Tre Facce dell Stessa Medaglia

coalescence, thermal production and the Koonin equation

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MICHIGAN STATE UNIVERSITY Founded 1855

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FRIB

GOALS:

— Review theory

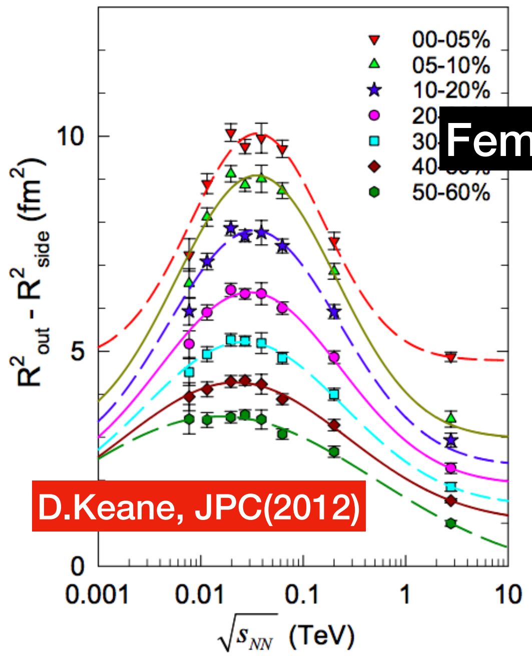
— Compare coalescence and Koonin Eq.

— Understand roles of Femtoscopic and Coalescence

— Wish list

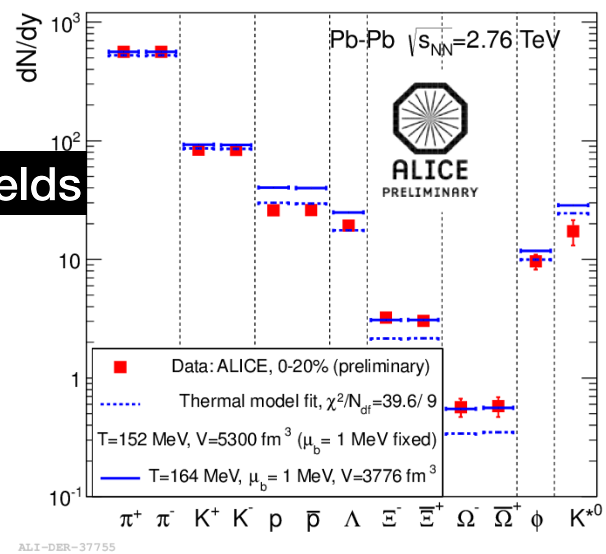
Three classes of measurement

Femtoscopy

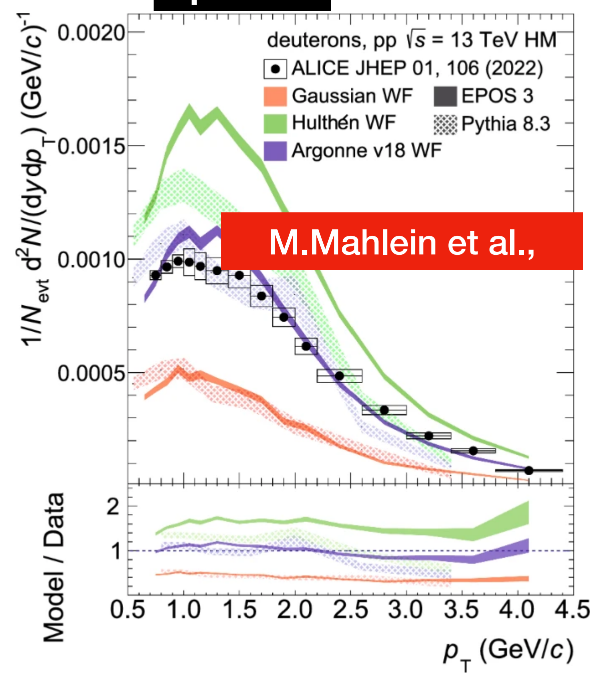


D.Keane, JPC(2012)

Yields

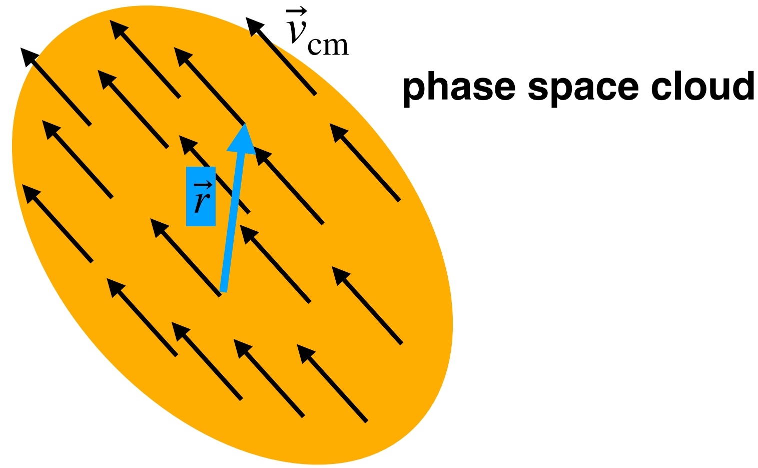


Spectra



M.Mahlein et al.,

Femtoscopy Theory



$$C(\vec{p}_1,\vec{p}_2) = \frac{P(\vec{p}_1,\vec{p}_2)}{P(\vec{p}_1)P(\vec{p}_2)}$$

$$= C(\vec{v}_{\rm cm},\vec{q})$$

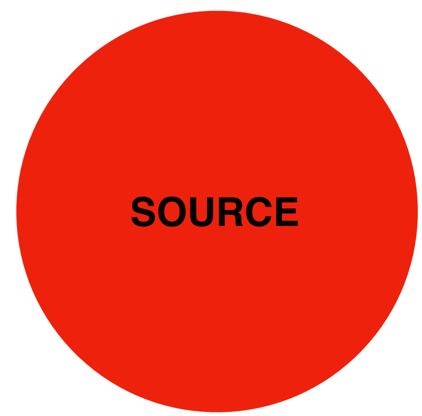
wave function

$$= \int d^3r\, S(\vec{v}_{\rm cm},\vec{r})\, |\phi_{\vec{q}}(\vec{r})|^2 ,$$

source function

$$S(\vec{v}_{\rm cm},\vec{r}) = \frac{\int d^3r_1 d^3r_2\ f_{\rm cm}(\vec{v}_{\rm cm},\vec{r}_1,t) f_{\rm cm}(\vec{v}_{\rm cm},\vec{r}_2,t)\delta(\vec{r}_1-\vec{r}_2-\vec{r})}{\int d^3r_1 d^3r_2\ f_{\rm cm}(\vec{v}_{\rm cm},\vec{r}_1,t) f_{\rm cm}(\vec{v}_{\rm cm},\vec{r}_2,t)}$$

“SOURCE FUNCTION” measures phase space cloud, not source!!!



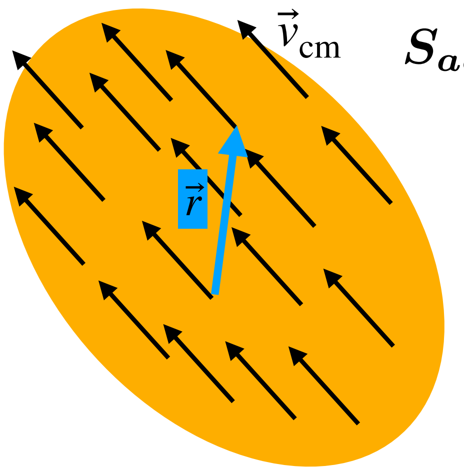
GOAL: Measure $C(\vec{p}_1,\vec{p}_2)$ **to infer** $S(\vec{v}_{\rm cm},\vec{r})$

For identical bosons: $|\phi|^2 = 1 + \cos(2\vec{q}\cdot\vec{r})$

Strong/Coulomb makes inversion more complicated

Coalescence vs Koonin Eq.

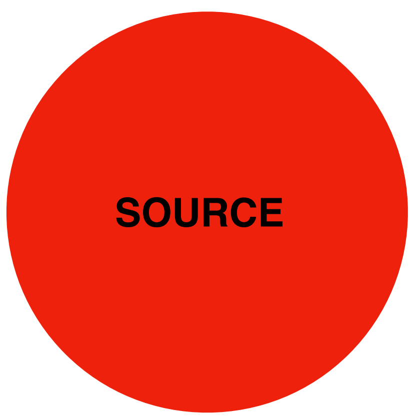
Both use same source function:



$$S_{ab}(\vec{v}_{\rm cm}, \vec{r}) = \frac{\int d^3r_a d^3r_b \; f_a(\vec{v}_{\rm cm}, \vec{r}_a, t) f_b(\vec{v}_{\rm cm}, \vec{r}_b, t) \delta(\vec{r}_a - \vec{r}_b - \vec{r})}{\int d^3r_a d^3r_b \; f_a(\vec{v}_{\rm cm}, \vec{r}_a, t) f_b(\vec{v}_{\rm cm}, \vec{r}_b, t)}$$

$\vec{S}_{ab}(\vec{v}_{\rm cm}, \vec{r})$ **is probability two particles of same $\vec{v}$ are separated by $\vec{r}$**

phase space cloud



will visit later

"Smoothness" approximation:
Sometimes $\vec{v}_{\rm cm} \rightarrow \vec{v}_{\rm cm} \pm \delta\vec{v}$
$\delta\vec{v}$ sometimes related to Wigner transform of wave function,
sometimes from $\vec{p}_a - \vec{p}_b$,
sometimes just some finite width distribution for statistics…

Coalescence vs Koonin Eq.

Both use same source function:

$$\frac{P_c(\vec{v}_{\rm cm})}{P_a(\vec{v}_{\rm cm})P_b(\vec{v}_{\rm cm})} = \int d^3r\, S_{ab}(\vec{v}_{\rm cm},\vec{r})\, |\phi_c(\vec{r})|^2 ,$$

source function — wave function

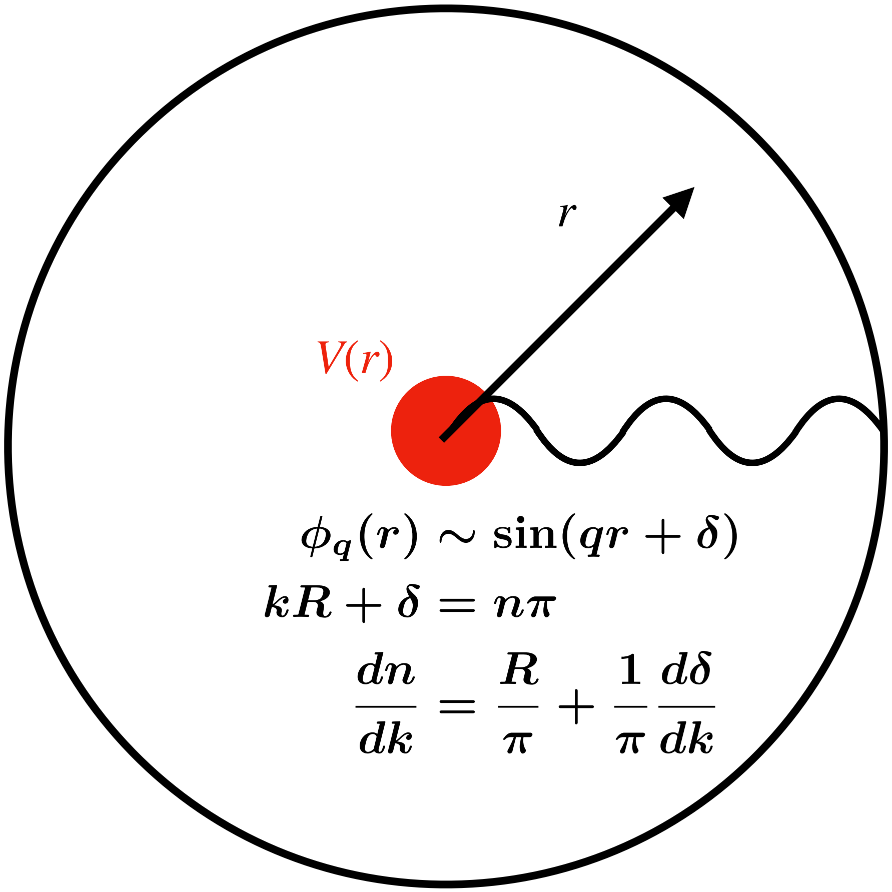
Coalescence

Femtoscopy

$$C_{ab}(\vec{v}_{\rm cm},\vec{q}) = \frac{P_{ab}(\vec{p}_1,\vec{p}_2)}{P_a(\vec{p}_1)P_b(\vec{p}_2)} = \int d^3r\, S_{ab}(\vec{v}_{\rm cm},\vec{r})\, |\phi_{\vec{q}}(\vec{r})|^2$$

source function — wave function

Wave Function and Density of States



Phase shifts:

a) **describe wave function outside range of** V

b) **give density of states**

Wave Function and Density of States

$$\phi_q(r) \sim \sin(qr + \delta)$$

$$kR + \delta = n\pi$$

$$\frac{dn}{dk} = \frac{R}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

$$\frac{1}{2\pi}\int dr \left\{|\phi_q(r)|^2 - |\phi_q^{(0)}(r)|^2\right\} = \frac{1}{\pi}\frac{d\delta}{dq}$$

$|\phi_q(r)|^2$ **describes contribution to density of states at** r

If $\Delta|\phi_q(r)|^2$ **is localized (compared to** $S(r)$**), Koonin formula becomes**

$$C_{ab}(\vec{v}_{\rm cm}, \vec{q}) = 1 + \frac{dn^{(\rm int)}_{\rm states}/dE}{dn^{(\rm free)}_{\rm states}/dE}$$

$$\frac{dn^{(free)}_{\rm states}}{dE} \sim \frac{1}{\rm volume}$$

Wave function and density of states

$$\frac{1}{2\pi}\int dr\left\{|\phi_q(r)|^2-|\phi_q^{(0)}(r)|^2\right\}=\frac{1}{\pi}\frac{d\delta}{dq}$$

$|\phi_q(r)|^2$ **describes contribution to density of states at** r

Femtoscopy:

If $\Delta|\phi_q(r)|^2$ **is localized (compared to** $S(r)$**), Koonin formula becomes**

$$C_{ab}(\vec{v}_{\rm cm},\vec{q})=1+\frac{dn^{(\rm int)}_{\rm states}/dE}{dn^{(\rm free)}_{\rm states}/dE}$$

$$\frac{dn^{(free)}_{\rm states}}{dE}\sim\frac{1}{\rm volume}$$

Coalescence:

If $|\phi_c(r)|^2$ **is localized, coalescence formula becomes**

$$P_c(\vec{v})=\frac{1}{(2\pi)^3}\int d^3r\; f_a(\vec{v},\vec{r})f_b(\vec{v},\vec{r})$$

Same as thermal model (aside from binding energy)

Wave function and density of states

BUT!!! wave function is not always localized

Deuteron r.ms. radius ≈ 3 **fm**

At small q**,** $\Delta\,|\,\phi_q(r)\,|^2$ **extends** $\approx 1/q$

Therefore:
a) Koonin/Coalescence formula necessary unless sources are large
b) Thermal model (which ignores extent of w.f.) is questionable for small sources
Example: $\pi^+\pi^-$ **bound (by Coulomb) states extend** ≈ 200 **fm**

Do you need a "good" wave function?

Femtoscopy:

Outside range of potential, $\phi_q(r) = \sin(qr + \delta)$

Overall strength of integrated w.f. is $(1/\pi) d\delta/dq$

Thus

all potentials give same answer if they have same δ **and** $d\delta/dq$

unless range of potential is large (e.g. *pd*)

Coalescence:

Outside range of potential, $\phi_c(r) = Ae^{-\sqrt{2\mu B}r}$

Thus

all potentials give same answer if they have same binding energy *B* and *A*.

unless range of potential is large

When are the approximations good?

Approximations are uncontrolled

Not out of control

The Four Approximations

The Truth:

$$P(p_a, p_b) = \sum_{f'} \left| \int dx_a dx_b T_{f'}(x_a, x_b) \phi_{f'}(x_a, x_b; p_a, p_b,) \right|^2$$

Sum over all "remainder" states f'

APPROXIMATIONS

1) $\phi(x_a, x_b; p_a, p_b)$ does not depend on f'

- **a) fails if phase space density is high (identical particles)**
  - **multi-particle symmetrization is important**
  - **otherwise, must calculate for all momenta, then integrate over all other particles**
- **b) fails if interaction with other particles lasts long time**
  - **at sufficiently small relative momentum, this is fine**
  - **Coulomb with other particles slowest other interaction**

$$\phi_f(x_a, x_b; p_a, p_b) \rightarrow \phi(x_a - x_b; p_a, p_b)$$

The Four Approximations

After first approximation:

$$P(p_a, p_b) = \sum_{f'} \left| \int dx_a dx_b T_{f'}(x_a, x_b) \phi(x_a, x_b; p_a, p_b,) \right|^2$$

APPROXIMATIONS

2) Emission ($T-$matrix) is independent.

Sum over f' and $T-$matrices must factorize

Ignores other correlations (energy/momentum/charge) conservation…

good at small relative momentum (other sources have longer characteristic scales)

$$\sum_{f'} \rightarrow \sum_{f'_a} \sum_{f'_b},$$

$$T_{f'}(x_a, x_b) \rightarrow T_{f'_a}(x_a) T_{f'_b}(x_b)$$

The Four Approximations

After approximations 1 & 2: **S.P. PRC 1997**

Define:

$$s_a(x,p) \equiv \sum_{f'_a} \int d\delta x \; T^*_{f'_a}(x+\delta x/2) T_{f'_a}(x-\delta x/2) e^{ip\cdot\delta x}$$

this gives

$$P_{ab}(p_a,p_b) = \int dx_a dx_b d\delta x d\tilde{q} s_a(\bar{P}_a + \tilde{q}, x_a) s_b(\bar{P}_b - \tilde{q}, x_b)$$
$$e^{i\tilde{q}\delta x} \phi^*_q(x_a - x_b + \delta x/2) \phi^*_q(x_a - x_b - \delta x/2)$$

3) Smoothness approximation:

a) Ignore $\tilde{q}$ **dependence in** $s_a(\bar{P}_a + \tilde{q}, x_a)$ **and** $s_b(\bar{P}_b - \tilde{q}, x_b)$**,**

b) replace $s_a(\bar{P}_a + \tilde{q}, x_a) s_b(\bar{P}_b + \tilde{q}, x_b)$ **with** $s_a(p_a, x_b) s_b(p_b, x_b)$ **or** $s_a(E_a, \vec{p}_{a,\mathrm{cm}} s_b(E_b, \vec{p}_{b,\mathrm{cm}})$

— good when emissions are thermal and matrices are broad

— questionable if relative momentum is small

— necessary if you don't know off-shell behavior of $s(p,x)$

— for coalescence you can add $e^{B/T}$ **factor**

The Four Approximations

Last approximation: non-simultaneous wave functions

$\phi_q(x_1 - x_2) = \phi(\Delta t = 0, \vec{x}_1 - \vec{x}_2)$ **in pair frame**

4) Non-simultaneous emission
— no problem for pure HBT
— should be fine for small relative momentum

Rearrangement Interactions

Sometimes interactions involve change of degrees of freedom:
No interaction through potential

Examples:
$K^+K^- \leftrightarrow \phi$
$\alpha d \leftrightarrow^6 \mathrm{Li}$

Wavefunction paradigm questionable — but thermal equilibrium still applies

When are the approximations good?

Femtoscopy:
- **Emission uncorrelated aside from FSI**
- **Relative motion is small, $q/\mu \lesssim 0.1$**
- **Phase space density not high**
  (as long as phase space densities $\lesssim 0.5$)
- **Range of interaction smaller than source size**
- **Rearrangement interactions, e.g. $K^+K^- \leftrightarrow \phi$**
  where wave function paradigm is questionable

Coalescence:
- **Same as above**
- **Wave function should not have high p components (low B)**
- **Should correct for binding energy: $e^{B/T}$**

Thermal:
- **Must be at freeze out!!**
- **Whenever wave function extent is $<<$ source size**
- **OK with rearrangement interactions**

Central H.I. Collisions:
— Usually very solid

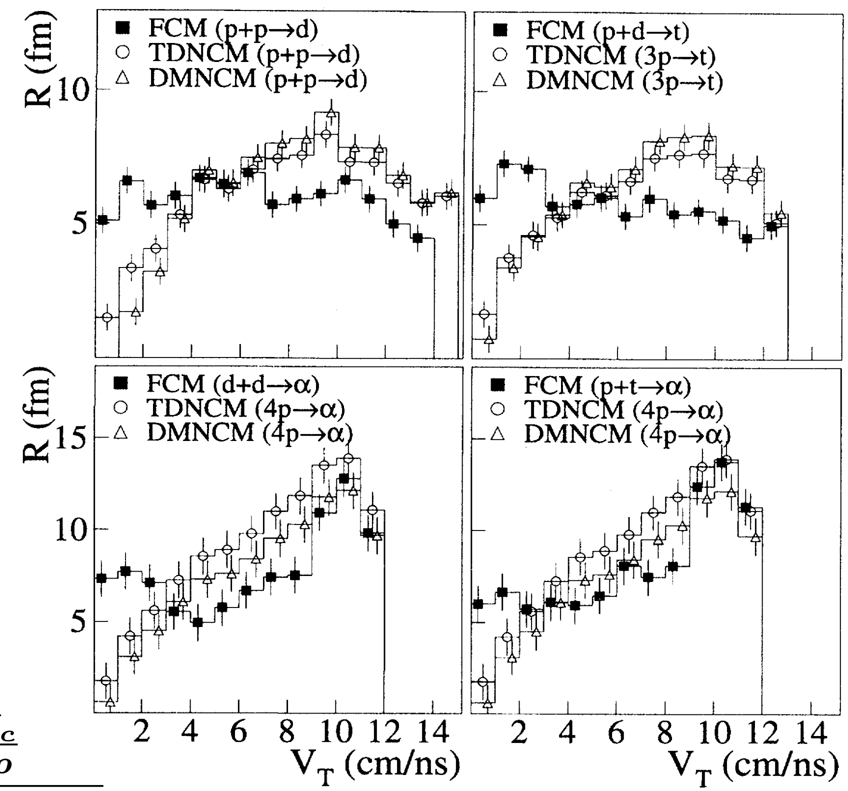
***pp* Collisions:**
— Be more careful

Rearrangment interactions
— Be careful

Complementary Roles

FROM FEMTOSCOPY:
$R_{\rm inv}, R_{\rm out}, R_{\rm side}, R_{\rm long}$ **+ plus radial form information**
all as function of y, p_t, ϕ

FROM COALESCENCE:
$R_{\rm inv}$ **or** $S_{ab}(r \approx 0)$
all as function of y, p_t, ϕ
deuteron spectra provides nothing additional beyond this



Llope et al., PRC 1995

For Gaussian:

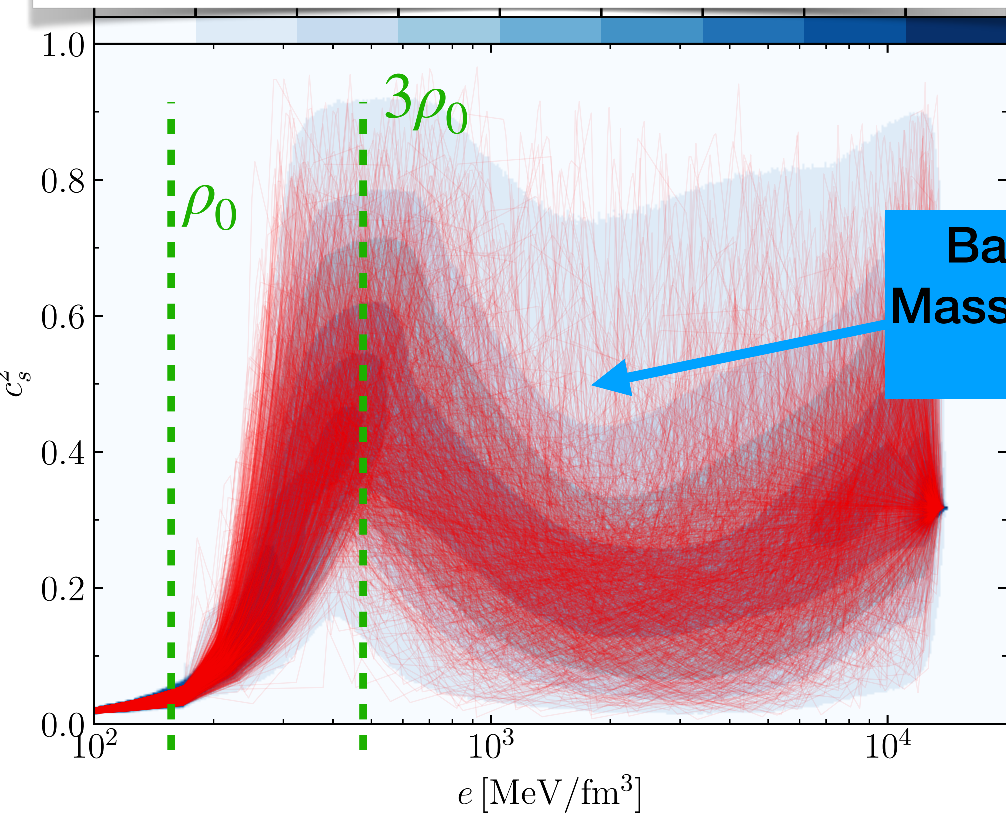
$$\frac{1}{(R^2+a^2/2)^{3/2}_{\rm inv}} = \frac{1}{\pi^{3/2}} \frac{(2S_c+1)}{(2S_a+1)(2S_b+1)} \frac{\frac{E}{m_c}\frac{dN_c}{d^3p}}{\frac{E}{m_a}\frac{dN_a}{d^3p}\frac{E}{m_b}\frac{dN_b}{d^3p}}$$

evaluated at same $\vec{v}$

Because you take third root — this is very accurate!

Scott's Femtoscopic Wish List

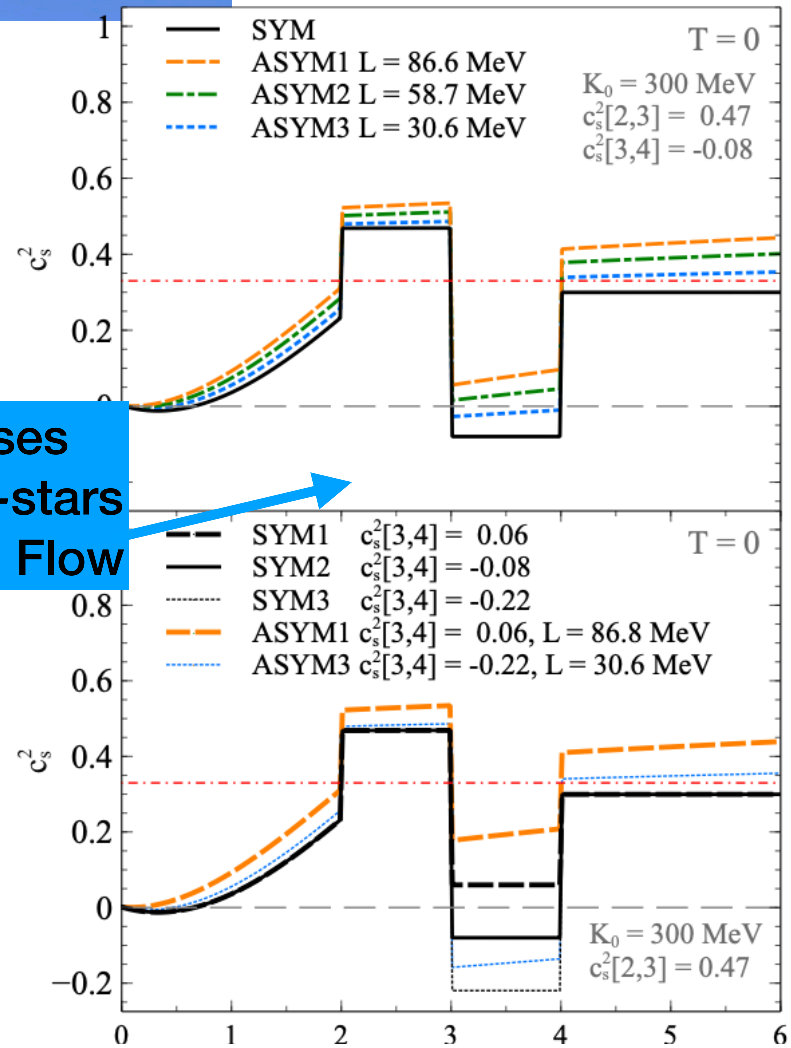
Focus on measurements related to bulk properties
Hyper-stiff EoS for $2\rho_0 < \rho_B < 4\rho_0$????
QCD Phase transition????



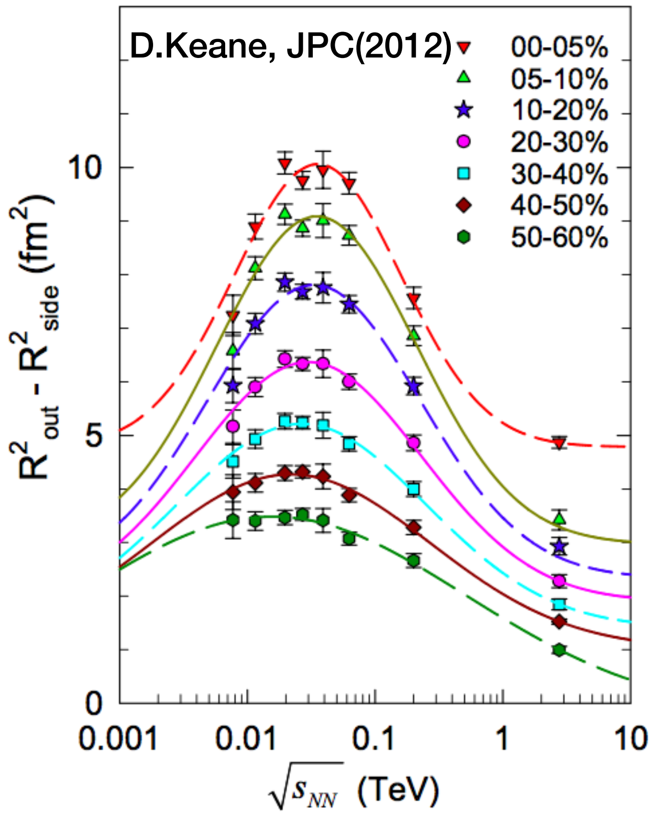
S.Altiparmak,C.Ecker,L.Rezzola, Ast.J.Lett. (2022)

Bayesian analyses
Mass-Radius of *n*-stars
H.I. Flow

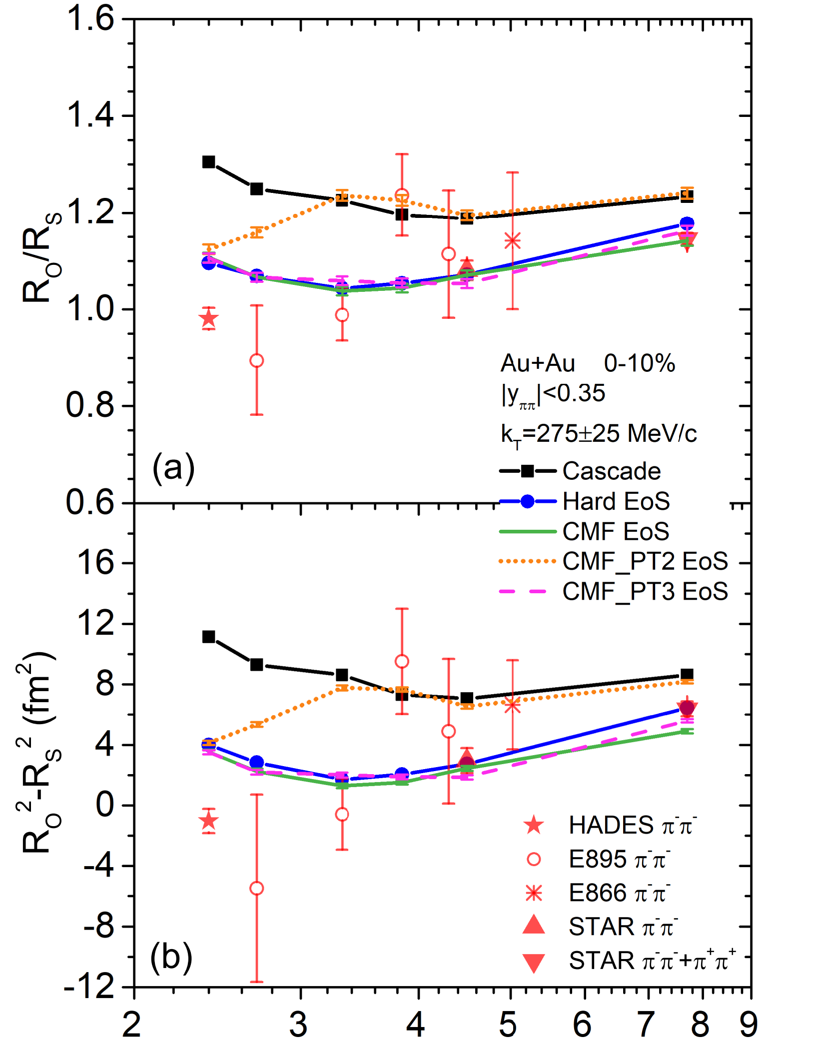
D.Oliinychenko,A.Sorensen,
V.Koch,L.McLerran
nucl-th 2208.11996



P.Li,J.Steinheimer,A.Kittiratpattana,M.Bleicher & Q.Li
Sci.China, Phys.Mech.Astron. (2023)

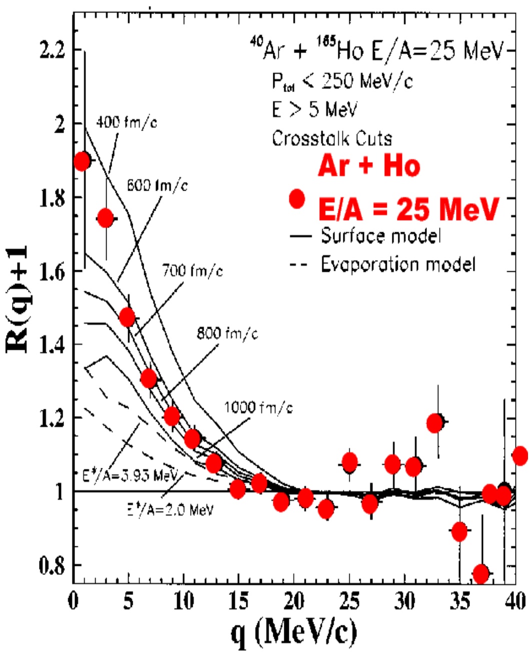


$\pi - \pi$ femtoscopy

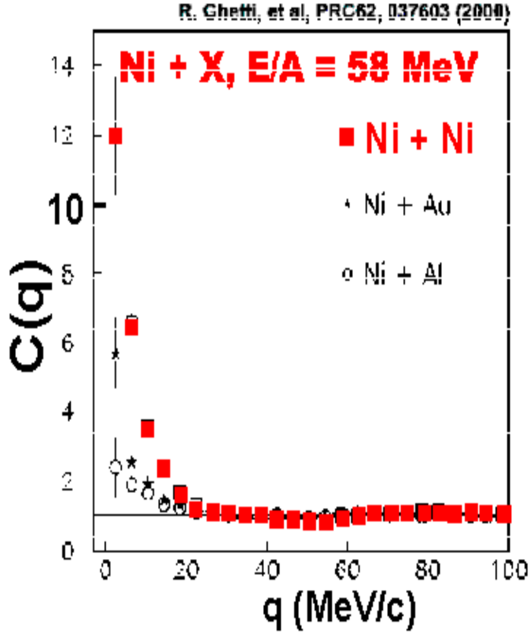


neutron-neutron correlations
25A MeV and 58A MeV

S.Gaff et al, PRC58, 2161 (98)



R. Ghetti, et al, PRC62, 037603 (2000)



Expand $\pi - \pi, K - K, p - p$
— full 6-dimensional
— lower energy: No boost invariance
— imaging
— relation to EoS

Coordinated analysis with *d* coalescence
— full 3-dimensional

Neutron-neutron femtoscopy
— no Coulomb
— shape

Non-identical femtoscopy
— relation fo EoS