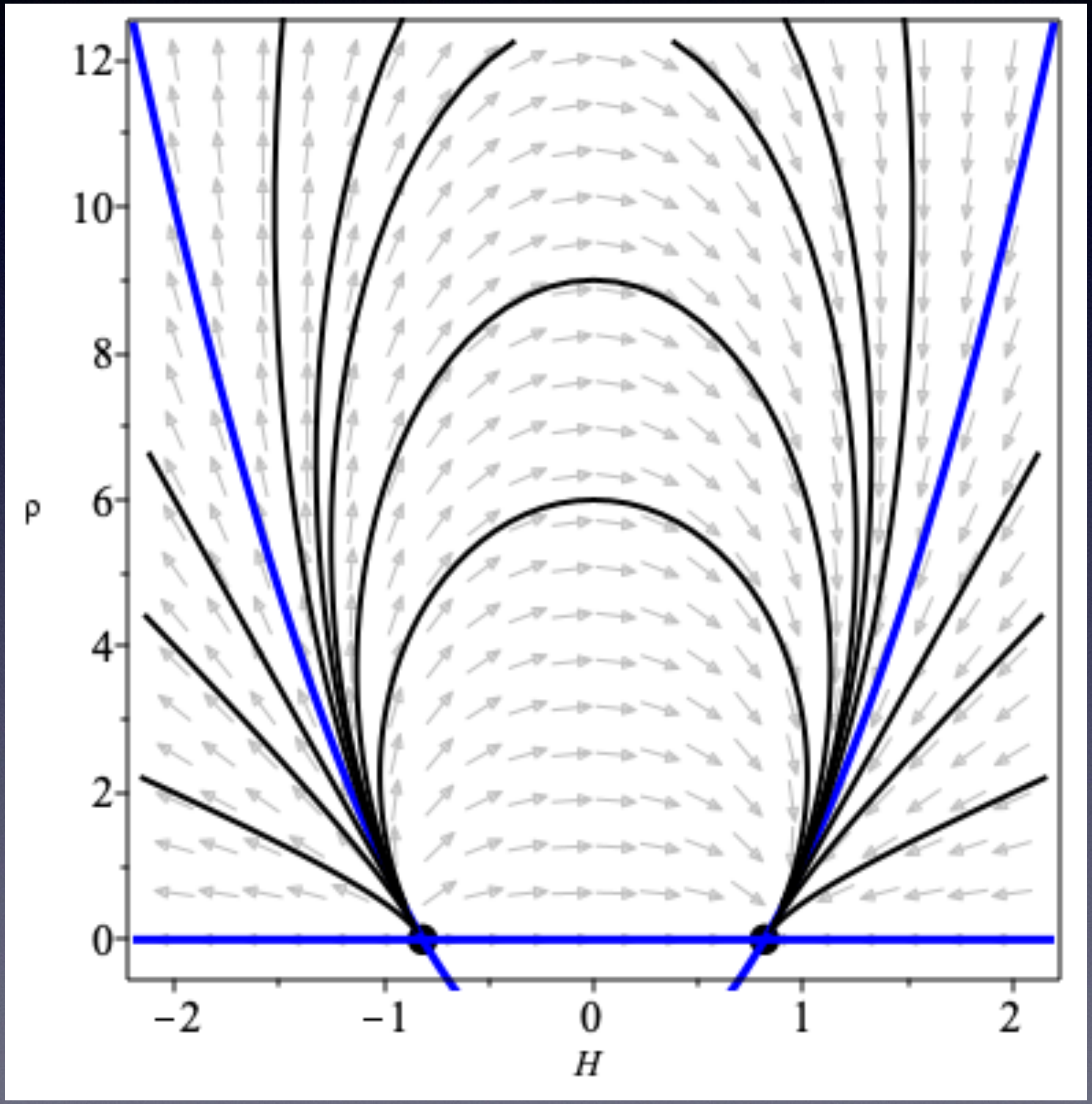
Emergent Universe

- General idea: the Universe is non singular but emerges from a static state in the past, represented by an Einstein model
- The problem is, that quite generally the Einstein model is either a saddle or a centre: for a given dynamics, only special initial conditions give trajectories in phase space emerging from the Einstein fixed point
- a true Emergent Universe should be asymptotic in the past to a non singular model for any initial conditions
- more in general, this non singular model should give rise to a transition phase between contraction and expansion, with a bounce in between

For Emergent Universe see: Ellis & Maartens 2003 gr-qc/0211082
Ellis Murugan & Tsagas 2004
Kobayashi & Ferreira 2018 arXiv:1801.09658

Λ and a w<-1/3 DE fluid

- DE violate the energy condition $\rho + 3P \geq 0$
- no Einstein fixed point
- all closed models have a bounce
- evolution of closed models between a contracting unstable and an expanding asymptotically stable de Sitter fixed point



General Theorem on Λ (Wald 1983)

Asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant

Robert M. Wald
Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637
(Received 5 July 1983)

We examine the late-time behavior of initially expanding homogeneous cosmological models satisfying Einstein's equation with a positive cosmological constant Λ. It is shown that such models of all Bianchi types except IX exponentially evolve toward the de Sitter solution, with time scale $(3/\Lambda)^{1/2}$. The behavior of Bianchi type-IX universes is similar, provided that Λ is sufficiently large compared with spatial-curvature terms. Thus, a positive cosmological constant provides an effective means of isotropizing homogeneous universes.

bottom line: de Sitter is usually an attractor,
FOR THE USUAL Λ in Einstein equations!

nonlinear equation of state

- a nonlinear equation of state $P=P(\rho)$ introduces one or more energy scales
- violation of standard energy conditions can be temporary
- one or more effective cosmological constants are possible
- effective cosmological constant: value of $\rho=\rho_\Lambda$ such that ρ_Λ is a fixed point of the energy conservation equation

$$P(\rho_\Lambda) = -\rho_\Lambda \Rightarrow \dot{\rho} = -\Theta[\rho + P(\rho)] \Rightarrow \dot{\rho} = 0$$

- asymptotic states, stable or unstable

▪Ananda & Bruni, Physical Review D, 74, 023523 (2006) [astro-ph/0512224]
▪Ananda & Bruni, Physical Review D, 74, 023524 (2006) [gr-qc/0603131]

Stable and unstable de Sitter vacua and Emergent Universe models

- effective cosmological constant: value of $\rho=\rho_\Lambda$ such that ρ_Λ is a fixed point of the energy conservation equation

$$P(\rho_\Lambda) = -\rho_\Lambda \Rightarrow \dot{\rho} = -\Theta[\rho + P(\rho)] \Rightarrow \dot{\rho} = 0$$

- each of these fixed points represents a de Sitter vacuum
- fixed points can be either asymptotically stable or unstable for $\Theta>0$
- new Emergent universe concept:
  - a model that in the past is asymptotic to a static unstable de Sitter vacuum (flat and open FLRW), or
  - emergence from a bounce off the unable de Sitter vacuum for closed models

quadratic equation of state

- conservation equation

$$\dot{\rho} = -3H(\rho + P),$$

- Raychaudhuri equation

$$\dot{H} = -H^2 - \frac{1}{6}(\rho + 3P),$$

- Hamiltonian (Friedmann) constraint

$$H^2 = \frac{1}{3}\rho - \frac{K}{a^2},$$

- equation of state

$$P = P_o + \alpha\rho + \beta\rho^2.$$

$$\beta = \frac{\epsilon}{\rho_c}.$$

P_o constant pressure,
α dimensionless, ρ_c energy scale

quadratic equation of state

- many different possibilities, depending on parameters
- two cosmological constants are possible if $\Delta>0$

$$\rho_{\Lambda,1} := \frac{1}{2\beta}[-(\alpha + 1) + \sqrt{\Delta}],$$

$$\rho_{\Lambda,2} := \frac{1}{2\beta}[-(\alpha + 1) - \sqrt{\Delta}],$$

$$\Delta := (\alpha + 1)^2 - 4\beta P_o$$

quadratic equation of state: dynamical system with dimensionless variables

$$x = \frac{\rho}{|\rho_c|}, \qquad y = \frac{H}{\sqrt{|\rho_c|}},$$

$$\eta = \sqrt{|\rho_c|}t, \qquad \nu = \frac{P_o}{\sqrt{|\rho_c|}}.$$

The system of equations then become:

$$x' = -3y(\nu + (\alpha + 1)x + \epsilon x^2),$$

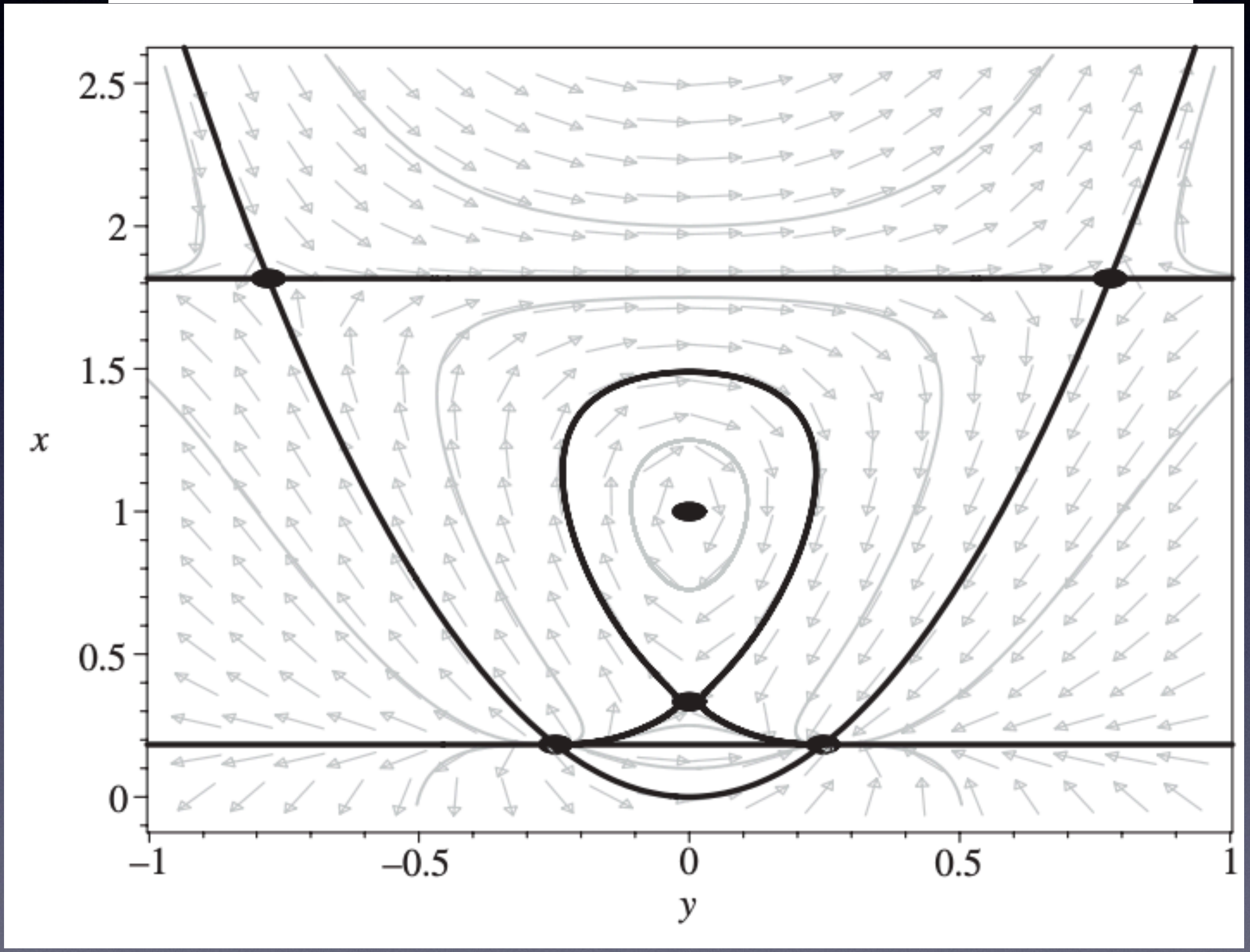
$\epsilon = \pm 1$

$$y' = -y^2 - \frac{1}{6}(3\nu + (3\alpha + 1)x + 3\epsilon x^2),$$

quadratic equation of state

- two effective cosmological constants
- two Einstein fixed points
- four de Sitter fixed points
- two are saddles, one is a repeller, one is an attractor
- cycle models
- bouncing models

$\epsilon = \pm 1, \quad \alpha > -1/3, \quad -\nu_1 < \nu < 0$



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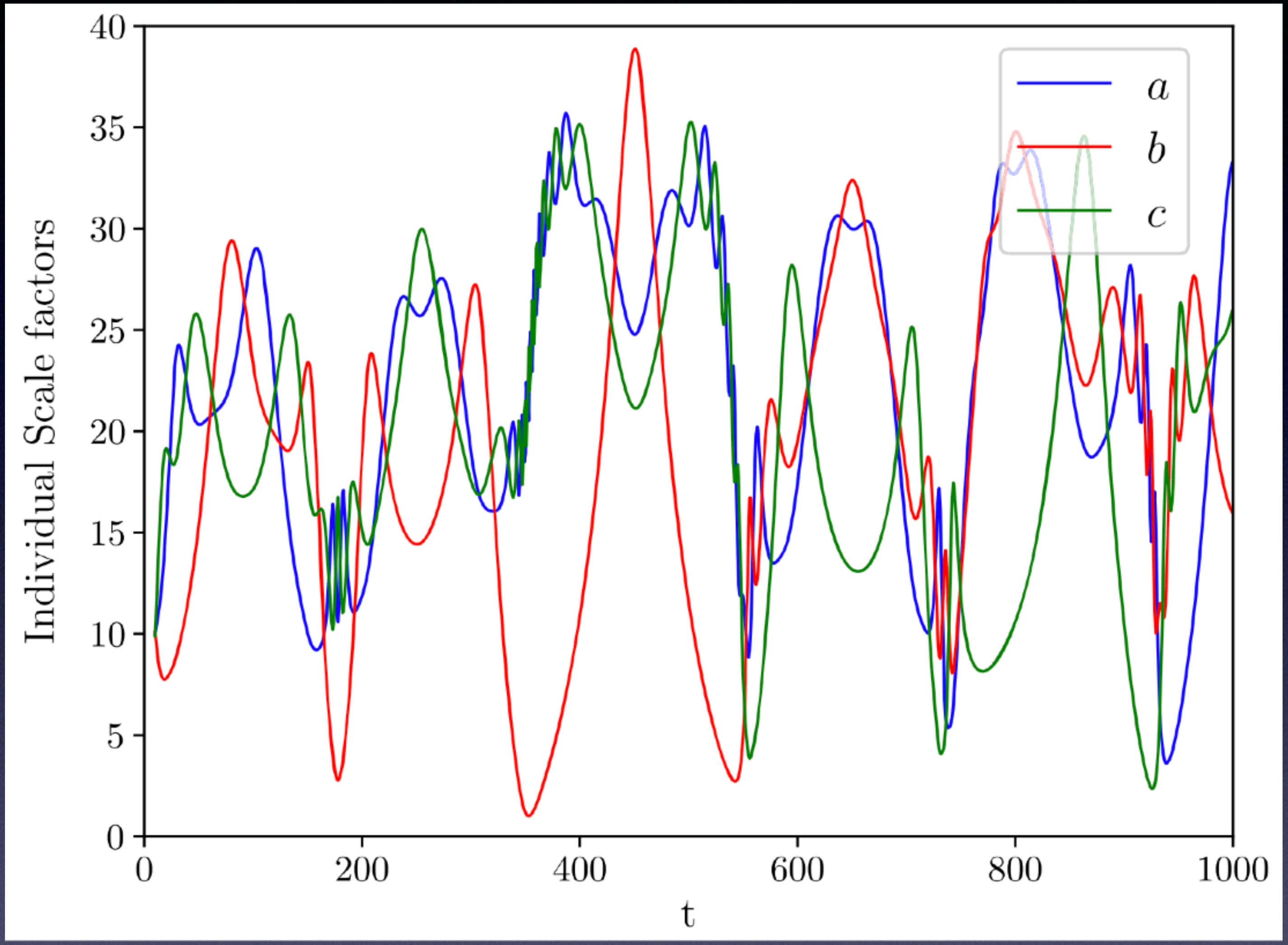
Biianchi IX with quadratic EoS

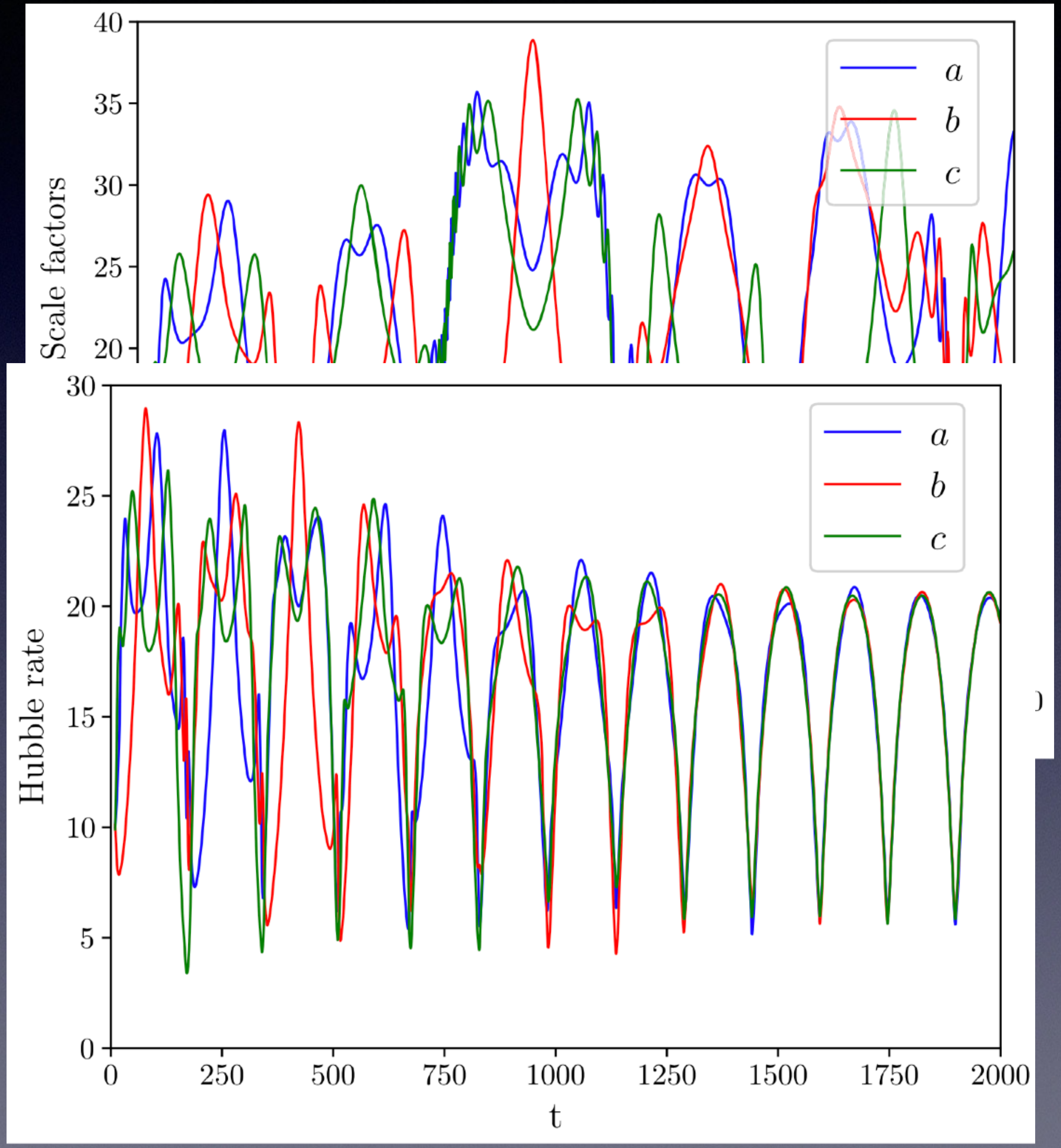
- Bianchi IX: most general homogenous anisotropic geometry
- general for super-horizon scales
- should emerge as average geometry
- focus on cycle case
- cycles survive by they are chaotic, i.e. in general always anisotropic
- isotropy emerges as an attractor if we introduce a friction anisotropic pressure term



C. Ganguly and M. Bruni, Physical Review Letters 123 201301 (2019). [arXiv:1902.06356]

Bianchi IX with quadratic EoS

- Bianchi IX: most general homogenous anisotropic geometry
- general for super-horizon scales
- should emerge as average geometry
- focus on cycle case
- cycles survive by they are chaotic, i.e. in general always anisotropic
- isotropy emerges as an attractor if we introduce a friction anisotropic pressure term



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DE with quadratic EoS:
1) + CDM and radiation
2) interacting DE & CDM

Burkmar and Bruni, PRD 107, 083533 (2023) 2302.03710
work with PhD student Molly Burkmar
+ work in progress

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See Molly's poster
for more details!

quadratic EoS + CDM and radiation

- we assume that two effective cosmological constants exist:

$$\dot{\rho_x} = -3H(\rho_x - \rho_\Lambda)\left(1 - \frac{\rho_x}{|\rho_c|}\right)$$

- Q: can bounces and cycles survive?

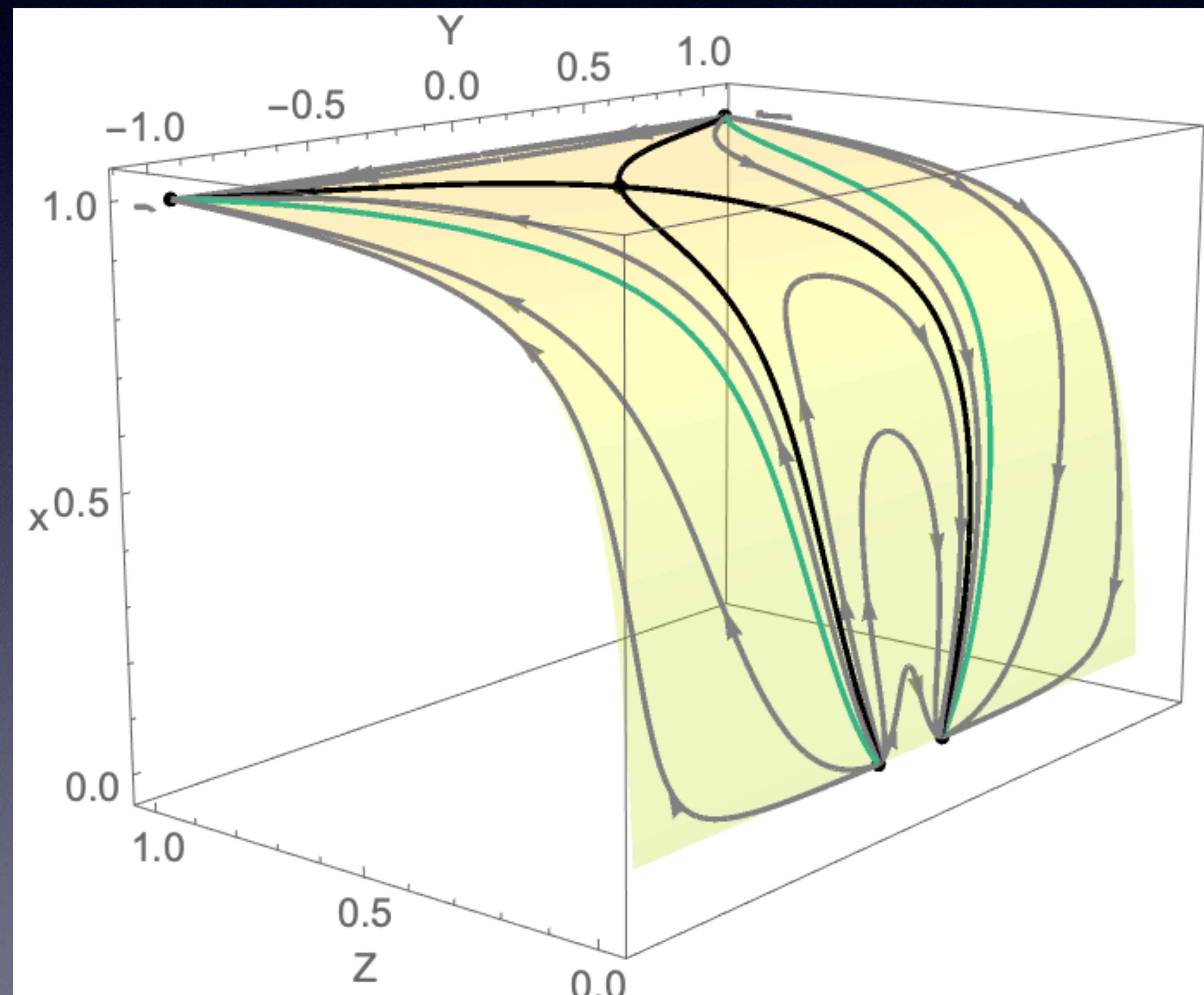
- use dimensionless variables

$$x = \frac{\rho_x}{|\rho_c|} \quad y = \frac{H}{\sqrt{|\rho_c|}} \quad z = \frac{\rho}{|\rho_c|} \quad v = \frac{\rho_v}{|\rho_c|} \quad R = \frac{\rho_\Lambda}{|\rho_c|} \quad \eta = \sqrt{|\rho_c|}t$$

- several different possibilities with R<x<1, depending on parameters; positive curvature crucial to have bounces and cycles, but some K>0 have a singularity

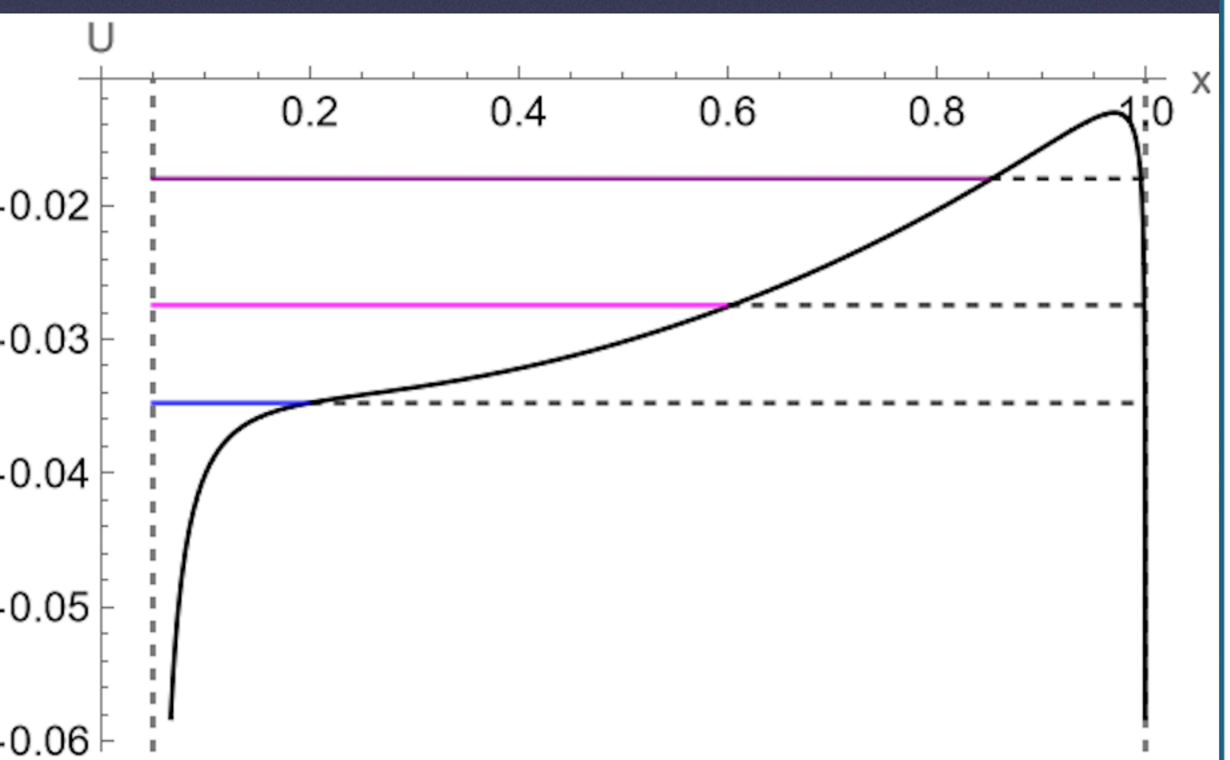
quadratic EoS + CDM and radiation

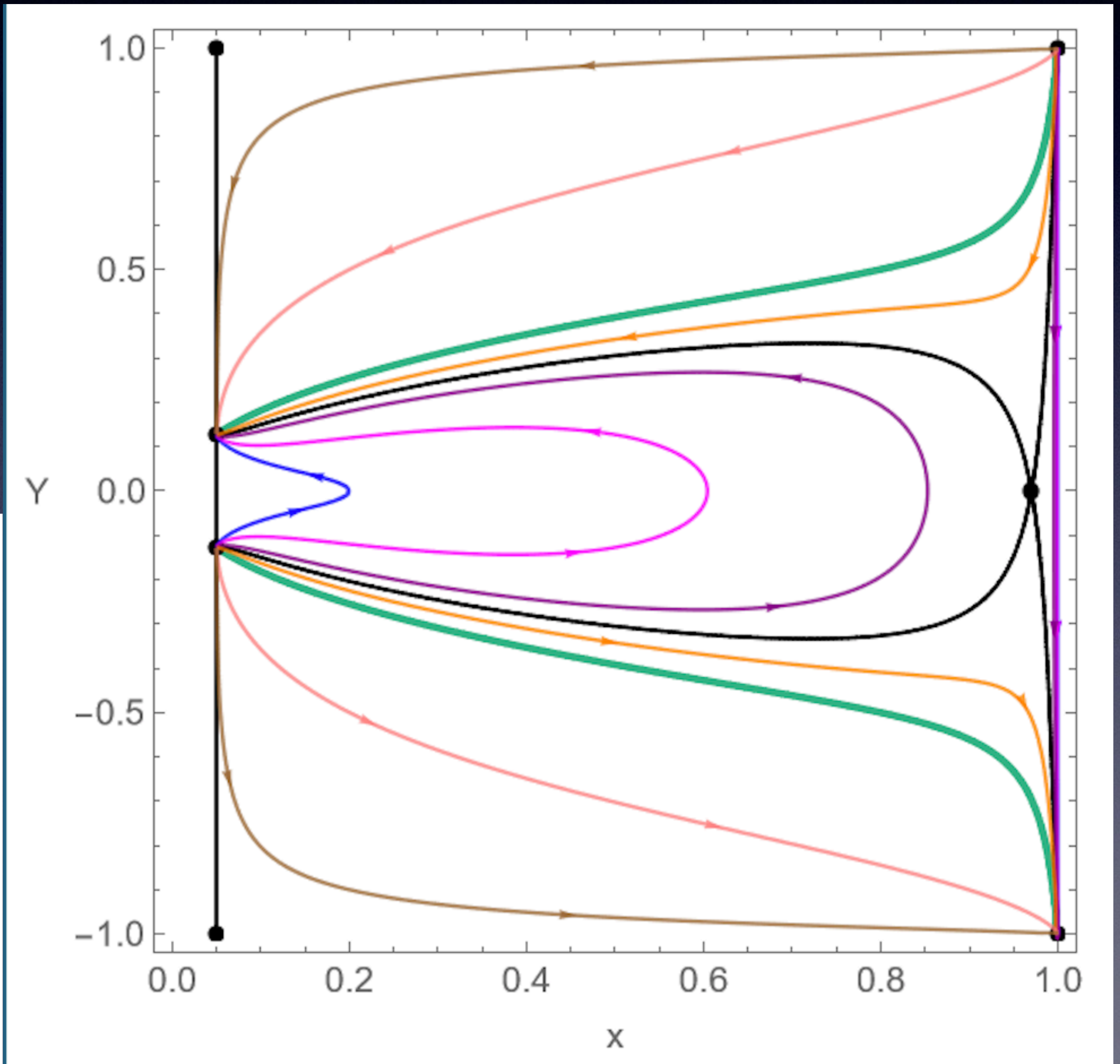
- with DE x, Hubble expansion rate y and a matter component z dynamics is in 3D, but first integral reduces motion to be 2D: projection on x-y plane
- phase space plot in terms of x and compactified variables Y and Z
- case with a single Einstein point: no cycle trajectories, only bounces



quadratic EoS + CDM and radiation

- finite potential barrier implies that there are closed-models trajectories with enough “energy” to have a past singularity





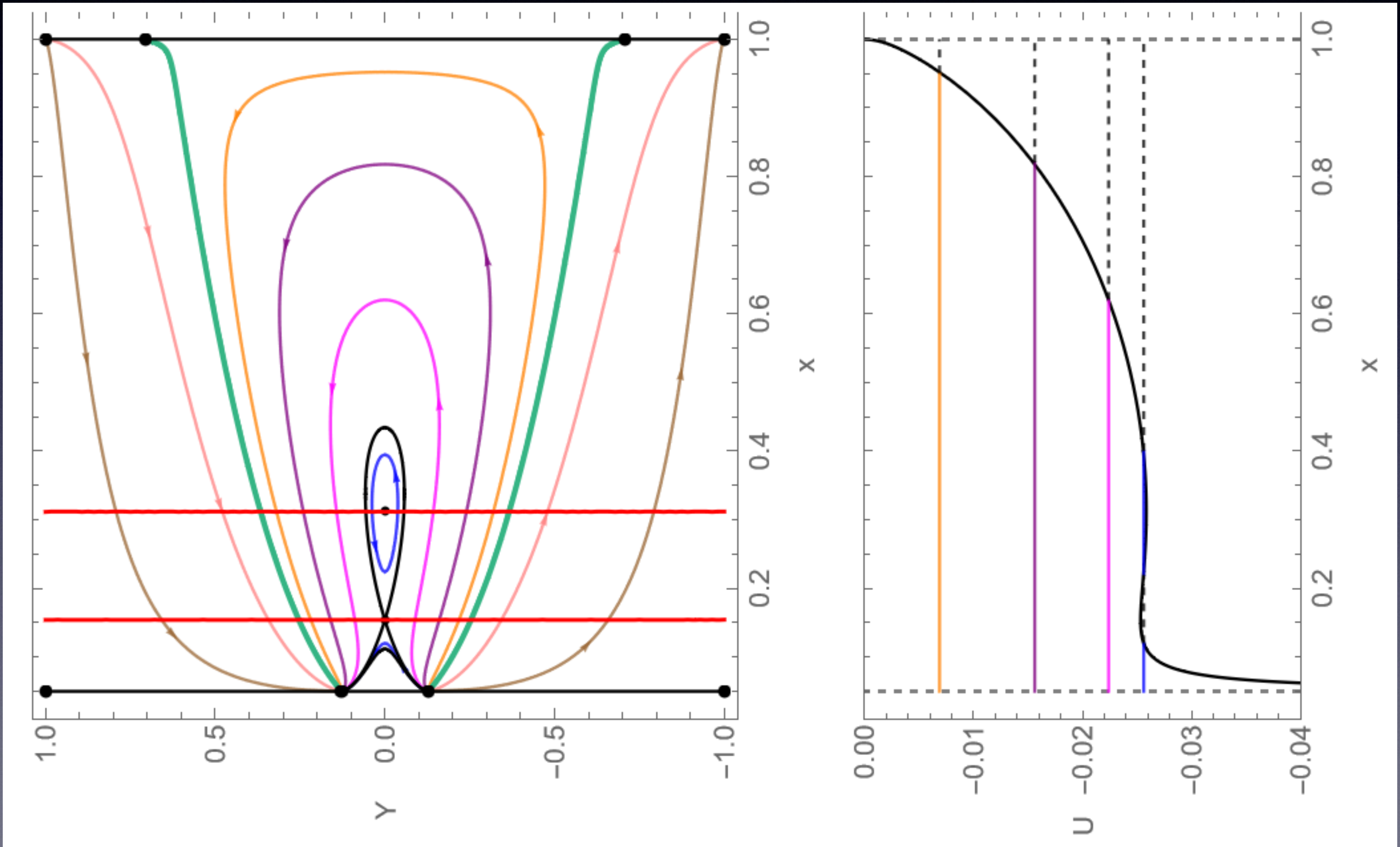
quadratic EoS + CDM and radiation

- for closed (positive curvature) models with quadratic EoS DE + unbounded CDM and radiation bounces are not generic

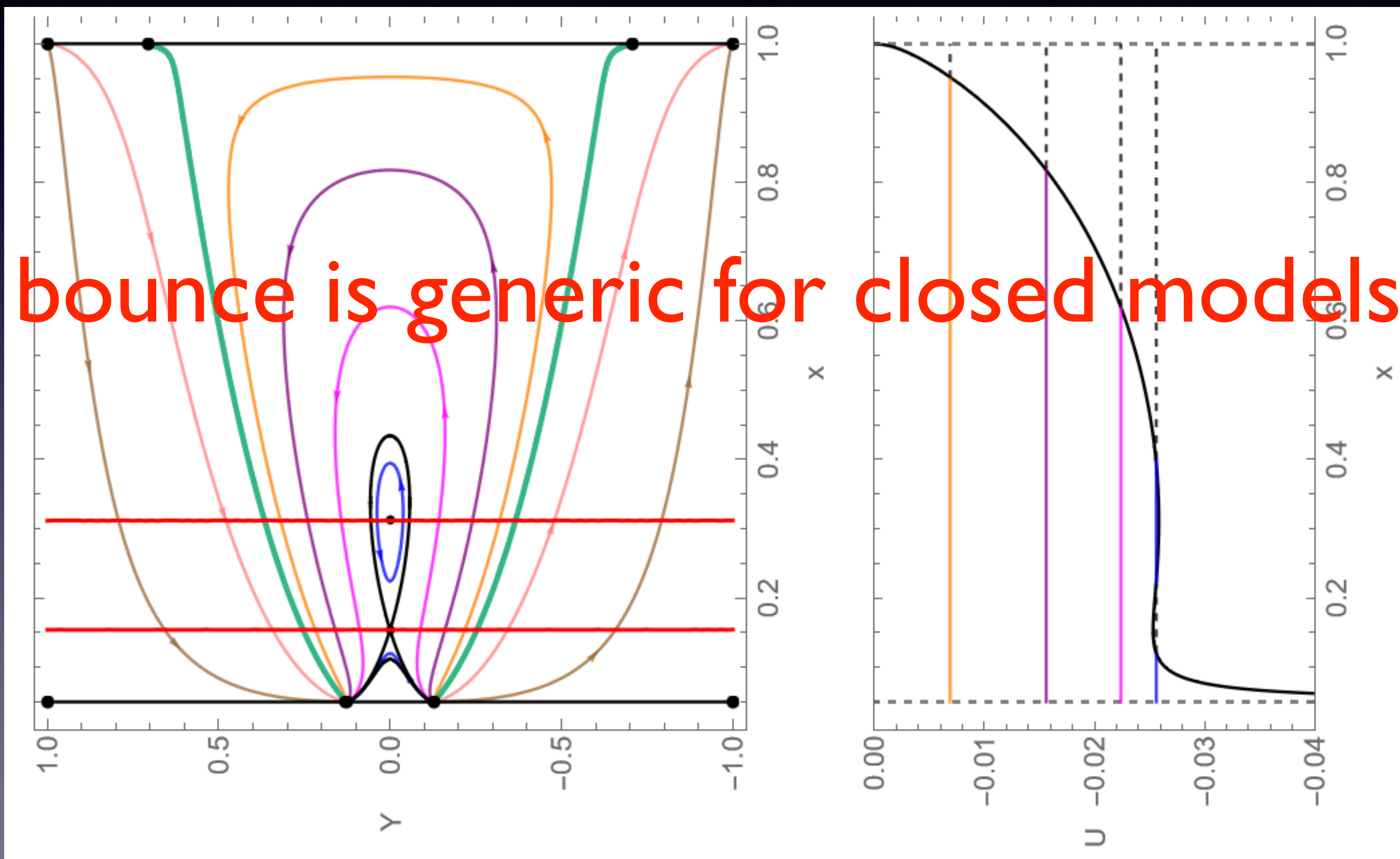
- in a realistic scenario for the early Universe radiation and CDM should arise at a later stage after the bounce, e.g. as in a standard post-inflation reheating phase

- in this light, we can then introduce an upper bound for CDM and radiation in our qualitative analysis

quadratic EoS DE + radiation and CDM with an upper bound



quadratic EoS DE + radiation and CDM with an upper bound



bounce is generic for closed models

coupled DE & CDM

- quadratic EoS DE now nonlinearly coupled to CDM
- work in progress with PhD student Molly Burkmar
- Dynamical System approach and dimensionless variables and parameters introduced as before

coupled DE & CDM

- Dynamical System approach and dimensionless variables and parameters introduced as before
- explicitly use w for the linear part of DE

$$\dot{\rho}_m = -3H\rho_m + \frac{qH\rho_x\rho_m}{\rho_i}$$

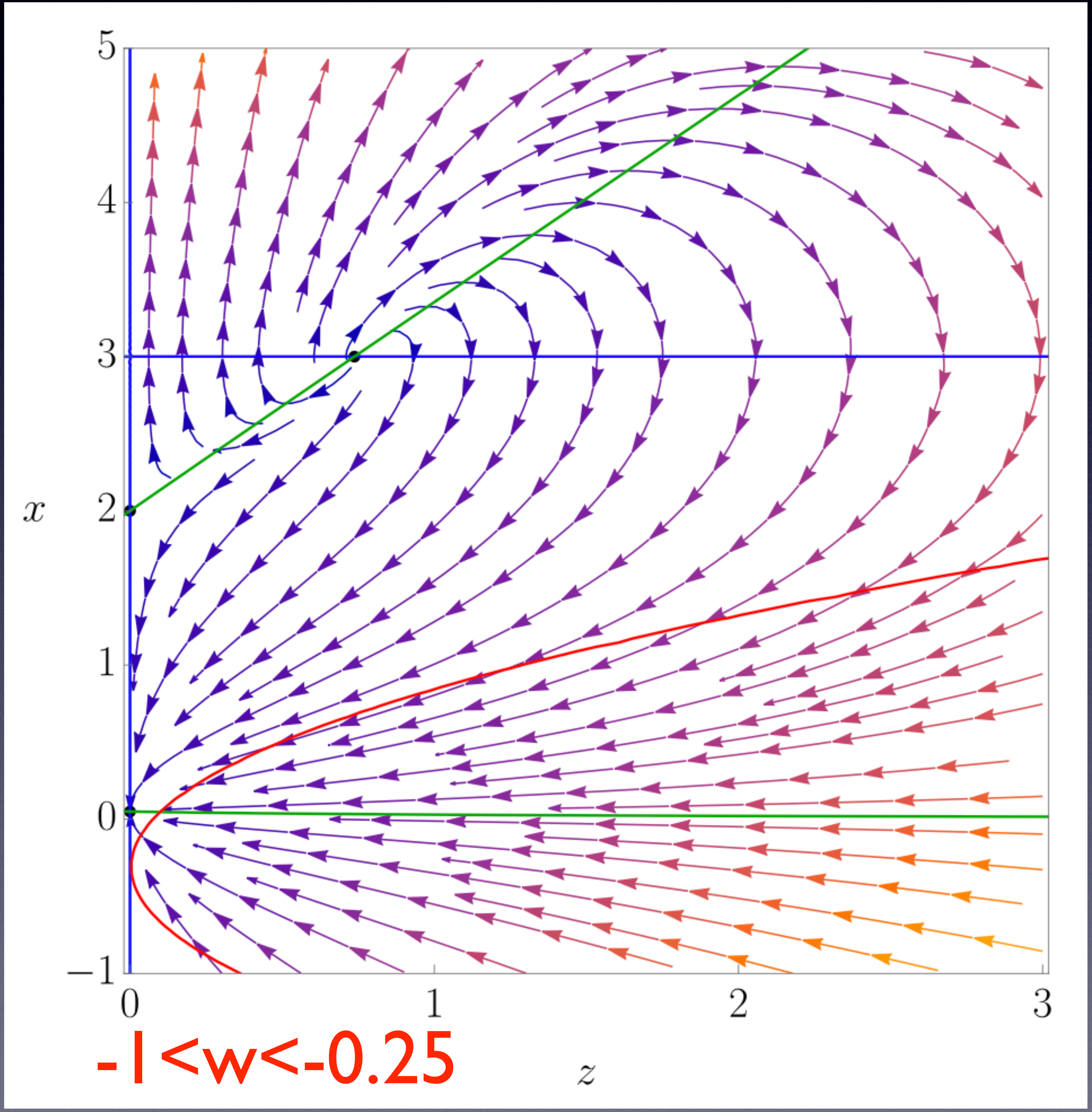
$$\dot{\rho_x} = -3H(\rho_x - \rho_\Lambda)\left(1 + w_x + \epsilon\frac{\rho_x}{\rho_*}\right) - \frac{qH\rho_x\rho_m}{\rho_i}$$

$$\dot{H} = -H^2 - \frac{1}{6}(\rho_m + \rho_x(1 + 3w_x - 3\epsilon\mathcal{R}) - 3\rho_\Lambda(1 + w_x) + 3\epsilon\frac{\rho_x^2}{\rho_*}) ,$$

$$H^2 = \frac{\rho_m}{3} + \frac{\rho_x}{3} - \frac{k}{a^2} .$$

DE+CDM dynamics

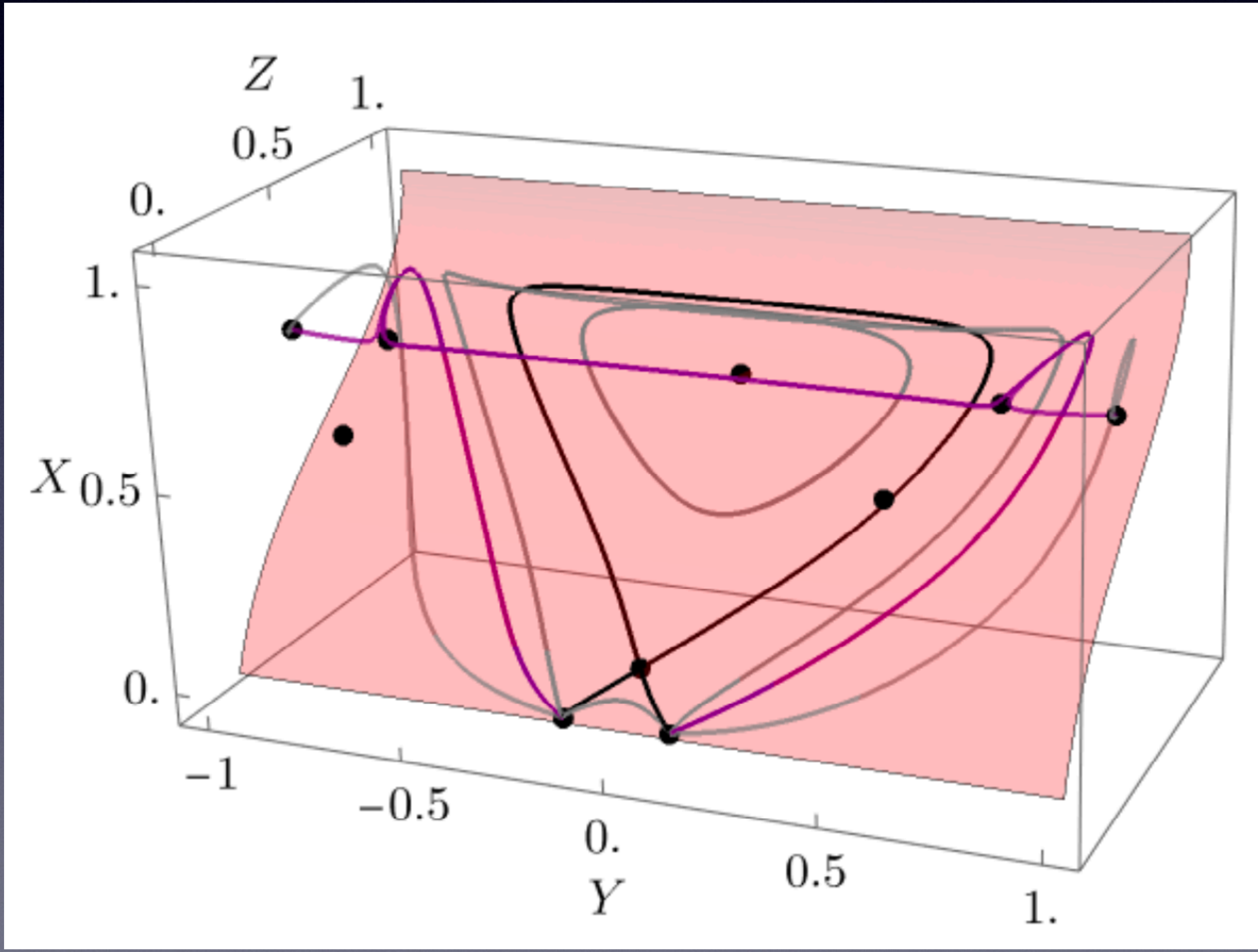
- assuming expansion H>0, eliminate H from continuity equations and use e-folding number N=ln(a) as time
- past attractor is a repelling spiral S, a high energy unstable "cosmological constant"
- all trajectories emerge from S
- all trajectories end in a low energy "cosmological constant" attractor
- the trick is to have trajectories that have a decelerated phase, below the red line



DE+CDM+H 3-D dynamics

purple trajectories represents curved models within Planck values for Ωκ, 4 de Sitter points and 2 Einstein points

- flat and open models EMERGE from the expanding de Sitter point, asymptotically in the past
- closed models EMERGE from the contracting de Sitter point, asymptotically in the past, go through a transient de Sitter phase with a bounce
- all go through deceleration and then a final accelerated phase toward the de Sitter future attractor

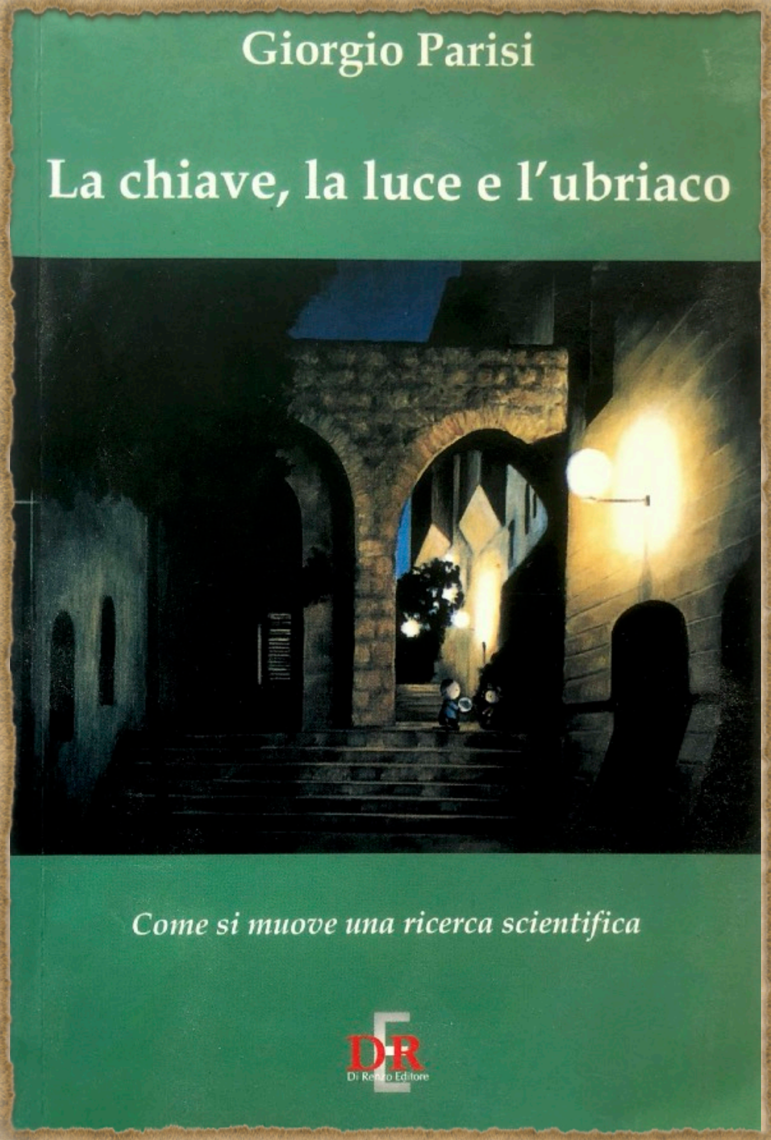


Summary and Outlook

- Licia's options are both good:
  - Option I: squeeze more from ΛCDM by going beyond FLRW + perturbations, nonlinearity and full GR
  - Option II: explore DE beyond Λ, both in the late and early Universe
- Numerical Relativity simulations can lead to new predictions in ΛCDM: field is in its infancy, more is needed
- DE, intended as any unknown component capable of acceleration, can produce bouncing models or models with a cycle; curvature can be important
- challenges: 1) to have the FLRW submanifold to be the attractor for Bianchi IX; 2) to look at perturbations

What is Cosmology?

- Cosmology is the branch of physical that studies the Universe as a whole
- the smallest bricks in cosmology are galaxies
- Aim of cosmology is to understand how the Universe we see came into being and developed in the course of time
- "a drunk man, in the night, is searching a key under a light. Someone arrives to help but, because nothing is found, she asks the drunk man if he is really sure the he lost the key there. He answers: No, I am not really sure , but it is here that there is light."



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