KLOE measurement of the $\sigma(e^+e^- \rightarrow \pi^+\pi^-(\gamma))$ with Initial State Radiation and its contribution to the muon (g-2)

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STORI11, LNF- FRASCATI, October 13th 2011

Outlook

* Motivation

* KLOE measurements of $\sigma(e^+e^- \rightarrow \pi^+\pi^-(\gamma))$

  and Evaluation of $a_\mu^{\pi\pi}$

  Normalized to Bhabha events:

  - Small (photon) angle measurements (KLOE05, KLOE08)
  - Large (photon) angle measurement (KLOE10)

  Normalized to $\mu\mu\gamma$ events (NEW!):

  - Small (photon) angle measurements(KLOE11, *preliminary*)

  Comparison btw KLOE11 and KLOE08, KLOE10

* Conclusion

Motivation

The muon anomaly $a_\mu = (g_\mu - 2)/2$ is one of the most precisely measured quantity in particle physics.

Latest BNL-2004 a_μ measurement shows a discrepancy of ~3σ with SM prediction

Is it this discrepancy the sign of "missing piece" - new physics?

This question stimulated many theoretical and experimental efforts !

The main uncertainty in the estimate of theoretical prediction is due to low energy hadron contribution

This uncertainty can be substantially reduced by hadronic cross section measurements in electron-positron annihilation at low energy.

a_μ comparison Exp & Theo

F. Jegerlehner, A. Nyffeler / Physics Reports 477 (2009) 1–110

Standard model theory and experiment comparison [in units 10^{-11}].

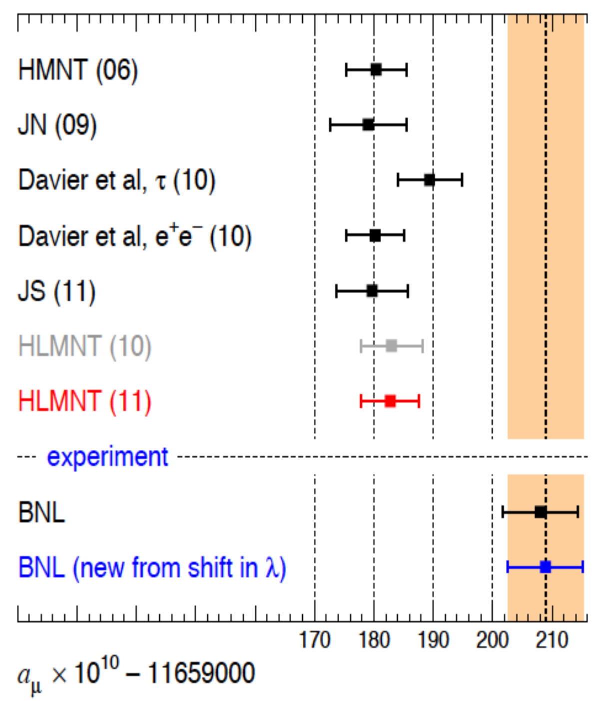
Contribution	Value	Error
QED incl. 4-loops + LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	−100.3	1.1
Hadronic light-by-light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116 591 790.0	64.6
Experiment	116 592 080.0	63.0
Exp. - The. 3.2 standard deviations	290.0	90.3

$(27.7 \pm 8.4)\ 10^{-10}$ Eidelman TAU08

$(24.6 \pm 8.0)\ 10^{-10}$ Davier *et al. arXiv:* 0908.4128

$$8.4 \approx 5_{HLO} \oplus 3_{LBL} \oplus 6_{BNL}$$

$$a_\mu^{Exp} - a_\mu^{Theo} = (27.6 \pm 8.7) \times 10^{-10} \sim 3.4\ \sigma$$

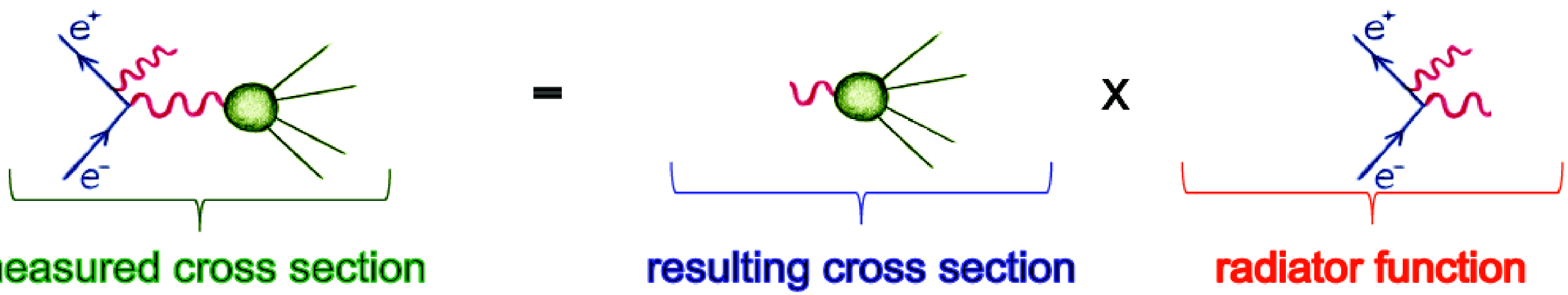


Is an hint of new physics ...

ISR: Initial State Radiation

Neglecting final state radiation (FSR):

$$\frac{d\sigma(e^+ e^- \to hadrons + \gamma)}{dM^2_{hadr}} = \frac{\sigma(e^+ e^- \to hadrons, M^2_{hadr})}{s} H(s, M^2_{hadr})$$



Theoretical input: precise calculation of the radiation function $H(s, M^2_{hadr})$

PHOKHARA MC Generator

Binner, Kühn, Melnikov; Phys. Lett. B 459, 1999

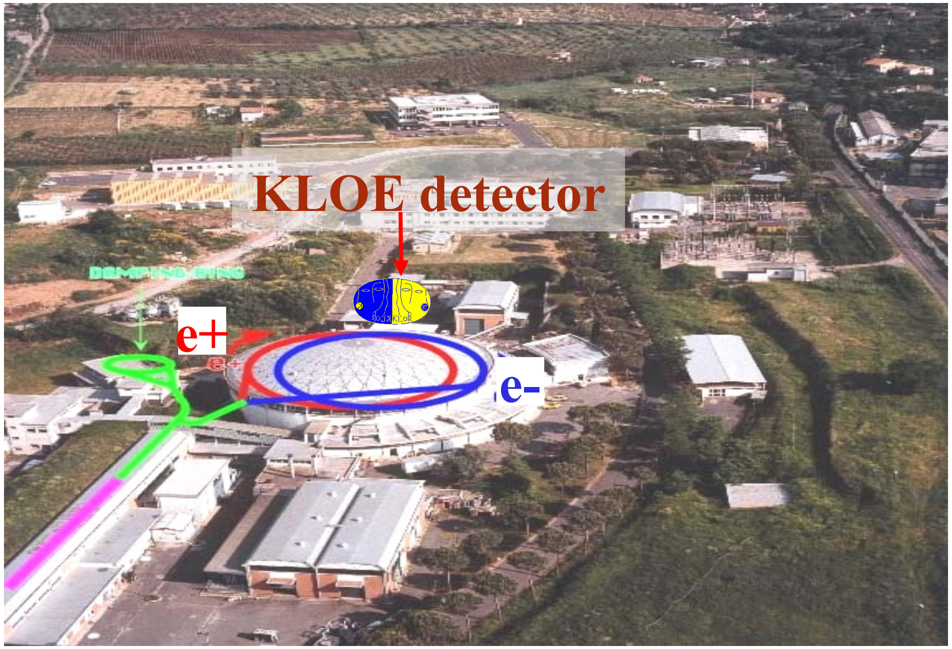
H. Czyż, A. Grzelińska, J.H. Kühn, G. Rodrigo, Eur. Phys. J. C 27, 2003

(exact next-to-leading order QED calculation of the radiator function)

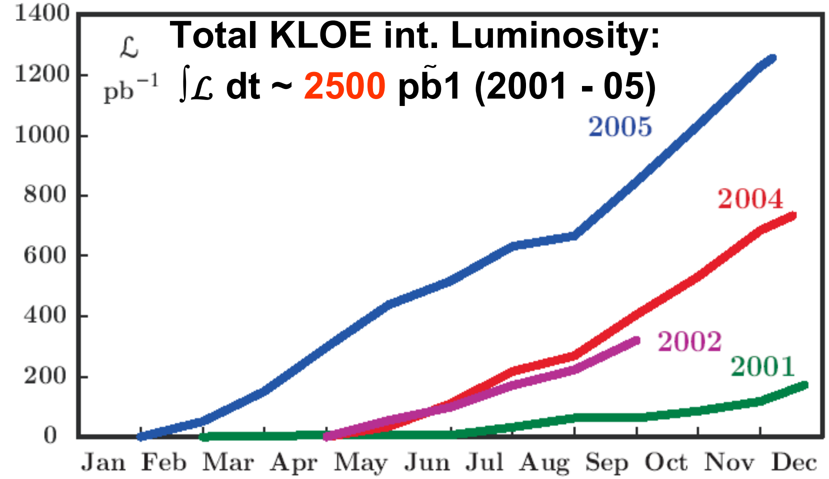
IN 2005 KLOE has published the first precision measurement of $\sigma(e^+e^-\to\pi^+\pi^-)$ with ISR using 2001 data (140pb^{-1}) PLB606(2005)12 ~3σ discrepancy btw a_μ^{SM} and a_μ^{exp}

DAΦNE: A Φ-Factory

e^+e^- - collider with $\sqrt{s} = m_\Phi \approx 1.0195$ GeV



Integrated Luminosity



Peak Luminosity Lpeak= 1.5 • 10^{32}cm^{-2}s^{-1}

KLOE05 measurement (PLB606(2005)12) was based on $140 pb^{-1}$ of 2001 data!

2006:

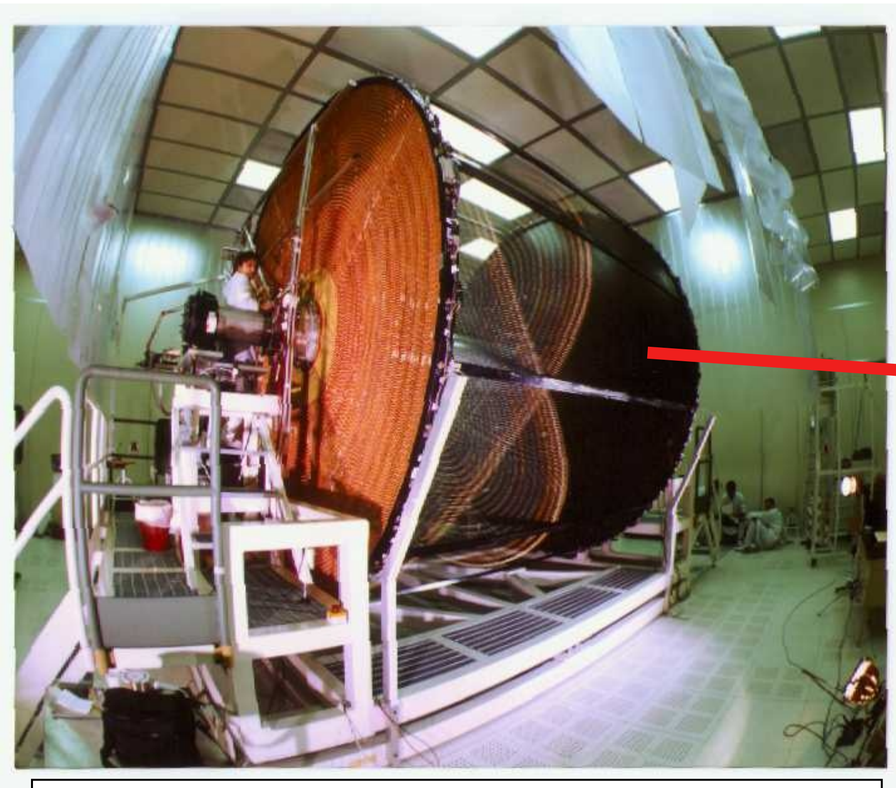
- **Energy scan (4 points around mΦ-peak)**
- **$240 pb^{-1}$ at = 1000 MeV (off-peak data)**

KLOE08 measurement (PLB670(2009)285) was based on $240 pb^{-1}$ from 2002 data!

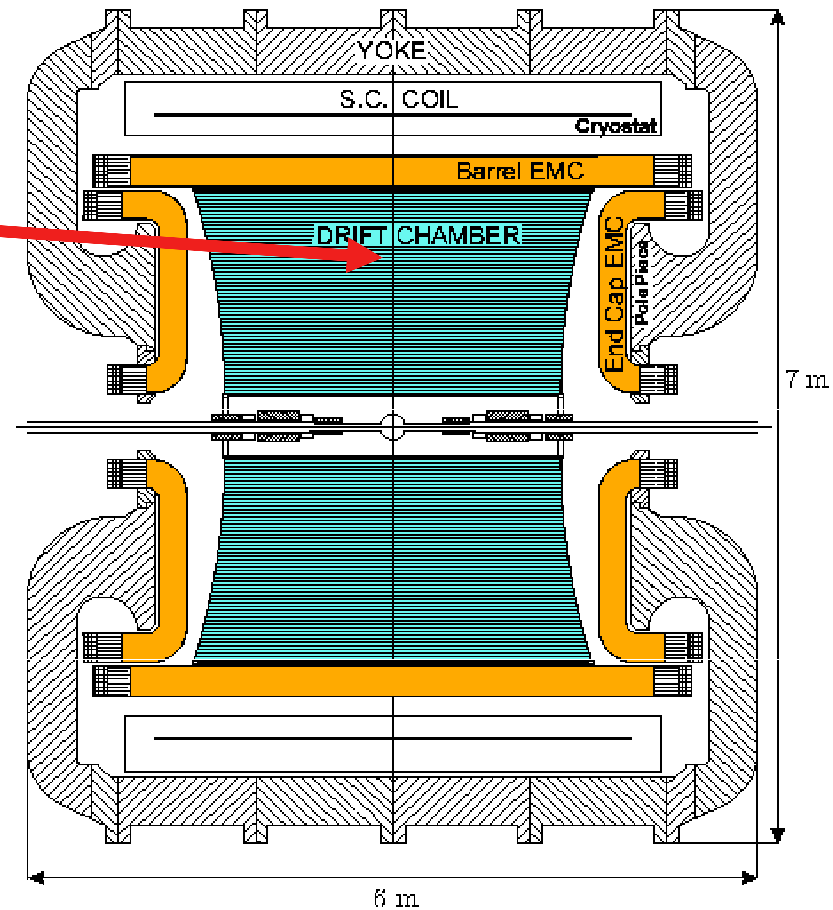
KLOE10 measurement (PLB700 (2011)102) based on 233 pb^{-1} of 2006 data (at 1 GeV, different event selection)

KLOE Detector

Drift chamber



Full stereo geometry, 4m diameter, 52.140 wires 90% Helium, 10% iC4H10



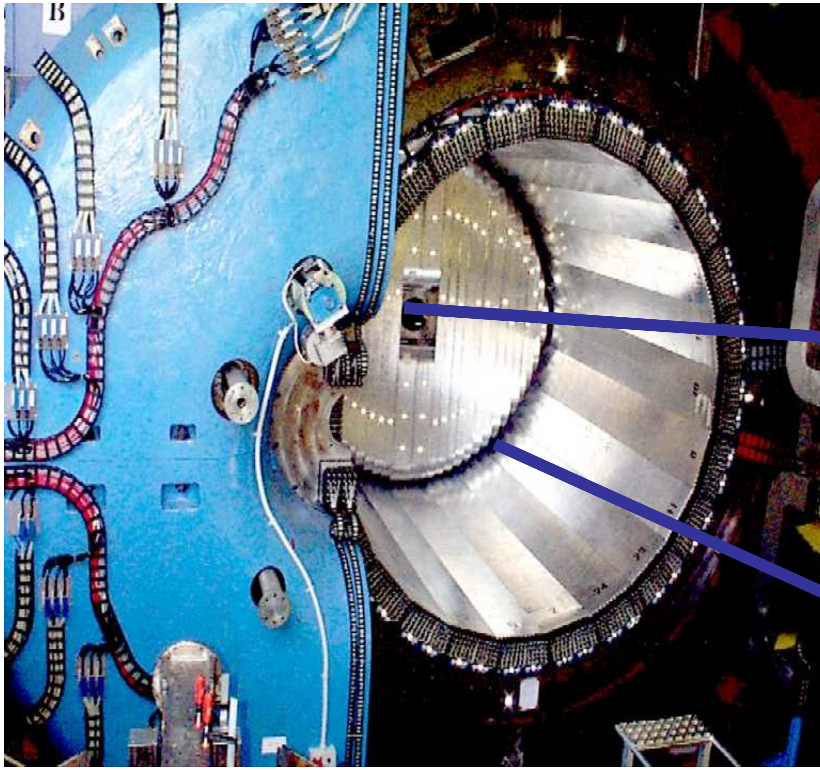
$\sigma_p/p = 0.4\%$ (for 90° tracks)

$\sigma_{xy} \approx 150$ mm, $\sigma_z \approx 2$ mm

Excellent momentum resolution

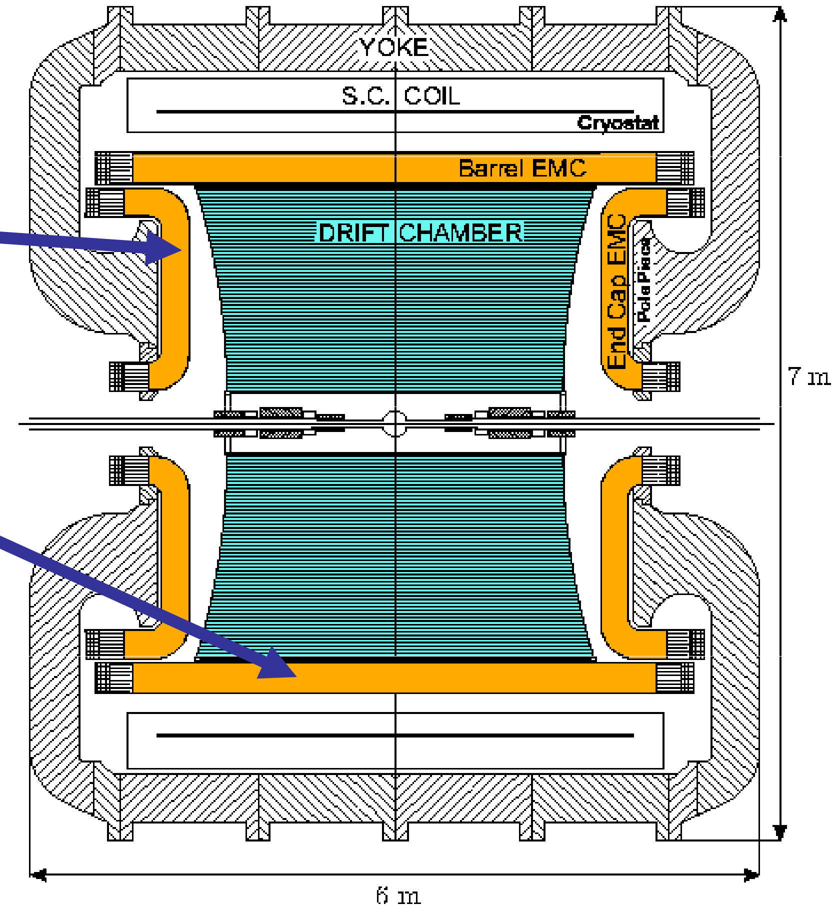
KLOE Detector

Electromagnetic Calorimeter



Pb / scintillating fibres (**4880 PMT**)

Endcap - Barrel - Modules



$\sigma_E/E = 5.7\% / \sqrt{E(GeV)}$

$\sigma_T = 54 \text{ ps} / \sqrt{E(GeV)} \oplus 100 \text{ ps}$

(Bunch length contribution subtracted from constant term)

Excellent timing resolution

Extracting $\sigma_{\pi\pi}$ and $|F_\pi|^2$ from $\pi\pi\gamma$ events

a) Via absolute Normalisation to Bhabha events (KLOE05,08,10):

1) $$\frac{d\sigma^{obs}_{\pi\pi\gamma(\gamma)}}{dM^2_{\pi\pi}} = \frac{\Delta N_{\text{Obs}} - \Delta N_{\text{Bkg}}}{\Delta M^2_{\pi\pi}} \cdot \frac{1}{\varepsilon_{\text{Sel}}} \cdot \frac{1}{\int L\,dt}$$

$d\sigma_{\pi\pi\gamma}(\gamma)/dM^2$ is obtained by subtracting background from observed event spectrum, divide by selection efficiencies, and ***int. luminosity***:

2) $$\sigma_{\pi\pi}(\mathrm{s}) \approx s\,\frac{d\sigma^{obs}_{\pi\pi\gamma(\gamma)}}{dM^2_{\pi\pi}} \cdot \frac{1}{H(\mathrm{s})}$$

Obtain $\sigma_{\pi\pi}$ from (ISR) - radiative cross section $d\sigma^{\pi\pi\gamma}(\gamma)/dM^2$ via theoretical radiator function H(s):

3) $$|\mathrm{F}_\pi|^2 = \frac{3s}{\pi\alpha^2\beta^3_\pi}\,\sigma_{\pi\pi}(\mathrm{s})$$

Relation between $|F_\pi|^2$ and the cross section $\sigma(e^+e^- \to \pi^+\pi^-)$

b) Via bin-by-bin Normalisation to rad. Muon events (New measurement!)

Radiative Corrections

Radiator-Function $H(s,s_p)$ (ISR):

- **ISR-Process calculated at NLO-level**

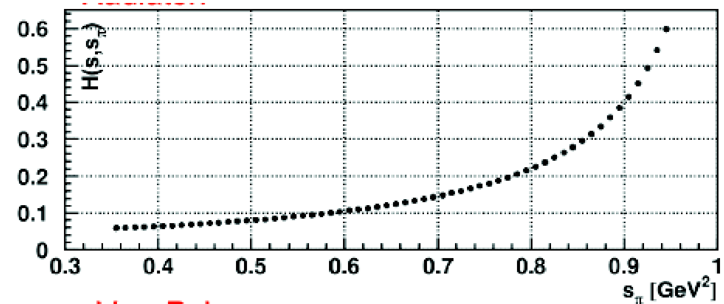
PHOKHARA generator

(H.Czyż, A.Grzelińska, J.H.Kühn, G.Rodrigo, EPJC27,2003)

Precision: 0.5%

$$s \cdot \frac{d\sigma_{\pi\pi\gamma}}{ds_\pi} = \sigma_{\pi\pi}(s_\pi) \times H(s,s_\pi)$$

Radiator:



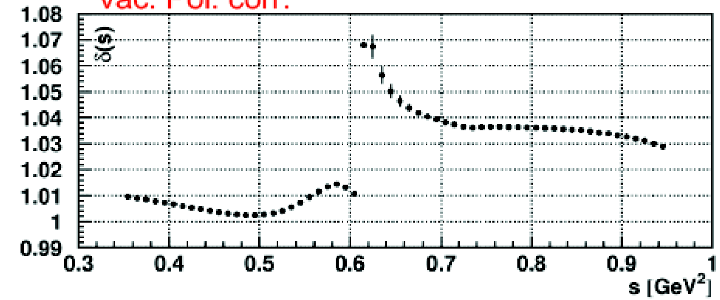
Radiative Corrections:

i) Bare Cross Section

divide by Vacuum Polarisation $d(s)=(a(s)/a(0))^2$

➔ from F. Jegerlehner

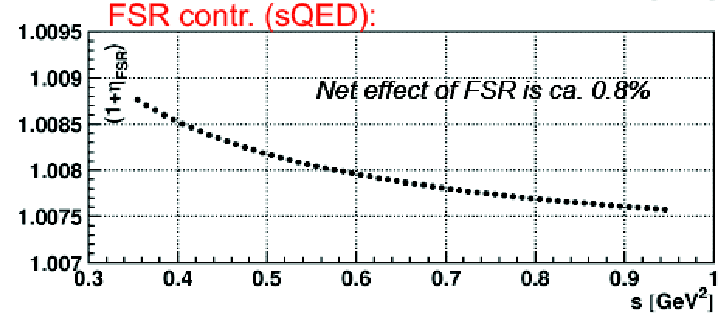
Vac. Pol. corr:



ii) FSR

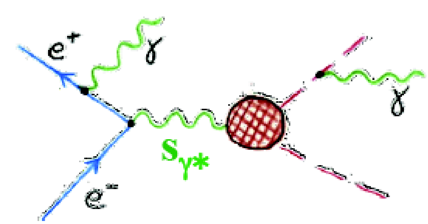
Cross section s_{pp} must be incl. for FSR for use in the dispersion integral of a_m

FSR contr. (sQED):



FSR corrections have to be taken into account in the efficiency eval. (Acceptance, M_{Trk}) and in the mapping $s_\pi \rightarrow s_{\gamma*}$

(H.Czyż, A.Grzelińska, J.H.Kühn, G.Rodrigo, EPJC33,2004)



$s_{\gamma*} > s_\pi$

SA Event Selection (KLOE08)

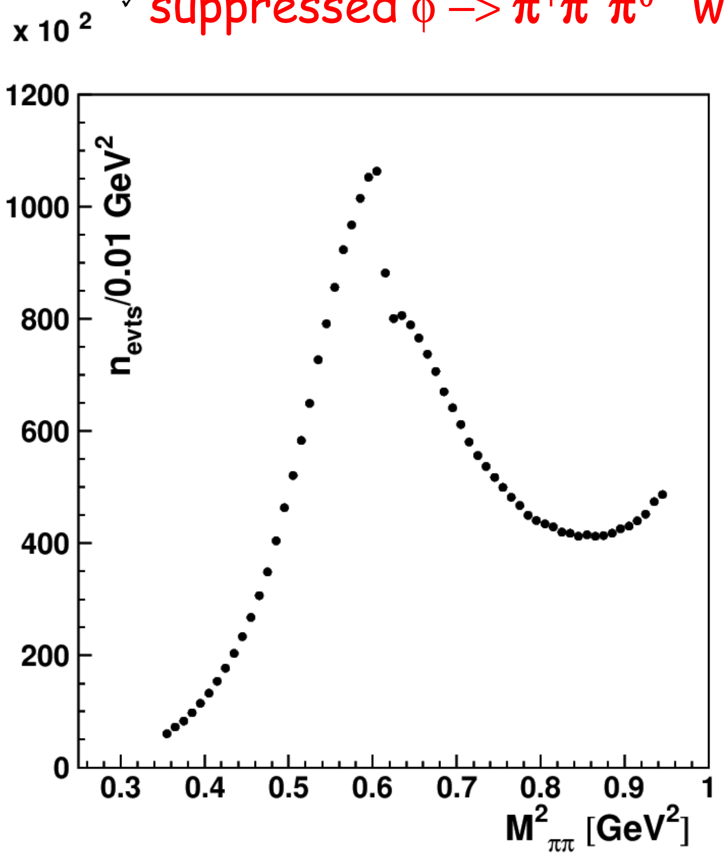
a) 2 tracks with $50° < \theta_{track} < 130°$

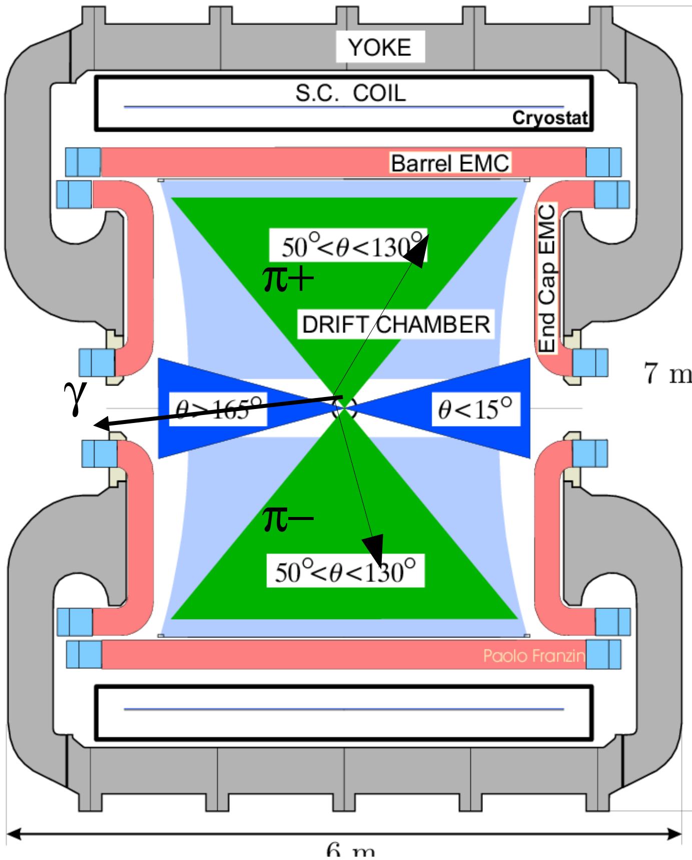
b) small angle (not detected) γ

c) ($\theta_{\pi\pi} < 15°$ or $> 165°$)

- ✓ high statistics for ISR
- ✓ low relative FSR contribution
- ✓ suppressed $\phi \to \pi^+\pi^-\pi^0$ wrt the signal

kinematics: $\vec{p}_\gamma = \vec{p}_{miss} = -(\vec{p}_+ + \vec{p}_-)$





statistics: 240pb^{-1} of 2002 data 3.1 Mill.

Events between 0.35 and 0.95 GeV^2

SA Kloe result (KLOE08)

Systematic errors on $a_\mu^{\pi\pi}$:

Reconstruction Filter	negligible
Background	0.3%
Trackmass/Miss. Mass	0.2%
p/e-ID and TCA	negligible
Tracking	0.3%
Trigger	0.1%
Acceptance ($\theta_{\pi\pi}$)	0.1%
Acceptance (θ_π)	negligible
Unfolding	negligible
Software Trigger	0.1%
$\sqrt{s}$ dep. Of H	0.2%
Luminosity($0.1_{th} \oplus 0.3_{exp}$)%	0.3%

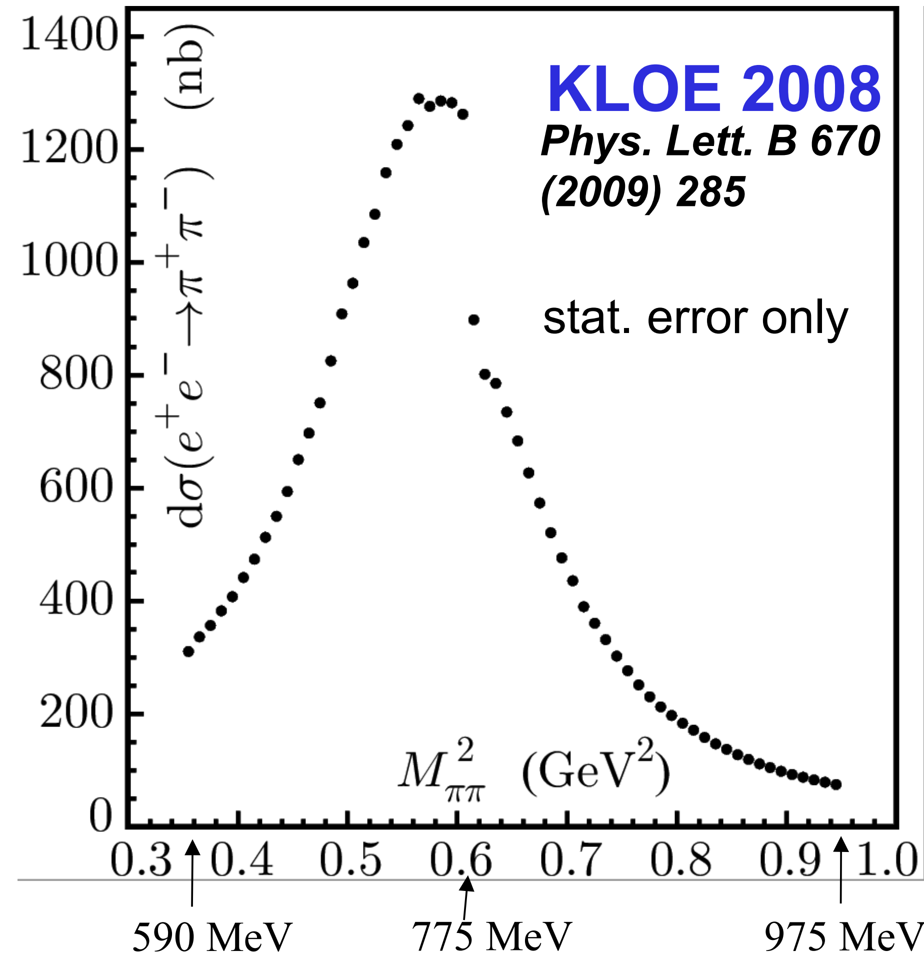
experimental fractional error on a_μ = 0.6 %

FSR resummation	0.3%
Radiator H	0.5%
Vacuum polarization	0.1%

theoretical fractional error on a_μ = 0.6 %

$$a_\mu^{\pi\pi} = \int_{x1}^{x2} \sigma_{ee\to\pi\pi}(s) K(s) ds$$

$\sigma_{\pi\pi}$, undressed from VP, inclusive for FSR as function of $(M_0^{\pi\pi})^2$



$a_\mu^{\pi\pi}(0.35\text{-}0.95\,\text{GeV}^2) = (387.2 \pm 0.5_{stat} \pm 2.4_{sys} \pm 2.3_{theo}) \cdot 10^{-10}$

LA Event Selection (KLOE10)

2 pion tracks at large angles
$50° < \theta_\pi < 130°$

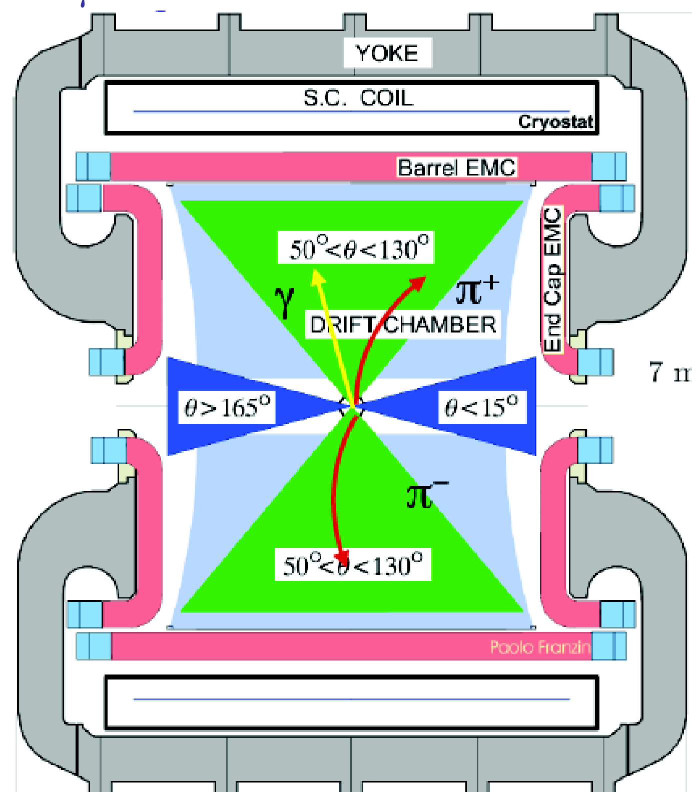
Photons at large angles
$50° < \theta_\gamma < 130°$

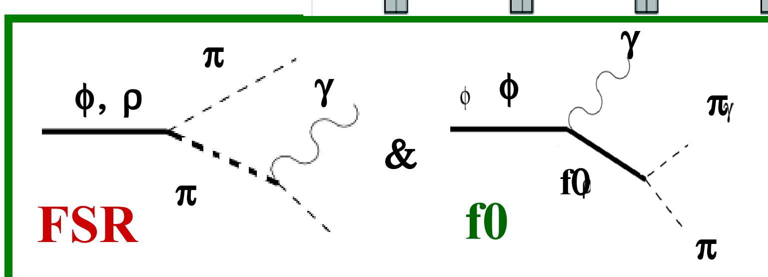
- ✓ **independent complementary analysis**
- ✓ **threshold region $(2m_\pi)^2$ accessible**
- ✓ **γISR photon detected (4-momentum constraints)**

- ✓ **lower signal statistics**
- ✓ **larger contribution from FSR events**
- ✓ **larger $\phi \rightarrow \pi^+\pi^-\pi^0$ background contamination**
- ✓ **irreducible background from ϕ decays ($\phi \rightarrow f^0\,\gamma \rightarrow \pi\pi\,\gamma$)**

Threshold region non-trivial due to irreducible FSR-effects, which have to be estimated from MC using phenomenological models (interference effects unknown)

At least 1 photon with $50° < \theta_\gamma < 130°$ and $E_\gamma > 20$ MeV => photon detected





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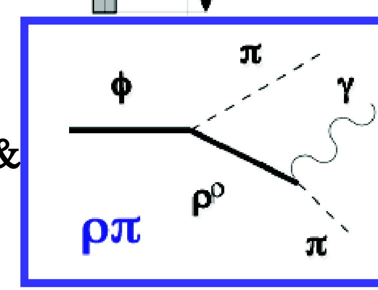
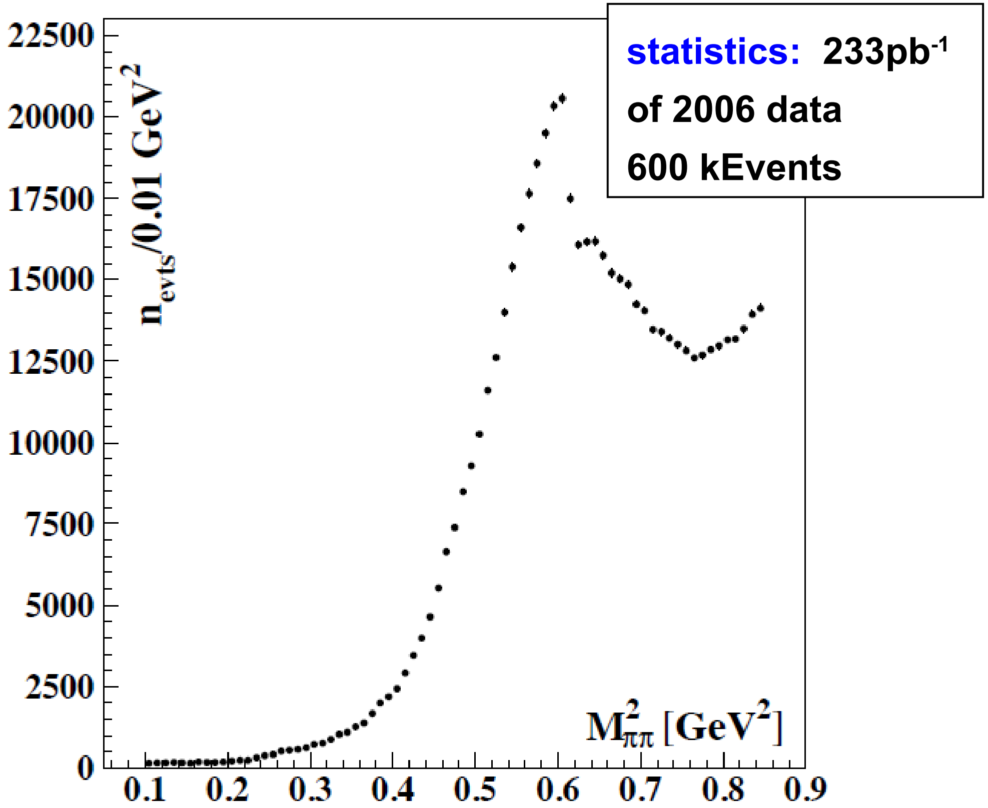


LA Event Selection (KLOE10)

2 pion tracks at large angles
$50° < \theta_\pi < 130°$

Photons at large angles
$50° < \theta_\gamma < 130°$

At least 1 photon with $50° < \theta_\gamma < 130°$ **and** E_γ **> 20 MeV -> photon detected**



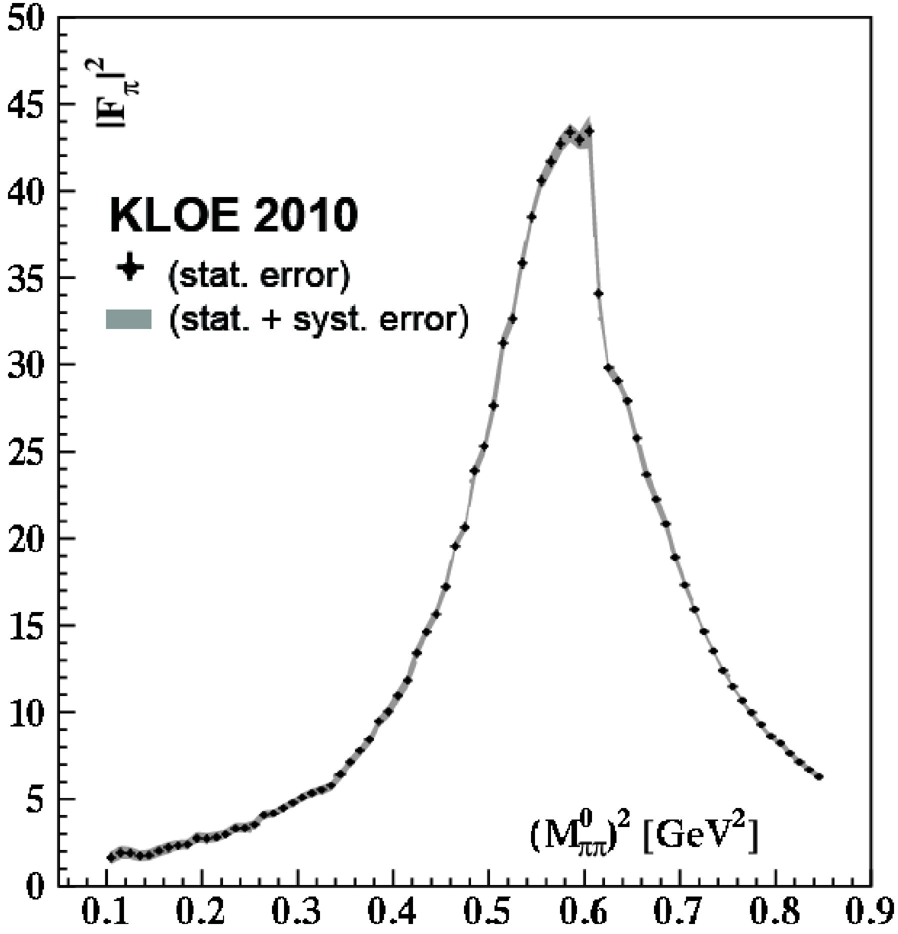
- ✓ independent complementary analysis
- ✓ threshold region $(2m_\pi)^2$ accessible
- ✓ γISR photon detected (4-momentum constraints)

- ✓ lower signal statistics
- ✓ larger contribution from FSR events
- ✓ larger $\phi \to \pi^+\pi^-\pi^0$ background contamination
- ✓ irreducible background from ϕ decays ($\phi \to f0\ \gamma \to \pi\pi\ \gamma$)

Use data sample taken at $\sqrt{s} \cong$ **1000 MeV**,
20 MeV below the ϕ-peak

KLOE10: Pion Form Factor

Phys. Lett. B 700 (2011) 102



Disp. Integral:

$$a_\mu^{\pi\pi} = \int_{x1}^{x2} \sigma_{ee\to\pi\pi}(s)K(s)ds$$

Table of systematic errors on $a_\mu^{\pi\pi}$(0.1-0.85 GeV^2):

Reconstruction Filter	negligible
Background	0.5%
f0+ρπ	0.4%
Ω cut	0.2%
Trackmass	0.5%
p/e-ID and TCA	negligible
Tracking	0.3%
Trigger	0.2%
Acceptance	0.5%
Unfolding	negligible
Software Trigger	0.1%
Luminosity($0.1_{th} \oplus 0.3_{exp}$)%	0.3%

experimental fractional error on a_μ = 1.0 %

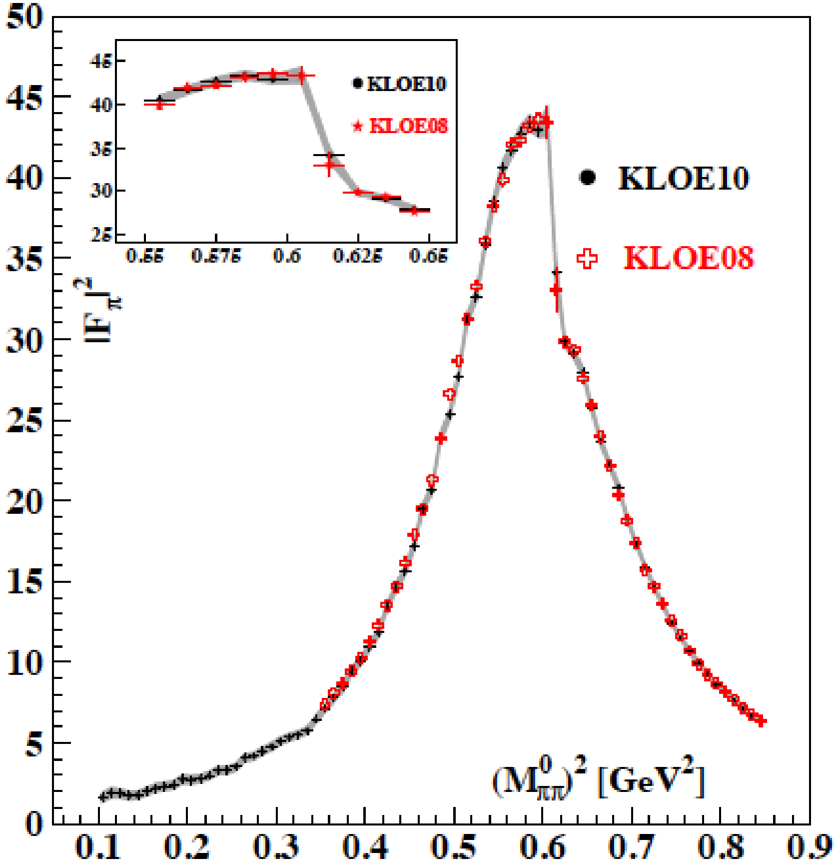
FSR treatment	0.8%
Radiator H	0.5%
Vacuum polarization	0.1%

theoretical fractional error on a_μ = 0.9 %

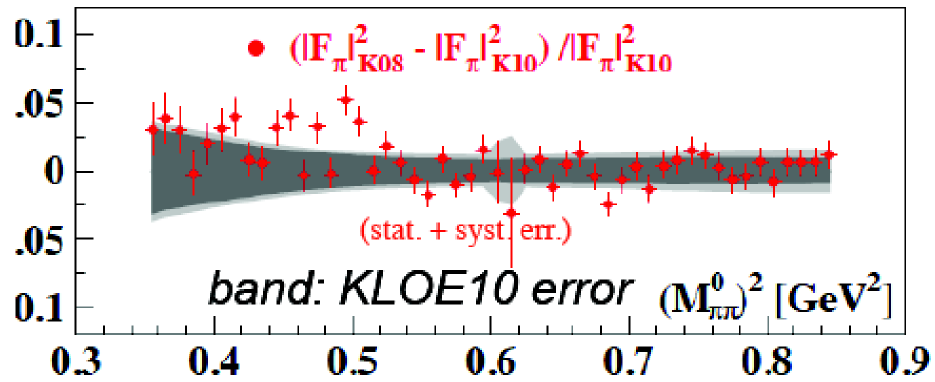
$a_\mu^{\pi\pi}$(0.1-0.85 GeV^2) = (478.5 ± 2.0 stat ± 5.0 sys ± 4.5 theo) · 10^{-10}

Comparison of results: KLOE10 vs KLOE08

KLOE08 result compared to KLOE10:



Fractional difference:



Excellent agreement with KLOE08, expecially above 0.5 GeV²

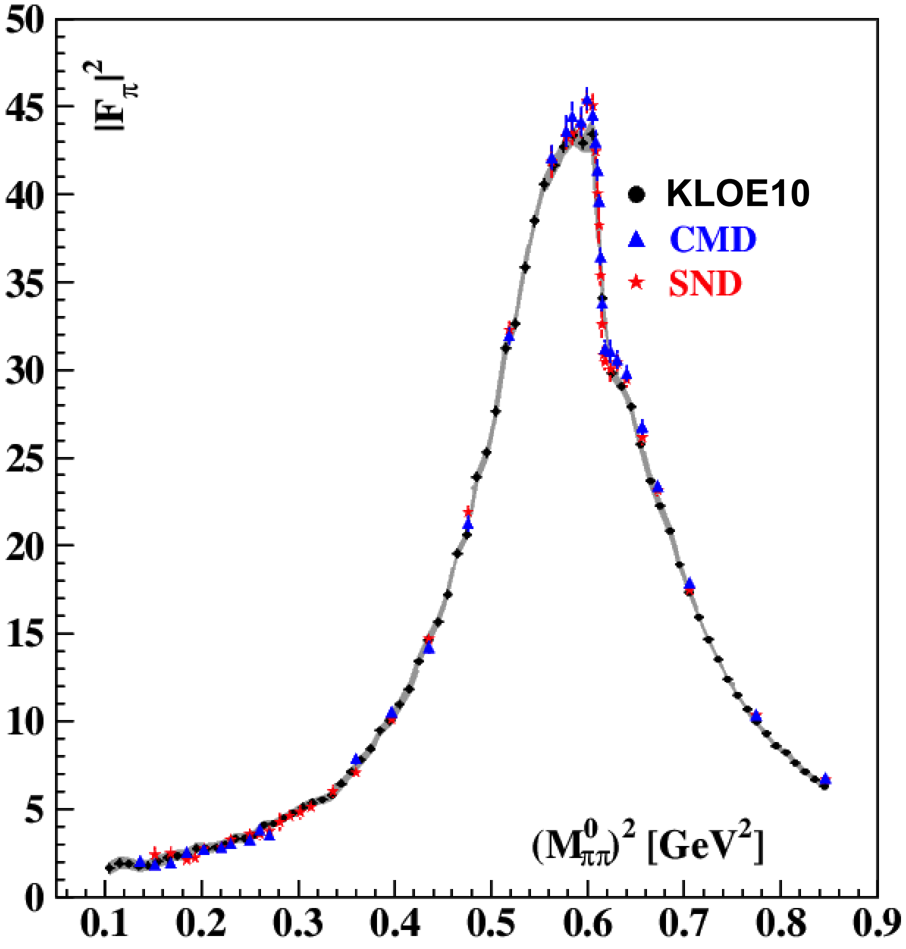
Combination of KLOE08 and KLOE10:

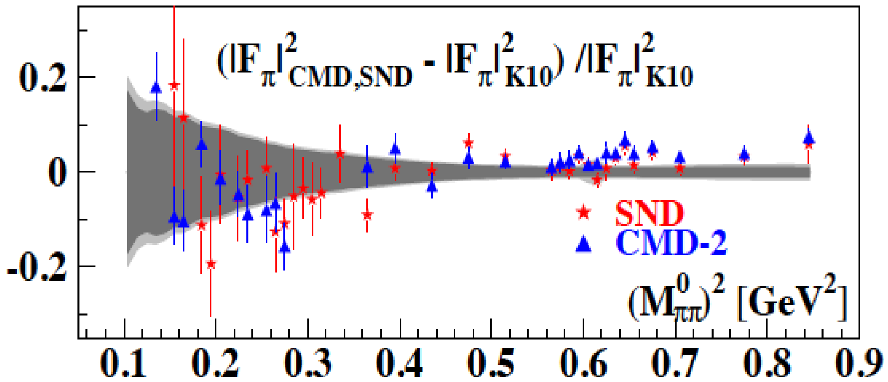
$a_\mu^{\pi\pi}$(0.1-0.95 GeV²) = (488.6±6.0) · 10^{-10}

KLOE covers ~70% of total a_μ^{HLO} with a fractional total error of 1.2%

Comparison of results: KLOE10 vs CMD-2/SND

CMD and SND results compared to KLOE10: Fractional difference





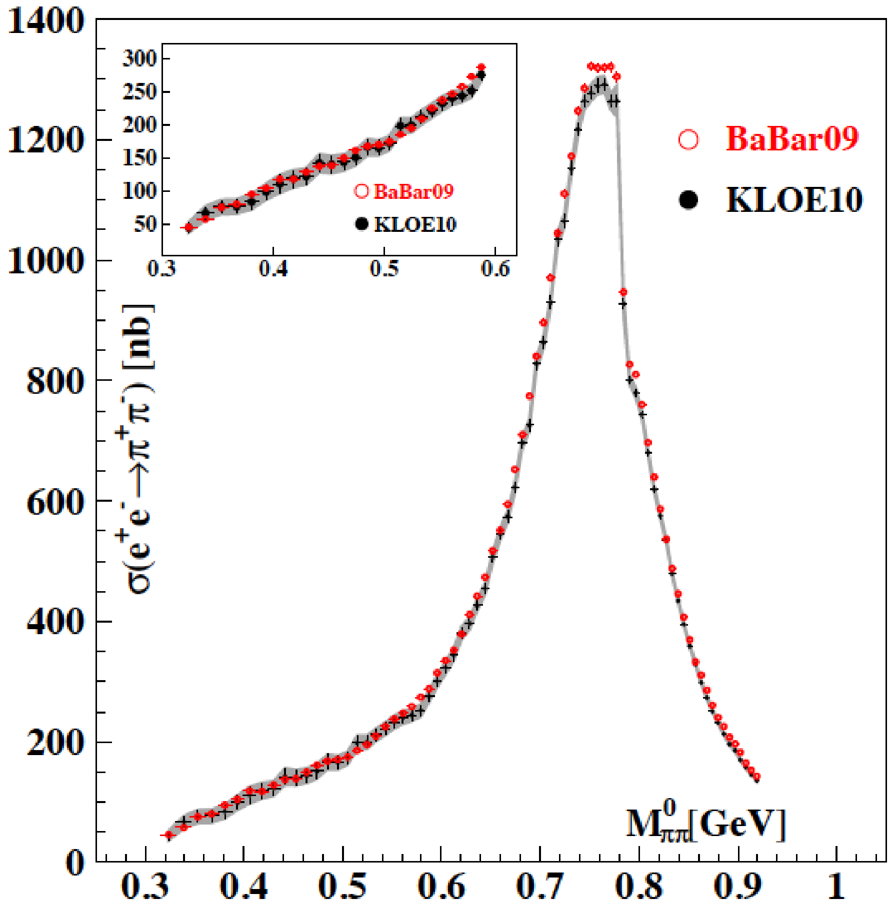
band: KLOE10 error

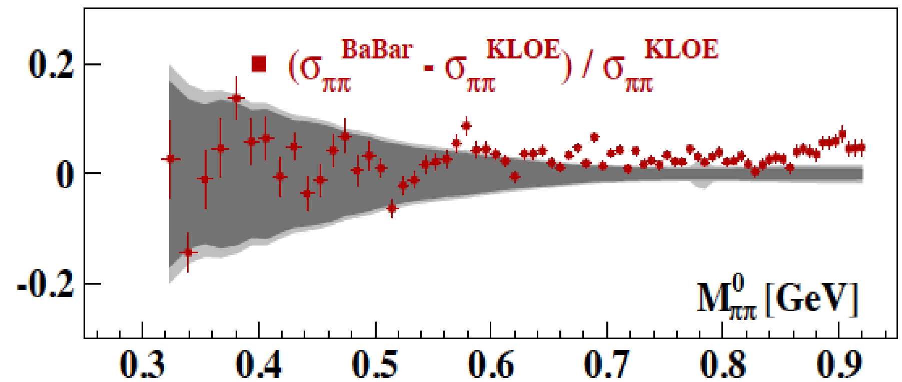
Below the ρ peak good agreement with CMD-2/SND.

Above the ρ peak KLOE10 slightly lower (as KLOE08)

Comparison of results: KLOE10 vs BaBar

BaBar results compared to KLOE10: Fractional difference





band: KLOE10 error

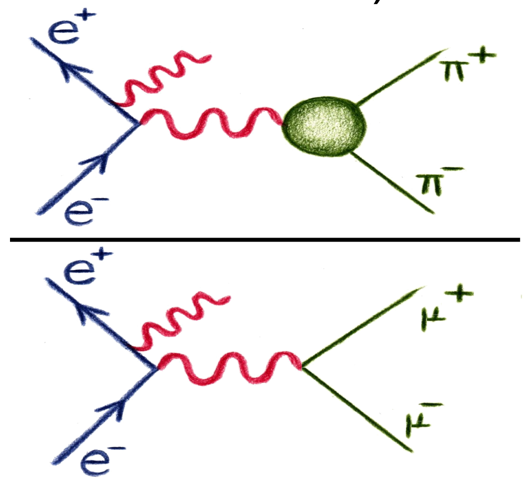
Agreement within errors below 0.6 GeV; BaBar higher by 2-3% above

New $\sigma_{\pi\pi}$ measurement from π/μ

An alternative way to obtain $|F_\pi|^2$ is the bin-by-bin ratio of pion over muon yields (instead of using absolute normalization with Bhabhas).

$$|F_\pi(s')|^2 \approx \frac{4(1+2m_\mu^2/s')\beta_\mu}{\beta_\pi^3} \frac{d\sigma_{\pi\pi\gamma}/ds'}{d\sigma_{\mu\mu\gamma}/ds'}$$

meas. quantities



Many radiative corrections drop out:

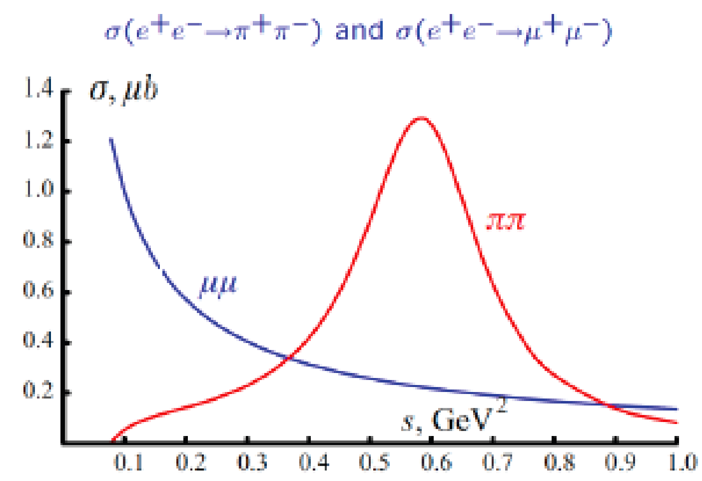
- **radiator function**
- **int. luminosity from Bhabhas**
- **Vacuum polarization**

$$\left(\sqrt{s} - \sqrt{|p_+|^2 + M_{trk}^2} - \sqrt{|p_-|^2 + M_{trk}^2}\right)^2 - (p_+ + p_-)^2 = 0$$

Separation btw $\pi\pi\gamma$ and $\mu\mu\gamma$ using M_{TRK}

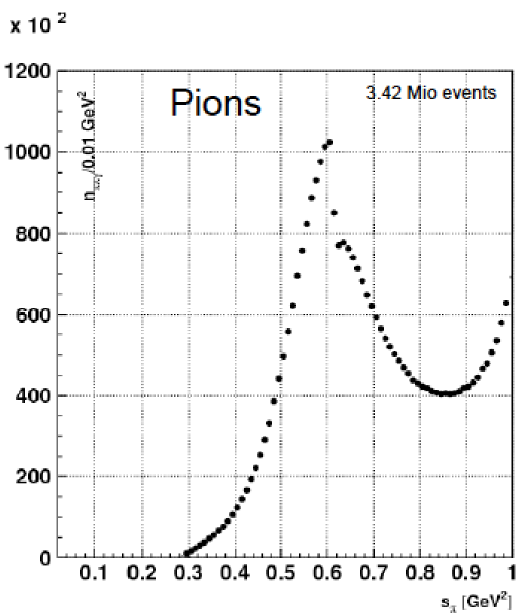
- *muons:* $M_{Trk} < 115$ *MeV*
- *pions :* $M_{Trk} > 130$ *MeV*

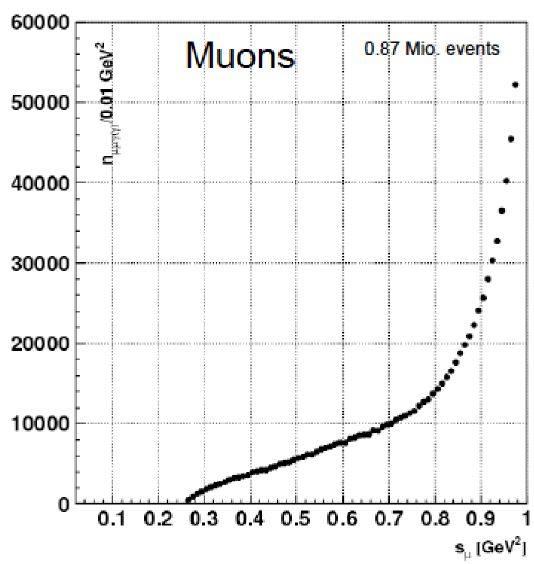
Very important control of π/μ separation in the ρ region! ($\sigma_{\pi\pi} >> \sigma_{\mu\mu}$)

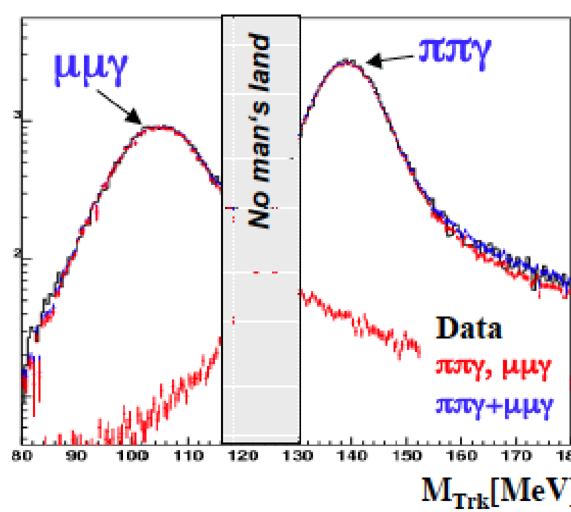


KLOE11: analysis of $\pi\pi\gamma/\mu\mu\gamma$

- 239.2 pb-1 of 2002 data sample (the same used in KLOE08 analysis), with photon at small angle : 0.87 Million $\mu\mu\gamma$ events (compared to 3.4 Million for $\pi\pi\gamma$)
- Careful work to achieve a control of ~1% in the muon selection, especially in the ρ region where $\pi\pi\gamma/\mu\mu\gamma \sim 10$. π/μ separation crosschecked in three different methods (MTRK fit, Kinematic fit, cut on σ_{MTRK})
- $\mu\mu\gamma$ (and $\pi\pi\gamma$) Efficiencies (Trk,Trg,PID) done on data
- Excellent data/MC agreement for many kinematic variables: M_{TRK}, track and γ polar angle, etc...



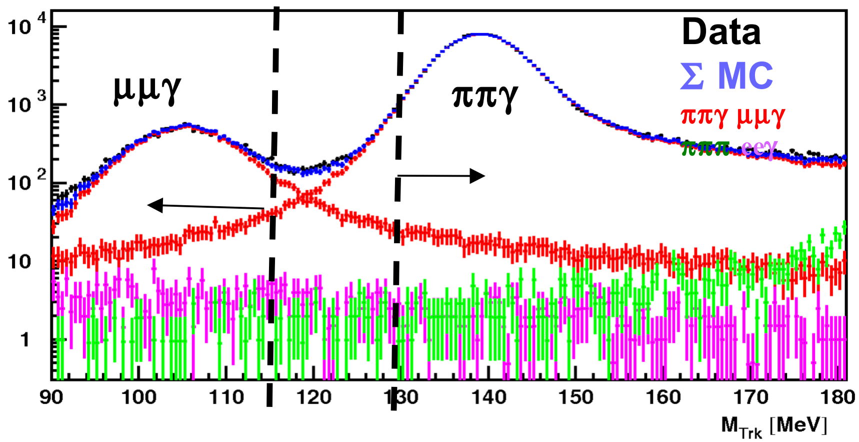




Background:

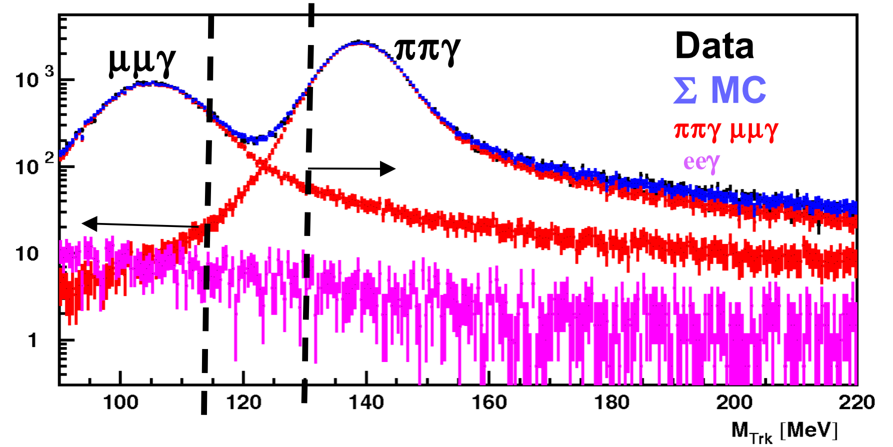
Main backgrounds estimated from MC shapes fitted to data distribution in MTrk ($\pi\pi\gamma/\mu\mu\gamma$, $\pi\pi\pi$, $ee\gamma$)

$0.60 < M_{\pi\pi}^2 < 0.62$ GeV2, χ^2/ndof = 158/180



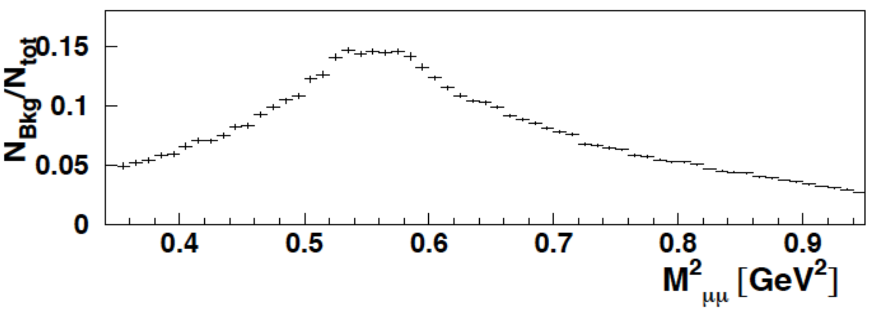
MTrk [MeV]

$0.84 < M_{\pi\pi}^2 < 0.86$ GeV2 χ^2/ndof = 179/258

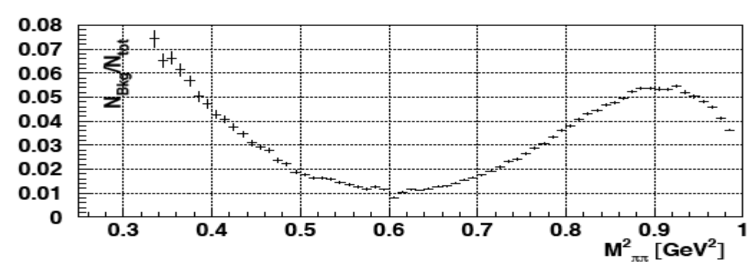


MTrk [MeV]

Tot % bckg to $\mu\mu\gamma$



Tot % bckg to $\pi\pi\gamma$

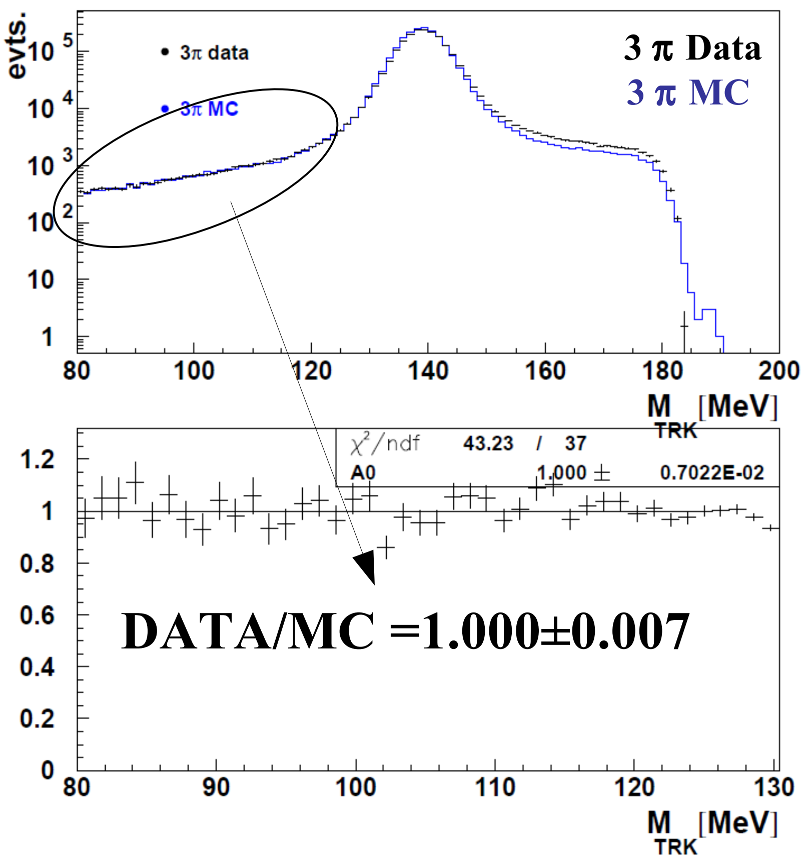


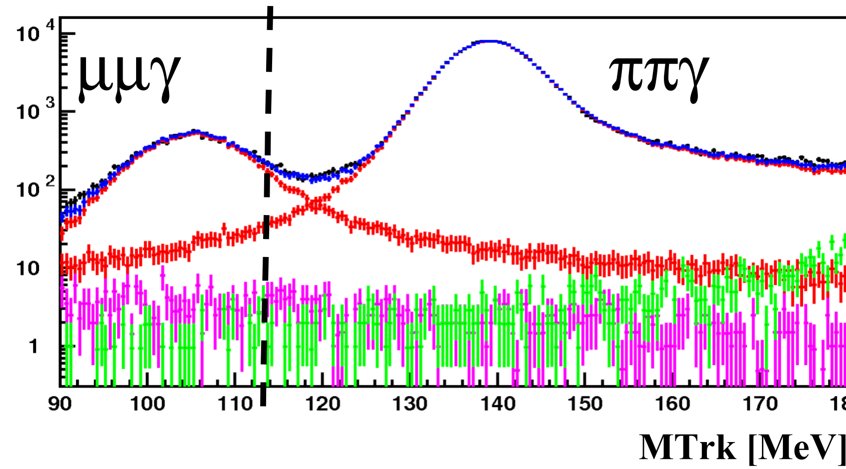
$M_{\pi\pi}^2$ (GeV2)

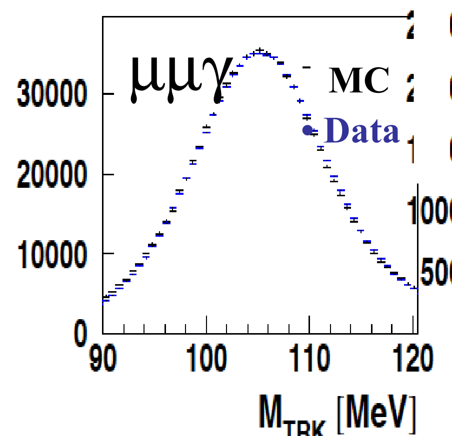
- Systematic error on $\mu\mu\gamma$ due to background~1% in the ρ peak

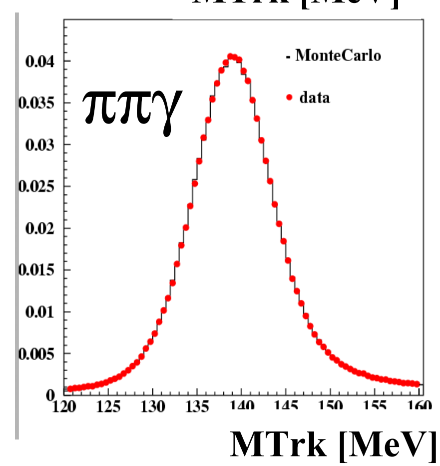
π/μ separation: control of $\pi\pi\gamma$ MTRK tail

- A careful work has been done to achieve a control of ~1% in the muon selection, especially ~0.6 GeV ^{2} (ρ peak) where $\pi/\mu \sim 10$.
- $\pi\pi\gamma$ % BG to $\mu\mu\gamma$ signal (MTRK<115 MeV) is ~10% ➔ $\pi\pi\gamma$ MTRK tail in the $\mu\mu\gamma$ region must be well under control.
- $\pi\pi\gamma$ MTRK tail tuned using $\phi\to\pi^+\pi^-\pi^0$ control sample. Excellent agreement on MTRK ($\pi\pi\gamma$ and $\mu\mu\gamma$) distributions





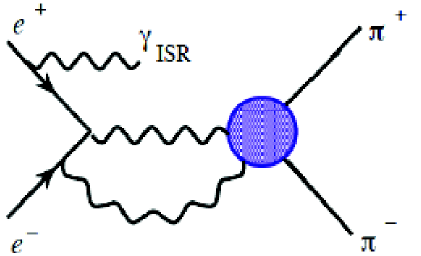




Final State Radiation (FSR)

The presence of not factorizable diagrams (like the 2 photon exchange) not present in Phokhara can lead to deviaton respect to:

$$\sigma^{0}_{\pi\pi} = \frac{4\pi\alpha^2}{3s'}(1+2m_\mu^2/s')\beta_\mu \frac{\left(d\sigma^{ISR+FSR}_{\pi\pi\gamma}(\theta^{\pi\pi}_{\Sigma}<15^o)/ds'\right)}{\left(d\sigma^{ISR}_{\mu\mu\gamma}(\theta^{\mu\mu}_{\Sigma}<15^o)/ds'\right)} \cdot Corr(\theta^{I+FSR}_{\Sigma}/\theta^{ISR}_{\Sigma})$$



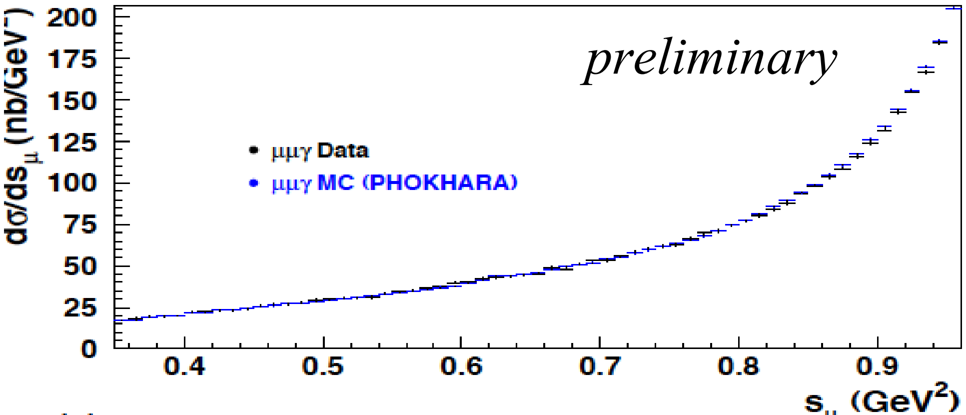
(c)

- For the ππγ this has been tested *in our previous publication and a validity within 0.2% was found (for pointlike pions)*
- For μμγ, we can assume the same 0.2% as conservative estimate of this missing contribution in the passage from I+FSR to ISR (in our small angle region)

We take the combined error of 0.3% for the uncertainty on the rel. FSR contribution

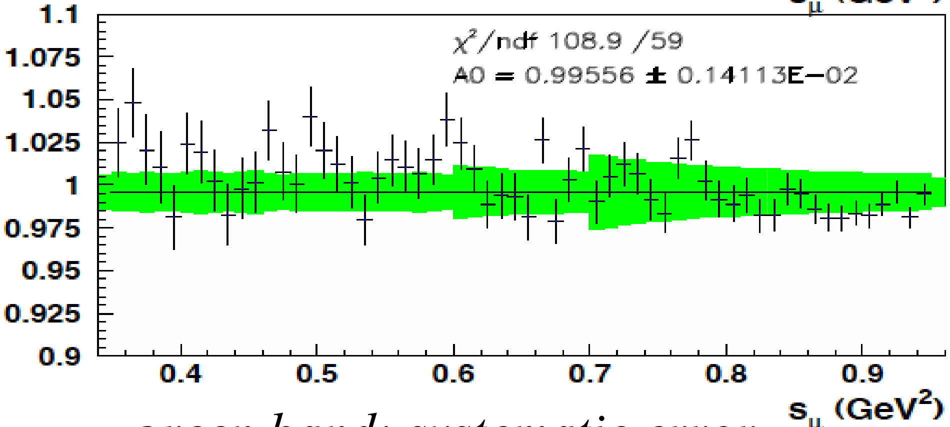
μμγ cross section: data/MC comparison

$$\frac{d\sigma^{obs}_{\mu\mu\gamma(\gamma)}}{dM^2_{\mu\mu}} = \frac{\Delta N_{Obs} - \Delta N_{Bkg}}{\Delta M^2_{\mu\mu}} \cdot \frac{1}{\varepsilon_{Sel}} \cdot \frac{1}{\int L dt}$$



$$\frac{d\sigma^{DATA}_{\mu\mu\gamma(\gamma)}}{d\sigma^{MC}_{\mu\mu\gamma(\gamma)}} = 0.996 \pm 0.001_{stat} \pm 0.01_{syst}$$

The systematic error has been averaged on s_μ



green band: systematic error

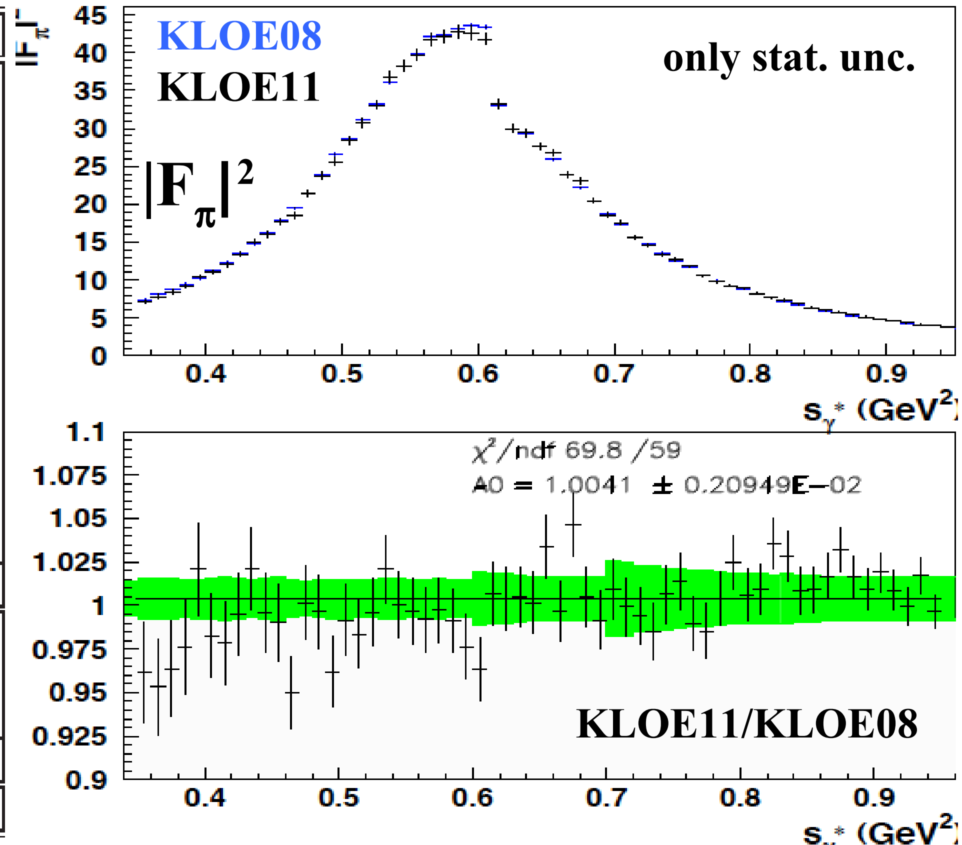
Good agreement with PHOKHARA MC

Consistency check of Radiator function, Luminosity, etc…

KLOE11 result on $|F_\pi|^2$ and comp. with KLOE08

	KLOE08	KLOE11
Syst. errors (%)	$\Delta^{\pi\pi}a_\mu$ abs	$\Delta^{\pi\pi}a_\mu$ ratio
Reconstruction Filter	negligible	negligible
Background subtraction	0.3	$0.8\ (0.3_{\pi\pi\gamma} \oplus 0.7_{\mu\mu\gamma})$
Trackmass	0.2	$0.4\ (0.2_{\pi\pi\gamma} \oplus 0.4_{\mu\mu\gamma})$
Particle ID	negligible	negligible
Tracking	0.3	$0.6\ (0.3_{\pi\pi\gamma} \oplus 0.5_{\mu\mu\gamma})$
Trigger	0.1	$0.1\ (0.1_{\pi\pi\gamma})$
Unfolding	negligible	negligible
Acceptance ($\theta_{\pi\pi}$)	0.2	negligible
Acceptance (θ_π)	negligible	negligible
Software Trigger (L3)	0.1	$0.1\ (0.1_{\pi\pi\gamma} \oplus 0.1_{\mu\mu\gamma})$
Luminosity	$0.3\ (0.1_{th} \oplus 0.3_{exp})$	-
$\sqrt{s}$ dep. of H	0.2	-
Total exp systematics	0.6	1.0
Vacuum Polarization	0.1	-
FSR treatment	0.3	0.3
Rad. function H	0.5	-
Total theory systematics	0.6	0.3
Total systematic error	0.9	1.1

preliminary



Good agreement btw the two measurements, especially in the ρ region!!!

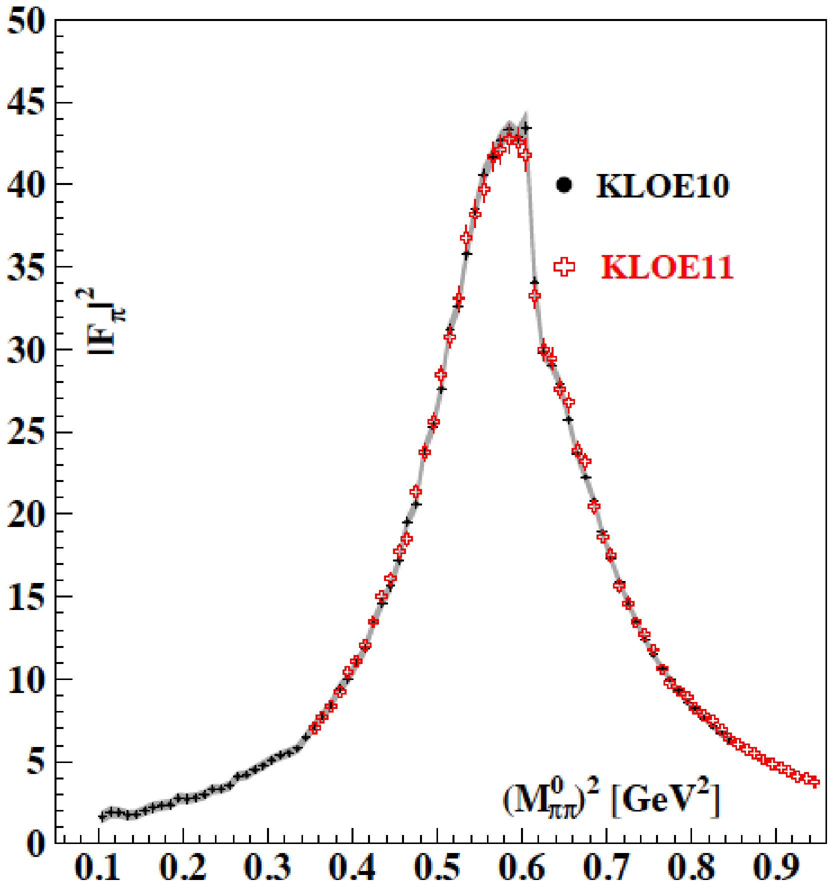
KLOE11: $a_\mu^{\pi\pi}$,(0.35-0.95 GeV^2) = (384.1 ± 1.2stat ± 4.0sys ± 1.2theo) · 10^{-10}

KLOE08: $a_\mu^{\pi\pi}$(0.35-0.95 GeV^2) = (387.2 ± 0.5stat ± 2.4sys ± 2.3theo) · 10^{-10}

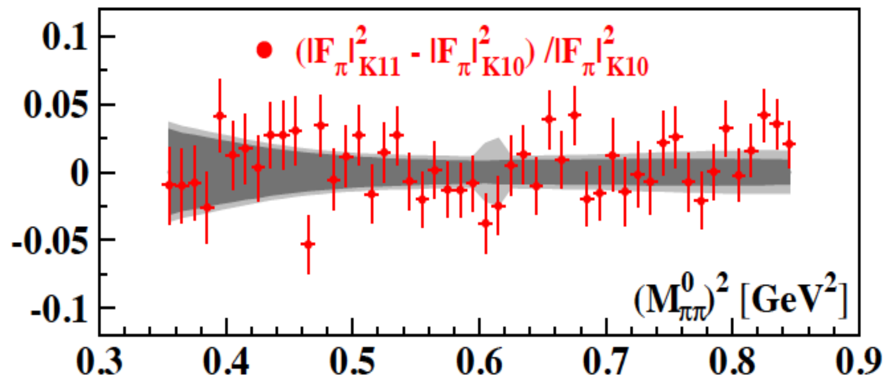
Comparison of results: KLOE11 vs KLOE10

KLOE11 result compared to KLOE10:

preliminary



Fractional difference:



band: KLOE10 error

Excellent agreement between the two measurements!

Comparison with other exp. in progress

KLOE11: $a_\mu^{\pi\pi}$,(0.35-0.85 GeV^2) = (376.4 ± 1.2stat ± 4.1sys tot) · 10^{-10}

KLOE10: $a_\mu^{\pi\pi}$,(0.35-0.95 GeV^2) = (376.6 ± 0.9stat ± 3.3sys tot) · 10^{-10}

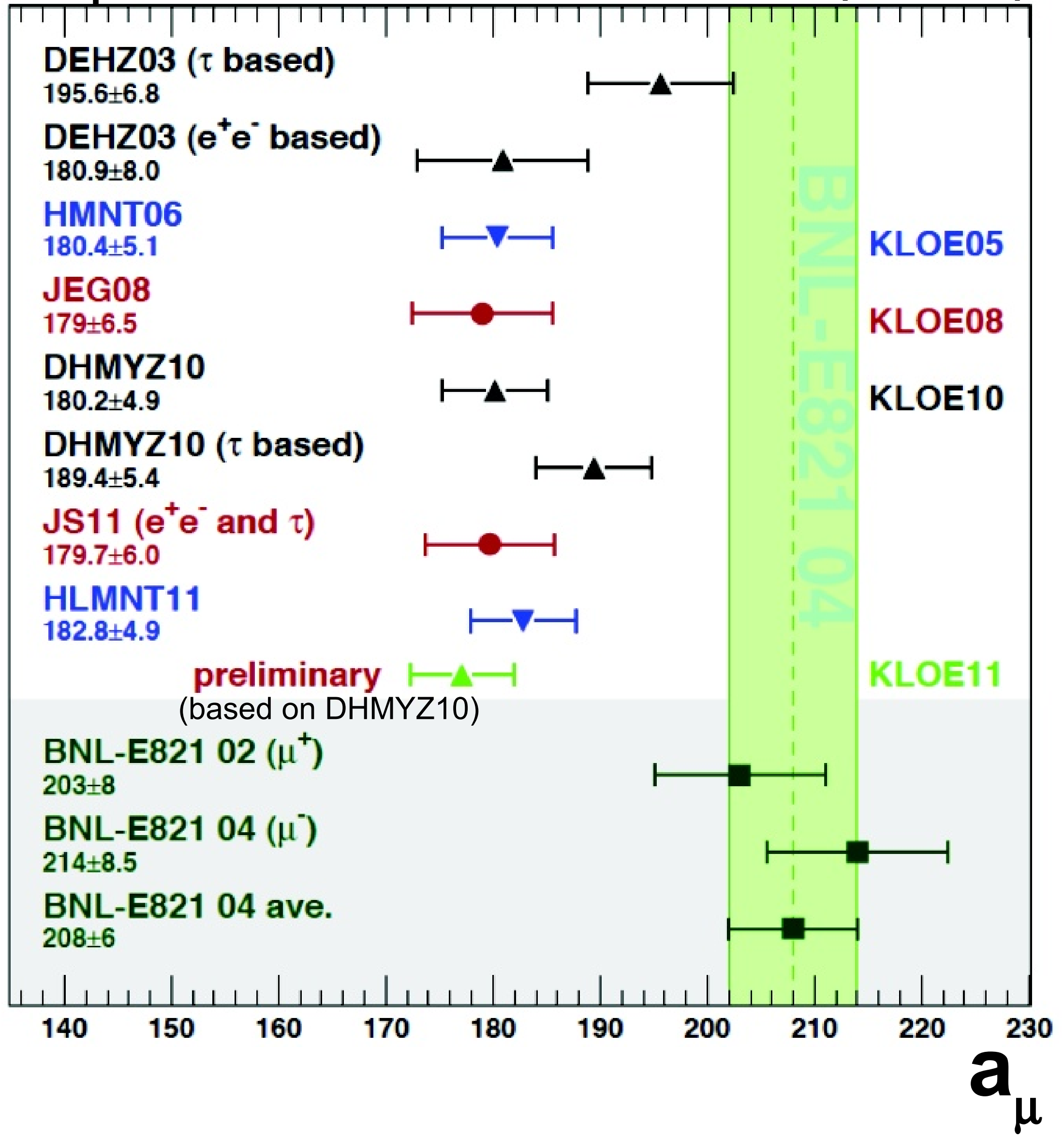
$a_\mu=(g_\mu-2)/2$:

Theoretical predictions compared to the BNL result (2009)

The latest inclusion of all e^+e^- data gives a discrepancy btw a_μ^{SM} and a_μ^{EXP} $\geq 3\,\sigma$

Preliminary KLOE11 in agreement with previous KLOE measurements and confirms this discrepancy!

Very important the new g-2 experiments (at FNAL and JPARC)!



Conclusion

During the last 10 years KLOE has performed a series of precision measurements with ISR which confirmed a 3σ discrepancy between a_μ^{SM} and the BNL measured value and allowed the measurement of $a_\mu^{\pi\pi}$ in the region 0.1-0.95 GeV^2(70% of a_μ^{HLO}) with 1.2% total error using KLOE data only.

A new (preliminary) measurement of $|F_\pi|^2$ from the $\pi\pi\gamma/\mu\mu\gamma$ ratio (based on 240 pb^{-1}) with 1.1% systematic error has been done. Preliminary results show good agreement for $\mu\mu\gamma$ cross section with PHOKHARA MC and for $|F_\pi|^2$ and $a_\mu^{\pi\pi}$ with previous KLOE published measurements.

Still more than1.5 fb^{-1} of KLOE data on tape. This is a ~4 improvement in statistics. We plan to analyse these data to improve $\sigma^{\pi\pi}$(and may be other channels) measurement.

In addition we expect about 25 fb^{-1} at KLOE-2 .
See (Balwierz and Giovannella's Talk)

Spares

Cross check of π/μ separation

- The π/μ separation has been crosschecked with two different (and independent) methods:

- A kinematic fit, in the hypothesis of 2 body+1γ (ISR) events.

- A cut on the quality of the fitted tracks, parametrized by σ_{MTRK}

$$\sigma^2_{M_{trk}} = \begin{pmatrix} \frac{\partial M_{trk}}{\partial k_1} & \frac{\partial M_{trk}}{\partial \cot\theta_1} & \frac{\partial M_{trk}}{\partial \varphi_1} & \frac{\partial M_{trk}}{\partial k_2} & \frac{\partial M_{trk}}{\partial \cot\theta_2} & \frac{\partial M_{trk}}{\partial \varphi_2} \end{pmatrix} \cdot$$

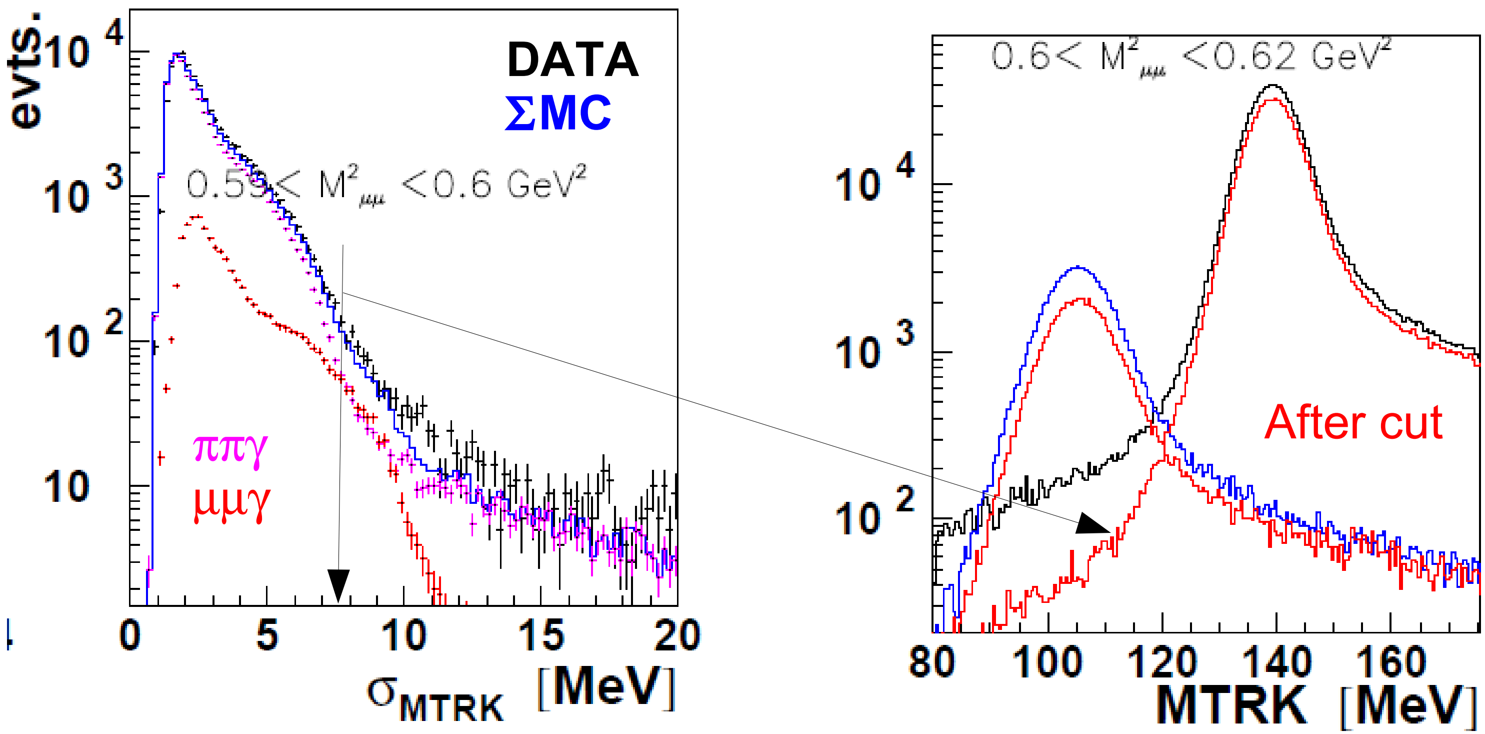
$$\begin{pmatrix} \sigma^2_{k_1} & \rho_{k_1 \cot\theta_1} & \rho_{k_1\varphi_1} & 0 & 0 & 0 \\ \rho_{\cot\theta_1 k_1} & \sigma^2_{\cot\theta_1} & \rho_{\cot\theta_1\varphi_1} & 0 & 0 & 0 \\ \rho_{\varphi_1 k_1} & \rho_{\varphi_1\cot\theta_1} & \sigma^2_{\varphi_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{k_2} & \rho_{k_2\cot\theta_2} & \rho_{k_2\varphi_2} \\ 0 & 0 & 0 & \rho_{\cot\theta_2 k_2} & \sigma^2_{\cot\theta_2} & \rho_{\cot\theta_2\varphi_2} \\ 0 & 0 & 0 & \rho_{\varphi_2 k_2} & \rho_{\varphi_2\cot\theta_2} & \sigma^2_{\varphi_2} \end{pmatrix} \begin{pmatrix} \frac{\partial M_{trk}}{\partial k_1} \\ \frac{\partial M_{trk}}{\partial \cot\theta_1} \\ \frac{\partial M_{trk}}{\partial \varphi_1} \\ \frac{\partial M_{trk}}{\partial k_2} \\ \frac{\partial M_{trk}}{\partial \cot\theta_2} \\ \frac{\partial M_{trk}}{\partial \varphi_2} \end{pmatrix}$$

- π/μ separation obtained with these methods well in agreement with the standard one.

Cross check of π/μ separation σ_{MTRK}

We have achieved an excellent Data/MC agreement for muons in many kinematic variables (as we did for pions)

BG reduction-Cut effect



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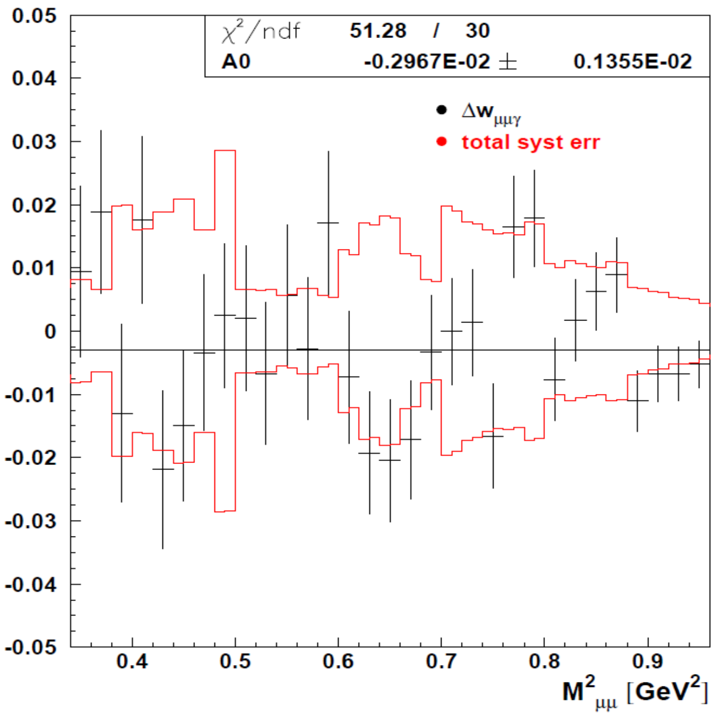
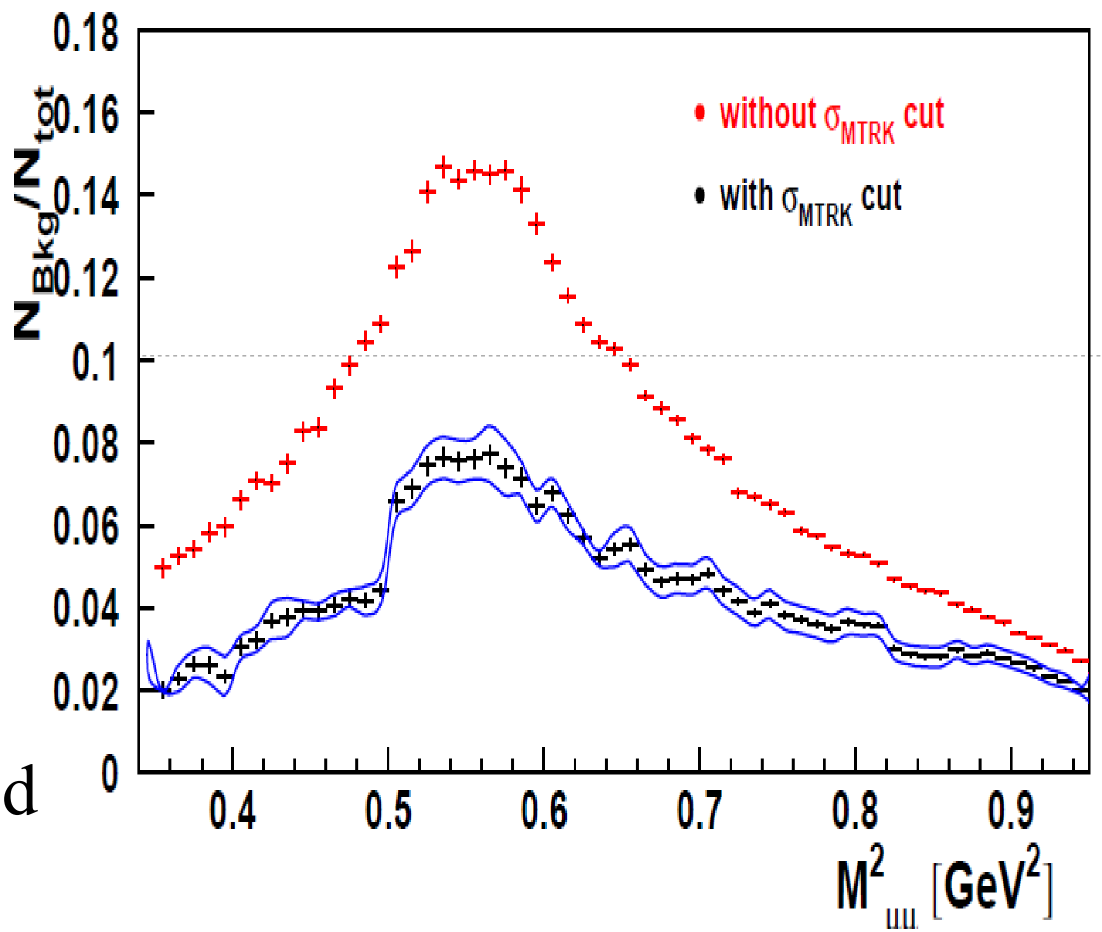
Cross check of π/μ separation σ_{MTRK}

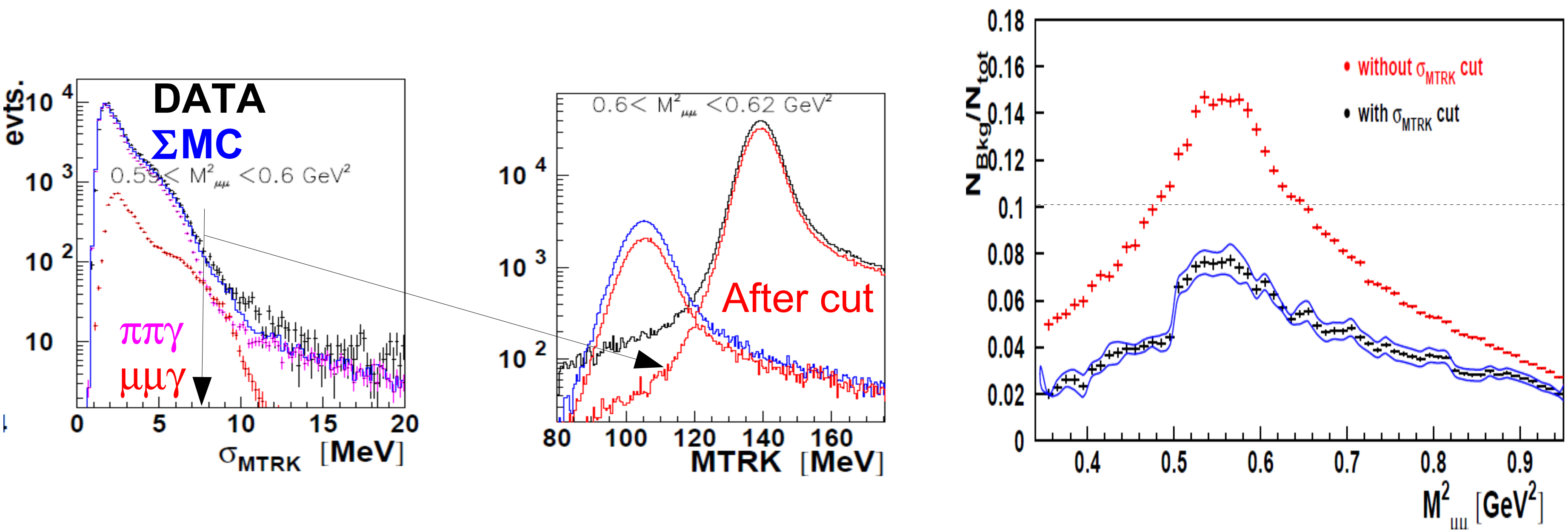




The $\mu^+\mu^-\gamma$ fraction obtained by the σ_{MTRK} cut are consistent with the standard procedure within the systematic error.

BG suppression under 10%

Cross check of π/μ separation σ_{MTRK}



- A cross check was realized by using a independent analysis based on a cut on the quality of the fitted tracks, parametrized by σ_{MTRK}
- The alternative analysis produced: a **BG contribution below 10%** (see figures); the $\mu^+\mu^-\gamma$ fraction determination consistent with the other procedure within the systematic error.