Charged lepton flavour violation in muons: theory overview

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Muons for the Future

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Flavour and CP violation: SM

Flavour in the **Standard Model:** interactions between *fermion* families (and the *Higgs*)

Y^u_{ij}, Y^d_{ij} and Y^ℓ_{ij} $\rightsquigarrow$ encode **flavour** dynamics (masses, mixings & CP violation)

flavour-universal gauge interactions

SM quark sector: 6 massive states

flavour violated in charged current interactions $V^{ij}_{\text{CKM}}\, W^\pm \bar{q}_i q'_j$

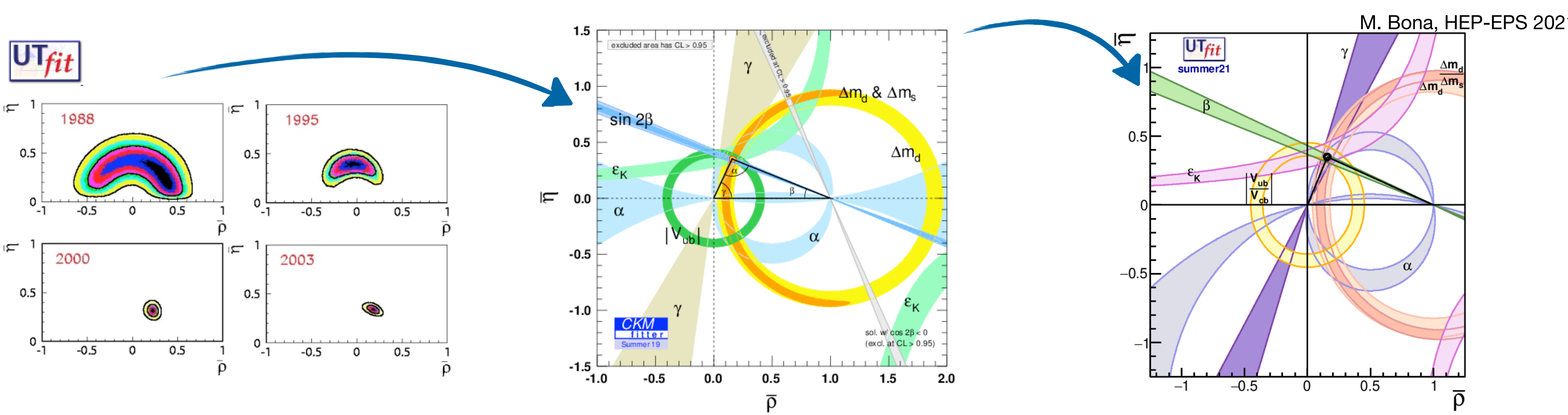
conservation of total baryon number in SM interactions

CP violation sources: δ_{CKM} and θ_{QCD}

(strongly constrained by **tiny** neutron **EDM**)

not enough to explain observed **BAU from baryogenesis**

Extensive probes of the **"CKM paradigm"**: meson oscillation and decays, CP violation...



Flavour and CP violation: SM

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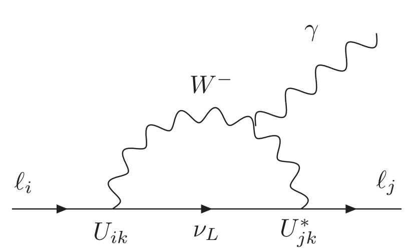
flavour-universal gauge interactions

SM lepton sector: (strictly) **massless neutrinos**
conservation of total lepton number and lepton flavours
lepton flavour universality preserved (only broken by Yukawas)
no intrinsic CPV sources - **tiny** leptonic **EDMs** (4-loop... $d_e^{\text{CKM}} \leq 10^{-38} e$ cm)

Extend the SM to accommodate $\nu_\alpha \leftrightsquigarrow \nu_\beta$: assume most **minimal extension** SM_{m_ν}

[SM_{m_ν}= "ad-hoc" m_ν (Dirac), U_{PMNS}]

In SM_{m_ν} : **flavour-universal** lepton couplings, total **lepton number conserved** (LNC)



cLFV possible... but **not observable!! BR($\mu \to e\gamma$)** $\sim 10^{-54}$

lepton **EDMs** still **beyond observation...**

▷ **cLFV, LNV, EDMs, ...**: observation of SM-forbidden leptonic modes and/or tensions with data
$\Rightarrow$ Discovery of New Physics! (Possibly before direct signal @ LHC!)

Flavour and CP violation: SM

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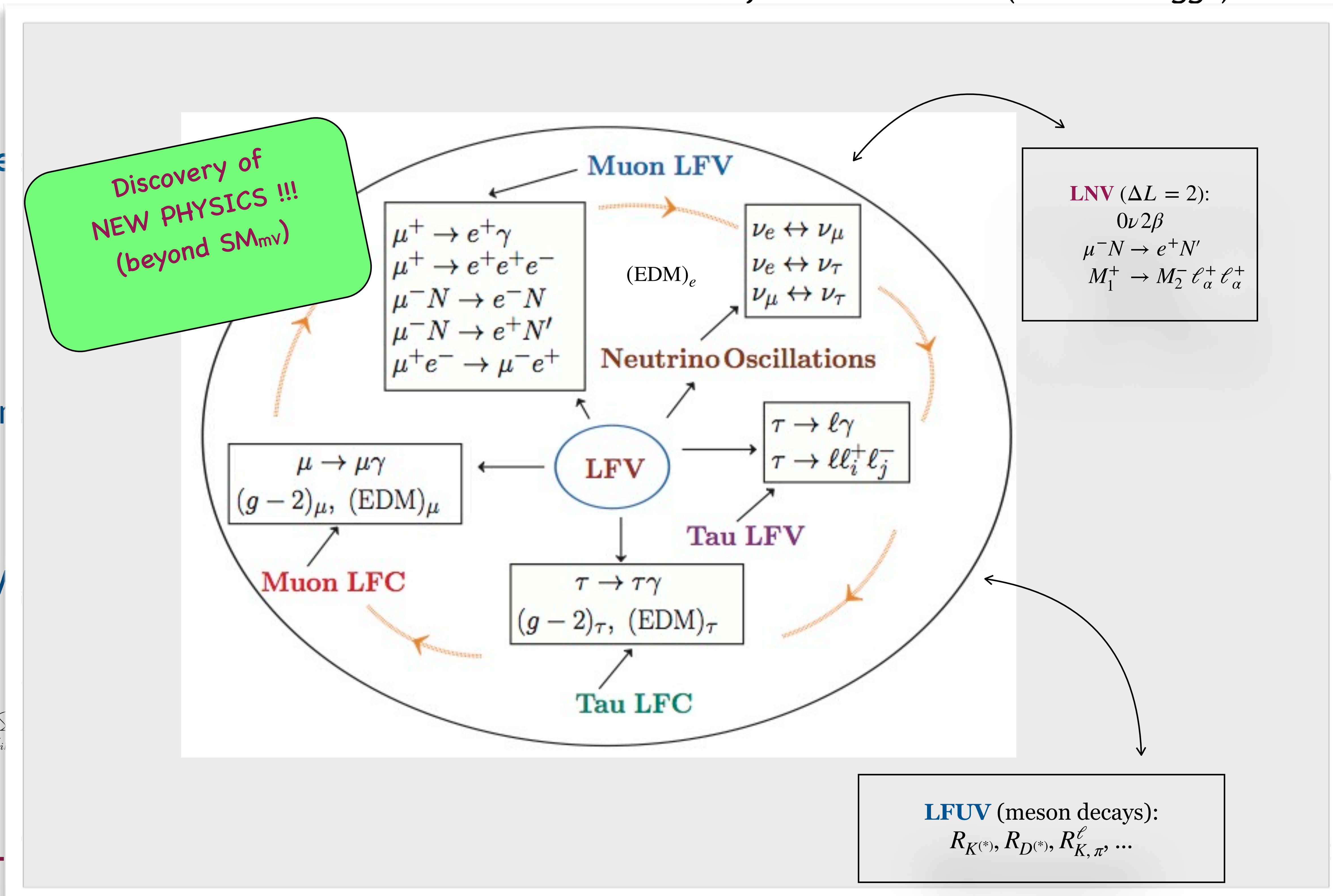

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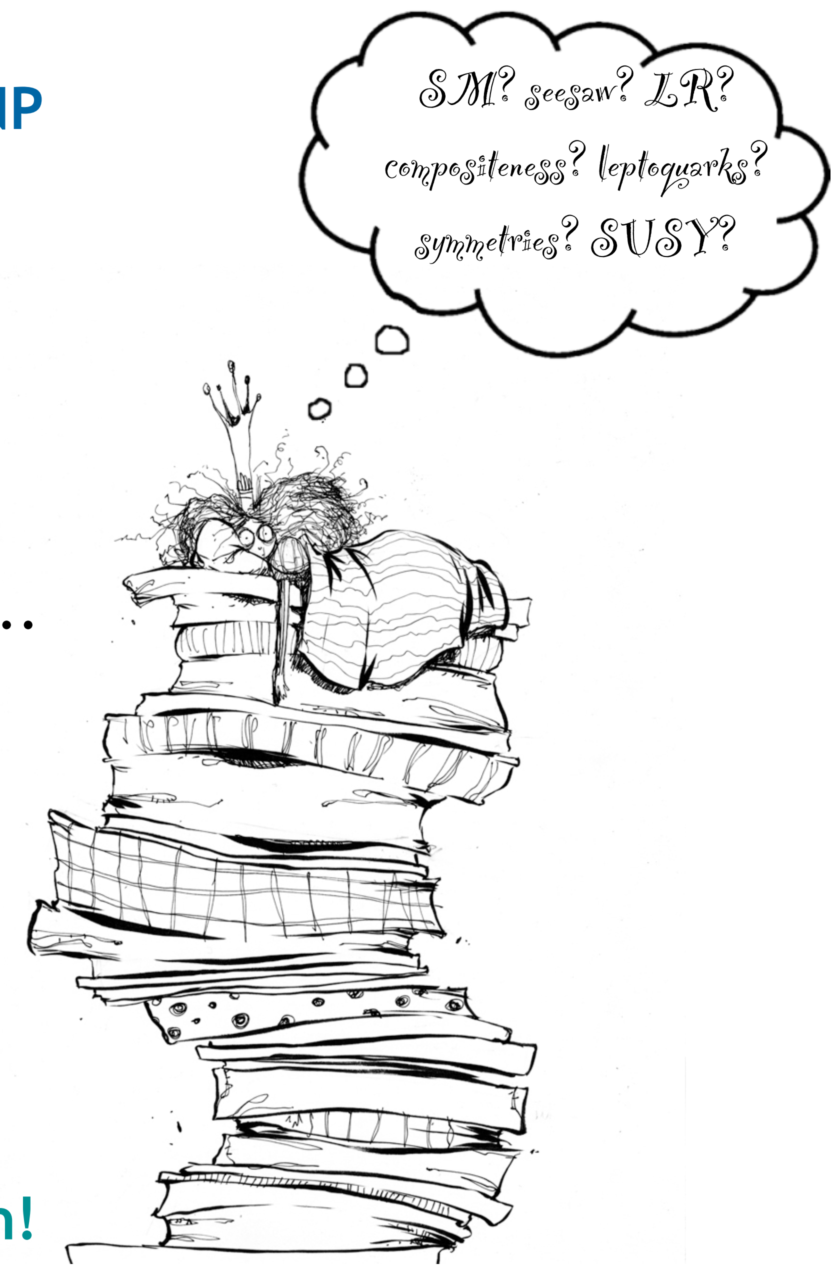
Flavour'ed paths: SM and beyond...

Strong arguments in favour of New Physics!

A number of theoretical caveats... and **observations unaccounted for in the** SM:
baryon asymmetry of the Universe, viable dark matter candidate, neutrino oscillations

- **Neutrino oscillations**: 1st laboratory ("flavoured") **evidence of NP**
  - ⇒ massive neutrinos and leptonic mixings $U^{\alpha i}_{\text{PMNS}}$
  - ⇒ New (Majorana) fields? New sources of CP violation?
    $\Delta L \neq 0$ and leptogenesis... (?)



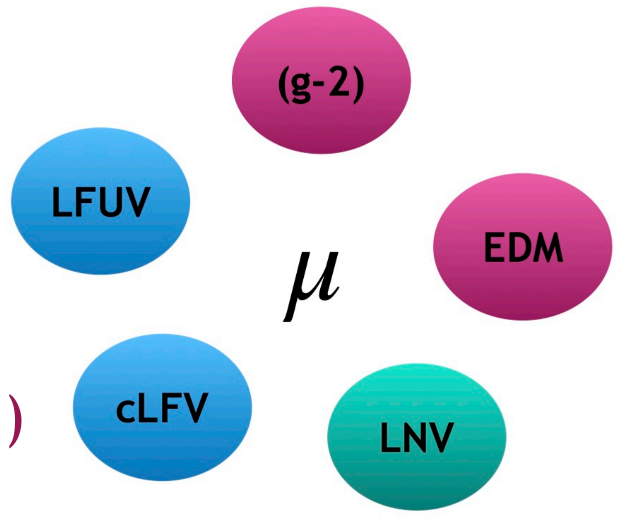
- **Tensions** (?) between **SM and observation: rooted in flavours!**
  - $(g-2)_\mu$, $(g-2)_e$, anomalies in atomic decays, (meson decays)...
  - ⇒ in close relation with the lepton sector
    (and often involving **muons!**)

Many **hints** and a clear **necessity** of **New Physics...**
Which NP model? Realised at which **scale** Λ_{NP}?
⇒ **Unique opportunities to search for NP in the lepton sector**
exploring connections to **mechanism of ν mass generation!**

Muon flavours to lead the way!

New Physics quests with muon cLFV

Why muon flavours?



A SM **muon ID: lepton, spin 1/2, charge -1**

First **discovered** in 1936 (cosmic radiation)

Mass m_μ = 105 MeV; **lifetime** $\tau_\mu = 2.1969811 \pm 0.0000022\,\mu$s

Michel decay $\mu^- \to e^- \bar{\nu} \nu$ (BR ≈ 100%) $\rightsquigarrow$ **determination of** G_F

Couplings to EW gauge bosons: $g_e = g_\mu = g_\tau \propto g$ (universal? or not!)

Electric dipole moment (from δ_{CKM}): $|d_\mu| \approx 10^{-36}$ e.cm (new CPV?)

Magnetic dipole moment $(g-2)_\mu$: an exciting adventure!

4.2σ **tension** with SM? (new physics expected to show up elsewhere? in **cLFV?**)

or in fair **agreement** with SM? (LQCD vs. data driven… impact for **EW fit?**)

and in comparison with $(g-2)_e$? (**LFU violating** new physics?)

See talk by P. Paradisi

"Bound states": **Muonium** $(\mu^+ e^-)$ $\rightsquigarrow$ **QED (and gravity!) tests;** $^\mu$H $\rightsquigarrow$ **proton radius**

Muonic atoms (1s bound state) $\rightsquigarrow$ **P violation,** cLFV, **and more!**

▷ Muons - ideal **probe for NP**: from lepton **flavour universality** tests,
to **anomalous magnetic moments**, … to cLFV!

cLFV muon observables

Muons - ideal **probe for NP**: from lepton **flavour universality** tests,
to **anomalous magnetic moments**, ... to **cLFV!**

Muon cLFV - extensive opportunities, numerous observables, at low- and high-energies

- **Leptonic decays:** radiative $\mu \to e\gamma$ and three-body $\mu \to 3e$
  muonic atoms $\mu^-(A,Z) \to e^-(A,Z)$ & LNV $\mu^-(A,Z) \to e^+(A,Z-2)^*$
  nuclear assisted Coulomb decays $\mu^-e^- \to e^-e^-$
  Muonium oscillations $\text{Mu}(\mu^+e^-) - \overline{\text{Mu}}(\mu^-e^+)$ and decays $\text{Mu}(\mu^+e^-) \to e^+e^-$
  Light "invisible" searches (e.g. $\mu \to e\phi$, ...)
  See talk by S. Renner

- **Semi-leptonic decays:** $\tau \to M\mu,\ M \to (M')\mu\ell$

- **At colliders:** $Z \to \mu\tau$, $H \to \mu\tau$ (e.g. FCC-ee, CEPC, ...);
  high p_T dilepton tails in $pp \to \mu\ell$ ...
  Numerous channels at a future muon collider!
  See talk by N. Craig

See talk by Y. Fujii

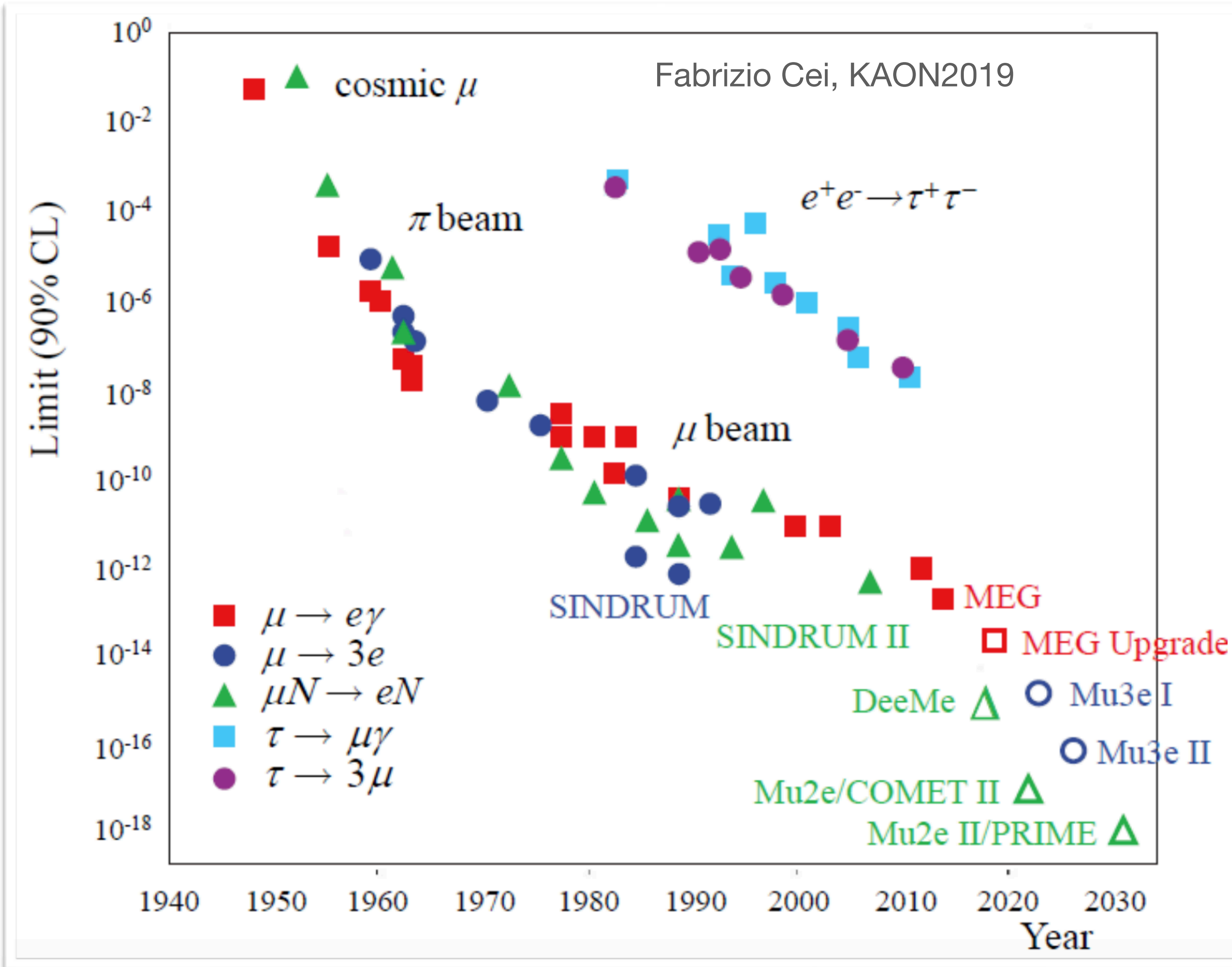
Muons: *lightest "unstables"* - clean objects, ideal & versatile probes for new physics searches
At the centre of a world-wide comprehensive programme - experiments *and* theory

cLFV muon observables

Searching for tiny μ-cLFV effects ⇒ high-intensity sources for excellent sensitivities

See talks by A. Papa & B. Bernstein

C. Voena and G. Pezzullo



⇒ **Need many many (really many!) muons:** excellent sensitivity with **current sources,**
amazing prospects with advent of **high-intensity beams** (PSI, FNAL, J-PARC)
and beyond?... ***Muon facility? Muon collider?***

cLFV muon observables

Searching for tiny μ-cLFV effects $\Rightarrow$ high-intensity sources for excellent sensitivities

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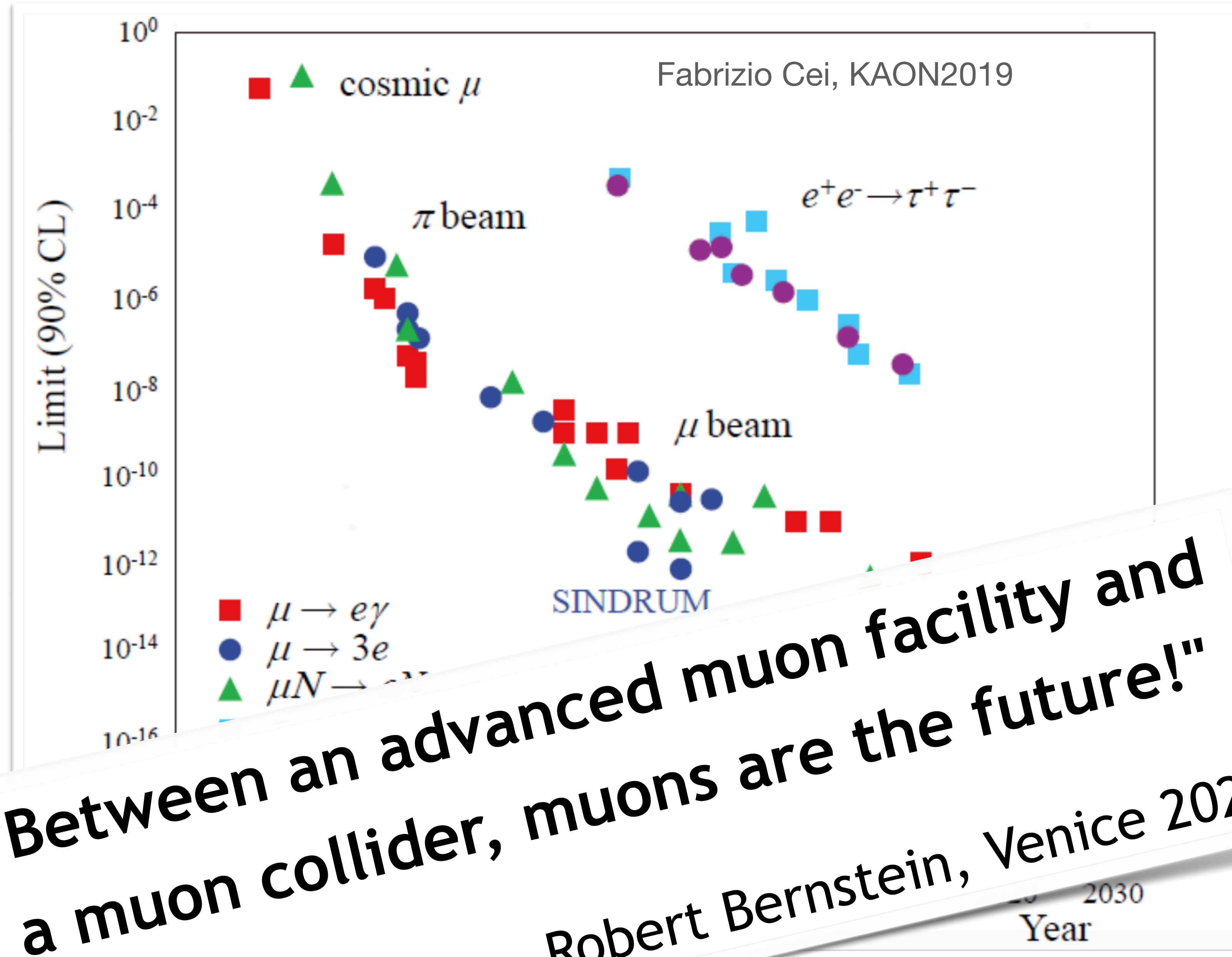


"Between an advanced muon facility and a muon collider, muons are the future!"

Robert Bernstein, Venice 2023

$\Rightarrow$ **Need many** ... **muons:** excellent sensitivity with **current sources**, ...zing prospects with advent of **high-intensity beams** (PSI, FNAL, J-PARC) and beyond?... ***Muon facility? Muon collider?***

Learning about New Physics from muon cLFV

aμazing sources of information on the NP models we do need!

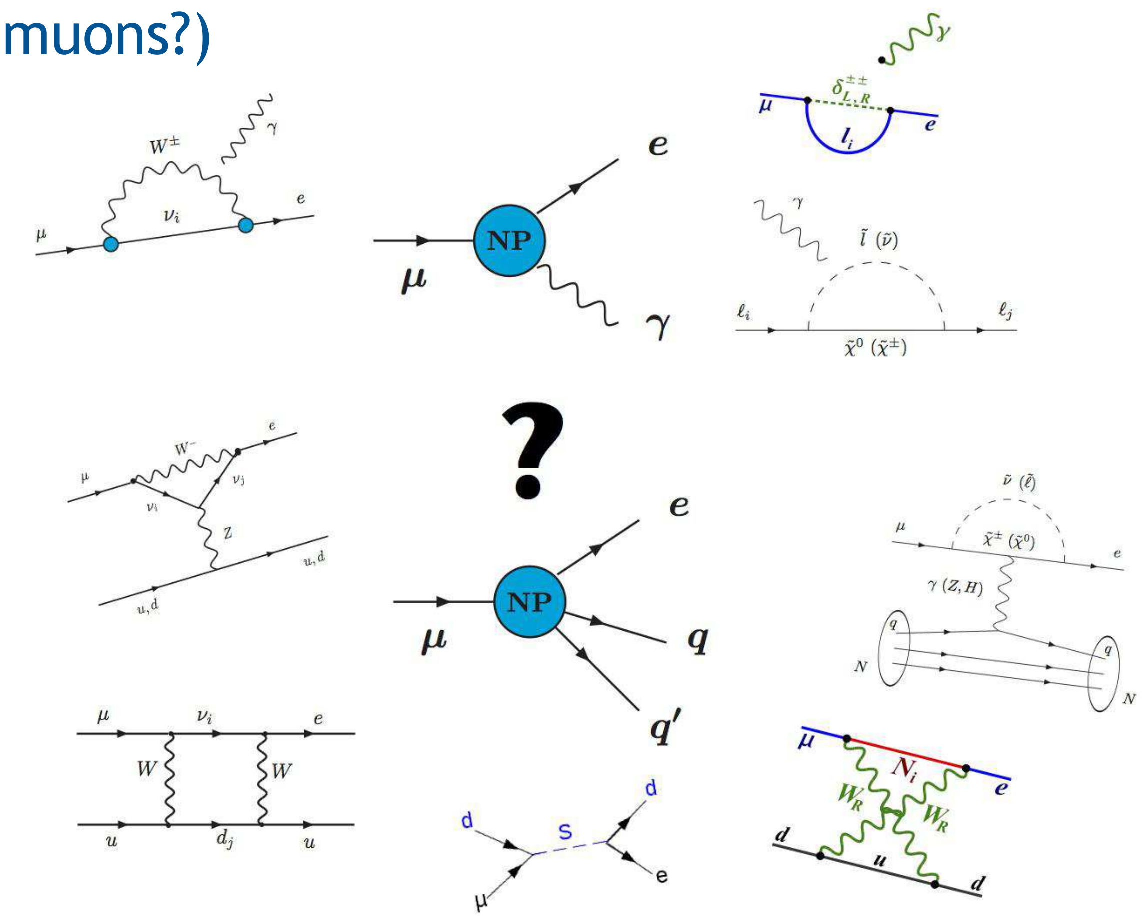
Muon cLFV: learning about New Physics

Aμazing prospects - so many experimental avenues, so many channels to study!

Near future: first hints of **New Physics** from **μ-cLFV**, or tighter constraints ... *but on which NP?*

In "theory", what are the methods to **interpret the data** - measurements or new bounds?

(What can we learn from all these muons?)



Towards the **full UV** complete **NP model:**
m_ν, BAU, DM, flavour & CP,
gauge unification, hierarchy, ...

Minimal NP models:
simple BSM

Constraining classes of
SM extensions: **EFT**
(model-independent)

Experimental data: ★
... **muon cLFV** ...

$\Rightarrow$ Two **phenomenological approaches** or
flavoured paths to New Physics:
Effective approach (model independent)
Model-specific (implications for a given BSM)

Learning about New Physics from muon cLFV: effective approach (EFT)

A brief survey of recent developments & ideas...

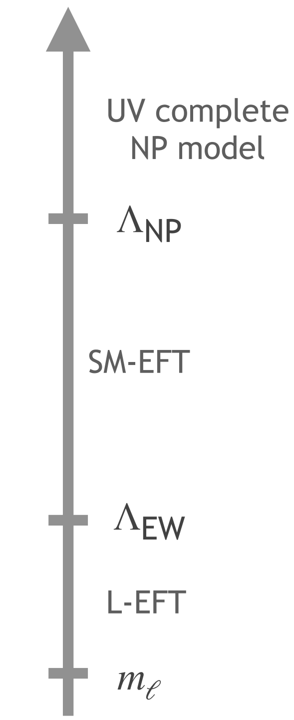
Muon cLFV: EFT approach to New Physics

SM interpreted as a low-energy limit of a (complete, yet unknown) NP model
⇒ **model-independent**, **effective approach (EFT)**

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \sum_{n\geq 5} \frac{1}{\boldsymbol{\Lambda}^{n-4}}\, \mathscr{C}^n(g, Y, \ldots)\, \mathscr{O}^n(\ell, q, H, \gamma, \ldots)$$

(unknown) NP **scale** — **effective** coefficients — effective **operators**

$\mathscr{O}^5 \rightsquigarrow$ Weinberg operator (m_ν)
$\mathscr{O}^6 \rightsquigarrow$ flavoured contributions
(among many others!)



Cast observables in terms of $\mathscr{C}_{ij}$ and Λ_{NP}; Apply **current data** (limits, ...)

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \ldots + \frac{\mathscr{C}_9 \mathscr{O}^9}{\Lambda'^5_{\text{LNV}}}(0\nu 2\beta) + \ldots$$

⇒ **Constrain** $\mathscr{C}_{ij}$ and/or **infer sensitivity** of process to large sets of $\mathscr{C}_{ij}$

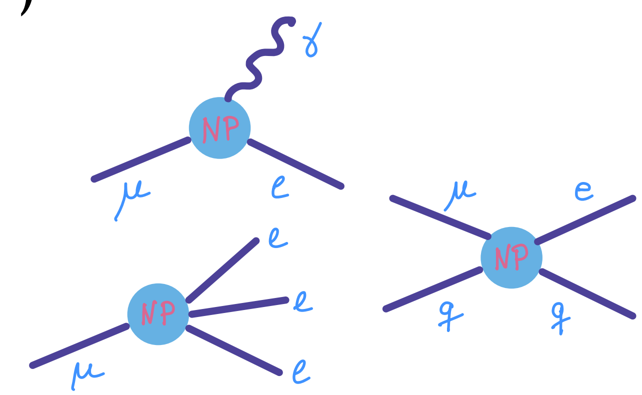
⇒ **Hints on** Λ_{NP} (and on **properties** of new states & **nature** of couplings)

Deceptively simple task... different new physics scales, numerous operators!
Technically very involved! **Many contributions in recent years (at all levels!)**

Muon cLFV: EFT approach to New Physics

Cast **current data** (limits, ...) in terms of $\mathscr{C}_{ij}$ and Λ_{NP}: cLFV operators ($\mathscr{O}^6$)

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \boxed{\frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta)} + \ldots$$



- **Simple examples:** at leading order one has

$$\text{BR}(\mu \to e\gamma) \simeq 384\pi^2 \frac{v^4}{\Lambda^4}\left(|C_{D,L}|^2 + |C_{D,R}|^2\right)$$

$$\text{BR}(\mu \to eee) \simeq \frac{v^4}{\Lambda^4}\left[\frac{1}{8}|C_{S,LL}|^2 + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)\,e|C_{D,L}|^2 + |C_{V,RL} + 4eC_{D,L}|^2\right] + (L \leftrightarrow R)$$

$\text{CR}(\mu - e, \text{N})$: *far more involved* (nuclear target effects, spin (in)-dependent contributions, ...)

Simple *"one-at-a-time"* limits: cLFV rates in terms of *one* $C_{D,S,\ldots}$:

	$\text{Br}(\mu^+ \to e^+\gamma)$		$\text{Br}(\mu^+ \to e^+e^-e^+)$		$\text{Br}^{\text{Au/Al}}_{\mu\to e}$	
	$4.2\cdot10^{-13}$	$4.0\cdot10^{-14}$	$1.0\cdot10^{-12}$	$5.0\cdot10^{-15}$	$7.0\cdot10^{-13}$	$1.0\cdot10^{-16}$
C_L^D	$1.0\cdot10^{-8}$	$3.1\cdot10^{-9}$	$2.0\cdot10^{-7}$	$1.4\cdot10^{-8}$	$2.0\cdot10^{-7}$	$2.9\cdot10^{-9}$
$C_{ee}^{S\,LL}$	$4.8\cdot10^{-5}$	$1.5\cdot10^{-5}$	$8.1\cdot10^{-7}$	$5.8\cdot10^{-8}$	$1.4\cdot10^{-3}$	$2.1\cdot10^{-5}$
$C_{\mu\mu}^{S\,LL}$	$2.3\cdot10^{-7}$	$7.2\cdot10^{-8}$	$4.6\cdot10^{-6}$	$3.3\cdot10^{-7}$	$7.1\cdot10^{-6}$	$1.0\cdot10^{-7}$
$C_{\tau\tau}^{S\,LL}$	$1.2\cdot10^{-6}$	$3.7\cdot10^{-7}$	$2.4\cdot10^{-5}$	$1.7\cdot10^{-6}$	$2.4\cdot10^{-5}$	$3.5\cdot10^{-7}$
$C_{\tau\tau}^{T\,LL}$	$2.9\cdot10^{-9}$	$9.0\cdot10^{-10}$	$5.7\cdot10^{-8}$	$4.1\cdot10^{-9}$	$5.9\cdot10^{-8}$	$8.5\cdot10^{-10}$
$C_{bb}^{S\,LL}$	$2.8\cdot10^{-6}$	$8.6\cdot10^{-7}$	$5.4\cdot10^{-5}$	$3.8\cdot10^{-6}$	$9.0\cdot10^{-7}$	$1.2\cdot10^{-8}$
$C_{bb}^{T\,LL}$	$2.1\cdot10^{-9}$	$6.4\cdot10^{-10}$	$4.1\cdot10^{-8}$	$2.9\cdot10^{-9}$	$4.2\cdot10^{-8}$	$6.0\cdot10^{-10}$
$C_{ee}^{V\,RR}$	$3.$[illegible]$\cdot10^{-5}$	$9.4\cdot10^{-6}$	$2.1\cdot10^{-7}$	$1.5\cdot10^{-8}$	$2.1\cdot10^{-6}$	$3.5\cdot10^{-8}$
$C_{\mu\mu}^{V\,RR}$	$3.0\cdot10^{-5}$	$9.4\cdot10^{-6}$	$1.6\cdot10^{-5}$	$1.1\cdot10^{-6}$	$2.1\cdot10^{-6}$	$3.5\cdot10^{-8}$
$C_{\tau\tau}^{V\,RR}$	$1.0\cdot10^{-4}$	$3.2\cdot10^{-5}$	$5.3\cdot10^{-5}$	$3.8\cdot10^{-6}$	$4.8\cdot10^{-6}$	$7.9\cdot10^{-8}$
$C_{bb}^{V\,RR}$	$3.5\cdot10^{-4}$	$1.1\cdot10^{-4}$	$6.7\cdot10^{-5}$	$4.8\cdot10^{-6}$	$6.0\cdot10^{-6}$	$1.0\cdot10^{-7}$
C_{bb}^{RA}	$4.2\cdot10^{-4}$	$1.3\cdot10^{-4}$	$6.5\cdot10^{-3}$	$4.6\cdot10^{-4}$	$1.3\cdot10^{-3}$	$2.2\cdot10^{-5}$
C_{bb}^{RV}	$2.1\cdot10^{-3}$	$6.4\cdot10^{-4}$	$6.7\cdot10^{-5}$	$4.7\cdot10^{-6}$	$6.0\cdot10^{-6}$	$1.0\cdot10^{-7}$

[Crivellin et al, 2017 (courtesy of M. Pruna)]

$\Rightarrow$ $\text{BR}(\mu \to e\gamma)$ depends on dipole C_D

(but **also sensitive** to scalar/tensor/vector contributions - RGE mixing, loops, ...)

Unexpected findings!

- Include as many observables & operators **as possible!** (e.g. $\mu e\gamma\gamma$ contact interactions, [Davidson et al, 2007.09612]
angular observables in polarised $\mu \to 3e$ decays [Bolton, Petcov, 2204.03468], ...)

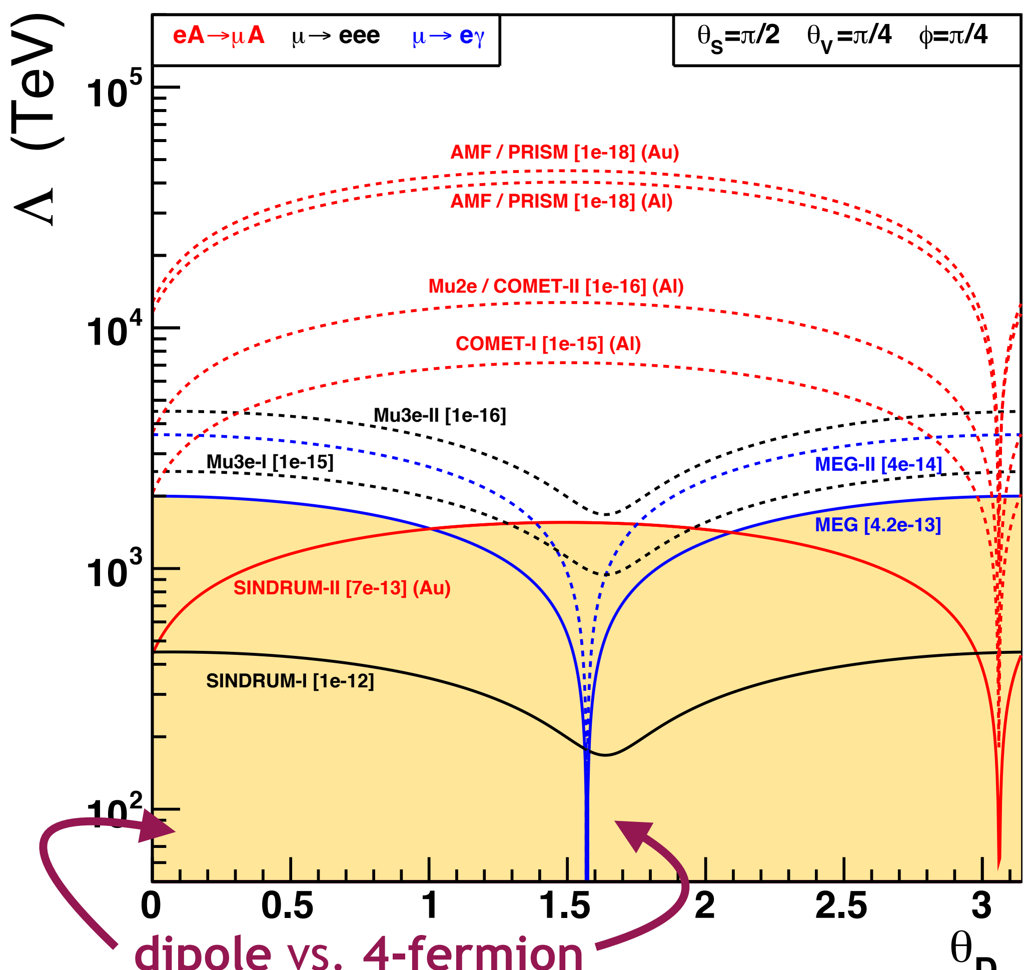
Muon cLFV: EFT approach to New Physics

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \dots$$

$\Rightarrow$ **cLFV data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to a given operator** $\mathscr{O}^6$)

- And results of a recent approach:

$$\mathscr{L}^{\text{eff}}_{\text{NP, cLFV}} = \frac{1}{\Lambda^2}\Big[C_D(\bar{e}\sigma^{\nu\rho}P_R\mu)F_{\nu\rho} + C_S(\bar{e}P_R\mu)(\bar{e}P_Re) + C_{VR}(\bar{e}\gamma^\nu P_L\mu)(\bar{e}\gamma_\nu P_Re) + C_{VL}(\bar{e}\gamma^\nu P_L\mu)(\bar{e}\gamma_\nu P_Le) + \\ + C_{\text{N-light}}\mathscr{O}_{\text{N-light}} + C_{\text{N-heavy}\perp}\mathscr{O}_{\text{N-heavy}\perp}\Big]$$



dipole vs. 4-fermion dominance

[Davidson & Echenard, 2204.00564]

$$\vec{C} = \{C_D, C_S, C_{VR}, C_{VR}, C_{VL}, C_{\text{N-light}}, C_{\text{N-heavy}\perp}\}$$

$$\Rightarrow \text{BR}(\mu \to e\gamma) \simeq 384\pi^2 \frac{v^4}{\Lambda^4} |\vec{C}\cdot \hat{e}_{DR}|^2 \rightsquigarrow \leq \text{BR}^{\text{exp (future)}}$$

and likewise for other observables...

Sensitivity to NP scales (current & future):

MEG $(\mu \to e\gamma)$ $\leftrightsquigarrow$ $\Lambda_{\text{cLFV}} \sim \mathscr{O}(10^3)$ **TeV** (dipole)

SINDRUM II $(\mu - e$, Au) $\leftrightsquigarrow$ $\Lambda_{\text{cLFV}} \sim \mathscr{O}(10^3)$ **TeV** (4f)

Mu2e/COMET II $(\mu - e$, Al) $\leftrightsquigarrow$ $\Lambda_{\text{cLFV}} \lesssim \mathscr{O}(10^4)$ **TeV** *(both dipole and 4f)*

Muon cLFV: EFT approach & conversion in nuclei

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \ldots$$

⇒ cLFV **data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to a given operator** $\mathscr{O}^6$)

- Fully exploring the potential of atomic (elastic) **muon-electron conversion,** CR($\mu - e$,N):

  Comparatively more involved theoretical approach!

  Important **nuclear effects** ("new" non-relativistic ET treatment: inclusion of intrinsic nucleon and muon velocities, lepton wave functions, full nuclear response - factored from cLFV physics ...)

  [see Cirigliano et al, 2203.09547; Hoferichter et al, 2204.06005 & Haxton et al, 2208.07945; among others ...]

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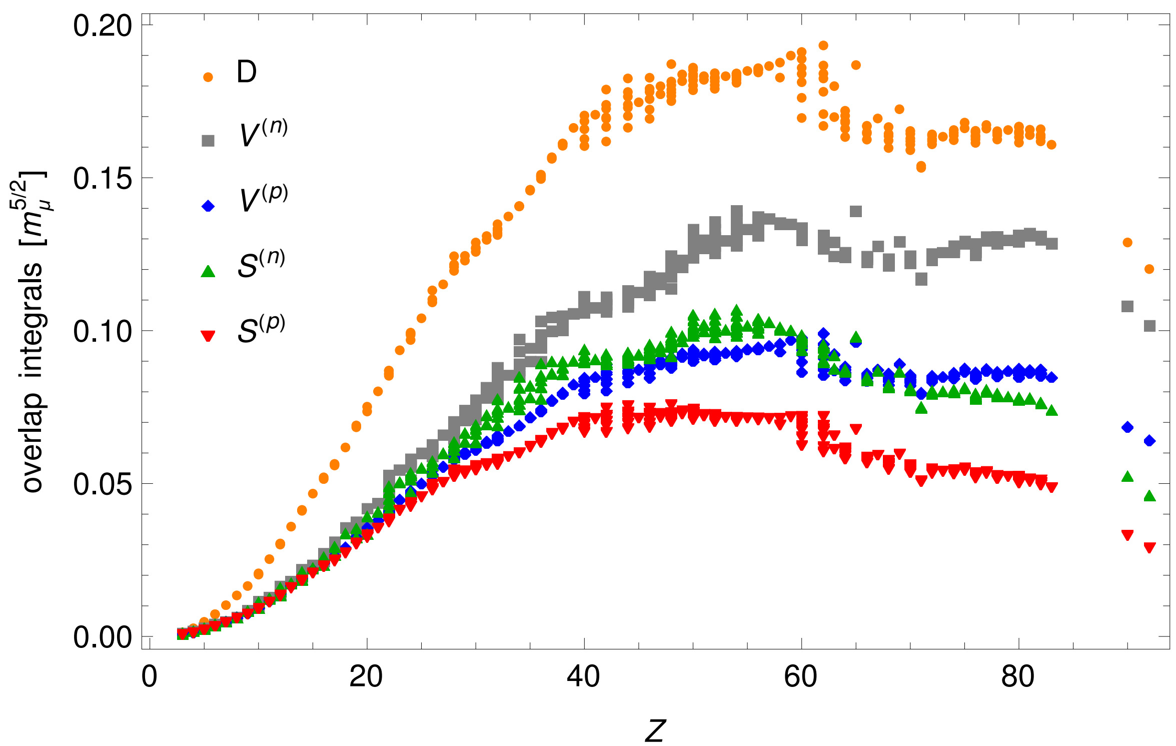
  Explore target-nucleus dependence to distinguish dominant operator (hint on **NP** model!)

  [extensive contributions since Kitano et al, 0203110! see Davidson et al, 1810.01884; Heeck et al, 2203.00702, ...]

  Choice of future targets offering the largest complementarity with respect to Aluminium (Mu2e, COMET)

$$\text{BR}_{\text{SI}}(\mu A \to e A) = \frac{32 G_F^2}{\Gamma_{\text{capture}}} \left[\left| C^{pp}_{V,R} V^{(p)} + C^{pp'}_{S,L} S^{(p)} + C^{nn}_{V,R} V^{(n)} + C^{nn'}_{S,L} S^{(n)} + C_{D,L} \frac{D}{4} \right|^2 + \{L \leftrightarrow R\} \right].$$

Overlap integrals:
more distinguishable at large Z !



[Heeck et al, 2203.00702]

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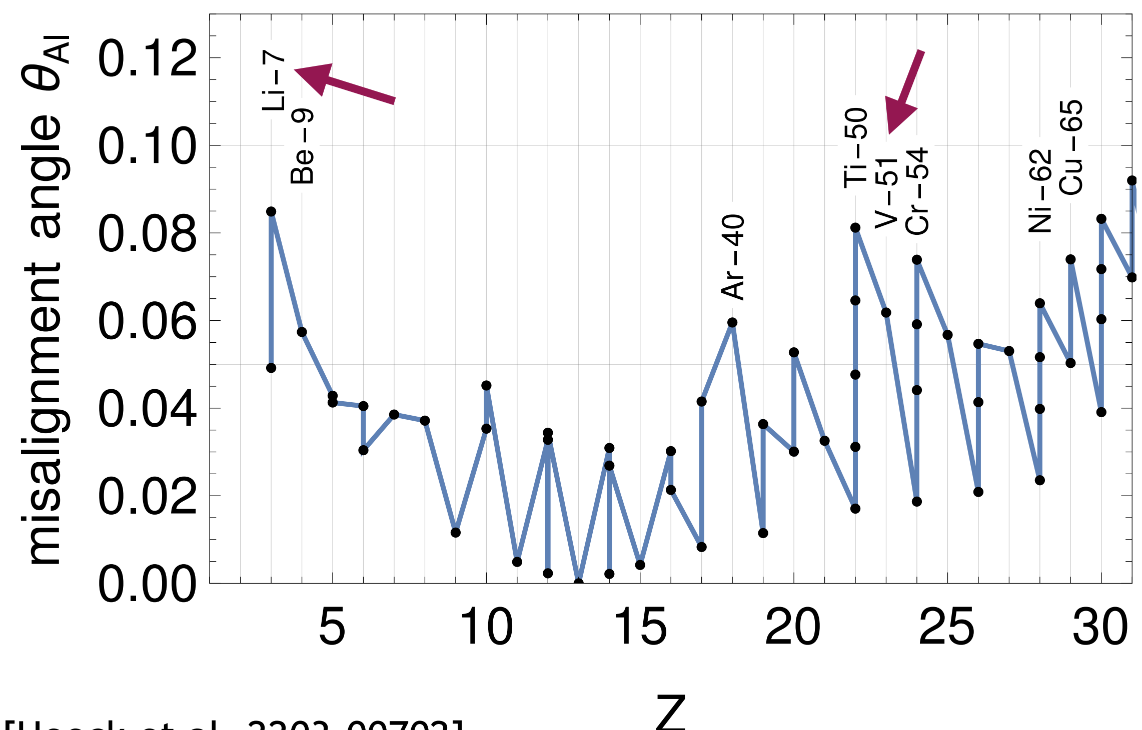


[Heeck et al, 2203.00702]

- **Heavier nuclei** (Au, Pb)! ... not feasible... *(AMF?)*
- Among experimental-friendly $Z \leq 25$ targets several (theoretically good) candidates Li-7, Ti-50, Ti-49, Cr-54, .., V-51

$\Rightarrow$ **Li-7 and/or V-51 :** preferable **"second" targets post CR($\mu - e$,Al) observation**

Muon cLFV: EFT approach & conversion in nuclei

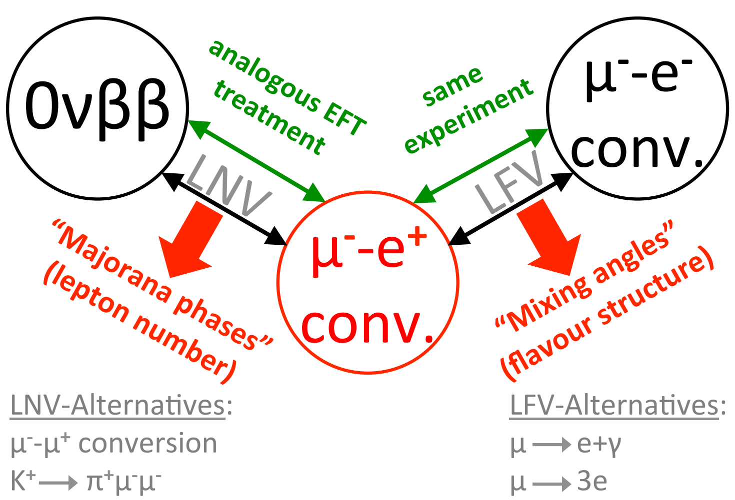
$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \ldots$$

$\Rightarrow$ **cLFV data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to a given operator** $\mathscr{O}^6$)

- Fully exploring the potential of atomic (elastic) **muon-electron conversion,** CR($\mu - e$,N):

  And of its lepton number violating counterpart, $\mu^- + (A,Z) \to e^+ + (A,Z-2)^{(*)}$

  A **unique connection** between **LNV** (in association with Majorana nature and possibly, neutrino mass generation) and **cLFV**



[see e.g. Geib et al, 1609.09088]

From a theoretical point of view, not straightforward!

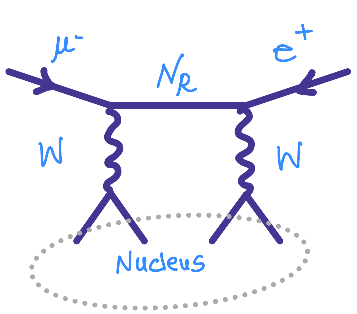
- Higher-dimension operators in $\mathscr{L}^{\text{eff}}$ (dim 6, 10, 14…)
- Nuclear matrix elements extremely hard to compute!

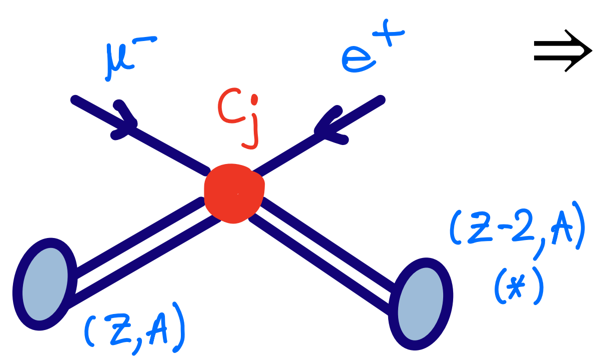
$$\Gamma^{\text{LNV}}_{\mu e} \approx \frac{G_F^4\, g_A^4}{32\pi^2} |\epsilon^2_{C_j}| \frac{m_e^2 m_\mu^2}{R^2} |F(Z-2,E_e)| < \phi_\mu >^2 |\mathscr{M}^{(\mu^-,e^+)}|^2$$

(only two $\mathscr{M}^{(\mu^-,e^+)}$ known, for Ti-48…)

[Domin et al, 0409033; Simkovic et al, 0103029]

$\Rightarrow$ Very hard to draw implications… **Must tackle NME!**





Best sensitivity: Ca, S and Ti (!?)

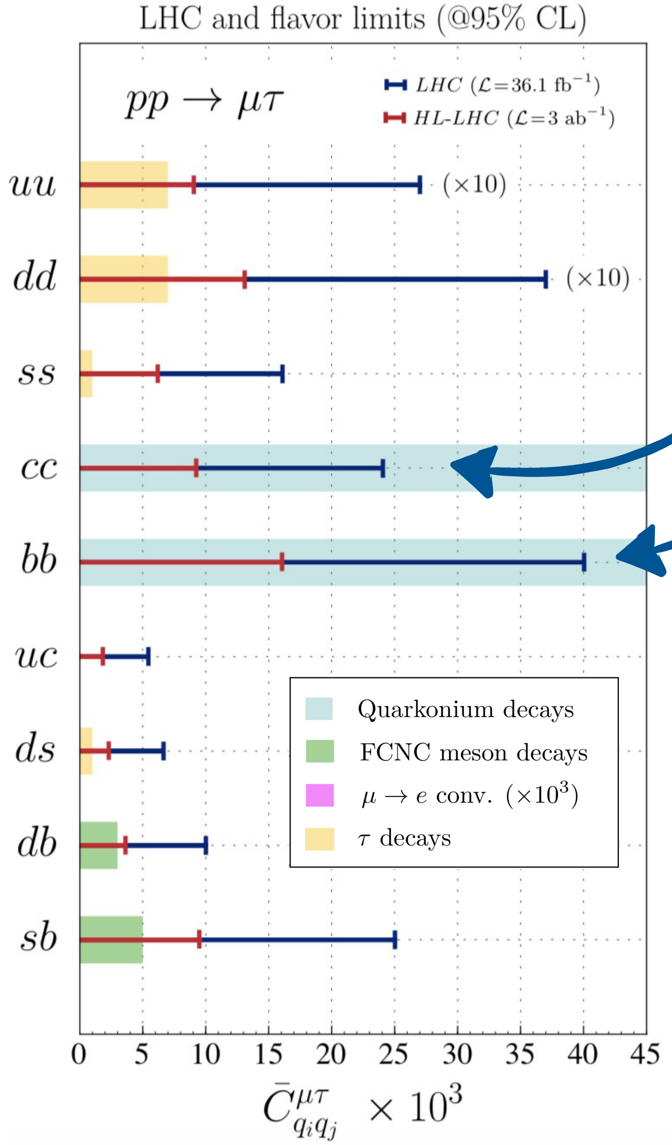
CR($\mu^- - e^+$, Ti) $< \mathscr{O}(10^{-15})$ [Yeo et al, 1705.07464]

Muon flavoured probes @ high scales

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \ldots$$

⇒ **cLFV data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to a given operator** $\mathscr{O}^6$)

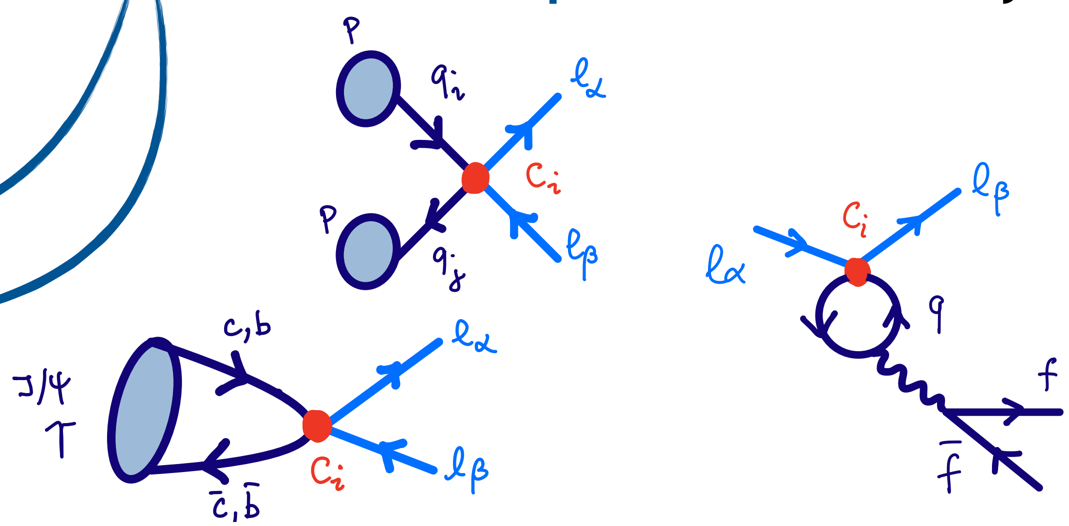
- **Semileptonic decays vs. Drell-Yan at the LHC:** lepton flavour'ed (violating) tails

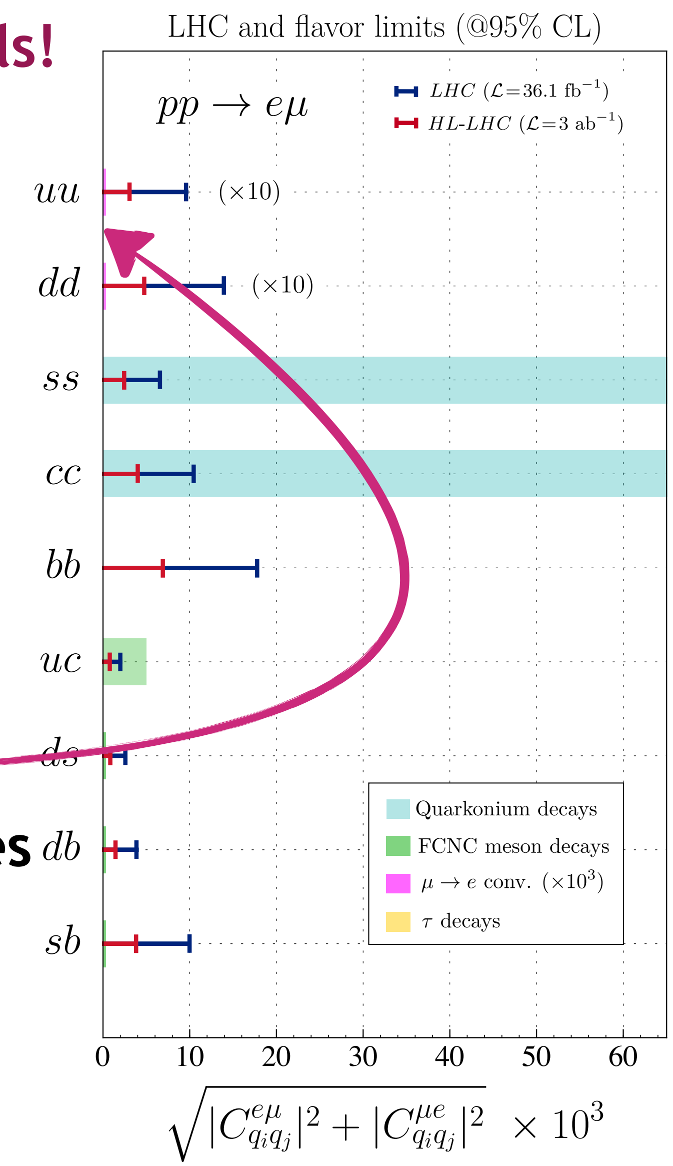


⇒ **High p_T di-lepton tails:** flavour probes!

LHC limits very competitive for **$\mu\tau$ tails!**

Better than quarkonium-decays





⇒ **Impossible** for **μe tails** **to out-perform cLFV searches**

($\mu - e$, N) conversion bounds

[Angelescu et al, 2002.05684]

[Further constraints from quarkonia decays, see e.g. Calibbi et al, 2207.10913]

Muon flavoured probes @ high scales

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \ldots$$

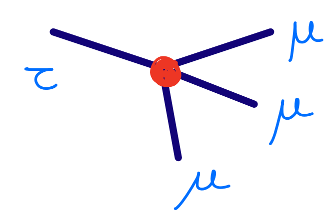
⇒ cLFV **data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to a given operator** $\mathscr{O}^6$)

- **Flavour physics (leptons!) at a Muon Collider**: what could we expect?

See talk by N. Craig

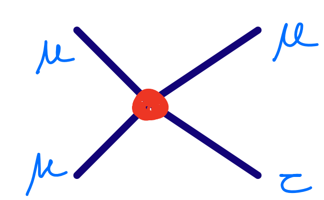
Extraordinary potential to probe NP at the source of **cLFV leptonic decays**

⇒ **Case of $\mu^+\mu^+$ collider:**



Recall **BR($\tau \to 3\mu$)** $= \frac{\text{BR}(\tau \to e\nu\bar{\nu})}{4G_F^2}[|C_{LL}|^2 + |C_{RR}|^2 + 1/2(|C_{LR}|^2 + |C_{RL}|^2)]$

$$\boldsymbol{\sigma(\mu^+\mu^+ \to \mu^+\tau^+)} = \frac{s}{4\pi}[|C_{LL}|^2 + |C_{RR}|^2 + 1/6(|C_{LR}|^2 + |C_{RL}|^2)]$$



Process	Current BR limit	$N(\mu^+\mu^+ \to \mu^+\tau^+)$
$\mu \to 3e$	$< 1.0 \times 10^{-12}$	$< 6.7 \times 10^{-2}$
$\tau \to 3e$	$< 2.7 \times 10^{-8}$	$< 1.0 \times 10^{4}$
$\tau \to \mu^+\mu^-e^-$	$< 2.7 \times 10^{-8}$	$< 1.0 \times 10^{4}$
$\tau \to e^+\mu^-\mu^-$	$< 1.7 \times 10^{-8}$	$< 6.4 \times 10^{3}$
$\tau \to e^+e^-\mu^-$	$< 1.8 \times 10^{-8}$	$< 6.8 \times 10^{3}$
$\tau \to \mu^+e^-e^-$	$< 1.5 \times 10^{-8}$	$< 5.7 \times 10^{3}$
$\tau \to 3\mu$	$< 2.1 \times 10^{-8}$	$< 7.9 \times 10^{3}$

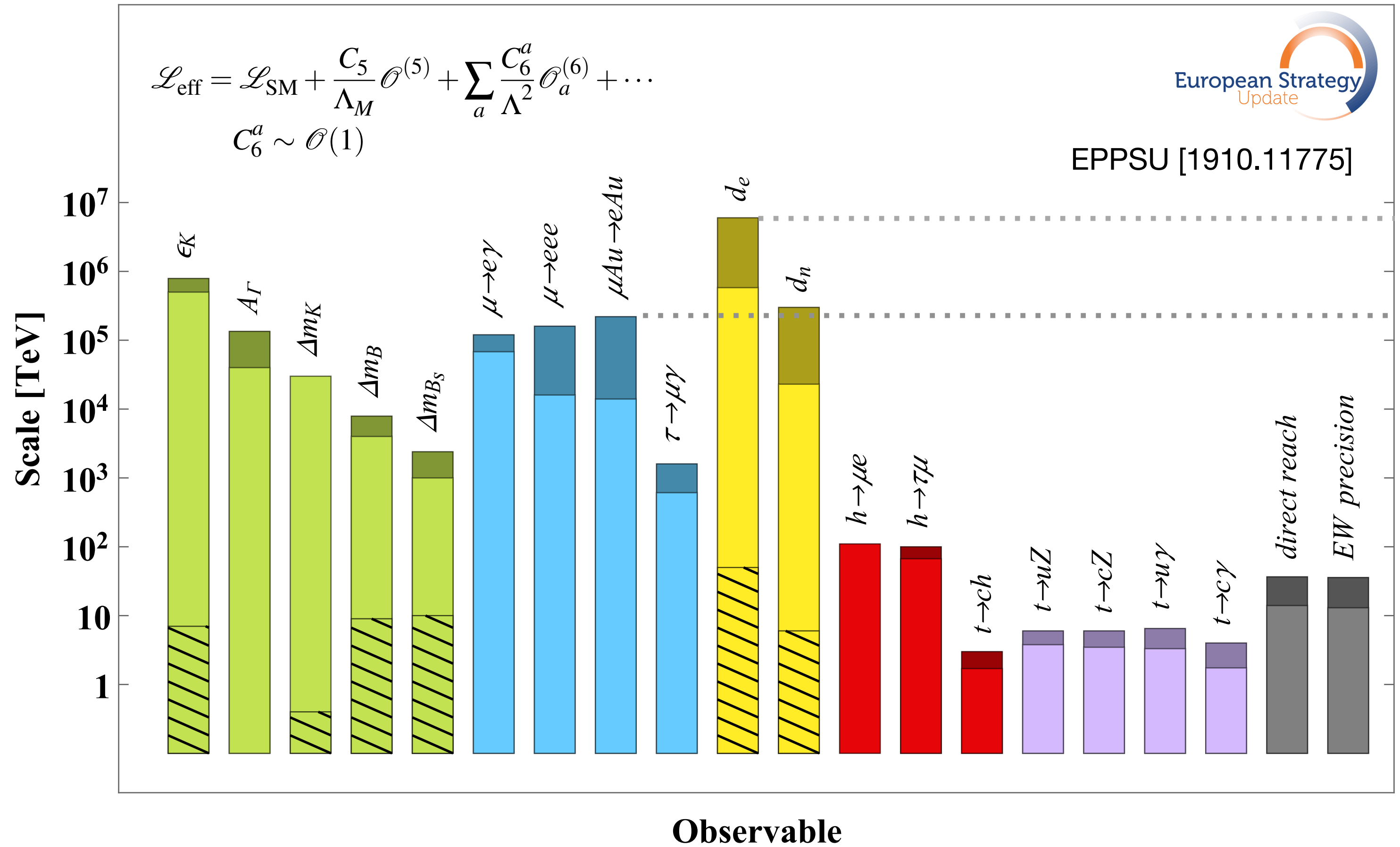
Number of events for a **muon collider** at $\sqrt{s}$ = 2 TeV and $\mathscr{L} = 1\ \text{ab}^{-1}$ (allowed from cLFV search limits)

cLFV transitions "observable", even under the **strong cLFV current bounds!**

And also for future sensitivities

[Fridell et al, 2304.14020]

The probing power of ~~flavour~~ muons



μ-Flavour observables:
probes sensitive to
NP scales

$\Lambda_{\text{NP}} \sim \mathcal{O}(10^5 \text{ TeV})$

beyond collider's reach!

Learning about New Physics from muon cLFV: SM extensions

Extremely active research field! A few examples of the constraining & probing power of muons amidst a vast array of well motivated constructions...

Muon cLFV: hinting towards the NP model

- Models of **New Physics** can change SM's predictions, introducing:
  - (i) **new sources** of **flavour violation** (corrections to SM vertices, new SM-NP interactions)
  - (ii) **new Lorentz structure** in the "four-fermion" interaction ⇒ new **effective operators**

  ⇒ **new cLFV couplings to SM and/or to new fields** in the Feynman diagrams!

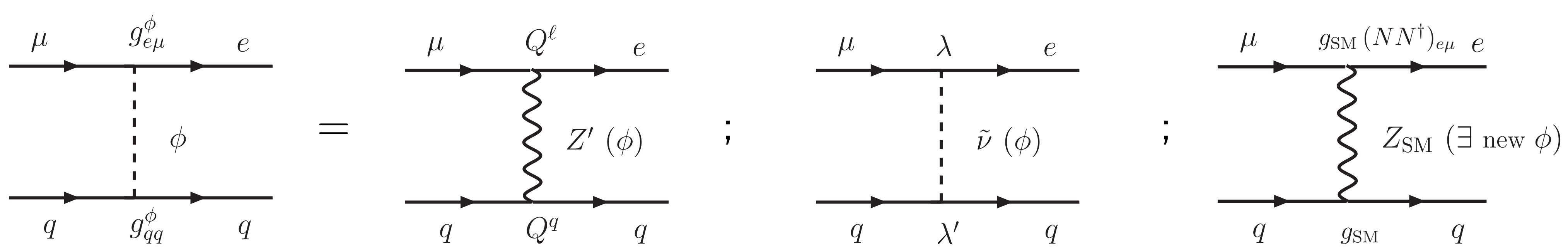
- **(A) cLFV couplings** ⇜⇝ extended **gauge/scalar** sectors

$$\mathcal{L}_\phi = g^\phi_{\ell\ell'}\,\bar{\ell}'\ell\,\phi + g^\phi_{qq'}\,\bar{q}'q\,\phi + \tilde{g}^\phi_{ff'}\,\bar{f}'f\,V_{\rm SM}$$

$$\rightsquigarrow\; g^\phi_{e\mu}\,\bar{\mu}\,e\,\phi \;+\; g^\phi_{qq}\,\bar{q}'\,q\,\phi \;+\; \tilde{g}^\phi_{e\mu}\,\bar{\mu}\,e\,Z_{\rm SM}$$

- **(A):** an example - **New contributions** to **$\mu - e$ conversion**

Tree level:



Additional U(1) | Rp violating SUSY | Extended Higgs

Higher order: many additional contributions! Boxes, penguins, ...

Muon cLFV: hinting towards the NP model

- Models of **New Physics** can change SM's predictions, introducing:
  (i) **new sources** of **flavour violation** (corrections to SM vertices, new SM-NP interactions)
  (ii) **new Lorentz structure** in the "four-fermion" interaction ⇒ new **effective operators**

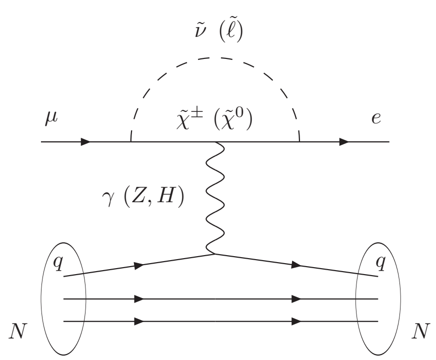
  ⇒ **new cLFV couplings to SM and/or to new fields** in the Feynman diagrams!

- **(B) cLFV couplings** ⇝ **new fermions & new scalar** fields   [ψ, ϕ carry lepton flavour & number]

$$\mathscr{L}_{\psi\phi} = h_{\ell\psi\phi}\,\bar{\ell}\,\psi\,\phi + h_{q\psi\phi}\,\bar{q}\,\psi\,\phi + h_{\ell q\phi}\,\bar{\ell}\,q\,\phi + \ldots$$
$$\rightsquigarrow h_{\mu\psi\phi}\,\bar{\mu}\,\psi\,\phi + h_{e\psi\phi}\,\bar{\psi}\,e\,\phi + h_{q\psi\phi}\,\bar{q}\,\psi\,\phi + h_{\mu q\phi}\,\bar{\mu}\,q\,\phi + h_{eq\phi}\,\bar{q}\,e\,\phi + \ldots$$

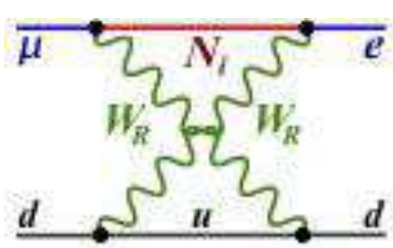
- **(B):** further **new contributions** to **$\mu - e$ conversion**

Typically loop:



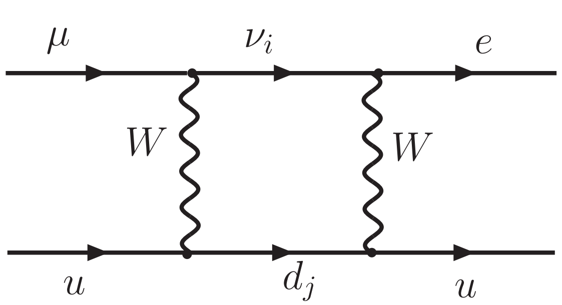
SUSY seesaw

;



Left-Right models

;



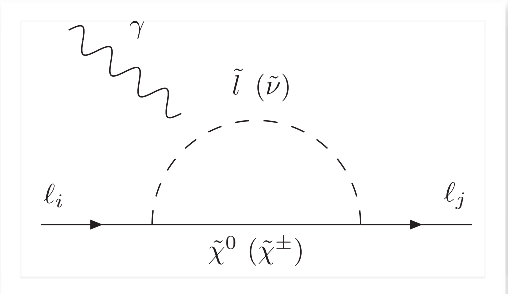
SM + sterile νs

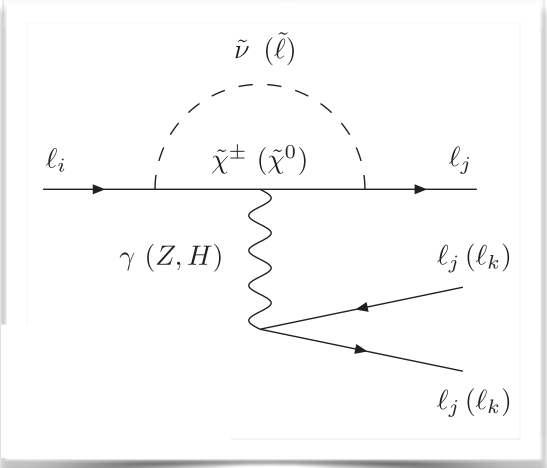
…

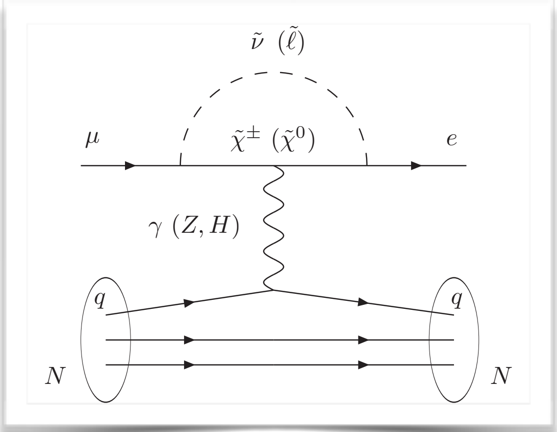
Hinting towards the NP model: peculiar patterns

- What can we then learn about **New Physics** from cLFV observables?

  ⇒ Let us consider a *"user-friendly" canvas*: the **type I SUSY seesaw**







Supersymmetry (SUSY):
fermions + scalar fermions $[(\ell, \nu) \leftrightsquigarrow (\tilde{\ell}, \tilde{\nu})]$
gauge/Higgs + fermionic "inos" $[(Z, W, H) \leftrightsquigarrow (\tilde{\chi}^0, \tilde{\chi}^\pm)]$
cLFV ⇒ contributions @ loop level

While $\mu \to e\gamma$ is a "dipole-dominated" transition, $\mu \to 3e$ and $\mu - e$ conversion receive contributions from anapole (long-distance), γ, Z & Higgs penguins, boxes, ...

***If* dominant contribution to $\mu - e$ conversion from:**

(i) Photon penguin ⇒ correlation between BR($\mu \to e\gamma$), BR($\mu \to 3e$) and CR($\mu - e$, N)

$$\text{CR}(\mu - e, \text{N}) \approx \mathbf{5 \times 10^{-3}}\, \text{BR}(\mu \to e\gamma)\ ;\ \text{BR}(\mu \to 3e) \approx \mathbf{6 \times 10^{-3}}\, \text{BR}(\mu \to e\gamma)$$

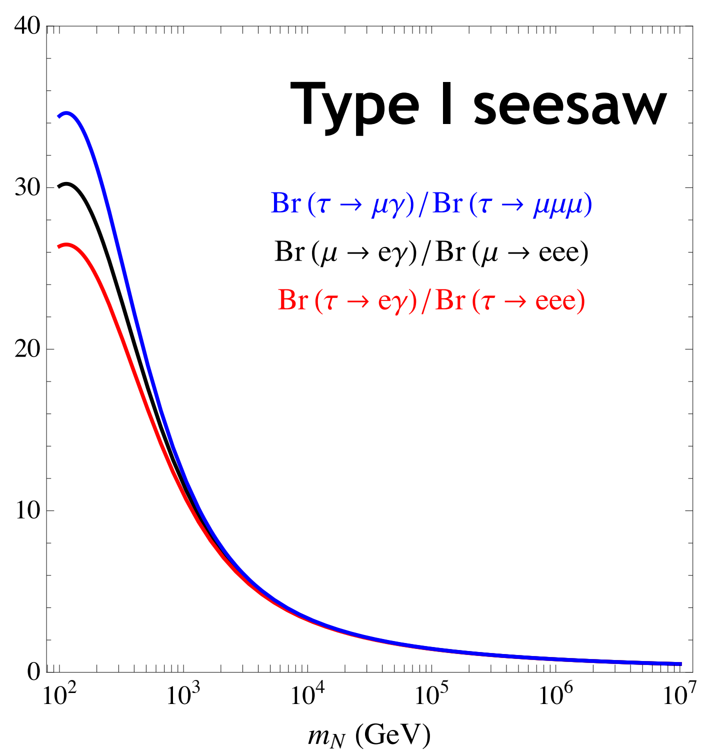
(ii) Z penguin ⇒ correlation between BR($\mu \to 3e$) and CR($\mu - e$, N) [and BR($Z \to e\mu$)]

(iii) Higgs penguin ⇒ unusual patterns! E.g. CR($\mu - e$, N) $\approx \mathbf{0.08 - 0.15}$ BR($\mu \to e\gamma$)

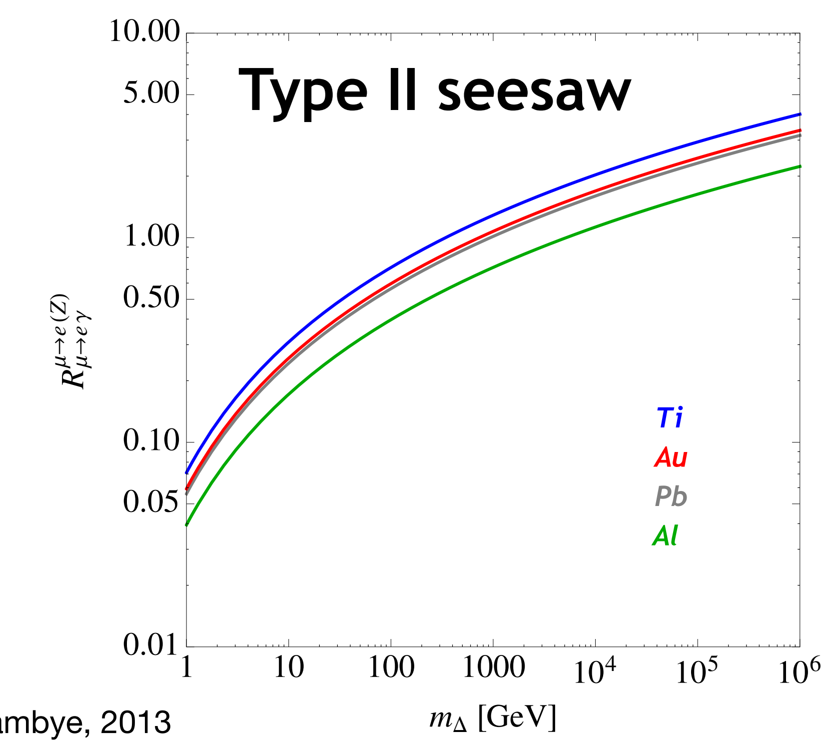
Peculiar patterns: disentangling seesaws

- What can we then learn about **New Physics** from cLFV observables?

Seesaw realisations: distinctive signatures for numerous **cLFV observables**
ratios of **observables** to **identify seesaw mediators** & constrain their **masses!**



Hambye, 2013

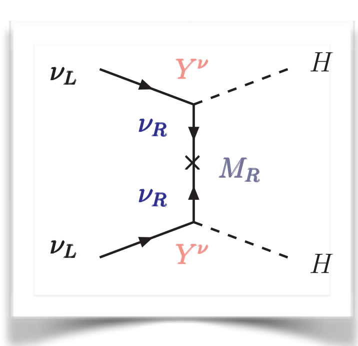


Type III seesaw

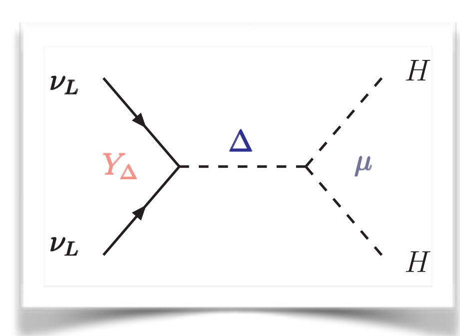
$$\frac{\text{BR}(\mu\to e\gamma)}{\text{BR}(\mu\to 3e)} = 1.3\times 10^{-3}$$

$$\frac{\text{BR}(\tau\to \mu\gamma)}{\text{BR}(\tau\to 3\mu)} = 1.3\times 10^{-3}$$

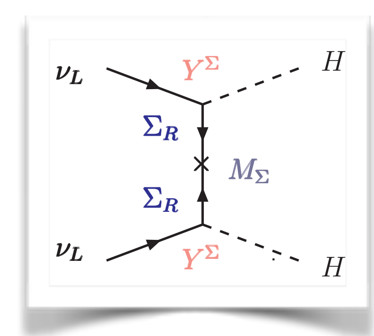
$$\frac{\text{BR}(\mu\to e\gamma)}{\text{CR}(e-\mu,\text{Ti})} = 3.1\times 10^{-4}$$



Type I (fermion singlet)



Type II (scalar triplet)



Type III (fermion triplet)

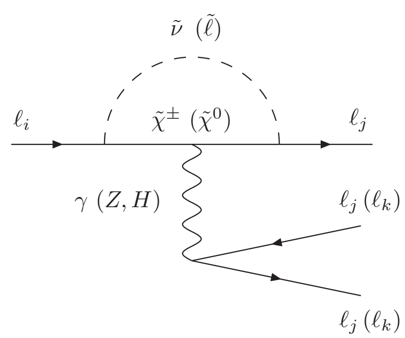
Hinting towards the NP model: peculiar patterns

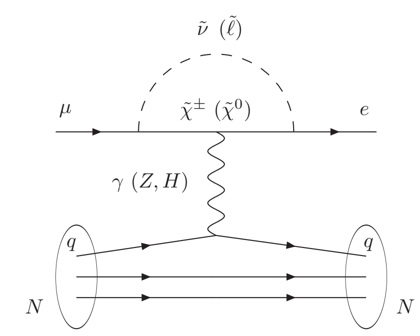
- What can we then learn about **New Physics** from cLFV observables?

  ⇒ **Most models of NP** predict/accommodate **extensive ranges for cLFV observables** (little info on scale and on the nature of couplings)

Model	$\mu \to eee$	$\mu N \to eN$	$\frac{\mathrm{BR}(\mu\to eee)}{\mathrm{BR}(\mu\to e\gamma)}$	$\frac{\mathrm{CR}(\mu N\to eN)}{\mathrm{BR}(\mu\to e\gamma)}$
MSSM	Loop	Loop	$\approx 6\times 10^{-3}$	$10^{-3} - 10^{-2}$
Type-I seesaw	Loop*	Loop*	$3\times 10^{-3} - 0.3$	$0.1 - 10$
Type-II seesaw	Tree	Loop	$(0.1-3)\times 10^{3}$	$\mathcal{O}(10^{-2})$
Type-III seesaw	Tree	Tree	$\approx 10^{3}$	$\mathcal{O}(10^{3})$
LFV Higgs	Loop†	Loop$^{*\,\dagger}$	$\approx 10^{-2}$	$\mathcal{O}(0.1)$
Composite Higgs	Loop*	Loop*	$0.05 - 0.5$	$2 - 20$

[adapted from Calibbi et al, 1709.00294]





Upon **experimental determination** of rates for **cLFV transitions:**

comparison of $\left.\frac{\mathrm{BR}(\mu\to 3e)}{\mathrm{BR}(\mu\to e\gamma)}\right|_{\mathrm{exp}}$ with $\left.\frac{\mathrm{BR}(\mu\to 3e)}{\mathrm{BR}(\mu\to e\gamma)}\right|_{\text{NP-th}}$

and of $\left.\frac{\mathrm{CR}(\mu-e,\mathrm{N})}{\mathrm{BR}(\mu\to e\gamma)}\right|_{\mathrm{exp}}$ with $\left.\frac{\mathrm{CR}(\mu-e,\mathrm{N})}{\mathrm{BR}(\mu\to e\gamma)}\right|_{\text{NP-th}}$

⇒ Probe NP model at the source of cLFV

Hinting towards the NP model: peculiar patterns

- What can we then learn about **New Physics** from cLFV observables?

  $\Rightarrow$ **Most models of NP** predict/accommodate **extensive ranges for cLFV observables** (little info on scale and on the nature of couplings)

Model	$\mu \to eee$	$\mu N \to eN$	$\frac{\text{BR}(\mu\to eee)}{\text{BR}(\mu\to e\gamma)}$	$\frac{\text{CR}(\mu N\to eN)}{\text{BR}(\mu\to e\gamma)}$
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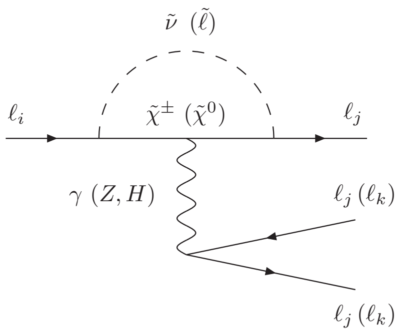
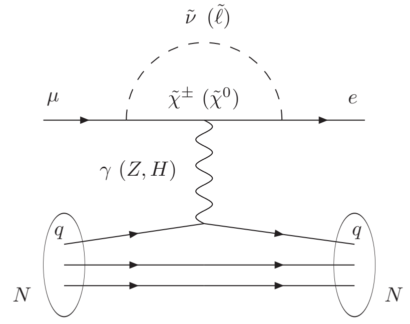
Composite Higgs	Loop*	Loop*	$0.05 - 0.5$	$2 - 20$

[adapted from Calibbi et al, 1709.00294]





In the absence of a direct discovery of the **new states**

$\Rightarrow$ **correlations of observables** might help **disentangling models of cLFV**

$\Rightarrow$ **hint** towards some **models, falsify** certain **realisations**

In all cases **provide complementary information to direct searches!**

(Well-motivated) NP models & μ cLFV

- What can we then learn about **New Physics** from **cLFV observables?**

  Not all models of NP necessarily lead to "experimentally observable" cLFV...

  (e.g. very heavy new states, symmetry-protection, ...)

  One can trivially compute contributions to cLFV observables, and thus exclude (or not) regimes of a given model... Or do much more!

- *A few illustrative examples* of **muon-cLFV** in **well-motivated BSM realisations:**

  from minimal SM extensions to UV-complete constructions,

  aiming at addressing SM observational problems and further "tensions" (anomalies)

  Addressing anomalies: extensions of SM's gauge sector and $(g-2)_\ell$

  cLFV from **LFUV (B-meson anomalies ...)**

  Mechanisms of neutrino mass generation (LFV in neutral sector!)

  GUT inspired, higher-order (**scotogenic**), ...

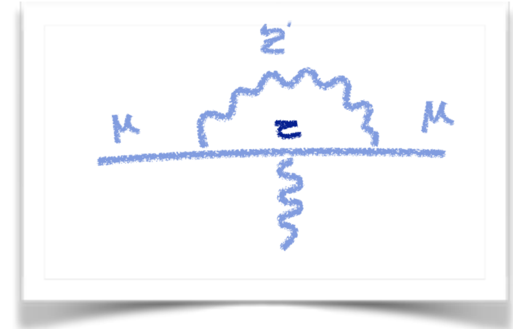
  Back to **seesaw** cLFV "signatures" - and how CP violating phases impacts them

  (Recall $m_\nu \Rightarrow$ LFV; LFV does not require $m_\nu \neq 0$!!)

- Extensive studies of SM extensions capable of addressing "anomalous" behaviours (i.e. tensions with SM expectation): $(g-2)_\ell$, and LFUV in B-meson decays

  Minimal, testable models (first step towards complete constructions)

  cLFV ⇝ LFUV (?)



- **Minimal SM extension** via **light vector Z'** (leptophilic **cLFV** couplings): explain both Δa_μ and Δa_e

$$\mathcal{L}^{\text{int}}_{Z'} = \sum_{\alpha,\beta} Z'_\mu \left[\bar{\ell}_\alpha \gamma^\mu (g_L^{\alpha\beta} P_L + g_R^{\alpha\beta} P_R)\ell_\beta + \bar{\nu}_\alpha \gamma^\mu (g_L^{\alpha\beta} P_L)\nu_\beta\right] + \text{H.c.}$$

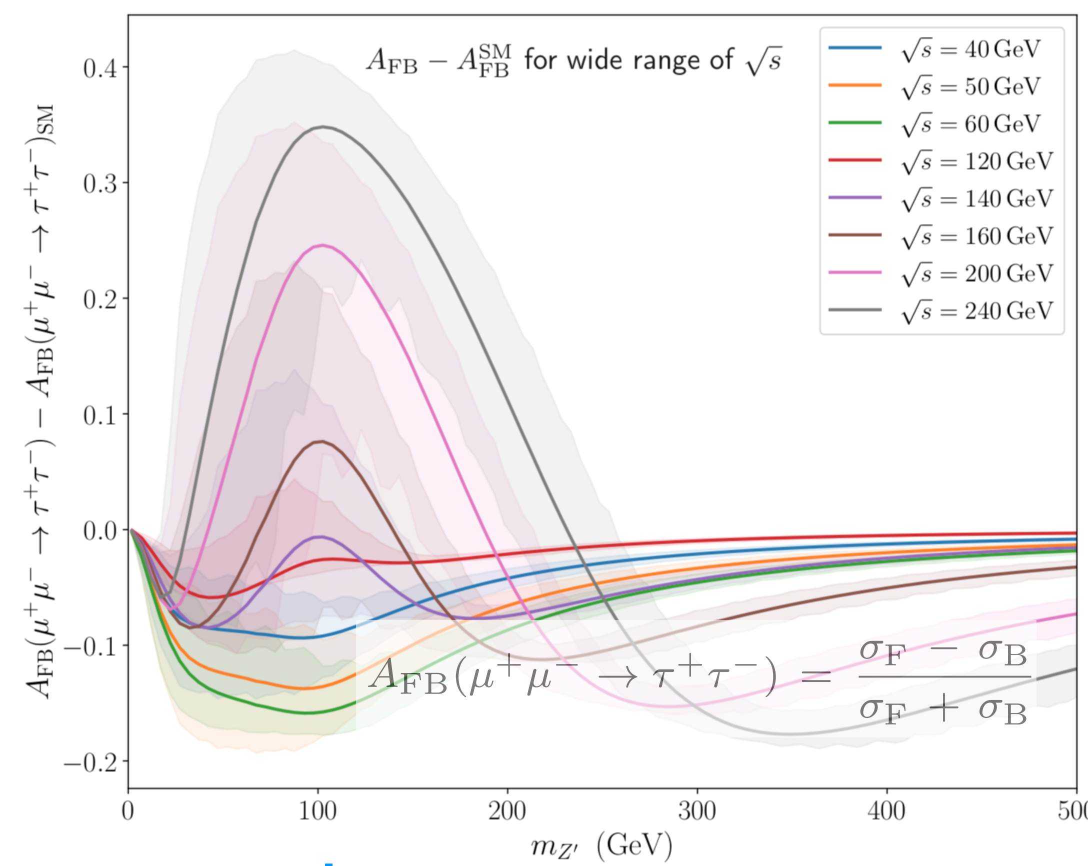
[Kriewald, Orloff, Pinsard, AMT, 2204.13134]

⇒ **Saturate** Δa_μ (Δa_e and Δa_τ strictly SM-like)

Very **stringent constraints** from

LFU tests ($Z \to \ell\ell$) and

cLFV ($\mu \to e\gamma$, $\tau \to \mu\bar{e}\mu$ and Mu – $\overline{\text{Mu}}$!)



Excellent prospects for future **muon collider**

$\mu^+\mu^- \to \tau^+\tau^-$ (SM s-channel$_{\gamma,Z}$, Z' t-channel)

⇒ **Cross section** very sensitive to presence of Z'

⇒ Clear **departure from SM** in **forward-backward asymmetry!**

See talk by N. Craig

Scotogenic models: neutrinos, DM and cLFV

- **Scotogenic models:** a link between **neutrino mass** generation and **dark matter!**

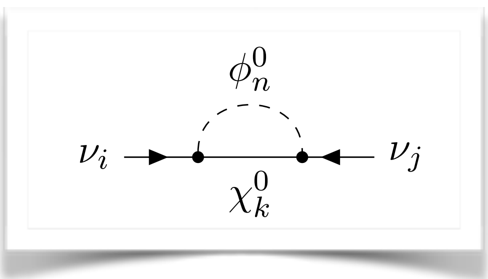
  [Ma, 2006] Additional Z_2 symmetry: stabilises dark matter candidate ... but

  $\Rightarrow$ neutrino masses @ 1-loop

Recent example: SM extended by $SU(2)_L$ **Weyl fermions, Majorana fermion singlets & scalars**

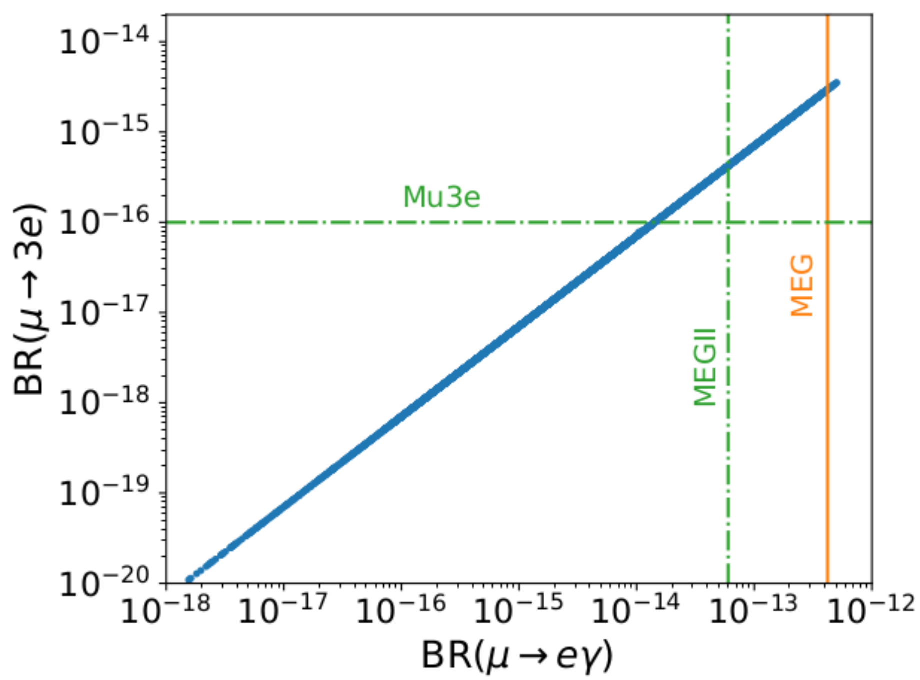
$\Rightarrow$ ν **mass generation, DM candidates,**

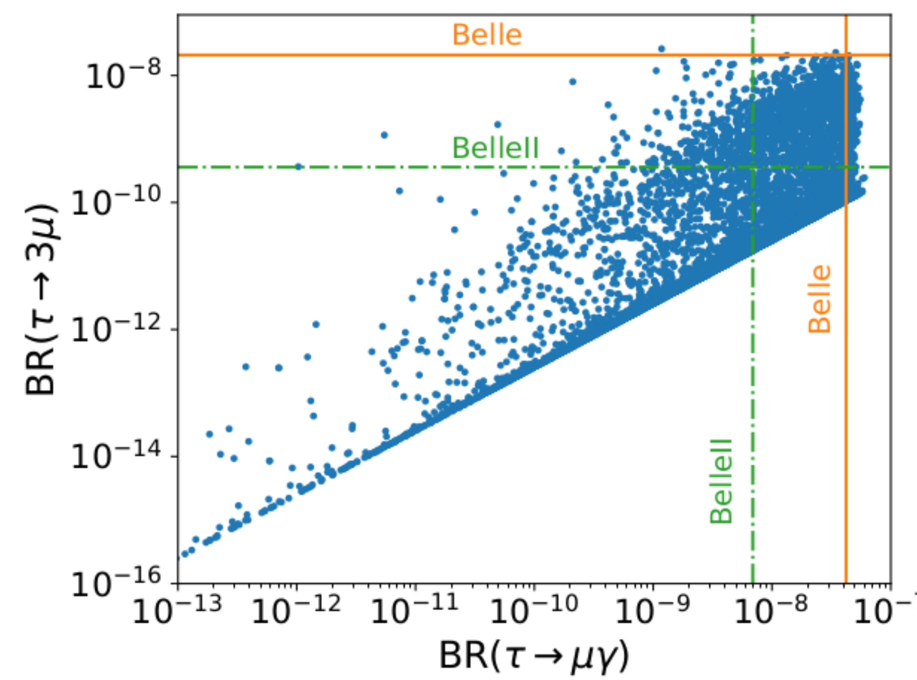
$(g-2)_\mu$ and **BAU** via leptogenesis!



	Ψ_1	Ψ_2	F_1	F_2	η	S
$SU(2)_L$	**2**	**2**	**1**	**1**	**2**	**1**
$U(1)_Y$	-1	1	0	0	1	0

[Alvarez et al, 2301.08485]





- **cLFV observables: strict correlation** between BR($\mu \to e\gamma$), BR($\mu \to 3e$) [dipole dominated]

  less manifest for τ decays [non-negligible box contributions]

  muon cLFV decays $\Rightarrow$ **falsify model** @ MEG II and Mu3e !

GUT models: type II seesaw (and cLFV)

- **Grand unified models:** several possibilities explored, from (SUSY) $SU(5)$ to $SO(10)$, ...

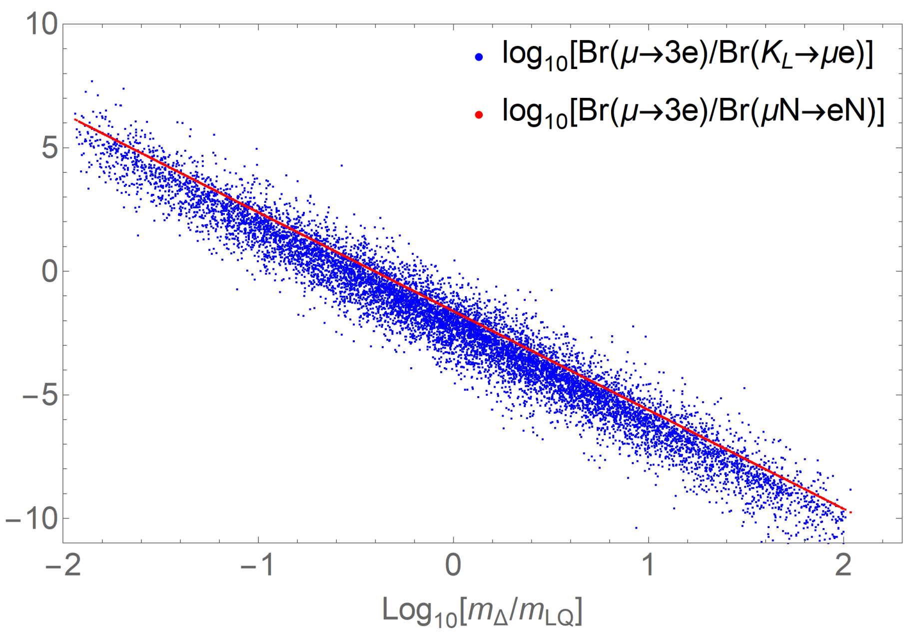
  Many realisations include mechanisms of ν **mass generation,** and open the door to **flavour violation** (at all levels)

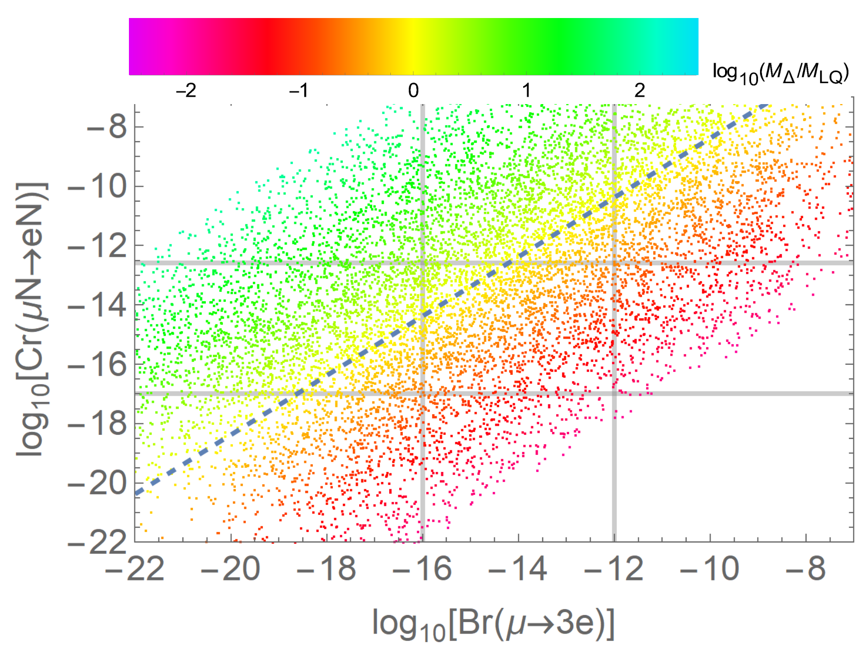
  Realised at **very high scales** (M_{GUT}) - how to probe and test them?

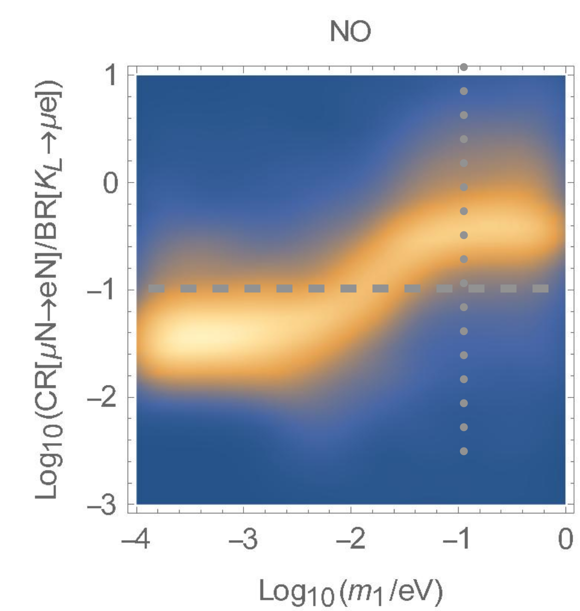
Illustrative example: non-susy **$SU(5)$ GUT & type II seesaw** [scalar triplet Δ, $SU(5)$ (LQ) partners in $\mathbf{15}$]

(variations to avoid "wrong" m_f relations)

[Calibbi and Gao, 2206.10682]







- **cLFV observables:** evident **correlation** between BR($\mu \to 3e$), BR($K_L \to \mu e$) and CR($e - \mu$, N),

  ratio dictated by masses of mediators (triplet Δ and $SU(5)$ leptoquark)

  future observation muon cLFV decays $\Rightarrow$ hint on m_Δ/m_{LQ}

  If BR($K_L \to \mu e$) $\geq 10 \times$ CR($e - \mu$, N) $\Rightarrow$ **disfavour IO**, $m_1^\nu \lesssim 10^{-2}$ eV

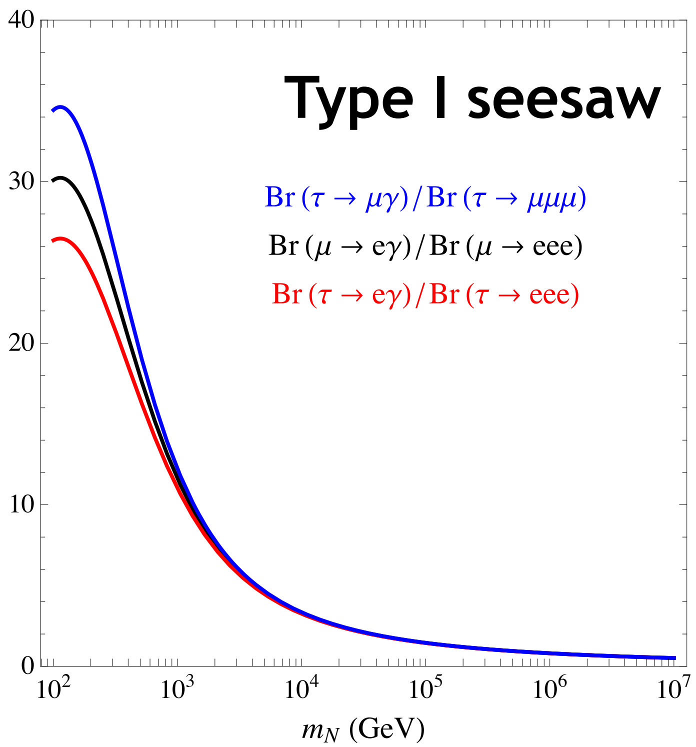
Flavoured probes of neutrino masses:
SM & sterile neutrinos (... and CPV phases!)

How leptonic CP violating phases can impact our expectations

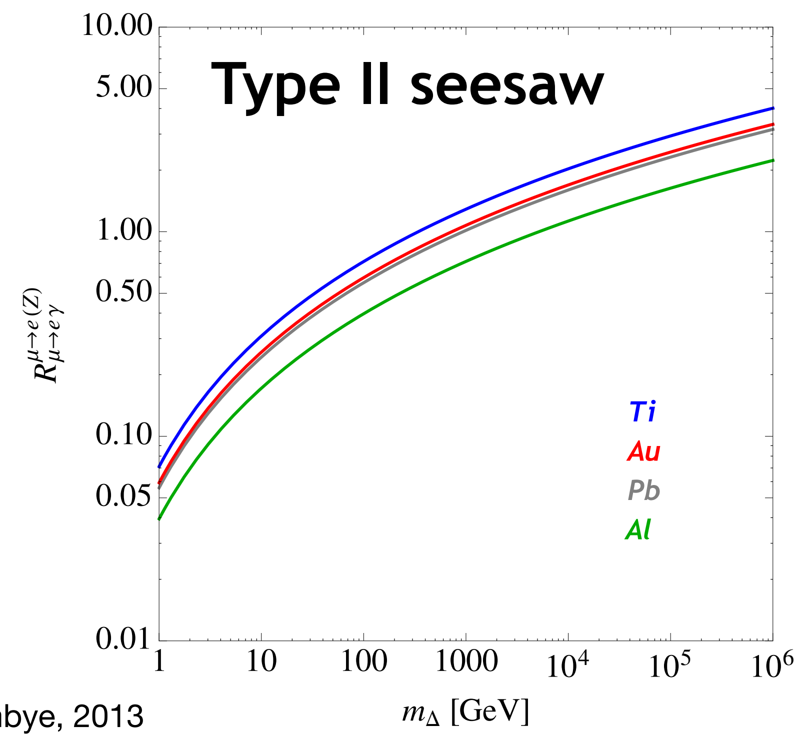
Peculiar patterns: disentangling seesaws

- **Models of NP** (and leptonic LFV) predict/accommodate **extensive ranges for cLFV...**

Seesaw realisations: distinctive signatures for numerous **cLFV observables**
ratios of observables to **identify seesaw mediators** & constrain their **masses!**



Hambye, 2013



Type III seesaw

$$\frac{BR(\mu\to e\gamma)}{BR(\mu\to 3e)} = 1.3\times 10^{-3}$$

$$\frac{BR(\tau\to \mu\gamma)}{BR(\tau\to 3\mu)} = 1.3\times 10^{-3}$$

$$\frac{BR(\mu\to e\gamma)}{CR(e-\mu,Ti)} = 3.1\times 10^{-4}$$

But! In general, such **studies assume CP conserving** couplings...
CP likely violated (PMNS, and other Dirac & Majorana NP phases ⇒ leptogenesis!)
Can **leptonic CP violating phases** impact our **naïve expectations for cLFV patterns?**

Toy models of massive Majorana neutrinos

Simplified "seesaw models" for phenomenology: SM + 2 heavy neutral leptons

- **Ad-hoc** (low-energy) **constructions: SM** extended via n_S **Majorana massive** states **(HNLs)**

  No assumption on **mechanism of mass generation**

  Well-defined interactions in physical basis

  Phenomenological low-energy limit of complete constructions (type I seesaw, ISS, ...)

Hypotheses: **3 active neutrinos** + **2 heavy neutral leptons** $n_L = (\nu_{Le}, \nu_{L\mu}, \nu_{L\tau}, \nu^c_s, \nu^c_{s'})^T$

interaction basis ⇝ **physical** basis $|n_L> = \mathcal{U}_{5\times5}|\nu_i>$

$n_s = 2$

Left-handed lepton mixing $\tilde{U}_{\text{PMNS}}$

3×3 sub-block, **non-unitary!**

Active-sterile mixing $\mathbf{U}_{\alpha\mathbf{i}}$

3×5 rectangular matrix

$$\mathcal{U}_{5\times5} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & U_{e5} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} & U_{\mu5} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} & U_{\tau5} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} & U_{s5} \\ U_{s'1} & U_{s'2} & U_{s'3} & U_{s'4} & U_{s'5} \end{pmatrix}$$

- **Non-unitary lepton mixing matrix: source of cLFV** in SM extensions via **heavy sterile ν**
- **Interference effects (CPV)** between heavier states can be present!

  ⇒ **Constructive & destructive interference effects** in **cLFV** decays (leptonic and boson)

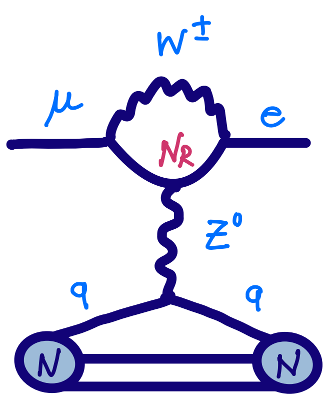
  ⇒ Impact to any **interpretation of experimental data**

Peculiar cLFV patterns… what if CPV & cLFV?

cLFV signatures: ratios of observables to identify mediators & constrain their masses!

But - CP violating phases do matter! *And impact naïve expectations…*

Consider "3+2" toy model (addition of 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times5}$, **CPV** phases)

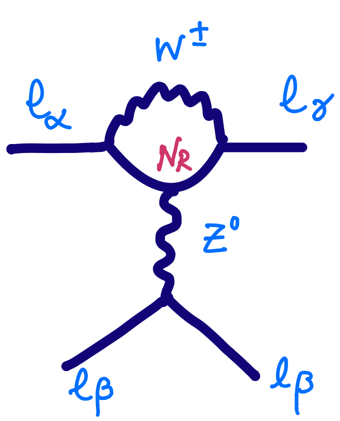


Observables dominated by common topology

$\mu - e$ **conversion in nuclei**

3-body muon decays ($\mu \to 3e$)

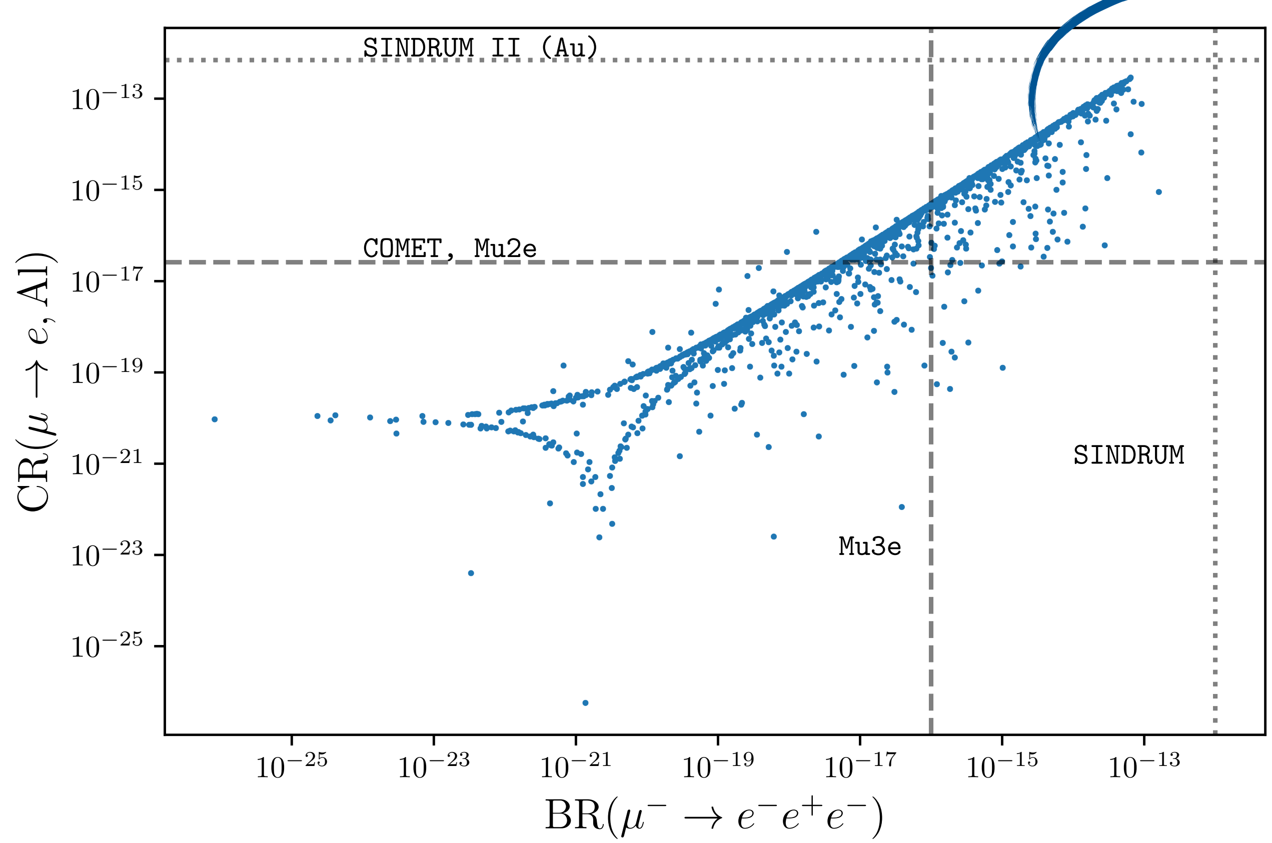
} *Z-penguin dominance*



$m_4 = m_5 = 1$ TeV

- CP conserving

[Abada, Kriewald, AMT, 2107.06313]



Strong correlation (CP conserving)

***Observation** of $\mu \to 3e$*

$\Rightarrow$ *observation of*

$\mu - e$ **conversion**

Peculiar cLFV patterns... what if CPV & cLFV?

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cLFV signatures: ratios of observables to identify mediators & constrain their masses!

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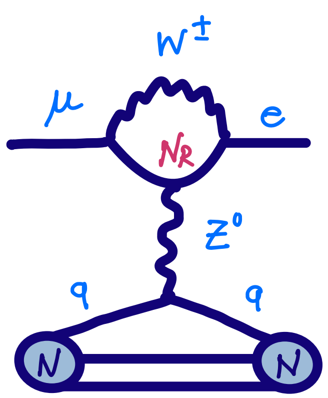

Observables dominated by common topology

- $\mu - e$ **conversion in nuclei**
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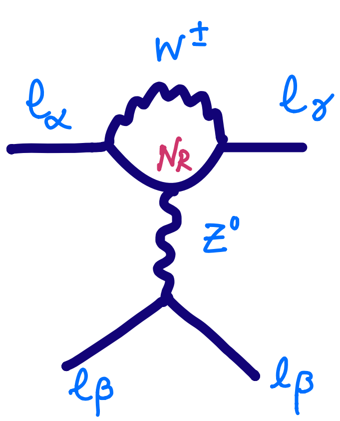
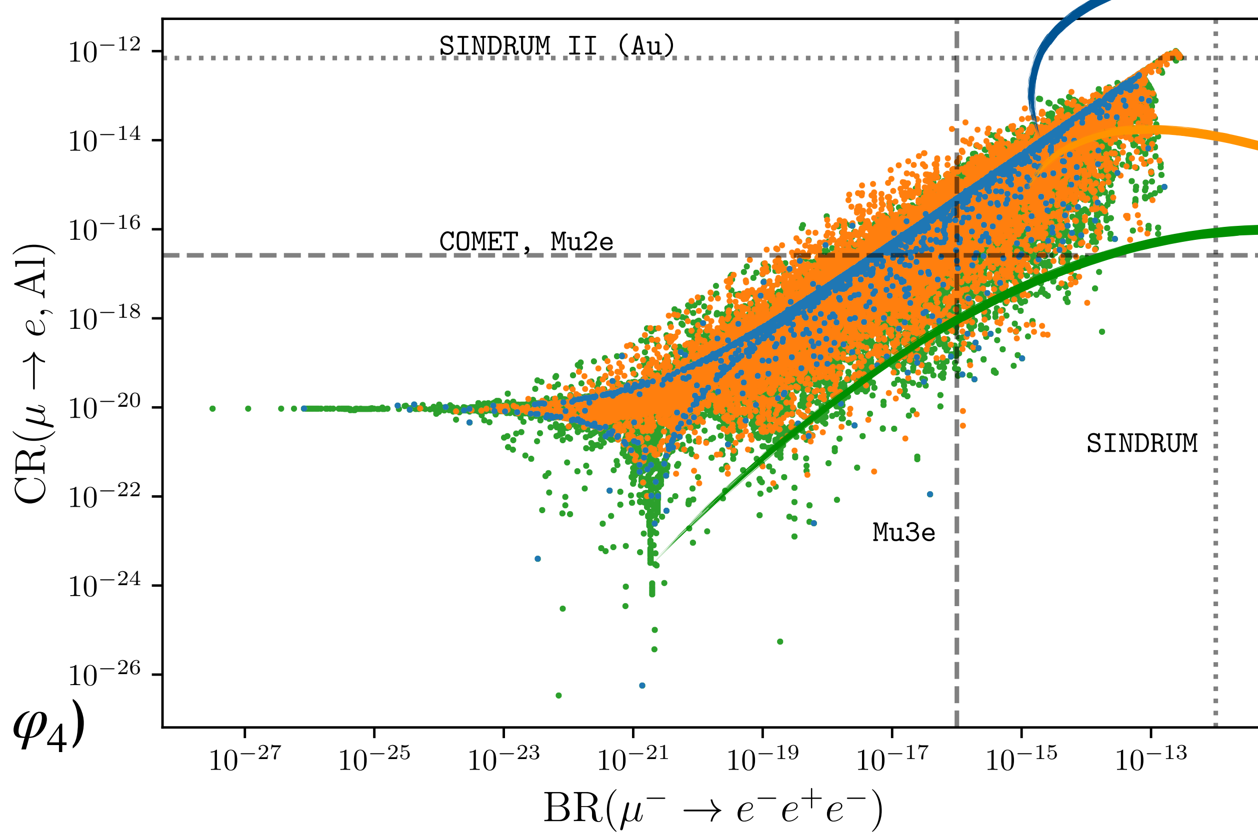


[Abada, Kriewald, AMT, 2107.06313]



Strong correlation (CP conserving)

Loss of correlation! (CP violating)

$m_4 = m_5 = 1$ TeV

- CP conserving
- CPV phases (random $\delta_{\alpha 4}, \varphi_4$)
- CPV phases (grid $n\pi/4$)

***Observation** of $\mu \to 3e$ $\not\Rightarrow$ observation of $\mu - e$ conversion*

Future cLFV data: what if CPV & cLFV?

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cLFV searches: future data can shed light on underlying NP model !

But - CP violating phases do matter! *And impact naïve theoretical expectations...*

Consider "3+2" toy model (addition of **2 heavy sterile** states; leptonic mixing $\mathcal{U}_{5\times5}$, **CPV** phases)

▶ **Impact** of **CPV phases** regarding **experimental prospects!**

Some *illustrative* **benchmark points** - **CP conserving (P_i)** and **CPV variants (P_i')**

	$\mathrm{BR}(\mu\to e\gamma)$	$\mathrm{BR}(\mu\to 3e)$	$\mathrm{CR}(\mu-e,\ \mathrm{Al})$	$\mathrm{BR}(\tau\to 3\mu)$	$\mathrm{BR}(Z\to\mu\tau)$
P_1	3×10^{-16} ○	1×10^{-15} ✓	9×10^{-15} ✓	2×10^{-13} ○	3×10^{-12} ○
P_1'	1×10^{-13} ✓	2×10^{-14} ✓	1×10^{-16} ✓	1×10^{-10} ✓	2×10^{-9} ✓
P_2	2×10^{-23} ○	2×10^{-20} ○	2×10^{-19} ○	1×10^{-10} ✓	3×10^{-9} ✓
P_2'	6×10^{-14} ✓	4×10^{-14} ✓	9×10^{-14} ✓	8×10^{-11} ✓	1×10^{-9} ✓
P_3	2×10^{-11} ✗	3×10^{-10} ✗	3×10^{-9} ✗	2×10^{-8} ✓	8×10^{-7} ✓
P_3'	8×10^{-15} ○	1×10^{-14} ✓	6×10^{-14} ✓	2×10^{-9} ✓	1×10^{-8} ✓

[Abada, Kriewald, AMT, 2107.06313]

P_2: only cLFV τ decays **within future reach**; **cLFV μ decays** beyond sensitivity...

What if one observes $\mu - e$ *and* $\mu\to 3e$? **Disfavour cLFV** from **HNL?** *or CPV...*

P_2': ***all*** considered **cLFV transitions within reach!**

(Non)-observation of cLFV observable(s) $\Rightarrow$ *not* necessarily ***disfavour*** **HNL extension!**

Future cLFV data: what if CPV & cLFV?

cLFV searches: future data can shed light on underlying NP model !

But - CP violating phases do matter! *And impact naïve theoretical expectations...*

Consider "3+2" toy model (addition of 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times 5}$, CPV phases)

▷ Impact of CPV phases regarding experimental prospects!

Some *illustrative* benchmark points - CP conserving (P_i) and CPV variants (P'_i)

	$\text{BR}(\mu\to e\gamma)$	$\text{BR}(\mu\to 3e)$	$\text{CR}(\mu-e,\text{ Al})$	$\text{BR}(\tau\to 3\mu)$	$\text{BR}(Z\to\mu\tau)$
P_1	3×10^{-16} ○	1×10^{-15} ✓	9×10^{-15} ✓	2×10^{-13} ○	3×10^{-12} ○
P'_1	1×10^{-13} ✓	2×10^{-14} ✓	1×10^{-16} ✓	1×10^{-10} ✓	2×10^{-9} ✓
P_2	2×10^{-23} ○	2×10^{-20} ○	2×10^{-19} ○	1×10^{-10} ✓	3×10^{-9} ✓
P'_2	6×10^{-14} ✓	4×10^{-14} ✓	9×10^{-14} ✓	8×10^{-11} ✓	1×10^{-9} ✓
P_3	2×10^{-11} ✗	3×10^{-10} ✗	3×10^{-9} ✗	2×10^{-8} ✓	8×10^{-7} ✓
P'_3	8×10^{-15} ○	1×10^{-14} ✓	6×10^{-14} ✓	2×10^{-9} ✓	1×10^{-8} ✓

[Abada, Kriewald, AMT, 2107.06313]

P_3: associated with large active-sterile mixings

CP conserving case excluded due conflict with bounds from cLFV μ decays

P'_3: suppression of rates from CPV phases (Dirac & Majorana)

reconcile large mixing regimes with observation!

CPV phases must be included to thoroughly assess viability of HNL regimes!

Future cLFV data: finding leptonic CPV

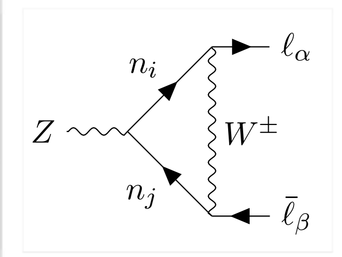
cLFV searches: future data can shed light on underlying NP model !

But - CP violating phases do matter! *And impact naïve theoretical expectations...*

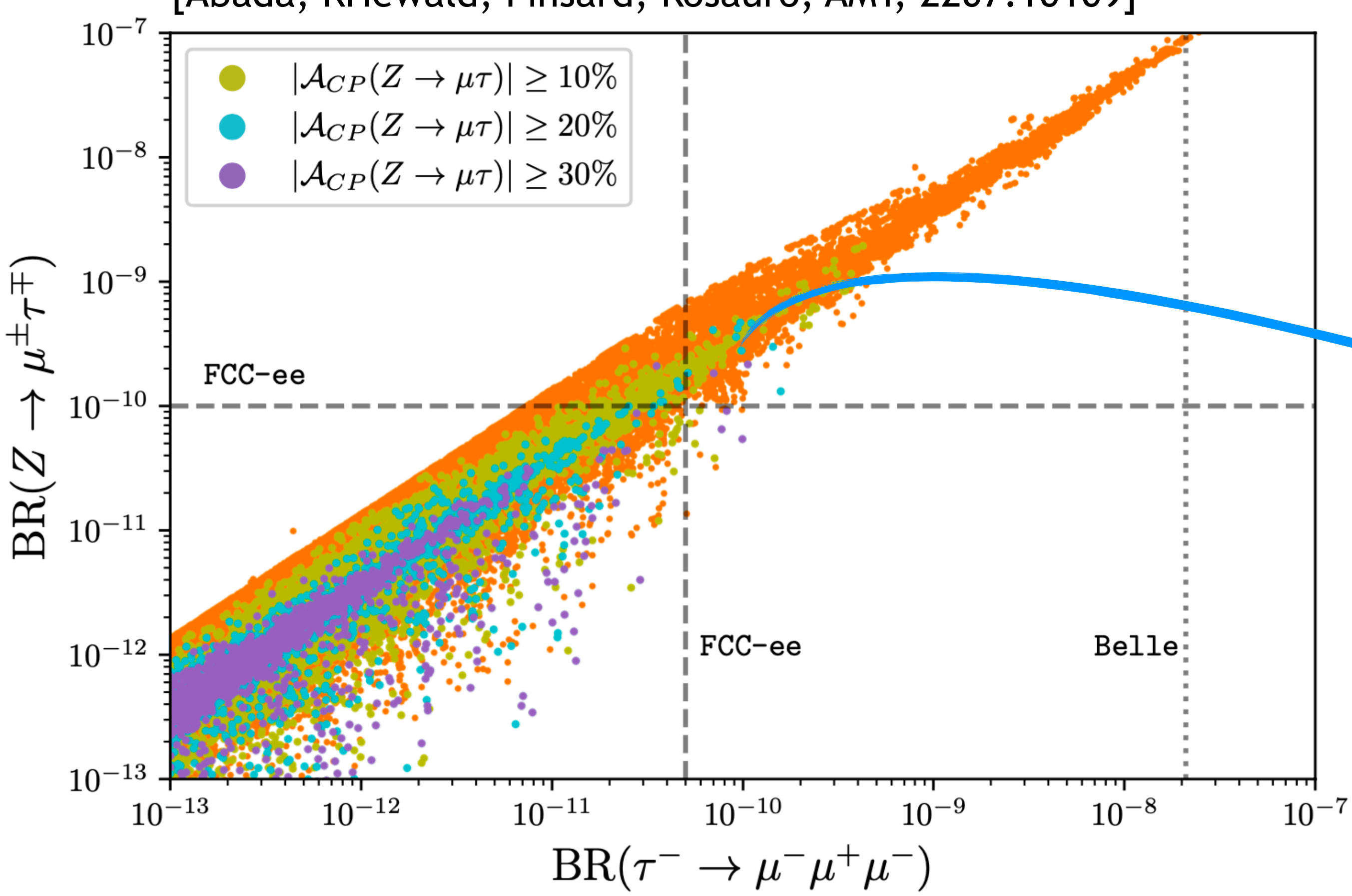
How to look for the **presence of new sources of leptonic CP violation,** in association with **cLFV?**

Consider further **observables: cLFV Z boson decays** and associated **CP asymmetries!**

$$\mathscr{A}_{CP}(Z \to \ell_\alpha \ell_\beta) = \frac{\Gamma(Z \to \ell_\alpha^- \ell_\beta^+) - \Gamma(Z \to \ell_\alpha^+ \ell_\beta^-)}{\Gamma(Z \to \ell_\alpha^- \ell_\beta^+) + \Gamma(Z \to \ell_\alpha^+ \ell_\beta^-)}$$



[Abada, Kriewald, Pinsard, Rosauro, AMT, 2207.10109]



$\Rightarrow$ For regimes with

$\mathrm{BR}(Z \to \mu\tau)$ and $\mathrm{BR}(\tau \to 3\mu)$

both **within future sensitivity**

$\mathscr{A}_{CP}(Z \to \mu\tau)$ as large as **20%!**

$\Rightarrow$ **Joint observation** *highly suggestive* of

SM extended by *at least* 2 heavy sterile ν

cLFV & CPV at work!

Outlook & conclusions

μ cLFV: overview

- **Confirmed observations** suggest the need to **go beyond the Standard Model**
  Other than ν masses, many experimental tensions appear to be "nested" in **lepton-related observables**

- **Lepton physics (muons!)** might offer **valuable hints** upon proposals of New Physics and **valuable probes** to **test the SM extensions!**

- **Experimental opportunities** with **muons:** near future discovery of New Physics, possibly before observation of new resonance at colliders

  $\Rightarrow$ **Theory efforts & theoretical approaches must be ready! (*on par!*)**

  - Consider as **many observables as possible** (proposal of new ones!)
  - Explore **distinct approaches: effective theories & NP models**
    Increase theoretical control: nuclear interactions in atomic $\mu - e$ conversion, running between scales, operator mixing effects, ...
    Include as many sources of data as possible, *synergies of observables & sectors*
  - Actively **contribute to prepare** next round of experiments (e.g. target nuclei, ...)

- **Lepton physics (muons!):** aμazing opportunities ahead!
  Calling upon joint **theory-experimental effort**

μ cLFV: overview

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- **Confirmed observations** suggest the need to **go beyond the Standard Model**
  Other than ν masses, many experimental tensions appear to be "nested" in **lepton-related observables**

- **Lepton physics (muons!)** might offer **valuable hints** upon proposals of New Physics and valuable probes to **test the SM extensions!**

- **Experimental opportunities** with **muons:** near future discovery of New Physics, possibly before observation of new resonance at colliders

  ⇒ **So much to be learnt from muon flavours...**

- Hint on New Physics couplings & new Lorentz structure (i.e. new interactions)
- Exclude regimes and regions in BSM parameter space
- **Falsify a model** (directly, or through correlations - cLFV patterns) and/or **reduce "ambiguities"** on other sectors...
  (and remember - CPV phases matter in flavours!)
- **Probe otherwise unreachable scales!**

"Leave no (flavoured) stone unturned" - leave no single grain of sand **unobserved,** or **muon-flavour unte(a)sted!** 😀

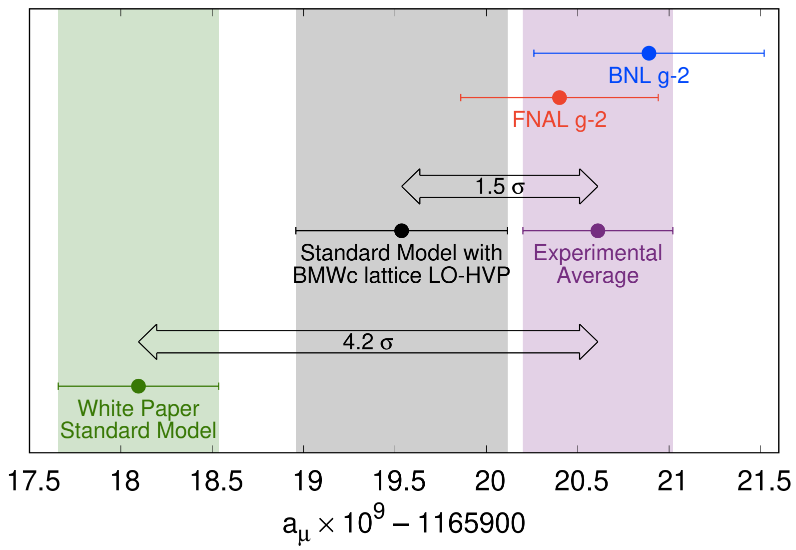
Additional material

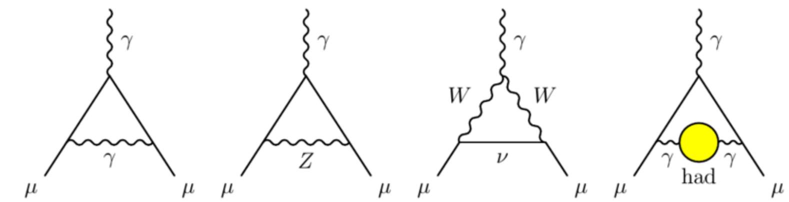
Anomalous magnetic moments: muons and electrons

IN2P3 Les deux infinis

- **Anomalous magnetic moment of the muon @ 2021:**

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = ?$$



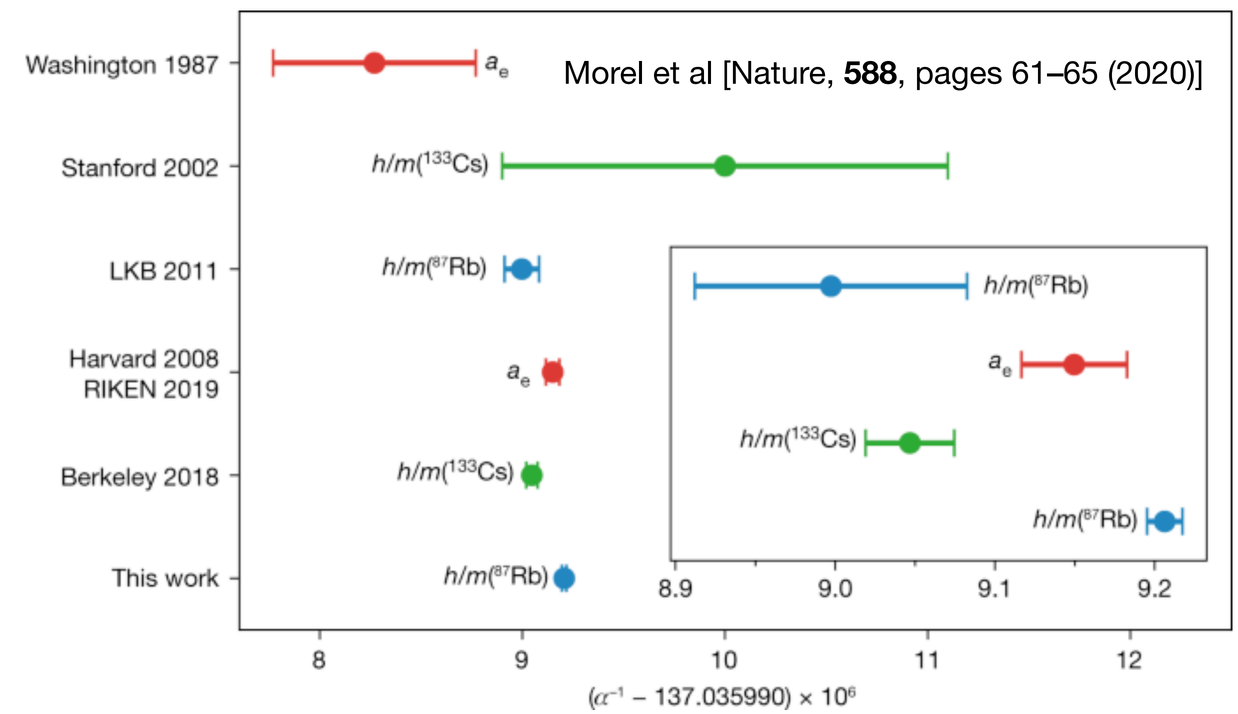


New Physics: badly needed? or not?

- **Anomalous magnetic moment of the electron**

(2018) $\Delta a_e^{\text{Cs}} = -0.88(36) \times 10^{-12} \quad \rightsquigarrow \mathbf{-2.3\sigma}$

(2020) $\Delta a_e^{\text{Rb}} = 0.48(30) \times 10^{-12} \quad \rightsquigarrow \mathbf{+1.7\sigma}$



Difference of 5.4σ in **determination of** α ?!
(SM input parameter!)

- **Two anomalies in** Δa_μ **and** Δa_e^{Cs} **?**

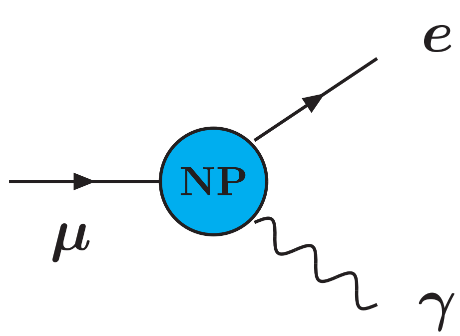
Possible hint of **lepton flavour universality violation?**

Lepton universality (MFV) naïvely suggests $\Delta a_e / \Delta a_\mu \approx m_e^2 / m_\mu^2 \sim +2.4 \times 10^{-5}$

but $\Delta a_e^{\text{Cs}} / \Delta a_\mu \sim -3.3 \times 10^{-4}$...

cLFV: modes & experimental prospects

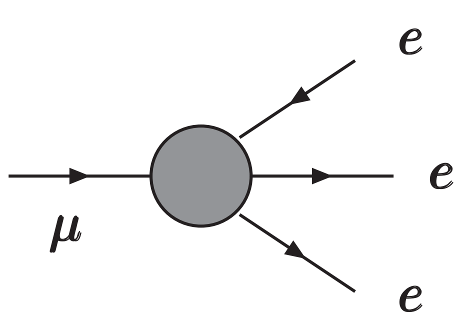
Radiative and 3-body muon decay channels



- cLFV decay: $\mu^+ \to e^+ \gamma$
- Event signature: $E_e = E_\gamma = m_\mu/2$ (~ 52.8 MeV)

  Back-to-back e^+ - γ ($\theta \sim 180°$); Time coincidence

- **Current status:** BR($\mu \to e\gamma$)$\lesssim 4.2 \times 10^{-13}$ [MEG, '16]
- **Future prospects:** MEG II @ PSI $\rightsquigarrow$ sensitivity 4×10^{-14}



- cLFV decay: $\mu^+ \to e^+ e^- e^+$
- Event signature: $\sum E_e = m_\mu$; $\sum \vec{P}_e = \vec{0}$

  common vertex; Time coincidence

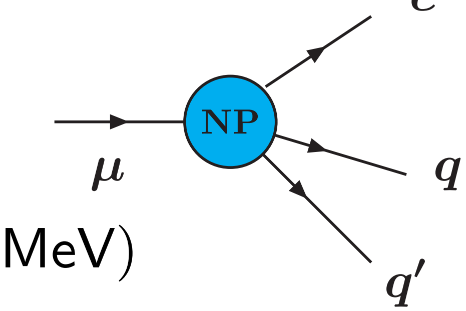
- **Current status:** BR($\mu \to eee$)$\lesssim 1.0 \times 10^{-12}$ [SINDRUM, '88]
- **Future prospects:** Mu3e @ PSI

  Phase I: 10^{-15} (πE5 μ source) $\Rightarrow$ **Phase II:** 10^{-16} (H.I. μ-beam)

cLFV: modes & experimental prospects

Nuclear assisted cLFV and Muonium channels

- cLFV $\mu^- - e^-$ conversion: $\mu^- + (A,Z) \to e^- + (A,Z)$
- **Event signature:** single mono-energetic electron, $E_{\mu e}^{\text{Al, Pb, Ti}} \approx \mathcal{O}(100\ \text{MeV})$



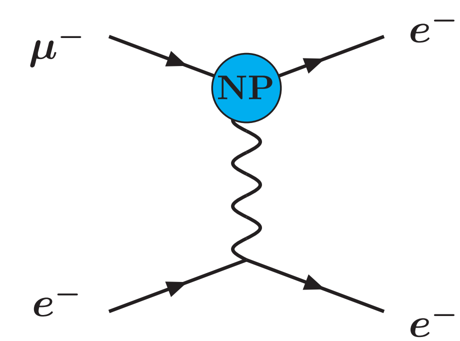
- **Current status:** **CR($\mu - e$, Au)$\lesssim 7 \times 10^{-13}$** [SINDRUM, '06]
- **Future prospects (Al):** **Mu2e @ FNAL I (II) $\sim 6 \times 10^{-17}$(few $\times 10^{-18}$);**
  COMET @ J-PARC I (II) $\sim 10^{-15}(10^{-18})$

- Coulomb enhaced muonic atom decay: $\mu^- e^- \to e^- e^-$

$$\Gamma(\mu^- e^- \to e^- e^-,\ \text{N}) \propto \sigma_{\mu e \to ee} v_{\text{rel}} \left[(Z-1)\,\alpha\, m_e\right]^3/\pi$$

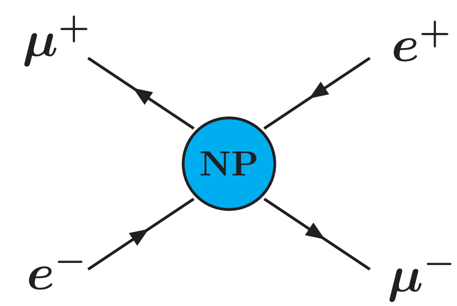
$\Rightarrow$ Consider large Z targets! Pb, U!?

- **Clean experimental signature:** back-to-back electrons, $E_{e-} \approx m_\mu/2$



- **Experimental status:** **New observable!**

- cLFV **Mu $-$ $\overline{\text{Mu}}$ conversion** $\Rightarrow$ Oscillation between $(e^-\mu^+) \leftrightsquigarrow (e^+\mu^-)$
- **Current status:** **P(Mu $-$ $\overline{\text{Mu}}$)$< 8.3 \times 10^{-11}$** [Willmann et al, 1999]
- **Future prospects:** **MUSE (J-PARC)? FNAL?**



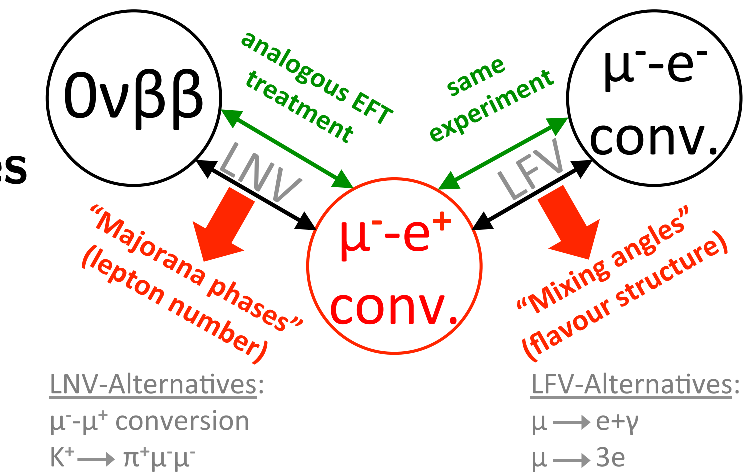
cLFV: modes & experimental prospects

LNV atomic conversion

- **LNV ($\Delta L = 2$) $\mu^- - e^+$ conversion:** $\mu^- + (A, Z) \to e^+ + (A, Z-2)^*$

  $\boldsymbol{\mu^- - e^-}$: coherent, single nucleon, nuclear ground state

  $\mu^- - e^+$: 2 nucleons ($\Delta Q = 2$), possibly **excited final states**



- **Event signature:** single positron - but *complex* E-spectrum

$$E^{N^*}_{\mu^- e^+} = m_\mu - E_B(A, Z) - E_R(A, Z) - \Delta_{Z-2^{(*)}}$$

$$E^{\text{Al,GDR}}_{\mu^- e^+} \approx \mathcal{O}(83.9\text{ MeV}) \quad [< \text{GDR}_{\text{Al}} > \sim 21.1\text{ MeV } (6.7\text{ MeV})]$$

[Geib et al, '16]

- **Experimental status - present bounds:**

Collaboration	year	Process	Bound
PSI/SINDRUM	1998	μ^-+Ti→ e^++Ca*	3.6×10^{-11}
PSI/SINDRUM	1998	μ^-+Ti→ e^++Ca	1.7×10^{-12}

- **Experimental status - future prospects:**

  Recent studies: **best sensitivity** associated with **Calcium**, **Sulphur** and **Titanium** **targets**

  CR($\mu^- - e^+$) < $\mathcal{O}$(few × 10^{-15}) for ^{48}Ti (both LNC and LNV searches) [Yeo et al, '17]

  For Aluminium targets improvement of current sensitivity maybe very hard (even factor 10)...

cLFV: modes & experimental prospects

Muon cLFV searches and axion-like particles

★ **Axion-like particles: very light, "invisible" (neutral or feebly coupled) states**

$\Rightarrow$ present in numerous **well motivated NP models;** role in **astrophysics & cosmology**

- If **ALPs** have **flavour violating couplings to leptons, dedicated cLFV searches**

  $\mu \to ea \quad \mu \to e\gamma a$, among many others (including τ modes)

- **Current limits:** BR($\mu^+ \to e^+ a$)$< 2.6 \times 10^{-6}$ [TRIUMF, '86]
  BR($\mu \to ea$)$< 5.8 \times 10^{-5}$ [TWIST, '14]
  BR($\mu \to ea\gamma$)$< 1.1 \times 10^{-9}$ [Crystal Box, '88]

- **Future reach (?):** BR($\mu \to ea$)$< 10^{-8}$ **@ Mu3e** [Perrevoort, '18]
  BR($\mu \to ea$)$< 2 \times 10^{-6}$ **@ COMET/Mu2e** [Garcia i Tomo et al, '11]
  Possibly at **MEG... BR($\mu \to ea$)$< 3 \times 10^{-7}$** (?)

- **Interesting synergy** between **rare muon decays** and **ALP searches!**

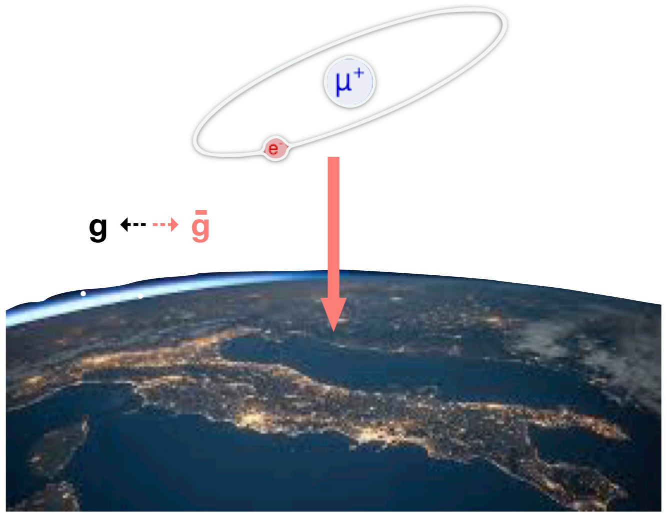
cLFV: modes & experimental prospects

Further muonic probes of New Physics

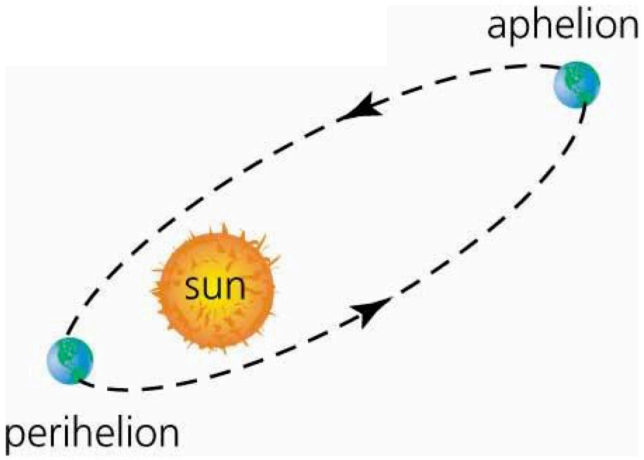
★ **Testing antimatter gravity with Muonium**

- **No direct test of gravity for antimatter ("heavy μ^+)**

  No test of weak equivalence principle (equivalence of gravitational and inertial masses) for **antimatter**

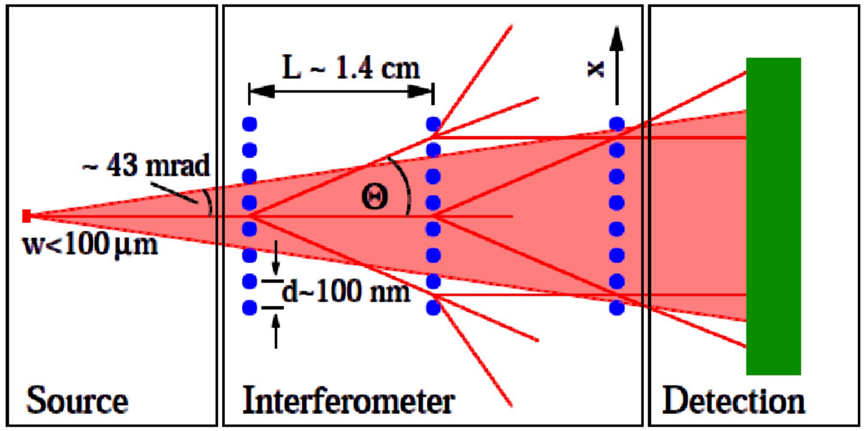


- **Methods: Annual modulation** of **Muonium 1s-2s transition** frequency



[Kirch, '07-'19]

Mach Zehnder atom interferometer



cLFV: modes & experimental prospects

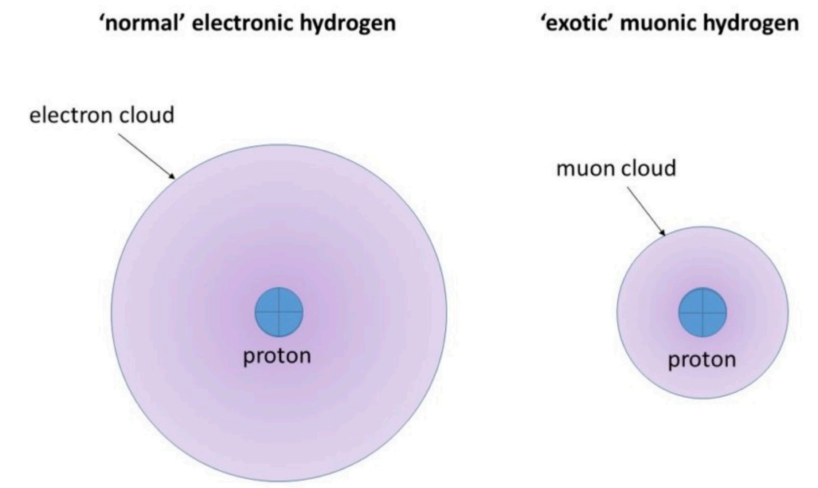
Further muonic probes of New Physics

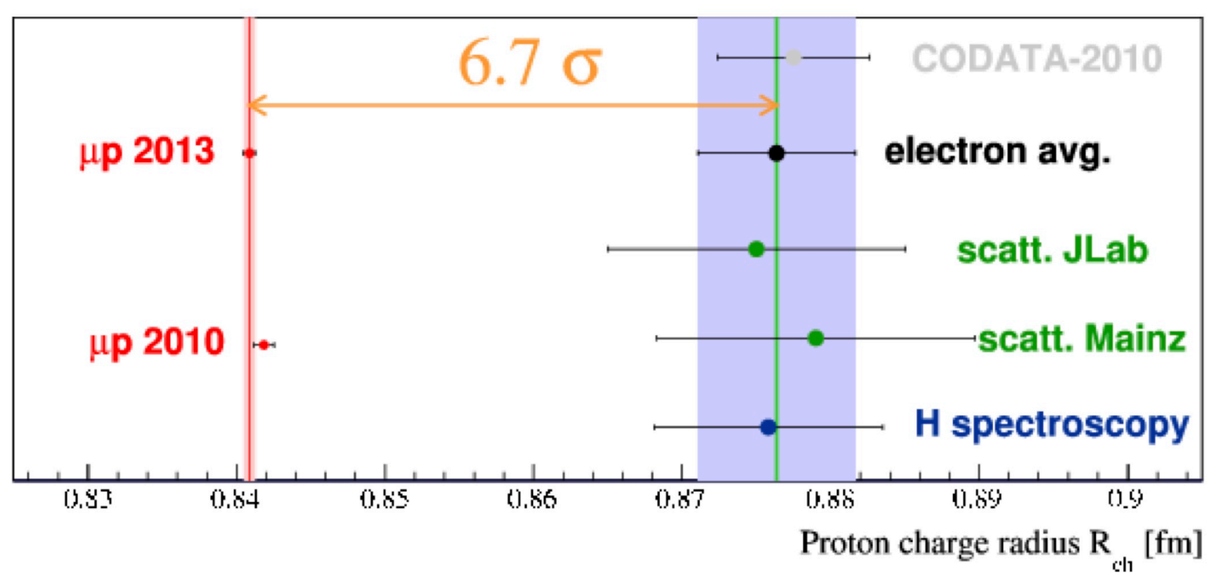
★ **Testing antimatter gravity with Muonium**

★ **The proton radius: experiments with muonic Hydrogen**

▶ Measurement of **proton radius** relies on **Lamb Shift** (sensitive to r_p)

▶ **Proton radius determined** with great accuracy for **muonic Hydrogen, $^{\mu}$H**





[1506.00873]

▶ An excess of $6\,\sigma$ deviations between r_p^{μ} and r_p^{e}! **New Physics?**

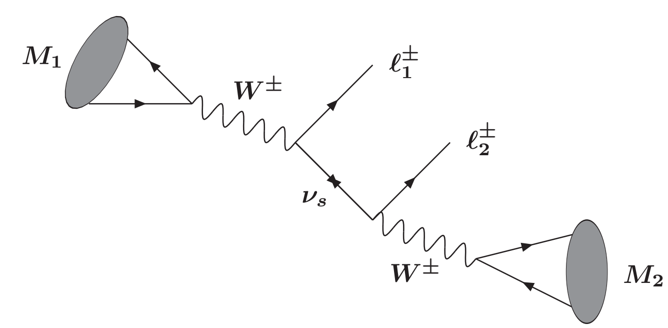
cLFV: modes & experimental prospects

Rare Lepton Number Violating ($\Delta L = 2$) decays

★ **LNV** suggests the presence of **Majorana states;** opens the door for leptogenesis...

- **Neutrinoless double beta decays:** $(A, Z) \to (A, Z+2) + 2e^-$

  Current status: $m_{\beta\beta} < (61 - 165)$ meV [Kamland-Zen, '16]



- **LNV in semileptonic tau and/or meson decays**

LNV decay	Current Bound	
	$\ell = e,\ \ell' = e$	$\ell = \mu,\ \ell' = \mu$
$K^- \to \ell^- \ell'^- \pi^+$	6.4×10^{-10}	1.1×10^{-9}
$D^- \to \ell^- \ell'^- \pi^+$	1.1×10^{-6}	2.2×10^{-8}
$D^- \to \ell^- \ell'^- K^+$	9.0×10^{-7}	1.0×10^{-5}
$B^- \to \ell^- \ell'^- \pi^+$	2.3×10^{-8}	4.0×10^{-9}
$B^- \to \ell^- \ell'^- K^+$	3.0×10^{-8}	4.1×10^{-8}

◄ ▼ **Experimental status:** BaBar, Belle

LNV decay	Current Bound	
	$\ell = e$	$\ell = \mu$
$\tau^- \to \ell^+ \pi^- \pi^-$	2.0×10^{-8}	3.9×10^{-8}
$\tau^- \to \ell^+ \pi^- K^-$	3.2×10^{-8}	4.8×10^{-8}
$\tau^- \to \ell^+ K^- K^-$	3.3×10^{-8}	4.7×10^{-8}

- **Future prospects:** LHCb (Upgrade I & II), Belle II (upgrade),

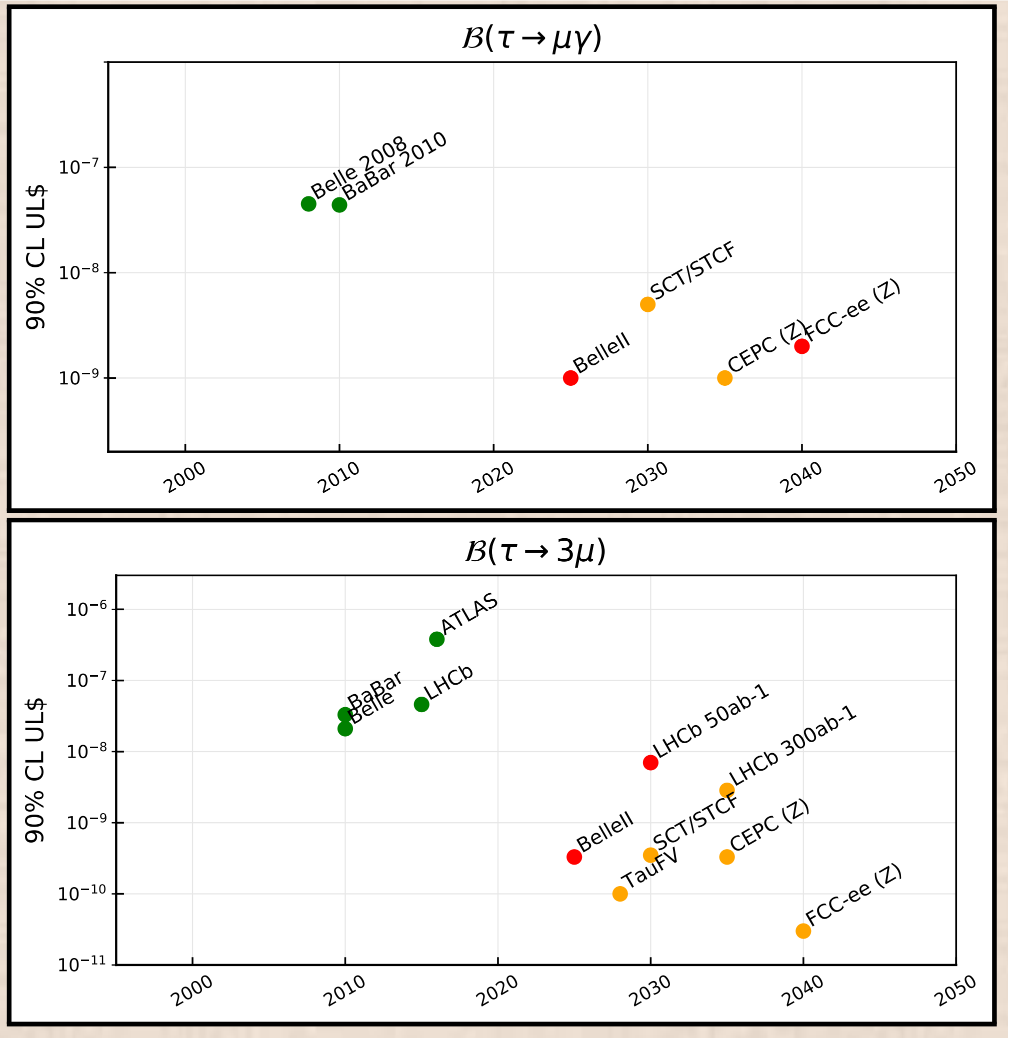
  TauFV, Super Charm-Tau factory... NA62, KOTO, KLEVER, ...

cLFV: modes & experimental prospects

Rare lepton processes: cLFV tau decays

- **Radiative decay:** $\tau^\pm \to \ell^\pm \gamma$
- **Event signature:** $E_{\text{final}} - \sqrt{s}/2 = \Delta E \sim 0$;
  $M_{\text{final}} = M_{\ell\gamma} \sim m_\tau$
- **Current status:** BR($\tau \to e\gamma$)$\lesssim 3.3 \times 10^{-8}$;
  BR($\tau \to \mu\gamma$)$\lesssim 4.4 \times 10^{-8}$ [BaBar, '10]

[Lusiani, EPPSU'19]



- **3-body decays:** $\tau^\pm \to \ell_i^\pm \ell_j^\mp \ell_k^\pm$
- **Event signature:** $E_{3\ell} - \sqrt{s}/2 \sim 0$;
  $M_{3\ell} \sim m_\tau$
- **Current status:** BR($\tau \to 3\ell$)$\lesssim \mathcal{O}(10^{-8})$
- **Future experimental prospects:**

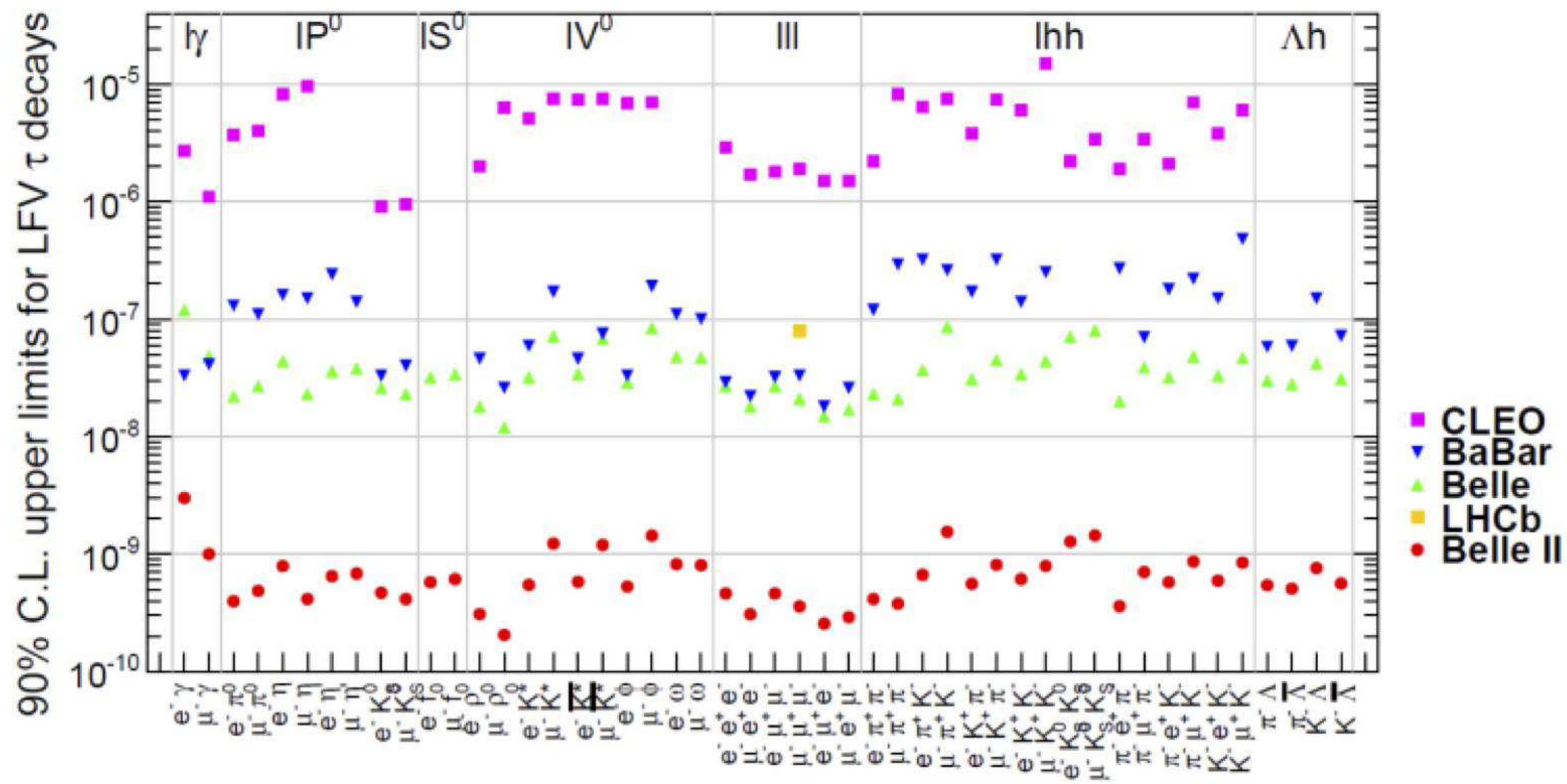
Belle II, LHCb Upgrades, ..., TauFV, (Super) Tau-Charm factories, FCC/CEPC

BR($\tau \to \ell\gamma$) $\leq 1 - 3 \times 10^{-9}$ BR($\tau \to 3\ell$) $\leq 1 - 2 \times 10^{-10}$

cLFV: modes & experimental prospects

cLFV semi-leptonic decays: tau leptons

▶ Meson(s) & charged lepton: $\tau \to \ell\, h^0$; $\tau \to \ell\, h_i\, h_j$



From Belle II physics book
arXiv:1808.10567

Meson decays: excellent testing grounds for lepton flavour dynamics - cLFV

▶ **K, D and B meson decays: abundant data [LHCb, BNL, KTeV, BaBar, Cleo, Belle, ...]**

$$\text{BR}(K_L \to \mu e) < 4.7 \times 10^{-12}; \quad \text{BR}(K^+ \to \pi^+ \mu^+ e^-) < 2.1 \times 10^{-11}$$

$$\text{BR}(D^0 \to \mu e) < 1.5 \times 10^{-8}; \quad \text{BR}(B \to \mu e) < 2.8 \times 10^{-9}, \ \ldots.$$

▶ **Future prospects:** BR$(B_{(s)} \to \mu e) < \mathcal{O}(10^{-10})$ LHCb'II, BR$(B \to X\tau e(\mu)) < \mathcal{O}(10^{-6})$ Belle II, ..

cLFV: modes & experimental prospects

cLFV decays at colliders

- **Z boson decays: $Z \to \ell_i \ell_j$** $\rightsquigarrow$ Zs abundantly produced at **LEP** and at the **LHC**
  - **Current bounds:** $\text{BR}(Z \to e\mu) < 7.5 \times 10^{-7}$ [ATLAS, 2014]

    $\text{BR}(Z \to \mu\tau) < 1.2 \times 10^{-5}$; $\text{BR}(Z \to e\tau) < 9.8 \times 10^{-6}$ [OPAL & DELPHI]

- **Higgs boson decays: $H \to \ell_i \ell_j$** $\rightsquigarrow$ "Higgs-factory" at LHC - study rare processes...
  - **Current data:** $\mathbf{BR(H \to \mu\tau) \lesssim 0.0025}$ [CMS]; $\mathbf{BR(H \to e\tau) \lesssim 0.0061}$ [CMS]

- **Production of "on-shell" NP states** $\Rightarrow$ **new interactions** induce cLFV **decays**

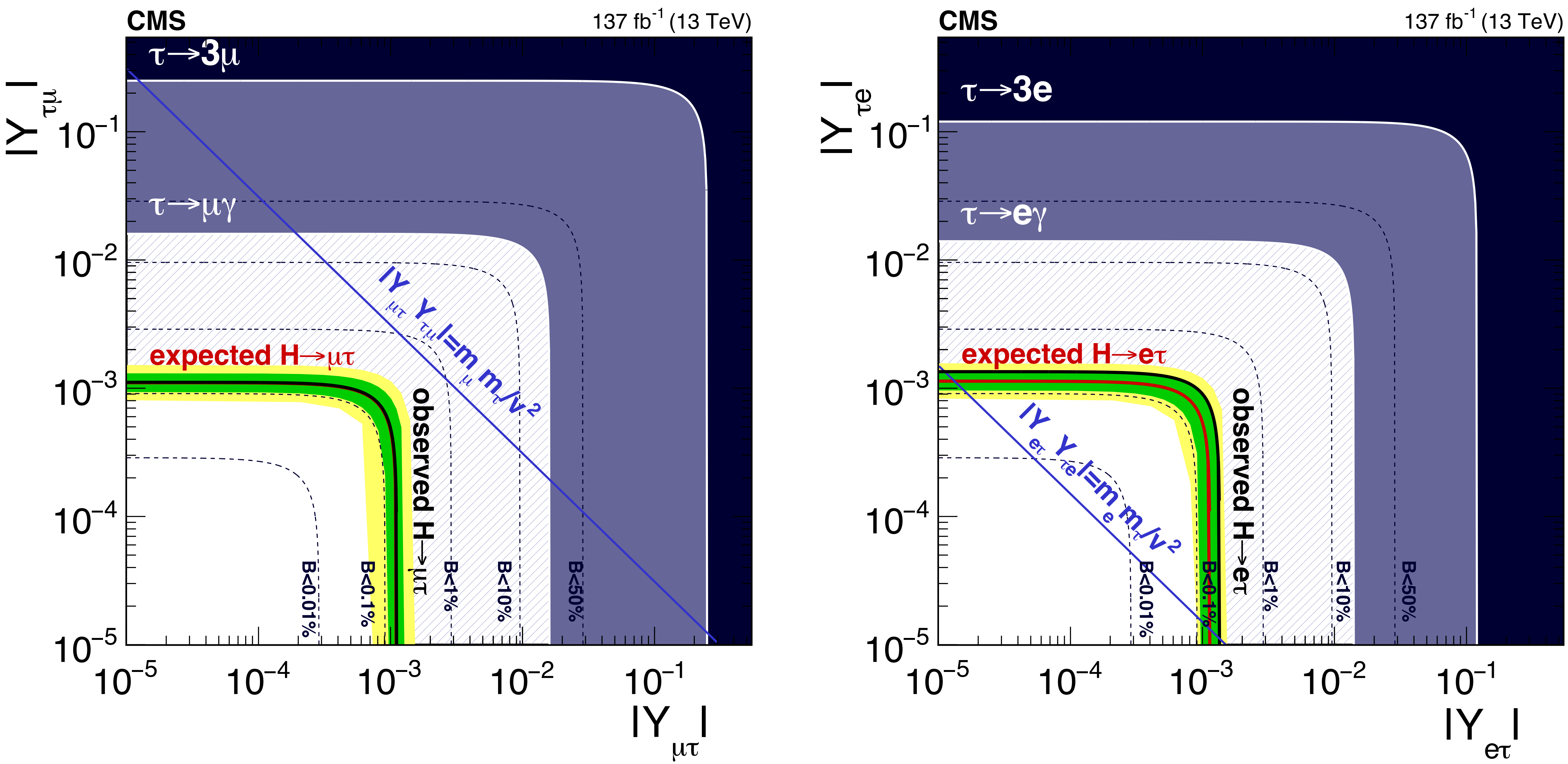
  Multiplicity, composition, E_{miss}, ...: properties of final state strongly model-dependent...

- **Future experimental prospects: LHC Run 2 !!**

  ... and a **Higgs factory** (linear/circular) ... and **FCC-ee** (at the Z pole)

cLFV: modes & experimental prospects

- cLFV decays at colliders



- **Limits on "effective" Yukawa couplings:** $$\Gamma(H\to\ell^\alpha\ell^\beta)=\frac{m_H}{8\pi}\left(|Y_{\ell^\alpha\ell^\beta}|^2+|Y_{\ell^\beta\ell^\alpha}|^2\right)$$

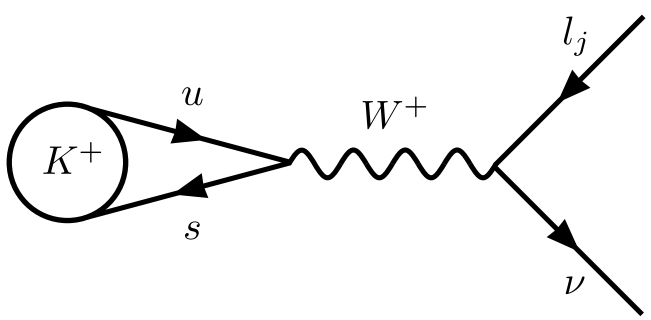
[CMS Coll, 2105.03007]

LFUV in kaon and pion leptonic decays

★ **In the SM, charged leptons have universal couplings to EW gauge bosons**

$g_e = g_\mu = g_\tau = g$ $\Rightarrow$ Studied and tested for π, Kaons and B-mesons

▶ Kaon sector: $R_K^\ell = \frac{\Gamma(K \to e\bar{\nu})}{\Gamma(K \to \mu\bar{\nu})} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_K^2 - m_e^2}{m_K^2 - m_\mu^2}\right)^2 (1 + \delta R_{\text{QED}})$



$R_K^{\text{SM}} = (2.477 \pm 0.001) \times 10^{-5}$ [Cirigliano et al, '07]

$R_K^{\text{exp}} = (2.488 \pm 0.009) \times 10^{-5}$ [NA62]

In the future, **NA62** & **TREK: $\mathcal{O}(0.2\%)$ precision!**

▶ Pion sector: $R^\pi_{e/\mu} = \frac{\Gamma(\pi \to e\bar{\nu}(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}(\gamma))}$

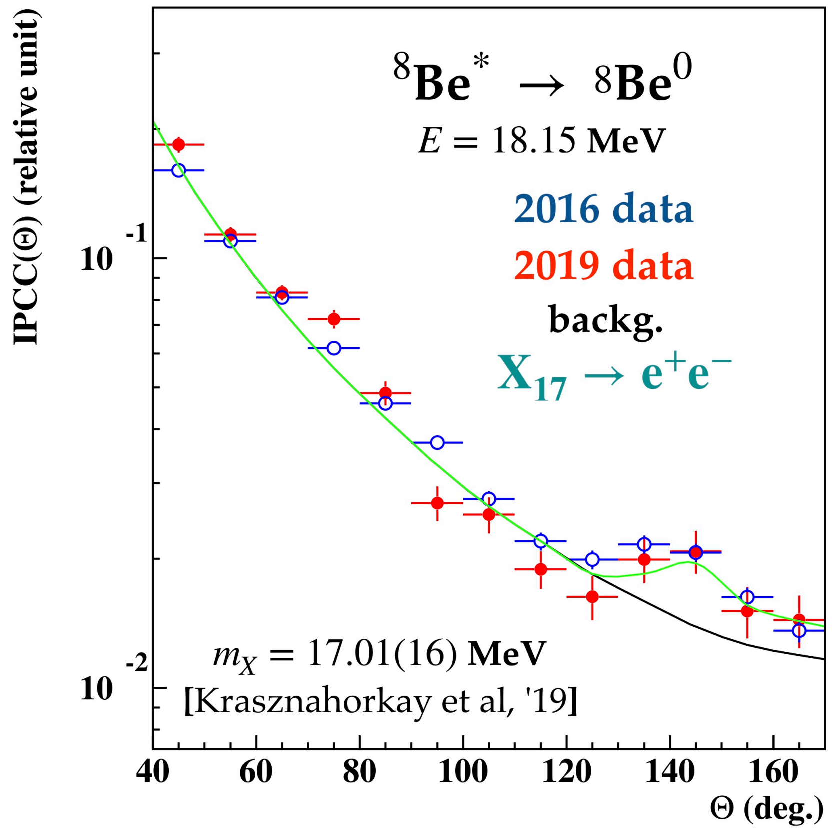
$R^\pi_{\text{SM}} = (1.2352 \pm 0.0002) \times 10^{-4}$ (± 0.16 pp mille)

$R^\pi_{\text{exp}} = (1.2344 \pm 0.00030) \times 10^{-4}$ (± 2.4 pp mille)

In the future, **PEN** and **PIENU** (final results): beyond **$\mathcal{O}(0.02\%)$ precision!**

Anomalous magnetic moments & more

A simultaneous explanation to Δa_μ and Δa_e^{Cs} and to ^8Be atomic decay "anomaly"?



⇒ Angular correlation of e^+e^- internal pair creation
$^8Be^*(j^\pi = 1^+, T=0) \to {}^8Be^0(j^\pi = 0^+, T=0)$ @ $5\sigma - 6\sigma$

⇒ Similar deviations in $^4He\ e^+e^-$ angular correlation
$^4He(0^- \to 0^+, E = 21.01$ MeV$)$ @ 7.2σ

⇒ Production and decay of (hypothetical) light vector boson
$m_X \sim 17$ MeV$, \Gamma_X/\Gamma_\gamma \sim \mathcal{O}(10^{-5})$ Feng et al [2006.01151]

Minimal framework: $SU(3)\times SU(2)\times U(1)\times U(1)_{B-L}$ $\rightsquigarrow Z'$

extra RH neutrinos, heavy vector-like leptons, scalar h_X

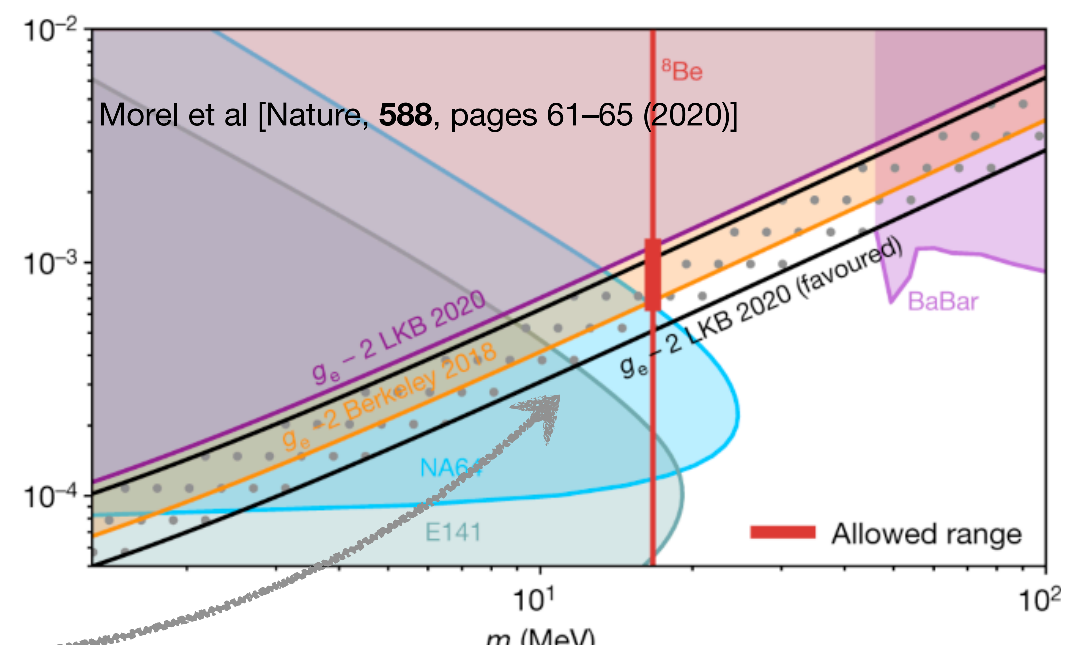
m_ν from mixings with N_R and L^0; **dynamical** $m_M = v_X y_M$

New **neutral currents** (Z' and h_X)

Cancellation of NP contributions: saturate Δa_μ and Δa_e^{Cs}

Constrained parameter space! ^8Be and $\Delta a_\mu \Rightarrow \Delta a_e$!

(Far more challenging with Δa_e^{Rb}...)

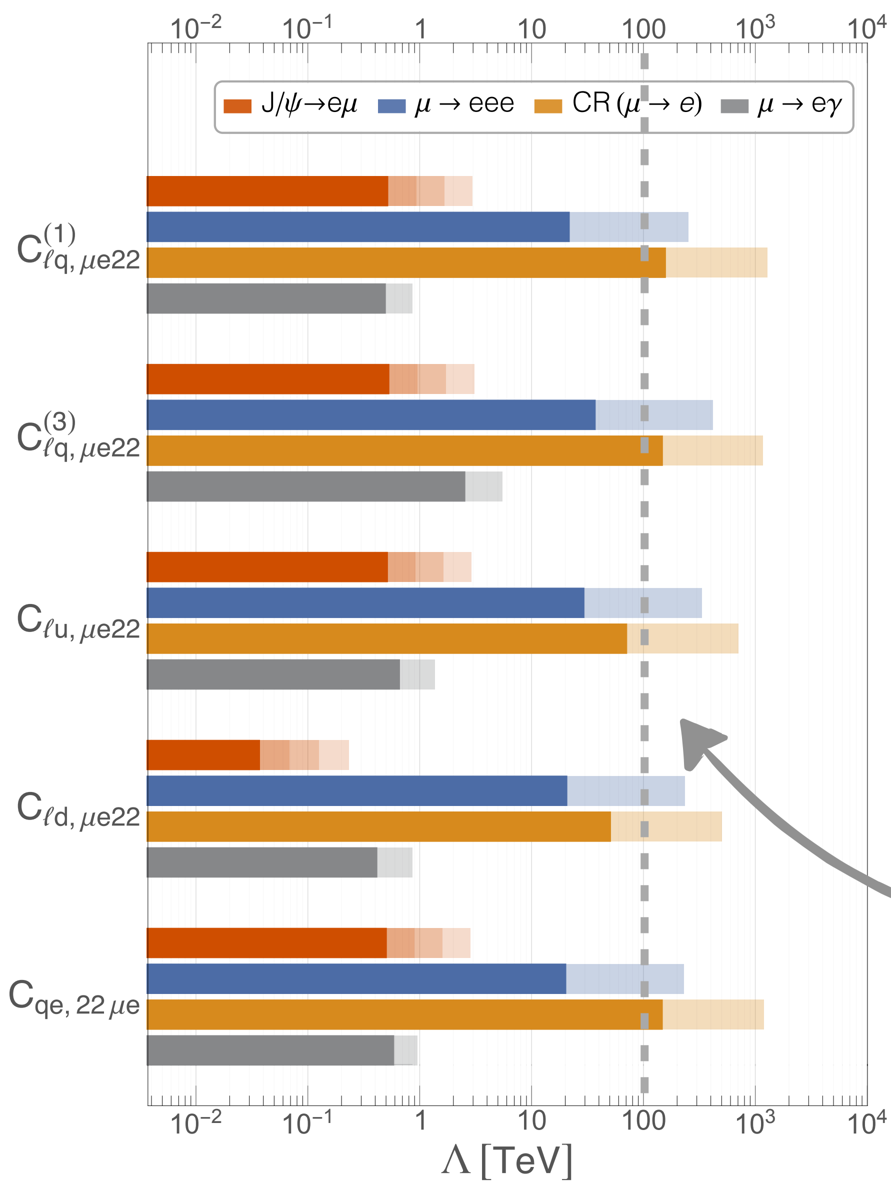


Muon cLFV: EFT approach to New Physics

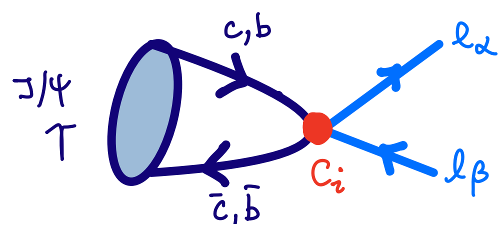
$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \ldots$$

⇒ **cLFV data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to a given operator** $\mathscr{O}^6$)

▷ **Comparative probing power** (simplified overview): **leptonic and (light) meson decays**



[Calibbi et al, 2207.10913]



- **Single** $\mathscr{O}^6_{2q2\ell}$ at a time (perturbative $\mathscr{C}_6$)
- Most **stringent** constraints: $\mu - e$ and $\mu \to 3e$
- **If NP** is such that

$$\frac{\mathscr{C}_6^{2q2\ell}}{\Lambda^2} \gtrsim \frac{1}{(10^3\ \text{TeV})^2} - \frac{1}{(10^2\ \text{TeV})^2}$$

⇒ **cannot observe** $J/\psi \to e\mu$ (or $\Upsilon(1S) \to e\mu$)

nor $\mu \to e\gamma$

(unless exotic NP at work!)

Minimal extensions of the SM: anomalies & μ-physics

- Extensive studies of SM extensions capable of addressing "anomalous" behaviours (i.e. tensions with SM expectation): $(g-2)_\ell$, and LFUV in B-meson decays
  Minimal, testable models (first step towards complete constructions)

  cLFV ⇜⇝ LFUV

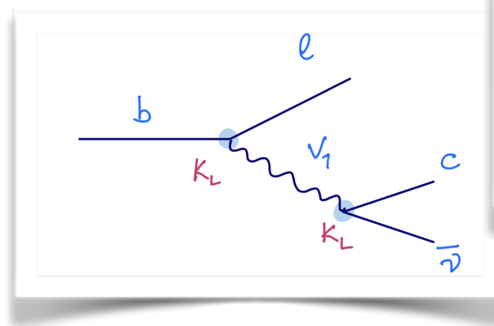
- **Minimal SM extension** via **single vector LQ (V_1^μ)** explain both $R_{K^{(*)}}$ and $R_{D^{(*)}}$ at tree-level

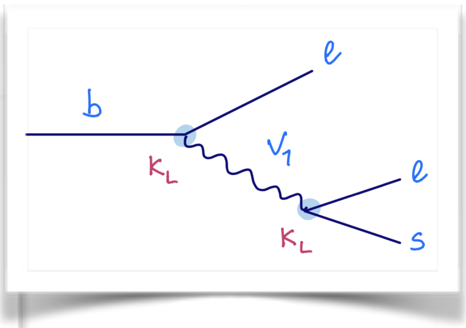
$$\mathcal{L} \supset V_1^\mu \left(\bar{d}_L^i \gamma_\mu \boldsymbol{K_L^{ik}} \ell_L^k + \bar{u}_L^j V_{ji}^\dagger \gamma_\mu \boldsymbol{K_L^{ik}} U_{kj}^{\mathrm{P}} \nu_L^j \right)$$

⇒ **Effective** $V_1\, q\, \ell$ couplings from **mixings** of **SM leptons** with **heavy vector-like lepton** ($SU(2)_L$ **doublets** to avoid $Z \to \ell\ell'$ decays)

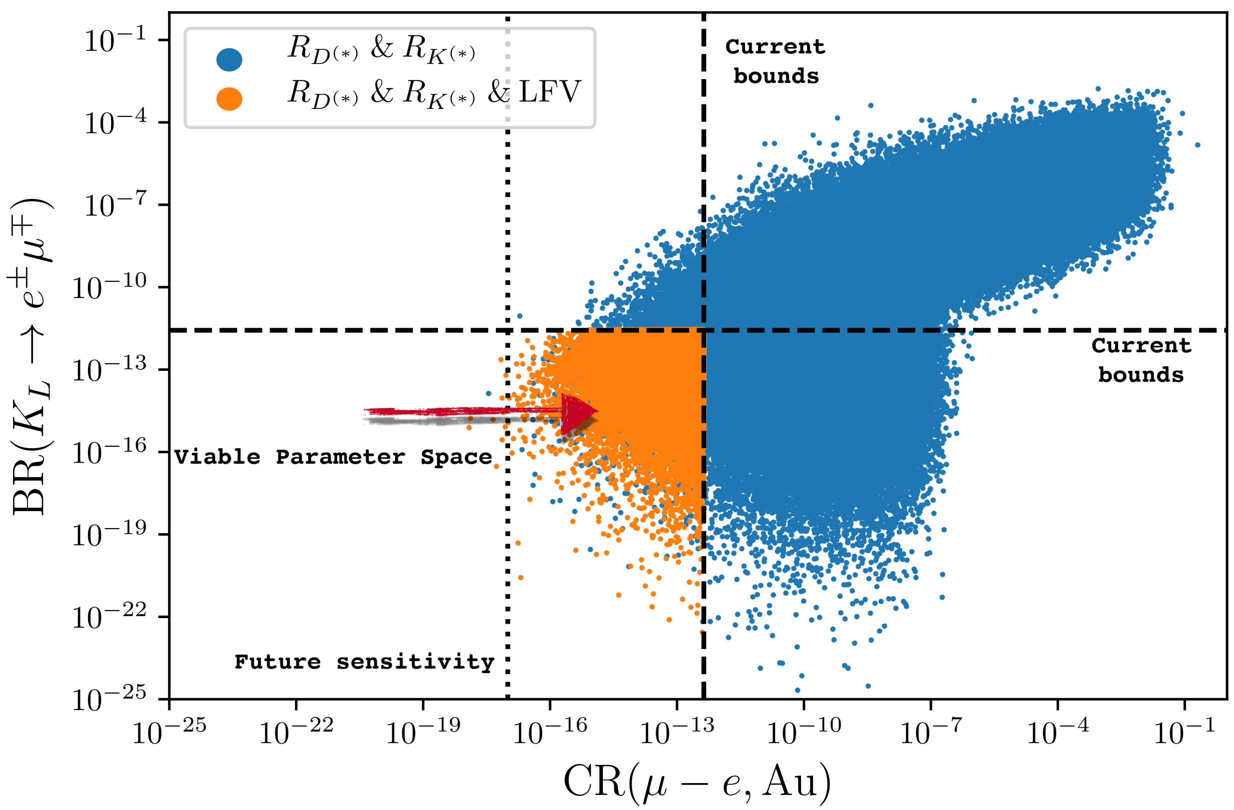
Most **constraining observables:**
$K_L \to e\mu$ and $\mu - e$ conversion in nuclei

⇒ viable regimes within **sensitivity** of **Mu2e** and **COMET**





[Hati, Kriewald, Orloff, AMT, 1907.05511]



Low-scale models of m_ν generation: type I seesaw

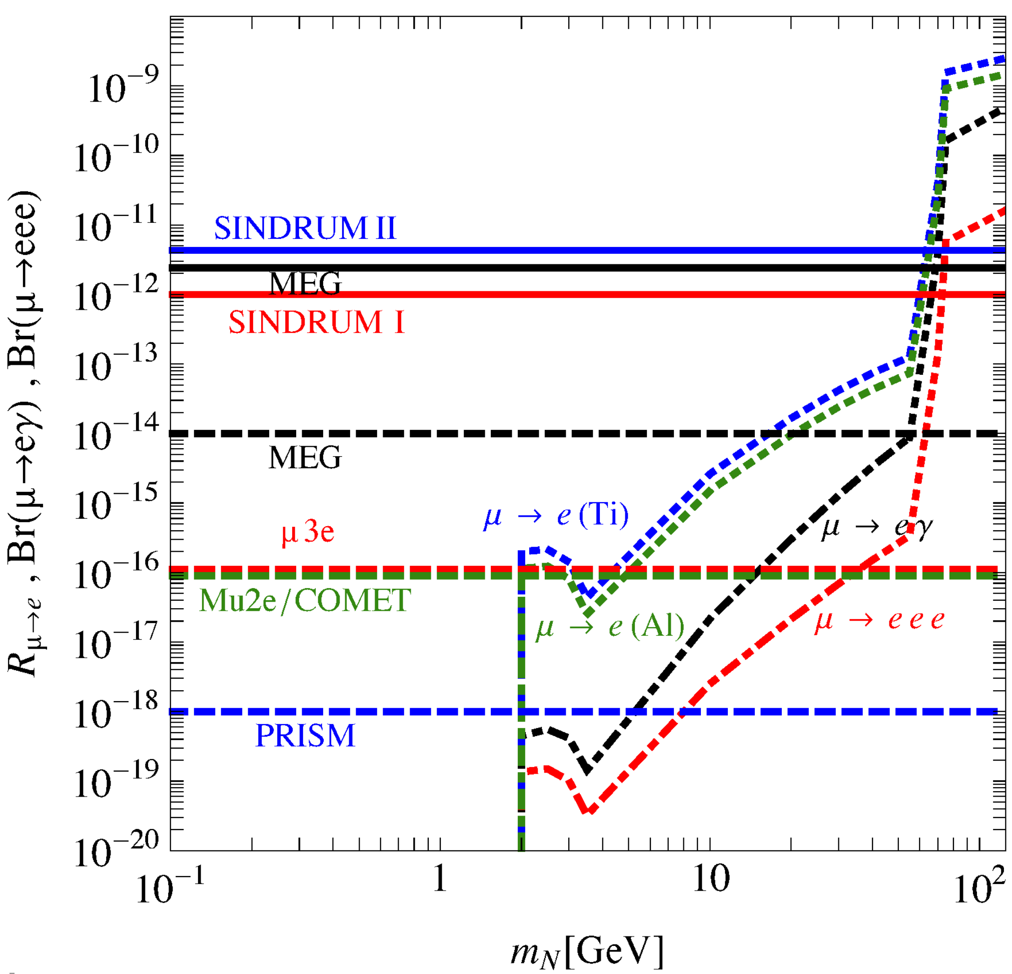
- Addition of **3 "heavy" Majorana RH neutrinos** to the SM: $\text{MeV} \lesssim m_{N_i} \lesssim 10^{\text{few}}\,\text{TeV}$

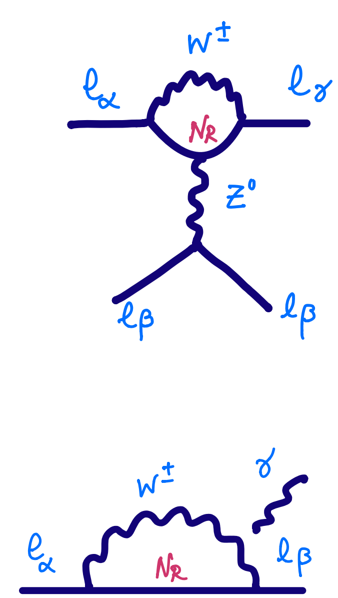
Spectrum & mixings: $m_\nu \approx -v^2 Y_\nu^T M_N^{-1} Y_\nu \qquad U^T \mathcal{M}_\nu^{6\times 6} U = \text{diag}(m_i)$

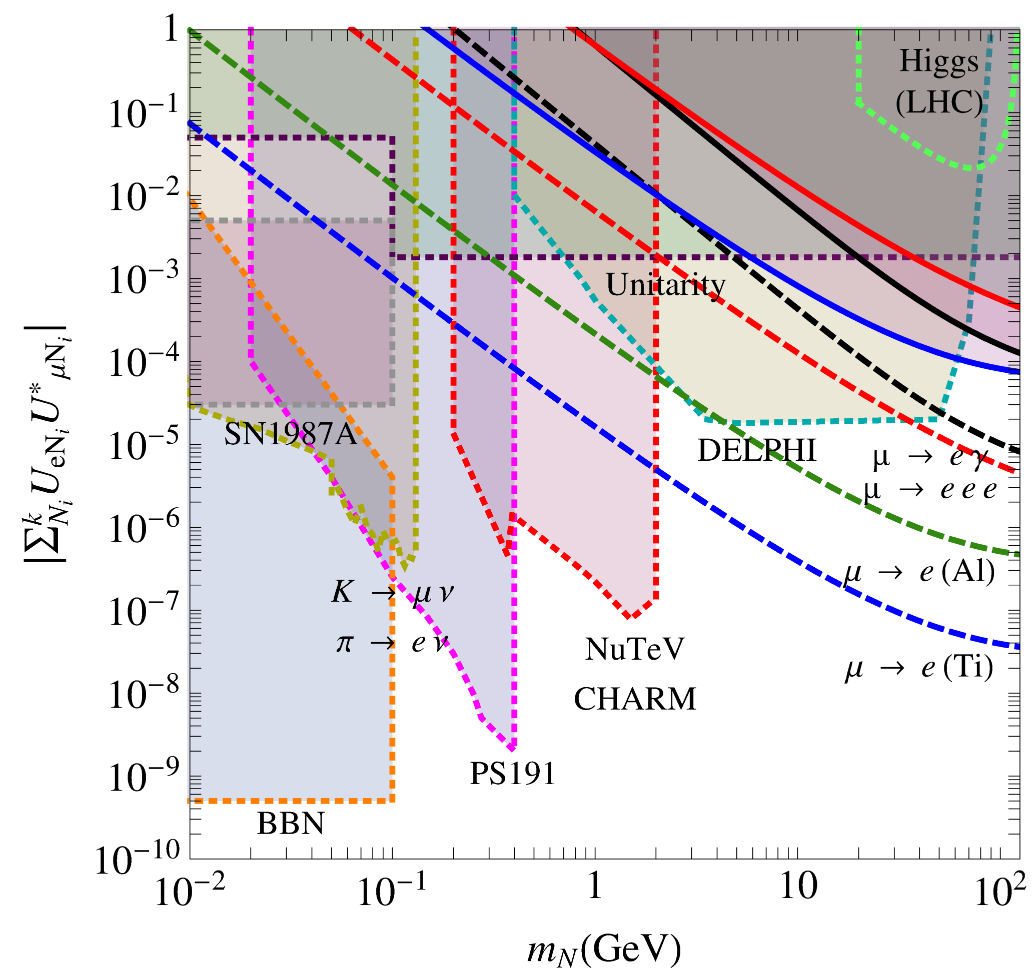
$$U = \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix} \qquad U_{\nu\nu} \approx (1-\varepsilon)\, U_{\text{PMNS}}$$

Heavy states do not decouple ⇒ modified neutral and charged leptonic currents

Rich phenomenology at **high intensities and at colliders**







Alonso, Dehn, Gavela, Hambye [1209.2679]

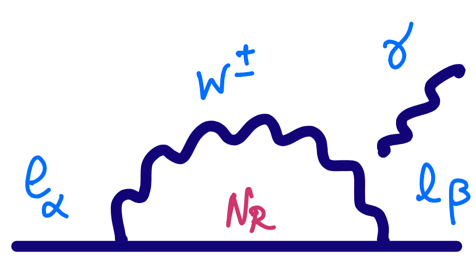
Leptonic cLFV decays: the role of CPV phases

cLFV processes mediated by sterile states at loop-level

Consider "3+2" toy model (addition of 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times5}$, CPV phases)

- **Sizeable contributions** to extensive leptonic **cLFV** observables

- **Interference effects (CPV)** between heavier states can be present!
  - $\Rightarrow$ **Constructive & destructive interference effects** in cLFV decays (leptonic and boson)
  - $\Rightarrow$ Impact to any **interpretation of** experimental **data**



- **Radiative decays:** BR$(\mu \to e\gamma) \propto |G_\gamma^{\mu e}|^2$

$$G_\gamma^{\mu e} = \sum_{i=4,5} \mathcal{U}_{ei}\,\mathcal{U}^*_{\mu i}\, G_\gamma\left(\frac{m^2_{N_i}}{m^2_W}\right)$$

$\mathcal{U}_{\alpha i}(\theta_{\alpha i}, \delta^{\mathrm{D}}_{\alpha i}, \varphi^{\mathrm{M}}_i)$

Assume (for *simplicity & illustrative purposes*): $m_4 \approx m_5$ and $\sin\theta_{\alpha 4} \approx \sin\theta_{\alpha 5} \ll 1$

$$|G_\gamma^{\mu e}|^2 \approx 4\,\sin^2\theta_{e4}\,\sin^2\theta_{\mu 4}\,\cos^2\left(\frac{\delta_{14}+\delta_{25}-\delta_{15}-\delta_{24}}{2}\right) G_\gamma\left(\frac{m^2_{N_i}}{m^2_W}\right)$$

$\Rightarrow$ **Radiative decays: rate** depends **only on Dirac phases**; full cancellation for $\Sigma\delta = \pi$

(Other form factors - more involved expressions, depend also on Majorana phases $\varphi_{4,5}$)

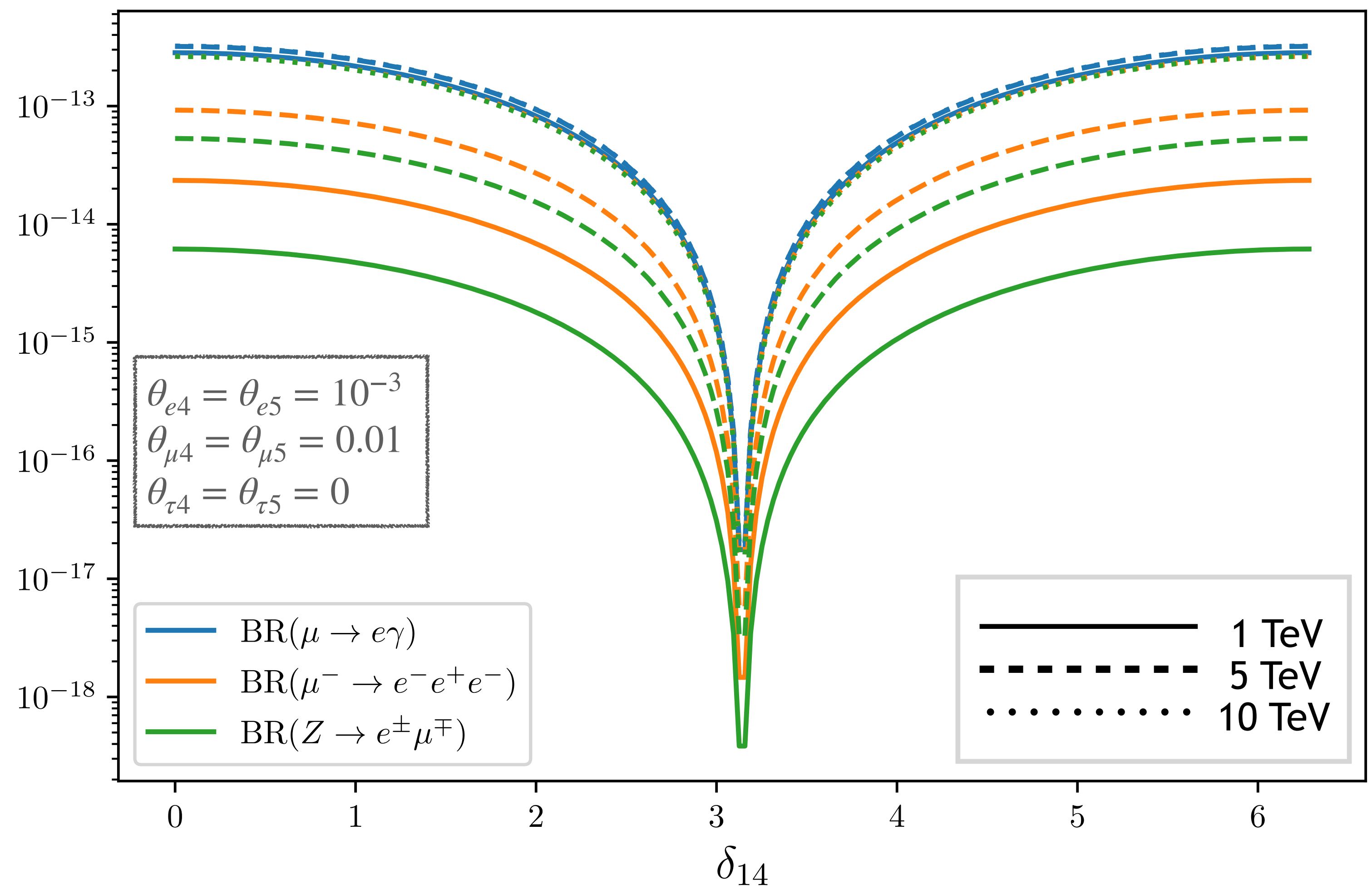
Leptonic cLFV decays: the role of CPV phases

cLFV processes: $\mu - e$ flavour transitions & Dirac phases

Consider "3+2" toy model (addition of 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times5}$, CPV phases)

Simplified approach: $\sin\theta_{\alpha4} = \sin\theta_{\alpha5}$; $m_4 = m_5 = (1, 5, 10)$ TeV

Abada, Kriewald, AMT [2107.06313]



- **Dirac: only $\delta_{14} \neq 0$**
  all other phases (Majorana & remaining Dirac) set to 0

$\Rightarrow$ **Strong cancellation** for

$\delta_{14} = \pi$

in all observables

(similar results for other Dirac phases)

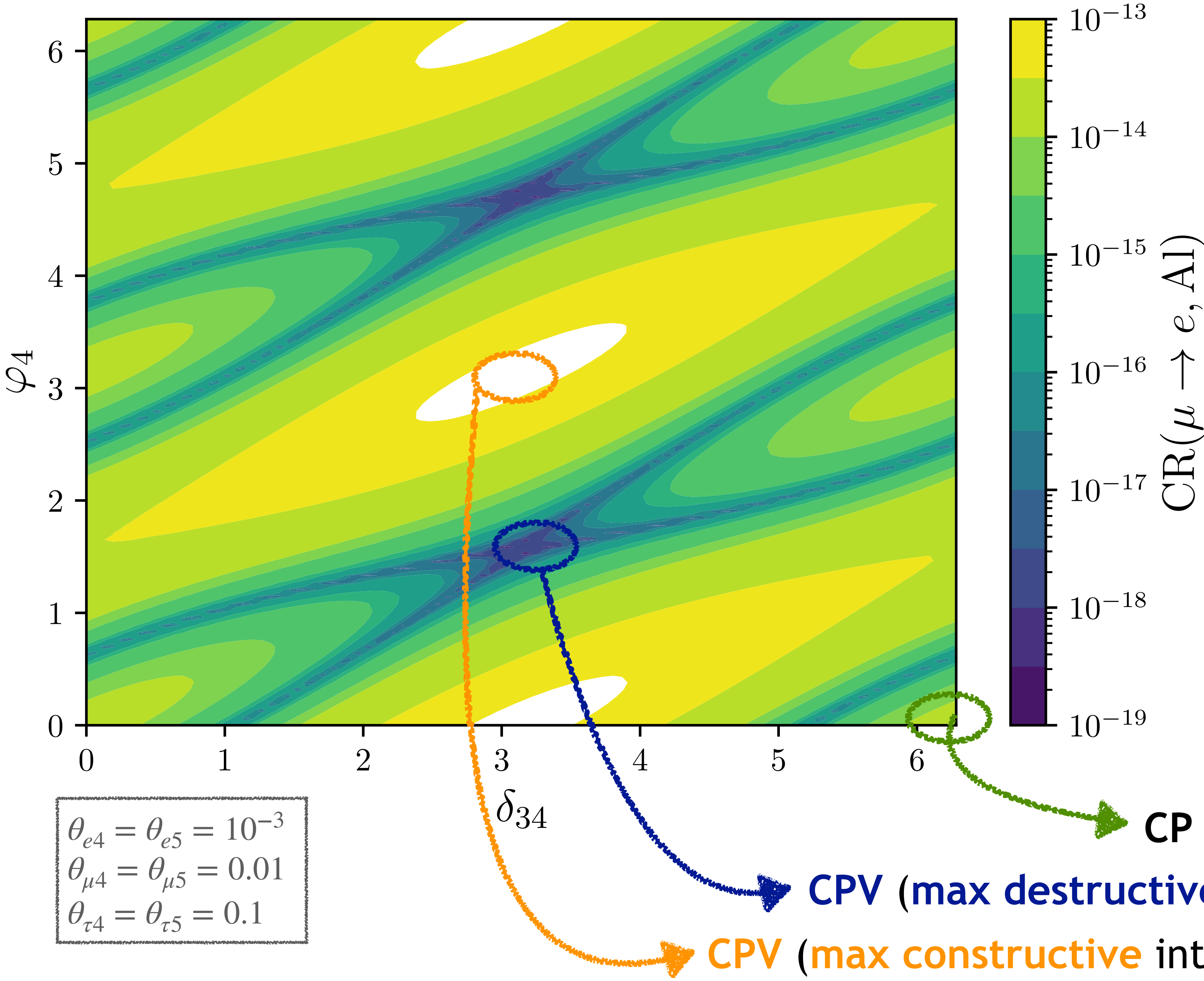
Leptonic cLFV decays: the role of CPV phases

cLFV processes: $\mu - e$ conversion and CPV Dirac / Majorana phases

Consider "3+2" toy model (addition of 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times5}$, CPV phases)

Simplified approach: $\sin\theta_{\alpha4} = \sin\theta_{\alpha5}$; $m_4 = m_5 = 1$ TeV

[Abada, Kriewald, AMT, 2107.06313]



- **Interference effects**
  Both destructive ***AND*** constructive!

  Joint effect of Dirac (δ_{34}) and Majorana (φ_4) CPV phases

  (all other dof's fixed)

  $\Rightarrow$ **From beyond experimental sensitivity...**
  to **within near future reach...**
  and even **already excluded!**

CPV & cLFV: phenomenological impact

CP violating phases do matter in cLFV observables!

- Consider "3+2" toy model (SM + 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times5}$, CPV phases)
- Phenomenological analysis: **experimental constraints** on **TeV-scale HNL extensions**
  - lepton flavour universality,
  - lepton number violation,
  - electroweak precision,
  - **cLFV!**, ...

  and further limits (e.g. η, perturbative unitarity, ...)

  conducting a **thorough survey of parameter space**

  $\rightsquigarrow$ random scans of mixings and phases, grid based, ...

$\Rightarrow$ **Impact for phenomenological studies (predictions) of cLFV observables**

$\Rightarrow$ Impact for **falsifiable scenarios**

$\Rightarrow$ More words of warning for **interpreting future data**

---

cLFV signatures: ratios of observables to **identify mediators** & constrain their **masses!**

But - CP phases (Dirac and/or Majorana) generically present in most **models of ν masses...**

And impact naïve expectations...

Peculiar cLFV patterns... what if CPV & cLFV?

IN2P3 Les deux infinis

cLFV signatures: ratios of observables to **identify mediators** & constrain their **masses!**

But - CP violating phases do matter! *And impact naïve expectations...*

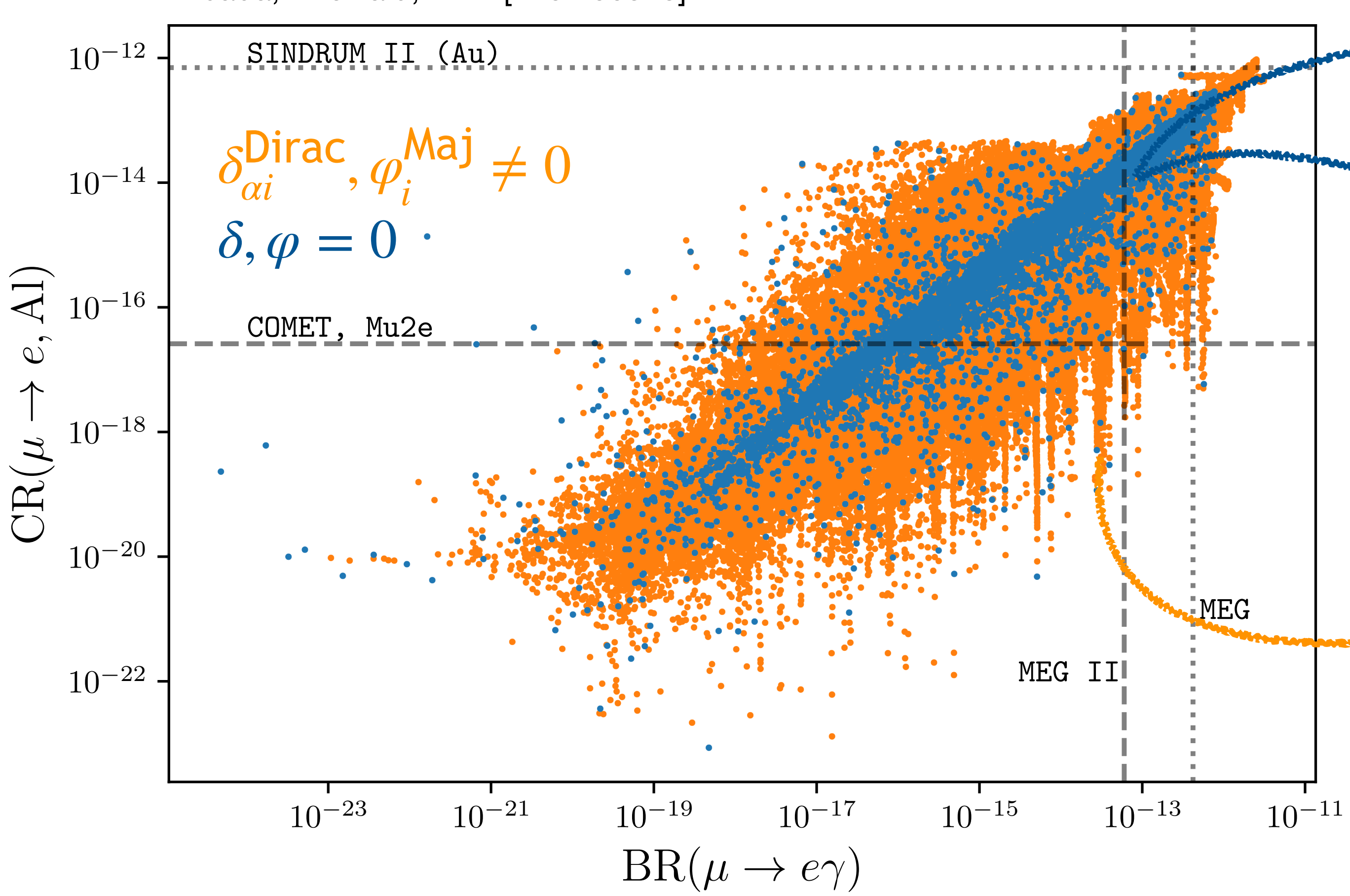
Consider "3+2" toy model (addition of **2 heavy sterile** states; leptonic mixing $\mathscr{U}_{5\times5}$, **CPV** phases)

Impact of **CPV phases** regarding **experimental prospects!**

General overview of parameter space: **all angles & CPV phases** *randomly* (independently) varied

Non-degenerate heavy states (mass around **TeV scale**)

Abada, Kriewald, AMT [2107.06313]



Strong correlation between observables (CP conserving case)

$\Rightarrow$ **Observation** of $\mu \to e\gamma$ *suggests* a *potential observation* of $\mu - e$ **conversion**

Loss of correlation between observables! (CP violating case)

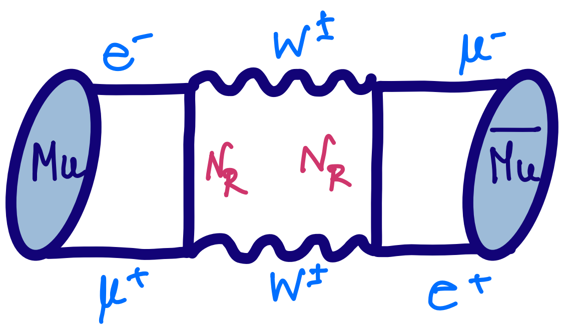
Peculiar cLFV patterns... what if CPV & cLFV?

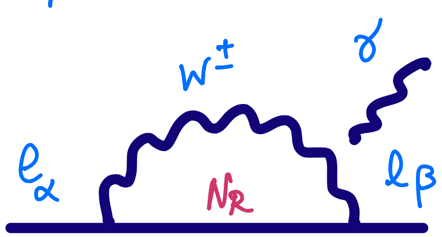
cLFV signatures: ratios of observables to identify mediators & constrain their masses!

But - CP violating phases do matter! *And impact naïve expectations...*

Consider "3+2" toy model (addition of **2 heavy sterile** states; leptonic mixing $\mathscr{U}_{5\times5}$, **CPV** phases)

Observables sensitive to **one unique source of flavour violation** (1-loop level) but with **distinct (dominant) topology**





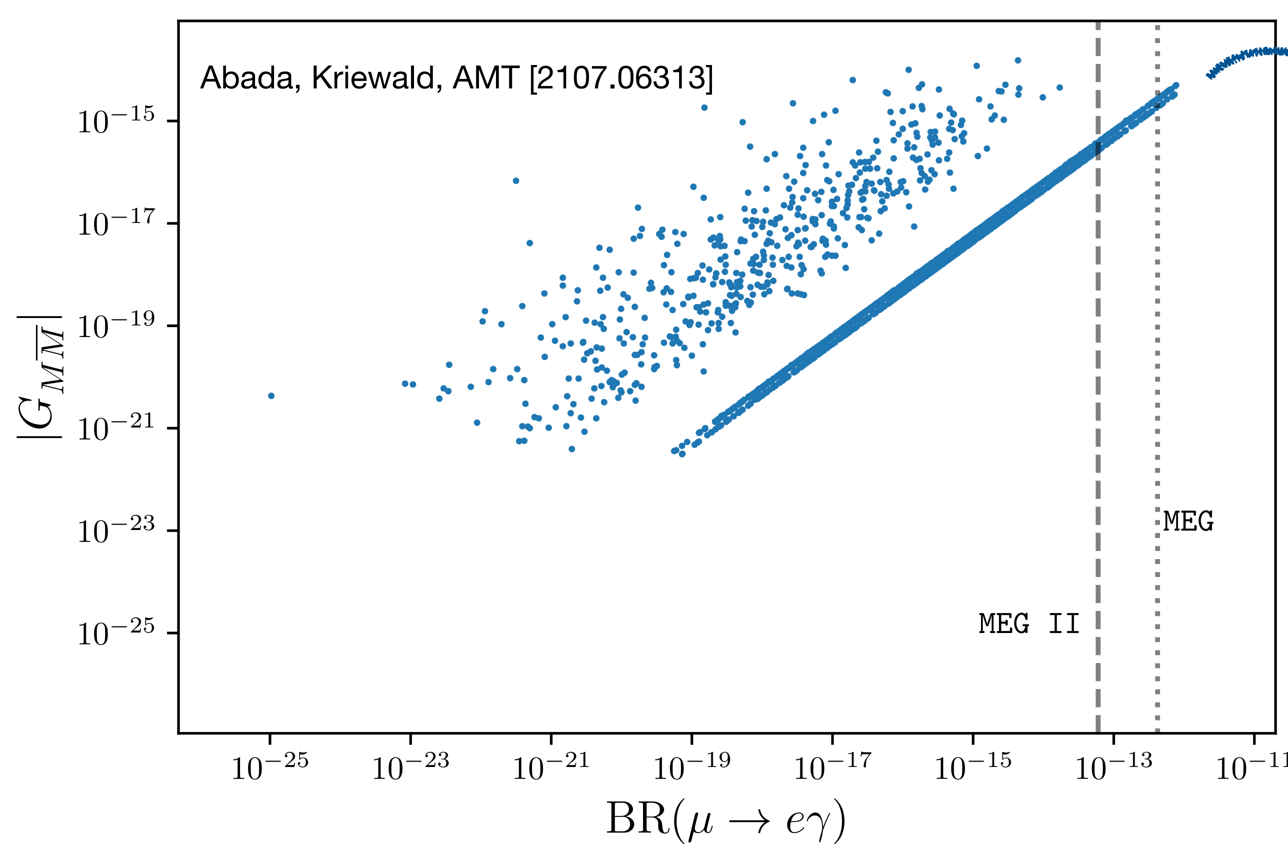
Muonium oscillations ($\mu^+e^- \to \mu^-e^+$): *box diagram*

Radiative muon decays ($\mu \to e\gamma$): *dipole*

} only depend on $\boldsymbol{\theta_{14(5)}}$ and $\boldsymbol{\theta_{24(5)}}$

$m_4 = m_5 = 1$ TeV

- CP conserving



Strong correlation for $\mu \to e\gamma$ and Muonium oscillations!

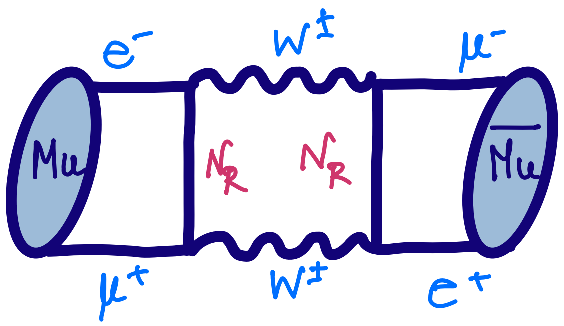
Peculiar cLFV patterns... what if CPV & cLFV?

cLFV signatures: ratios of observables to identify mediators & constrain their masses!

But - CP violating phases do matter! *And impact naïve expectations...*

Consider "3+2" toy model (addition of **2 heavy sterile** states; leptonic mixing $\mathcal{U}_{5\times5}$, **CPV** phases)

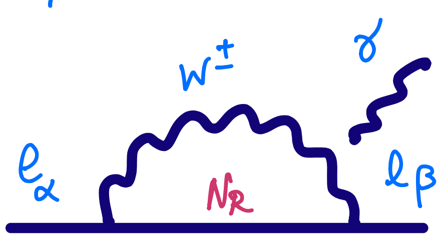
Observables sensitive to **one unique source of flavour violation** (1-loop level) but with **distinct (dominant) topology**



Muonium oscillations ($\mu^+e^- \to \mu^-e^+$): *box diagram*

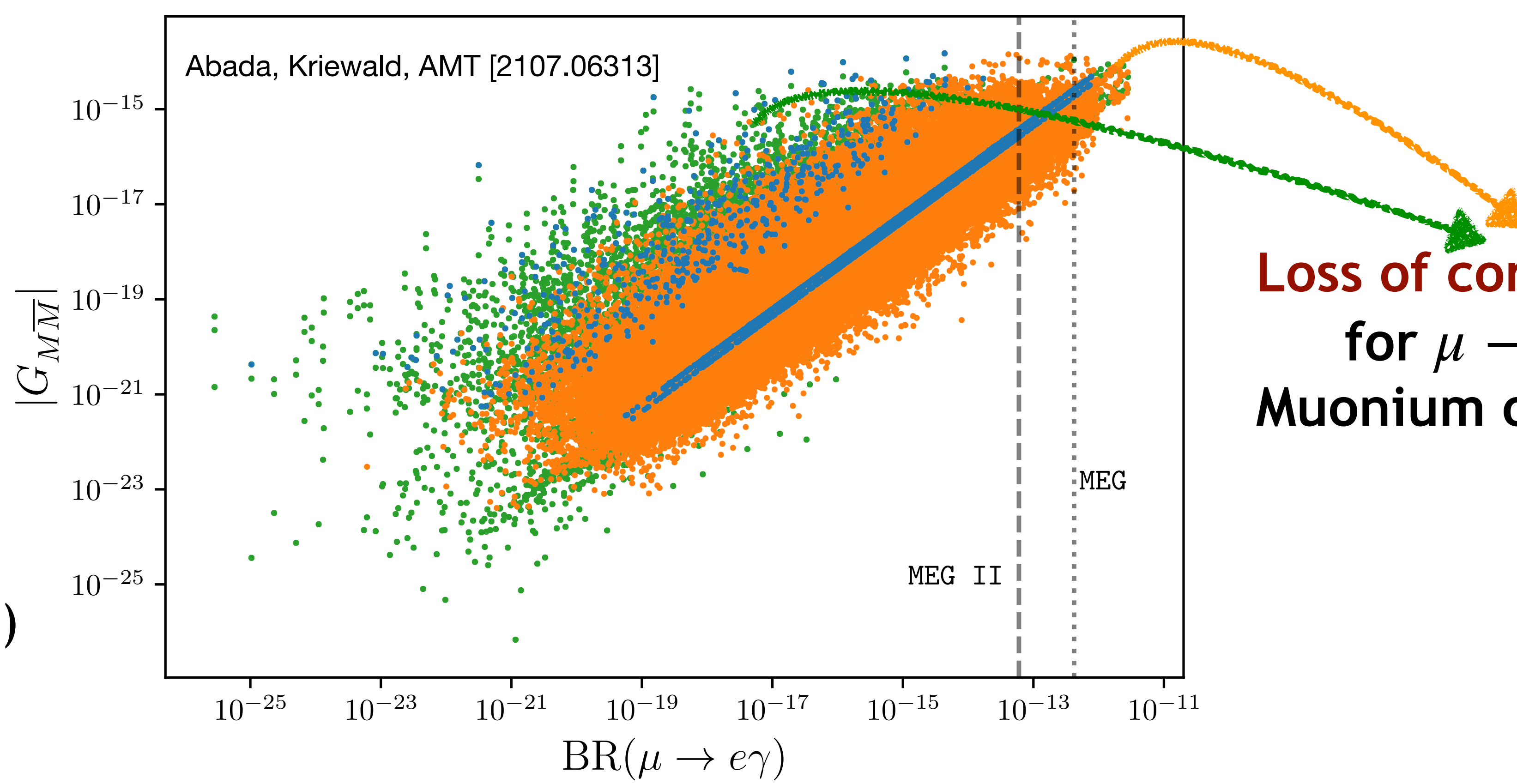
Radiative muon decays ($\mu \to e\gamma$): *dipole*

} only depend on $\boldsymbol{\theta_{14(5)}}$ and $\boldsymbol{\theta_{24(5)}}$



$m_4 = m_5 = 1$ TeV

- CP conserving
- CPV phases (random $\delta_{\alpha4}, \varphi_4$)
- CPV phases (grid $n\pi/4$)



Loss of correlation for $\mu \to e\gamma$ and Muonium oscillations!

Future cLFV data: what if CPV & cLFV?

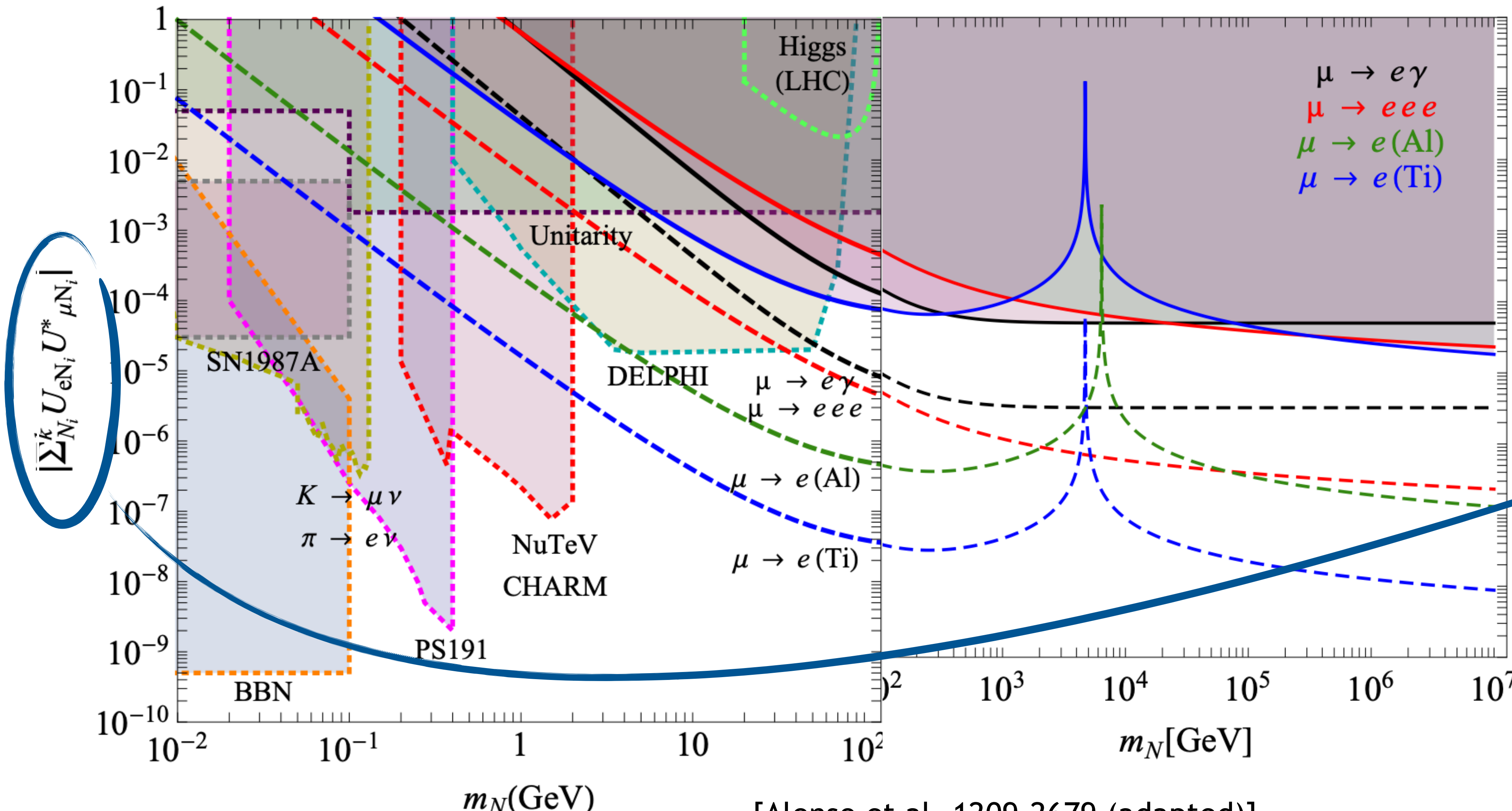
cLFV searches: future data can shed light on underlying NP model !

But - CP violating phases do matter! *And impact naïve theoretical expectations...*

Consider "3+2" toy model (addition of 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times5}$, **CPV** phases)

▷ **Impact** of **CPV phases** regarding **experimental prospects!**

In general, **numerous constraints** on **SM extensions via sterile fermions** from (negative searches for) **flavour violating transitions:**



Bounds on combinations of **active-sterile mixings**

$$\Rightarrow |\sum_i U_{ei} U^*_{\mu i}|(m_N) \lesssim 10^{-n}$$

[Alonso et al, 1209.2679 (adapted)]

Future cLFV data: what if CPV & cLFV?

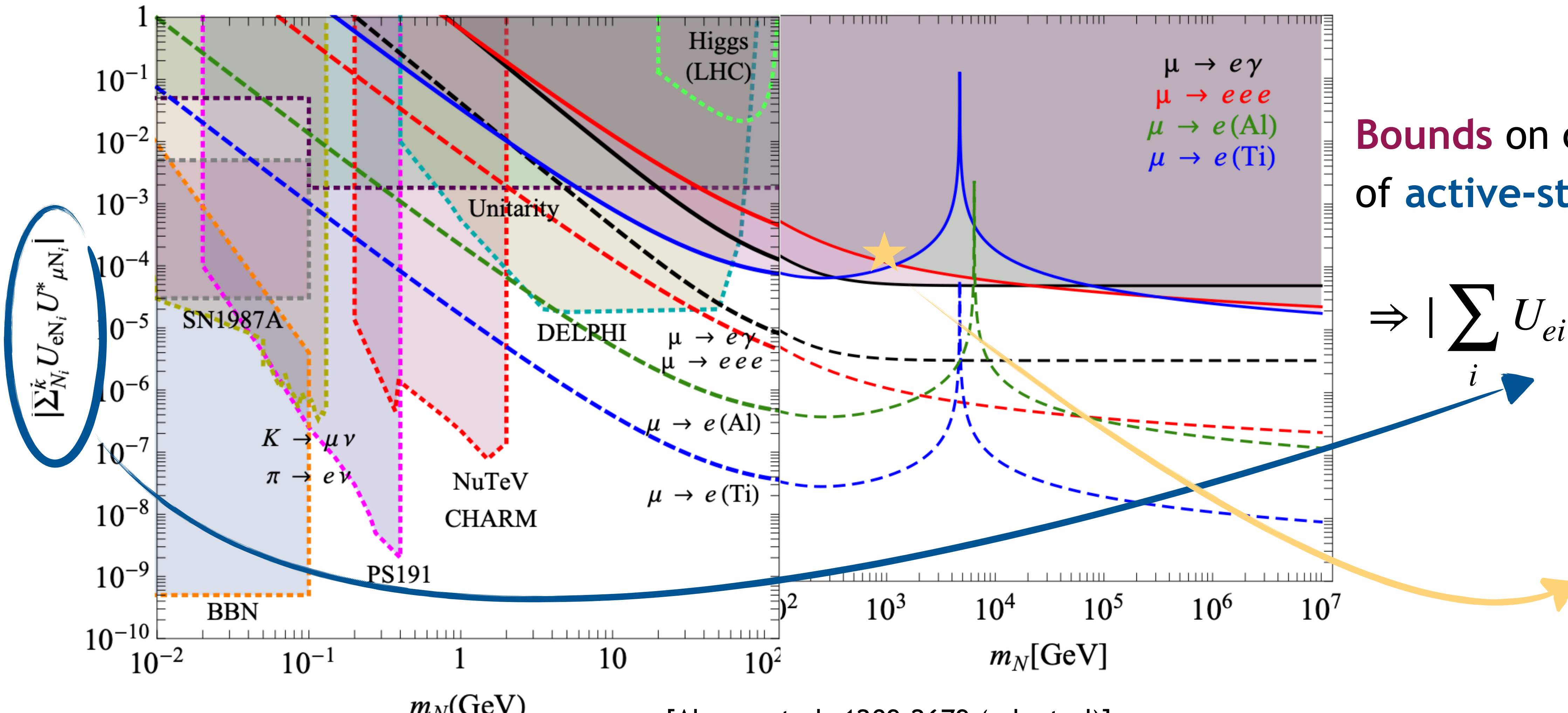
cLFV searches: future data can shed light on underlying NP model !

But - CP violating phases do matter! *And impact naïve theoretical expectations...*

Consider "3+2" toy model (addition of 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times5}$, CPV phases)

▶ **Impact** of **CPV phases** regarding **experimental prospects!**

In general, numerous constraints on **SM extensions via** sterile fermions from (negative searches for) flavour violating transitions:



[Alonso et al, 1209.2679 (adapted)]

Bounds on combinations of **active-sterile mixings**

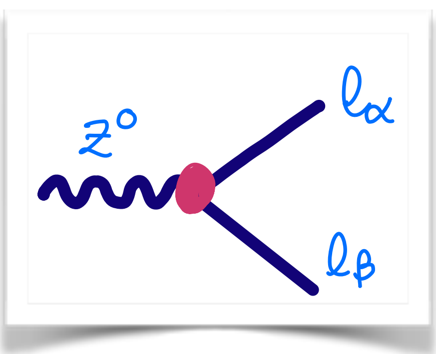
$$\Rightarrow |\sum_i U_{ei} U^*_{\mu i}|(m_N) \lesssim 10^{-n}$$

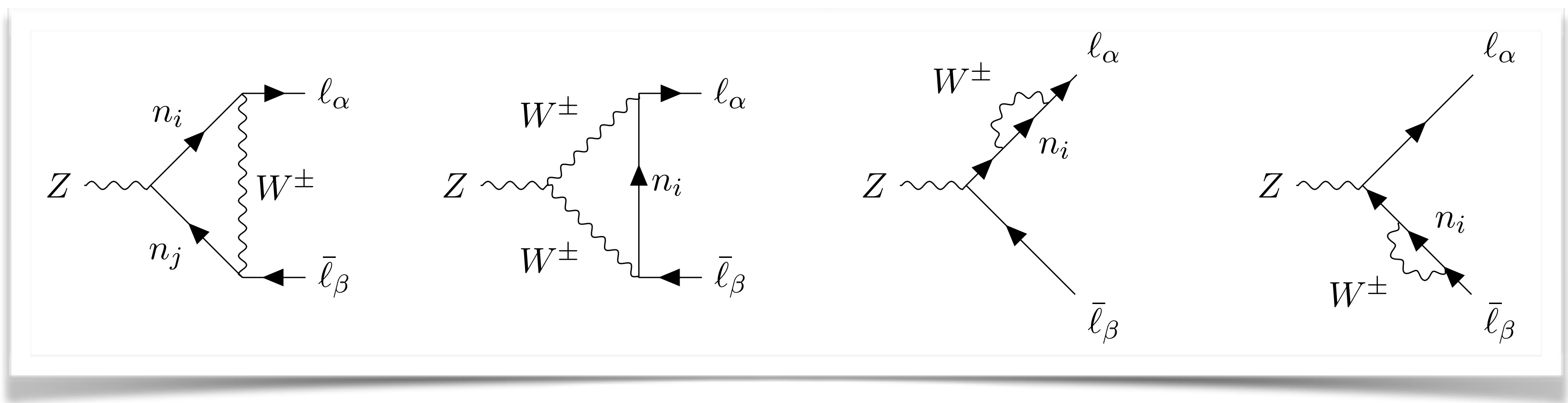
cLFV boson decays and heavy neutral leptons

cLFV processes: $H \to \ell_\alpha \ell_\beta$, $Z \to \ell_\alpha \ell_\beta$ and CPV Dirac / Majorana phases

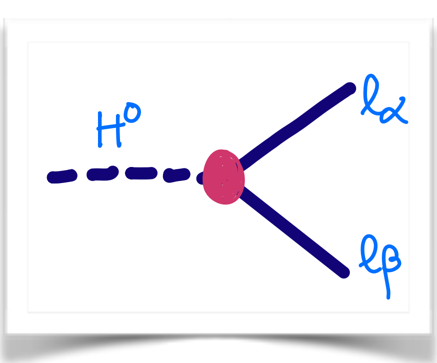
Scalar and vector boson cLFV decays sensitive to additional heavy sterile states

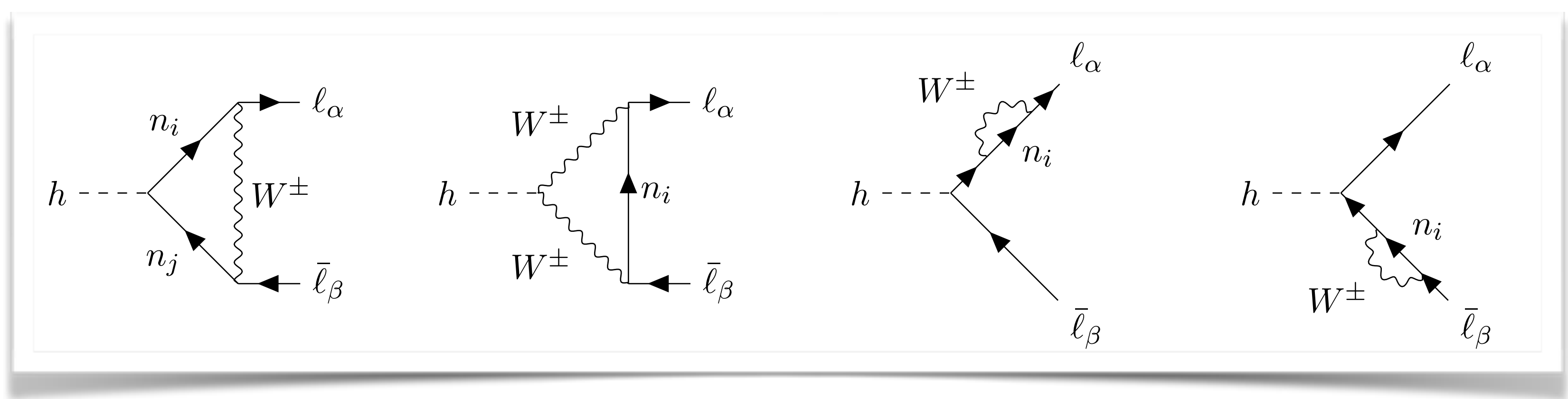
See also extensive contributions by several groups: for instance
[9403398], …, [1405.4300], [1412.6322], [1503.04159], [1607.05257], [1612.0929], [1703.00896], [1710.02510], [1807.01698], [1912.13327], [2005.11234], … *among many others!*





[Abada, Kriewald, Pinsard, Rosauro, AMT, 2207.10109]





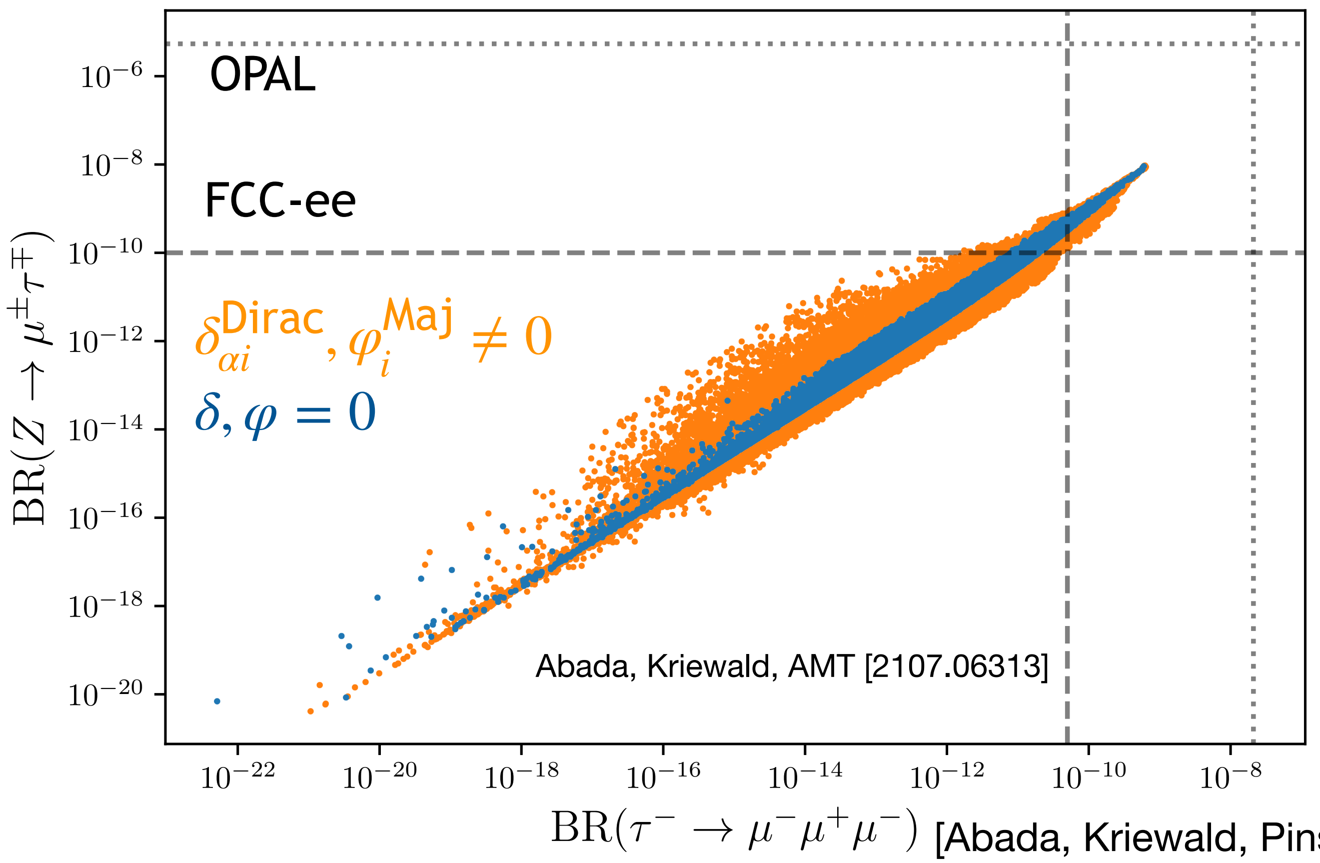
Full computation (no approximation) of cLFV widths;
both unitary & Feynman gauges for complete models with HNL (type I seesaw, ISS, …)

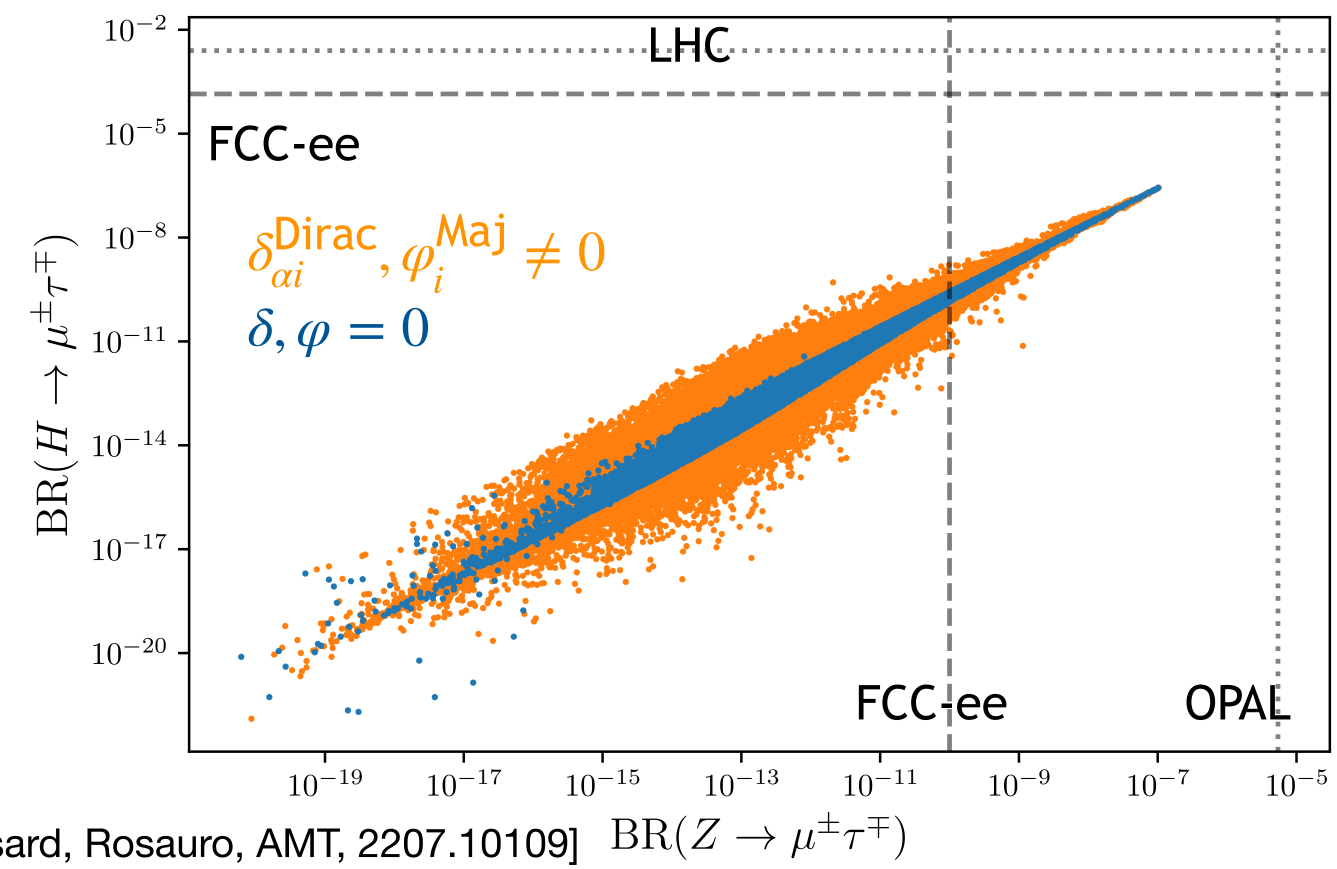
cLFV boson decays and heavy neutral leptons

cLFV processes: $H \to \ell_\alpha \ell_\beta$, $Z \to \ell_\alpha \ell_\beta$ and CPV Dirac / Majorana phases

Consider "3+2" toy model (addition of 2 heavy sterile states; leptonic mixing $\mathcal{U}_{5\times5}$, CPV phases)

All angles & CPV phases *randomly* (independently) varied; **non-degenerate** heavy states **(TeV)**





[Abada, Kriewald, Pinsard, Rosauro, AMT, 2207.10109]

⇒ **Important contributions of sterile fermions to cLFV Higgs and Z decays!**

($H \to \mu\tau$ most promising, but still beyond "observation", even FCC-ee...)

⇒ **Clear effect of Majorana and Dirac phases on decay rates:**

Constructive and destructive interferences

Milder **loss of correlation** with respect to CP conserving case than **cLFV leptonic decays**