Higher-twist contributions of SIDIS in Target Fragmentation Region

Xuan-Bo Tong The Chinese University of HongKong(ShenZhen)

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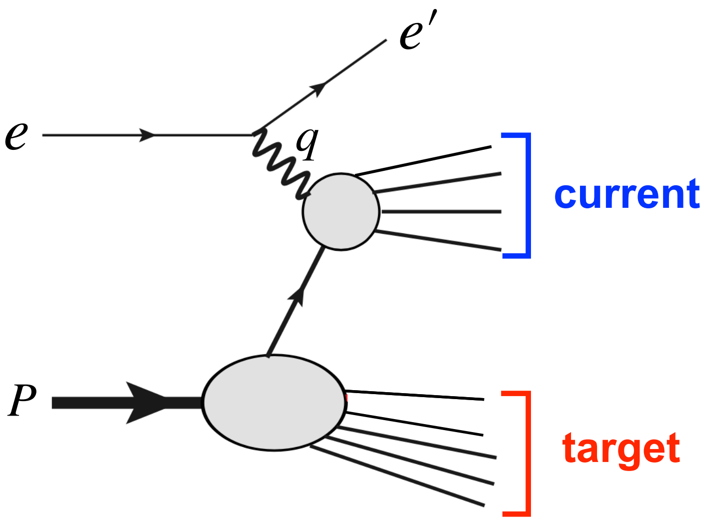


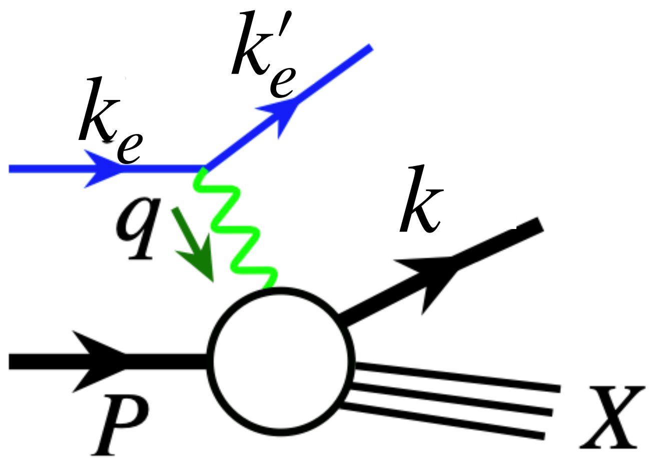
- **Introduction**
  - Factorizations and Kinematic regions of SIDIS
- **Higher-twist contributions in TFR**
  - at Small transverse momentum
  - at Large transverse momentum

Based on JHEP 11 (2021) 038 and work in preparation with Kai-Bao Chen, Jian-Ping Ma

Introduction

Semi-Inclusive Deep Inelastic Scattering

$$e(k_e, \lambda_e) + h(P, s) \rightarrow e(k_e') + h'(k) + X$$



$$Q^2 = -q^2 \gg M^2$$

$$x_B = \frac{Q^2}{2p \cdot q}$$

- Various probes to the partonic structure of the hadrons:

  TMD distributions, nucleon spin partition, target fragmentation..

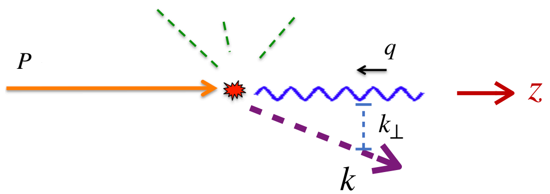
- Experiments: HEMERS, COMPASS, JLab, U.S. EIC, EICc…

- Cornerstone: QCD Factorizations by approximations in specific kinematics
- Different kinematics & Approximation accuracy ⇨ Different factorizations and probes.

Factorizations and Kinematic regions of SIDIS

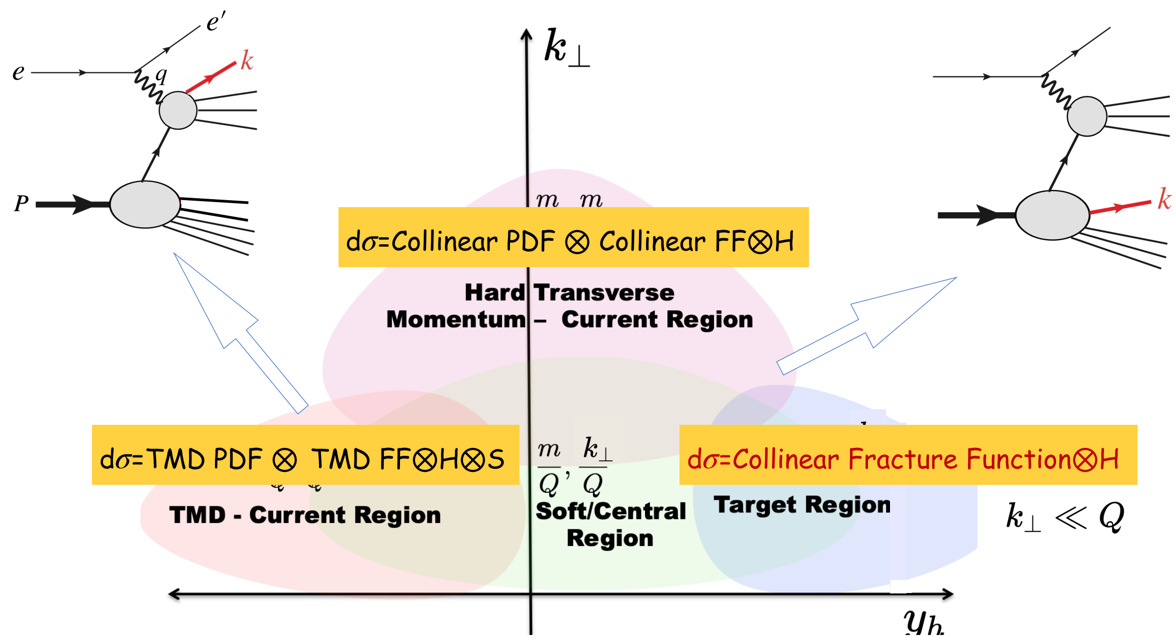
- Different factorizations apply in different regions of $(y_h, k_\perp)$

Photon-proton frame:



y_h : Rapidity

$k_\perp$: Transverse momentum



[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

- Targer fragmentation region

Fracture functions(diffractive parton distributions)

- Conditional distributions: describe the partonic structure of the target nucleon when the detected hadron is generated from the target remnants.

e.g. Trentadue, Veneziano, Phys. Lett. B323 (1994) 201. Graudenz, Nucl. Phys. B432 (1994) 351
Berera, Soper, Phys.Rev.D 53 (1996) 6162-6179. Collins, Phys. Rev. D57 (1998) 3051
Grazzini, Trentadue, Veneziano, Nucl. Phys. B519 (1998) 394

Factorizations and Kinematic regions of SIDIS

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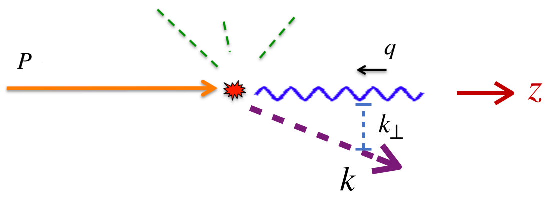
Photon-proton frame:



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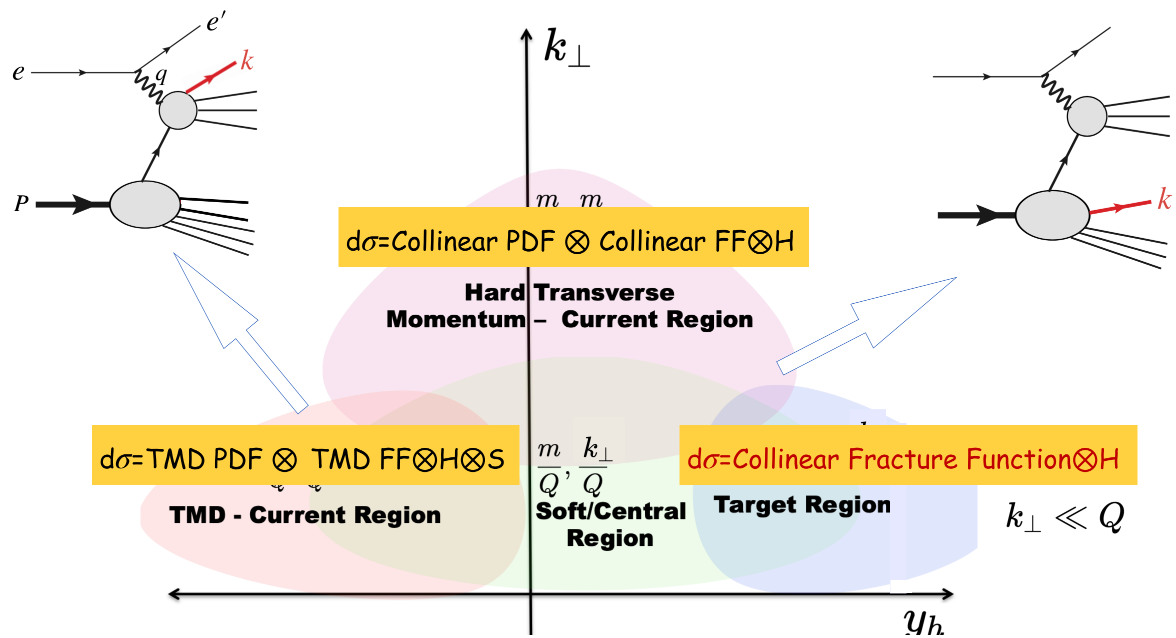


[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

- Two scale $k_\perp, Q$ defines different power expansions

  - Small transverse momentum: $k_\perp \sim M \ll Q$

$$\sigma(Q) = \sigma_2 + \left(\frac{M}{Q}\right)\sigma_3 + \left(\frac{M}{Q}\right)^2 \sigma_4 + \cdots$$

  - Large transverse momentum: $M \ll k_\perp \ll Q$

$$\sigma_2(k_\perp) = \left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right)\sigma_2^{(2)} + \left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right)^2 \sigma_2^{(3)} + \cdots$$

- Factorization theorem differs at different powers.
  1. Improve the predictions
  2. Generate unique physical effects

Higher-twist contributions at Small transverse momentum

- **Available TFR studies focus on the leading power of M/Q.**

$$\sigma(Q) = \sigma_2 + \left(\frac{M}{Q}\right)\sigma_3 + \left(\frac{M}{Q}\right)^2 \sigma_4 + \cdots$$

- What happens at $1/Q,\ 1/Q^2$ levels? $(k_\perp \sim M)$
- Twist-3/4 collinear fracture functions contribute?

Twist-2	$F_{UU,T} = x u_1$ $F_{LL} = x l_{1L}$ $F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^{\perp} \lvert k_\perp \rvert / M$ $F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} \lvert k_\perp \rvert / M$
Higher-twist	$F_{UU}^{\cos\phi_h}, F_{LU}^{\sin\phi_h}, F_{LL}^{\cos\phi_h}, F_{UL}^{\sin\phi_h}$ $F_{LT}^{\cos(2\phi_h-\phi_s)}, F_{UT}^{\sin(2\phi_h-\phi_s)}, F_{UT}^{\sin\phi_s}, F_{LT}^{\sin\phi_s}$

⇒ Only 4 of 18 structure functions at twist-2 at tree level
[Anselmino-Barone-Kotzinian, Phys. Lett. B 699 (2011) 108].

⇒ At least 4 of the other 14 structure functions are already accessible with data collected at CLAS12

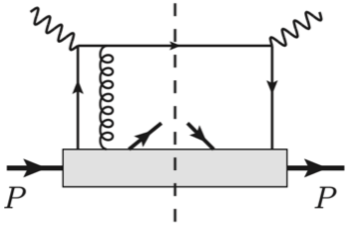
Initial hadron: spin-1/2;
Final hadron: unpolarized

What partonic structures can we learn?

- Twist-3/4 contributions of SIDIS in TFR at small $k_\perp$

$$\sigma(Q) = \sigma_2 + \left(\frac{M}{Q}\right)\sigma_3 + \left(\frac{M}{Q}\right)^2\sigma_4 + \cdots$$

Multi-parton correlations (quantum interference)



$$\sigma_3 \sim \sum_X \langle P, s\,|\bar{\psi}|h, X\rangle\langle X, h|\Gamma D_\perp \psi|\,P, s\rangle$$

$$D_\perp = \partial_\perp + ig_s A_\perp$$

e.g. parton "transverse motion", quark-gluon correlation in presence of a final hadron

Technique

Collinear expansion

—-widely applied in the CFR SIDIS or other hard processes

Higher-twist effects

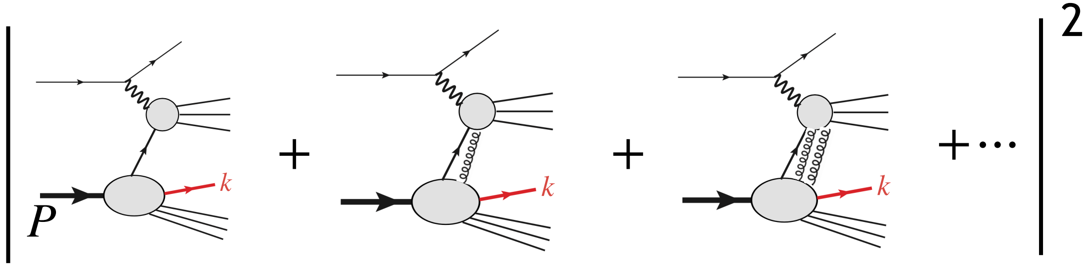
e.g. Ellis-Furmanski-.Petronzio, Nucl.Phys.B21229(1983);
Nucl.Phys.B207.1(1982).
Qiu, Phys.Rev.D42, 30(1990).
Qiu-Sterman, Nucl.Phys.B353, 105(1991);
Nucl.Phys.B353, 137(1991).

Color gauge invariance.

Twist-3 contributions of SIDIS in TFR

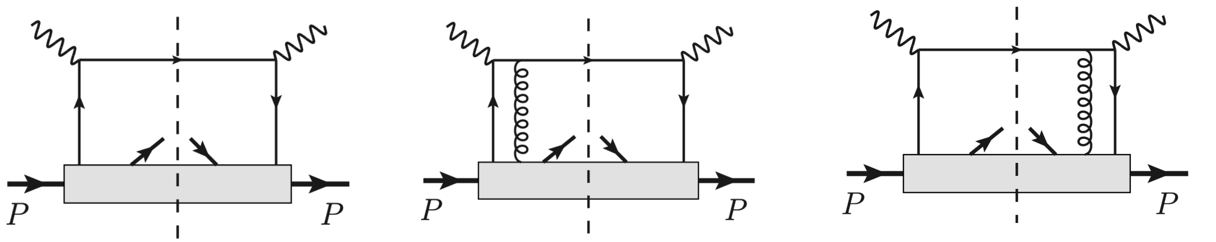
- Differential cross-section

$d\sigma \propto$



$$\frac{d\sigma}{dx_B dy d\xi d\psi d^2k_\perp} = \frac{\alpha_{\rm em}^2 y}{2Q^4} L_{\mu\nu} W^{\mu\nu} \qquad \xi = \frac{k^+}{P^+}$$

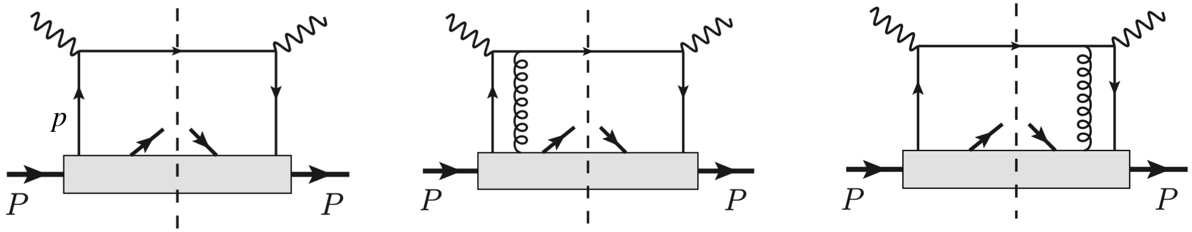
- Hadron tensor(tree level) in TFR



- Partonic scatterings are akin to the inclusive DIS e.g. Z.T. Liang and X.N. Wang (2007)
- Similar observation was given for the twist-2 case to all-order-α_s, see Collins, Phys. Rev. D57 (1998) 3051
- Difference: the correlation matrix, which have a detected hadron!

Analysis

Hadron tensor(tree level) in TFR



- Clear roles of the gluon field at next-leading power of Q:

$$A^{\mu} \sim (\underset{+}{1},\ \underset{-}{\lambda^2},\ \underset{\perp}{\lambda})Q \qquad \lambda = \frac{M}{Q}$$

(1) A^{+}: gauge link only

(2) $A^{\perp}$: "physical" degree , $\sum_X \langle P\,|\bar{\psi}|h, X\rangle\langle X, h|\Gamma A^{\perp}\psi|\,P\rangle \sim \lambda$ —— not gauge invariant

- Quarks are not exactly parallel to the incoming hadron beam, $p^{\mu} \sim (1,\ \lambda^2,\ \lambda)Q$

$$\langle p_{\perp}\rangle \sim \sum_X \langle P\,|\bar{\psi}|h, X\rangle\langle X, h|\Gamma\partial_{\perp}\psi|\,P\rangle \sim \lambda$$

Combined with $A^{\perp}$(Ward identity),

$$\sum_X \langle P, s|\bar{\psi}|h, X\rangle\langle X, h|\Gamma D_{\perp}\psi|P, s\rangle$$

- Other sub-leading contribution from Γ matrix,

$$\sum_X \langle P, s|\bar{\psi}|h, X\rangle\langle X, h|\Gamma\psi|P, s\rangle \qquad \Gamma = \gamma_{\perp}^{\mu}, \gamma_{\perp}^{\mu}\gamma_5$$

Correlations

- After the collinear expansion and organizations,

$$W_{\mu\nu} = W_{\mu\nu}^{(0)} + W_{\mu\nu}^{(1)}$$

$$W_{\mu\nu}^{(0)} = \mathrm{Tr}\left[h_{\mu\nu}^{(0)}\mathcal{M}\right] \ ,$$

$$h_{\mu\nu}^{(0)} = \gamma_\mu\gamma^+\gamma_\nu/2 \ ,$$

$$\mathcal{M}_{ij}(x,\xi,k_\perp) = \frac{1}{2\xi(2\pi)^3}\int\frac{d\xi^-}{2\pi}e^{-ixP^+\xi^-}\sum_X\langle P,s|\bar{\psi}_j(\xi)\mathcal{L}_n^\dagger(\xi)|k;X\rangle\langle k;X|\mathcal{L}_n(0)\psi_i(0)|P,s\rangle\Big|_{\xi^+,\xi_\perp=0}$$

$$W_{\mu\nu}^{(1)} = \mathrm{Tr}\left[h_{\mu\nu,\alpha}^{(1)}\varphi_{ij}^\alpha\right] + \left(\mathrm{Tr}\left[h_{\nu\mu,\alpha}^{(1)}\varphi_{ij}^\alpha\right]\right)^*$$

$$h_{\mu\nu,\alpha}^{(1)} = \frac{1}{4q^-}\gamma_\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma_\nu$$

$$\varphi_{ij}^\alpha(x,\xi,k_\perp) = \frac{1}{2\xi(2\pi)^3}\int\frac{d\xi^-}{2\pi}e^{-ixP^+\xi^-}\sum_X\langle P,S|\bar{\psi}_j(\xi)\mathcal{L}_n^\dagger(\xi)|k;X\rangle\langle k;X|\mathcal{L}_n(0)(-i\overrightarrow{D}_\perp^\alpha)(0)\psi_i(0)|P,S\rangle\Big|_{\xi^+,\xi_\perp=0}$$

- Two types of collinear correlations (Twist-3, chiral even)

$$\mathcal{M}^{[\gamma^\mu]} = \frac{1}{P^+}\left[k_\perp^\mu u^h - s_L\tilde{k}_\perp^\mu u_L^h - \tilde{s}_\perp^\mu M u_T - \frac{k_\perp^{\langle\mu}k_\perp^{\nu\rangle}}{M}\tilde{s}_{\perp\nu}u_T^h\right] ,$$

$$\mathcal{M}^{[\gamma^\mu\gamma_5]} = \frac{1}{P^+}\left[\tilde{k}_{\perp\mu}l^h + s_L k_\perp^\mu l_L^h + s_\perp^\mu M l_T - \frac{k_\perp^{\langle\mu}k_\perp^{\nu\rangle}}{M}s_{\perp\nu}l_T^h\right] ,$$

$$\varphi_\mu^{[\gamma^+]} = k_{\perp\mu}u_d^h - s_L\tilde{k}_{\perp\mu}u_{dL}^h - M\tilde{s}_{\perp\mu}u_{dT} - \frac{k_{\perp\langle\mu}k_{\perp\beta\rangle}}{M}\tilde{s}_\perp^\beta u_{dT}^h \ ,$$

$$\varphi_\mu^{[\gamma^+\gamma_5]} = i\left(\tilde{k}_{\perp\mu}l_d^h + s_L k_{\perp\mu}l_{dL}^h + M s_{\perp\mu}l_{dT} - \frac{k_{\perp\langle\mu}k_{\perp\beta\rangle}}{M}s_\perp^\beta l_{dT}^h\right)$$

Parametrizations are similar to TMD PDFs $(f \to u,\ g \to l,\ \perp \to h)$

$$\tilde{a}^\mu \equiv \epsilon_\perp^{\mu\nu}a_\nu$$

$$k_{\perp\langle\mu}k_{\perp\beta\rangle} \equiv k_{\perp\mu}k_{\perp\nu} - \tfrac{1}{2}k_\perp^2 g_{\mu\nu}^\perp$$

e.g. Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, (2007)

Equation of motion

- However. these distributions are not independent, one can use equation of motion $i\not{D}\psi = 0$ to relate

$$\mathcal{M}^{[\gamma^\mu]} = \frac{1}{P^+}\left[k_\perp^\mu u^h - s_L \tilde{k}_\perp^\mu u_L^h - \tilde{s}_\perp^\mu M u_T - \frac{k_\perp^{\langle\mu}k_\perp^{\nu\rangle}}{M}\tilde{s}_{\perp\nu} u_T^h\right],$$

$$\mathcal{M}^{[\gamma^\mu\gamma_5]} = \frac{1}{P^+}\left[\tilde{k}_{\perp\mu} l^h + s_L k_\perp^\mu l_L^h + s_\perp^\mu M l_T - \frac{k_\perp^{\langle\mu}k_\perp^{\nu\rangle}}{M} s_{\perp\nu} l_T^h\right],$$

$$\varphi_\mu^{[\gamma^+]} = k_{\perp\mu} u_d^h - s_L \tilde{k}_{\perp\mu} u_{dL}^h - M\tilde{s}_{\perp\mu} u_{dT} - \frac{k_{\perp\langle\mu}k_{\perp\beta\rangle}}{M}\tilde{s}_\perp^\beta u_{dT}^h,$$

$$\varphi_\mu^{[\gamma^+\gamma_5]} = i\left(\tilde{k}_{\perp\mu} l_d^h + s_L k_{\perp\mu} l_{dL}^h + M s_{\perp\mu} l_{dT} - \frac{k_{\perp\langle\mu}k_{\perp\beta\rangle}}{M} s_\perp^\beta l_{dT}^h\right)$$

- Unified formulas:

$$x u_S^K = \mathrm{Re}\left(l_{dS}^K - u_{dS}^K\right),$$

$$x l_S^K = \mathrm{Im}\left(l_{dS}^K - u_{dS}^K\right)$$

$S = \text{null},\ L \text{ or } T$

$K = \text{null or } \perp$

- The structure functions can be expressed in terms of the distributions with or without the covariant derivative —> Neat forms

Twist-3 contributions in TFR

- Eight structure functions contribute at twist-3(M/Q) in TFR, which are all missing at twist-2. [Work in preparation, K.B Chen, J.P. Ma, X.B. Tong]

$$F_{UU}^{\cos(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} u(x_B,\xi,k_\perp),$$

$$F_{LU}^{\sin(\phi_h)} = 2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} l(x_B,\xi,k_\perp)$$

$$F_{LL}^{\cos(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} l_L(x_B,\xi,k_\perp)$$

$$F_{UL}^{\sin(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} u_L(x_B,\xi,k_\perp)$$

$$F_{LT}^{\cos(2\phi_h-\phi_s)} = -\sum_a e_a^2\, x_B^2 \frac{|k_\perp^2|}{QM} l_T^h(x_B,\xi,k_\perp)$$

$$F_{UT}^{\sin(2\phi_h-\phi_s)} = -\sum_a e_a^2\, x_B^2 \frac{|k_\perp^2|}{QM} u_T^h(x_B,\xi,k_\perp)$$

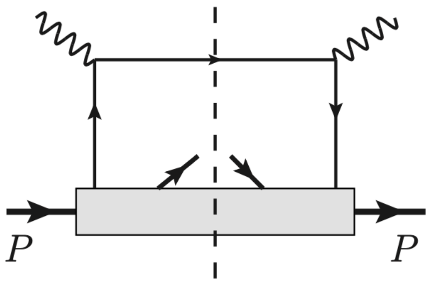
$$F_{UT}^{\sin(\phi_s)} = -2\sum_a e_a^2\, x_B^2 \frac{M}{Q} u_T^h(x_B,\xi,k_\perp)$$

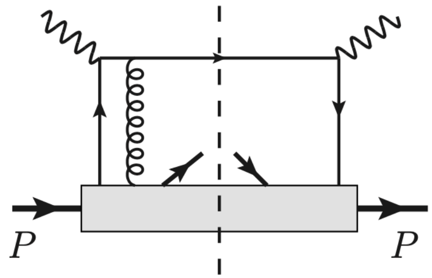
$$F_{LT}^{\sin(\phi_s)} = -2\sum_a e_a^2\, x_B^2 \frac{M}{Q} l_T^h(x_B,\xi,k_\perp)$$

- Probe eight twist-3 collinear quark fracture functions at tree level
  - Two with unpolarized target; Two with longitudinally polarized target; Four with transversely polarized target;
  - Four of them (red, blue) are already accessible with data collected at CLAS12. [see Hayward's talk]
  - Functions without the covariant derivative
  - Have no simple probability interpretation, but are fundamental ingredients in QCD

Twist-4 contributions in TFR

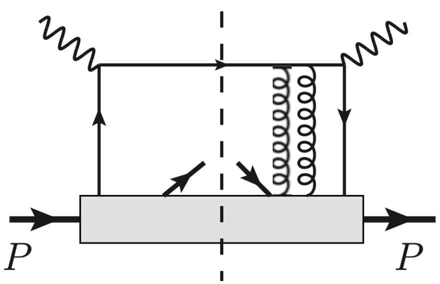
- Correlations

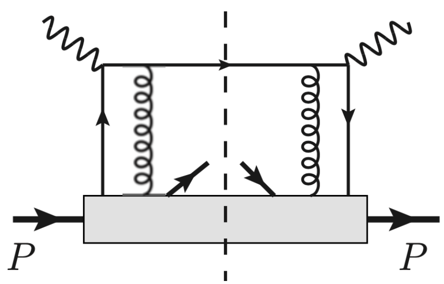




$$\mathcal{M} \sim \sum_X \langle P|\bar{\psi}_j(\xi)\mathcal{L}_n^\dagger(\xi)t|k;X\rangle\langle k;X|\mathcal{L}_n(0)\psi_i(0)|P\rangle$$

$$\varphi_\rho^{(1)} \sim \sum_X \langle P|\bar{\psi}_j(\xi)\overleftarrow{D}_{\rho\perp}\mathcal{L}_n^\dagger(\xi)|k;X\rangle\langle k;X|\mathcal{L}_n(0)\psi_i(0)|P\rangle$$





$$\varphi_{\rho\alpha}^{(2)} \sim \sum_X \langle P|\bar{\psi}_j(\xi)\overleftarrow{D}_{\rho\perp}\overleftarrow{D}_{\alpha\perp}\mathcal{L}_n^\dagger(\xi)|k;X\rangle\langle k;X|\mathcal{L}_n(0)\psi_i(0)|P\rangle$$

$$\varphi_{\rho\alpha}^{(2,M)} \sim \sum_X \langle P|\bar{\psi}_j(\xi)\overleftarrow{D}_{\rho\perp}\mathcal{L}_n^\dagger(\xi)|k;X\rangle\langle k;X|\mathcal{L}_n(0)\overrightarrow{D}_{\alpha\perp}\psi_i(0)|P\rangle$$

- Much more fracture function are involved in the parametrizations at twist-4;
- QCD qquation of motions also provide simplification.

Akin to the TMDs in e.g. S.Y. Wei et al Phys. Rev. D 95, 074017 (2017)

Twist-4 contributions in TFR

- Results up to twist-4

Twist-2	$F_{UU,T} = xu_1 + 4x^2\kappa^2 u_{+3dd}$ $F_{LL} = xl_{1L} + 4x^2\kappa^2 u_{+3ddL}$ $F_{LT}^{\cos(\phi-\phi_S)} = K_\perp \left(xl_{1T}^\perp + 4x^2\kappa^2 u_{+3ddT}^{\perp 3}\right)$ $F_{UT,T}^{\sin(\phi-\phi_S)} = K_\perp \left(xu_{1T}^\perp + 4x^2\kappa^2 u_{+3ddT}^{\perp}\right)$
Twist-4	$F_{UU,L} = 8x^3\kappa^2 u_3$ $F_{UU}^{\cos 2\phi} = -2x^2 K_\perp^2 \kappa^2 u_{-3d}^\perp$ $F_{UL}^{\sin 2\phi} = 2x^2 K_\perp^2 \kappa^2 u_{+3dL}^\perp$ $F_{UT,L}^{\sin(\phi-\phi_S)} = 8x^3 K_\perp \kappa^2 u_{3T}^\perp$ $F_{UT}^{\sin(\phi+\phi_S)} = -x^2 K_\perp^3 \kappa^2 \left(u_{+3dT}^{\perp 4} + u_{-3dT}^{\perp 2}\right)$ $F_{UT}^{\sin(3\phi-\phi_S)} = -x^2 K_\perp^3 \kappa^2 \left(u_{+3dT}^{\perp 4} - u_{-3dT}^{\perp 2}\right)$

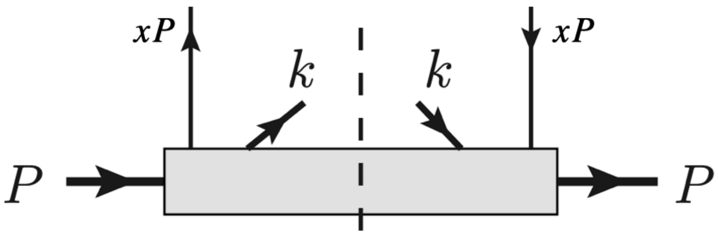
Twist-3
$F_{UU}^{\cos\phi} = -2x^2 K_\perp \kappa u^\perp$
$F_{UL}^{\sin\phi} = -2x^2 K_\perp \kappa u_L^\perp$
$F_{LU}^{\sin\phi} = 2x^2 K_\perp \kappa l^\perp$
$F_{LL}^{\cos\phi} = -2x^2 K_\perp \kappa l_L^\perp$
$F_{UT}^{\sin\phi_S} = -2x^2 \kappa u_T$
$F_{UT}^{\sin(2\phi-\phi_S)} = -x^2 K_\perp^2 \kappa u_T^\perp$
$F_{LT}^{\cos\phi_S} = -2x^2 \kappa l_T$
$F_{LT}^{\cos(2\phi-\phi_S)} = -x^2 K_\perp^2 \kappa l_T^\perp$

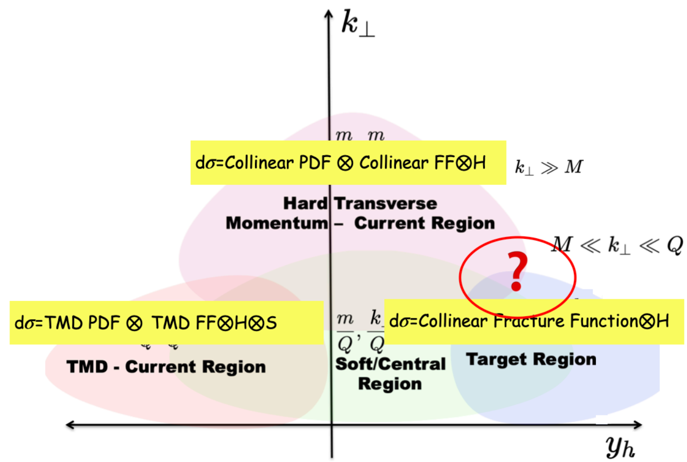
$$K_\perp \equiv \frac{|\vec{k}_\perp|}{M}, \quad \kappa \equiv \frac{M}{Q}$$

- All 18 structure functions are nonzero up to twist-4;
- All leading twist structure functions have twist-4 addenda
- Six structure functions are at twist-4.
- Have twist-4 fracture functions with $D_\perp$
- Twist-3 parts are all separated from twist -2/4 parts

Higher-twist contributions at Large transverse momentum

- Higher-twist effects can also generated at $M \ll k_\perp \ll Q$ in TFR





[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

- At $k_\perp \sim M$, the fracture functions are treated as entirely non-perturbative.
- However, when $M \ll k_\perp \ll Q$, they can be further factorized.
- $k_\perp$-dependence of FrF: perturbatively calculable with expansion in $M/k_\perp$

$$\mathcal{F}(k_\perp) = \left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right)\mathcal{F}_2 + \left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right)^2\mathcal{F}_3 + \cdots$$

Target spin: U/L T

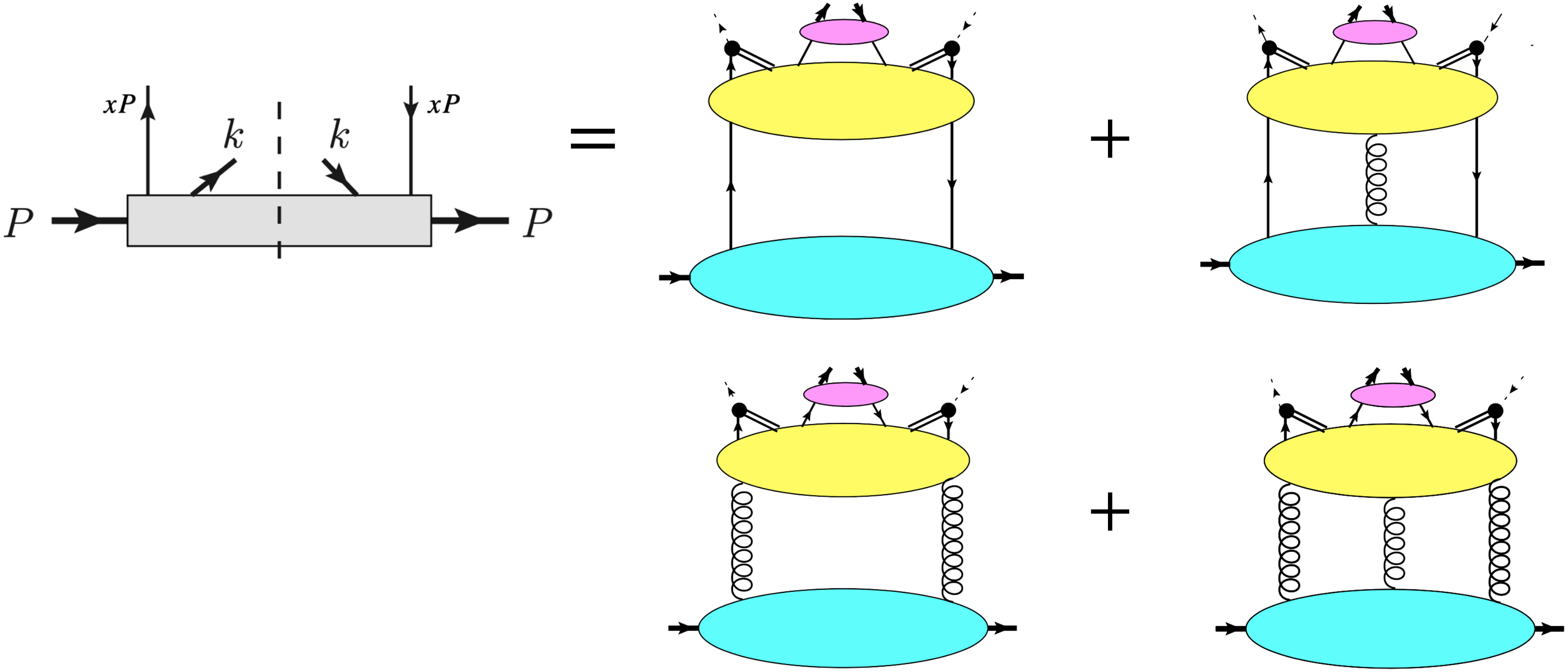
Power behavior depends on the target polarization

Twist-2 collinear fracture function

$$F_{UU,T} = x u_1$$
$$F_{LL} = x l_{1L}$$
$$F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^{\perp} |k_\perp|/M$$
$$F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} |k_\perp|/M$$

Higher-twist contributions at Large transverse momentum

- Higher-twist effects can also generated at $M \ll k_\perp \ll Q$ in TFR



- **Re-factorization at large transverse momentum:**

Collinear Fracture functions≈Collinear (multi)parton distribution⊗H($k_\perp$)⊗ Collinear FFs

[K.B Chen, J.P. Ma, X.B. Tong JHEP 11 (2021) 038]

Target transverse spin effects are presented at Sud-leading Power with Twist-3 PDF .

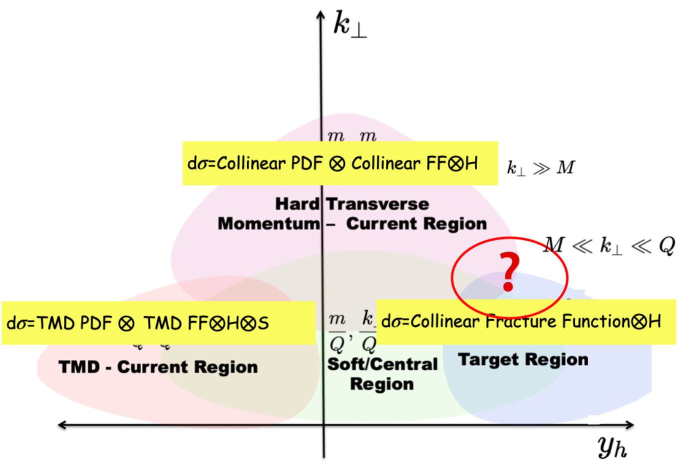
Higher-twist contributions at Large transverse momentum

- Higher-twist effects can also generated at $M \ll k_\perp \ll Q$ in TFR

- **Re-factorization at large transverse momentum:**

Collinear Fracture functions≈Collinear (multi)parton distribution⊗H($k_\perp$)⊗ Collinear FFs

[K.B Chen, J.P. Ma, X.B. Tong JHEP 11 (2021) 038]



[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

- Unify different factorizations in the intermediate $k_\perp$ region;
- Help understand the transitions between different hadron production mechanisms
- Help resum large-logarithm; Build models of fracture functions

Factorizations of the twist-2 collinear FrFs

- Focus on twist-2 quark collinear FrFs

$F_{UU,T} = x u_1$ Unplorized

$F_{LL} = x l_{1L}$ Double spin asymmetry (L, L)

$F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^{\perp} |k_\perp| / M$ Single transverse spin asymmetry(SSA):

$F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} |k_\perp| / M$ Double spin asymmetry(L,T)

- u_1, l_1 : Unpolarized or longitudinal spin

$\frac{1}{k_\perp^2}$ $\otimes$ twist-2 FF $\otimes$ twist-2 parton distributions

- $u_{1T}^h, \, l_{1T}^h$: Transverse spin

$\frac{1}{(k_\perp^2)^2}$ $\otimes$ twist-2 FF $\otimes$ twist-3 multi-parton distributions

Helicity conservation -> No twist-3 FF $\otimes$ twist-2 distributions

Factorizations of the twist-2 collinear FrFs

	$k_\perp \sim M$	$M \ll k_\perp \ll Q$
$F_{UU,T} = x u_1$ $F_{LL} = x l_{1L}$	Twist-2 effect	Twist-2 effect
$F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^\perp \lvert k_\perp \rvert / M$ $F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^\perp \lvert k_\perp \rvert / M$	Twist-2 effect	Twist-3 effect

Factorizations: transversely polarized target

Set of Twist-3 distributions

A pair of quark fields: e.g., Chen-Ma- Zhang, Phys. Lett. B 754 (2016)

$$q_T(x)s_\perp^\mu = P^+ \int \frac{d\lambda}{4\pi} e^{-ix\lambda P^+} \left\langle h_A \middle| \bar{\psi}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma_\perp^\mu \gamma_5 \mathcal{L}_n(0)\psi(0) \middle| h_A \right\rangle$$

$$-iq_\partial(x)s_\perp^\mu = \int \frac{d\lambda}{4\pi} e^{-ix\lambda P^+} \left\langle h_A \middle| \bar{\psi}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma^+ \gamma_5 \partial_\perp^\mu (\mathcal{L}_n\psi)(0) \middle| h_A \right\rangle$$

A pair of quark fields with one gluon field strength tenor(Chirality even):

Qiu-Sterman, Phys. Rev. D 59 (1999) 014004

$$T_F(x_1,x_2)\,\epsilon_\perp^{\mu\nu} s_{\perp\nu} = g_s \int \frac{d\lambda_1 d\lambda_2}{4\pi} e^{-i\lambda_2(x_2-x_1)p^+ - i\lambda_1 x_1 p^+} \langle h_A | \bar{\psi}(\lambda_1 n)\,\gamma^+ G^{+\mu}(\lambda_2 n)\,\psi(0) | h_A \rangle$$

$$T_\Delta(x_1,x_2)\,s_\perp^\mu = -ig_s \int \frac{d\lambda_1 d\lambda_2}{4\pi} e^{-i\lambda_2(x_2-x_1)p^+ - i\lambda_1 x_1 p^+} \langle h_A | \bar{\psi}(\lambda_1 n)\,\gamma_5 \gamma^+ G^{+\mu}(\lambda_2 n)\,\psi(0) | h_A \rangle$$

Factorizations: transversely polarized target

Set of Twist-3 distributions

Set of twist-3 gluon distributions: Ji, Phys. Lett. B 289, 137 (1992).

$$\frac{i^3 g_s}{P^+}\int\frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi}e^{i\lambda_1 x_1 P^+ + i\lambda_2(x_2-x_1)P^+}\langle h_A|G^{a,+\alpha}(\lambda_1 n)G^{c,+\gamma}(\lambda_2 n)G^{b,+\beta}(0)|h_A\rangle$$
$$=\frac{N_c}{(N_c^2-1)(N_c^2-4)}d^{abc}O^{\alpha\beta\gamma}(x_1,x_2)-\frac{i}{N_c(N_c^2-1)}f^{abc}N^{\alpha\beta\gamma}(x_1,x_2),$$

F-type

From the Bose-symmetry and covariance Beppu-Koike-Tanaka-Yoshida, Phys. Rev. D 82 (2010) 054005

$$O^{\alpha\beta\gamma}(x_1,x_2)=-2i\Big[O(x_1,x_2)g_\perp^{\alpha\beta}\tilde{s}_\perp^\gamma+O(x_2,x_2-x_1)g_\perp^{\beta\gamma}\tilde{s}_\perp^\alpha+O(x_1,x_1-x_2)g_\perp^{\gamma\alpha}\tilde{s}_\perp^\beta\Big],$$

$$N^{\alpha\beta\gamma}(x_1,x_2)=-2i\Big[N(x_1,x_2)g_\perp^{\alpha\beta}\tilde{s}_\perp^\gamma-N(x_2,x_2-x_1)g_\perp^{\beta\gamma}\tilde{s}_\perp^\alpha-N(x_1,x_1-x_2)g_\perp^{\gamma\alpha}\tilde{s}_\perp^\beta\Big],$$

All the other twist-3 gluon distributions can be determined by

$$N(x_1,x_2)\qquad O(x_1,x_2)$$

Factorizations: transversely polarized target

$$F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^h(x_B, \xi, k_\perp)|k_\perp|/M$$

- Single transverse-spin asymmetry (T-odd effect)

Time reversal invariance implies that none-zero SSA is induced by the interference of the amplitude of different phase

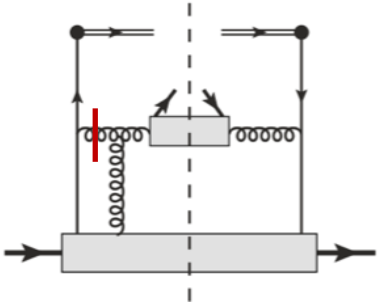
ETQS mechanism

Qiu-Sterman, Phys. Rev. D 59 (1999) 014004
Phys. Rev. Lett. 67 (1991) 2264
Efremov-Teryaev, Phys. Lett. B 150 (1985) 383

- Perturbative way to generate the phase in TFR:

$$\frac{1}{p^2 \pm i\epsilon} = P\frac{1}{p^2} \mp i\pi\delta(p^2)$$

"pole"



Some propagators in FrFs' diagrams should go on shell

- Ward identity
- Constrain the partonic momenta: soft, hard

- Two-parton correlations do not contribute to SSA

SSA: Quark-gluon correlations

Hard pole: $$u_{1T}^h(x_B,\xi,k_\perp)/M\Big|_{HP} = g_s^2\frac{N_c}{(k_\perp^2)^2}\int\frac{dz}{z^2}d_g(z)\frac{z^2}{y}\left(\xi T_\Delta(y,x)-(\xi+2xz)T_F(y,x)\right)$$

$$y = x+\xi/z.$$

Soft-fermion pole: $$u_{1T}^h(x_B,\xi,k_\perp)/M\Big|_{SFP} = g_s^2\frac{1}{N_c}\frac{1}{(k_\perp^2)^2}\int\frac{dz}{z^2}d_g(z)\frac{x\xi z}{y^4}\Big[(xz-\xi)T_F(y,0) -(\xi+xz)T_\Delta(y,0)\Big]$$

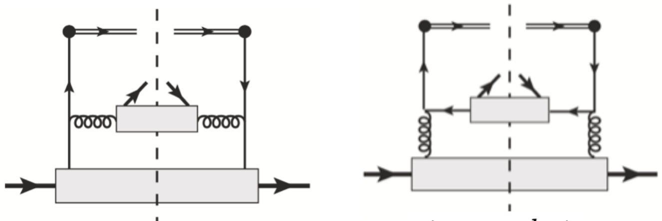
Soft gluon pole: $$u_{1T}^h(x_B,\xi,k_\perp)/M\Big|_{SGP} = \frac{g_s^2 N_c}{(k_\perp^2)^2}\int\frac{dz}{z^2}d_g(z)\frac{1}{y^3}\Big[z^3(y^3+3x^2y-2x^3)T_F(y,y) - y\xi z^2(y^2+x^2)\frac{\partial T_F(y,y)}{\partial y}\Big].$$

One integration on the twist-3 distribution

Factorizations: transversely polarized target

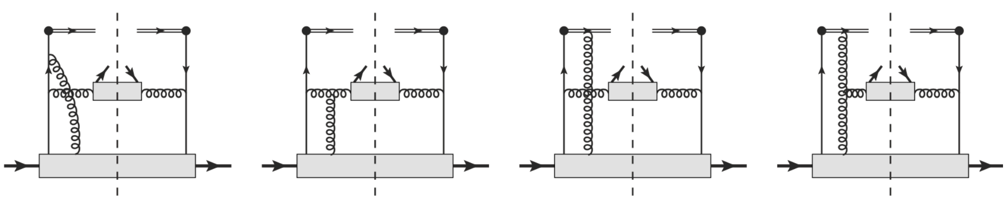
$$F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} |k_\perp|/M$$

- Double spin asymmetry (T-even effect)
  - Do not need a phase to get none-zero results
  - Two-parton correlations contribute

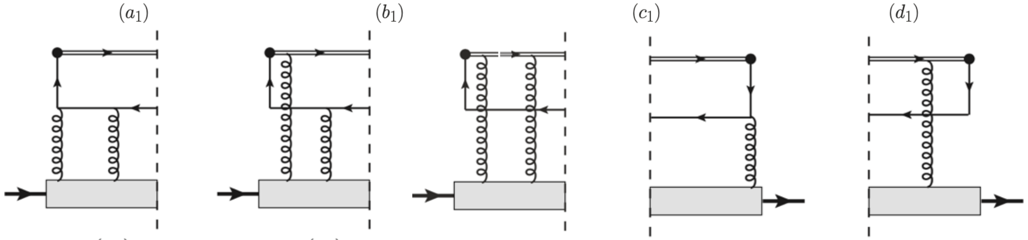


$\langle \psi(0)\bar{\psi}(x) \rangle$ $\quad$ $\langle G^{a,\mu} G^{b,\nu} \rangle$

- A gauge invariant results are obtained with QCD equation of motions



(a_1) $\quad$ (b_1) $\quad$ (c_1) $\quad$ (d_1)



(a_1) $\quad$ (a_2) $\quad$ (b_1) $\quad$ (b_2)

Factorizations: transversely polarized target

DSA: Quark-gluon correlations

$$l_{1T}^h(x,\xi,k_\perp)/M \Big|_{q\bar{q}+qG\bar{q}} = \frac{1}{2(k_\perp^2)^2}\int \frac{dz}{z^2} d_g(z)\Big[\Big(H_{2p,T}(x,\xi)q_T(y) + H_{2p,\partial}(x,\xi)q_\partial(y)\Big) + \frac{2}{\pi}\int dx_2 \Big(T_F(y,x_2)H(x,\xi,x_2) + T_\Delta(y,x_2)H_A(x,\xi,x_2)\Big)\Big],$$

DSA: Pure Gluonic correlations

$$\begin{aligned} l_{1T}^h(x,\xi,k_\perp)/M\Big|_{2G+3G} &= \frac{-16\pi\alpha_s\xi^4}{(k_\perp^2)^2}\int \frac{dz dx_2}{z^2 y^3} d_{\bar{q}}(z)\Big\{\frac{xz}{\pi y}T_F(x_2,x_2+y) - \frac{2xz}{y(y-x_2)}\Big[N(y,x_2) \\ &- N(y-x_2,y) + 2N(y-x_2,-x_2)\Big] + \frac{1}{x_2^2(y-x_2)}\Big(x_2\xi\Big[O(y-x_2,y) \\ &- N(y-x_2,y)\Big] + (2\xi y + yx_2 z - y^2 z - \xi x_2)\Big[N(y,x_2) + O(y,x_2)\Big] \\ &+ y(z(y+x_2) - 2\xi)\Big[N(y-x_2,-x_2) + O(y-x_2,-x_2)\Big]\Big)\Big\}. \end{aligned}$$

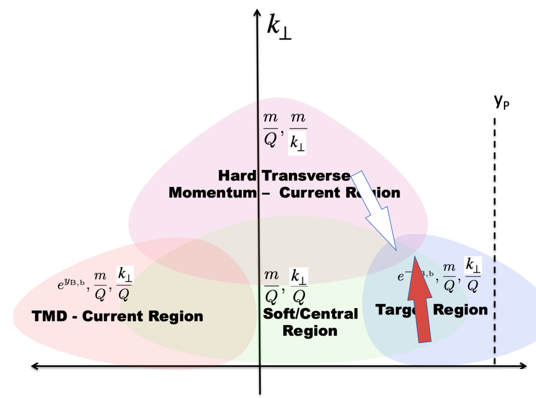
2 Integration on the twist-3 distribution, no derivative term

Summary&Outlook

- **Twist-3/4 contributions of SIDIS in TFR at small $k_\perp$**
  - All 18 structure functions are nonzero up to twist-4; can be expressed in term of twist-3/4 fracture function
  - All leading-twist structure functions have twist-4 addenda
  - Twist-3 parts are all separated from the other .
  - Evolution of the twist-3,4 fracture function?

- **Factorization of twist-2 fracture functions**
  - Studied the $\Lambda_{QCD} \ll k_\perp \ll Q$ for SIDIS in the TFR at tree level.
  - One-loop extension.
  - To full understand the interplay between the target and current regions, further study on the current region at large forward rapidity is needed.
  - Numerical/Phenomenological studies;
  - May help the kinematic regime estimation in SIDIS.

    e.g. M. Boglione et al, arXiv: 2201.12197;JHEP 10 (2019) 122;



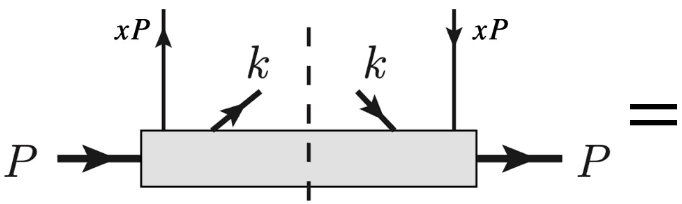
[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

	$k_\perp \sim M$	$M \ll k_\perp \ll Q$
$F_{UU,T} = x u_1$ $F_{LL} = x l_{1L}$	Twist-2 effect	Twist-2 effect
$F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^{\perp} \lvert k_\perp \rvert / M$ $F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} \lvert k_\perp \rvert / M$	Twist-2 effect	Twist-3 effect

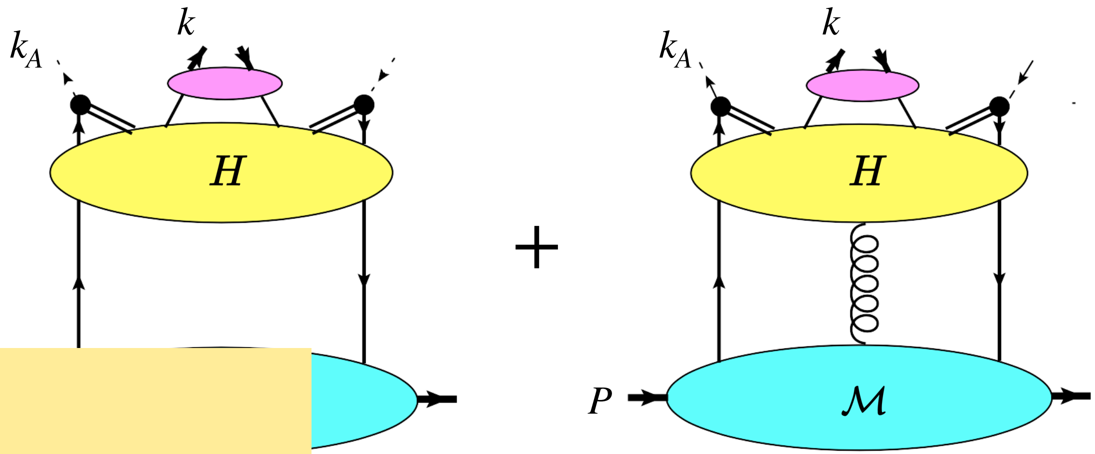
Backup

Transversely polarized Fracture Function

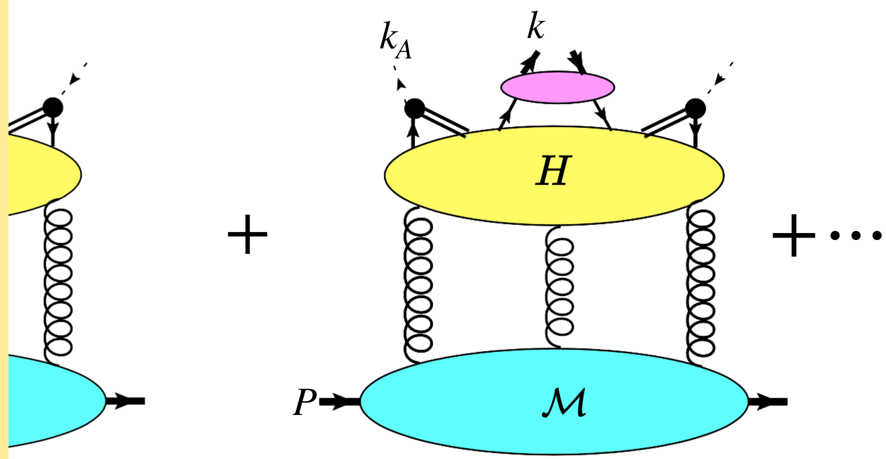
$\mathcal{M}_{Fij}(x, \xi, k_\perp)$



Two-parton correlations | Three-parton correlations



$\langle \psi(0) A_a^\mu(y) \bar{\psi}(x) \rangle$



No twist-3 FF ⊗ twist-2 distributions

Chirality even

twist-2 quark/gluons distributions:

$$\int \frac{dy}{2\pi} e^{-iyxP^+} \langle h_A | \left(\bar{\psi}(yn)\mathcal{L}_n^\dagger(yn)\right)_\beta \left(\mathcal{L}_n(0)\psi(0)\right)_\alpha | h_A \rangle$$
$$= \frac{1}{2N_c}\left[q(x)\gamma^- + s_L \Delta q(x)\gamma_5\gamma^- + h_1(x)\gamma_5 \gamma \cdot s_\perp \gamma^- \right]_{\alpha\beta} + \cdots$$

$$\frac{1}{xP^+}\int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle h_A | \left(G^{+\mu}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\right)^a \left(\mathcal{L}_n(0) G^{+\nu}(0)\right)^a | h_A \rangle$$
$$= -\frac{1}{2} g_\perp^{\mu\nu} g(x) - \frac{i}{2}\epsilon_\perp^{\mu\nu} s_L \Delta g(x) + \cdots .$$

$\langle A_a^\mu(x) A_b^\nu(y) A_c^\sigma(0) \rangle$

Fracture Function

- Density matrix of collinear quark fracture function(FrF)

$$\mathcal{M}_{Fij}\left(x,\xi,k_\perp\right)$$
$$=\int\frac{d\lambda}{2\pi}e^{-ixP^+\lambda}\sum_X\langle h_A(P,s)|[\bar{\psi}(\lambda n)\mathcal{L}_n^\dagger(\lambda n)]_j|Xh_B(k)\rangle\,\langle h_B(k)X\,|[\mathcal{L}_n(0)\psi(0)]_i|\,h_A(P,s)\rangle$$

- Conditional parton distribution;
- "Hybrids" between the PDF and FF
- Kinematics variables

$$x=\frac{p_a^+}{P^+}\quad \xi=\frac{k^+}{P^+}\quad \boldsymbol{k}_\perp$$

- Color gauge invariance

$$\mathcal{L}_n(x)=\mathrm{P}\exp\left\{-ig_s\int_0^\infty d\lambda G^+(\lambda n+x)\right\}$$

- Same evolution as the PDF—DGLAP Eq

- $k_\perp$integrated FrF: Trentadue-Veneziano Phys. Lett. B 323 (1994) 201
- $k_\perp$un-integrated FrF: Berera-Soper Phys. Rev. D 53 (1996) 6162
- TMD quark fracture function: Anselmino-Barone-Kotzinian, Phys. Lett. B
- Other applications: hadron collisions

e.g. Chen-Ma-Tong, JHEP 10 (2019) *285*

Ceccopieri -Trentadue, Phys. Lett. B 668 (2008) 319

Ceccopieri, Phys. Lett. B 703 (2011) 491