Fracture Function Formalism: longitudinal target asymmetries

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Outlook

- **LO (twist-2) nucleon structure in SIDIS**
- **Non-perturbative inputs Spin and Transverse Momentum Dependent (STMDs)**
  - Parton Distribution Functions in nucleon:
    - SIDIS, DY => STMD PDFs
  - Parton Fragmentation Functions STMD FF:
    - Hadron production in e^+e^- annihilation (SIA), SIDIS, high p_T hadron production in pp collisions
  - STMD Fracture Functions to describe hadron production in the target fragmentation region (TFR) of SIDIS

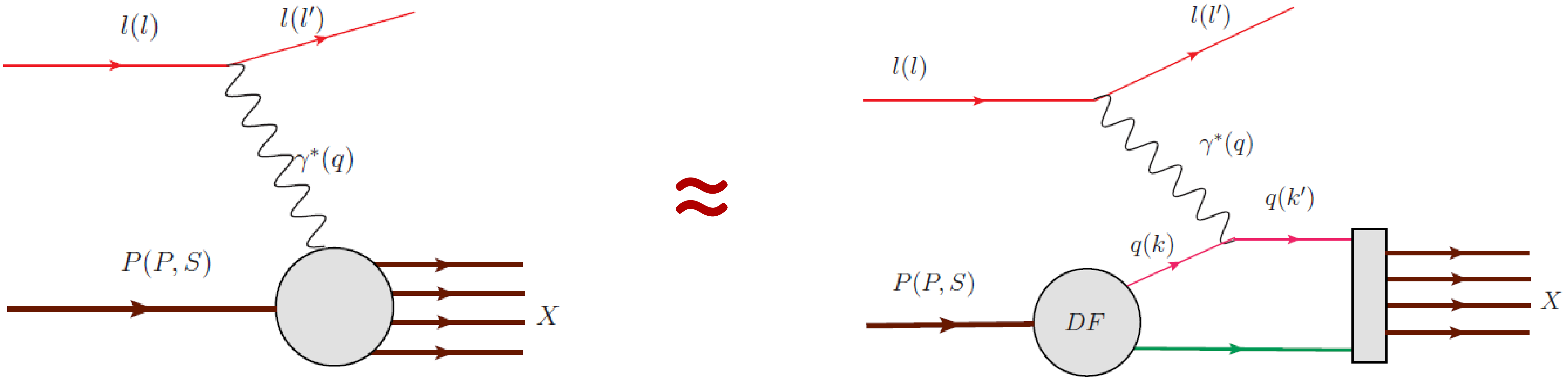
Twist-2 TMD PDFs

Nucleon Polarization \ Quark polarization	U	L	T
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij} k_T{}^j}{M} h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{[\mathbf{k}_T \times \mathbf{S}_T]_3}{M} f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2) + \frac{\mathbf{k}_T}{M} \frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x,k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

pQCD factorization: DIS

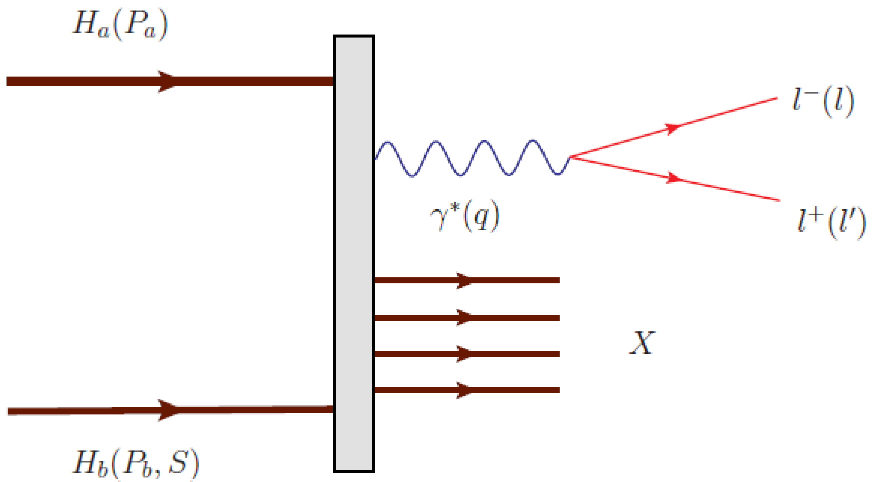
At large $Q^2 = -q^2$ the DIS can be described using QED lepton quark scattering cross section and nonperturbative input – colinear PDF $f_1^q(x)$: $d\sigma^{lN\to lhX} \sim \sum_q f_q\left(x, k_T^2\right) \otimes d\sigma^{lq\to l'q'}$



Access to nucleon unpolarized $f_1^{q+\bar{q}}(x)$ and longitudinally polarized $g_1^{q+\bar{q}}(x)$

leading twist colinear (transverse momentum integrated) PDFs

pQCD TMD factorization: DY processes



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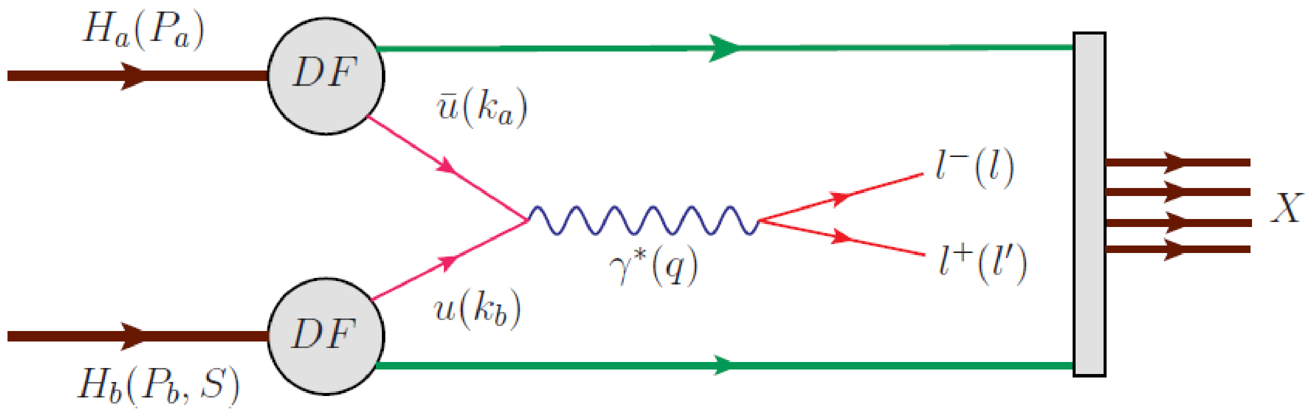
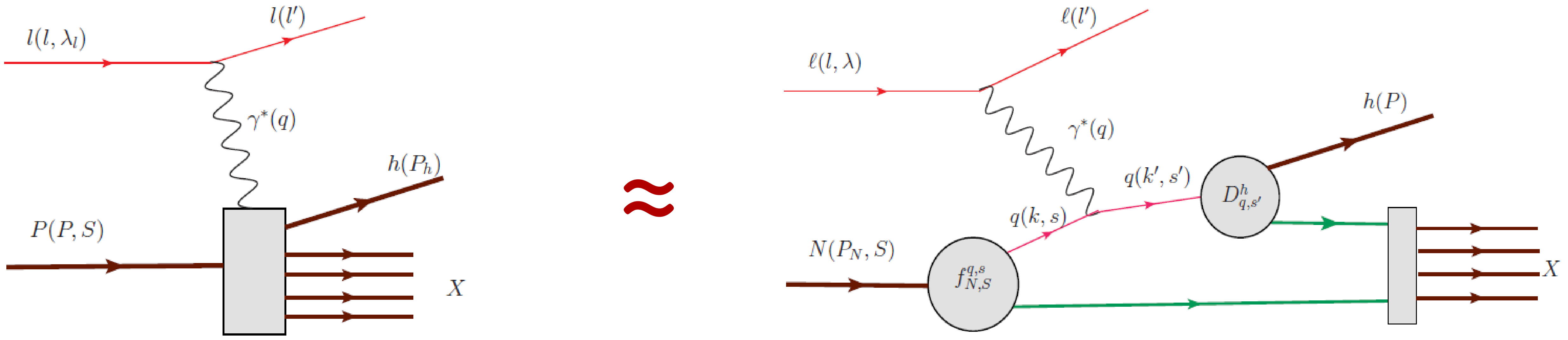


We can access to nucleon, pion and kaon TMD PDFs

$f_1(x, k_T^2)$, $g_1(x, k_T^2)$, $h_1(x, k_T^2)$ and $h_1^{\perp}(x, k_T^2)$ leading twist PDFs

if we do not intergrate over transverse momentum of virtual photon

pQCD TMD factorization: SIDIS in CFR



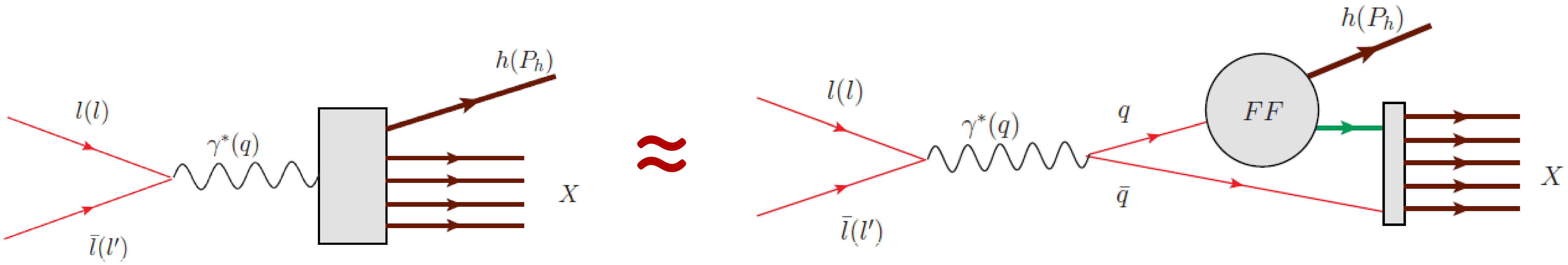
$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

Access to nucleon PDFs $f_1^q(x,k_T^2)$, $g_1^q(x,k_T^2)$ and $h_1^q(x,k_T^2)$, ... leading twist TMD PDFs.

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s'}_T}{m_h} H_1(z,p_T^2)$$

Measured in e^+e^- semi-inclusive annihilation (SIA) to 2 back-to-back jets $e^+e^- \to h_1h_2 + X$

QCD TMD factorization in semi-inclusive e+e- annihilation (SIA)



Access to $q+\bar{q}$ fragmentation finctions $D^h_{q+\bar{q}}(z, p^2_\perp)$

Two hadron production in opposite hemispheres: acces to Collins FF $H^h_{1q}(z, p^2_\perp)$

LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

$$\times\left[\begin{array}{l} F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+\\ S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+\\ S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\sin(\phi_h-\phi_S)+D_{nn}(y)\left(\begin{array}{l}F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+\\ F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+\\ \lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\end{array}\right]$$

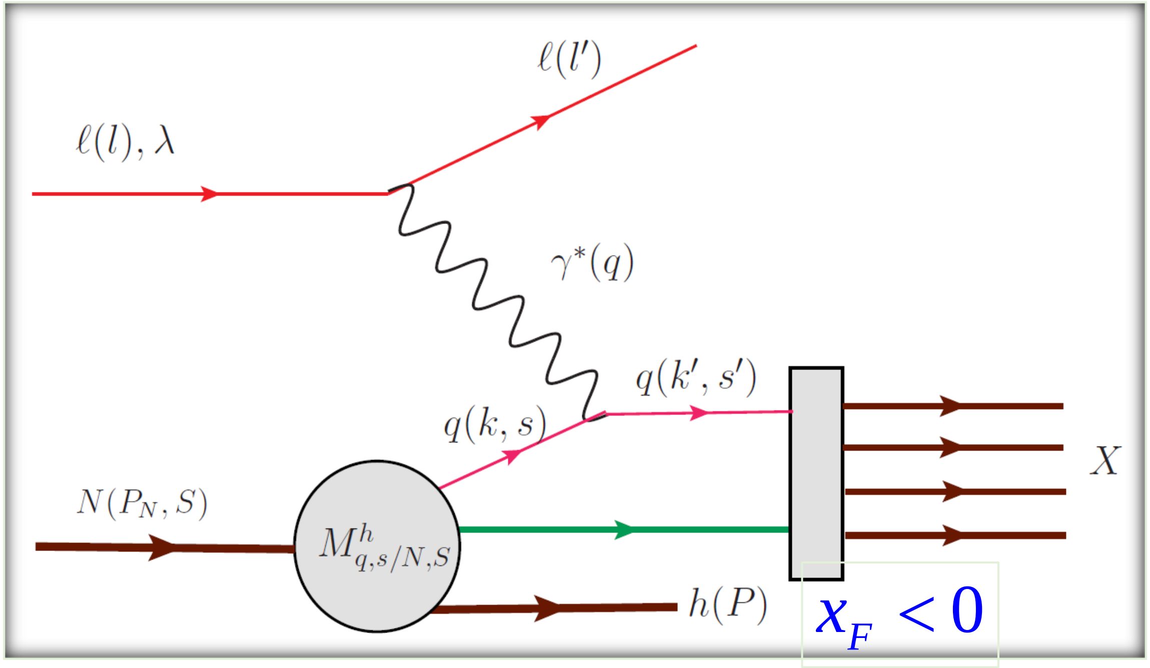
Sivers

Collins

$$D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2},\qquad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}$$

At LO only 8 terms contributes out of 18 Structure Functions entering in the general expression of SIDIS cross section
6 azimuthal modulations, 4 terms are generated by Collins effect in fragmentation

SIDIS: TFR



Trentadue, Veneziano 1994
Graudenz 1994
Collins 1998, 2000, 2002
de Florian, Sassot 1997, 1998
Grazzini, Trentadue, Veneziano 1998
Ceccopieri, Trentadue 2006, 2007, 2008
Sivers 2009
Ceccopieri , Mancusi 2013
Ceccopieri 2013
……..

$$\frac{d\sigma^{\ell(l)+N(P_N)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\zeta} = M^h_{q/N}\left(x,Q^2,\zeta\right)\otimes\frac{d\sigma^{\ell(l)+q(k)\to\ell(l')+q(k')}}{dQ^2},$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Fracture function *M* is a Conditional Probability Distribution Function (CPDF) to observe the hadron h produced in target nucleon momentum direction in γ^*P CMS when hard probe interacts with parton carrying fraction x of nucleon momentum.

Collinear Frac.Func.: application to HERA data, 1

D. de Florian, R. Sassot, Leading Proton Structure Function. PRD 58, 054003 (1998)

$$ep \to e'p'X: \quad \frac{d^3\sigma^p_{target}}{d\beta dQ^2 dx_{\mathrm{P}}} = \frac{4\pi\alpha^2}{\beta Q^4}\left(1-y+\frac{y^2}{2}\right) M_p^h\left(\beta, Q^2, x_{\mathrm{P}}\right), \quad \beta = \frac{x}{1-\zeta}, \quad \zeta = \frac{p_h^+}{p_N^+} \quad x_{\mathrm{P}} = \zeta$$

$$x M_q^{p/p}(\beta, Q_0^2, x_{\mathrm{P}}) = N_s \beta^{a_s}(1-\beta)^{b_s}\{C_{\mathrm{P}}\beta x_{\mathrm{P}}^{\alpha_{\mathrm{P}}} + C_{\mathrm{LP}}(1-\beta)^{\gamma_{\mathrm{LP}}}[1 + a_{\mathrm{LP}}(1-x_{\mathrm{P}})^{\beta_{\mathrm{LP}}}]\}$$

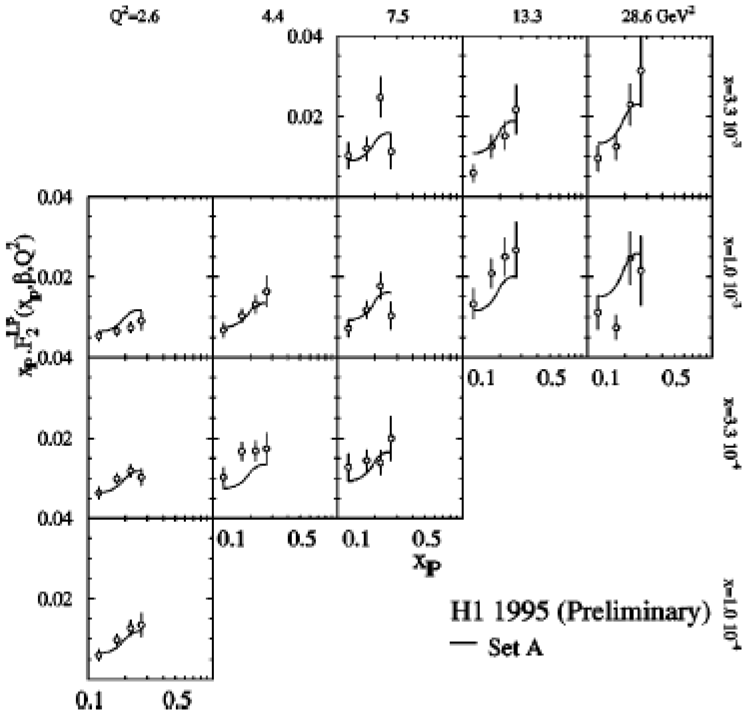


FIG. 2. H1 leading-proton data against the outcome of the fracture function parametrization (solid lines).

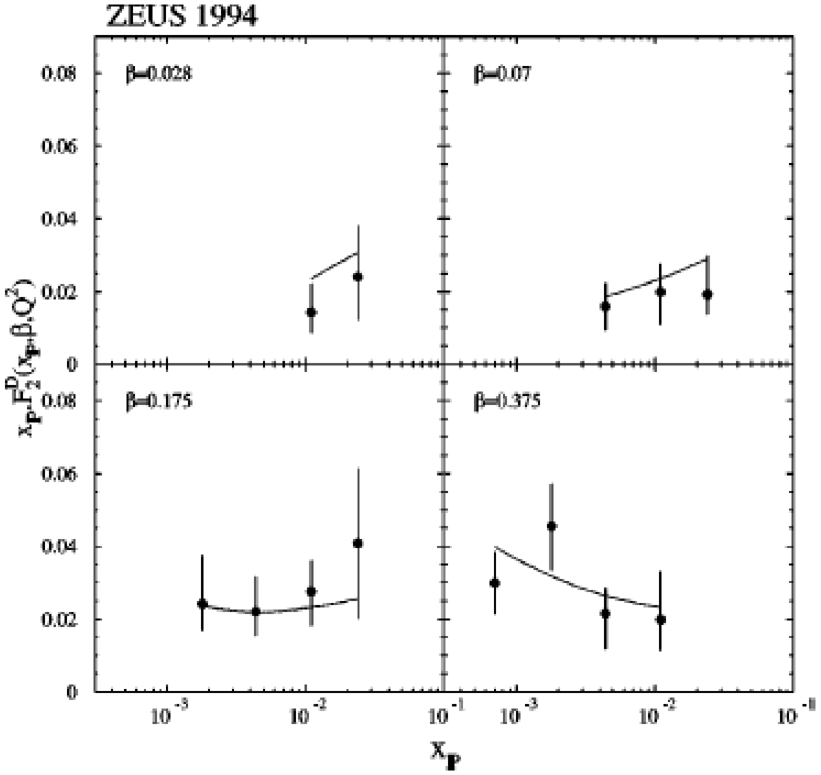
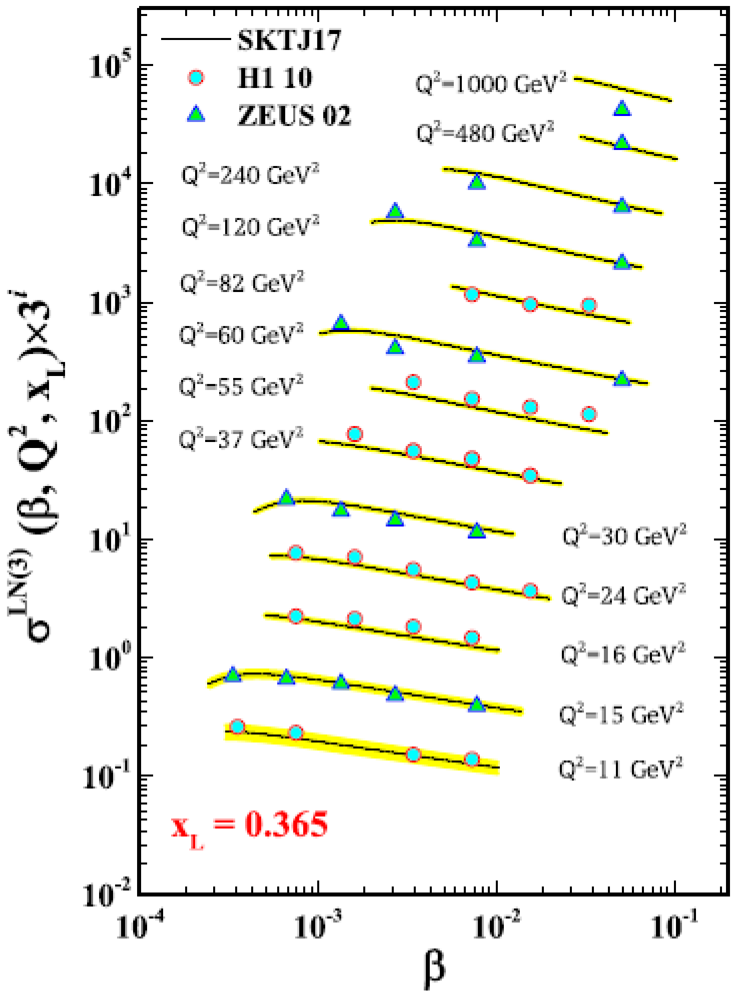


FIG. 8. ZEUS diffractive data, against the expectation coming from the fracture function parametrization (fit A).

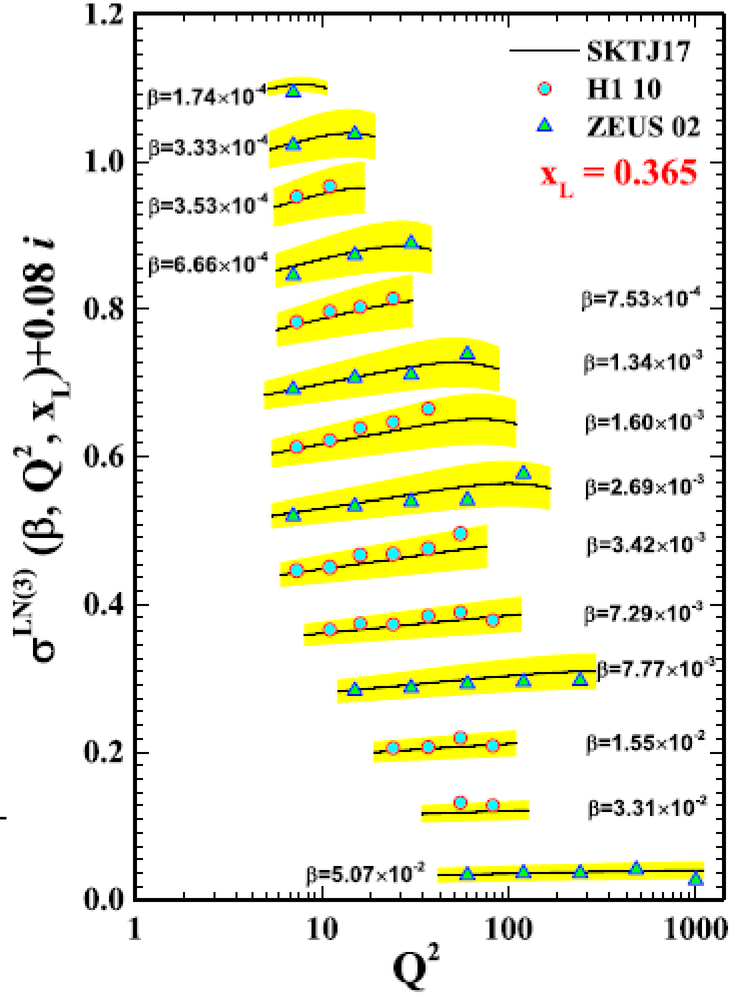
Collinear Frac.Func.: application to HERA data, 2

Shoeibi *et al*, Neutron fracture functions. PRD 95, 074011 (2017)

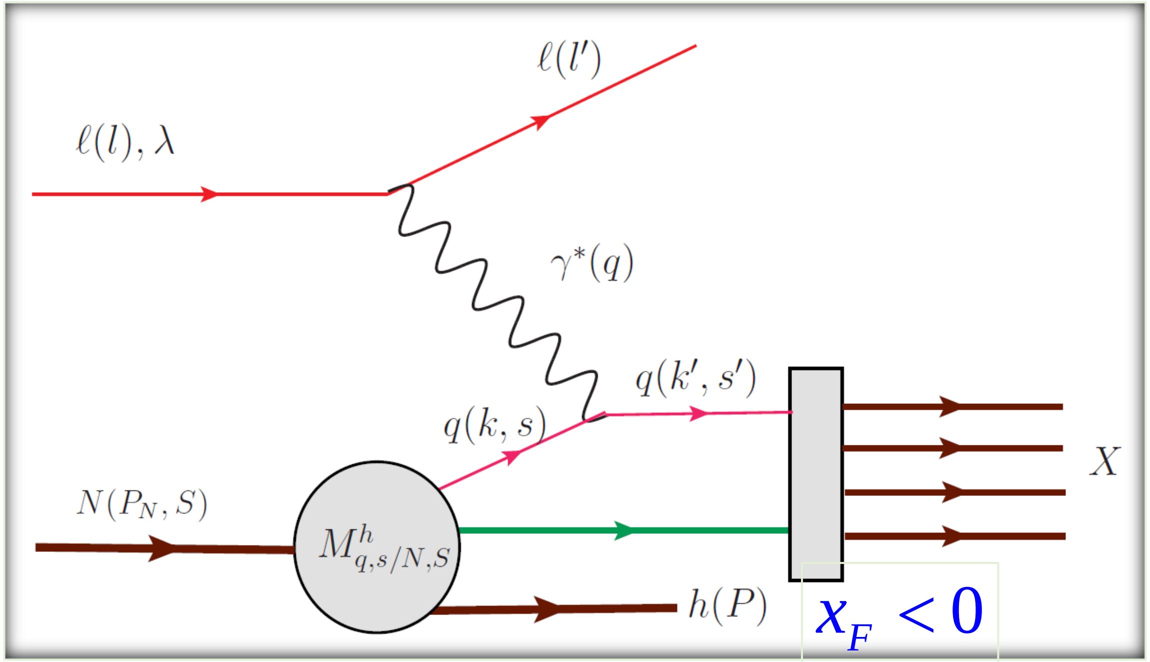
$$ep \to enX$$



$$x_L \simeq \frac{E_h}{E_P}, \quad \beta = \frac{x}{1-x_L}$$



SIDIS TFR: Spin & TMD (STMD) Fracture Functions



Anselmino, Barone and AK, **PL B 699** (2011)108; **706** (2011)46; **713** (2012)317

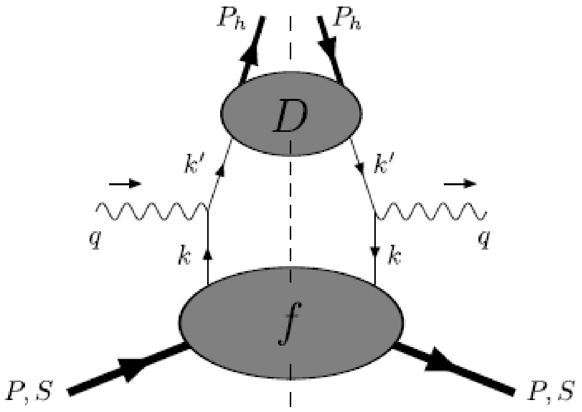
Nucleon and quark polarization are included, produced hadron and quark transverse momentum are not integrated over. Classification of twist-two Fracture Functions and cross sections expressions.

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T)\otimes\frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

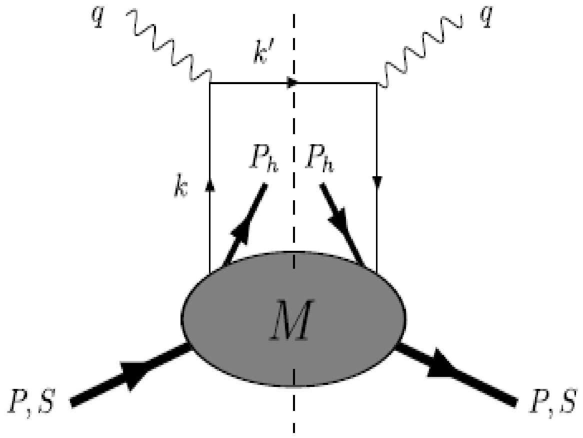
$$\mathbf{k}_T\cdot\mathbf{P}_T = k_T P_T\cos(\phi_h-\phi_q),\qquad \zeta=\frac{P^-}{P_N^-}\approx x_F(1-x)$$

Quark correlator

SIDIS



CFR



TFR

$$\mathcal{M}^{[\Gamma]}(x_B,\vec{k}_\perp,\zeta,\vec{P}_{h\perp}) = \frac{1}{4\zeta}\int\frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6}e^{i(x_B P^-\xi^+ - \vec{k}_\perp\cdot\vec{\xi}_\perp)}\sum_X\int\frac{d^3P_X}{(2\pi)^3 2E_X}\times$$

$$\times\langle P,S|\bar{\psi}(0)\Gamma|P_h,S_h;X\rangle\langle P_h,S_h;X|\psi(\xi^+,0,\vec{\xi}_\perp)|P,S\rangle$$

$$\Gamma = \gamma^-,\quad \gamma^-\gamma_5,\quad i\sigma^{i-}\gamma_5$$

Probabilistic interpretation at LO:

the conditional probabilities to find an unpolarized ($\Gamma = \gamma^-$), a longitudinally polarized ($\Gamma = \gamma^-\gamma_5$) or a transversely polarized ($\Gamma = \sigma^{i-}\gamma_5$) quark with longitudinal momentum fraction x_{Bj} and transverse momentum $\boldsymbol{k}_\perp$ inside a nucleon fragmenting into a hadron carrying a fraction ζ of the nucleon longitudinal momentum and a transverse momentum $\boldsymbol{P}_{h\perp}$.

STMD Fracture Functions for spinless hadron production

Nucleon Polarization \ Quark polarization	U	L	T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N\, m_h}\hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^{\,j}}{m_h}\hat{t}_1^{h} + \frac{\epsilon_T^{ij} k_T^{\,j}}{m_N}\hat{t}_1^{\perp}$
L	$\frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N\, m_h}\hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h}\hat{t}_{1L}^{h} + \frac{S_L \mathbf{k}_T}{m_N}\hat{t}_{1L}^{\perp}$
T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h}\hat{u}_{1T}^{h} + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N}\hat{u}_{1T}^{\perp}$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h}\hat{l}_{1T}^{h} + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N}\hat{l}_{1T}^{\perp}$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T(\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2}\hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T(\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2}\hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T(\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h}\hat{t}_{1T}^{\perp h}$

At twist-2 there are 16 independent Fracture Functions depending on quark and TFR hadron momenta

$$x,\, k_T^2,\, \zeta,\, P_T^2,\, k_T P_T \cos(\phi_h - \phi_q)$$

Azimuthal dependences for different nucleon and quark polarizations appears not only in prefactors, as it was in the case of SIDIS in CFR, but also in the argument of fracture functions

The terms which contains the same prefactors as in SIDIS in CFR are marked in red

STMD Fracture Functions for spinless hadron production

		Quark polarization		
		U	L	T
Nucleon Polarization	U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N\, m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^{\ j}}{m_h} \hat{t}_1^{h} + \frac{\epsilon_T^{ij} k_T^{\ j}}{m_N} \hat{t}_1^{\perp}$
	L	$\frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N\, m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^{h} + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^{\perp}$
	T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^{h} + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^{\perp}$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^{h} + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^{\perp}$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T(\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T(\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T(\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

$\hat{u}_1 \rightarrow$ unintegrated twist-2 fracture functions

U, L, T subscrupts $\rightarrow$ unpolarized, longitudinal and transversely polarized nucleon

$\perp, h \rightarrow$ dependence on transverse momenum of quark and produced hadron

Quark transverse momentum integrated Fracture Functions

In single hadron production in TFR NO access to final quark transverse momentum and polarization

Quark transverse momentum integrates fracture functions market by tilde:

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{u}_1\left(x_B,k_T^2,\zeta,P_{T1}^2,\mathbf{k}_T\cdot\mathbf{P}_{T1}\right)$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{u}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^{\perp}\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{l}_{1L}$$

$$\tilde{l}_{1T}^h(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{l}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^{\perp}\right\}$$

LO cross-section of single hadron production in TFR

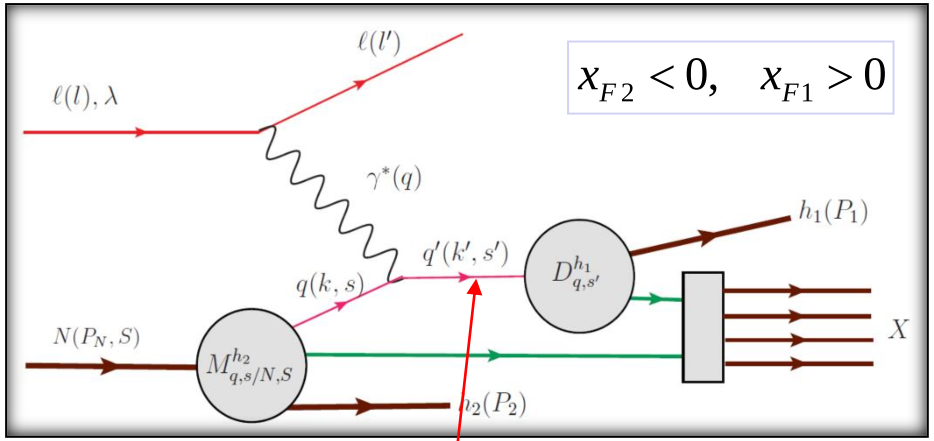
$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = \frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2 \times$$

$$\times\left[\begin{array}{l}\tilde{u}_1\left(x,\zeta,P_T^2\right) - S_T \boxed{\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)} + \\ \lambda y(2-y)\left(S_L \tilde{l}_{1L}\left(x,\zeta,P_T^2\right) + S_T\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\end{array}\right]$$

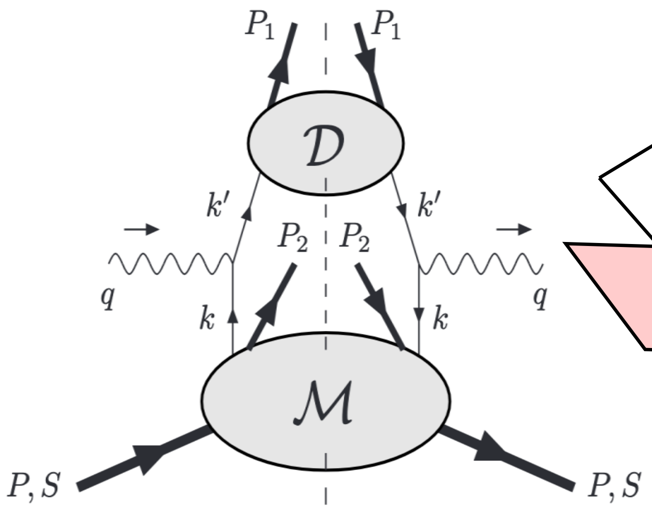
At LO (twist 2) only 4 terms out of 18 Structure Functions in SIDIS,
Only 2 azimuthal modulations

In single hadron production in TFR NO access to final quark transverse momentum and polarization → No Collins-like $\sin(\phi_h+\phi_S)$ modulation

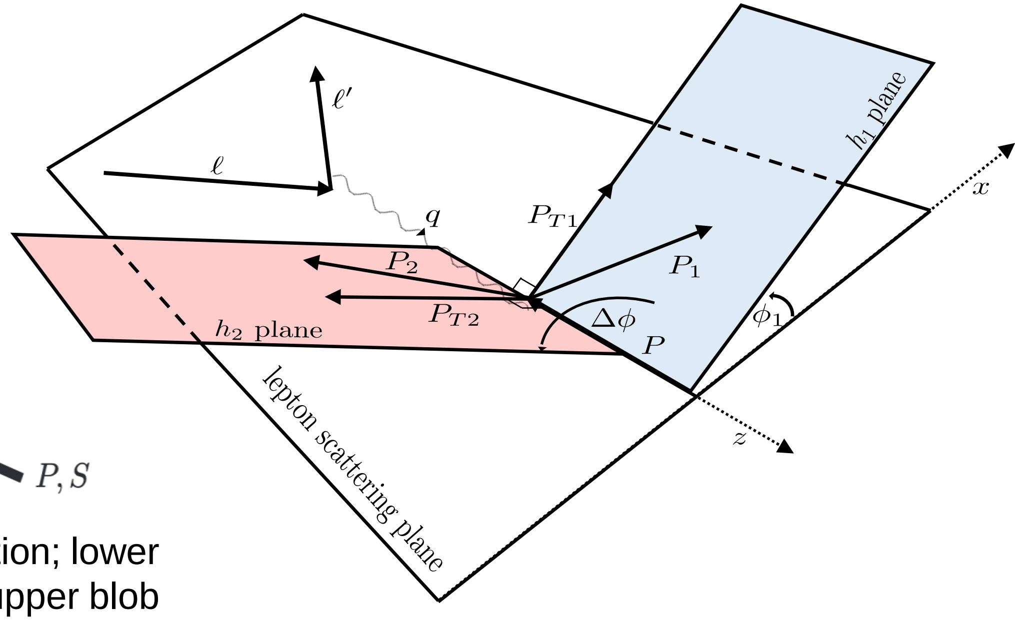
Double hadron production in DIS (DSIDIS): TFR & CFR



Access to final quark transverse momentum and polarization



Handbag diagram for dihadron production; lower blob contains Fracture Functions and upper blob contains the FFs.



$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2), \quad \mathbf{s}'_T \text{ - fragmenting quark transverse polarization}$$

Unintegrated DSIDIS LO cross-section: accessing quark polarization

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\hat{u}^{h_2}\otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{l}^{h_2}\otimes D_1^{h_1} + \hat{t}^{h_2}\otimes\frac{\mathbf{p}_T\times\mathbf{s}'_T}{m_{h_1}}H_1^{h_1}\right)$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{LT}\right)\right)$$

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms as in 1h SIDIS cross section.

$$D_{ll}(y) = \frac{y(2-y)}{1+(1-y)^2}$$

Back-to-back two hadrons production provides access to all 16 twist-2 k_T-unintegrated fracture functions (see additional slides)

DSIDIS azimuthal modulations

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1}\cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1}\cos(\phi_1+\phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right)\cos(2\phi_2) \right)$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D \frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2} \right]$$

$$C[\hat{M}\cdot D w] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1})\hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2})D_a(z,p_T^2)w$$

Structure functions $F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi),\ \text{with } \Delta\phi = \phi_1 - \phi_2$$

A_{LU} asymmetry, 1

Anselmino, Barone and AK, PLB **713** (2012) 317

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1} \cos(\phi_1+\phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right) \cos(2\phi_2) \right)$$

Nucleon Polarization \ Quark polarization	U	L	T
U		*	
L			
T			

Nucleon Polarization \ Quark polarization	U	L	T
U	*		*
L			
T			

$F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi)$, with $\Delta\phi = \phi_1 - \phi_2$

Choosing as independent angles $\Delta\phi$ and ϕ_2 $(\phi_1 = \Delta\phi + \phi_2)$

and integrating σ_{UU} over ϕ_2 we eliminate all terms

proportional to $D_{NN} \Rightarrow$

A_{LU} asymmetry, 2

$$\mathcal{A}_{LU} = -\frac{y(1-\frac{y}{2})}{(1-y+\frac{y^2}{2})}\frac{\mathcal{F}_{LU}^{\sin\Delta\phi}}{\mathcal{F}_{UU}}\sin\Delta\phi$$

$$= -\frac{|\boldsymbol{P}_{1\perp}||\boldsymbol{P}_{2\perp}|}{m_N m_2}\frac{y(1-\frac{y}{2})}{(1-y+\frac{y^2}{2})}\frac{\mathcal{C}[w_5\hat{l}_1^{\perp h}D_1]}{\mathcal{C}[\hat{u}_1 D_1]}\sin\Delta\phi,$$

$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = -\frac{P_{T1}P_{T2}}{m_2 m_N}\frac{F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x,z,\zeta,P_{T1}^2,P_{T2}^2,\cos(\Delta\phi)\right)}{F_0^{\hat{u}\cdot D_1}\left(x,z,\zeta,P_{T1}^2,P_{T2}^2,\cos(\Delta\phi)\right)}\sin(\Delta\phi)$$

Expected leading-twist asymmetry is proportional to $\sin(\Delta\phi)$ and $P_{T1}P_{T2}$

A_{LU} @ CLAS12, (1)

H. Avakian, T. B. Hayward and A. Kotzinian et al, CLASS Collaboration. arXiv:2208.05086v1 [hep-ex] to be published in PRL

- Observed non-zero asymmetries are the first experimental confirmation of possible spin-orbit correlations between hadrons produced simultaneously in the CFR and TFR.
- Observed linear dependence on the product of transverse momenta is consistent with expectations.

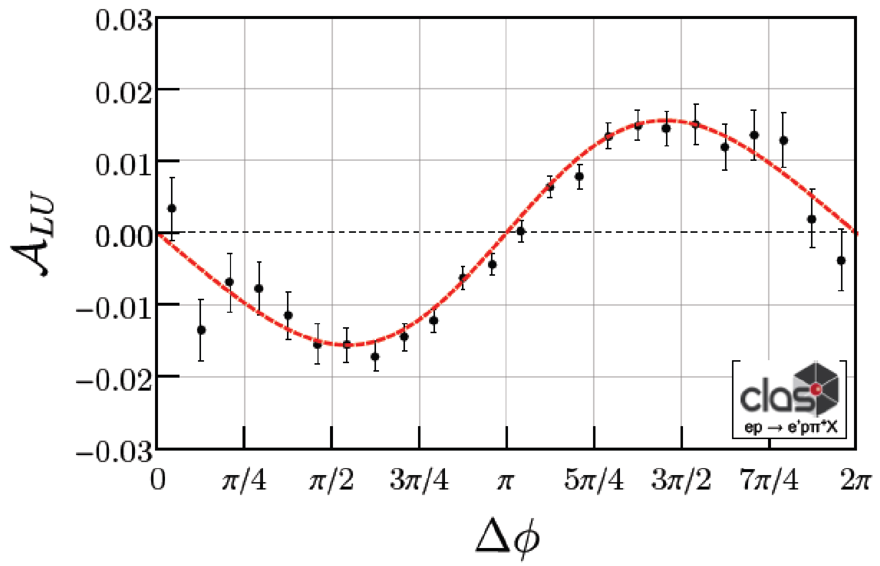


FIG. 2. The beam spin asymmetry, $\mathcal{A}_{LU}$, as a function of $\Delta\phi$ and integrated over all other kinematics for the entire data set. A clear $\sin(\Delta\phi)$ dependence is observed with small $\sin(2\Delta\phi)$ contributions.

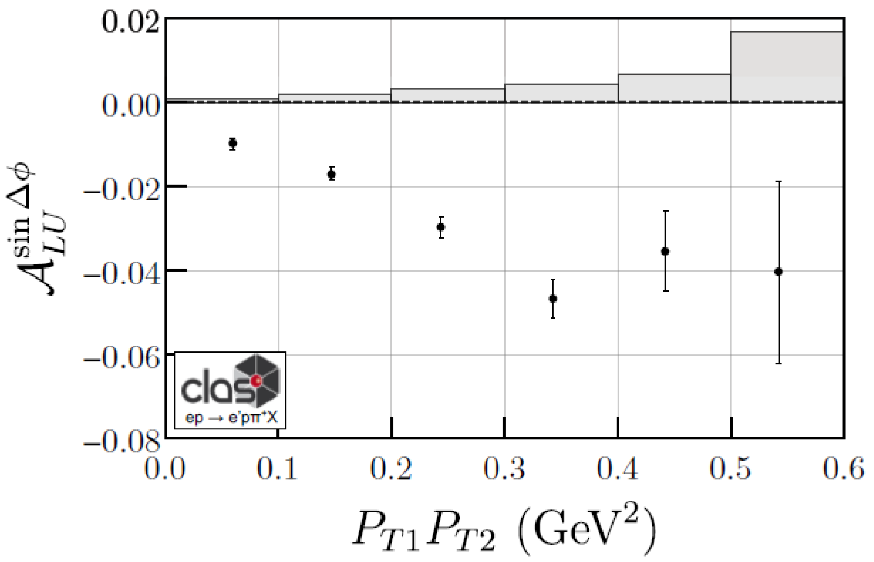


FIG. 3. The measured $\mathcal{A}_{LU}^{\sin\Delta\phi}$ asymmetry as a function of $P_{T1}P_{T2}$. Thin black bars indicate statistical uncertainties and wide gray bars represent systematic uncertainties.

Unpolarized beam and longitudinally polarized target: σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$+D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1} \right) \sin(2\phi_2) \right)$$

More complicated azimuthal dependence compared to σ_{LU}.
But here also, as for σ_{UU}, after integration over ϕ_2 at fixed $\Delta\phi = \phi_1 - \phi_2$ only the first contribution survives.

Nucleon Polarization \ Quark polarization	U	L	T
U		*	*
L			
T			

Unpolarized beam and longitudinally polarized target: A_{UL}

$$A_{UL} = \frac{\int d\phi_2 \sigma_{UL}}{\int d\phi_2 \sigma_{UU}} = -\frac{P_{T1} P_{T2}}{m_2 m_N} \frac{F_{k1}^{\hat{u}_{1L}^{\perp h} \cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)}{F_0^{\hat{u} \cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)} \sin(\Delta\phi)$$

Very similar expression to A_{LU} but gives access to another fracture function

JLab A_{LU} and A_{UL} double hadron production provide access to fracture functions in empty boxes for SIDIS reactions

Nucleon Polarization	Quark polarization: U	Quark polarization: L	Quark polarization: T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^j}{m_h} \hat{t}_1^h + \frac{\epsilon_T^{ij} k_T^j}{m_N} \hat{t}_1^{\perp}$
L	$\frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^h + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^{\perp}$
T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^h + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^{\perp}$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^h + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^{\perp}$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T(\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T(\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T(\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

Conclusions

- Frac. Funs: A new members of the polarized TMDs family -- 16 LO STMD fracture functions
- For hadron produced in the TFR of SIDIS, only 4 k_T-integrated fracture functions of unpolarized and longitudinally polarized quarks are accessible at twist-two
  - SSA contains only a Sivers-type modulation $\sin(\phi_h-\phi_S)$ but no Collins-type $\sin(\phi_h+\phi_S)$ or $\sin(3\phi_h-\phi_S)$. The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long-range correlations between the struck quark polarization and P_T of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms. Access to all 16 STMD fracture functions.
  - The first b2b σ_{LU} asymmetry measurement at JLAB12 shows significant effect
- Polarized SIDY cross section ($p + p \to l^+l^- + h + X$) at LO contains 2 azimuthal independent, 20 lepton plane azimuthal angle independent and 52 lepton plane azimuthal angle dependent terms. In total – 74 terms. Access to all 16 STMD fracture functions. See additional slides.