Teleparallel Cosmology
Towards a framework to tackle cosmic tensions

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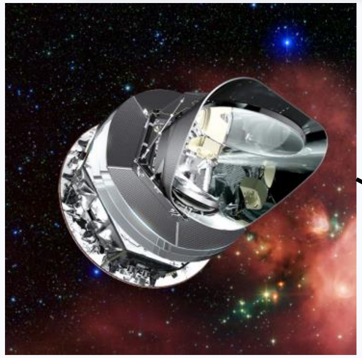
Outline

- **Standard gravity** and the motivation for modified gravity
- **Modified Gravity** through other branches of physics
- The **Teleparallel Gravity** (TG) formalism
- **Extended TG cosmology** observations
- **Model-independent cosmology**
- Concluding Remarks

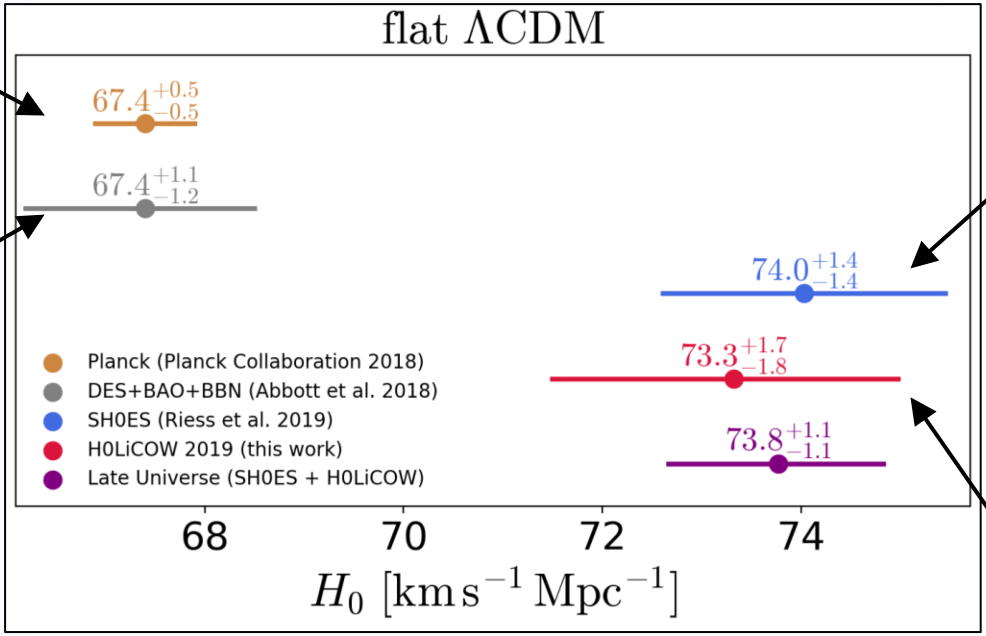
The H_0 Tension



Planck Mission (predictions from the CMB)

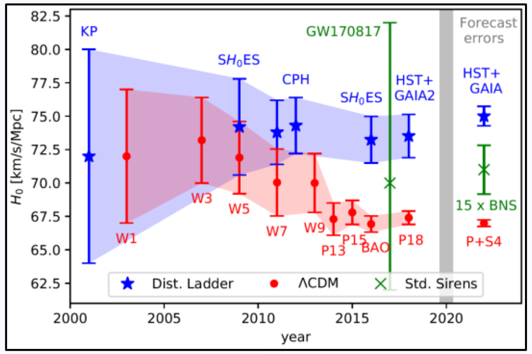
THE DARK ENERGY SURVEY

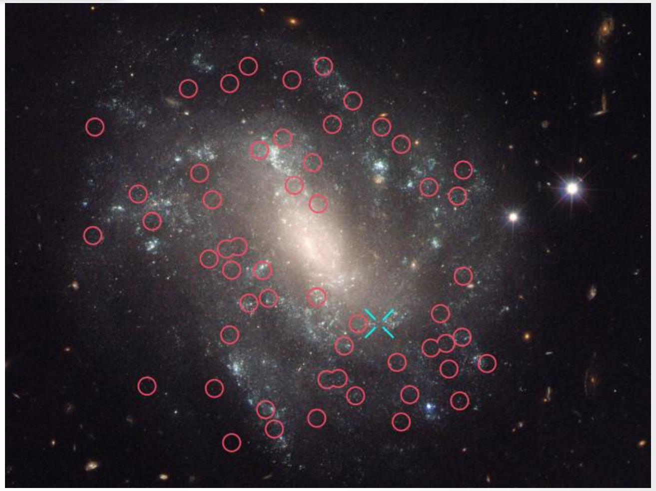
(visible and near-infrared survey)



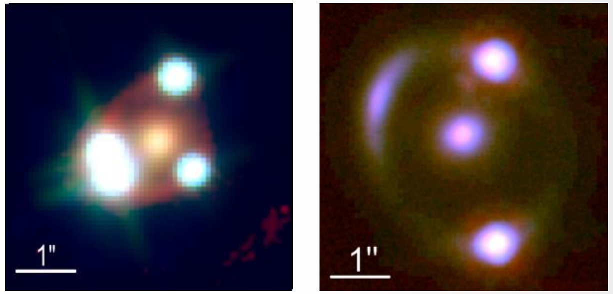
Wong et al. MNRAS 498, 1 (2020)

Cosmic Tension > 5σ



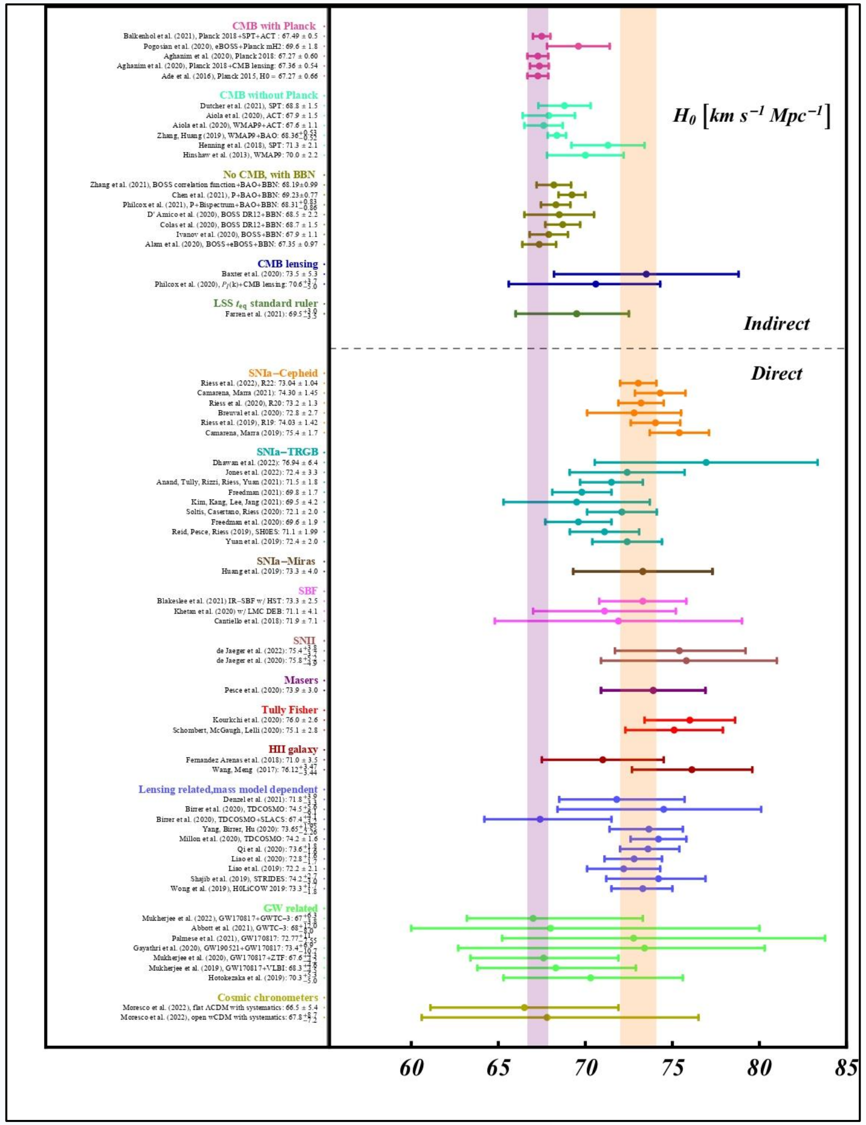


The SH0ES Project (calibrated with Cepheid variables)



H0LICOW (strong lensing - cosmography)

The H_0 Tension



Di Valentino et al. CQG, 38 (15) (2021)

Cosmology Intertwined, JHEAp. 2204, 002 (2022)

Modified Gravity through Lovelock's Theorem

Adding new fields (scalar, vector, tensor)

Adding more than second-order derivatives of the metric

Use more/less than four dimensions of spacetime

Lovelock's Theorem

For **second order** vacuum field equations $E_{\mu\nu} = 0$, if $E_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$, then

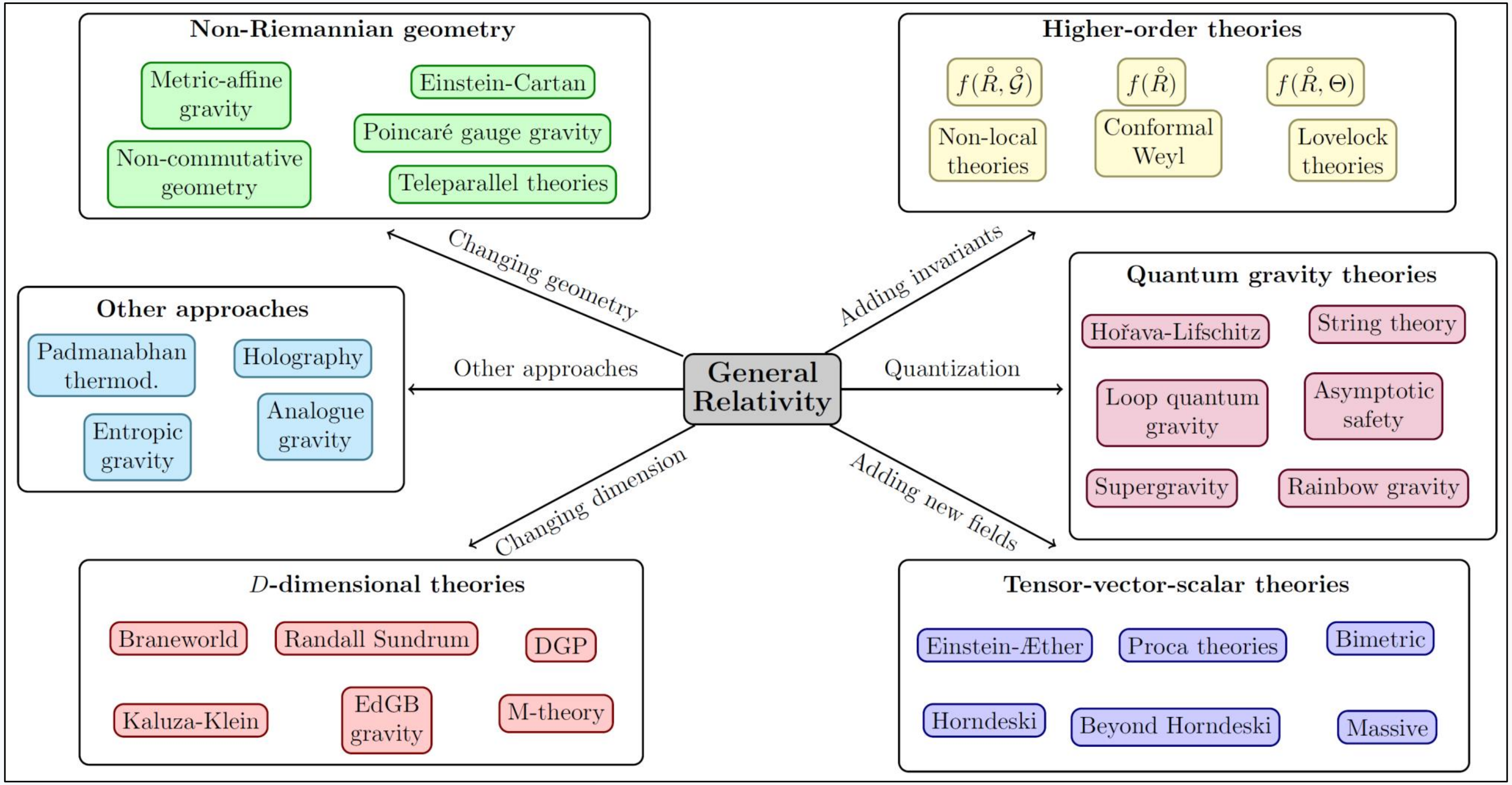
$$E_{\mu\nu} \coloneqq G_{\mu\nu} + \Lambda g_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R} g_{\mu\nu} + \Lambda g_{\mu\nu}$$

Consider non-local terms

Take an emergent form of gravity

Changing the gravitational connection

The Modified Gravity Landscape



What inspiration can we get from other branches of physics?

Inspiration from Particle Physics

- **Gauge Principle**: Replace **global symmetries** by **local ones**
- **Group generators** produce compensating **fields**
- This results in the **standard model forces**

Can we apply this to gravity?

Gauge theory of gravity

- Formulating a **gauge theory of gravity (1956 onwards)**
- Starting from **special relativity (SR)**
  - Applying **Yang-Mills theory** to **SR**
  - Result is **Poincaré gauge theory** (**curvature** and **torsion** appear as field strengths)
- **Torsion** is the **field strength** of the **translation group**

Hehl et al. Phys.Rept. 258, 1 (1995) [arXiv:gr-qc/9402012]

Modified Gauge Gravity

- One can always **modify gravity** (supergravity, conformal, metric affine,…)
- In all of them, **torsion** is related to the **gauge structure** of the theory
- Here, torsion opens the possibility of having a **quantum theory** of gravity

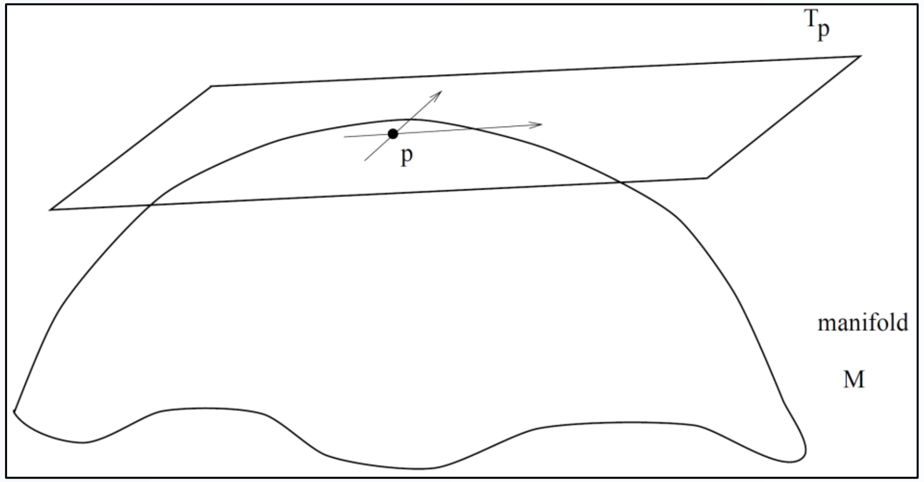
Modifying Gravity

- **Accelerating Universe (1998)**: Thousands of works in modified gravity ($f(\mathcal{R})$, Horndeski, Galileon, Lovelock, massive, Weyl,…)
- These are almost all **curvature-based**
- Can we **modify gravity** using **torsion**?

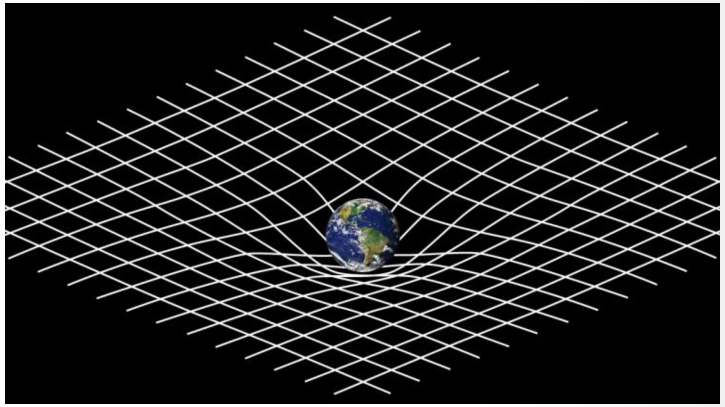
Saridakis et al. [The Cantata Consortium], Modified Gravity and Cosmology: An Update by the CANTATA Network. Springer, Cham (2021) [arXiv:2105.12582]

Rethinking the connection

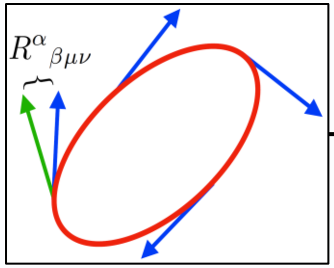
Spacetime tells matter how to move; matter tells spacetime how to curve

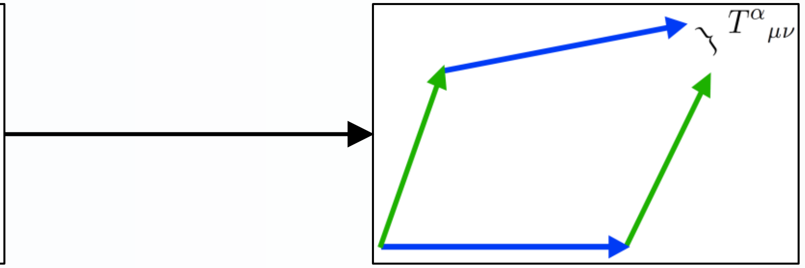


John Archibald Wheeler



Connection of gravity: **Curvature** is a property of the **connection**, not of the **spacetime**





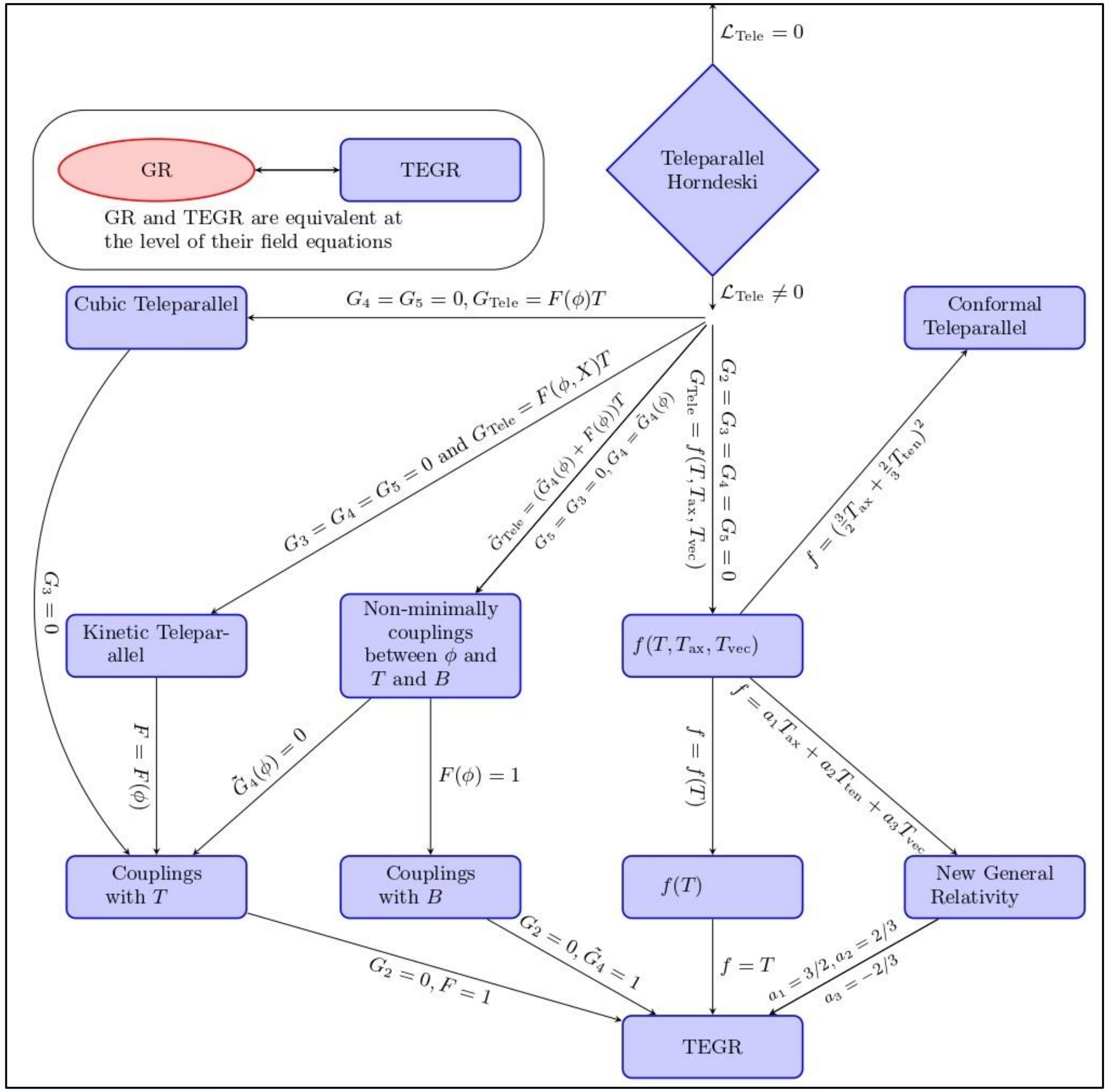
The Teleparallel Equivalent of GR (TEGR)

- **TEGR**: This is the simplest torsional theory of gravity
- Tetrad ($e^a{}_\mu$): Relate the tangent space ($g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$)
- Use the **teleparallel connection** ($\Gamma^\sigma_{\mu\nu} = e_a{}^\sigma \partial_\nu e^a{}_\mu + e_a{}^\sigma \omega^a{}_{\nu\mu}$) instead of the **Levi-Civita connection** (Christoffel symbols)
- **Torsion tensor**: Measures torsion ($T^\sigma{}_{\mu\nu} = \Gamma^\sigma_{\nu\mu} - \Gamma^\sigma_{\mu\nu}$)
- TEGR Action:

$$\boxed{S = -\frac{1}{16\pi G}\int d^4x\, e[T]}$$

where $T \equiv \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}{}^{\rho} T^{\nu\mu}{}_{\nu}$

Modified Teleparallel Gravity



Bahamonde et al. RoPP 86 026901 (2023) [arXiv:2106.13793]

$f(T)$ Teleparallel Gravity

- **Curvature-Torsion Relation**: $\mathcal{R} = -T + B$

$\boxed{B \propto \nabla^{\mu} T^{\lambda}{}_{\lambda\mu}}$

- **$f(T)$ Gravity**: Inspire by $f(\mathcal{R})$ gravity

$$S = \frac{1}{16\pi G} \int d^4x\, e[-T + f(T)] + S_{\text{mat}}$$

- Taking a flat (**FLRW**) cosmology: $g_{\mu\nu} = \text{diag}\left(-1, a(t)^2, a(t)^2, a(t)^2\right)$

- **Friedmann equations**:

$$H^2 = \frac{8\pi \mathrm{G}}{3}\rho_m - \frac{f(T)}{6} + \frac{T}{3} f_T$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 - f_T - 2T f_{TT}}$$

$$\boxed{\begin{aligned} T &= 6H^2 \\ &= 6\left(\frac{\dot{a}}{a}\right)^2 \end{aligned}}$$

Scalar Perturbations

- $\boldsymbol{f(T)}$ **gravity** leaves imprints at the **perturbative level**

$$e^0{}_\mu = \delta^0_\mu(1+\psi)\,,e^i{}_\mu = \delta^i_\mu a(1-\phi) \;\Rightarrow\; ds^2 = (1+2\psi)dt^2 - a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

- **Matter over-density perturbations** also contribute through

$$\delta_m = \frac{\delta\rho_m}{\rho_m}$$

- **Matter perturbation evolution** equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m + 4\pi G_{\text{eff}}\rho_m\delta_m = 0$$

Identifying the **effective gravitational constant**

$$G_{\text{eff}} = \frac{G_N}{1+f_T}$$

$f(T,B)$ Gravity

$$B \propto \nabla^{\mu} T^{\lambda}{}_{\lambda\mu}$$

- **Curvature-Torsion Relation**: $\mathcal{R} = -T + B$
- Naturally, $f(T) \neq f(-T+B) = f(\mathcal{R})$
- $f(T,B)$ organically decouples **second-order** (T) and **fourth-order** (B) contributions
- Only **linear** B is a boundary term
- $\boldsymbol{f(T,B)}$ **Gravity**: Generalizing $f(\mathcal{R})$ gravity

$$S = \frac{1}{16\pi G}\int d^4x\, e[-T + \boldsymbol{f(T,B)}] + S_{\mathrm{mat}}$$

$f(T,B)$ Background Cosmology

- As before, the **Friedmann equations** are

$$3H^2 = 8\pi G(\rho_m + \rho_{\text{eff}})$$
$$3H^2 + \dot{H} = -8\pi G(p_m + p_{\text{eff}})$$

- Therefore, the **effective fluid** will have

$$8\pi G\rho_{\text{eff}} = 3H^2(3f_B + 2f_T) - 3H\dot{f}_B + 3\dot{H}f_B - \frac{1}{2}f$$
$$8\pi G p_{\text{eff}} = \frac{1}{2}f - \left(3H^2 + \dot{H}\right)(3f_B + 2f_T) - 2H\dot{f}_T + \ddot{f}_B$$

- Giving an **effective EoS**

$$\omega_{\text{eff}} = -1 + \frac{\ddot{f}_B - 3H\dot{f}_B - 2\dot{H}f_T - 2H\dot{f}_T}{3H^2(3f_B + 2f_T) - 3H\dot{f}_B + 3\dot{H}f_B - \frac{1}{2}f}$$

arXiv:1508.05120
arXiv:1609.08373
arXiv:1909.10328

$$T = 6H^2$$
$$B = 6\left(3H^2 + \dot{H}\right)$$
$$\mathcal{R} \equiv -T + B = 6\left(2H^2 + \dot{H}\right)$$

Horndeski Gravity

Horndeski Gravity: Produces the most **general second-order theory** that contains only **one scalar field** (in **standard gravity**)

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right]$$

where

$$\mathcal{L}_2 = \mathrm{G}_2(\phi, X)$$
$$\mathcal{L}_3 = \mathrm{G}_3(\phi, X)\Box\phi$$
$$\mathcal{L}_4 = \mathrm{G}_4(\phi, X)\mathcal{R} + \mathrm{G}_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$
$$\mathcal{L}_5 = \mathrm{G}_5(\phi, X)\mathcal{G}_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}\mathrm{G}_{5,X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]$$

Teleparallel Horndeski Gravity (TeleDeski)

- **TeleDeski Goal**: What is the **TG analog** of **Horndeski theory**?
- **Conditions**: (i) Field equations must be second-order; (ii) terms cannot be parity-violating; (iii) contributions can be at most quadratic in torsion
- **Extra contribution**: $\mathcal{L}_{\text{Tele}} = G_{\text{Tele}}(\phi, X, T, T_{Ax}, T_{vec}, I_2, J_i)$ [I_2 - linear coupling with matter, J_i - quadratic coupling with matter]

Bahamonde et al. PRD 100, 064018 (2019)

Tensor Perturbations

- Taking **tensor perturbations** for **tetrads fields**

$$e^{0}{}_{\mu} = \delta^{0}_{\mu}\,, e^{i}{}_{\mu} = \delta^{i}_{\mu} + \frac{1}{2}\delta^{j}_{\mu}\delta^{ki}h_{jk} \Rightarrow \quad ds^2 = dt^2 - a^2\left(\delta_{ij} + h_{ij}\right)dx^i dx^j$$

- Produces a **gravitational wave propagation equation** (GWPE)

$$\ddot{h}_{ij} + (3 + \alpha_M)H\dot{h}_{ij} - (1 + \alpha_T)\frac{k^2}{a^2}h_{ij} = 0$$

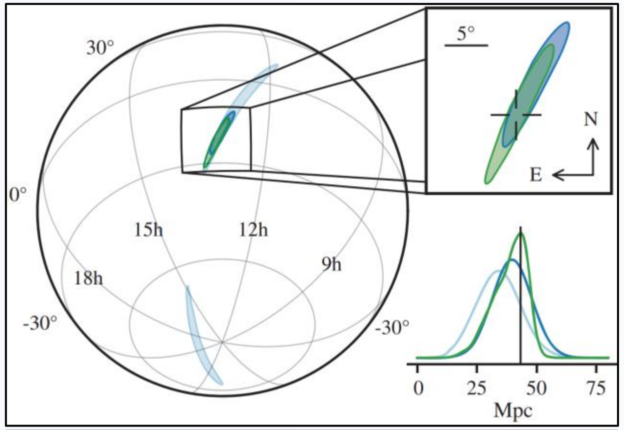
in the Fourier domain

- $\alpha_T = c_T^2 - 1$ is the **tensor excess speed** and $\alpha_M = \frac{1}{HM_*^2}\frac{dM_*^2}{dt}$ is the **Planck mass run rate** (M_*^2 is the effective **Planck mass**)

The Era of Multi-messenger Astronomy

GW170817

LIGO-Virgo localization



Virgo observatory

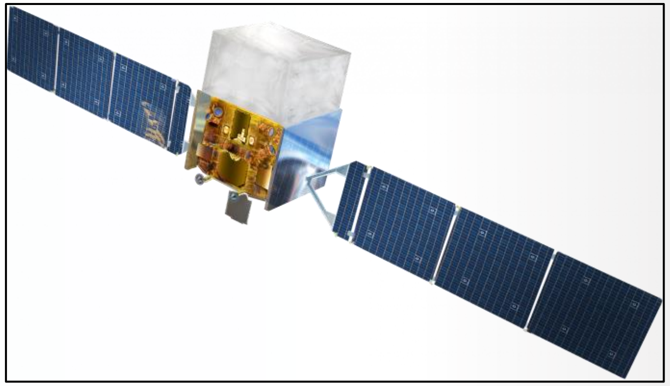
$$M_{\text{Tot}} = 2.74^{+0.04}_{-0.01} M_\odot$$

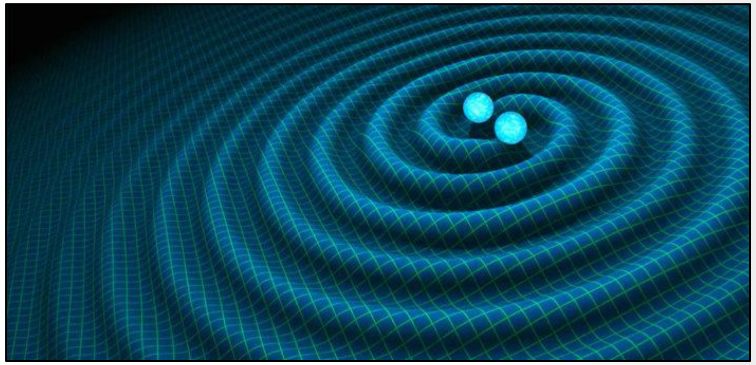
$$\Delta T = 1.7s$$

$$c_T = c^{+7\times10^{-16}}_{-3\times10^{-15}}$$

GRB170817A

Fermi Telescope





GWs in TeleDeski

$$\ddot{h}_{ij} + (3 + \alpha_M) H \dot{h}_{ij} - (1 + \alpha_T) \frac{k^2}{a^2} h_{ij} = 0$$

- **TeleDeski GWPE:**

$$\alpha_T = \frac{2X}{M_*^2} \left(2G_{4,X} - 2G_{5,\phi} - G_{5,X}(\ddot{\phi} - \dot{\phi}H) - 2G_{\text{Tele,J}_8} - \frac{1}{2} G_{\text{Tele,J}_5} \right) = 0$$

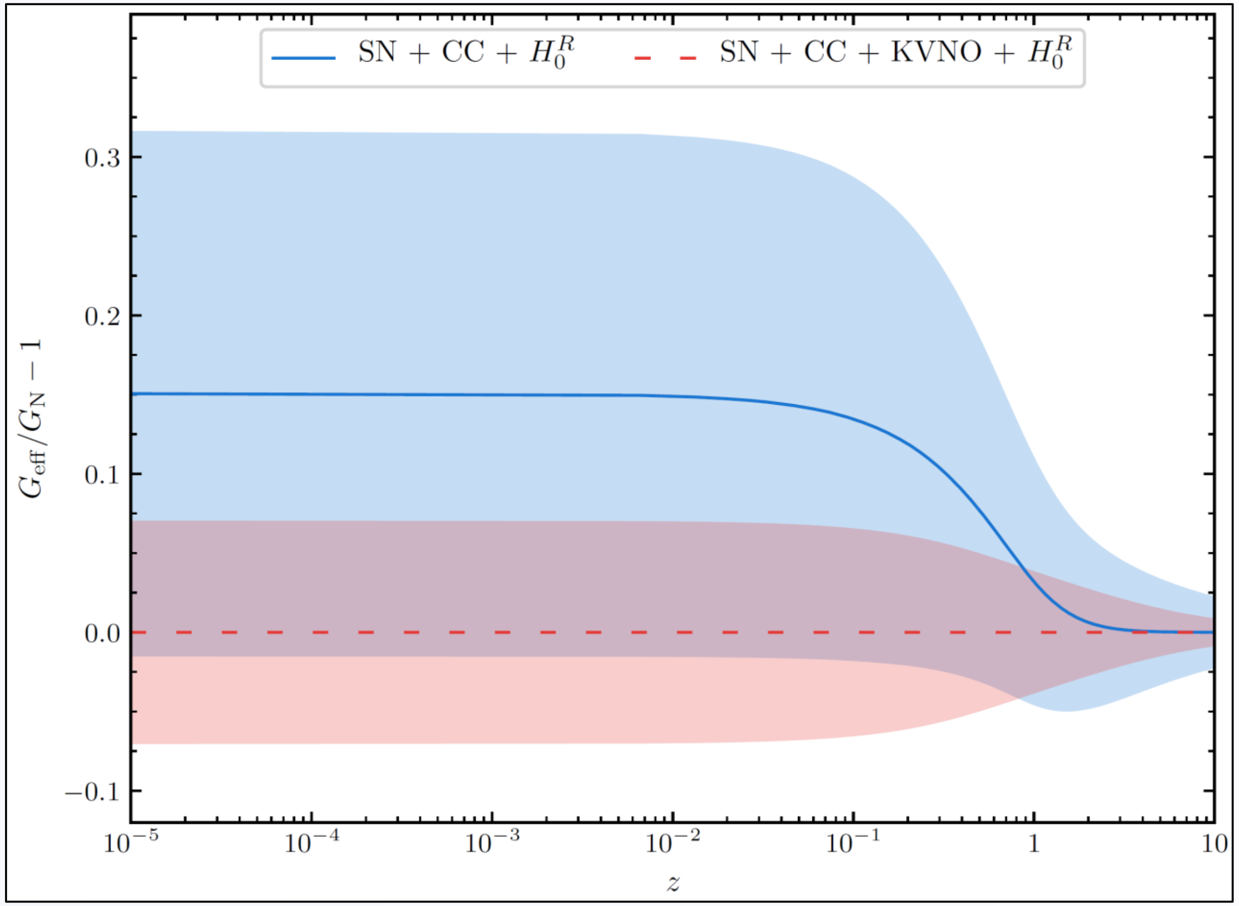
where $M_*^2 = 2\left(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X} + 2XG_{\text{Tele,J}_8} + \frac{1}{2}XG_{\text{Tele,J}_5} - G_{\text{Tele},T}\right)$

- **Running Planck mass**: Continues to observe $\alpha_M = \frac{1}{HM_*^2} \frac{dM_*^2}{dt}$
- **New possibilities**: Opens new possibilities for reviving Horndeski gravity

Bahamonde et al. PRD 101, 084060 (2020)

What do observations tell us about modified teleparallel gravity?

$f_{\mathrm{PLM}}(T)$ Model



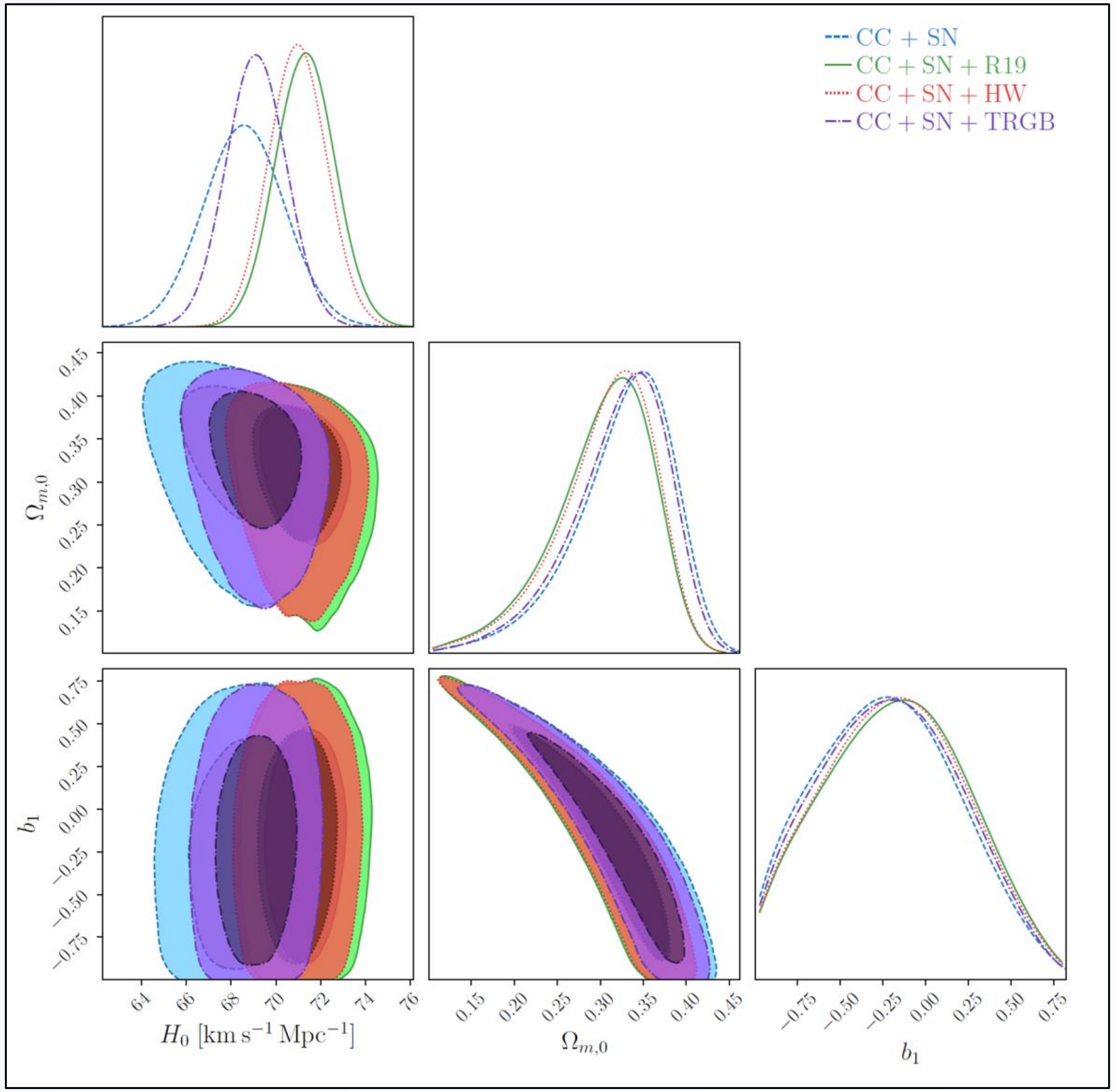
Model: $f_{\mathrm{PLM}} = \alpha_1 T^{b_1}$

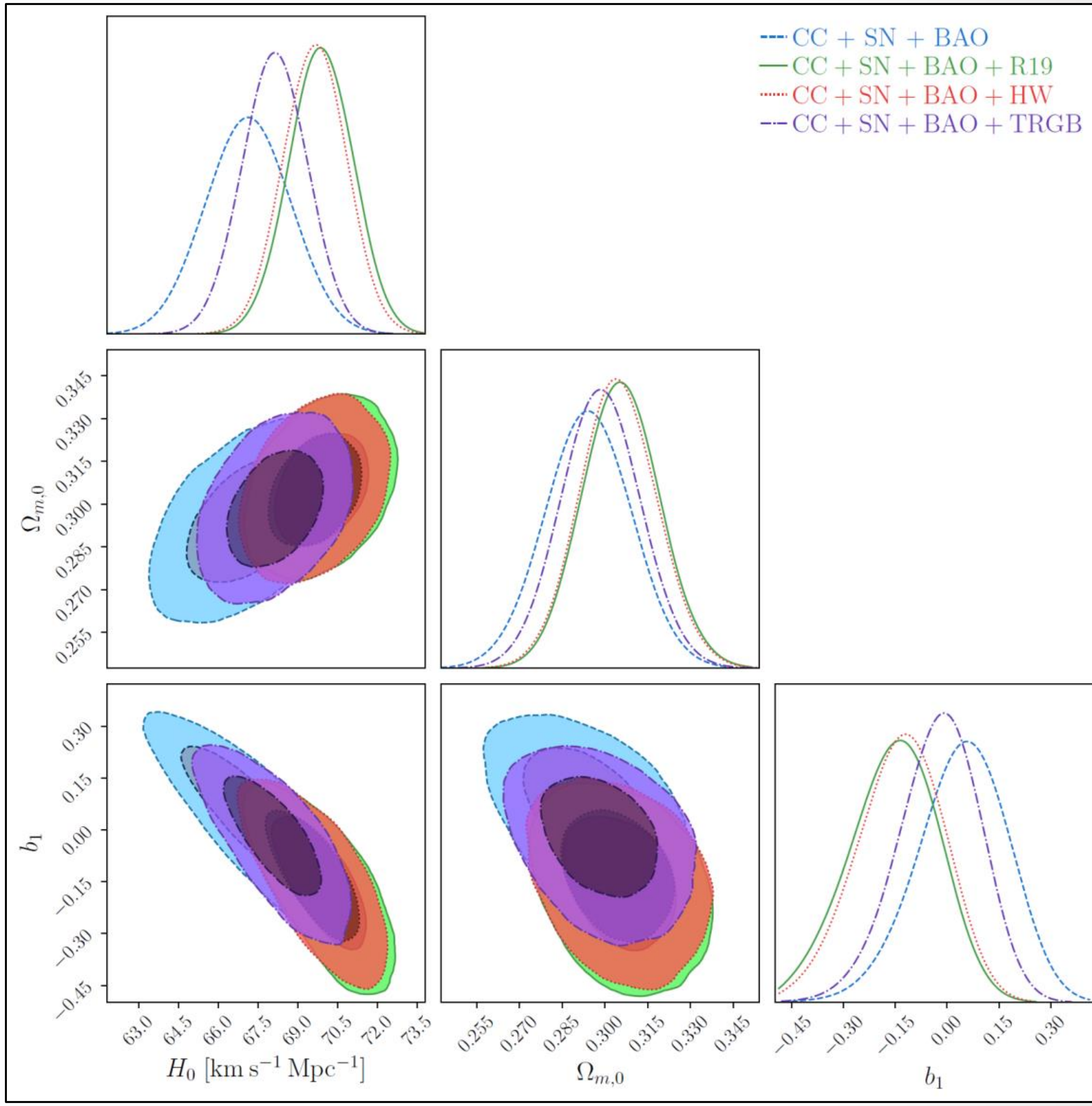
where $\alpha_1 = \left(6H_0^2\right)^{1-b_1} \dfrac{1-\Omega_0^{\mathrm{m}}}{2b_1-1}$

$G_{eff}(z \simeq 0) \simeq G_N$

KVNO - $\Delta\alpha/\alpha$ from quasar data

Precision Cosmology Constraints for f_{PLM}CDM





Results for $f_{\mathbf{PLM}}$CDM

Data Sets	H_0 $\left[\mathrm{km\ s^{-1}\ Mpc^{-1}}\right]$	$\Omega_{\mathrm{m},0}$	b_1	ΔAIC	ΔBIC
CC+SN	68.5 ± 1.8	$0.350^{+0.045}_{-0.064}$	$-0.22^{+0.41}_{-0.48}$	1.45	6.43
CC+SN+R19	$\mathbf{71.3^{+1.3}_{-1.4}}$	$\mathbf{0.326^{+0.045}_{-0.065}}$	$\mathbf{-0.13^{+0.40}_{-0.50}}$	$\mathbf{1.51}$	$\mathbf{6.50}$
CC+SN+HW	71.0 ± 1.3	$0.329^{0.045}_{0.062}$	$-0.16^{+0.41}_{-0.48}$	1.51	6.50
CC+SN+TRGB	$69.1^{+1.4}_{-1.3}$	$0.344^{+0.045}_{-0.063}$	$-0.20^{+0.42}_{-0.47}$	1.87	6.85
CC+SN+BAO	$\mathbf{67.1 \pm 1.6}$	$\mathbf{0.294 \pm 0.015}$	$\mathbf{0.06 \pm 0.13}$	$\mathbf{1.68}$	$\mathbf{6.68}$
CC+SN+BAO+R19	69.9 ± 1.2	$0.305^{+0.014}_{-0.013}$	$-0.14^{+0.12}_{-0.13}$	0.56	5.56
CC+SN+BAO+HW	69.7 ± 1.2	$0.304^{+0.014}_{-0.012}$	$-0.12^{+0.12}_{-0.13}$	0.89	5.89
CC+SN+BAO+TRGB	68.1 ± 1.2	0.298 ± 0.014	$-0.01^{+0.11}_{-0.12}$	2.00	7.00

$$\boxed{\mathrm{AIC} = 2k - 2\ln L}$$

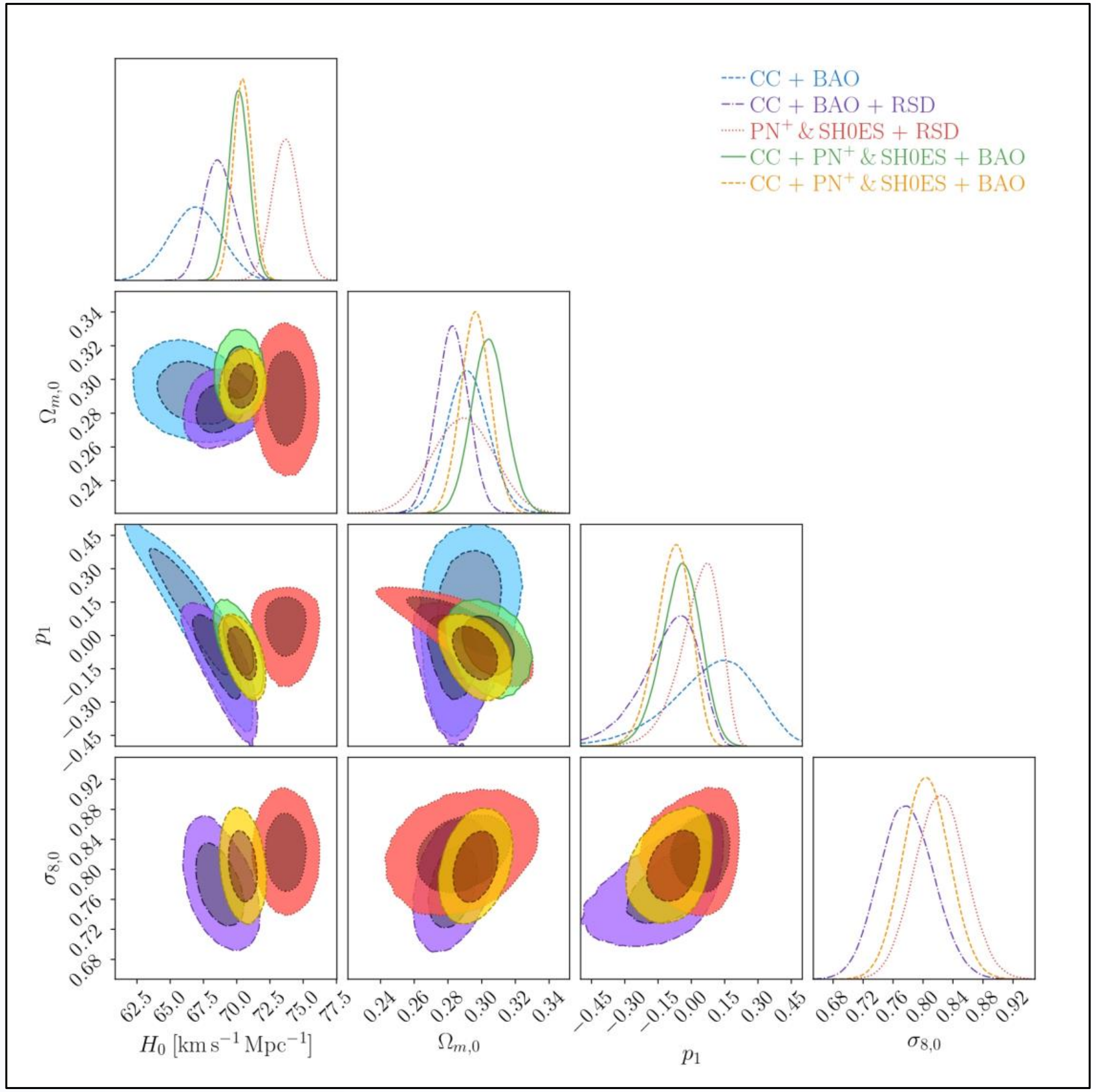
Number of
Model parameters

Maximum likelihood

$$\boxed{\mathrm{BIC} = k\ln n - 2\ln L}$$

Number of
points in a data set

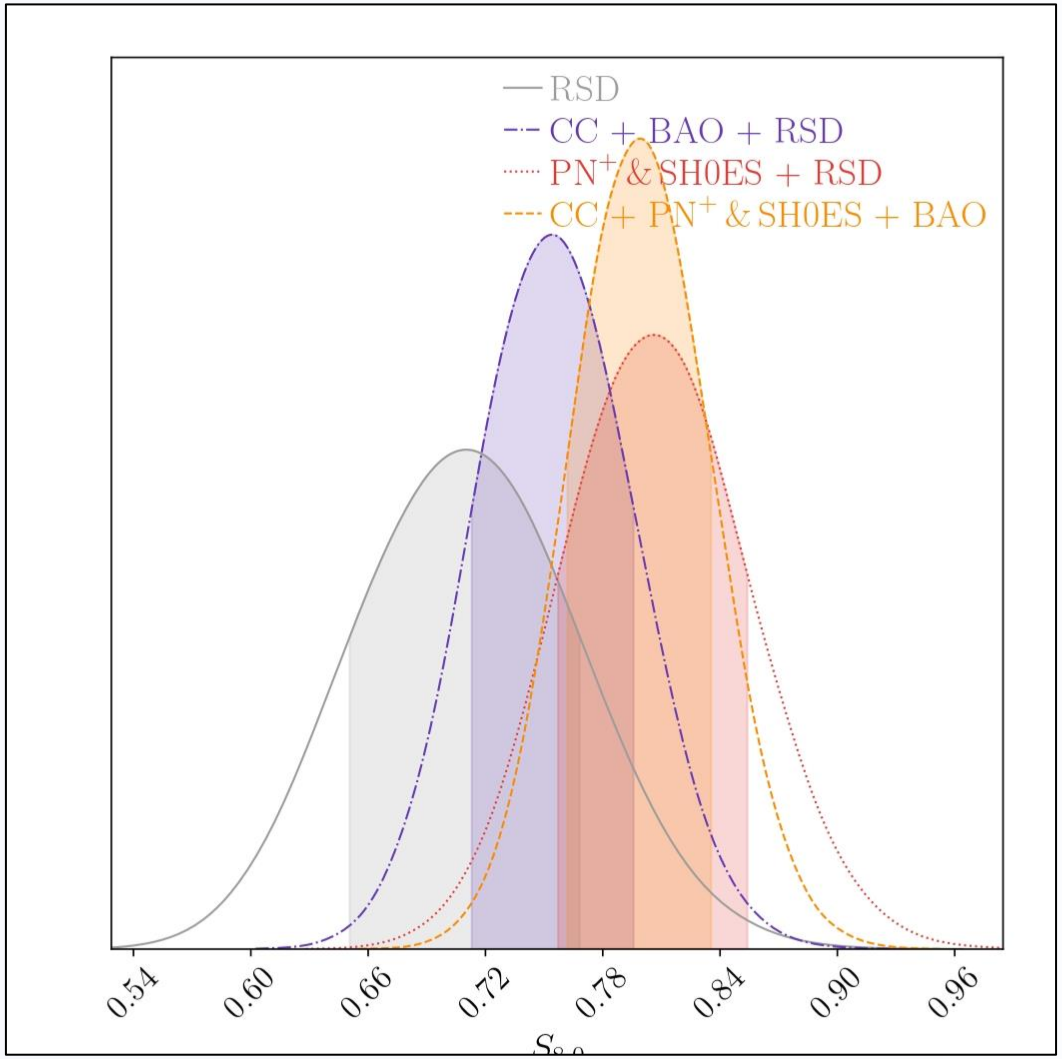
f_{PLM}CDM at perturbative level



Results for $f_{\mathbf{PLM}}$CDM

Data Sets	$H_0\ [\mathrm{km\ s^{-1}\ Mpc^{-1}}]$	$\Omega_{\mathrm{m},0}$	b_1	$\sigma_{8,0}$
CC+BAO	$\mathbf{66.9^{+2.0}_{-1.9}}$	$\mathbf{0.291^{+0.013}_{-0.012}}$	$\mathbf{0.14^{+0.17}_{-0.21}}$	$-$
CC+BAO+RSD	$68.5^{+1.3}_{-1.1}$	$0.283^{+0.009}_{-0.009}$	$-0.046^{+0.100}_{-0.150}$	$0.778^{+0.036}_{-0.037}$
PN⁺+RSD	$\mathbf{73.7 \pm 1.00}$	$\mathbf{0.289 \pm 0.019}$	$\mathbf{0.071^{+0.073}_{-0.102}}$	$\mathbf{0.825^{+0.032}_{-0.036}}$
CC+PN⁺+BAO	$70.10^{+0.82}_{-0.68}$	$0.305^{+0.010}_{-0.010}$	-0.040 ± 0.087	$-$
CC+PN⁺+BAO+RSD	$70.44^{+0.72}_{-0.71}$	$0.296^{+0.009}_{-0.009}$	$-0.068^{+0.073}_{-0.084}$	$0.803^{+0.032}_{-0.084}$

Precision Cosmology Constraints for $f_{\mathbf{PLM}}$CDM



$$S_8 = \sigma_8 \sqrt{\frac{\Omega_m}{0.3}}$$

Data Sets	$S_{8,0}$
RSD	$\mathbf{0.710^{+0.058}_{-0.060}}$
CC+BAO+RSD	$0.755^{+0.041}_{-0.042}$
PN⁺+RSD	$\mathbf{0.806^{+0.048}_{-0.049}}$
CC+PN⁺+BAO+RSD	0.799 ± 0.037

$f_{\mathrm{PM}}(T, B)$ Model

Recall:

$$S = \frac{1}{16\pi G}\int d^4x\, e[-T + \boldsymbol{f(T,B)}] + S_{\mathrm{mat}}$$

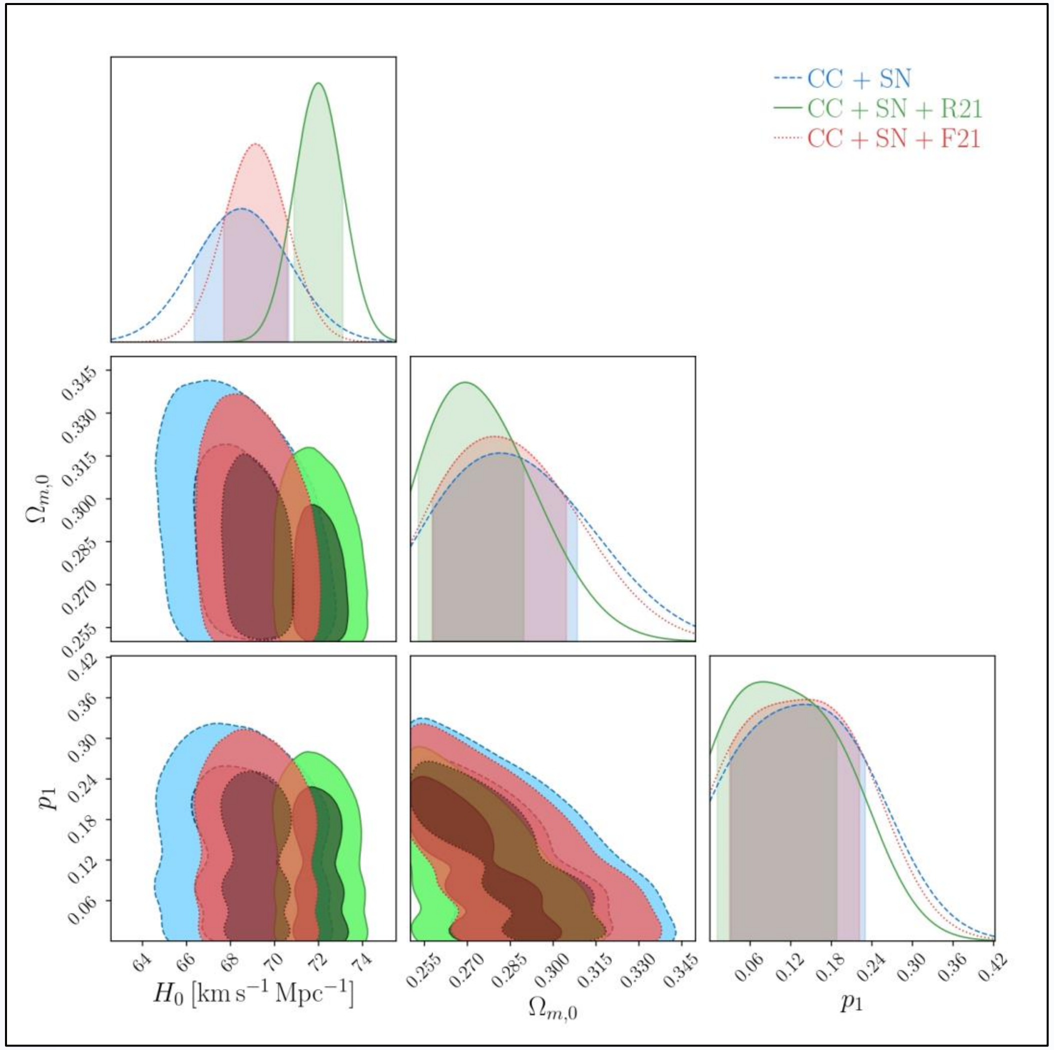
<u>Model:</u>

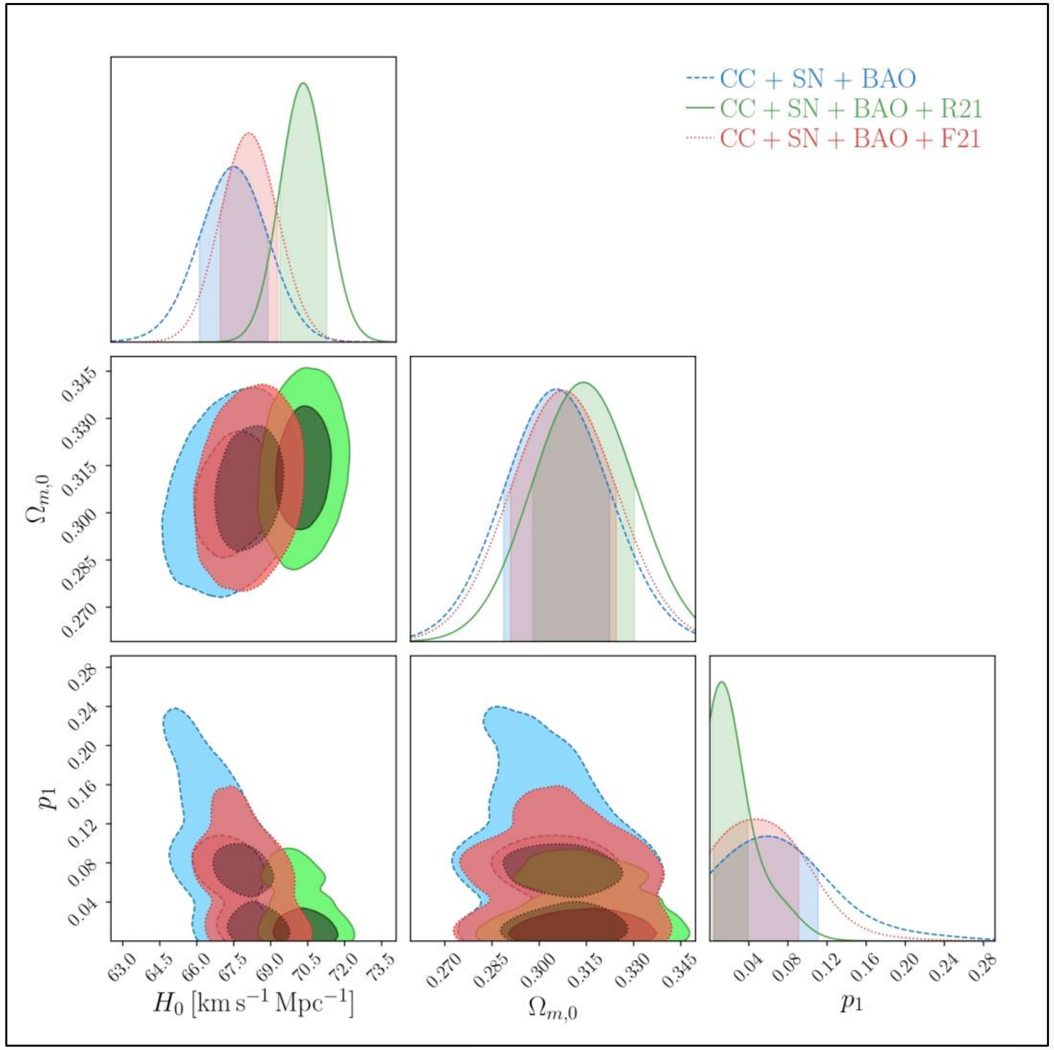
$$f_{\mathrm{PM}}(T,B) = \alpha_1 (B)^{p1}$$

where

$$\alpha_1 = \frac{6H_0^2\left(\Omega_{\mathrm{m},0} + \Omega_{\mathrm{r},0} - 1\right)}{(p_1 - 1)(B_0)^{p_1} + 6H_0\left(p_1(p_1 - 1)(B_0)^{p1-2}\dot{B}|_{t=t_0}\right)}$$

Precision Cosmology Constraints for f_{PM}CDM





Results for f_{PM}CDM

Data Sets	H_0 $\left[\mathrm{km\ s^{-1}\ Mpc^{-1}}\right]$	$\Omega_{\mathrm{m},0}$	p_1	ΔAIC	ΔBIC
CC+SN	$68.5^{+2.1}_{-2.2}$	$0.281^{+0.027}_{-0.023}$	$0.140^{+0.090}_{-0.109}$	1.21	6.19
CC+SN+R21	$\mathbf{72.0 \pm 1.1}$	$\mathbf{0.269^{+0.021}_{-0.016}}$	$\mathbf{0.080^{+0.108}_{-0.068}}$	$\mathbf{1.04}$	$\mathbf{6.02}$
CC+SN+F21	$69.1^{+1.5}_{-1.4}$	$0.279^{+0.025}_{-0.022}$	$0.143^{+0.079}_{-0.113}$	1.10	6.08
CC+SN+BAO	$\mathbf{67.5 \pm 1.4}$	$\mathbf{0.305 \pm 0.017}$	$\mathbf{0.058 \pm 0.053}$	$\mathbf{-0.04}$	$\mathbf{4.96}$
CC+SN+BAO+R21	$70.32^{+0.95}_{-0.94}$	0.314 ± 0.016	$0.012^{+0.027}_{-0.009}$	0.79	5.79
CC+SN+BAO+F21	68.1 ± 1.2	0.308 ± 0.017	$0.046^{+0.045}_{-0.042}$	0.13	5.12

$$\boxed{\mathrm{AIC} = 2k - 2\ln L}$$

$$\boxed{\mathrm{BIC} = k\ln n - 2\ln L}$$

Can we do this in a model-independent way?

Gaussian Processes Regression

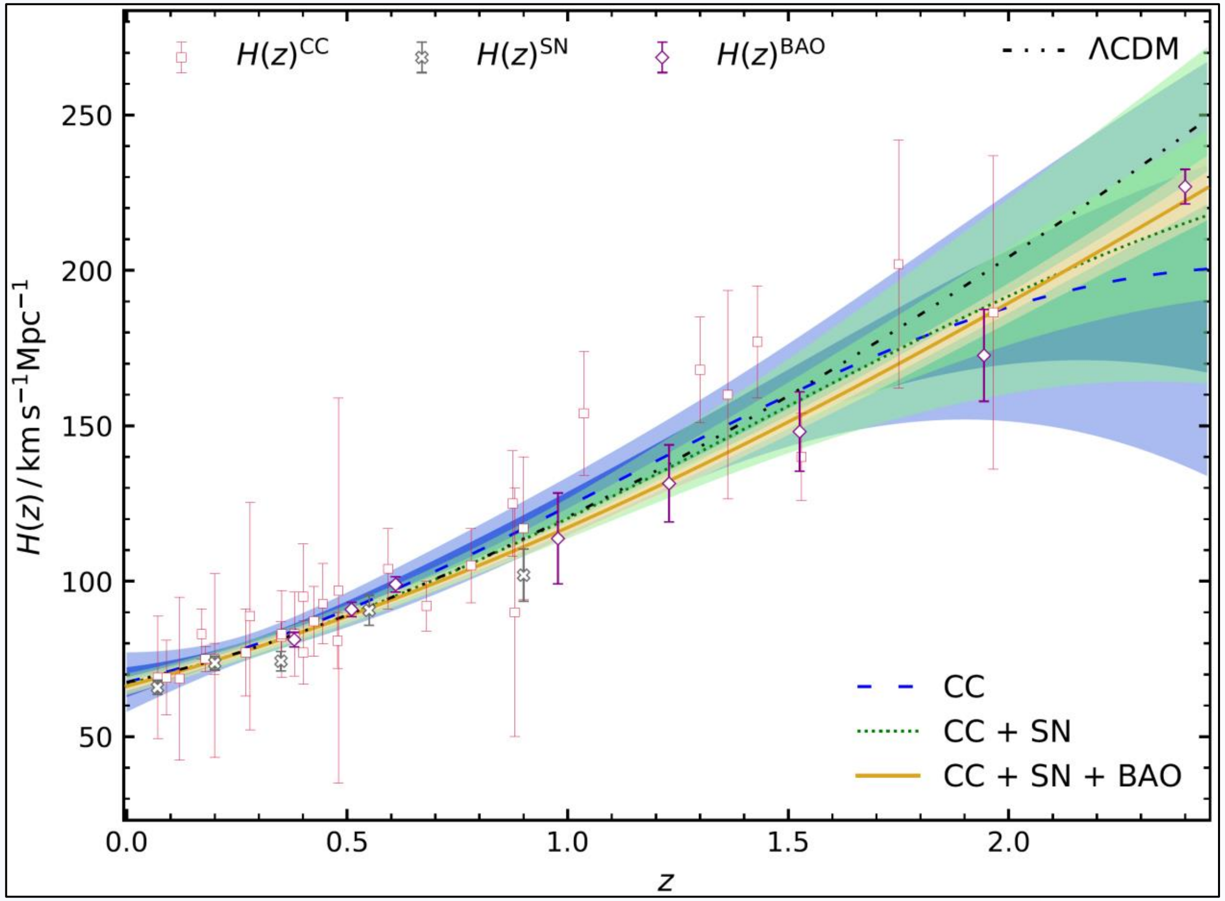
- The covariance function contains **non-physical hyperparameters** θ which define the distribution $k(\theta, x, x')$
- Iterating over these values using **Bayesian inference** (or others) can produce better hyperparameters
- The result is a (physics) **model independent reconstruction** of the behavior of some parameter
- This is superior to regular fitting because it is nonparametric and so **assumes no physical model** whatsoever

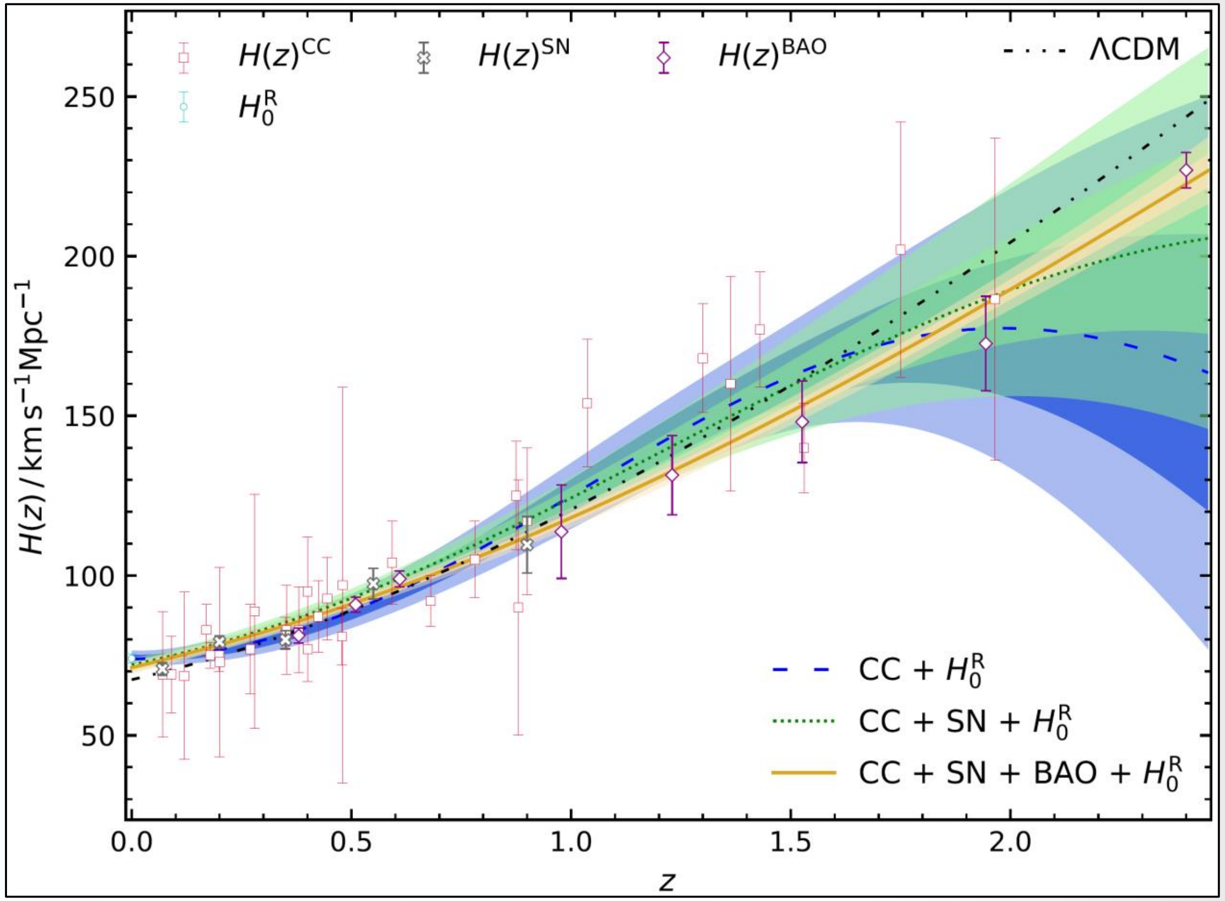
The Covariance Functions

Squared Exponential (Gaussian or RBF):

$$k(x, x') = \sigma_f^2 \ \mathrm{Exp}\left[-\frac{1}{2}\left(\frac{x - x'}{l_f}\right)^2\right]$$

Square Exponential H_0 GP





$H_0 = 67.539 \pm 4.772 {\rm km\ s^{-1} Mpc^{-1}}$

$H_0 = 67.001 \pm 1.653 {\rm km\ s^{-1} Mpc^{-1}}$

$H_0 = 66.197 \pm 1.464 {\rm km\ s^{-1} Mpc^{-1}}$

$H_0 = 73.782 \pm 1.374 {\rm km\ s^{-1} Mpc^{-1}}$

$H_0 = 72.022 \pm 1.076 {\rm km\ s^{-1} Mpc^{-1}}$

$H_0 = 71.180 \pm 1.025 {\rm km\ s^{-1} Mpc^{-1}}$

Square Exponential Covariance for H_0

Distance (in σ units) between the H_0 arguments:

$$d\left(H_{0,i}, H_{0,j}\right) = \frac{H_{0,i} - H_{0,j}}{\sqrt{\sigma_i^2 + \sigma_j^2}}$$

Data set(s)	$H_0\left[\mathrm{km\ s^{-1}\ Mpc^{-1}}\right]$	$d\left(H_0, H_0^{\mathrm{R19}}\right)$	$d\left(H_0, H_0^{\mathrm{TRGB}}\right)$	$d\left(H_0, H_0^{\mathrm{HW}}\right)$
CC	67.539 ± 4.772	-1.304	-0.441	-1.133
CC+$\boldsymbol{H_0^{\mathrm{R}}}$	$\mathbf{73.782 \pm 1.374}$	**-0.126**	**1.711**	**0.217**
CC+SN	67.001 ± 1.653	-3.225	-1.118	-2.617
CC+SN+H_0^{R}	72.022 ± 1.076	-1.128	1.026	-0.622
CC+SN+BAO	$\mathbf{66.197 \pm 1.464}$	**-3.841**	**-1.513**	**-3.113**
CC+SN+BAO+H_0^{R}	71.18 ± 1.025	-1.628	0.645	-1.046

Boundary Conditions

ΛCDM (or $f(T) = \Lambda$) works at late times

This implies that

$$f_T(z \simeq 0) \simeq 0$$

$$\Rightarrow f(z \simeq 0) = 6H_0^2\left(\Omega_{m_0} - 1\right)$$

Briffa et al. CQG 38 055007 (2020)

$$S = \frac{1}{16\pi G}\int d^4x\, e[-T + \boldsymbol{f(T)}] + S_{\text{matter}}$$

Propagating $f\big(T(z)\big)$

- The Friedmann equation contains f_T which **need to be eliminated finite difference** methods
- Using a **central differencing** approach (error $\sim \mathcal{O}\big(\Delta z^2\big)$), we can assume

$$f'(z_i) \simeq \frac{f(z_{i+1}) - f(z_{i-1})}{z_{i+1} - z_{i-1}}$$

- Therefore, we can remove the $f_T(T) = f'(z)/T'(z)$

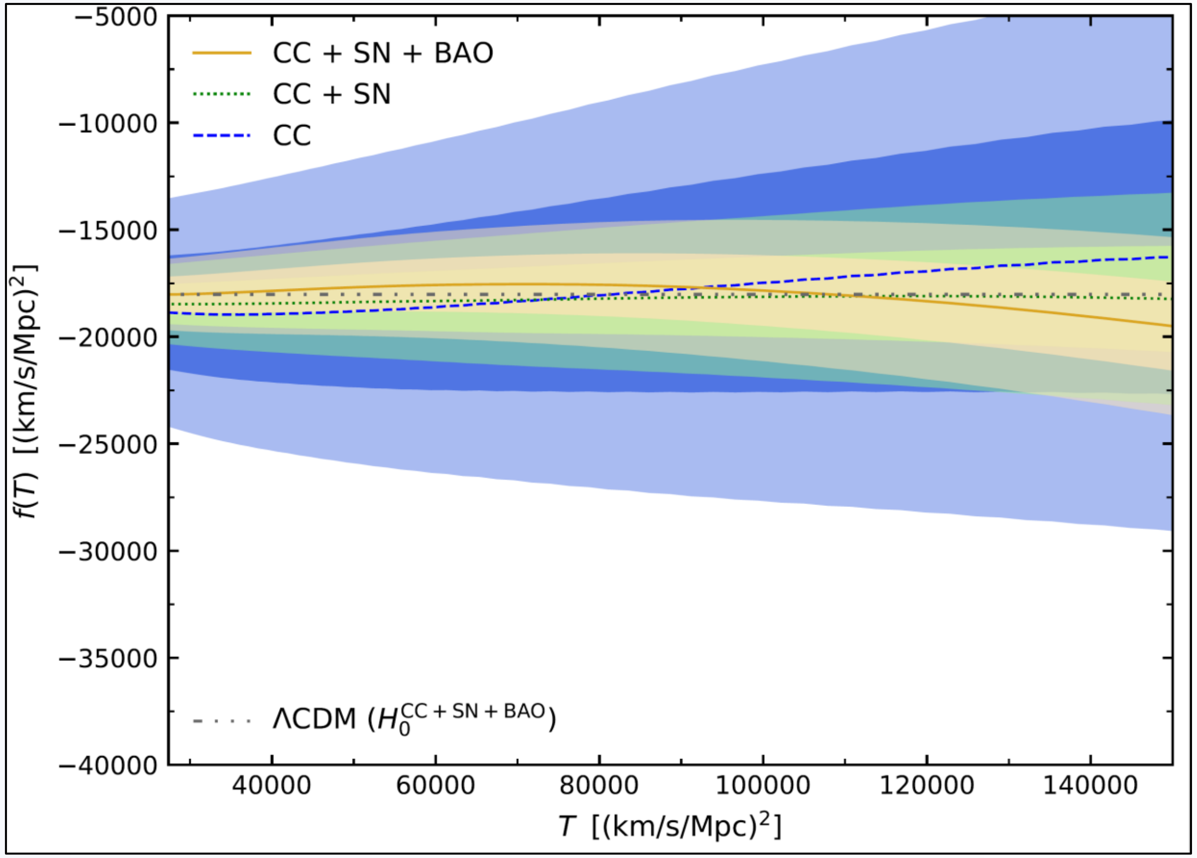
$$\boxed{H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} + \frac{T}{3}\boldsymbol{f_T}}$$

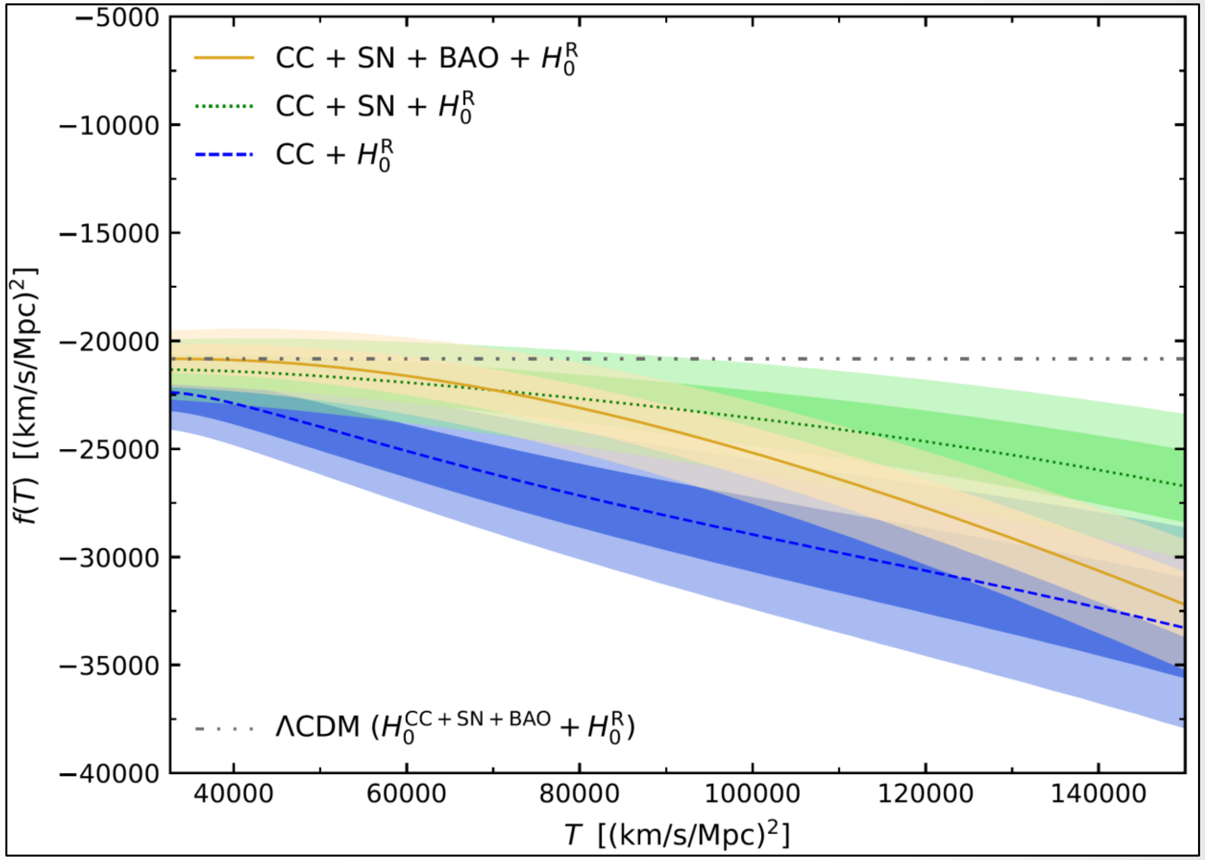
- This then gives a **propagation equation**

$$f(z_{i+1}) = f(z_{i-1}) + 2(z_{i+1} - z_{i-1})\frac{H'(z_i)}{H(z_i)}\left(3H(z_i)^2 + \frac{f(z_i)}{2} - 3H_0^2\Omega_{m_0}(1+z_i)^3\right)$$

- Using **forward differencing**, we can produce a second boundary condition

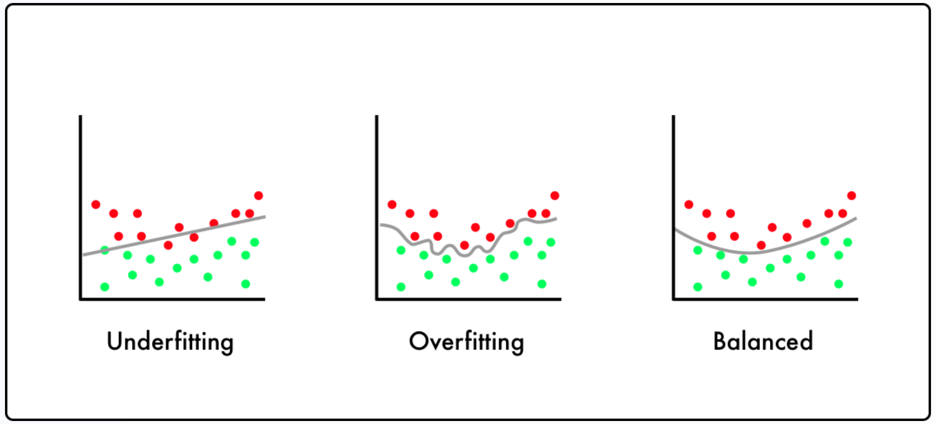
Square Exponential $f(T)$ GP



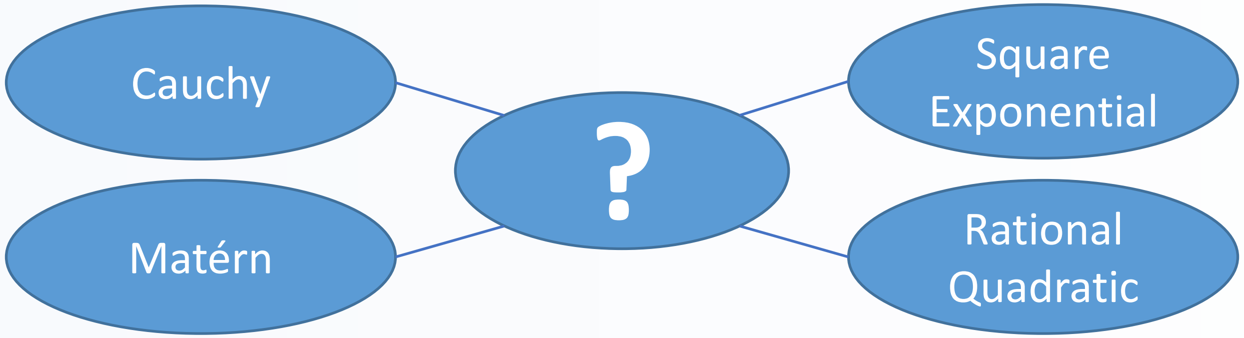


Open Problems with GP Reconstructions

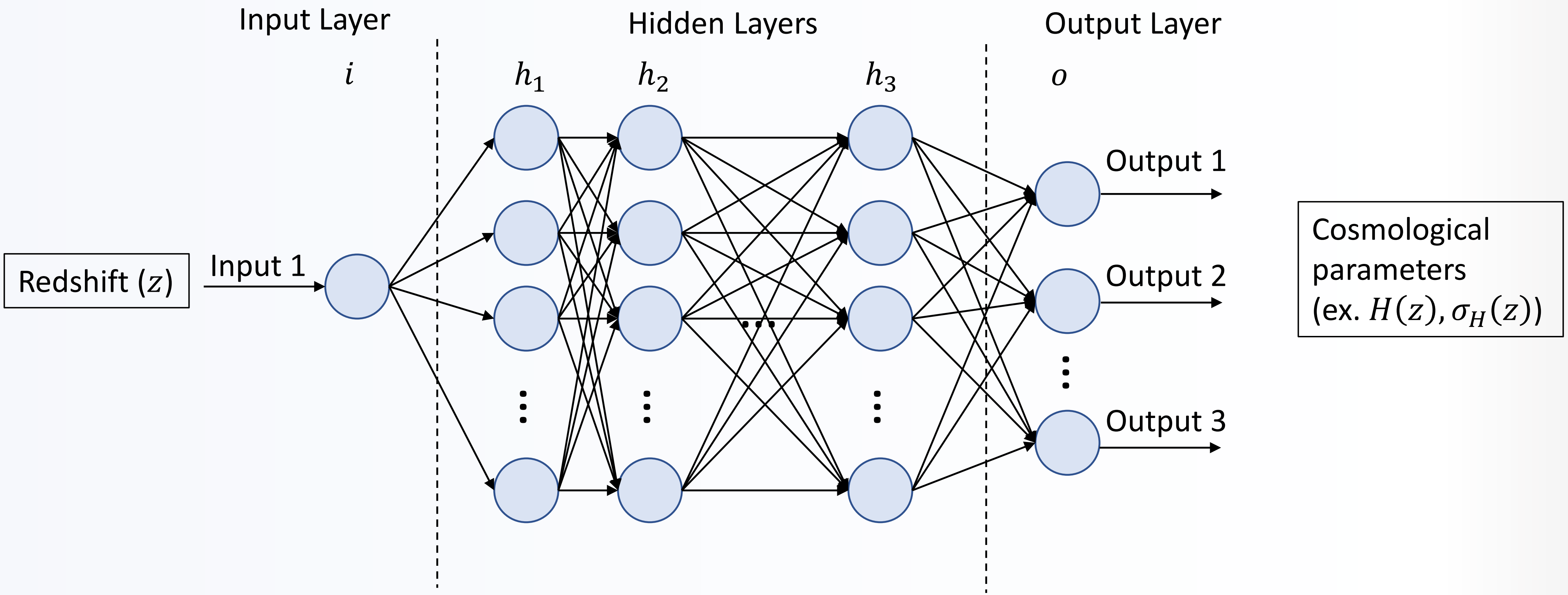
- **Overfitting at origin**: GP is very prone to overfitting for small data sets, which is especially pronounced at the origin, i.e. Hubble constant



- **Kernel Selection Problem**: There is no natural kernel for cosmology

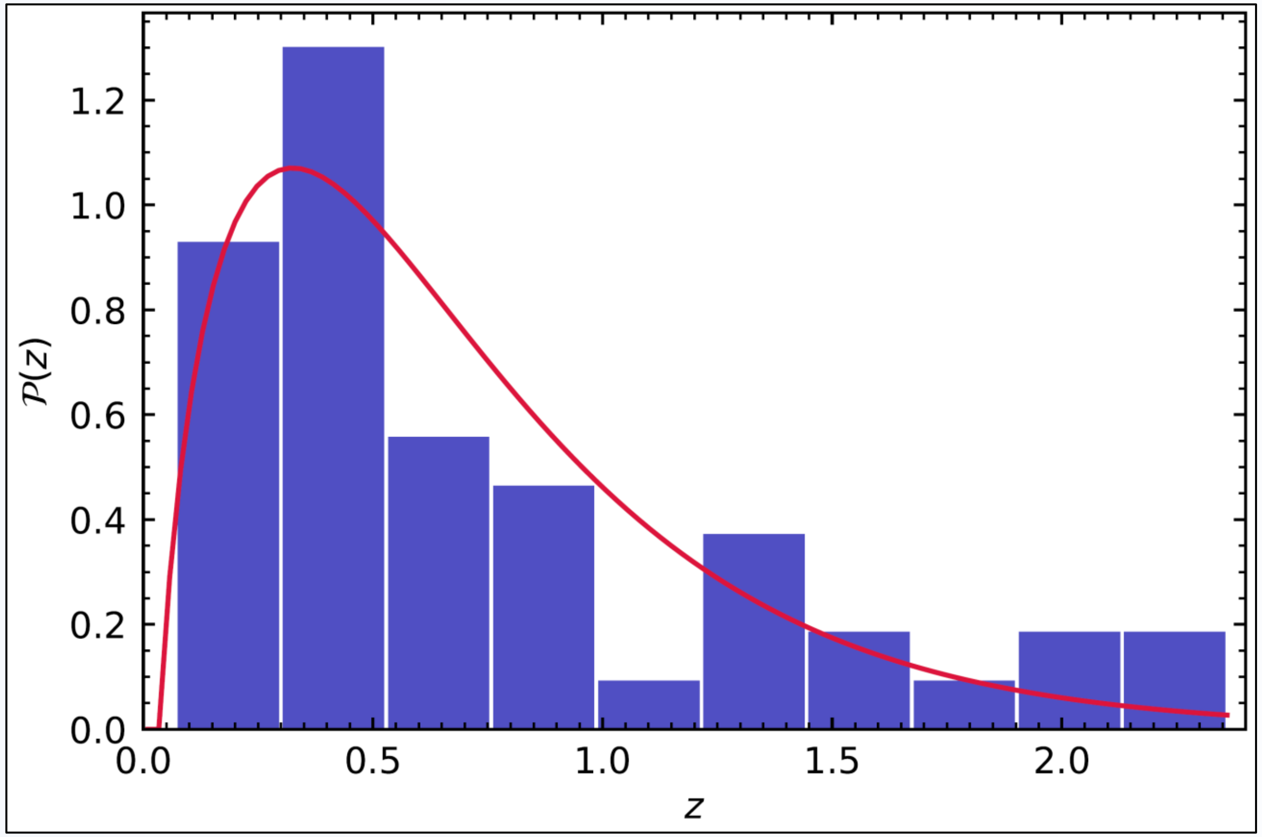


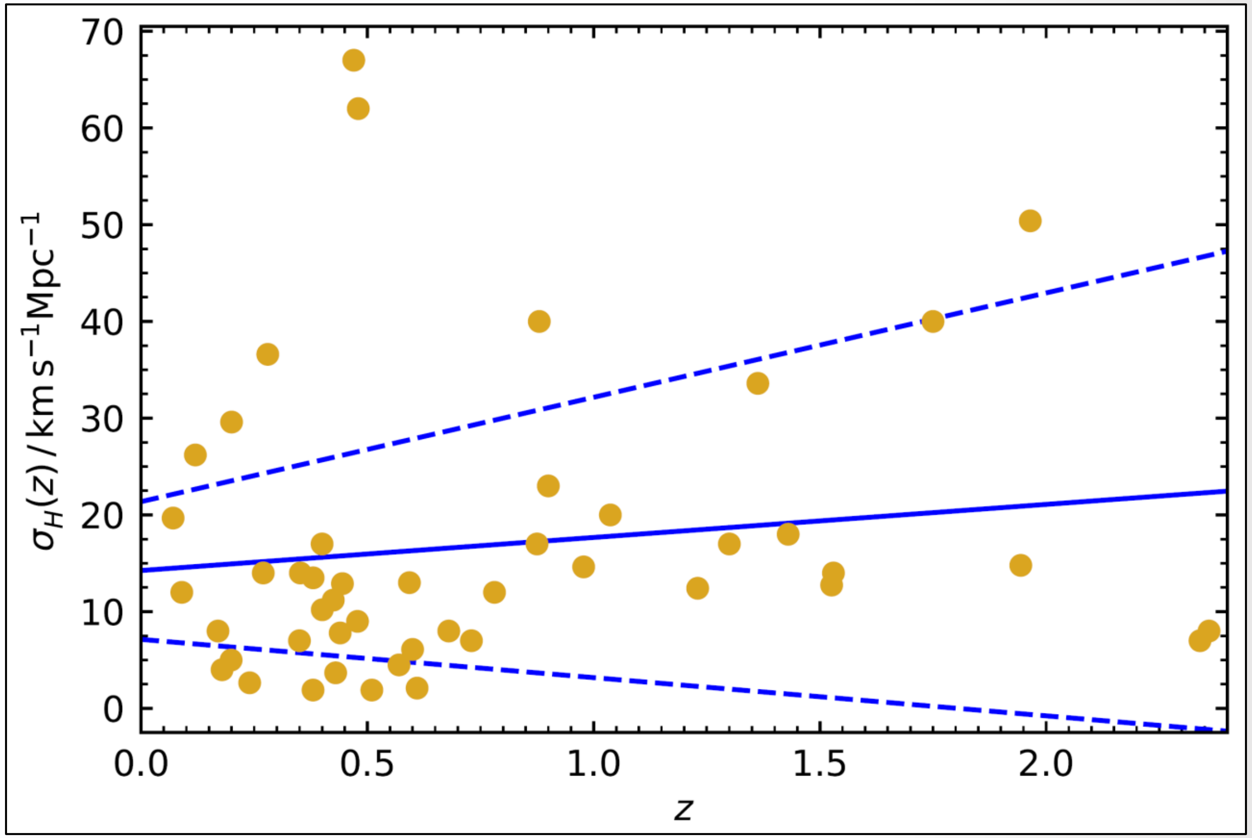
Artificial Neural Networks (ANNs)



Training Data for the ANN

CC+BAO dataset





This observes the gamma distribution:

$$\mathcal{P}(z,\alpha,\lambda) = \frac{\lambda^\alpha}{\Gamma(\alpha)} z^{\alpha-1} e^{-\lambda z}$$

Mean: $\sigma_H = 14.25 + 3.42z$
Upper error: $\sigma_H = 21.37 + 10.79z$
Lower error: $\sigma_H = 7.14 - 3.95z$

Designing the ANN

- **Risk** – Optimizes the **number of hidden layers and neurons** in an ANN

$$\text{risk} = \sum_{i=1}^{N} (\text{Bias}_i^2 + \text{Variance}_\text{i}) = \sum_{i=1}^{N} \left(\left[H_{Obs}(z_i) - H_{pred}(z_i) \right]^2 + \sigma_H^2(z_i) \right)$$

- **Loss** – Balances the **number of iterations** a system needs to predict the observational data

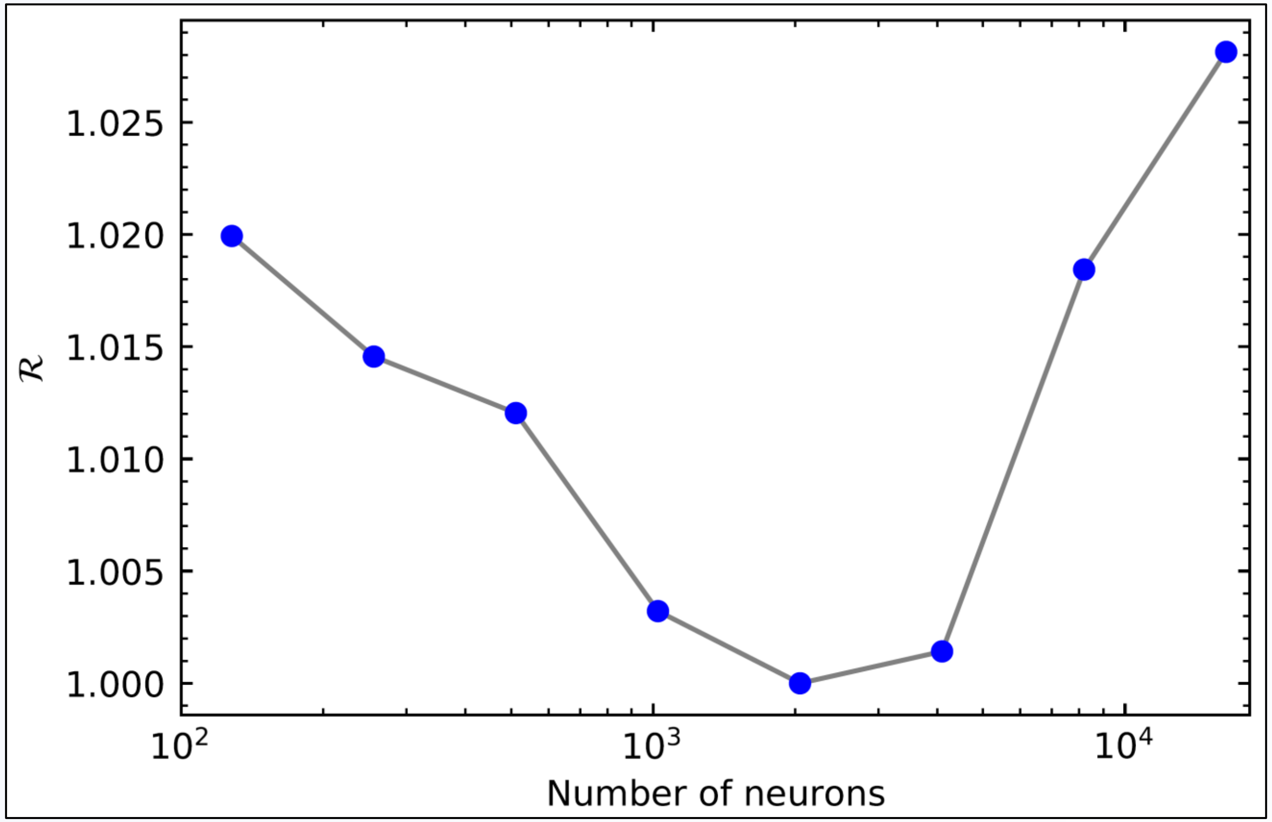
1. **L1** (Least absolute deviation)

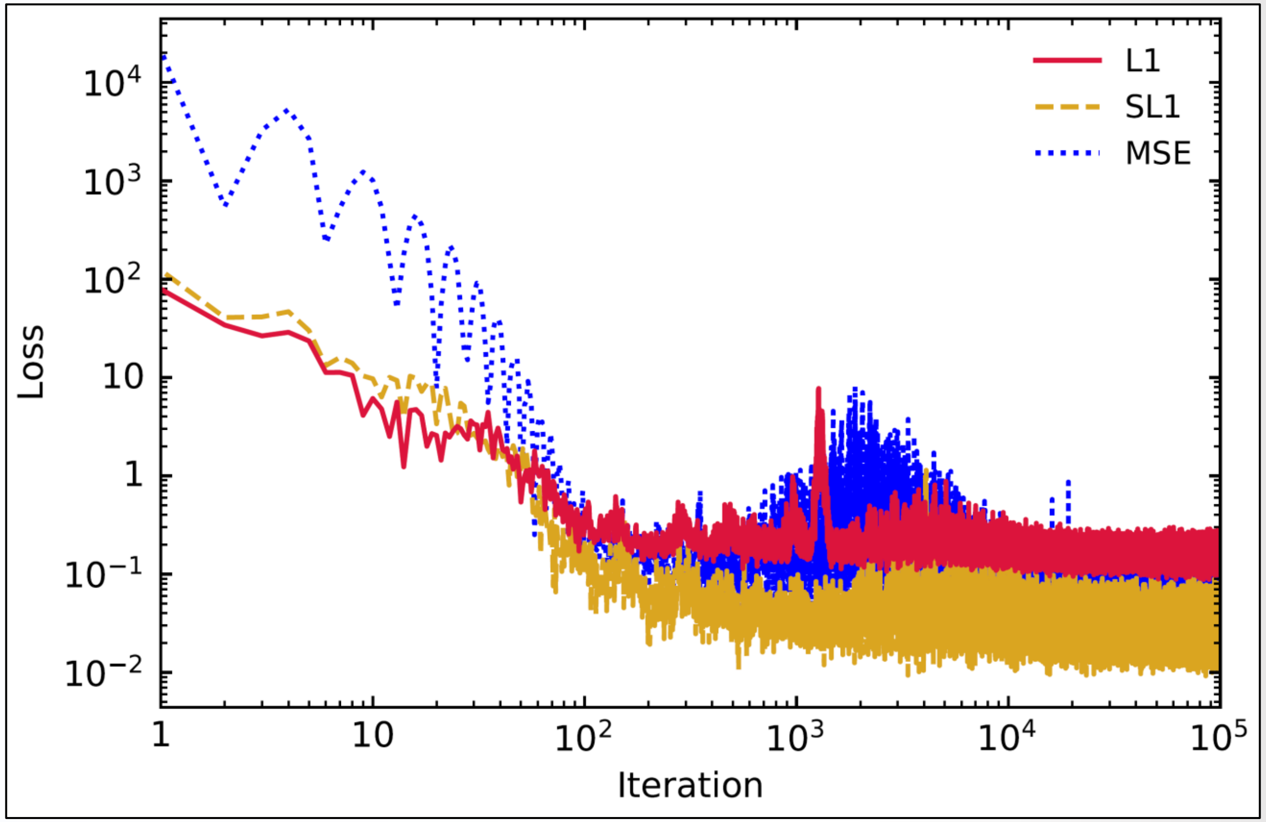
$$\text{L1} = \sum_{i=1}^{N} \left| H_{Obs}(z_i) - H_{pred}(z_i) \right|$$

2. Smoothed L1 (**SL1**)
3. Mean Square Error (**MSE**)

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} \left(H_{Obs}(z_i) - H_{pred}(z_i) \right)^2$$

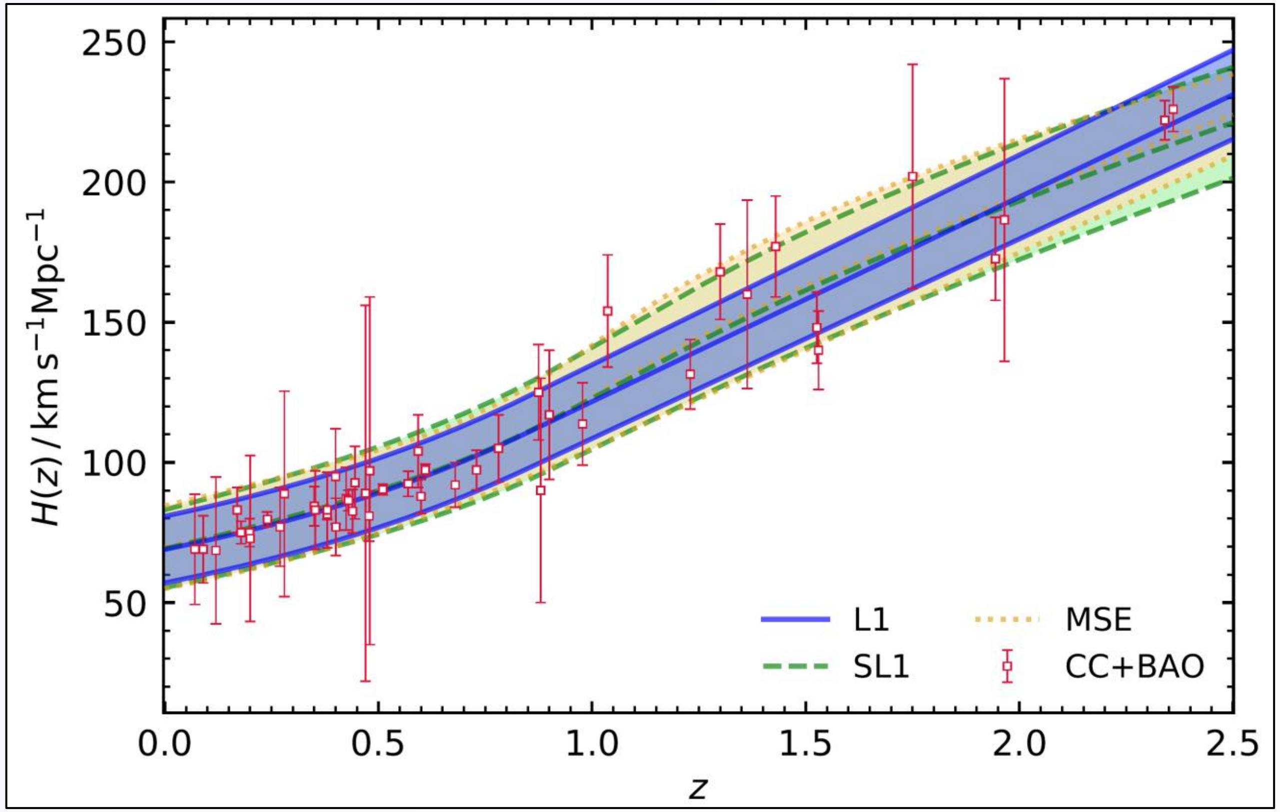
Building the ANN

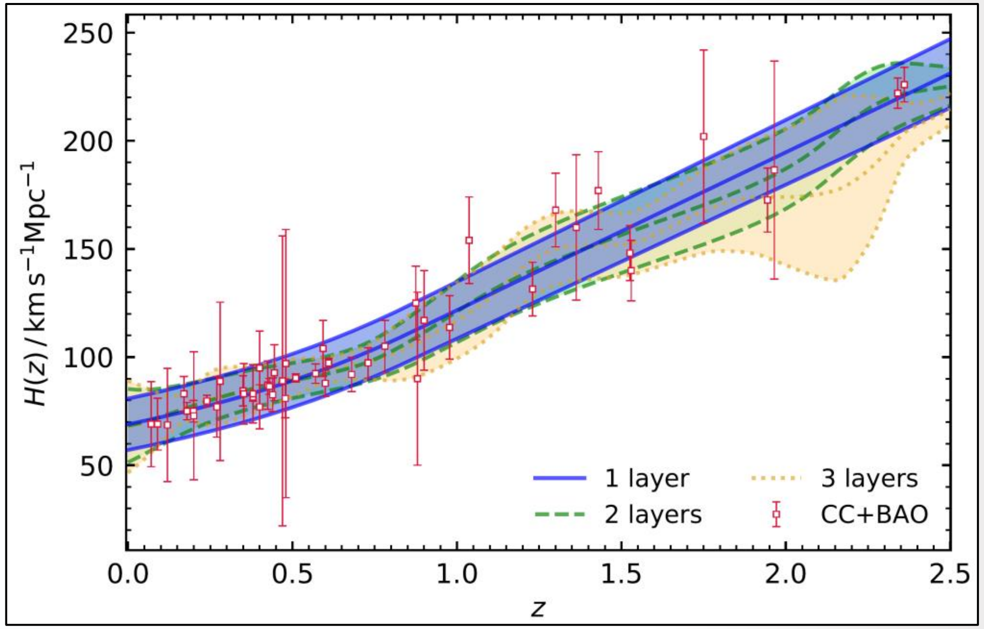




Risk function for **one layer** ($number\ of\ neurons = 2^n$
$n \in \{7, .. 14\}$)

Using the ANN





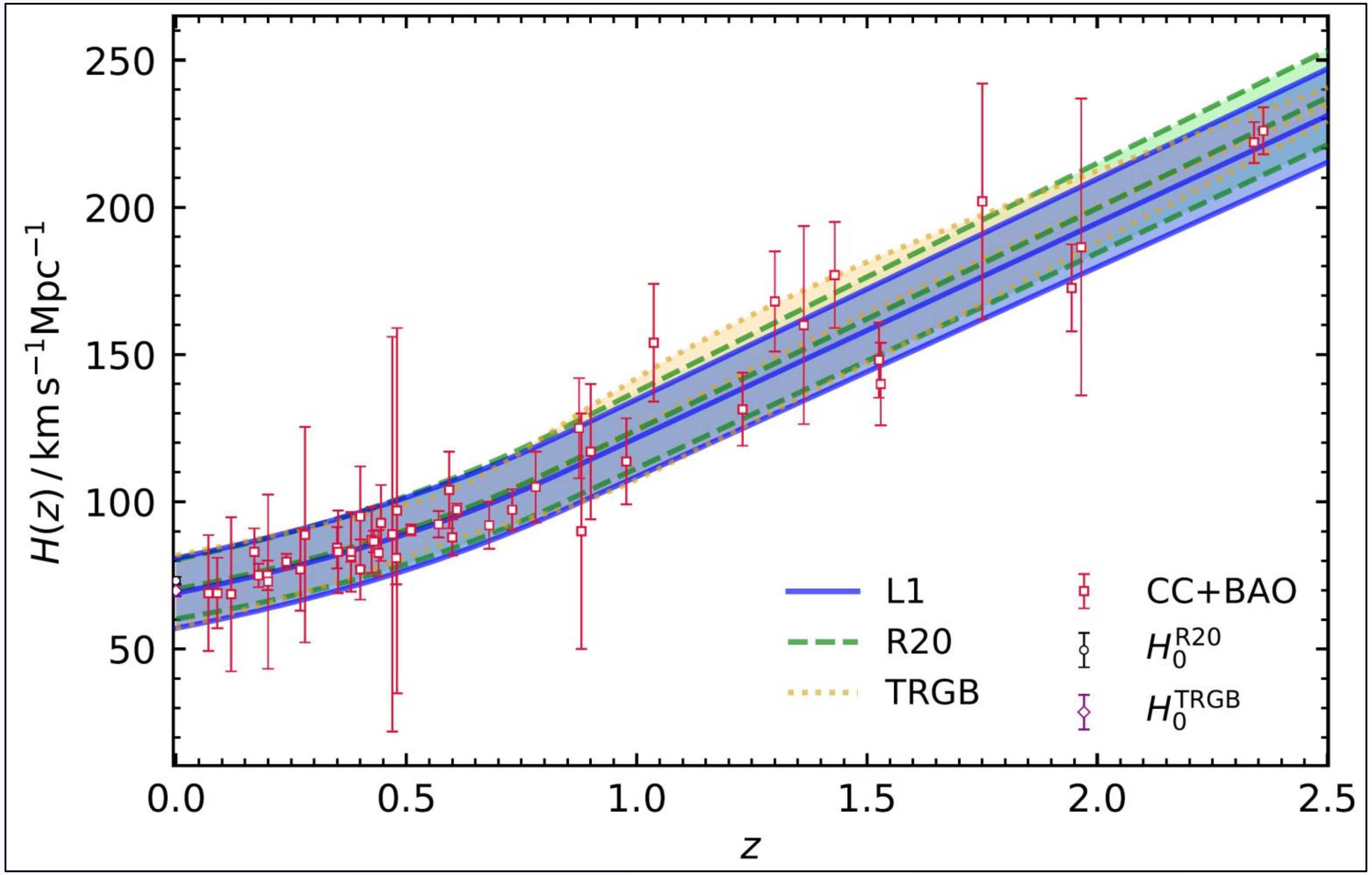
One layer is preferred

MSE: $H_0 = 69.76 \pm 14.82\ \mathrm{km\ s^{-1}Mpc^{-1}}$

L1: $H_0 = 68.93 \pm 11.90\ \mathrm{km\ s^{-1}Mpc^{-1}}$

SL1: $H_0 = 69.18 \pm 13.92\ \mathrm{km\ s^{-1}Mpc^{-1}}$

What about priors?



Priors:

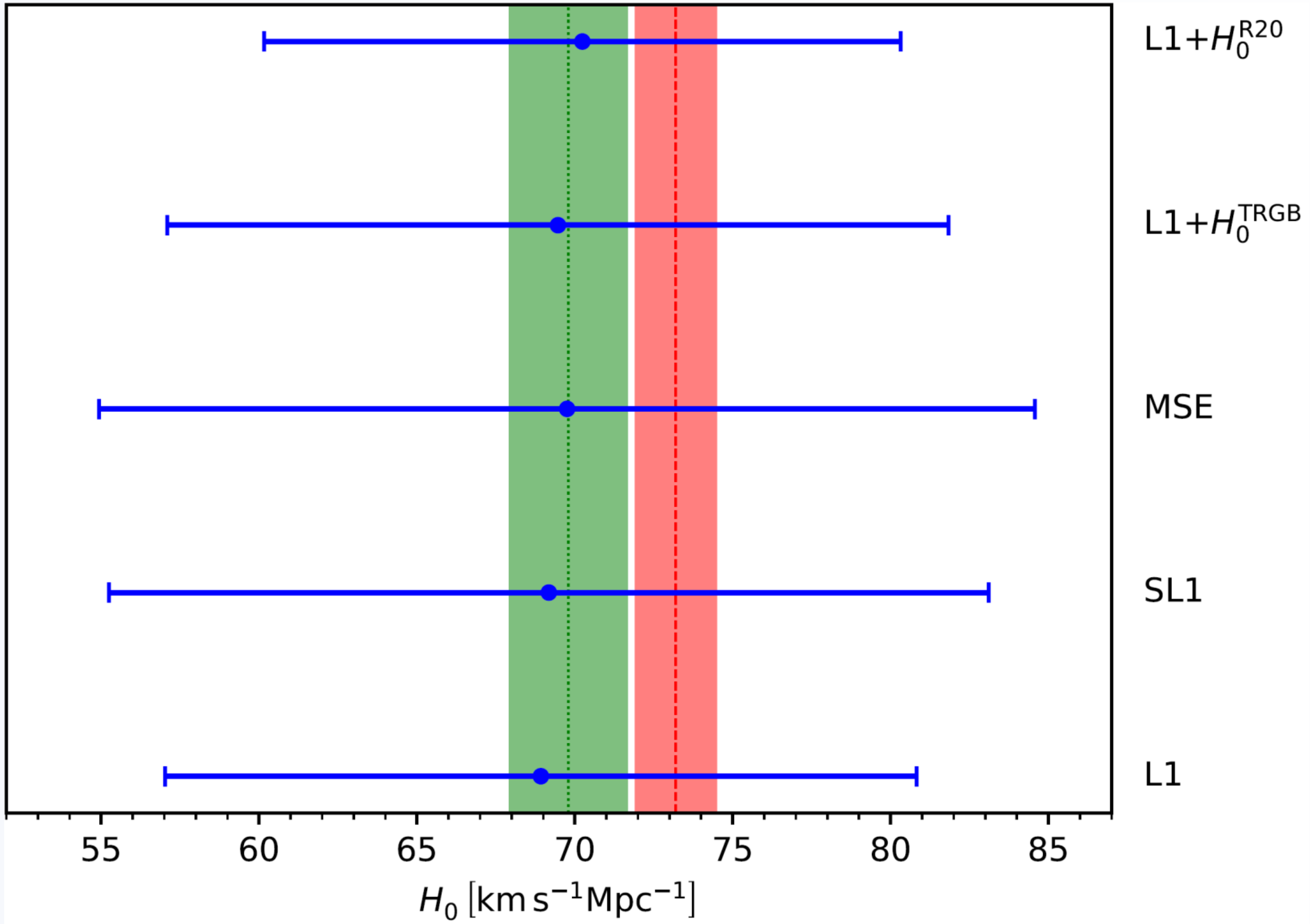
$H_0^{\mathrm{R20}} = 73.2 \pm 1.3\ \mathrm{km\,s^{-1}Mpc^{-1}}$

$H_0^{\mathrm{TRGB}} = 69.8 \pm 1.9\ \mathrm{km\,s^{-1}Mpc^{-1}}$

$\boldsymbol{H_0^{\mathrm{R20}}}$: $H_0 = 70.24 \pm 10.08\ \mathrm{km\,s^{-1}Mpc^{-1}}$

$\boldsymbol{H_0^{\mathrm{TRGB}}}$: $H_0 = 69.47 \pm 12.37\ \mathrm{km\,s^{-1}Mpc^{-1}}$

Whisker Plot of Results



<u>Priors</u>:

$H_0^{\mathrm{R20}} = 73.2 \pm 1.3\ \mathrm{km\ s^{-1} Mpc^{-1}}$

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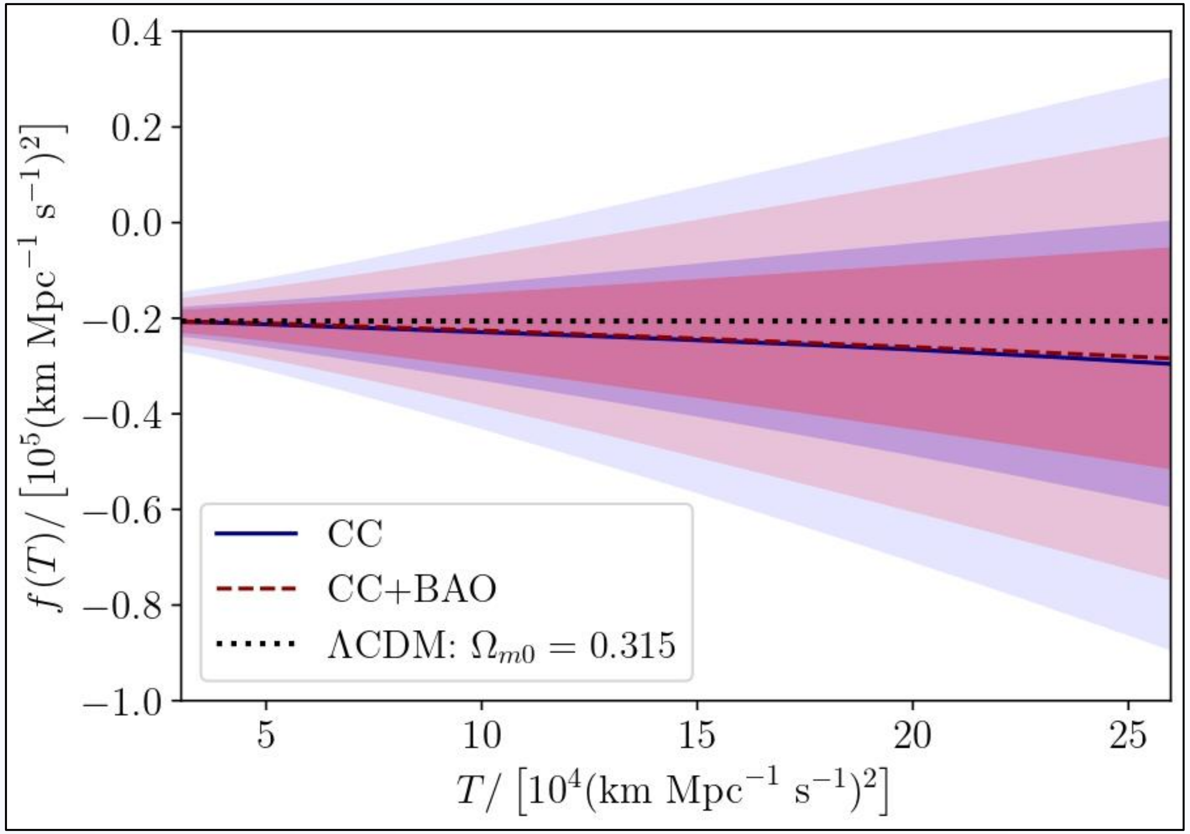
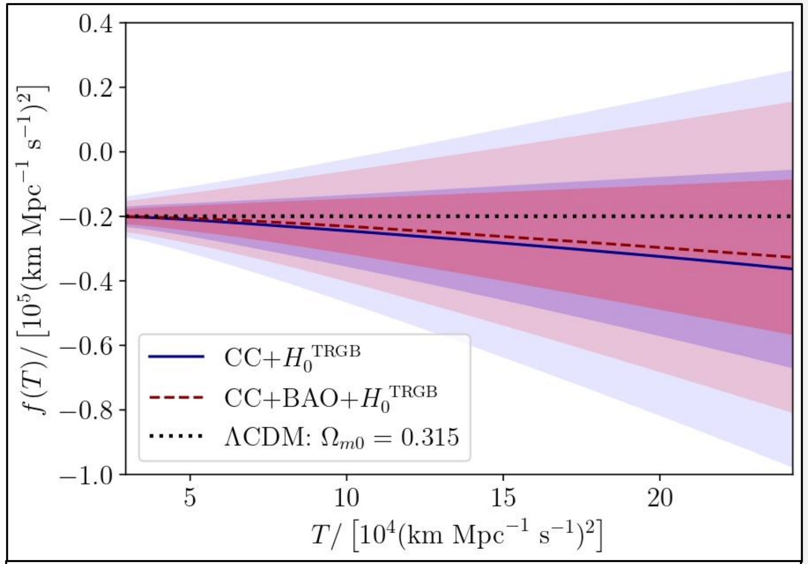
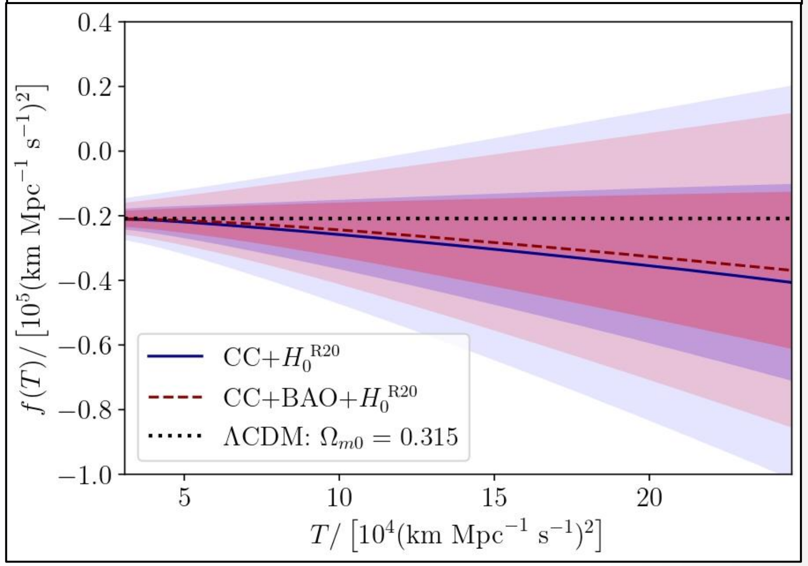
Propagating f(T)CDM







Conclusion

- **TG offers an interesting alternative** to traditional ways to modify gravity
- TG satisfies a number of preliminary **observational tests**, and offers a more consistent picture of modified gravity
- TG is compatible with novel methods being developed in conjunction with **machine learning**

Thank You