Production of Resonances in Partial Chemical Equilibrium

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Motivation - why Partial Chemical Equilibrium (PCE)?

- Statistical production: hadron abundances and spectra
- (Simple) statistical model of interacting hadrons: interactions via inclusion of (free) resonance states
  [R. Dashen, S.K. Ma, H.J. Bernstein, Phys. Rev. 187 (1969) 345]

Chemical freeze-out

- Hadron abundances set by three (four) parameters: V, T_{ch}, μ_B, (γ_s)
- $T \sim 140 - 160$ MeV ($\sqrt{s_{NN}}$ dependent)

Kinetic freeze-out

- Sets the p_T spectra
- need transverse expansion
- slope due to T_k and $\langle v_t \rangle$
- $T_k \sim 80 - 120$ MeV (also higher)

How to build a scenario with chemical and kinetic freeze-out?

- need to freeze the effective numbers of stable hadrons—projected numbers after decays of all resonances $N_h^{eff} = \sum_r p_{r\to h} \langle N_r \rangle$
- Assumption: at T_{ch} inelastic collisions stop and elastic continue

Production of resonances

Chemical equilibrium

- chemical potentials due to conserved (B, S, Q)
- resonances suppressed due to their higher mass

Changes wrt equilibrium

- daughter particles not reconstructed
- by interactions, resonances are restored (to what extent?)

Observed abundance of resonances bears information about the microscopic dynamics within the hadronic fireball.

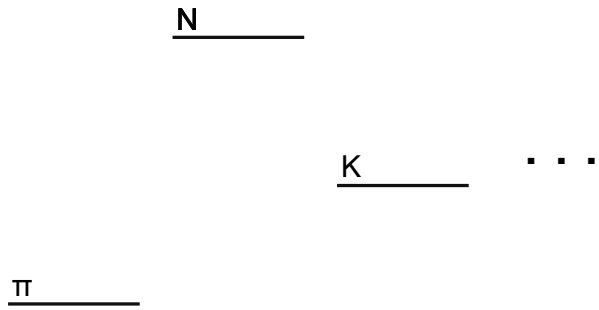
Partial chemical equilibrium

- At every T, resonances are in equilibrium with their own daughters
- predictions on the ratios N_R^{eff}/N_h^{eff}; they depend on T
- effective number of resonances includes contributions from decays of heavier resonances $N_R^{eff} = \sum_r p_{r\to R} N_r$

Can PCE reproduce data on resonance production?
Can we extract T from N_R^{eff}/N_h^{eff}?

The chemical potentials

- ground state species do not change one into other $\Rightarrow$ chemical potential for each



[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B 378 (1992) 95]

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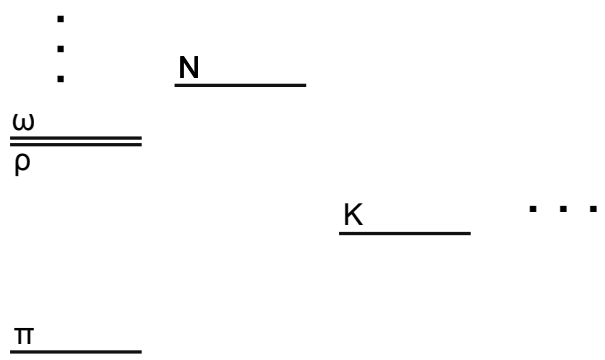
- ground state species do not change one into other $\Rightarrow$ chemical potential for each
- resonances equilibrated with ground state
  $\Rightarrow$ no extra energy to produce or decay resonance
- resonance chemical potentials from those of stable hadrons, e.g.
  $\mu_\rho = 2\mu_\pi$, $\mu_\omega = 3\mu_\pi$



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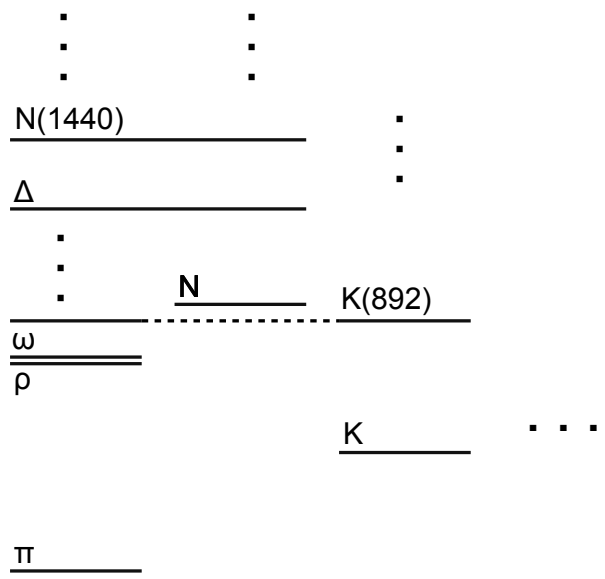
- resonances that decay into two different stable species, e.g.
  $\mu_\Delta = \mu_N + \mu_\pi$, $\mu_{K(892)} = \mu_\pi + \mu_K$



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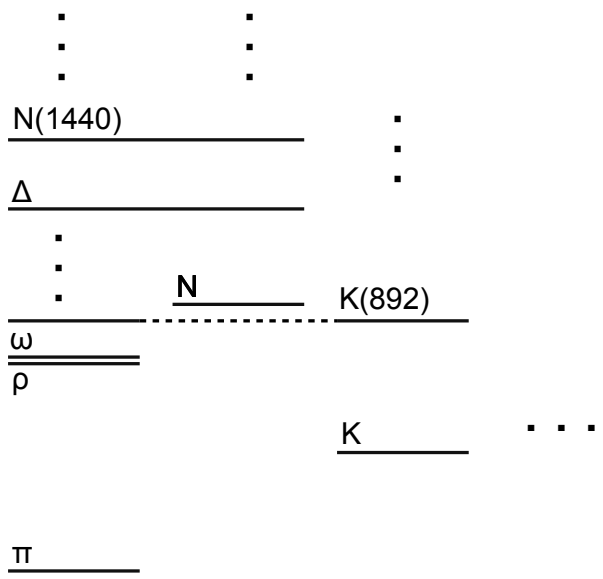
- resonances that decay into two different stable species, e.g.
  $\mu_\Delta = \mu_N + \mu_\pi$, $\mu_{K(892)} = \mu_\pi + \mu_K$
- Resonances with more decay channels, chain decays:

$$\mu_R = \sum_h p_{R\to h}\mu_h$$



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Evolution of chemical potentials

Keep the (effective stable) particle numbers constant, as a function of temperature!

$$\langle N_h^{eff} \rangle = \sum_r p_{r\to h} V(T) n_r(T, \{\mu(T)\}), \qquad \frac{\mathrm{d}\langle N_h^{eff} \rangle}{\mathrm{d}T} = 0$$

$$-\frac{\frac{\mathrm{d}V}{\mathrm{d}T}}{V} \sum_r p_{r\to h} n_r(T) = \sum_r p_{r\to h} \frac{\mathrm{d}n_r(T)}{\mathrm{d}T}$$

Obtain the derivative of volume from entropy conservation:
$0 = \mathrm{d}S/\mathrm{d}T = \mathrm{d}(sV)/\mathrm{d}T$

$$-\frac{\frac{\mathrm{d}V}{\mathrm{d}T}}{V} = \frac{\frac{\mathrm{d}s}{\mathrm{d}T}}{s}$$

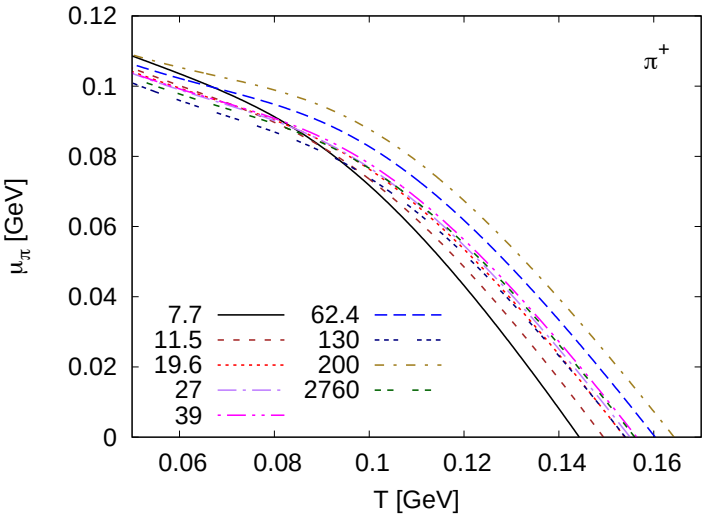
Equations for the evolution of chemical potentials

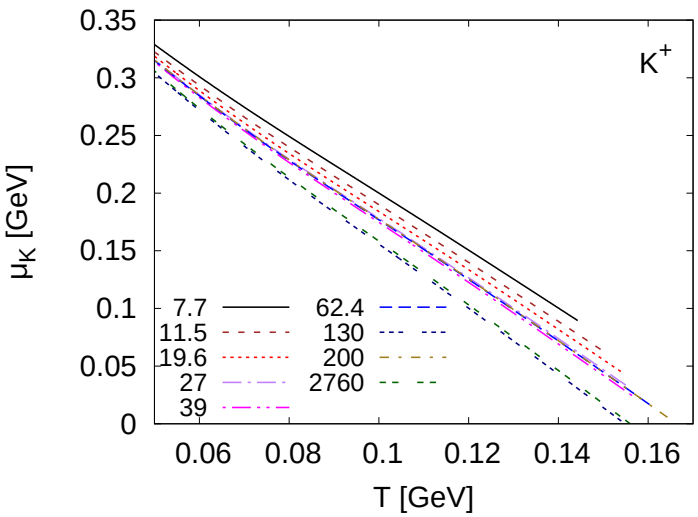
$$\frac{\sum_r p_{r\to h} \frac{\mathrm{d}n_r(T,\{\mu(T)\})}{\mathrm{d}T}}{\mathrm{d}s/\mathrm{d}T} = \frac{1}{s} \sum_r p_{r\to h} n_r(T, \{\mu(T)\})$$

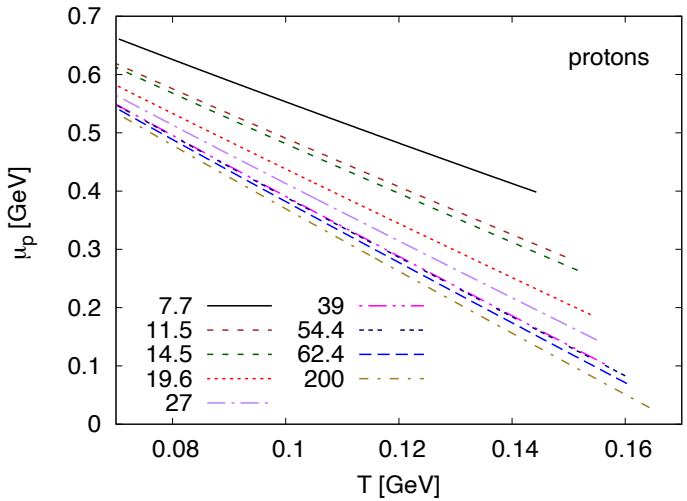
Evolution of chemical potentials: results

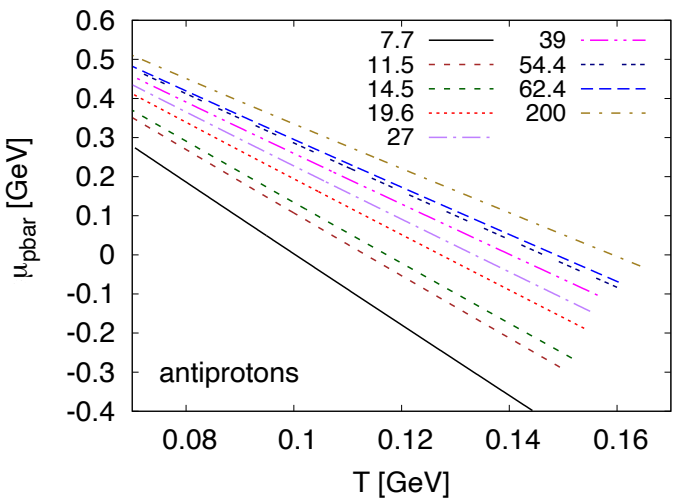
Start the evolution of chemical potentials at the chemical freeze-out

[STAR collab., Phys. Rev. C 96 (2017) 044904 and ALICE collab., Nucl. Phys. A 904-905 (2013) 531c]

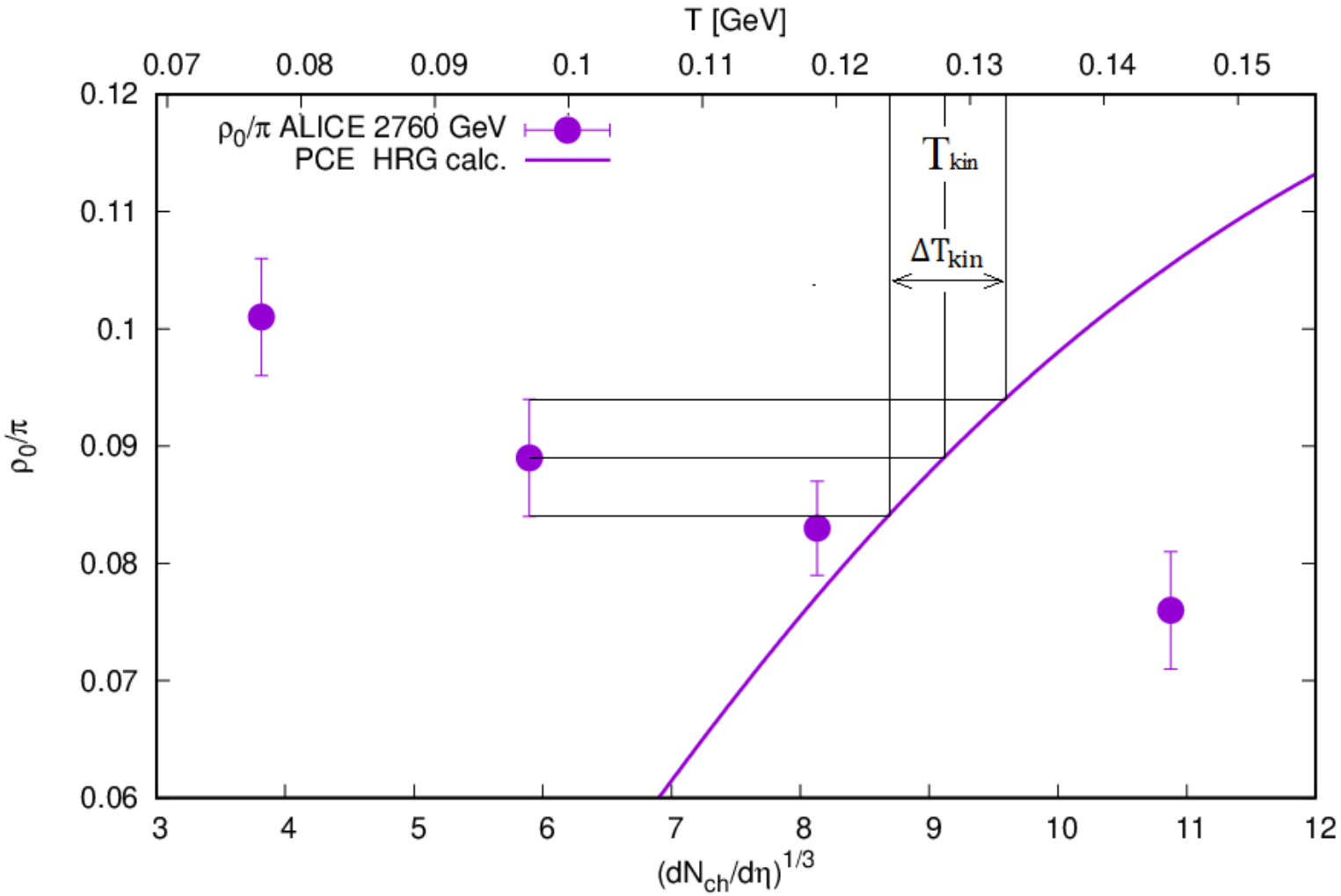








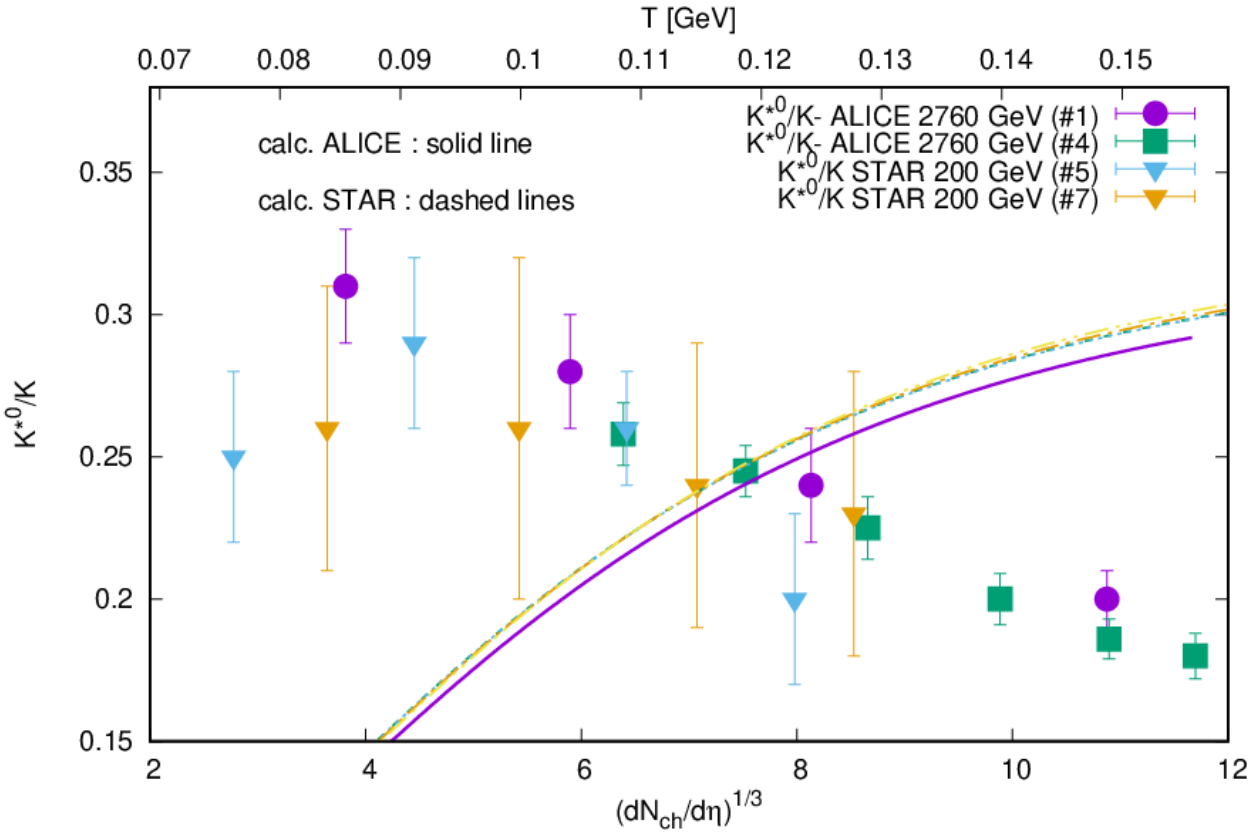
Ratio ρ_0/π (and how to read the results)



[ALICE collab., Phys. Rev. C 99 (2019) 064901] [Figure: Sándor Lökös]

Ratio K^*/K

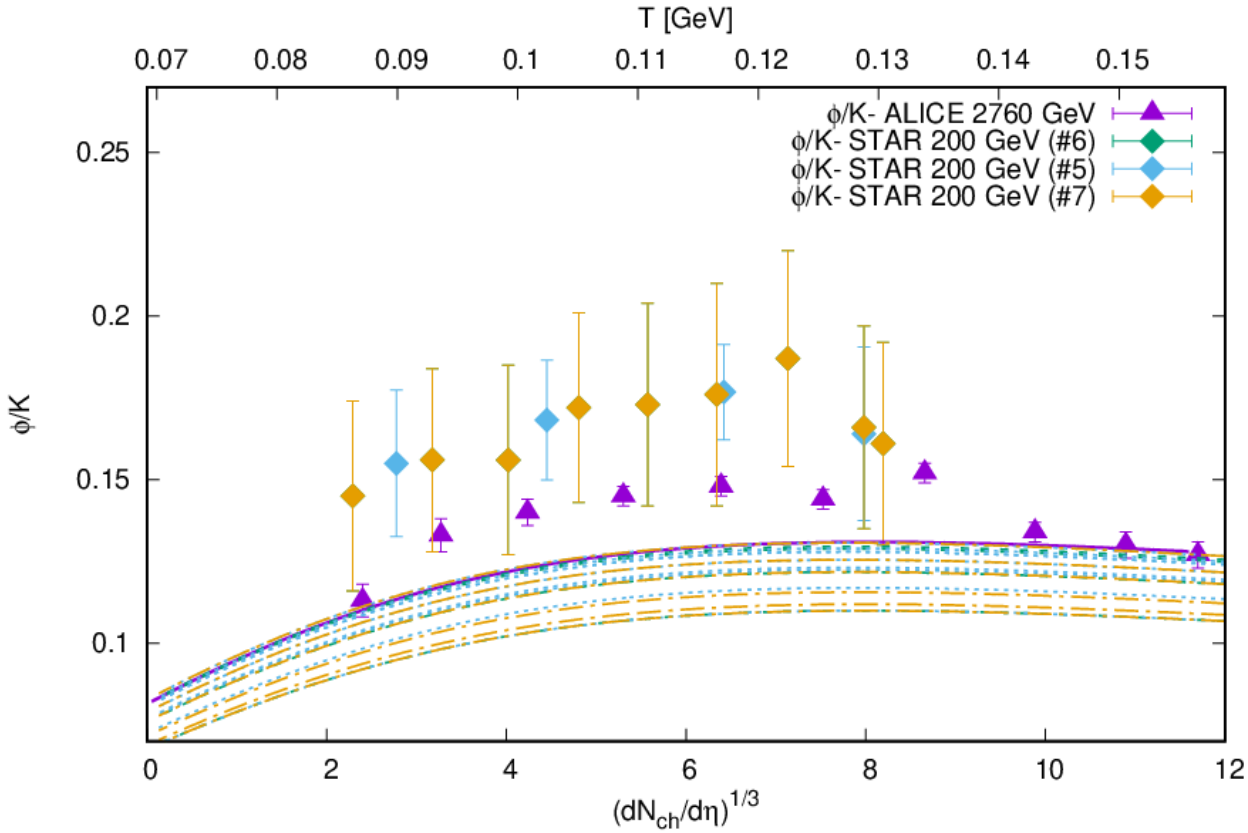
- indicated lower freeze-out temperature in central collisions
- $T_{kin} > 95$ MeV



[ALICE collab., Phys. Rev. C 91 (2015) 024609] [ALICE collab., Phys. Rev. C 95 (2017) 064606]
[STAR collab., Phys. Rev. C 84 (2011) 034909] [STAR collab., Phys. Rev. Lett. 97 (2006) 032301]
[Figure: Sándor Lökös]

Ratio ϕ/K

- data in mid central collisions above the PCE calculations



[ALICE collab., Phys. Rev. C 91 (2015) 024609] [STAR collab., Phys. Rev. C 70 (2009) 064903] [STAR collab., Phys. Rev. C 84 (2011) 034909] [STAR collab., Phys. Rev. Lett. 97 (2006) 032301]

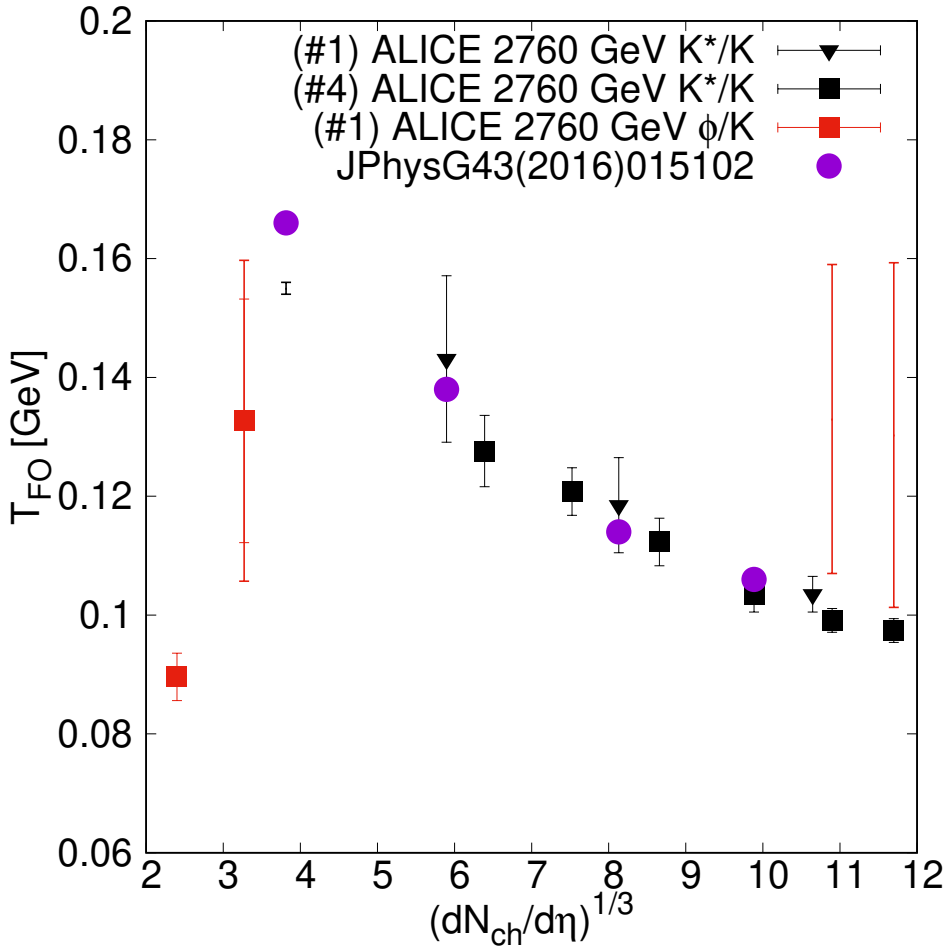
[Figure: Sándor Lökös]

K^*/K: temperatures compared to FO from spectra

Pb+Pb, $\sqrt{s_{NN}} = 2.76$ TeV

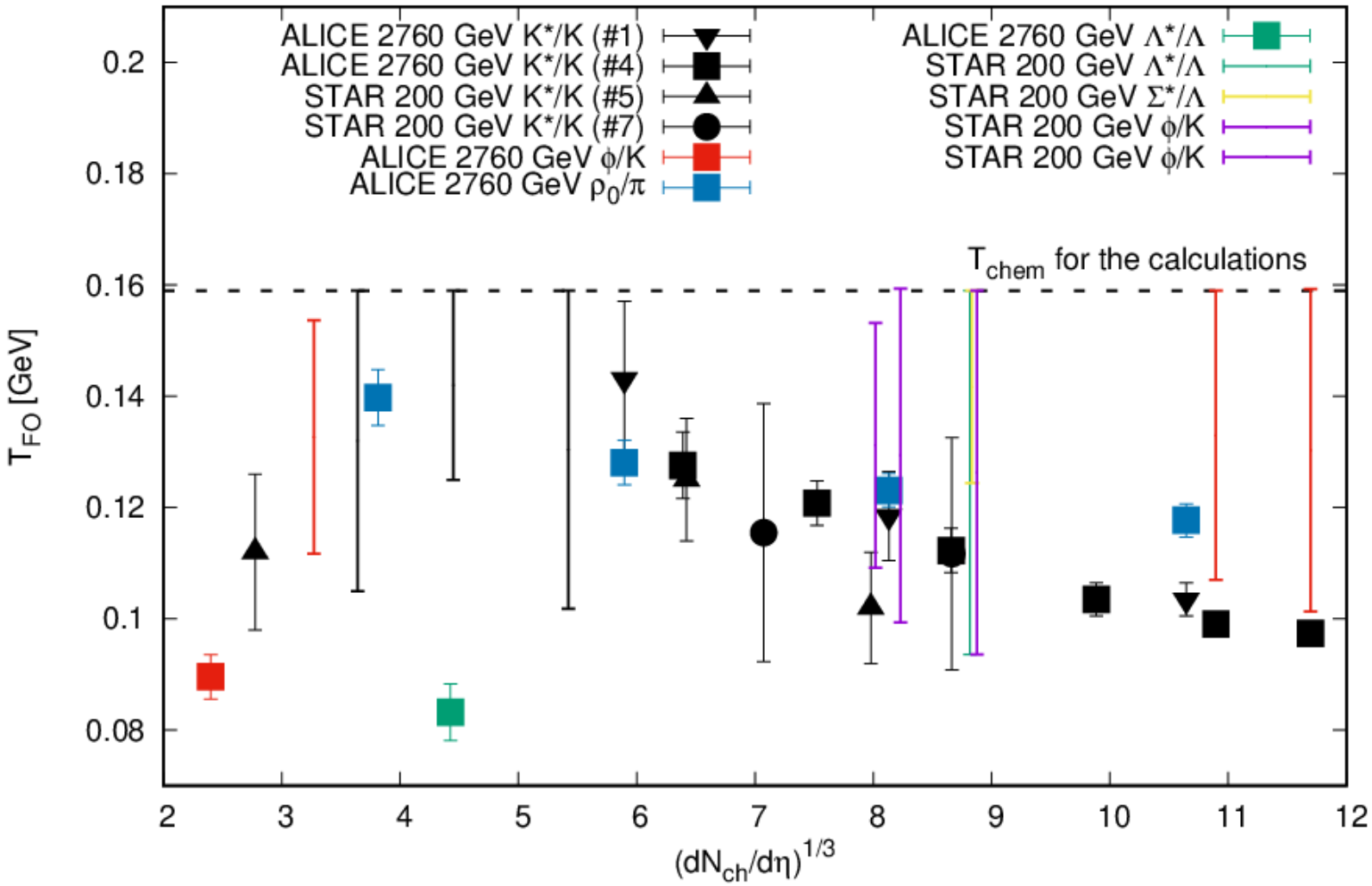
T_{kin} from p_t spectra π, p, K, Λ
fit with blast-wave model
PCE, resonances included

[I. Melo, B. Tomášik: J.Phys. G43 (2016) 015102]



Temperatures from p_t spectra and K^*/K ratios agree very well.
ϕ/K does not fit into this agreement.

Summary of extracted temperatures



[Figure: Sándor Lökös]

Conclusions

- Partial Chemical Equilibrium
  - keeps effective numbers of stable species independent of temperature
  - no reactions that would change one stable species into another (these would lead off equilibrium)
  - resonances in equilibrium with stable species
- Results for resonance production
  - ρ^0/π and K^*/K qualitatively: lower FO temperature in central collisions, quantitatively disagreement with kinetic FO fits
  - ϕ/K does not follow this trend, some centralties have too many ϕs
- Possible improvements to PCE
  - take into account entropy production
  - include hadron interactions via phase shifts

[S. Lökös, B. Tomášik, Physical Review C 106 (2022) 034912, 2206.11300]

BONUS

Net-Proton number fluctuations in Partial Chemical Equilibrium

Net-proton number fluctuations from PCE

- Not calculable as derivatives of the partition function!
  - $\partial^i \log Z / \partial \mu^i$: only fluctuations due to exchange with the heat bath
  - decays of resonances may randomize proton number (even at fixed B)
- Net-proton number cumulants obtained via

$$\left\langle (\Delta N_{p-\bar{p}})^l \right\rangle_c = \left\langle (\Delta N_p)^l \right\rangle_c + (-1)^l \left\langle (\Delta N_{\bar{p}})^l \right\rangle_c$$

- individual cumulants via derivatives of the generating function

$$\left\langle (\Delta N)^l \right\rangle_c = \left. \frac{\mathrm{d}^l K(i\xi)}{\mathrm{d}(i\xi)^l} \right|_{\xi=0} \qquad K(i\xi) = \ln \sum_{N=0}^{\infty} e^{i\xi N} P(N)$$

$$K(i\xi) = \sum_R \ln \left\{ \sum_{N_R=0}^{\infty} P_R(N_R) \left(e^{i\xi} p_R + (1 - p_R) \right)^{N_R} \right\}$$

- $P_R(N_R)$: number probability of resonance R, from PCE model
  p_R: average number of protons from resonance R

Net-proton number fluctuations from PCE, part 2

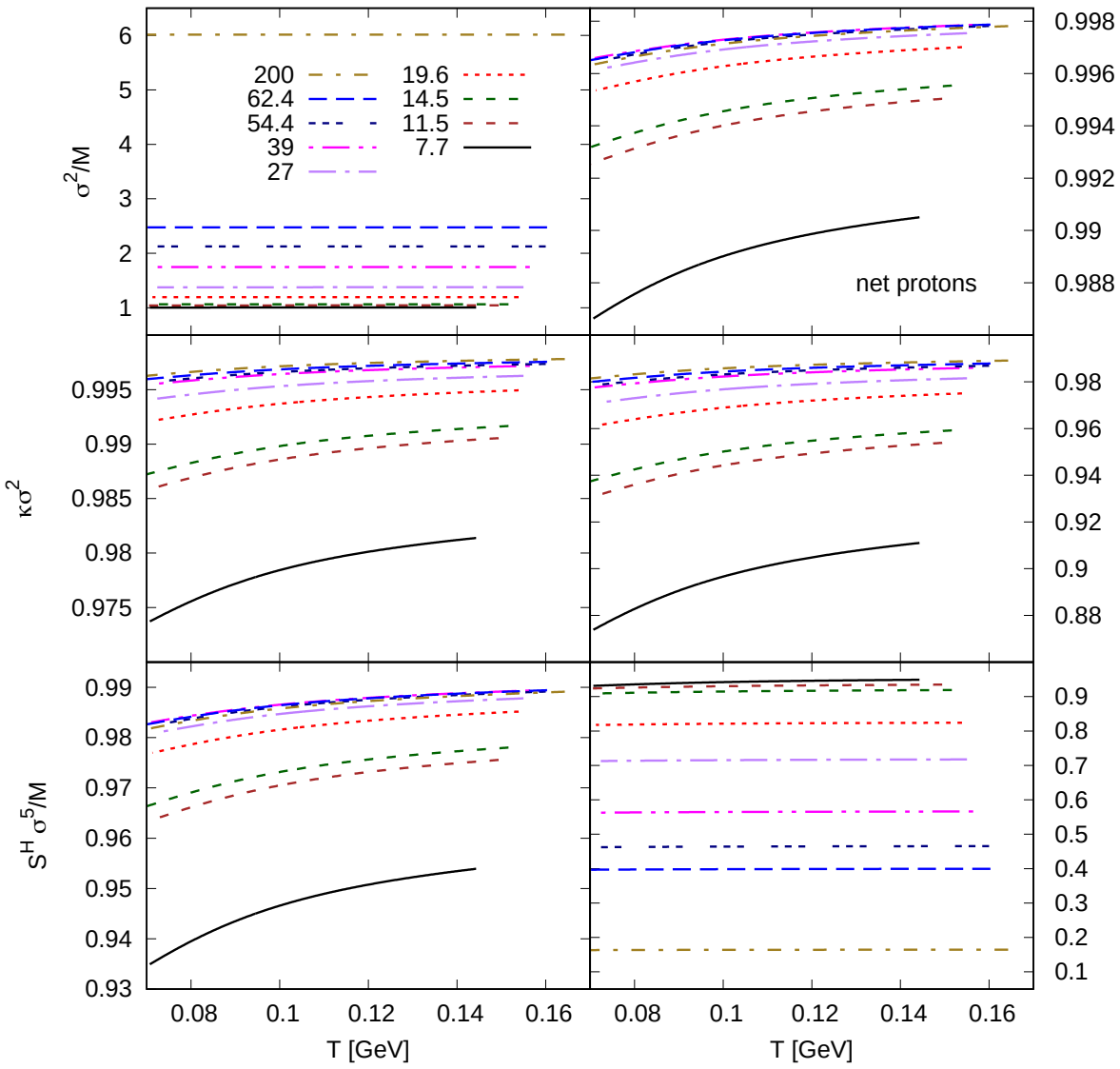
- Cumulants of the resonance number distributions

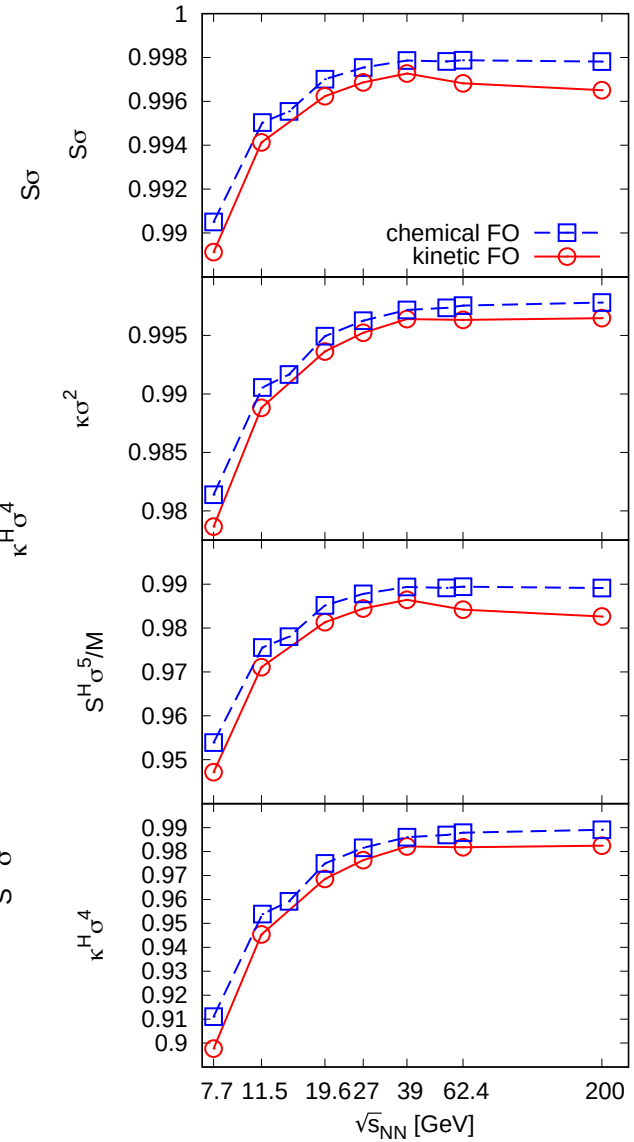
$$\langle N_R \rangle_c = \frac{g_R V}{2\pi^2} m_R^2 T \sum_{j=1}^{\infty} \frac{(\mp 1)^{j-1}}{j} e^{j\mu_R/T} K_2\left(\frac{j m_R}{T}\right),$$

$$\left\langle (\Delta N_R)^l \right\rangle_c = \frac{g_R V}{2\pi^2} m_R^2 T \sum_{j=1}^{\infty} (\mp 1)^{j-1} j^{l-2} e^{j\mu_R/T} K_2\left(\frac{j m_R}{T}\right).$$

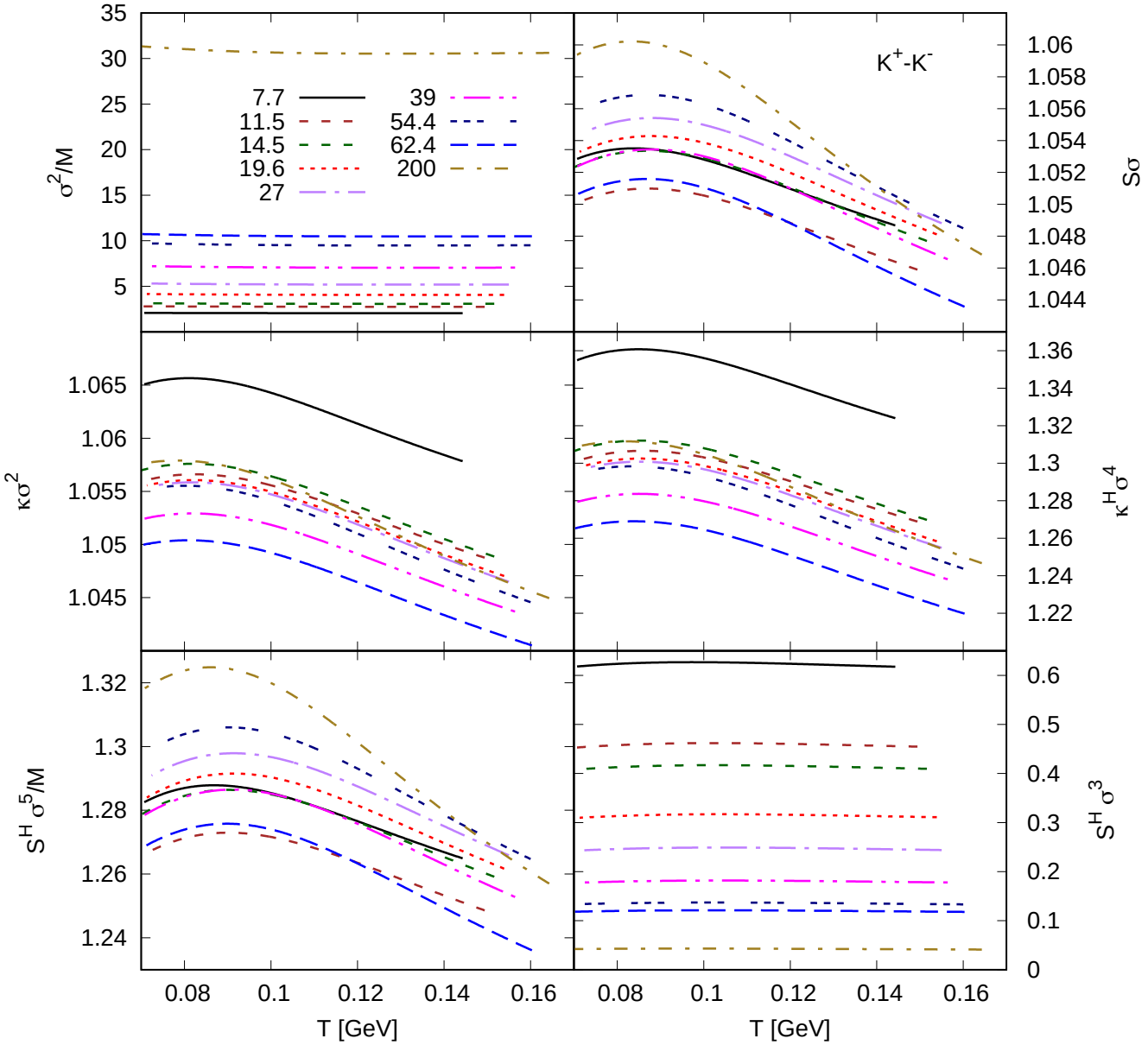
- first terms in the sums correspond to Boltzmann approximation (not BE or FD)
- In Boltzmann approximation, cumulants of all orders are the same!

Results for net-proton cumulants in PCE





Results for $K^+ - K^-$ cumulants in PCE



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  - ρ^0/π and K^*/K qualitatively: lower FO temperature in central collisions, quantitatively disagreement with kinetic FO fits
  - ϕ/K does not follow this trend, some centralties have too many ϕs
- Results from PCE on net-proton number fluctuations
  [B. Tomášik, P. Hillmann, M. Bleicher, Physical Review C 104 (2021) 044907]
  - volume-independent ratios of cumulants of net-proton number are almost temperature independent $\Rightarrow$ they reflect values at chemical freeze-out
  - experimental data on cumulants at low energies are not reproduced
- Possible improvements to PCE
  - take into account entropy production
  - include hadron interactions via phase shifts