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Two-particle interferometry with Lévy-stable sources in $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions at **STAR**



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Eötvös University, Budapest



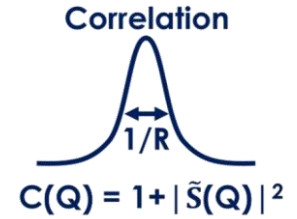
Catania (Italy), November 6-10, 2023

Part I. Introduction, Motivation

Catania (Italy), November 6-10, 2023



Basic definitions of femtoscopic correlation functions



• Single particle momentum distribution: $N_1(p) = \int d^4x S(x, p)$ phase-space density

• Pair momentum distribution: $N_2(p_1, p_2) = \int d^4x_1 d^4x_2 S(x_1, p_1) S(x_2, p_2) |\psi_{p_1, p_2}(x_1, x_2)|^2$

• Correlation function: $C(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1) N_2(p_2)}$ pair separation

• Pair source/spatial correlation: $D(r, K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$ pair center-of-mass

relative pair momentum
average pair momentum*

Pair wave function, containing FSI!

$$C(Q, K) = \int d^4r D(r, K) |\psi_Q(r)|^2$$

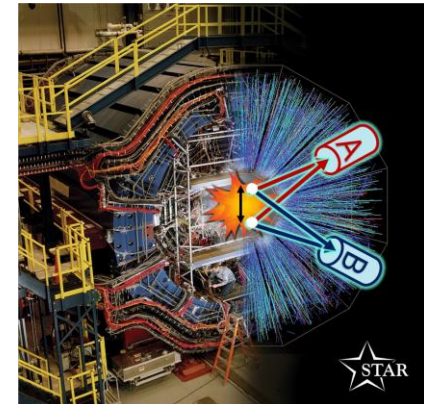
*Instead of K , m_T is often used:
 $m_T = \sqrt{k_T^2 + m_\pi^2}, k_T = \sqrt{K_x^2 + K_y^2}$

• Experiments: measuring $C(Q) \rightarrow$ information about $D(r)$ and FSI

• Experimental (and phenomenological) indications:
power-law tail for pions, **non-Gaussianity?**

D. A. Brown and P. Danielewicz. In: Phys. Lett. B 398 (1997), pp. 252–258.

S. S. Adler et al., PHENIX Coll. In: Phys. Rev. Lett. 98 (2007), p. 132301.



$$S(\mathbf{r}) = \mathcal{L}(\alpha, R; \mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3q e^{i\mathbf{q}\mathbf{r}} e^{-\frac{1}{2}|\mathbf{q}^T \mathbf{R} \mathbf{q}|^{\alpha/2}}$$

spherical sym.: $R_{ij} = R^2 \delta_{ij}$

What is the shape of the source? Gaussian vs. Lévy distributions in heavy-ion physics

Csörgő, Hegyi, Zajc, Eur.Phys.J.C 36 (2004) 67

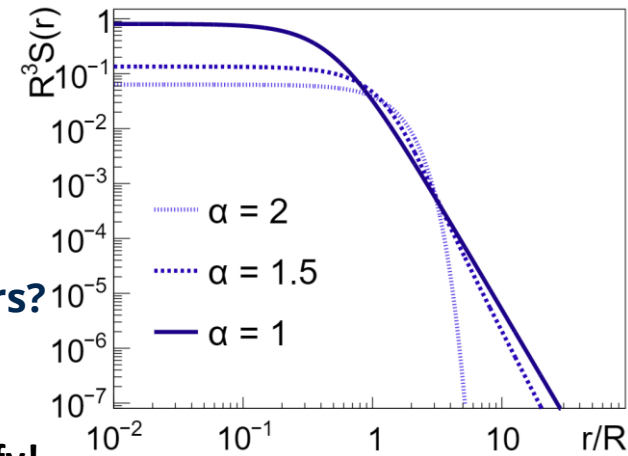
• Symmetric Lévy-stable distribution

- From generalized central limit theorem, power-law tail (if $\alpha < 2$) $\sim r^{-(1+\alpha)}$
- $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy
- Retains the same α under convolution

$$S(r) = \mathcal{L}(\alpha, R; r)$$

↓

$$D(r) = \mathcal{L}(\alpha, 2^{1/\alpha} R; r)$$



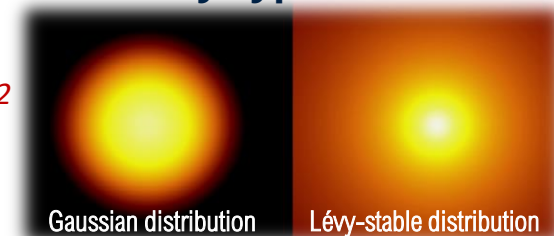
• Indications in heavy ion coll. – Lévy source for pion pairs?

- RHIC (PHENIX, STAR) *Phys.Rev. C97 (2018) no.6, 064911*
- LHC (CMS) *Phys.Part.Nucl. 51 (2020) 3, 267-269*
- SPS (NA61/SHINE) *arXiv:2306.11574 (CMS-HIN-21-011)*
- + LEP (L3) in e^+e^- *Eur.Phys.J.C 83 (2023) 10, 919*

See talk of
M. Csanád
and **B. Pórfy!**

• Some of the possible (competing) reasons for the appearance of Lévy-type sources:

- Jet fragmentation *Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36*
- Critical behavior *Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828*
- Event averaging *Cimerman, Tomasik, Plumberg, Phys.Part.Nucl. 51 (2020) 3, 282*
- Resonance decays *Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308*
- Anomalous diffusion *Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002;*



• Lévy in 1D: not because of 3D→1D conversion! (see backup)

New data analyses and phenomenological investigations are needed to gain a better understanding!

$$S(\mathbf{r}) = \mathcal{L}(\alpha, R; \mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3q e^{i\mathbf{q}\mathbf{r}} e^{-\frac{1}{2}|\mathbf{q}^T \mathbf{R} \mathbf{q}|^{\alpha/2}}$$

spherical sym.: $R_{ij} = R^2 \delta_{ij}$

Lévy source at top RHIC energy

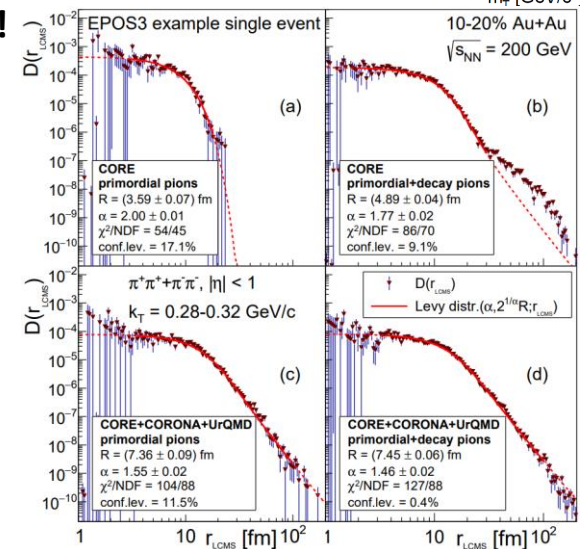
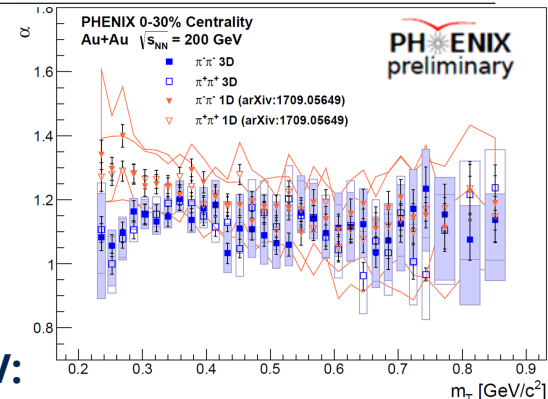
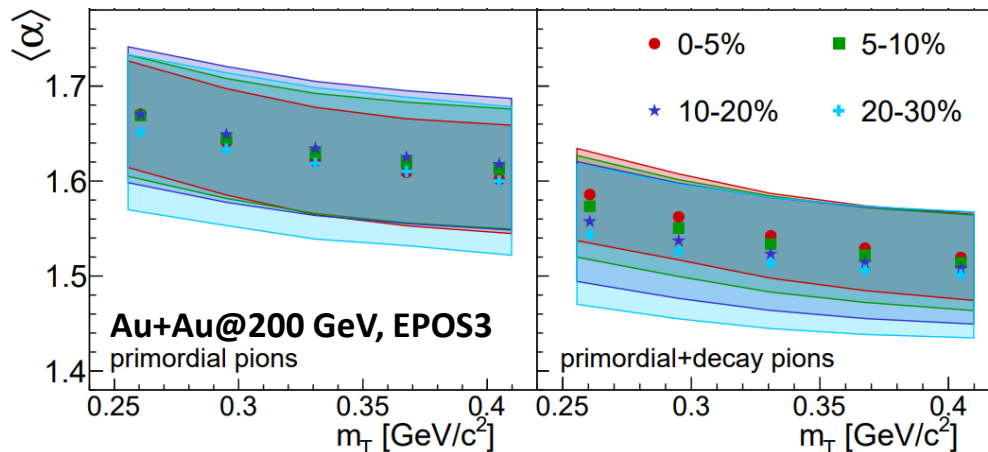
- Not spherically sym. source: 3D vs. 1D α compatible!
 $\alpha < 2$ in 1D analyses not because of angle averaging!

Kurgyis, Acta Phys. Pol. B Proc. Suppl. vol. 12 (2), 477 (2019)

$$\mathbf{R} = \begin{pmatrix} R_{out}^2 & 0 & 0 \\ 0 & R_{side}^2 & 0 \\ 0 & 0 & R_{long}^2 \end{pmatrix} \quad \mathbf{q} = \begin{pmatrix} q_{out} \\ q_{side} \\ q_{long} \end{pmatrix}$$

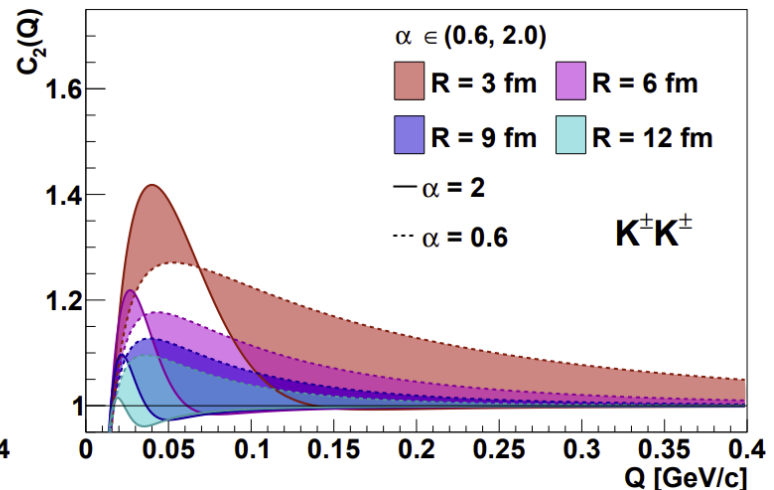
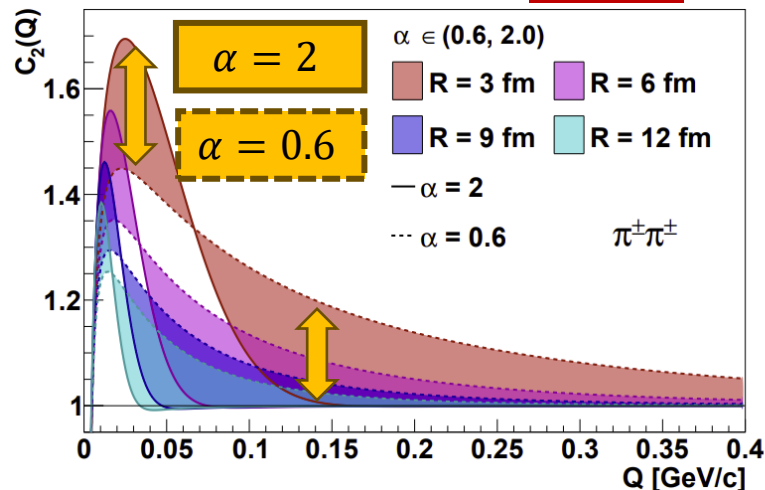
- EPOS event-by-event pion pair-source analysis at 200 GeV:
Decays and hadronic rescattering (UrQMD) play an important role!

Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308



Recent phenomenological developments

- **Coulomb Corrections for Bose-Einstein Correlations from One- and Three-Dimensional Lévy-Type Source Functions** *Kurgyis, Kincses, Nagy, Csanád, Universe 9 (2023) 7, 328*
- **Event-by-Event Investigation of the Two-Particle Source Function in Heavy-Ion Collisions with EPOS** *Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308* **See talk of M. Csanád!**
Kórodi, Kincses, Csanád, Phys.Lett.B (accepted) arXiv:2212.02980
- **A novel method for calculating Bose-Einstein correlation functions with Coulomb final-state interaction** *Nagy, Purzsa, Csanád, Kincses, Eur.Phys.J.C (accepted) arXiv:2308.10745* **See talk of M. Nagy!**



Part II.

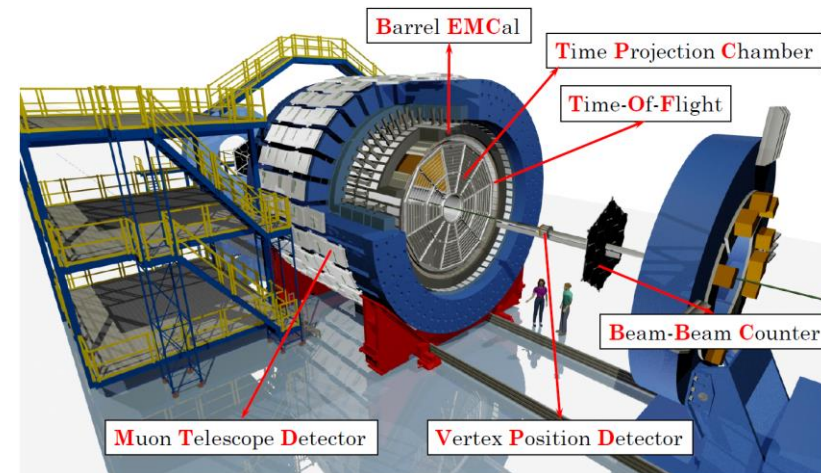
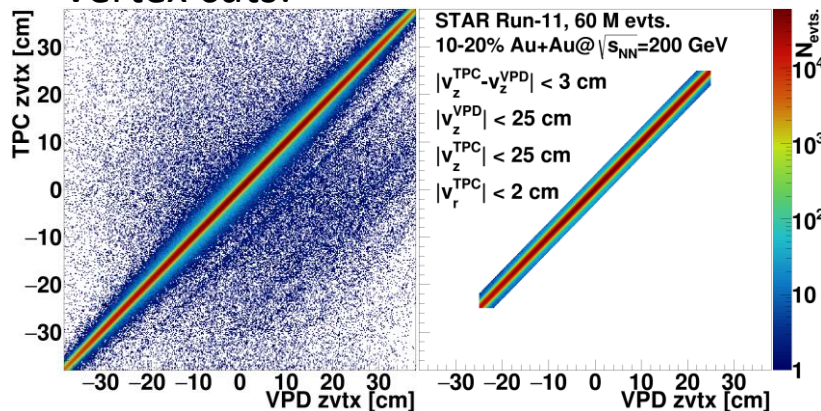
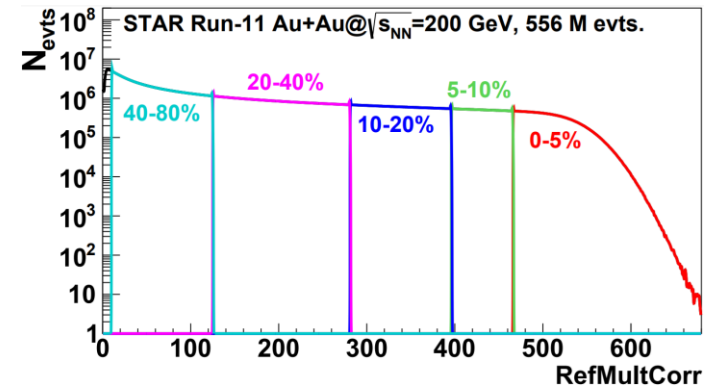
Measurement and fitting of correlation functions



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Lévy HBT analysis at STAR, Au+Au @ 200 GeV

- **STAR Run-11 data analyzed**
After trigger cuts and bad run cuts: **550M events**
- **Detectors used for the analysis:**
 - **BBC, TPC, VPD:** centrality, vertex position
 - **TPC:** tracking, dE/dx Particle Identification (PID)
 - **TOF:** time-of-flight PID
- **Event selection:**
 - Pile-up cuts using TOF vs. TPC multiplicity
 - Vertex cuts:



Measurement of two-pion correlation functions

• Track-selection criteria

- Combined PID using TPC $N\sigma$ (based on dE/dx) and TOF $N\sigma$ (based on time-of-flight)
- Further single-track cuts on TPC number of hits, p_T , η , Distance of Closest Approach (DCA)

• Pair-selection criteria

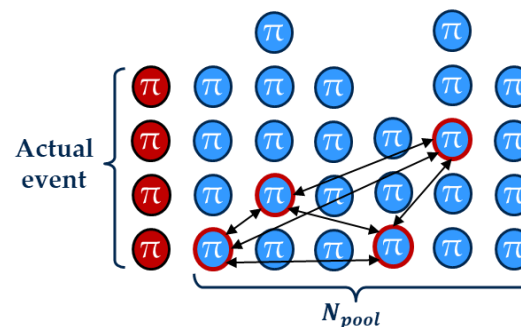
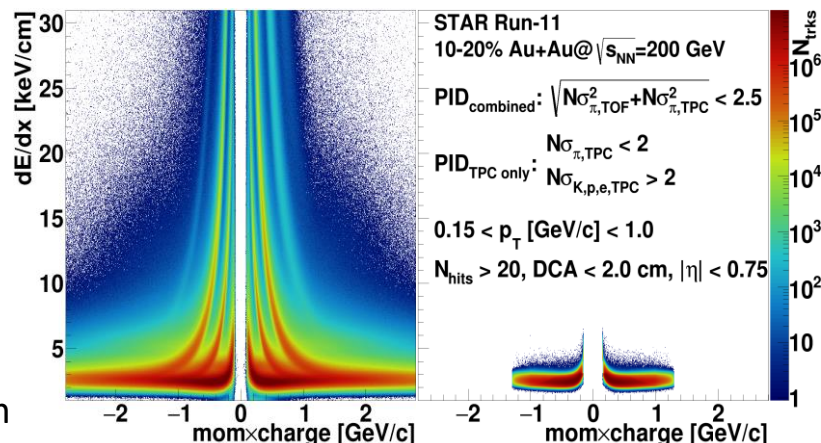
- Splitting level (SL) < 0.6 *J. Adams et al. (STAR Coll.), Phys. Rev. C 71, 044906 (2005)*
- Fraction of Merged Hits (FMH) < 5%
- Average pair-separation (on TPC pad rows) $\Delta r > 3$ cm

• Event mixing

- Similarly to *PHENIX Coll., Phys.Rev. C97 (2018) no.6, 064911*
- 2 cm wide z vertex bins, 5% wide centrality bins
- A(Q): pions from the same event
- B(Q): pions from different events
- C(Q)=A(Q)/B(Q), appropriately normalized

• Centrality and k_T selection:

- 21 k_T bins, (0.175 - 0.750) GeV/c
- 4 centrality bins (0-10%, 10-20%, 20-30%, 30-40%)



Fitting process with Lévy parametrization

- Lévy parametrization without final state effects:

$$C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^\alpha}$$

LCMS three-momentum difference $Q = |q_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$

Lévy exponent α
 Lévy scale parameter R
 Intercept parameter (correlation strength) λ

- Formula used for fitting procedure:

$$C(Q) = \underbrace{(1 - \lambda + \lambda \cdot K(Q; \alpha, R))}_{\text{Coulomb correction}} \cdot \underbrace{(1 + e^{-|RQ|^\alpha})}_{\text{Possible linear background (usually negligible)}} \cdot N \cdot (1 + \epsilon Q)$$

- Coulomb-correction:

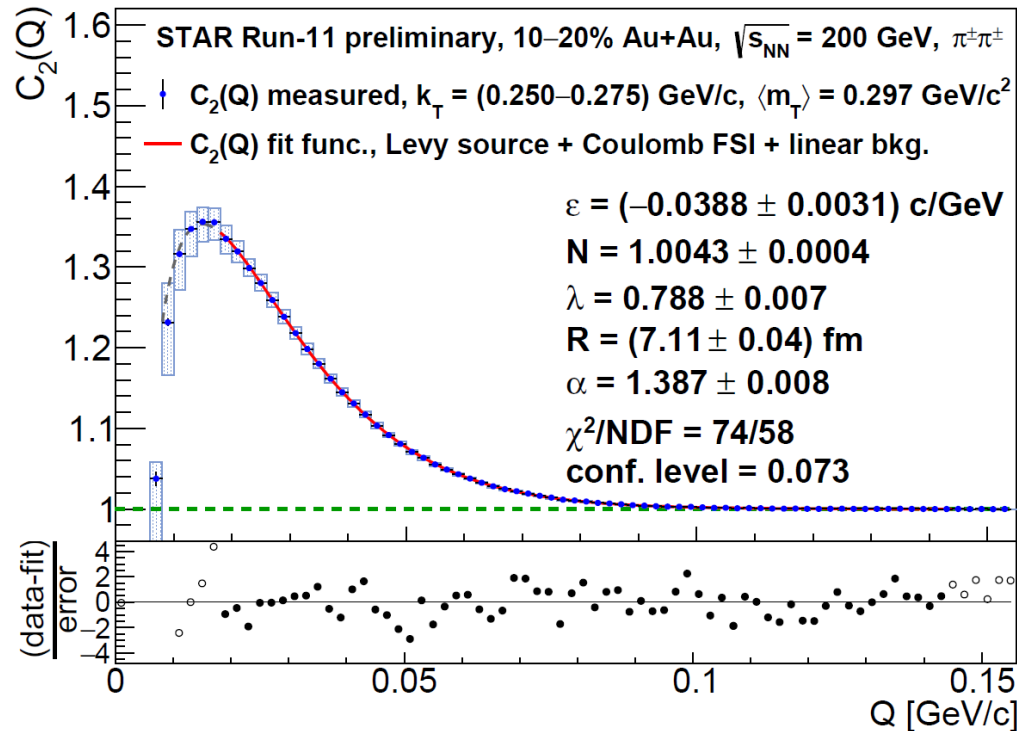
$$K(Q; \alpha, R) = \frac{\int D(\mathbf{r}) |\psi^{Coul}(\mathbf{r})|^2 d\mathbf{r}}{\int D(\mathbf{r}) |\psi^{(0)}(\mathbf{r})|^2 d\mathbf{r}}$$

Two-particle wave function (with Coulomb interaction) ψ^{Coul}
 Two-particle wave function (plane wave) $\psi^{(0)}$
 Spatial correlations $D(\mathbf{r})$
 calculated numerically
Kincses, Nagy, Csanád, Phys.Rev.C 102 (2020) 6, 064912

$$C(Q) = (1 - \lambda + \lambda \cdot K(Q; \alpha, R) \cdot (1 + e^{-|RQ|^\alpha})) \cdot N \cdot (1 + \varepsilon Q)$$

An example fit to a two-pion correlation function

- Example fit in the 10-20% centrality class, at $k_T = (0.250-0.275)$ GeV/c
- **Iterative fitting method**, Coulomb FSI + Lévy-source
- Track and pair **sys. uncertainties illustrated with boxes**
- Low-Q points left out from fit, **fit range study included in total systematic uncertainties**
- **Fits converged, conf.level > 0.1%**
- Confidence levels approx. uniformly distributed
- Similar fits done in all 4 centrality class and 21 k_T bins, **centrality and m_T dependence of the source parameters investigated**



Part III.
Preliminary results,
Summary, outlook

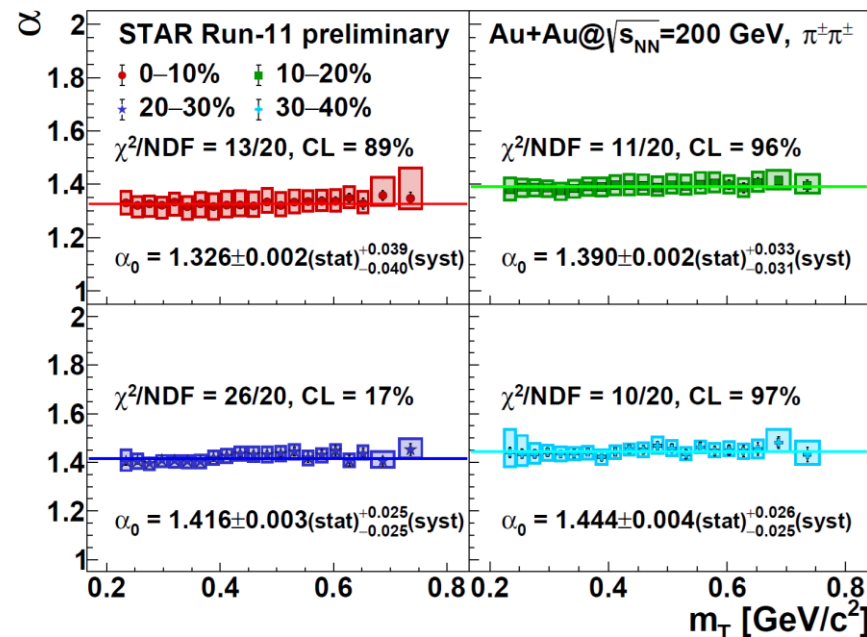
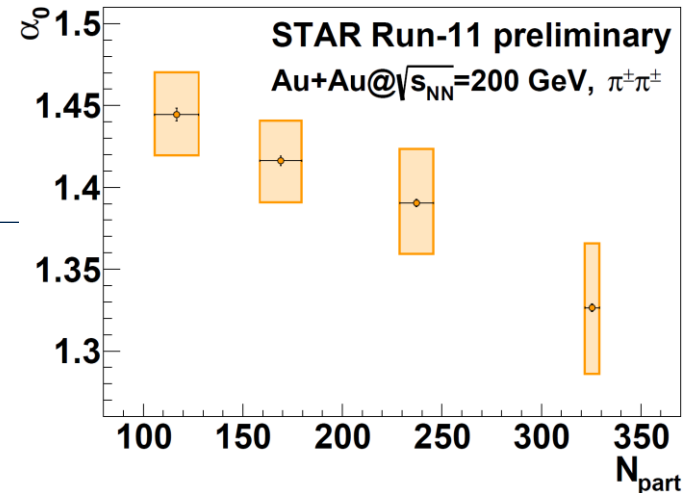
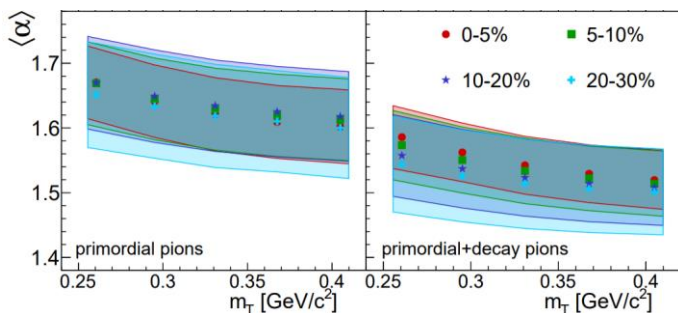


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Lévy exponent $\alpha(m_T, \text{centrality})$

- Non-gaussian values ($\alpha \ll 2$)
- No dependence on m_T , slight centrality dep.
- α_0 vs N_{part} (m_T average values from constant fit)
 - Decreasing trend due to anti-correlation with λ and R ?
 - CMS observed opposite trend, see talk of M. Csanád
- EPOS model: slightly higher α values

D. Kincses, M. Stefaniak, M. Csanád, Entropy 24 (2022) 3, 308

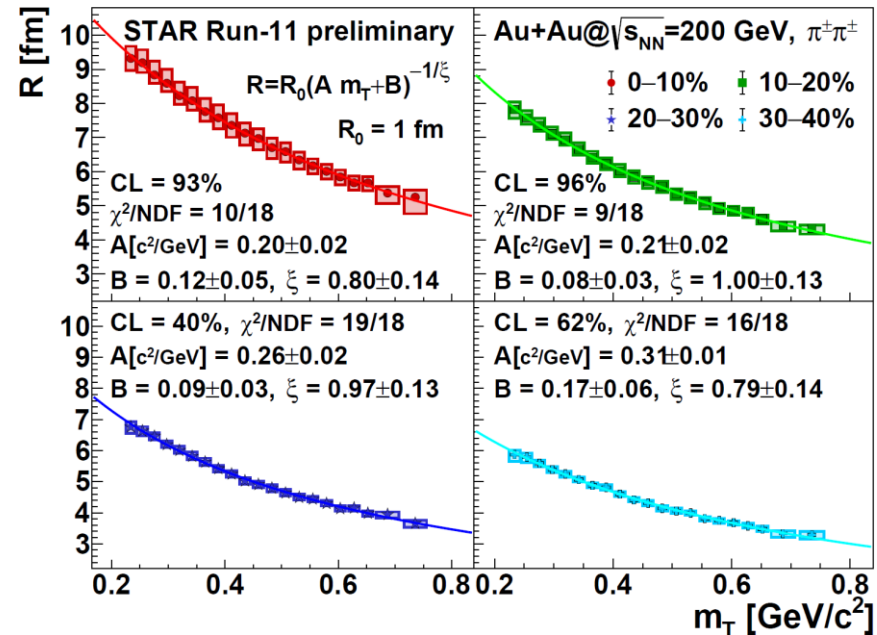
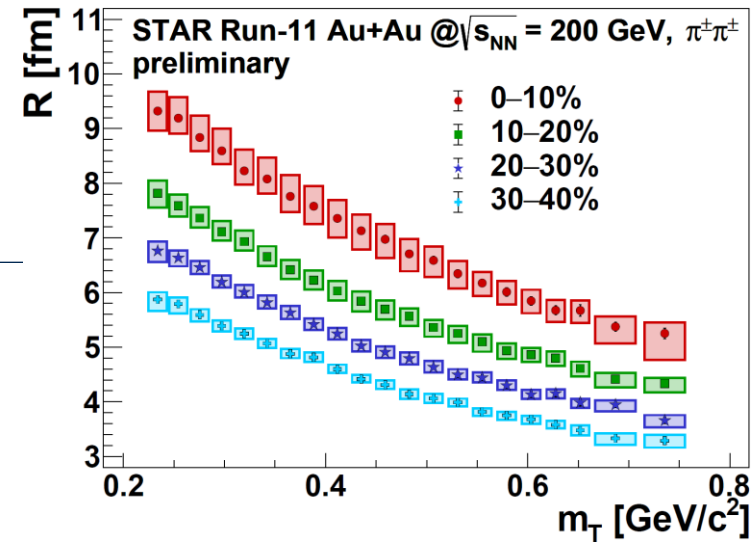


Lévy scale $R(m_T, \text{centrality})$

- Decreasing trend with m_T , also with centrality
- Connection to flow and initial geometry?
- Fits with $R = R_0(Am_T + B)^{-1/\xi}$, $R_0 = 1$ fm
- Hydro calculations (R_{Gauss}): $\xi = 2$

Makhlin, Sinyukov, Z. Phys. C 39, 69 (1988)
Csörgő, Lörstad, Phys. Rev. C 54, 1390 (1996)
Chapman, Scotto, Heinz, Acta Phys.Hung.A 1 (1995) 1-31
Csanád, Csörgő, Lörstad, Ster, J. Phys. G 30, S1079 (2004)

- Our case (R_{Levy}): ξ close to 1



Correlation strength $\lambda(m_T, \text{centrality})$

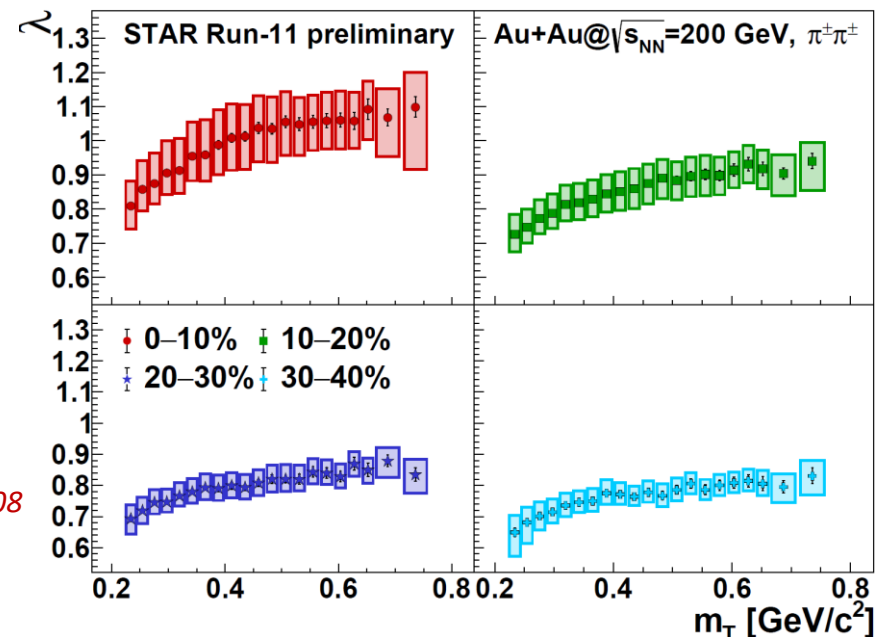
- Without FSI: $C_2^{(0)}(Q = 0) = 2 \rightarrow \lambda \equiv \lim_{Q \rightarrow 0} C_2^{(0)}(Q) - 1$, experimentally often $\lambda < 1$
- Core-halo picture** - two-component source: $S = S_{core} + S_{halo} \Rightarrow D = D_{(c,c)} + D_{(c,h)} + D_{(h,h)}$
 - Core: primordial + decays of short-lived resonances
 - Halo: decays of long-lived resonances \rightarrow large R \rightarrow small Q \rightarrow measurement limited

$$\lambda = N_{core}^2 / (N_{core} + N_{halo})^2$$

Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497

Bolz et al, Phys.Rev. D47 (1993) 3860-3870

- For power-law sources, more complicated picture!**
- Increase from low to high m_T**
 - More decay products at low m_T ?
 - In-medium mass modification of η '?
 - Partially coherent particle emission?
- Vance, Csörgő, Kharzeev, Phys. Rev. Lett.81 (1998), pp. 2205–2208*
- Bolz et al., Phys. Rev. D47 (1993), pp. 3860–3870*
- Decrease from central to peripheral**



Summary

- 1-dim. two-pion correlation functions investigated
- Lévy-source + Coulomb FSI → good description
- Further syst. uncertainty investigations underway
- Similar analyses already ongoing:
 - Pions in 3D (out,side,long) @200 GeV
 - Kaons in 1D @200 GeV
 - Pions in 1D at lower energies

Stay tuned!

Next updates at:

ZIMÁNYI SCHOOL 2023

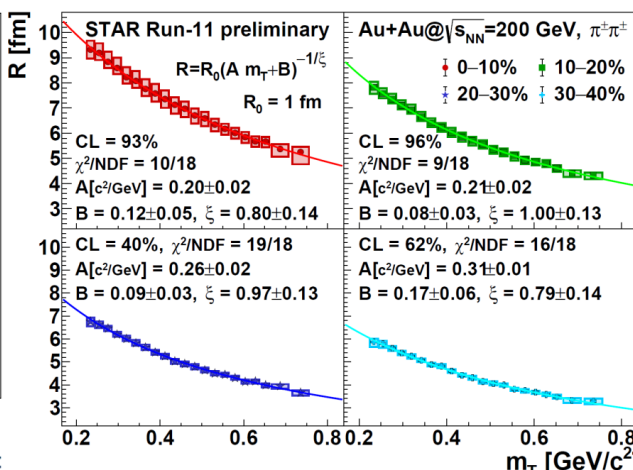
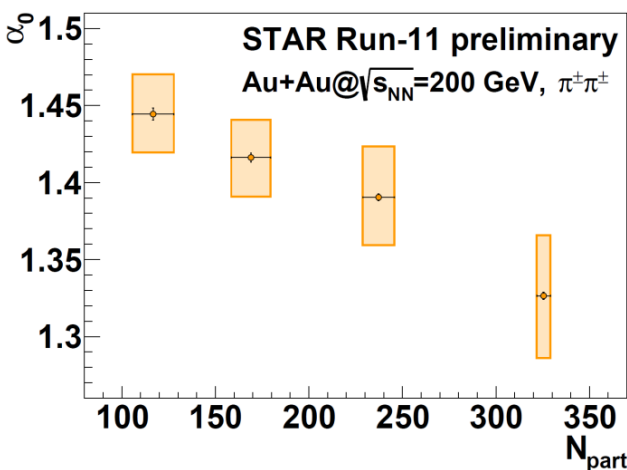
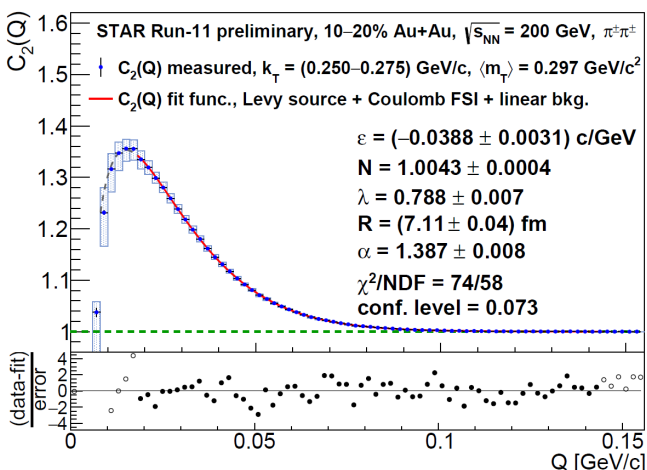


23rd ZIMÁNYI SCHOOL
WINTER WORKSHOP
ON HEAVY ION PHYSICS

December 4-8, 2023
Budapest, Hungary



József Zimányi (1931 - 2006)



Further details, backup slides



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Systematic uncertainties

- **Systematic uncertainties already investigated:**
 - Single track- and pair-cut variations, fit limits
- **Systematic uncertainties to be investigated:**
 - Purity correction, momentum smearing, more detailed fit limit study, strong interaction

TABLE II. Sources of systematic uncertainties.

n	source of uncertainty	settings ($j = 0, 1, \dots$)
0	NHitsFit cut	18, 20, 22
1	DCA cut	1.5 cm, 2 cm, 2.5 cm
2	$ \eta $ cut	0.5, 0.75, 1.0
3	PID $N\sigma$	default, loose, strict
4	SL cut	0.5, 0.6, 0.7
5	FMH cut	0%, 5%, 10%
6	$\Delta u, \Delta z$ limits	default, loose, strict
7	Lower fit limit in Q	default, +1 bin, -1 bin
8	Higher fit limit in Q	default, +5 bin, -5 bin

TABLE III. m_T averaged asymmetric systematic uncertainties [%].

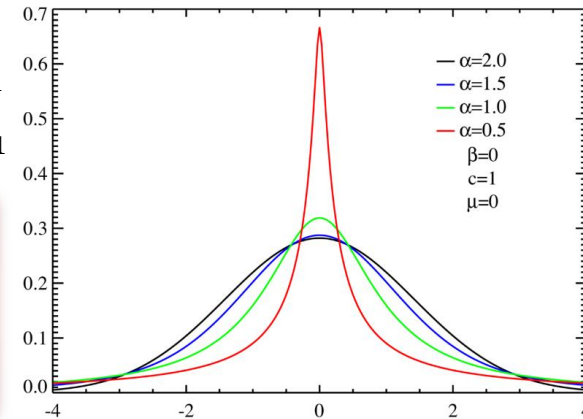
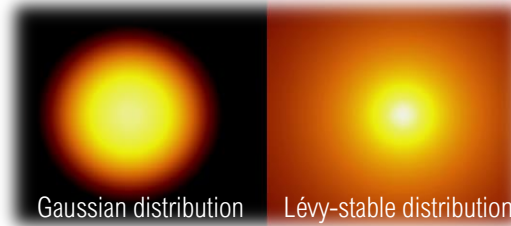
		λ								R								α							
n		0–10%		10–20%		20–30%		30–40%		0–10%		10–20%		20–30%		30–40%		0–10%		10–20%		20–30%		30–40%	
		$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$	$\langle\delta\uparrow\rangle$	$\langle\delta\downarrow\rangle$
0		3.4	2.1	1.6	1.1	0.7	0.5	0.3	0.2	1.4	0.9	0.7	0.5	0.3	0.2	0.2	0.1	0.8	1.2	0.4	0.7	0.2	0.3	0.1	0.2
1		4.2	2.9	3.6	2.6	3.3	2.5	3.0	2.2	0.7	0.4	0.4	0.3	0.3	0.3	0.3	0.2	0.2	0.4	0.2	0.3	0.2	0.2	0.2	0.2
2		1.2	1.1	0.4	0.6	0.7	0.8	0.6	1.5	0.6	0.7	0.2	0.4	0.4	0.5	0.3	0.9	0.7	0.6	0.4	0.3	0.4	0.4	1.0	0.4
3		1.3	0.4	1.3	0.6	1.3	0.6	1.7	1.0	0.7	0.3	0.6	0.3	0.6	0.4	0.8	0.6	0.2	0.6	0.2	0.6	0.3	0.6	0.6	0.8
4		0.2	0.1	0.1	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.0
5		5.4	6.0	4.0	4.4	3.0	3.1	2.5	2.8	2.6	3.1	2.0	2.2	1.5	1.5	1.2	1.3	2.5	1.8	1.9	1.6	1.4	1.3	1.3	1.2
6		3.5	4.2	2.5	3.0	1.9	2.1	1.7	1.9	1.7	2.1	1.3	1.5	0.9	1.0	0.8	0.9	1.7	1.3	1.3	1.0	1.0	0.8	0.9	0.8
7		0.5	0.5	0.7	0.7	0.5	0.4	0.9	0.6	0.3	0.3	0.4	0.4	0.3	0.2	0.5	0.3	0.2	0.3	0.4	0.4	0.2	0.3	0.4	0.6
8		1.4	1.1	0.7	0.8	0.7	0.8	1.1	1.2	0.8	0.6	0.4	0.5	0.3	0.4	0.3	0.3	0.7	0.9	0.6	0.5	0.6	0.5	0.8	0.7
	Σ	8.7	8.3	6.4	6.2	5.2	4.7	4.9	4.6	3.7	4.0	2.6	2.8	2.0	2.0	1.8	1.9	3.3	2.9	2.5	2.2	1.9	1.8	2.2	1.9

Properties of univariate stable distributions

- **Univariate stable distribution:** $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(q) e^{-ixq} dq$, where the characteristic function:

- $\varphi(q; \alpha, \beta, R, \mu) = \exp(iq\mu - |qR|^\alpha (1 - i\beta \operatorname{sgn}(q)\Phi))$
- α : index of stability
- β : skewness, symmetric if $\beta = 0$
- R : scale parameter
- μ : location, equals the median,
if $\alpha > 1$: $\mu = \text{mean}$

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), & \alpha \neq 1 \\ -\frac{2}{\pi} \log|q|, & \alpha = 1 \end{cases}$$



- **Important characteristics of stable distributions:**

- Retains same α and β under convolution of random variables
- Any moment greater than α isn't defined

In 3D: $\mathcal{L}(\mathbf{r}; \alpha, R) = \frac{1}{(2\pi)^3} \int d^3q e^{i\mathbf{q}\mathbf{r}} e^{-\frac{1}{2} |qRq|^\alpha}$

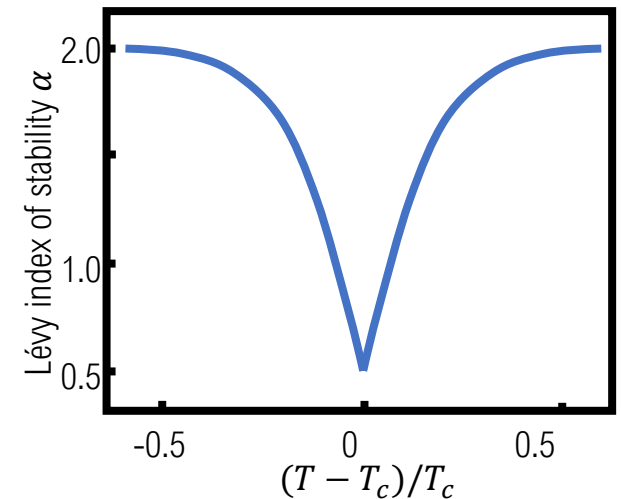
$$R_{\sigma\nu}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & R_{\text{out}}^2 & 0 & 0 \\ 0 & 0 & R_{\text{side}}^2 & 0 \\ 0 & 0 & 0 & R_{\text{long}}^2 \end{pmatrix}$$

Second order phase transition?

- Second order phase transitions: **critical exponents**
 - **Near the critical point**
 - Specific heat $\sim ((T - T_c)/T_c)^{-\alpha}$
 - Order parameter $\sim ((T - T_c)/T_c)^{-\beta}$
 - Susceptibility/compressibility $\sim ((T - T_c)/T_c)^{-\gamma}$
 - Correlation length $\sim ((T - T_c)/T_c)^{-\nu}$
 - **At the critical point**
 - Order parameter $\sim (\text{source field})^{1/\delta}$
 - **Spatial correlation function** $\sim r^{-d+2-\eta}$
 - Ginzburg-Landau: $\alpha = 0, \beta = 0.5, \gamma = 1, \eta = 0.5, \delta = 3, \nu = 0$
- QCD \leftrightarrow 3D Ising model
- Can we measure the η power-law exponent?
- Depends on spatial distribution: measurable with femtoscopy!
- **What distribution has a power-law exponent? Levy-stable distribution!**

Lévy index as critical exponent?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class \leftrightarrow 3D Ising
Halasz et al., Phys.Rev.D58 (1998) 096007
Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
 - Random field 3D Ising: $\eta = 0.50 \pm 0.05$
Rieger, Phys.Rev.B52 (1995) 6659
 - 3D Ising: $\eta = 0.03631(3)$
El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- **Change in α_{Levy} - proximity of CEP?**



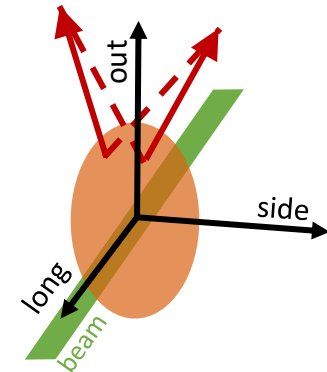
- Modulo finite size/time and non-equilibrium effects
- Other possible reasons for Lévy distributions: anomalous diffusion, QCD jets, ...

Kinematic variables of the correlation function I.

- Smoothness approximation ($p_1 \approx p_2 \approx K$): $S(x_1, K - q/2) S(x_2, K + q/2) \approx S(x_1, K) S(x_2, K)$
 - $C_2(q, K) = \int d^4r D(r, K) \left| \psi_q^{(2)}(r) \right|^2$
 - Without any FSI $\left| \psi_q^{(2)}(r) \right|^2 = 1 + \cos(qr)$
- $$\left. \begin{array}{l} C_2(q, K) = \int d^4r D(r, K) \left| \psi_q^{(2)}(r) \right|^2 \\ \text{Without any FSI } \left| \psi_q^{(2)}(r) \right|^2 = 1 + \cos(qr) \end{array} \right\} C_2^{(0)}(q, K) \simeq 1 + \frac{\tilde{D}(q, K)}{\tilde{D}(0, K)}, \text{ where } \tilde{D}(q, K) = \int D(x, K) e^{iqx} d^4x$$
- **HBT correlation function in direct connection with Fourier transform of the pair-source function**
 - Important to determine the nature and dimensionality of the correlation function
 - Lorentz-product of $q = (q_0, \mathbf{q})$ and $K = (K_0, \mathbf{K})$ is zero, i.e.: $qK = q_0 K_0 - \mathbf{q}\mathbf{K} = 0$
 - Energy component of q can be expressed as $q_0 = \mathbf{q} \frac{K}{K_0}$
 - If the energy of the particles are similar, K is approximately on shell
 - **Correlation function can be measured as a function of three-momentum variables**

Kinematic variables of the correlation function II.

- $C_2(\mathbf{q}, \mathbf{K})$ as a function of three-momentum variables
- \mathbf{K} dependence is smoother, \mathbf{q} is the main kinematic variable
- Close to mid-rapidity one can use $k_T = \sqrt{K_x^2 + K_y^2}$, or $m_T = \sqrt{k_T^2 + m^2}$
- For any fixed value of m_T , the correlation function can be measured as a function of \mathbf{q} only
- Usual decomposition: **out-side-long or Bertsch-Pratt (BP) coordinate-system**
 - $\mathbf{q} \equiv (q_{out}, q_{side}, q_{long})$
 - long: beam direction
 - out: k_T direction
 - side: orthogonal to the others
 - Essentially a rotation in the transverse plane
- Customary to use a Lorentz-boost in the long direction and change to the **Longitudinal Co-Moving System (LCMS)** where the average longitudinal momentum of the pair is zero



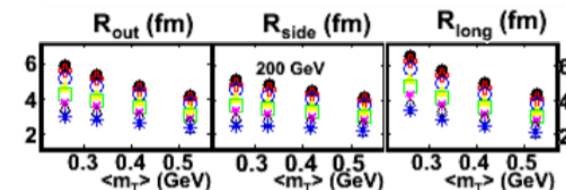
Kinematic variables of the correlation function III.

- Drawback of a 3D measurement: lack of statistics, difficulties of a precise shape analysis
- **What is the appropriate one-dimensional variable?**
- Lorentz-invariant relative momentum: $q_{inv} \equiv \sqrt{-q^\mu q_\mu} = \sqrt{q_x^2 + q_y^2 + q_z^2 - (E_1 - E_2)^2}$
- Equivalent to three-mom. diff. in Pair Co-Moving System (PCMS), where $E_1 = E_2$: $q_{inv} = |q_{PCMS}|$
- In LCMS using BP variables: $q_{inv} = \sqrt{(1 - \beta_T)^2 q_{out}^2 + q_{side}^2 + q_{long}^2}$ $\beta_T = 2k_T / (E_1 + E_2)$
- **Value of q_{inv} can be relatively small even when q_{out} is large!**
- Experimental indications: **in LCMS source is \approx spherically symmetric**
- Correlation function boosted to PCMS will not be spherically symmetric
- Let us introduce the following variable invariant to Lorentz boosts in the beam direction:

$$Q \equiv |q_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{z,LCMS}^2}$$

$$\text{where } q_{z,LCMS}^2 = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}.$$

STAR, Phys.Rev.C 92 (2015) 1, 014904



Kinematic variables of the correlation function IV.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

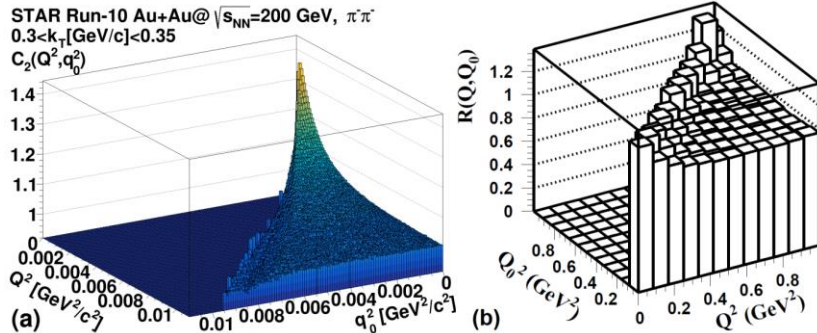


Figure 3.4: Example two-dimensional pion correlation functions for $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions (a) and $\sqrt{s} = 91$ GeV e^+e^- collisions (b). The latter figure is taken from the thesis of Tamás Novák [161].

Q dep. corr.func.

q_{inv} dep. corr.func.

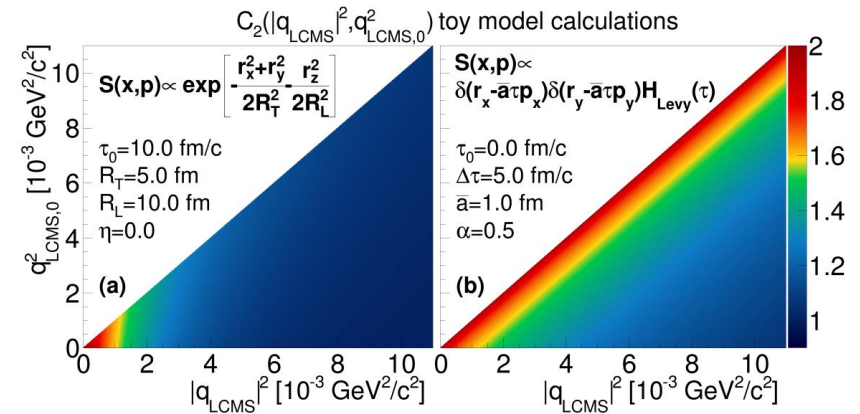
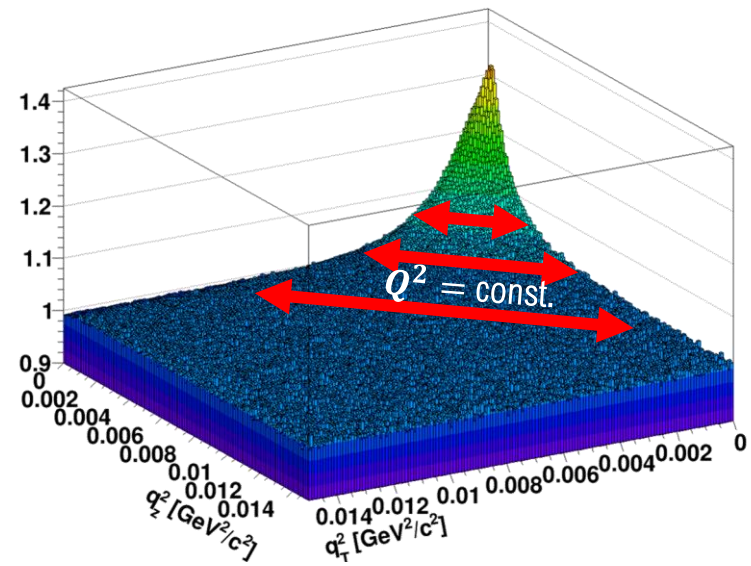
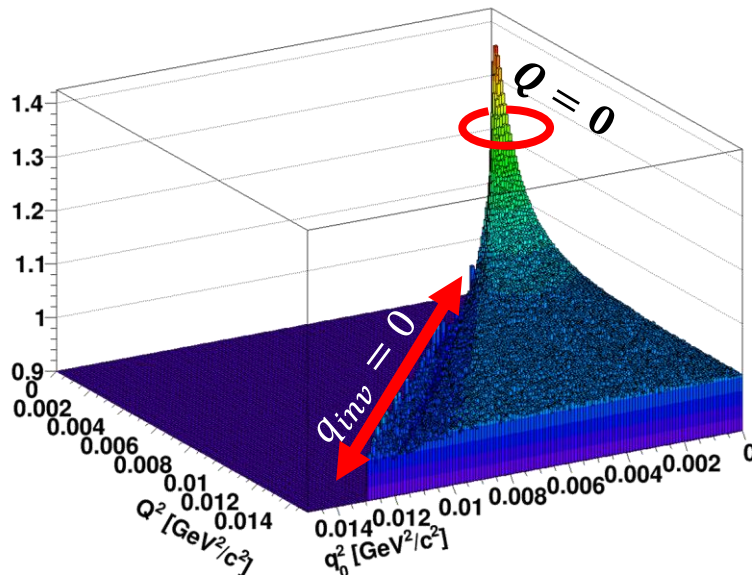


Figure 3.5: Toy model calculation for two different types of source functions. Taking a Gaussian source in both space and time leads to a correlation function that depends mostly on $|q_{LCMS}|$ (a), while a source that shows strong space-time and momentum space correlation leads to a q_{inv} dependent correlation function (b).

Kinematic variables of the correlation function V.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

$$Q = |\mathbf{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$$



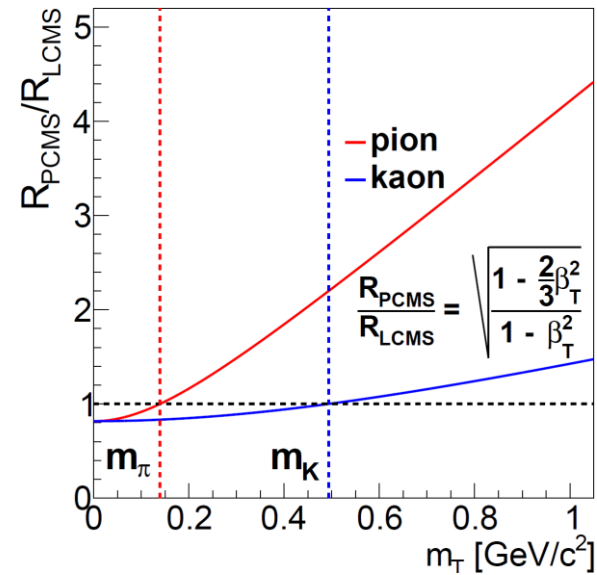
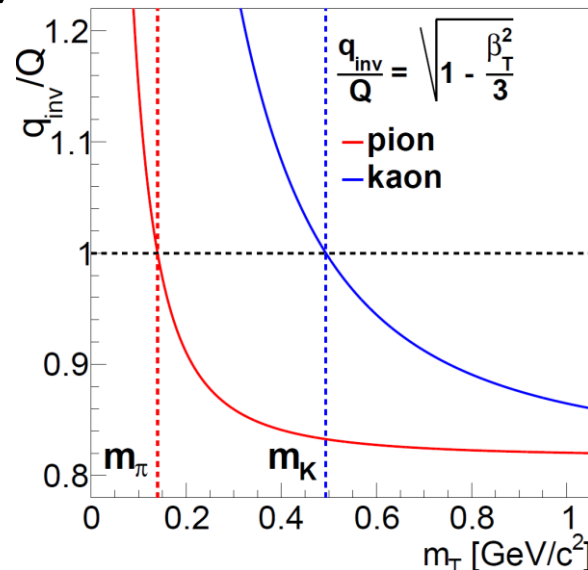
D. Kincses, Ph.D. thesis, [10.15476/ELTE.2022.164](https://doi.org/10.15476/ELTE.2022.164)

Kinematic variables of the correlation function VI.

- Correlation function measured in LCMS, Coulomb effect calculated in PCMS
- Approximation:
- (Note $m_T < m$
not physical of course)

$$q_{inv} \equiv q_{PCMS} \approx q_{LCMS} \cdot \sqrt{1 - \beta_T^2/3}$$

$$R_{PCMS} \approx R_{LCMS} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$



Coulomb correction and fitting of the corr. function

- Core-Halo model, Bowler-Sinyukov method: $C_2(Q, k_T) = 1 - \lambda + \lambda \int d^3r D_{(c,c)}(\mathbf{r}, k_T) |\Psi_Q^{(2)}(\mathbf{r})|^2$
- Neglecting FSI and using a Lévy-stable source function: $C_2^{(0)}(Q, k_T) = 1 + \lambda e^{-|RQ|^\alpha}$
- **Using numerical integral calculation as fit function results in numerically fluctuating χ^2 landscape**
- Treat FSI as correction factor: $K(Q, k_T) = \frac{C_2(Q, k_T)}{C_2^{(0)}(Q, k_T)}$
- An iterative method can be used: $C_2^{(fit)}(Q; \lambda, R, \alpha) = C_2^{(0)}(Q; \lambda, R, \alpha) \cdot K(Q; \lambda_0, R_0, \alpha_0)$
- Procedure continued until $\Delta_{iteration} = \sqrt{\frac{(\lambda_{n+1} - \lambda_n)^2}{\lambda_n^2} + \frac{(R_{n+1} - R_n)^2}{R_n^2} + \frac{(\alpha_{n+1} - \alpha_n)^2}{\alpha_n^2}} < 0.01$
- **Iterations usually converge within 2-3 rounds, fit parameters can be reliably extracted**

Coulomb correction and fitting of the corr. function

- Lévy-type correlation function without final state effects: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^\alpha}$

- Bowler-Sinyukov method:

$$C(Q_{LCMS}; \lambda, R_{LCMS}, \alpha) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{PCMS})\right) \cdot \left(1 + e^{-|R_{LCMS} Q_{LCMS}|^\alpha}\right) \cdot N \cdot (1 + \varepsilon Q_{LCMS})$$

Intercept parameter (correlation strength)
Lévy scale parameter
Possible linear background (usually negligible)

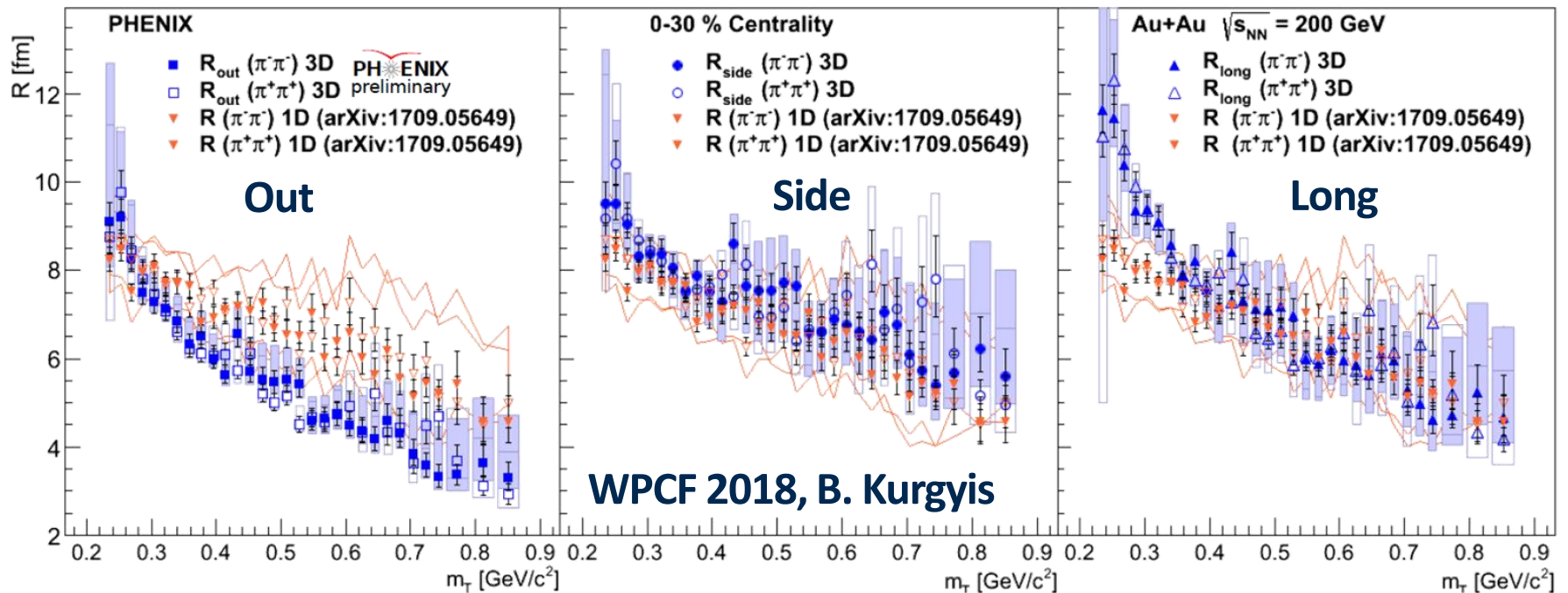
Coulomb correction
Lévy exponent

- Coulomb-correction calculated numerically (in PCMS)

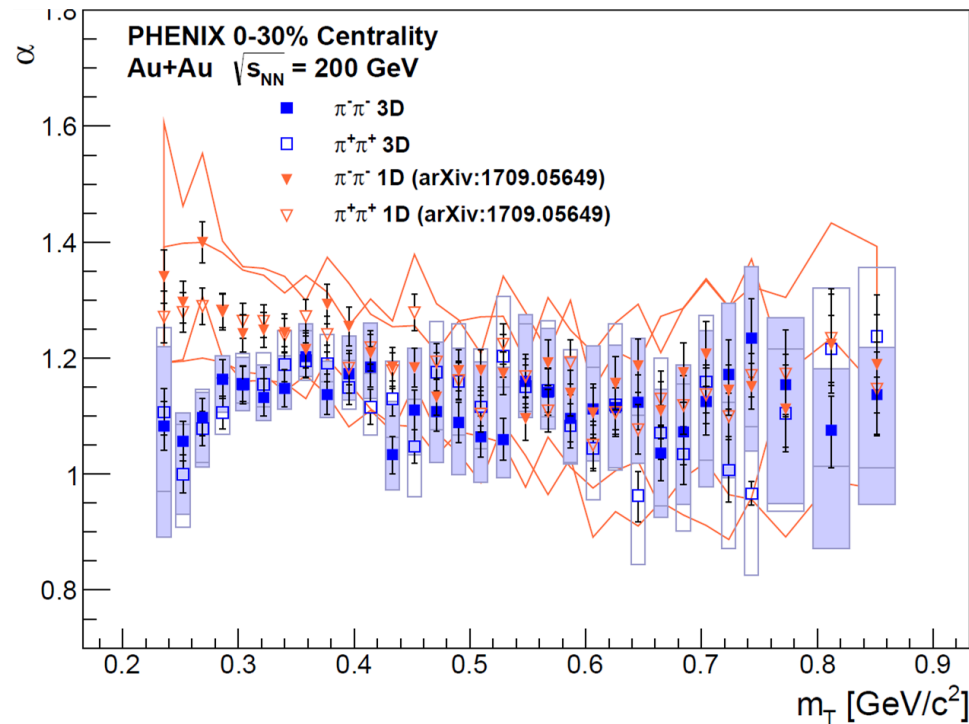
$$q_{inv} \equiv q_{PCMS} \approx q_{LCMS} \cdot \sqrt{1 - \beta_T^2/3} \qquad R_{PCMS} \approx R_{LCMS} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$

Cross-check with 3D analysis – PHENIX preliminary

$$C(Q) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{inv}) \cdot \left(1 + e^{-|R_o^2 q_o^2 + R_s^2 q_s^2 + R_l^2 q_l^2|^{\alpha/2}} \right) \right) \cdot N \cdot (1 + \varepsilon Q)$$



Cross-check with 3D analysis – PHENIX preliminary



- **Compatible with 1D (Q_{LCMS}) measurement** of Phys. Rev. C 97, 064911 (2018)
- Small discrepancy at small m_T : due to large R_{long} at small m_T ?

3D Gaussian vs 1D Levy

- **3D Gaussian does not result in 1D Levy**
 - Difference: several percent
 - Available experimental precision: much better than this difference

