Femtoscopic correlations of lightest nuclei

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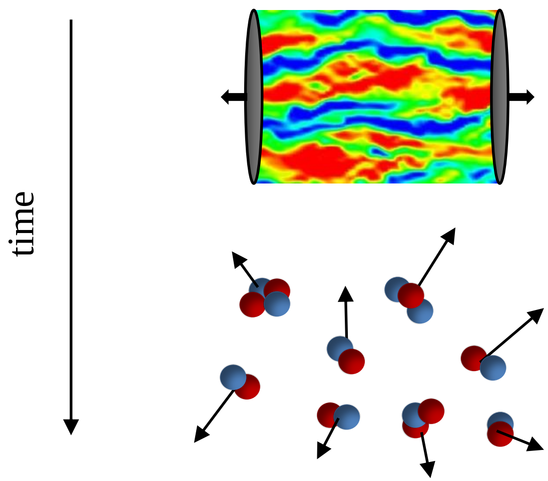
Why femtoscopy of light nuclei is interesting?

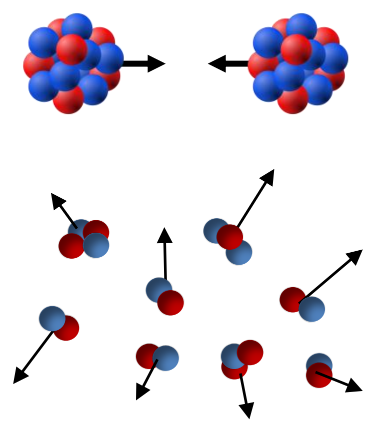
- Femtoscopy is sensitive to production mechanism of light nuclei.

Two very different situations:

Genuine production in high-energy collisions

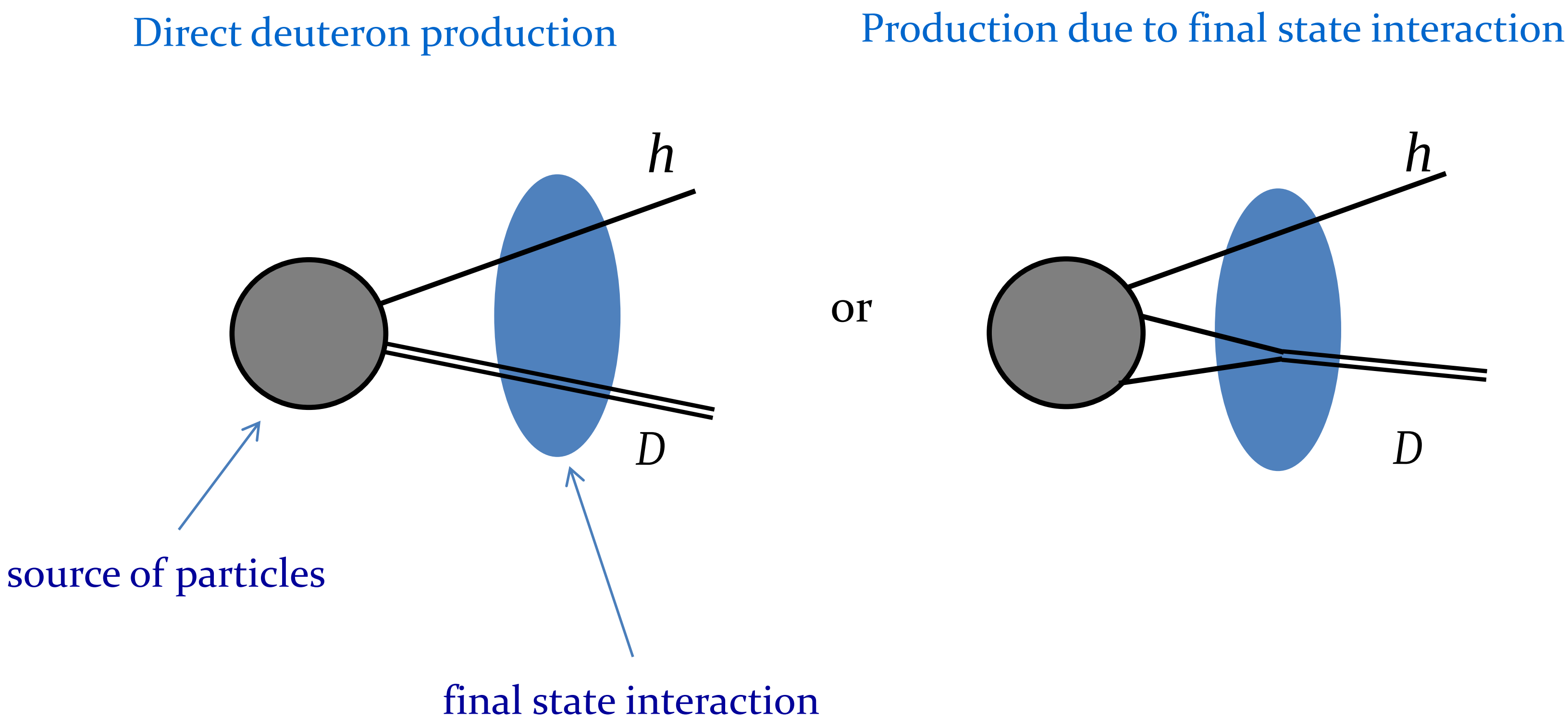


Shattering of incoming nuclei collisions



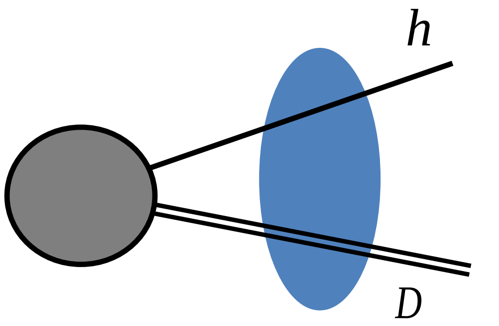
- Femtoscopy is sensitive to interaction, including nulcear resonances.

Hadron-deuteron correlations



Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle



Direct production

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h \, d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D) \frac{dN_h}{d\mathbf{p}_h} \frac{dN_D}{d\mathbf{p}_D}$$

Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) = \int d^3 r_h \, d^3 r_D \, D(\mathbf{r}_h) \, D(\mathbf{r}_D) \left| \psi(\mathbf{r}_h, \mathbf{r}_D) \right|^2$$

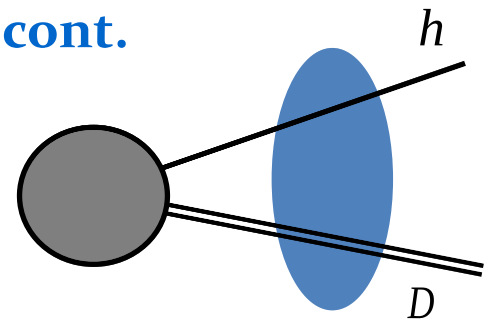
distribution of emission points

h-*D* wave function

S.E. Koonin, Phys. Lett. B **70**, 43 (1977)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.



Separation of CM and relative motion

$$\begin{cases} \mathbf{R} \equiv \dfrac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$R(\mathbf{q}) = \int d^3 r \; D_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$

„Relative" source function

$$D_r(\mathbf{r}) \equiv \int d^3 R \; D\left(\mathbf{R} - \frac{m_D}{m_D + m_h} \mathbf{r} \right) D\left(\mathbf{R} + \frac{m_h}{m_D + m_h} \mathbf{r} \right) = \left(\frac{1}{4\pi R_s^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2} \right)$$

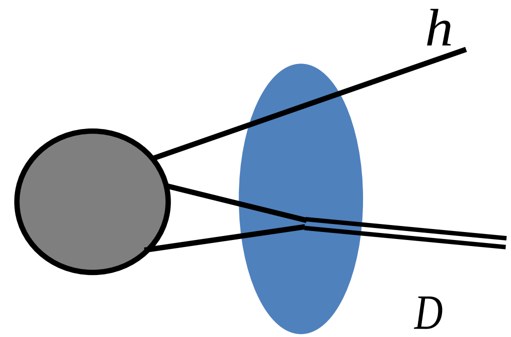
$$D(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2} \right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D)\, W_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) W_D = \int d^3r_h\, d^3r_n\, d^3r_p\, D(\mathbf{r}_h)\, D(\mathbf{r}_n)\, D(\mathbf{r}_p) \left|\psi_{hD}(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p)\right|^2$$

Deuteron formation rate

$$\frac{dN_D}{d\mathbf{p}_D} = W_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p} \qquad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

spin factor

$$W_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_n d^3\mathbf{r}_p\, D(\mathbf{r}_n) D(\mathbf{r}_p) \left|\psi_D(\mathbf{r}_n, \mathbf{r}_p)\right|^2 = \frac{3}{4}(2\pi)^3 \int d^3r_{np}\, D_r(\mathbf{r}_{np}) \left|\phi_D(\mathbf{r}_{np})\right|^2$$

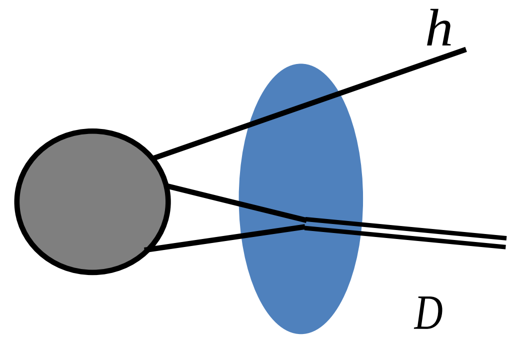
$$\psi_D(\mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_D(\mathbf{r}_{np})$$

St. Mrówczyński & P. Słoń, Acta Physica Polonica B **51**, 1739 (2020)

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion



$$\begin{cases} \mathbf{R} \equiv \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})$$

$$R(\mathbf{q}) = \frac{1}{W_D} \int d^3R\, d^3r_{np}\, d^3r\, D(\mathbf{r}_h) D(\mathbf{r}_n)\, D(\mathbf{r}_p) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2 \left|\varphi_D(\mathbf{r}_{np})\right|^2$$

For Gaussian source

$$R(\mathbf{q}) = \int d^3r\, D_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$

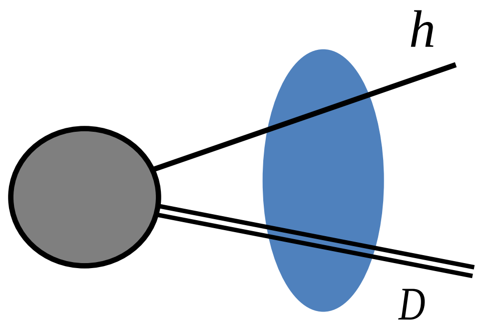
$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

For non-Gaussian source, W_D remains in the correlation function!

Hadron-deuteron correlation function

Direct production

$$R(\mathbf{q}) = \int d^3 r\, D_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



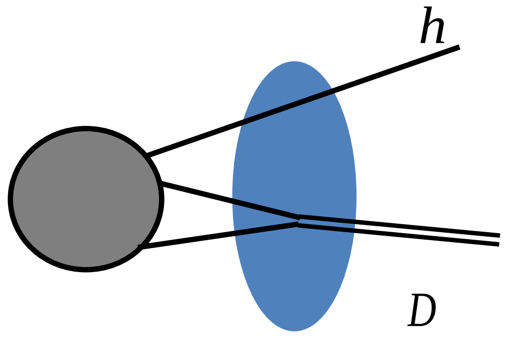
$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$\sqrt{\frac{4}{3}} \approx 1.15$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

Final state interaction

$$R(\mathbf{q}) = \int d^3 r\, D_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



Hadron-deuteron correlation function

The wave function in scattering asymptotic state

$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{qr}} + f(\mathbf{q})\frac{e^{iqr}}{r}$$

The *s*-wave amplidtue

$$f(\mathbf{q}) = -\frac{a}{1-iqa}$$

a – scattering length

Coulomb interaction via Gamow factor

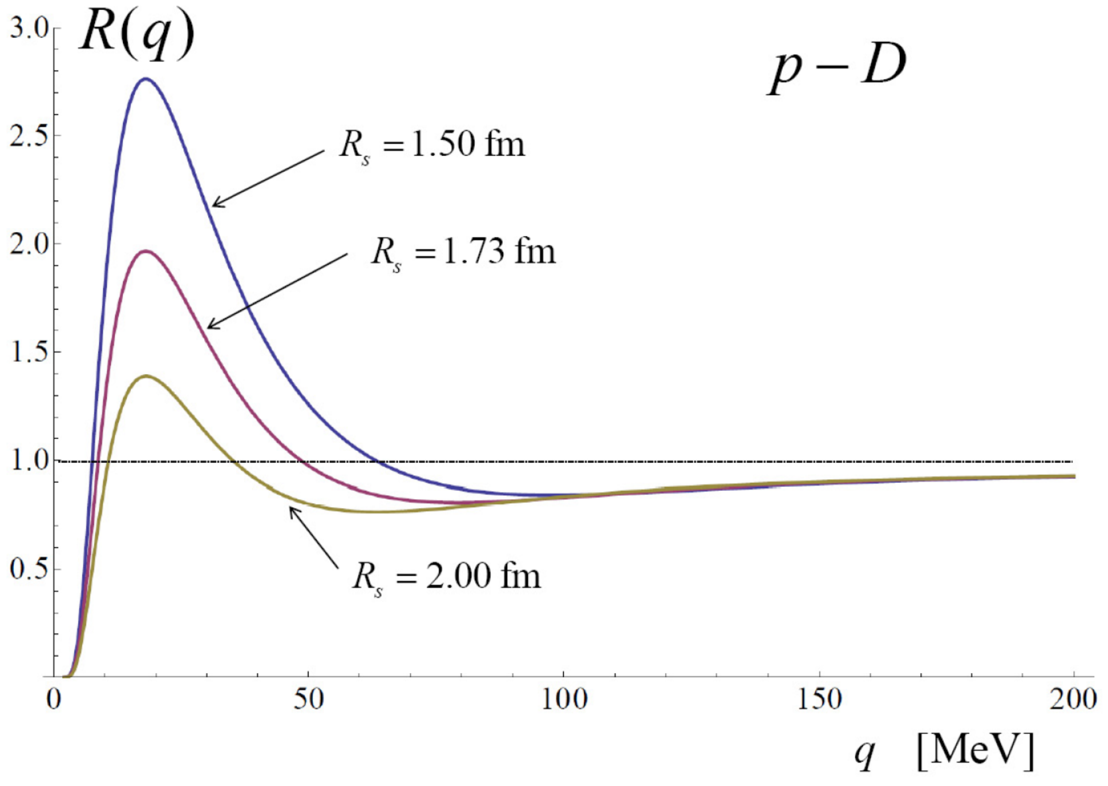
$$G(q) = \pm\frac{2\pi}{a_B q}\frac{1}{\exp\left(\pm\frac{2\pi}{a_B q}\right)-1}$$

$a_B = \frac{1}{\mu\alpha}$ – Bohr radius

Interference of strong and Coulomb interaction ignored!

R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

p-D correlation function



spin 1/2 spin 3/2

$$R(q) = \frac{1}{3} R_{1/2}(q) + \frac{2}{3} R_{3/2}(q)$$

$$a_{1/2} = 4.0 \text{ fm}$$

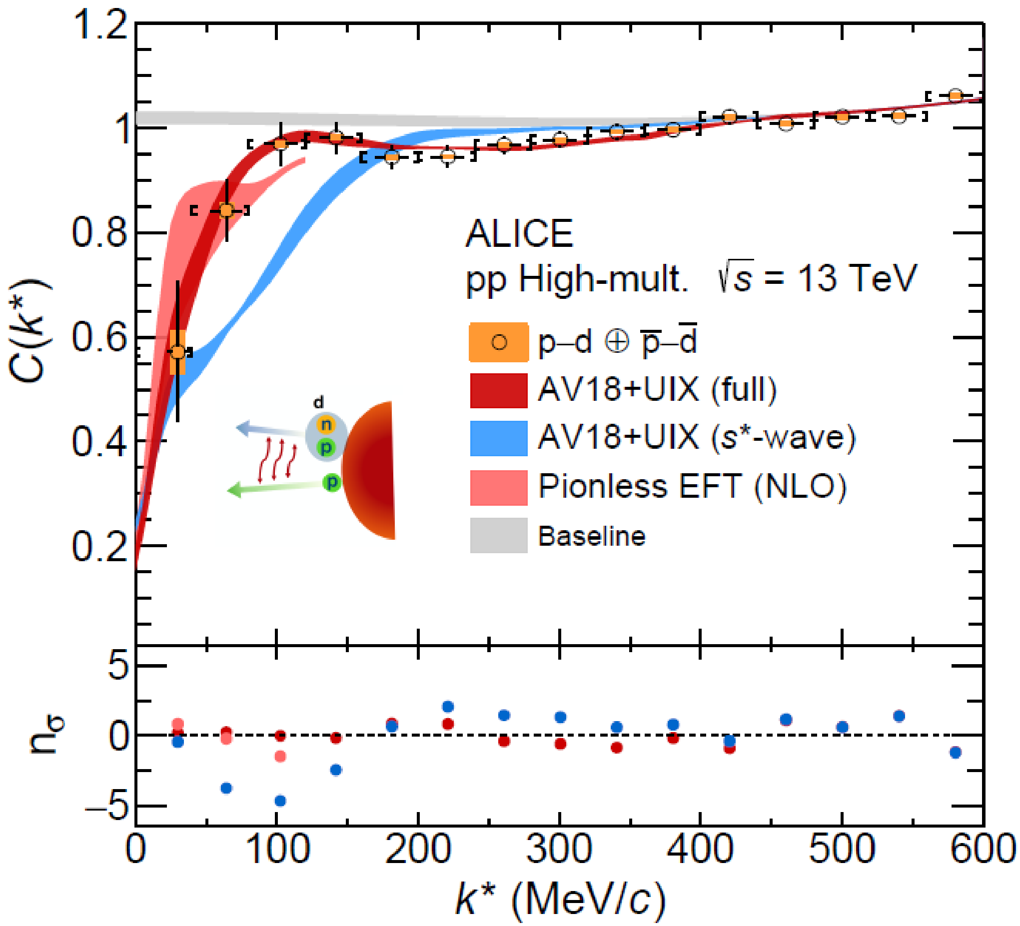
$$a_{3/2} = 11.0 \text{ fm}$$

$$2.00 = \sqrt{\frac{4}{3}} 1.73 = \frac{4}{3} 1.50$$

R_s from *p-D* correlation function vs. R_s from *p-p* correlation function

St. Mrówczyński & P. Słoń, Acta Physica Polonica B **51**, 1739 (2020)

p-D correlation function



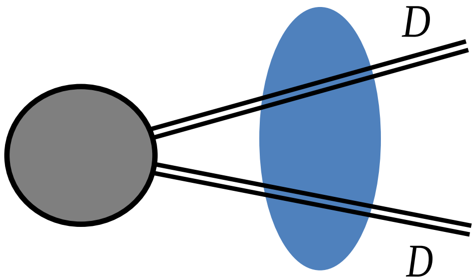
$R_s = 1.43 \pm 0.16$ fm

ALICE arXiv:2308.16120

M. Viviani et al, arXiv:2306.02478 – full three body calculations

Deuteron-deuteron correlation function

1) Deuteron is treated as an elementary particle



Experimental definition

$$\frac{dN_{DD}}{d\mathbf{p}_1 d\mathbf{p}_2} = R(\mathbf{p}_1, \mathbf{p}_2) \frac{dN_D}{d\mathbf{p}_1} \frac{dN_D}{d\mathbf{p}_2}$$

Theoretical formula

$$R(\mathbf{p}_1, \mathbf{p}_2) = \int d^3 r_1 \, d^3 r_2 \, D(\mathbf{r}_1) \, D(\mathbf{r}_1) \left| \psi_{DD}(\mathbf{r}_1, \mathbf{r}_2) \right|^2$$

distribution of emission points

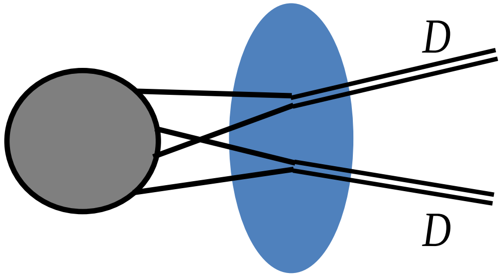
D-*D* wave function

Deuteron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{DD}}{d\mathbf{P}_1^D\, d\mathbf{P}_2^D} = R(\mathbf{P}_1^D, \mathbf{P}_2^D)\, W_D^{\ 2}\, \frac{dN_n}{d\mathbf{p}_1} \frac{dN_p}{d\mathbf{p}_2} \frac{dN_n}{d\mathbf{p}_3} \frac{dN_p}{d\mathbf{p}_4}$$



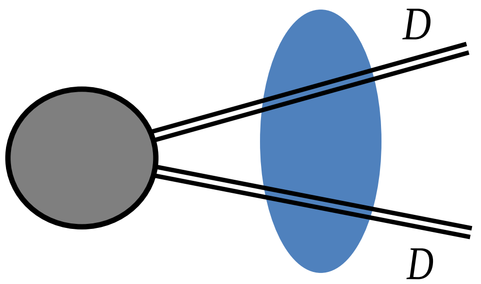
Theoretical formula

$$R(\mathbf{P}_1^D, \mathbf{P}_2^D)\, W_D^{\ 2} = \int d^3 r_1^p d^3 r_1^n d^3 r_2^p d^3 r_2^n\ D(\mathbf{r}_1^p)\, D(\mathbf{r}_1^n)\ D(\mathbf{r}_2^p) D(\mathbf{r}_2^n)$$
$$\times \left| \psi_{DD}(\mathbf{r}_1^p, \mathbf{r}_1^n, \mathbf{r}_2^p, \mathbf{r}_2^n) \right|^2$$

Deuteron-deuteron correlation function

Direct production

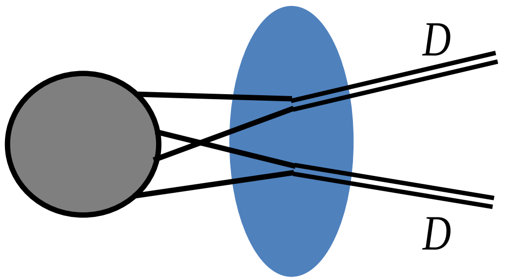
$$R(\mathbf{q}) = \int d^3r \, D_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$



$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

$\sqrt{2} \approx 1.41$

$$D_{4r}(\mathbf{r}) = \left(\frac{1}{2\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2} \right)$$

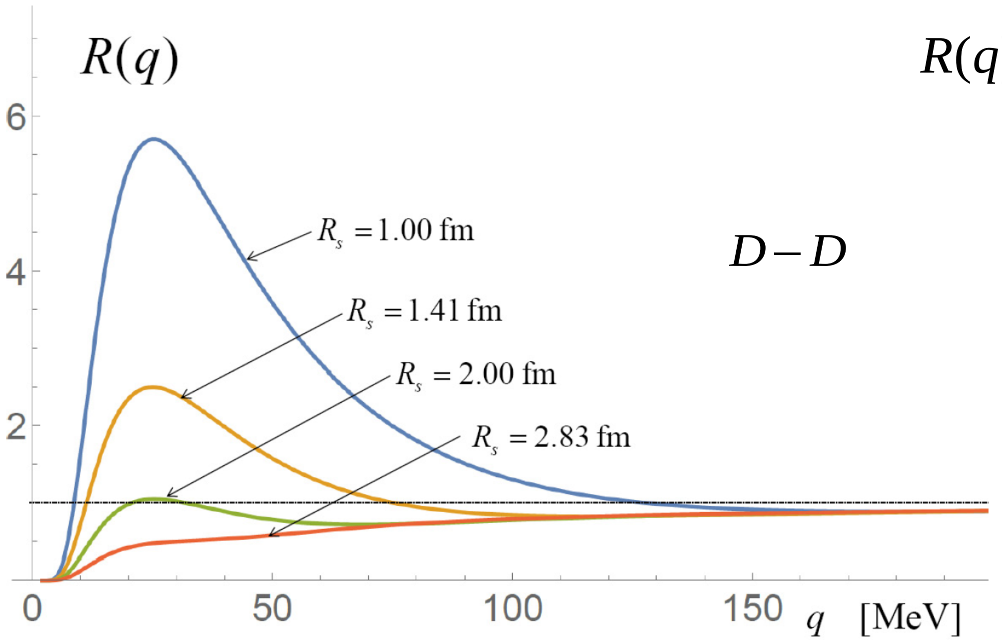


Final state interaction

$$R(\mathbf{q}) = \int d^3r \, D_{4r}(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$

St. Mrówczyński & P. Słoń, Physical Review C **104**, 024909 (2021)

Deuteron-deuteron correlation function



spin 0   spin 1   spin 2

$$R(q) = \frac{1}{9} R_0(q) + \frac{3}{9} R_1(q) + \frac{5}{9} R_2(q)$$

$$a_0 = (10.2 + 0.2i)\ \text{fm}$$

$$a_2 = 7.5\ \text{fm}$$

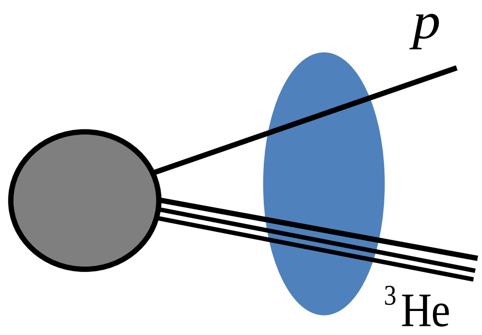
$$2.83 = \sqrt{2} \cdot 2.00 = \left(\sqrt{2}\right)^2 \cdot 1.41 = \left(\sqrt{2}\right)^3 \cdot 1.00$$

R_s from *D-D* correlation function vs. R_s from *p-p* & *p-D* correlation function

Proton-^{3}He correlation function

Direct production

$$R(\mathbf{q}) = \int d^3r \, D_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



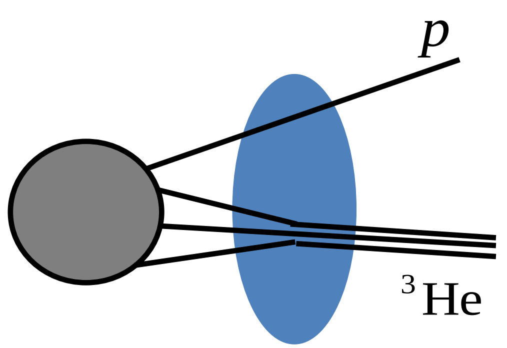
$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$\sqrt{\frac{3}{2}} \approx 1.22$$

$$D_{4r}(\mathbf{r}) = \left(\frac{3}{8\pi R^2}\right)^{3/2} \exp\left(-\frac{3\mathbf{r}^2}{8R^2}\right)$$

Final state interaction

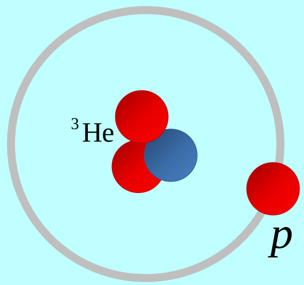
$$R(\mathbf{q}) = \int d^3r \, D_{4r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

Resonance ^{4}Li

^{4}Li



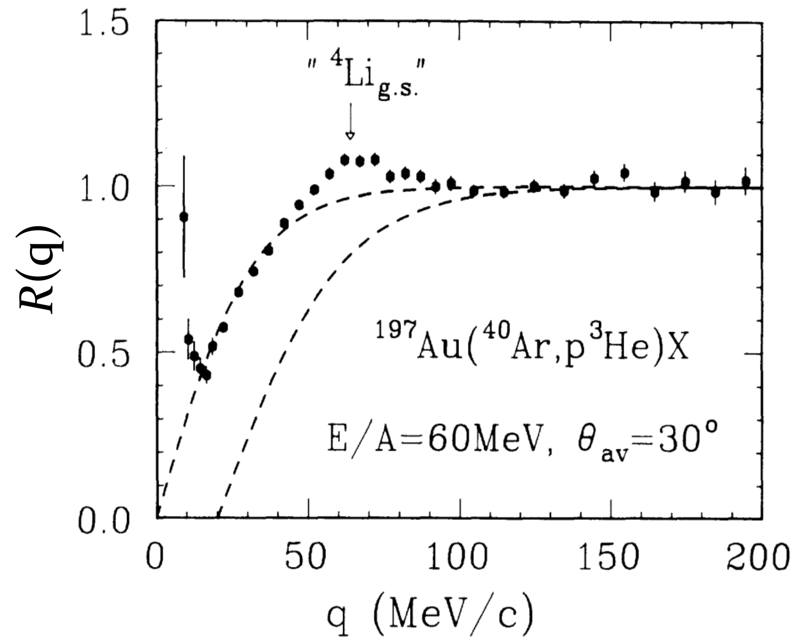
$^4\mathrm{Li} \rightarrow {}^3\mathrm{He} + p$

$\Gamma = 6\ \mathrm{MeV}$

$m = m_{^3\mathrm{He}} + m_p + 4.1\ \mathrm{MeV}$

$m = 3749.7\ \mathrm{MeV}$

$s = 2$

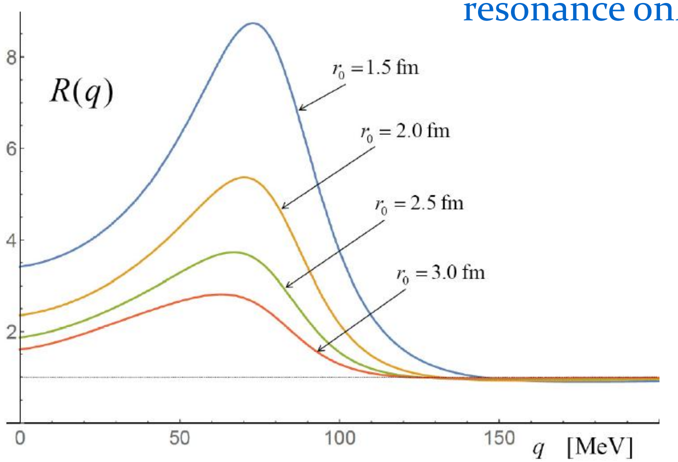


J. Pochodzala et al. Phys. Rev. C **35**, 1695 (1987)

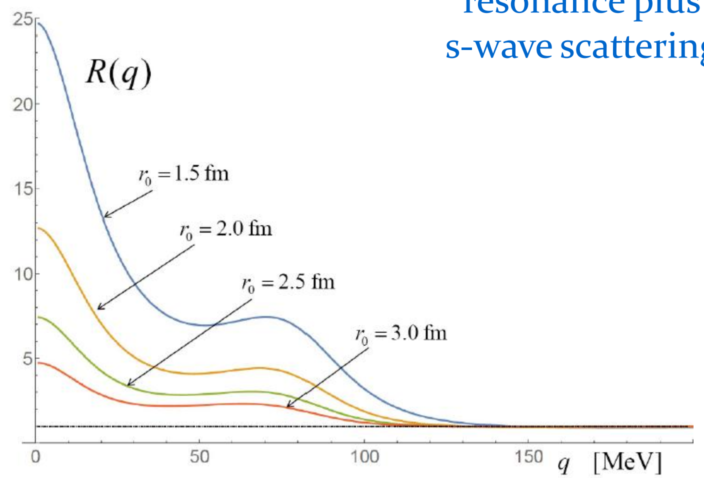
HADES data presented by M. Stefaniak

Correlation function p-^{3}He

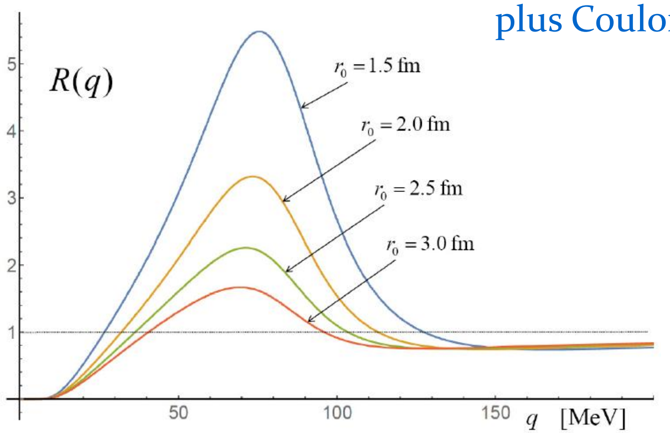
resonance only



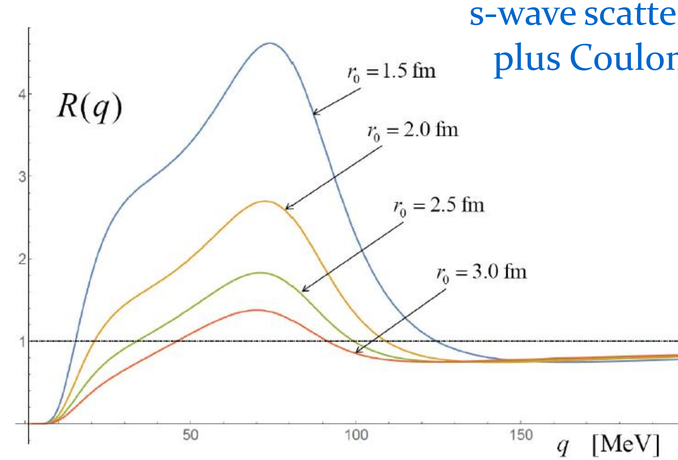
resonance plus
s-wave scattering



resonance
plus Coulomb



resonance plus
s-wave scattering
plus Coulomb



S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

How to measure yield of ^{4}Li

$$\frac{dN_{\text{Li}}}{d\mathbf{p}} = S_R \frac{dN_p}{d\mathbf{p}} \frac{dN_{^3\text{He}}}{d\mathbf{p}}$$

p - momentum per nucleon

$$S_R \equiv \int d^3q \, R_R(\mathbf{q})$$

correlation function where only the ^{4}Li resonance contributes

Conclusions

- The correlation function of light nuclei depends on whether the nuclei are produced directly or due to final state interactions.

- The correlation function of light nuclei is sensitive to nuclear interactions, can reveal an existence of nuclear resonances.