



WPCF 2023 - XVI Workshop on Particle Correlations and  
Femtoscopia & IV Resonance Workshop 2023

**Charm and Bottom quarks dynamics in Heavy-Ion Collisions: anisotropic flows  $v_n$  and their correlations with Event-Shape Engineering technique.**

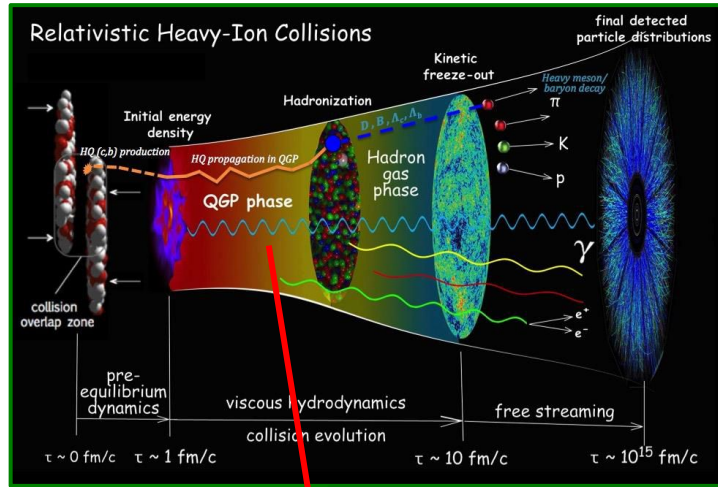
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**In collaboration with: Y. Sun, V. Minissale, S. Plumari, V. Greco**

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INFN - Laboratori Nazionali del Sud (LNS)

# Basic scales of charm and bottom quarks



One of the main probes to signal the Quark-Gluon Plasma properties:  
**Heavy Quarks**

Charm  $M_c \approx 1.3$  GeV and Bottom  $M_b \approx 4.2$  GeV

- $m_{c,b} \gg \Lambda_{QCD}$   
pQCD initial production
- $m_{c,b} \gg T_{RHIC,LHC}$   
negligible thermal production
- $\tau_0 < 0,08$  fm/c  $\ll \tau_{QGP}$
- $\tau_{th} \approx \tau_{QGP} \gg \tau_{g,q}$

They experience the full evolution of the QGP.

They carry more informations with respect to their light counterparts.

Initial production  
 $\tau_0 < 0.1$  fm/c

Dynamics in  
QGP

B, D,  $\Lambda_c$

b, c

$\bar{b}, \bar{c}$

$\bar{B}, \bar{D}, \bar{\Lambda}_c$

Adapted from  
Rapp & Greco

Hadronization:  
Final hadron  
Spectra and  
observables

Reviews:

1. X.Dong, V. Greco Prog. Part. Nucl. Phys. 104 (2019),
2. A.Andronic EPJ C76 (2016), 3) R.Rapp, F.Prino J.Phys. G43 (2016)

# **CATANIA MODEL: QUASI-PARTICLE MODEL AND TRANSPORT THEORY**

**$R_{AA}, v_n$  and  $v_n - v_m$  correlations**  
in charm sector

# Quasi Particle Model (QPM) fitting IQCD

**Non perturbative dynamics**  $\rightarrow$  M scattering matrices (q,g  $\rightarrow$  Q)  
evaluated by Quasi-Particle Model fit to **IQCD thermodynamics**

$N_f=2+1$   
Bulk:  
u,d,s

$$m_g^2(T) = \frac{2N_c}{N_c^2 - 1} g^2(T) T^2$$

$$m_q^2(T) = \frac{1}{N_c} g^2(T) T^2$$



**Thermal masses of gluons  
and light quarks**

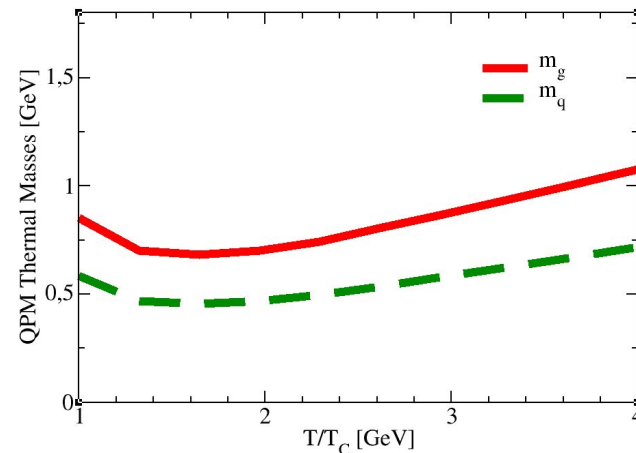
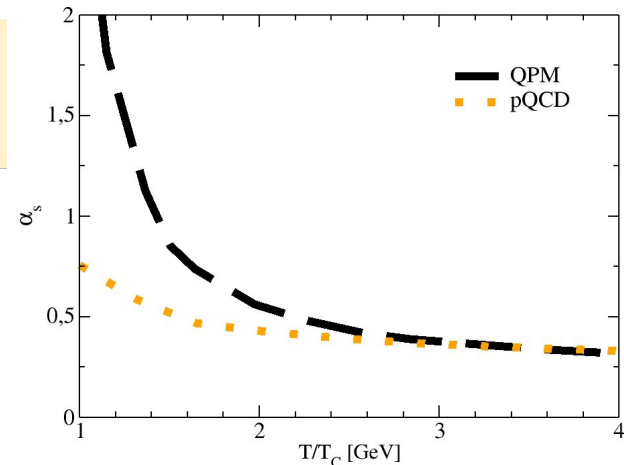
$g(T)$  from a fit to  $\epsilon$  from IQCD data  $\rightarrow$  good reproduction of P,  $\epsilon$ -3P

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left[ \lambda \left( \frac{T}{T_c} - \frac{T_s}{T_c} \right) \right]^2}$$

$$\lambda=2.6$$

$$T_s=0.57 T_c$$

Larger than pQCD especially as  $T \rightarrow T_c$



# Relativistic Boltzmann equation at finite $\eta/s$

## Bulk evolution

$$p^\mu \partial_\mu f_q(x, p) + m(x) \partial_\mu^x m(x) \partial_p^\mu f_q(x, p) = C[f_q, f_g]$$

$$p^\mu \partial_\mu f_g(x, p) + m(x) \partial_\mu^x m(x) \partial_p^\mu f_g(x, p) = C[f_q, f_g]$$

Free-streaming

field interaction

$$\varepsilon - 3p \neq 0$$

Collision term  
gauged to some  $\eta/s \neq 0$

Equivalent to  
viscous hydro at  $\eta/s \approx 0.1$

## HQ evolution

$$p^\mu \partial_\mu f_Q(x, p) = C[f_q, f_g, f_Q]$$

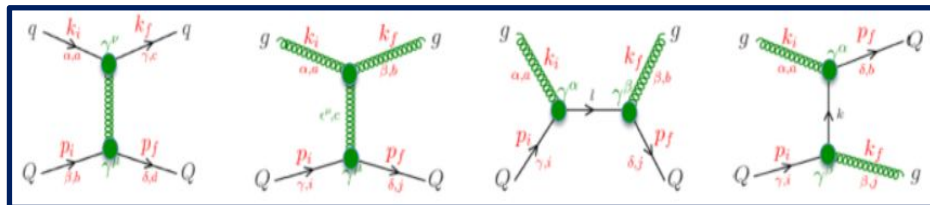
$$C[f_q, f_g, f_Q] = \frac{1}{2E_1} \int \frac{d^3 p_2}{2E_2 (2\pi)^3} \int \frac{d^3 p_1'}{2E_1' (2\pi)^3}$$

$$\times [f_Q(p_1') f_{q,g}(p_2') - f_Q(p_1) f_{q,g}(p_2)]$$

$$\times |M_{(q,g) \rightarrow Q}(p_1 p_2 \rightarrow p_1' p_2')|$$

$$\times (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2')$$

Feynman diagrams at first order pQCD for HQs-bulk interaction:



Scattering matrices  $M_{g,q}$  by QPM fit to IQCD thermodynamics

**HADRONIZATION: hybrid Coalescence + fragmentation**

For details: S. Plumari talk

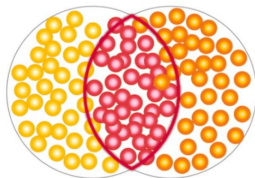
# Information from non-equilibrium: anisotropic flows $v_n(p_T)$

M.L. Sambaturo et al., *Eur.Phys.J.C* 82 (2022) 9, 833

$$E \frac{d^3N}{dp_T} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left\{ 1 + \sum_{i=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)] \right\}$$

## Elliptic flow $v_2$

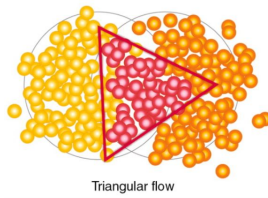
- asymmetry between the in-plane and out-of-plane directions



Elliptic flow

## Triangular flow $v_3$

- event-by-event fluctuations in the initial distributions of nucleons



Triangular flow

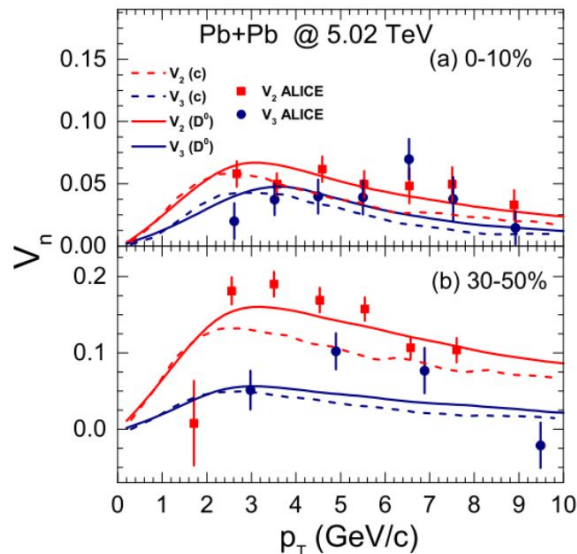
$$e_n = \frac{\langle r_{\perp}^n \cos[n(\varphi - \Phi_n)] \rangle}{\langle r_{\perp}^n \rangle} \quad \Phi_n = \frac{1}{n} \arctan \frac{\langle r_{\perp}^n \sin(n\varphi) \rangle}{\langle r_{\perp}^n \cos(n\varphi) \rangle}$$

$$r_{\perp} = \sqrt{x^2 + y^2}, \quad \varphi = \arctan(y/x)$$



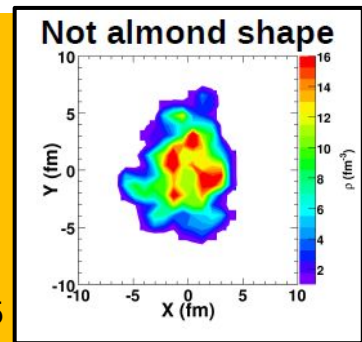
Azimuthal anisotropies depend on

- the interaction and coupling of heavy quarks with the medium;
- the initial conditions of the system, i.e. geometry of the collision;
- the fluctuations in the distributions of nucleons and gluons within the nuclei



Monte Carlo Glauber for initial condition of partons

S.Plumari et al, *Phys.Rev.C* 92 (2015) 5



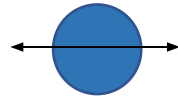


# Event-Shape-Engineering technique

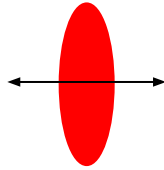
Selection of events with the **same centrality** but different **initial geometry** on the basis of the magnitude of the second-order harmonic reduced flow vector  $q_2$ .

$$q_2 = |\vec{Q}_2|/\sqrt{M}$$

$$\vec{Q}_2 = \sum_{j=1}^M e^{i2\varphi_j}$$



20 % small  $q_2$

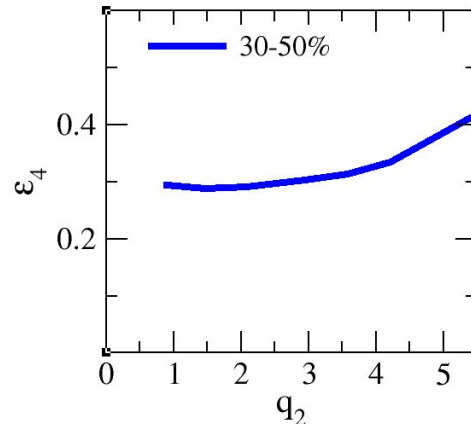
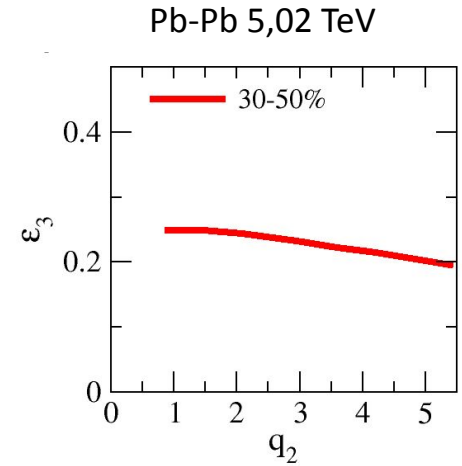
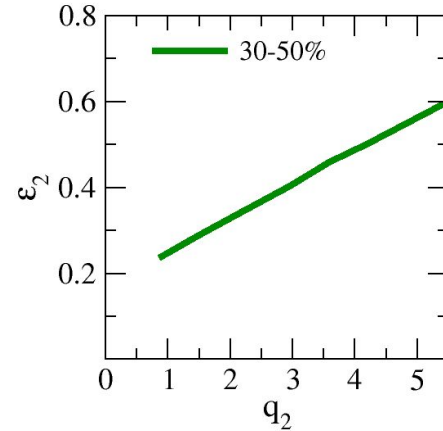


20 % large  $q_2$

Large  $q_2 \rightarrow$  large  $\epsilon_2$

$$\epsilon_n = \frac{\langle r_{\perp}^n \cos[n(\varphi - \Phi_n)] \rangle}{\langle r_{\perp}^n \rangle} \quad \Phi_n = \frac{1}{n} \arctan \frac{\langle r_{\perp}^n \sin(n\varphi) \rangle}{\langle r_{\perp}^n \cos(n\varphi) \rangle}$$

$$r_{\perp} = \sqrt{x^2 + y^2}, \quad \varphi = \arctan(y/x)$$

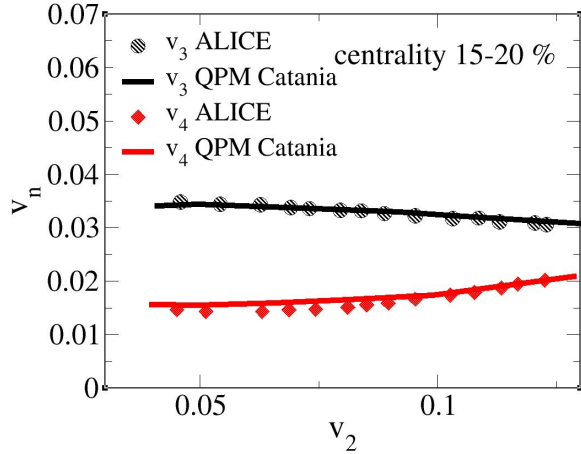


Anti-correlation between  $\epsilon_2$  and  $\epsilon_3$

Non-linear correlation between  $\epsilon_2$  and  $\epsilon_4$

# ESE: $v_n - v_m$ correlations

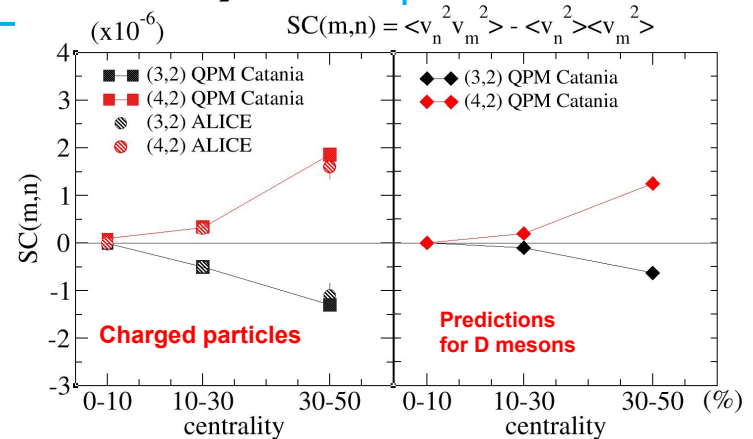
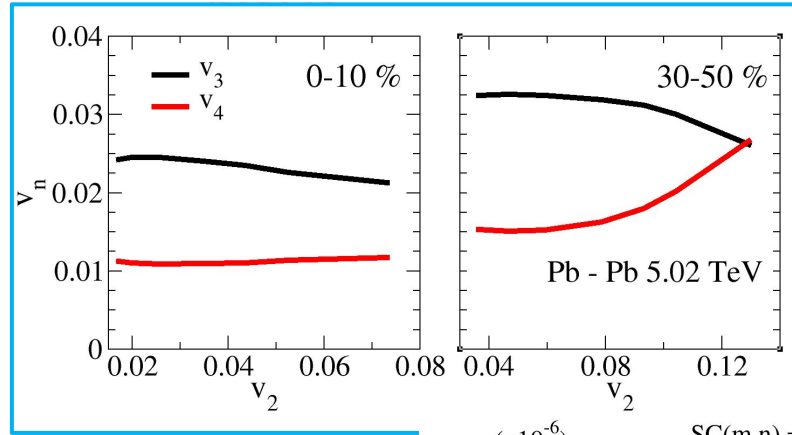
## Charged particles



Correlations between the  $\epsilon_n$  and  $\epsilon_m$  present in the initial geometry  $\rightarrow$  correlations between flow harmonics different orders, i.e. correlations  $v_n$  and  $v_m$

- $\triangleright$  Good description of  $v_n - v_m$  correlation for bulk
- $\triangleright$  Prediction for similar and weaker correlation between soft and hard particles

## Predictions for D



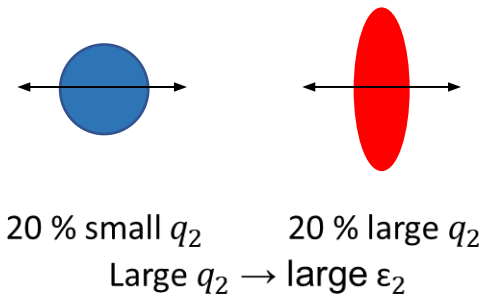


# Event-shape-engineering

Selection of events with the **same centrality** but different **initial geometry** on the basis of the magnitude of the second-order harmonic reduced flow vector  $q_2$ .

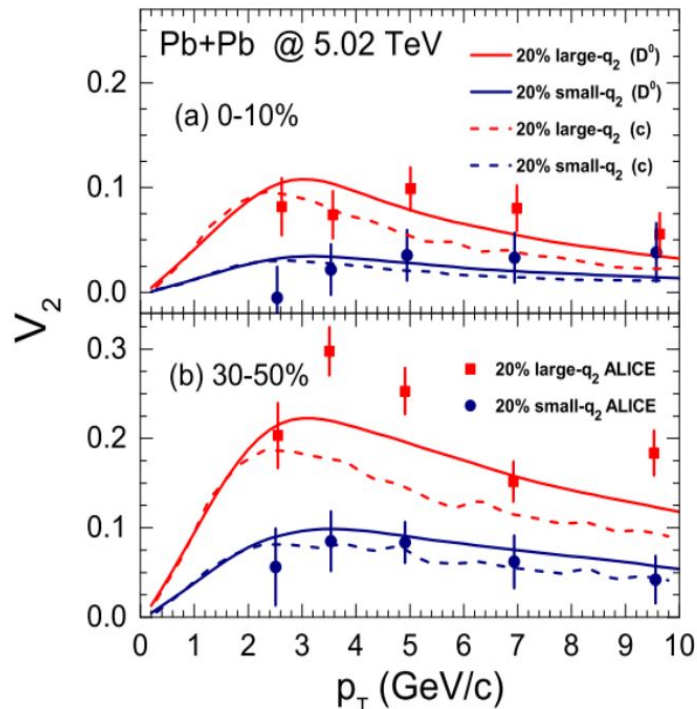
$$q_2 = |\vec{Q}_2|/\sqrt{M}$$

$$\vec{Q}_2 = \sum_{j=1}^M e^{i2\phi_j}$$



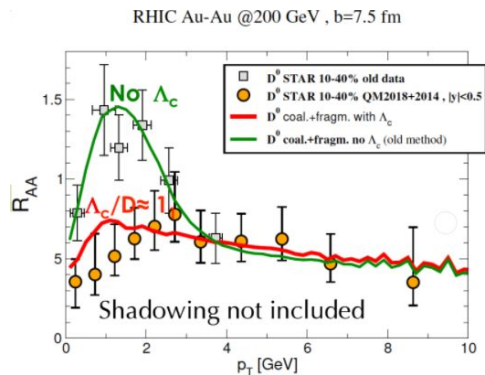
$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)] \rangle}{\langle r_\perp^n \rangle} \quad \Phi_n = \frac{1}{n} \arctan \frac{\langle r_\perp^n \sin(n\varphi) \rangle}{\langle r_\perp^n \cos(n\varphi) \rangle}$$

$$r_\perp = \sqrt{x^2 + y^2}, \quad \varphi = \arctan(y/x)$$

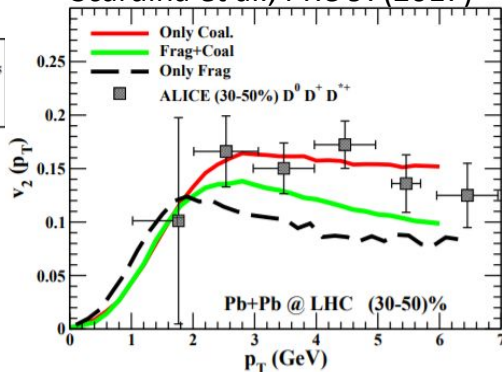


Discrepancy between selected  $v_2$  and unbiased one  $\sim 50\%$

# Catania QPM: some prediction for charm...



Scardina et al., PRC 97(2017)



Good description of

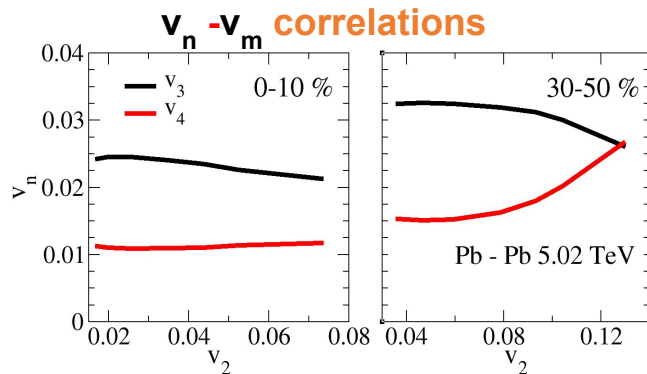
$R_{AA}, v_2$  at RHIC & LHC energies  
within error bars

Monte Carlo Glauber for initial  
condition of partons

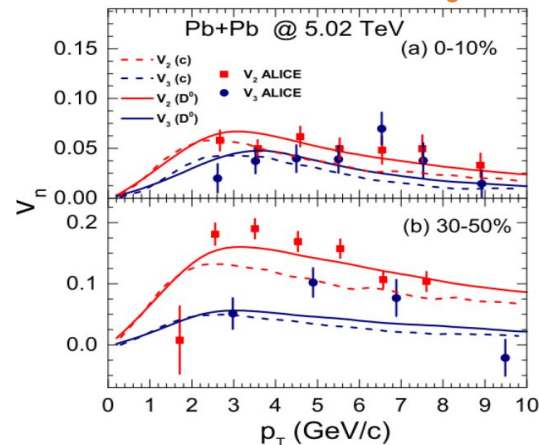
S.Plumari et al, *Phys.Rev.C* 92 (2015) 5

Predictions for D mesons

- Event-Shape Engineering Technique: Prediction for similar correlation for hard particles wrt bulk



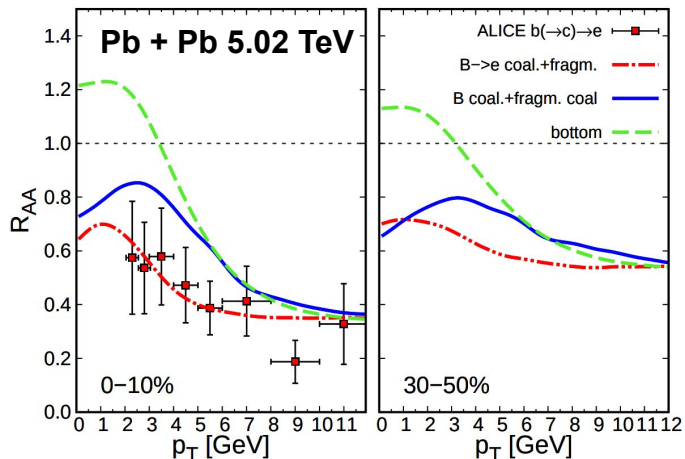
Triangular flow  $v_3$



# Extension to bottom dynamics: $R_{AA}$ and $v_{(n=2,3)}$

Hadronization with coalescence + fragmentation model

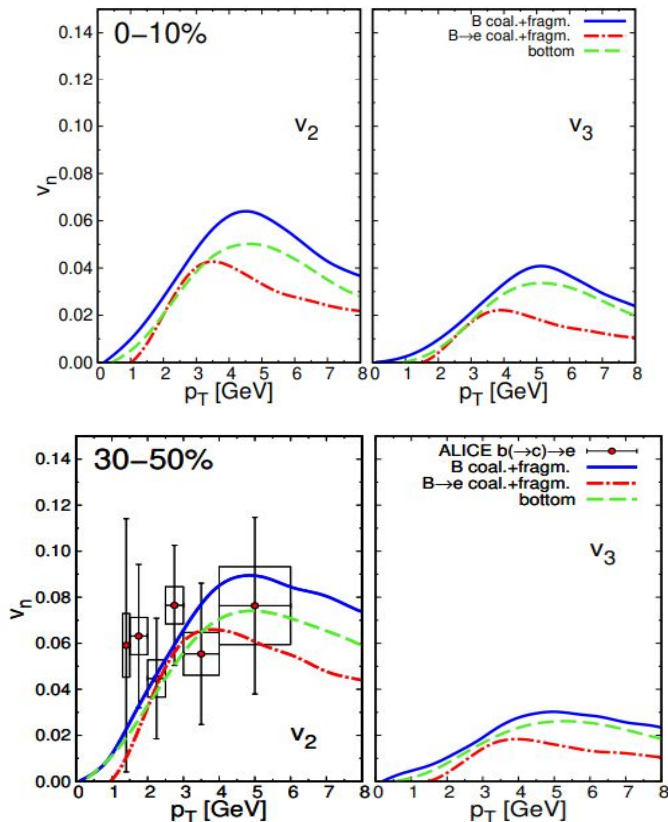
- Prediction for B meson  $R_{AA}$  and  $v_n$
- $R_{AA}$  and  $v_n$  of electrons from semileptonic B meson decay



No parameters changed with respect to charm dynamics

## Compared to charm quark:

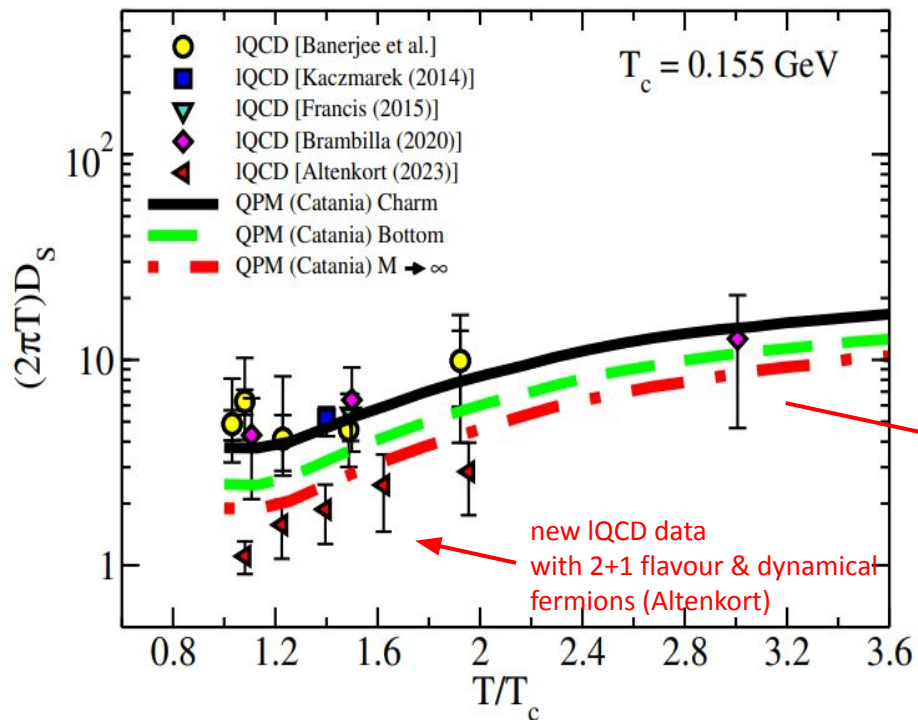
- Efficiency of conversion of  $\epsilon_2$  :  
15% smaller for  $v_2$  in most central collisions.  
40% smaller for  $v_2$  at 30–50% centrality.
- Efficiency of conversion of  $\epsilon_3$  :  
30% smaller for  $v_3$  at both 0–10% and 30–50% centralities.



M.L. Sambaturo et al., e-Print: 2304.02953

Data from: ALICE coll., arxiv:2211.13985

# $(2\pi T)D_s$ : Charm quark vs Bottom quark



- IQCD data are in  $M_Q \rightarrow \infty$ , so the  $D_s$  evaluated is mass independent + quenched medium (data until 2020)
- QPM use finite mass and includes dynamical fermions –  $D_s$  saturates only for  $M > 10$  GeV.

$$D_s = \frac{T}{M \gamma} = \frac{T}{M} \tau_{th}$$

Calculations with fictitious super-heavy quark staying in the  $M_Q \rightarrow \infty$  limit to correctly compare to new IQCD data

From  $D_s$  we obtain ( in the  $1-2T_c$  range):

- $\tau_{th}(c) \sim 5$  fm/c
- $\tau_{th}(b) \sim 11$  fm/c

**Good agreement with the new IQCD data including dynamical fermions which are the more pertinent one to compare to.**

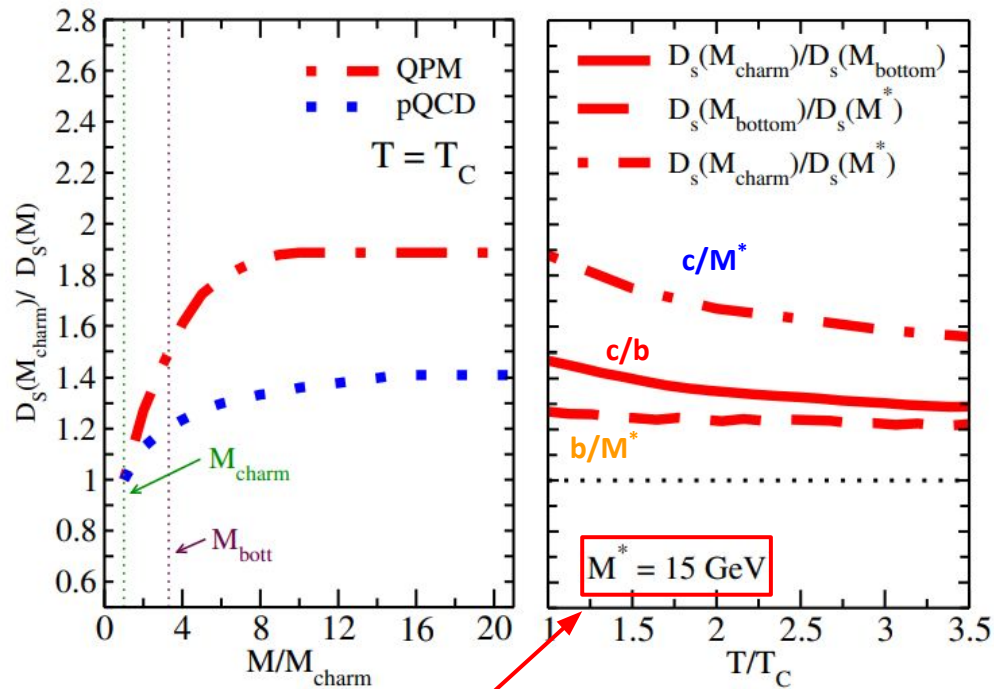
# Conclusions

- **Catania Quasi-Particle model for charm quark:**
  - Good agreement with experimental data for  $R_{AA}$  and  $v_2$
  - Extension to higher order anisotropic flows  $v_n$  : good description also for triangular flow  $v_3$
  - **Event-Shape Engineering technique:** prediction for significant  $v_n - v_m$  correlations for hard particles, similar correlation between soft and hard particles.
  
- **Extension to bottom quark dynamics:**
  - Good description of  $R_{AA}$  and  $v_2$  of electrons from semileptonic B meson decay
  - Prediction for  $v_3$  different from zero in central and semi-peripheral collisions.
  
- **Spatial diffusion coefficient  $D_s(T)$  in good agreement with the new IQCD data which include dynamical fermions - the more pertinent one to compare to.**

**Thanks for the attention!**



# $(2\pi T)D_s$ ratios: Charm quark vs Bottom quark



➤  $D_s(M_{\text{charm}})/D_s(M)$  as a function of  $M/M_{\text{charm}}$  at  $T_C$ :

**Saturation scale of  $D_s$  for  $M_Q \sim 8 M_{\text{charm}} \gtrsim 10 \text{ GeV}$**   
 $D_s(M_{\text{charm}})/D_s(M \rightarrow \infty) = 1.9$  for QPM.

$D_s(M_{\text{charm}})/D_s(M \rightarrow \infty) \approx 1.4$  for pQCD.

➤ Ratios at fixed mass as a function of  $T$ :

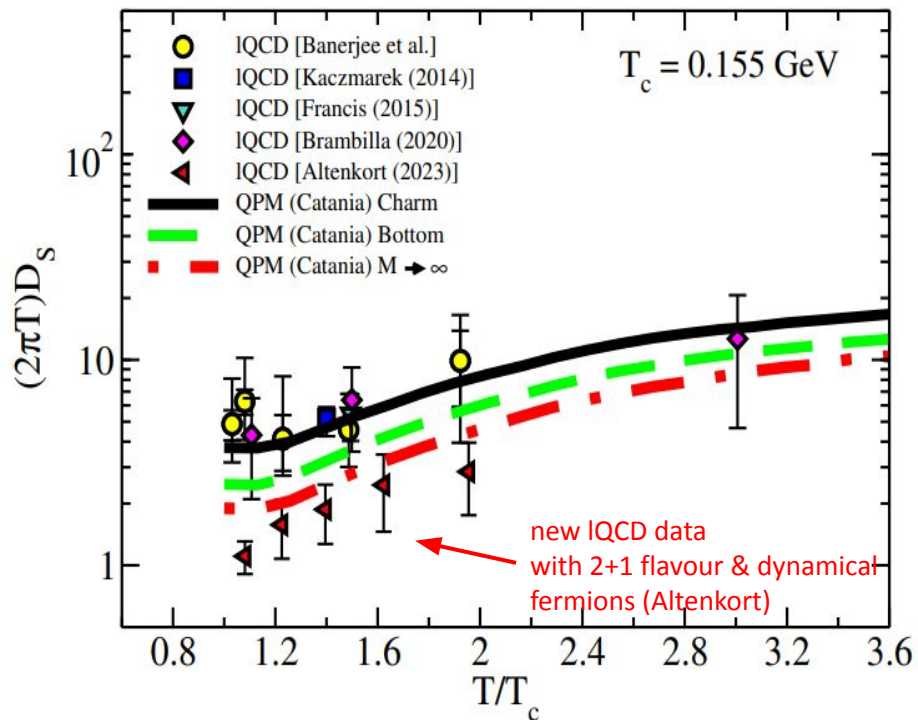
-  $b/M^*$ : about 25% in all  $T$  range

-  $c/b$ : about 50% at  $T_C$  and not smaller than 30%

-  $c/M^*$ : factor 1.5-2

fictitious super-heavy quark staying in the  $M_Q \rightarrow \infty$  limit

# $(2\pi T)D_s$ : Charm quark vs Bottom quark



From  $D_s$  we obtain ( in the  $1-2T_c$  range):

- $\tau_{th}(c) \sim 5 \text{ fm}/c$
- $\tau_{th}(b) \sim 11 \text{ fm}/c$  breaking w.r.t. the relation:  
 $\tau_{th}(b) = (M_b/M_c)\tau_{th}(c) \sim 3.3 \tau_{th}(c) \sim 16.5 \text{ fm}/c$

- IQCD data are in  $M_Q \rightarrow \infty$ , so the  $D_s$  evaluated is mass independent + quenched medium (data until 2020)
- QPM use finite mass and includes dynamical fermions

$$D_s = \frac{T}{M \gamma} = \frac{T}{M} \tau_{th}$$

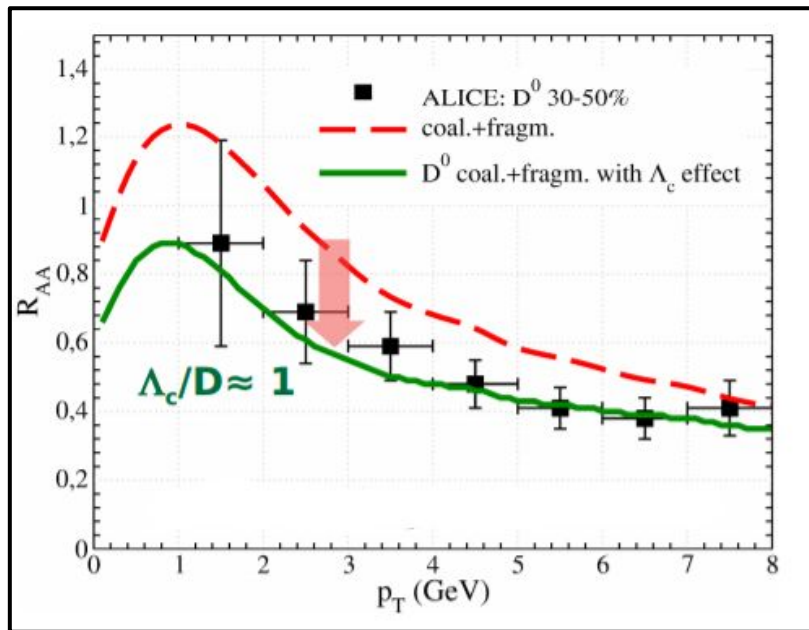
From kinetic theory is expected that:

$$\tau_{th}(b)/\tau_{th}(c) \approx \gamma_c/\gamma_b \approx M_b/M_c$$

In QPM approach  $\rightarrow D_s(c)$  is 30-40% larger than  $D_s(b)$  (no mass independence)

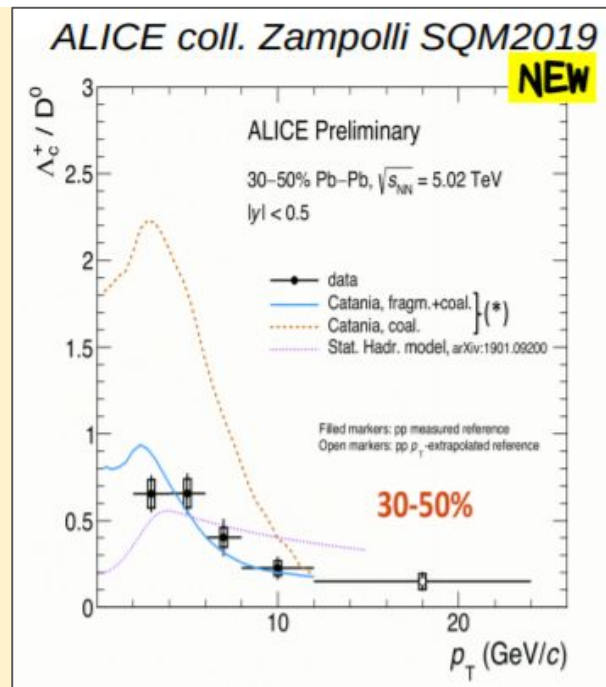
$M \rightarrow \infty$  limit is not reached for charm

# D meson: Impact of large $\Lambda_c$ production on $R_{AA}$



$D_s(T)$  of charm quark that reproduces  $R_{AA}$  and  $v_2$  gives good description of

- Impact of  $\Lambda_c/D^0$
- Triangular flow  $v_3(p_T)$ .
- $q_2$  selected anisotropic flow and spectra.



- With the same coalescence plus fragmentation model we describe the  $\Lambda_c/D^0$

S. Plumari, et al.,  
Eur. Phys. J. C78 no. 4, (2018) 348

# Numerical solution of Boltzmann Equation

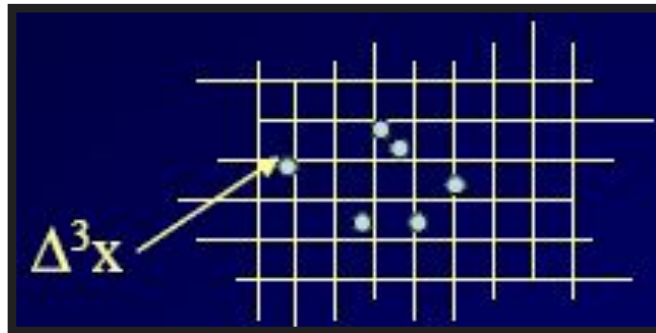
- Use Test-Particle Method to sample the phase space distribution function

$$f(\vec{x}, \vec{p}, t) = \omega \sum_{i=1}^{N_{test}} \delta^{(3)}(\vec{x} - \vec{r}_i(t)) \delta^{(3)}(\vec{p} - \vec{p}_i(t))$$

$F_i$  solution of Boltzmann eq.

→ Test particles solve classical Hamilton eq. of motion

$$\begin{cases} \vec{p}_i(t + \Delta t) = \vec{p}_i(t - \Delta t) + 2\Delta t \cdot \left( \frac{\partial \vec{p}_i}{\partial t} \right)_{coll} \\ \vec{r}_i(t + \Delta t) = \vec{r}_i(t - \Delta t) - 2\Delta t \cdot \left[ \frac{\vec{p}_i(t)}{E_i(t)} \right] \end{cases}$$



- Collision Integral mapped through a Stochastic Algorithm

$$P_{22} = \frac{\Delta N_{coll}^{2 \rightarrow 2}}{\Delta N_1 \Delta N_2} = v_{rel} \sigma_{22} \frac{\Delta t}{\Delta^3 x}$$

$\Delta t \ll 0$  and  $\Delta^3 x \ll 0$  : exact solution

Final phase-space of HQ + bulk parton scattering sampled according to  $|M_{QCD}|^2$  □ code test through simulations in a “box”

[Scardina, Colonna, Plumari, and Greco PLB v.724, 296 (2013)]

[Xu and Greiner PRC v. 71, (2005)]

# Hybrid Hadronization Model for HQs

✓ **COALESCENCE**: Formula developed for the light sector [**Greco, Ko, Levai PRL 90 (2003)**]

$$\frac{dN_H}{d^2\mathbf{P}_T} = g_H \int \prod_{i=1}^n \frac{d^3p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i f_{q_i}(x_i, p_i) f_W(x_1 \dots x_n; p_1 \dots p_n) \delta\left(\mathbf{P}_T - \sum_i^n p_{T,i}\right)$$

Statistical Factor  
Color-spin-isospin

Parton Distribution Functions  
(after Boltzmann evolution)

Hadron Wigner Function

(parameters fix according to quark model)

C.-W. Hwang, *EPJ C*23, 585 (2002)

C. Albertus et al., *NPA* 740, 333 (2004)

✓ **FRAGMENTATION**: HQs that do not undergo to Coalescence

$$\frac{dN_H}{d^2\mathbf{P}_T} = \sum_f \int dz \frac{dN_f}{d^2p_T} \frac{D_{f \rightarrow H}(z)}{z^2}$$

We use Peterson parametrization:  $D_H(z) \propto \left[ z \left( 1 - \frac{1}{z} - \frac{\epsilon_c}{1-z} \right)^2 \right]^{-1}$  *Peterson et al. PRD 27 (1983) 105*

Parameter  $\epsilon_c$  tuned to reproduce *D* and *B* meson spectra in pp collisions.

# Non-perturbative effects: impact of off-shell dynamics

## QPM vs. DQPM

□ Partons are dressed by non-perturbative spectral functions:

$$A_i^{BW}(m_i) = \frac{2}{\pi} \frac{m_i^2 \gamma_i^*}{(m_i^2 - M_i^2)^2 + (m_i \gamma_i^*)^2}$$

$$C[f] = \int dm_i A(m_i) \int dm_f A(m_f) \times \frac{1}{2E_p} \int \frac{d^3 \mathbf{q}}{2E_q (2\pi)^3} \int \frac{d^3 \mathbf{q}'}{2E_{q'} (2\pi)^3} \int \frac{d^3 \mathbf{p}'}{2E_{p'} (2\pi)^3} \times \frac{1}{\gamma_Q} \sum (\mathcal{M}_Q)^2 (2\pi)^4 \delta^4(p + q - p' - q') \times [f(\mathbf{p}') \hat{f}(\mathbf{q}', m_f) - f(\mathbf{p}) \hat{f}(\mathbf{q}, m_i)]$$

For references: W. Cassing, Nucl.Phys. A831, 215  
E. Bratkovskaya, Nucl.Phys. A856, 162  
H. Berrehrah, Phys. Rev. C 89(5), 054901  
M.L. Sambaturo et al., Eur.Phys.J.C 80 12, 1140

Evaluated in DQPM approach

Off-shell  $\approx$  PHSD but also larger widths!

BOX CALCULATION [T=200 MeV] FOR CHARM

Bulk is not with the same energy density

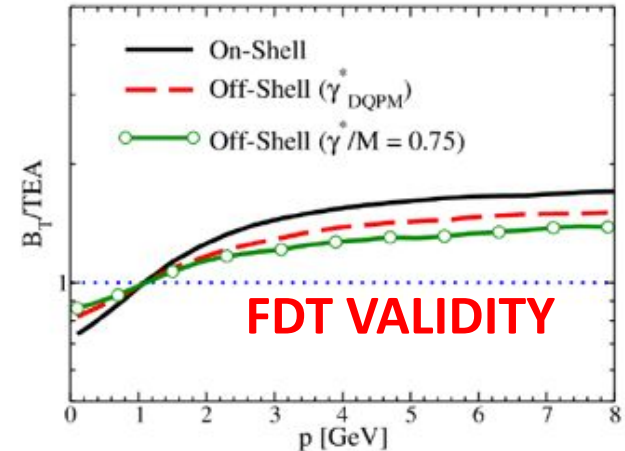
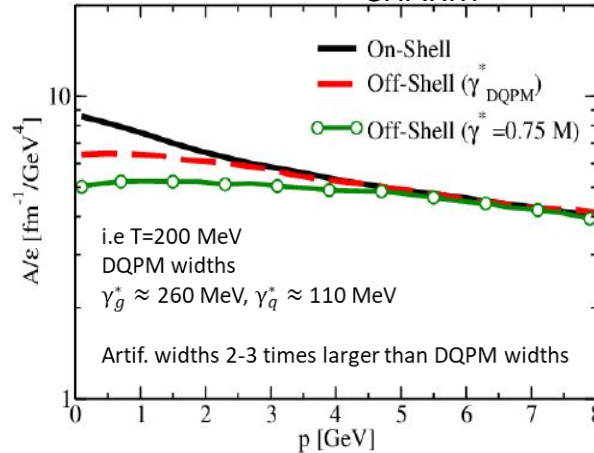
The energy density of off-shell case is smaller

➤ Transport coefficient scales with energy density of the system  $\epsilon$

➤ Larger breaking for low  $p$  region ( $p \lesssim 2-3$  GeV/c)

→ larger off-shell effects

→ 30-40% decreasing drag





# On-shell vs Off-shell energy loss

BOX CALCULATION [T=200 MeV] FOR CHARM

□ Partons are dressed by non-perturbative spectral functions:

$$A_i^{BW}(m_i) = \frac{2}{\pi} \frac{m_i^2 \gamma_i^*}{(m_i^2 - M_i^2)^2 + (m_i \gamma_i^*)^2}$$

Boltzmann equation and off-shell extension

$$p^\mu \partial_\mu f_Q = C[f_Q, f_g, f_q]$$

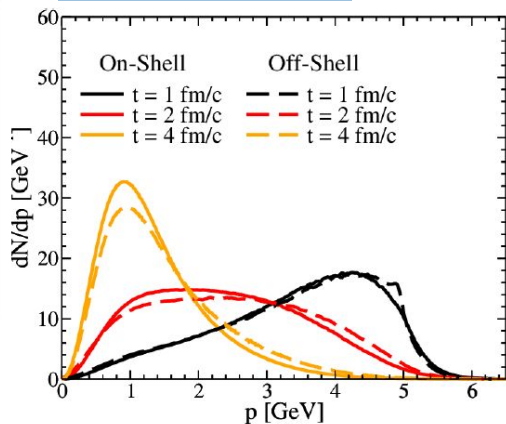
Plasma uniform  $\rightarrow p^0 \partial_0 f_Q = C[f_Q, f_g, f_q]$

$$\frac{\partial f_Q}{\partial t} = \frac{1}{E_Q} C[f_Q, f_g, f_q]$$

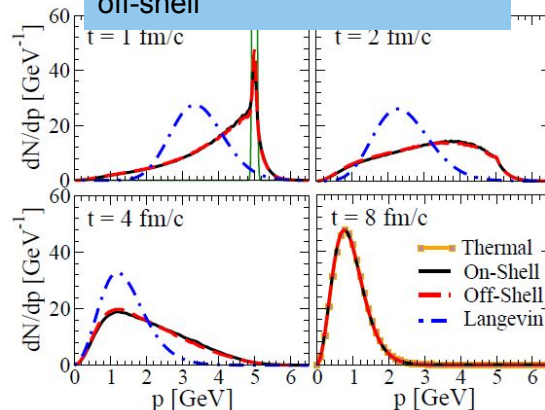
$$f(t + \Delta t, p) = f(t, p) + \frac{1}{E_Q} C[f]$$

$C[f_q, f_g, f_Q]$  Collision integral calc. both in on-shell and off-shell mode

$$\epsilon_{onshell} = \epsilon_{offshell}$$



+k(p) making the Drag on-shell=Drag off-shell



The difference between on-shell and off-shell mode can be adsorbed by multiplying scattering matrix for a  $k$  factor