Hadronic Vacuum Polarization on lattice

Z. Fodor

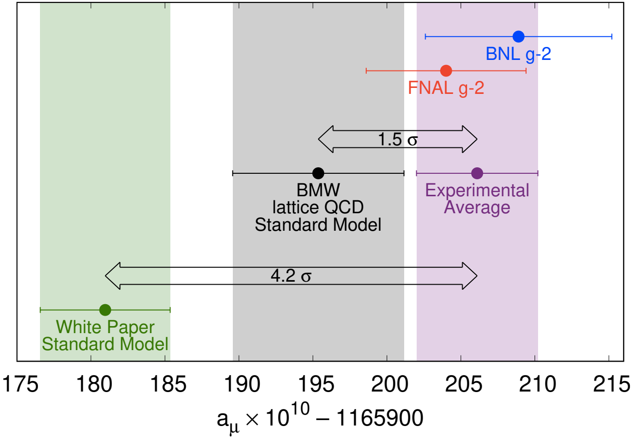
Penn State, Univ. Wuppertal, FZ Juelich, Univ. Budapest, UCSD

Budapest–Marseille–Wuppertal collaboration (BMW)

Borsanyi, Fodor, Guenther, Hoelbling, Katz, Lellouch, Lippert, Miura, Parato, Szabo, Stokes, Toth, Torok, Varnhorst

Venice, Italy, May 30, 2023

Tensions in $(g-2)_\mu$: take-home message



[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

[Muon g-2 coll., Phys. Rev. Lett. 126, 141801 (2021)]

Outline

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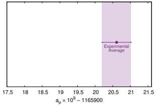
1. 
Experimental Average
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$a_\mu \times 10^9 - 1165900$

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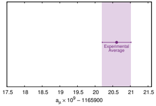
1. 

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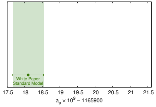
2. 

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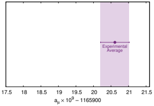
1. 
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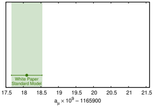
2. 
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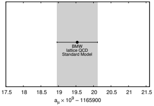
3. 
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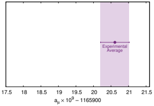
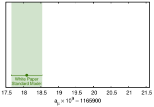
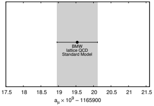
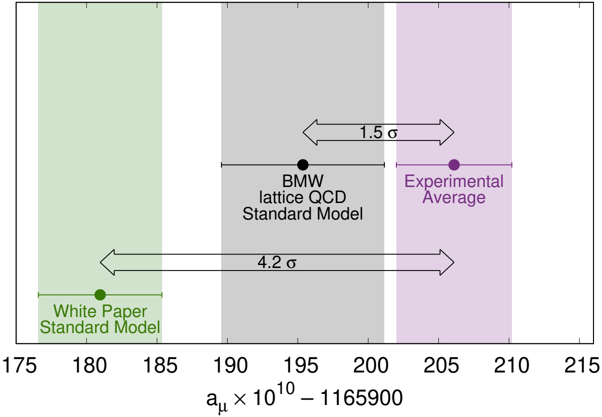
1. 

2. 

3. 

4. Summary



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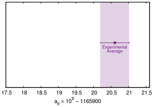
1. 

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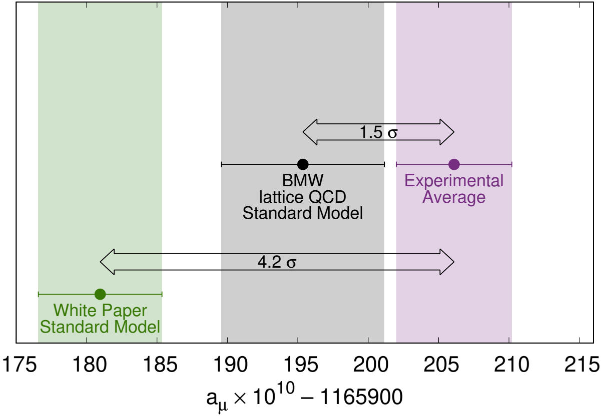

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Experimental result

- Newly announced result at Fermilab

$$a_\mu(\mathrm{FNAL}) = 11\,659\,204.0(5.4)\cdot 10^{-10} \quad (0.46\,\mathrm{ppm})$$

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- Target uncertainty: (1.6)

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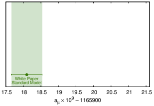
2. 
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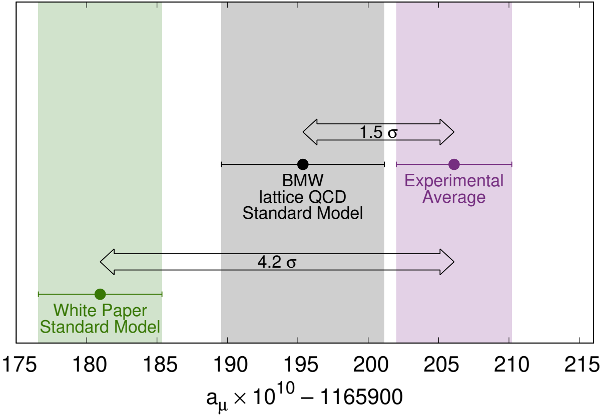
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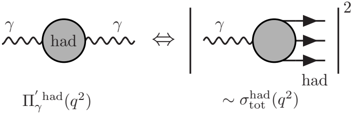
HVP from R-ratio

- Optical theorem



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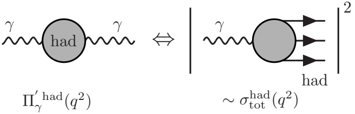

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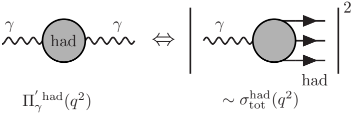

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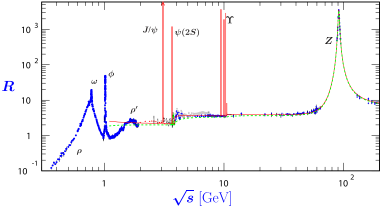

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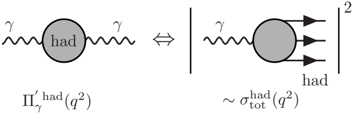

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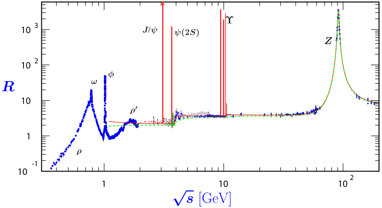

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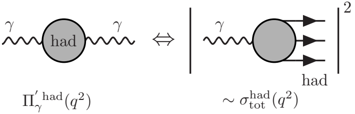

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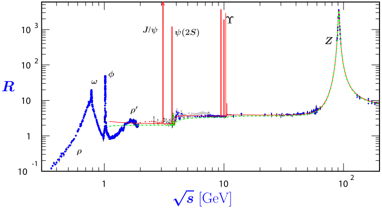

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Systematic uncertainty: ≈4 times larger than the statistical error (e.g. Davier et al.)

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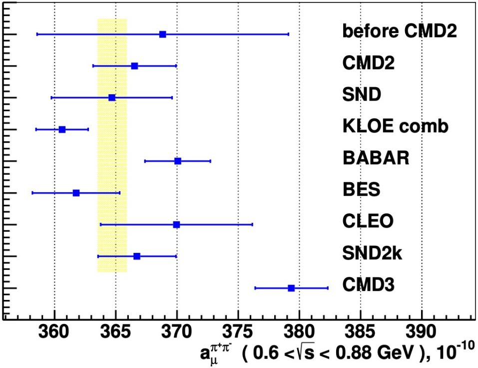

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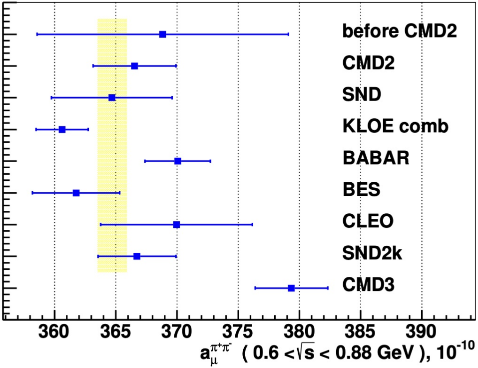

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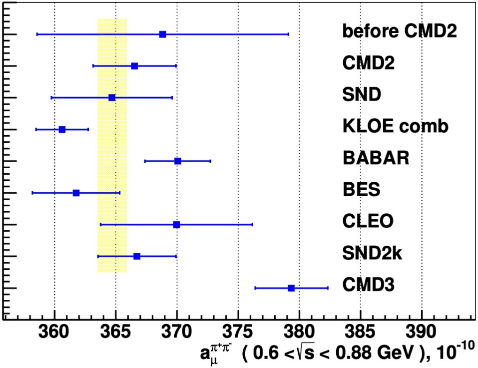

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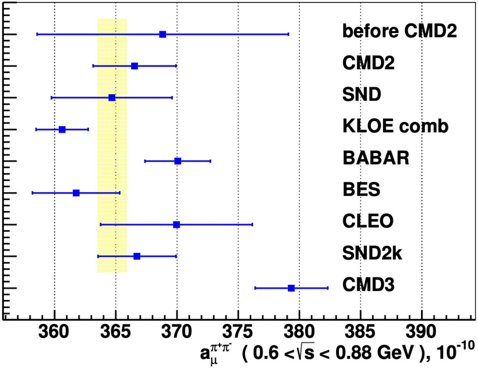

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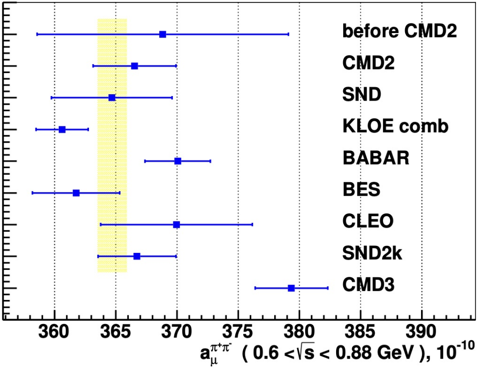

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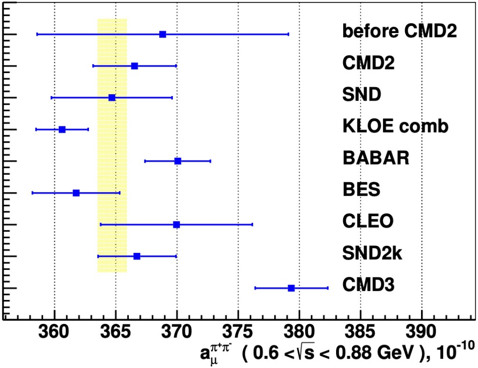

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White Paper must further inflate errors: less chance for new physics?

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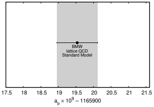
3. 
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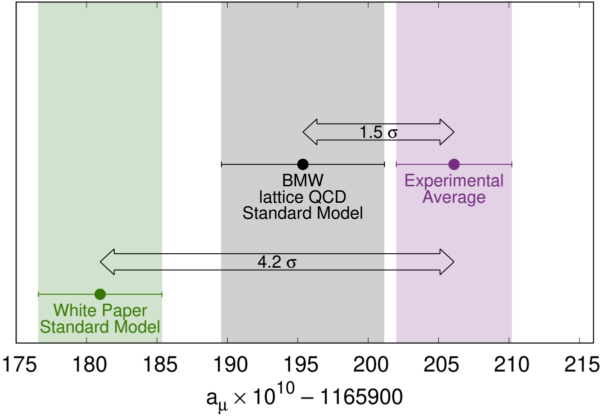
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$a_\mu^{\text{LO-HVP}}$ from lattice QCD Nature 593 (2021) 7857, 51

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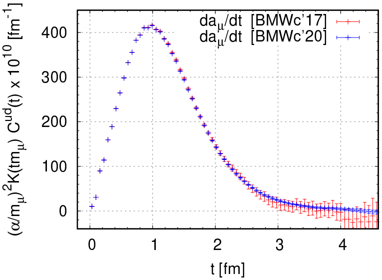
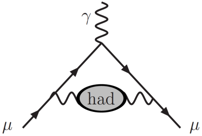
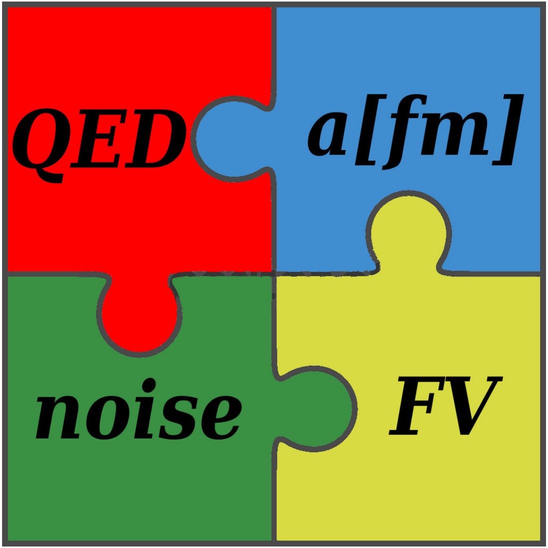
$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt\, K(t)\, C(t)$$



$K(t)$ describes the leptonic part of diagram



New challenges



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2. For separation of isospin breaking effects: w_0 scale setting
   - Moderate m_q dependence
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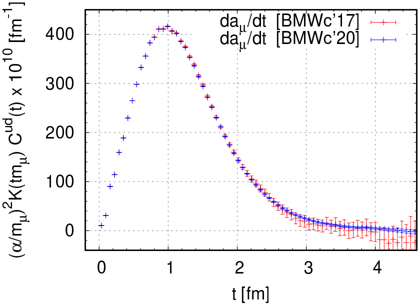
   - No experimental value
     $\longrightarrow$ Determine value of w_0 from $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70]\ \text{fm}$$

Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances



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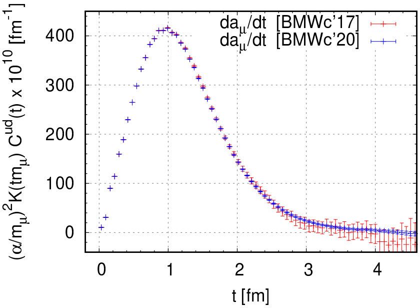

- Low Mode Averaging: use exact (all2all) quark propagator in IR and stochastic in UV
- decrease noise by replacing $C(t)$ by upper/lower bounds above t_c

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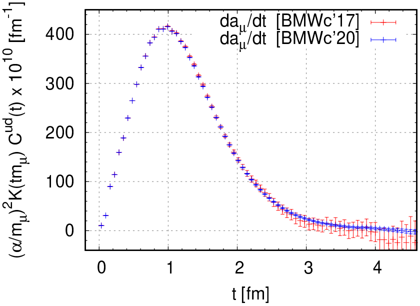

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$\longrightarrow$ few permil level accuracy on each ensemble

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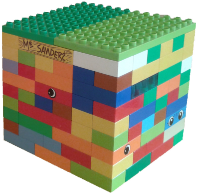

$L_{\text{big}} = 10.752\,\text{fm}$

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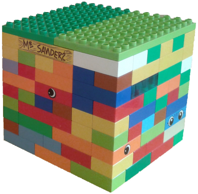

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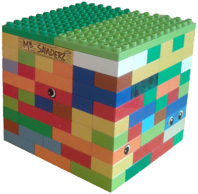
$L_{\mathrm{ref}} = 6.272\,\mathrm{fm}$   $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$

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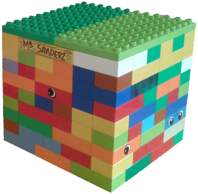

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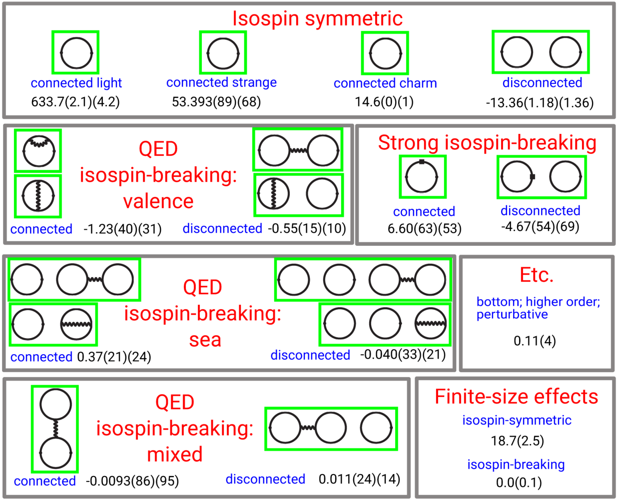
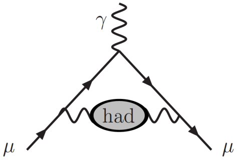
Isospin breaking effects

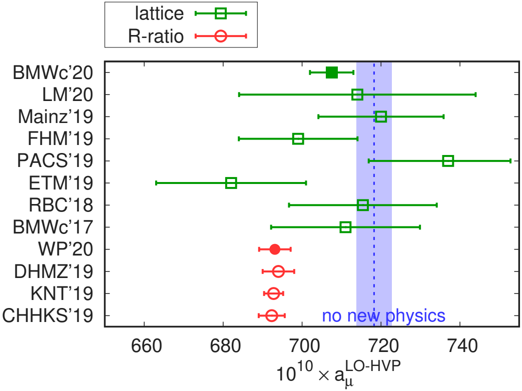
- Include leading order IB effects: $\mathcal{O}(e^2)$, $\mathcal{O}(\delta m)$



$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

Final result for LO-HVP (hadronic vacuum polarization)





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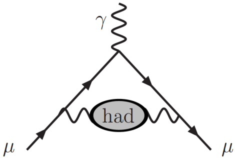

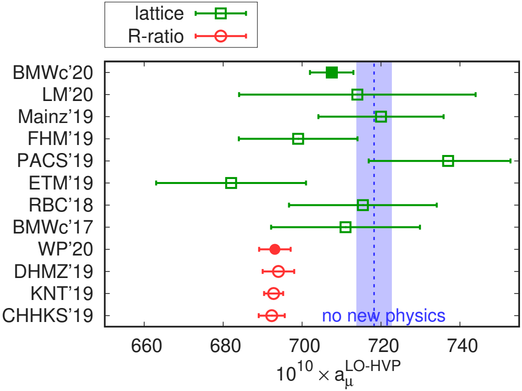

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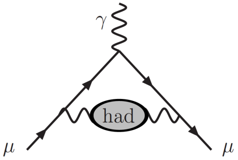

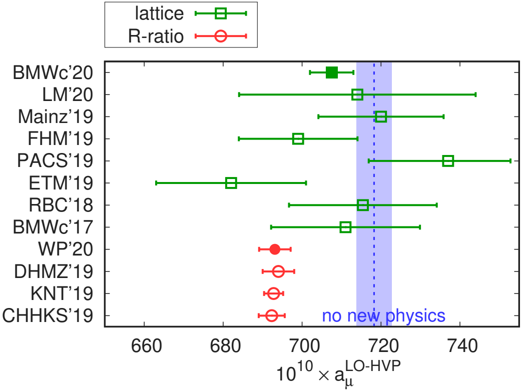

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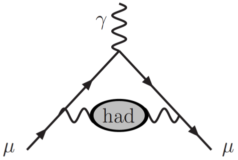

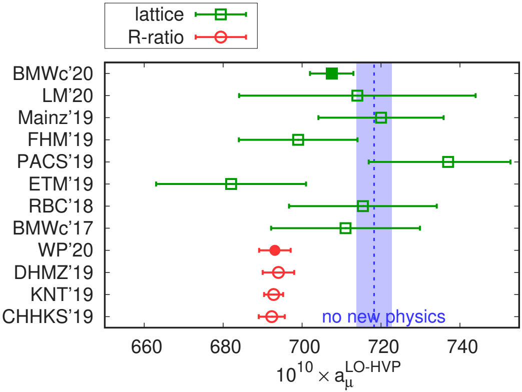

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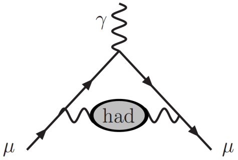

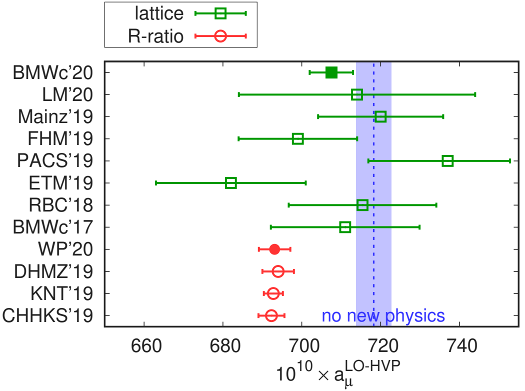

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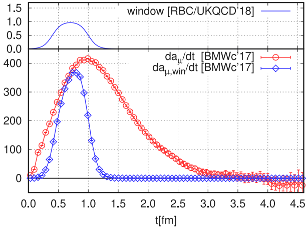
[RBC/UKQCD'18]



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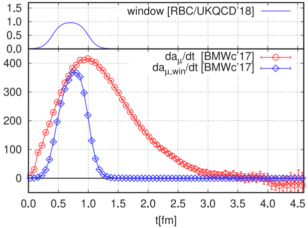

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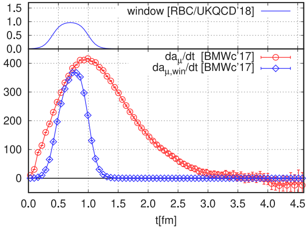

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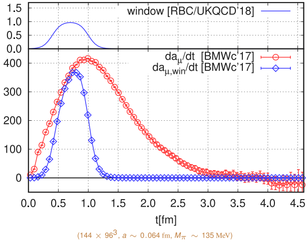

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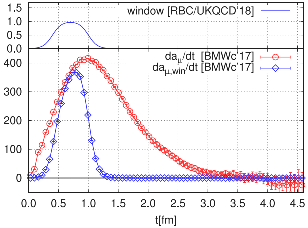

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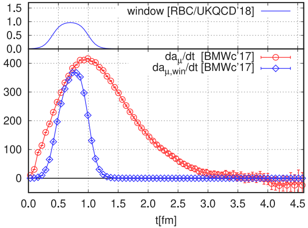

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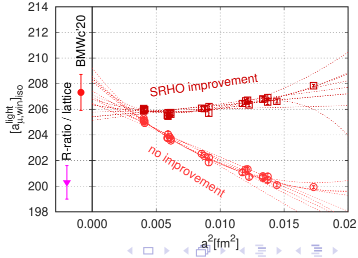
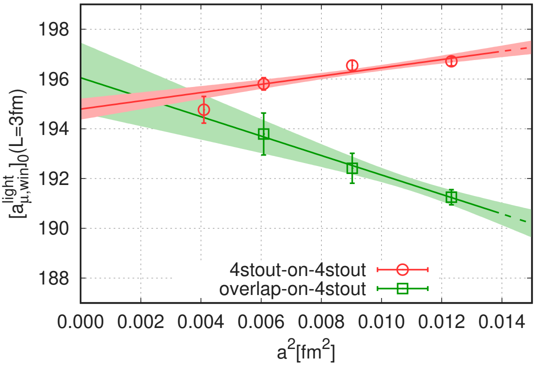


Crosscheck – overlap



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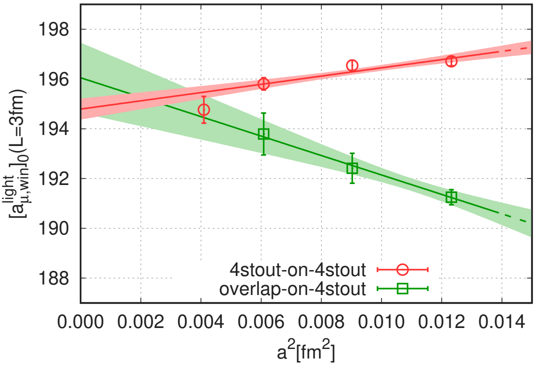
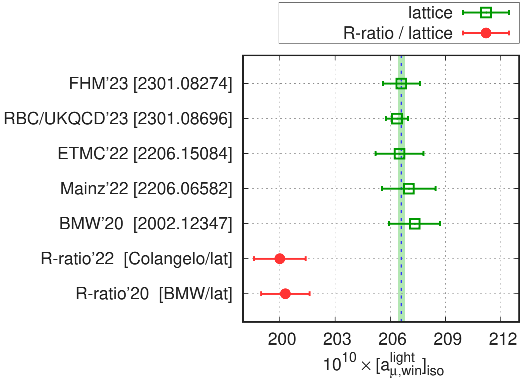


- compute $a_{\mu,\text{win}}$ with overlap valence
- local current instead of conserved $\longrightarrow$ had to compute Z_V
- cont. limit in $L = 3$ fm box consistent with staggered valence

Tension in the window observables



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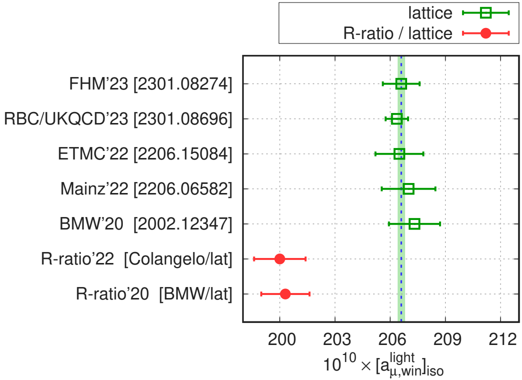

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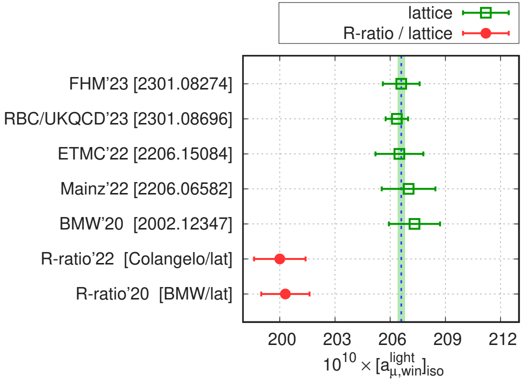

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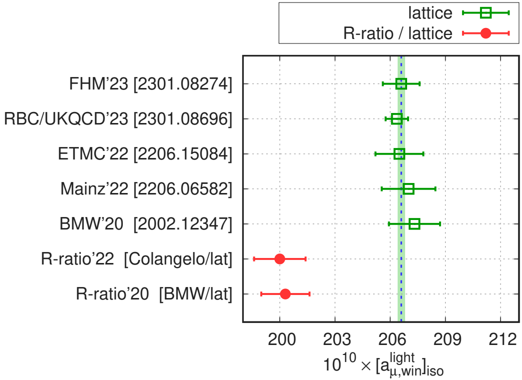

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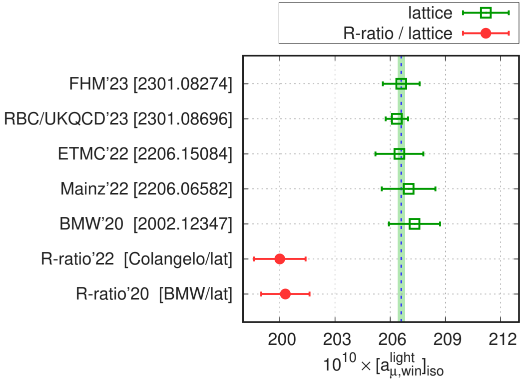

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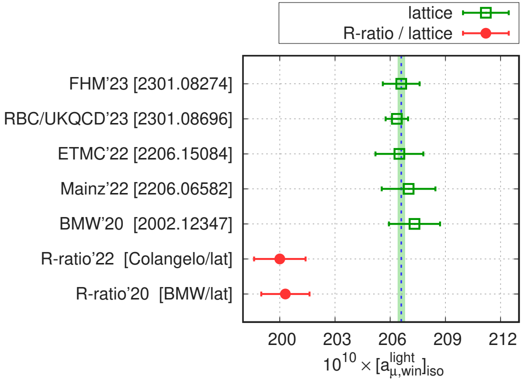

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either new physics
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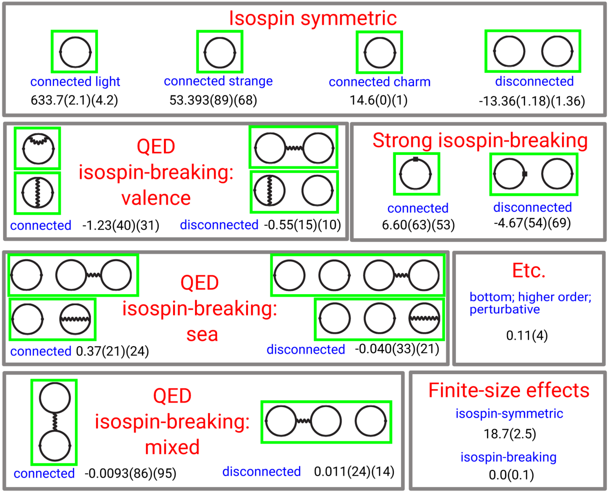
Outline

5. Summary

Final result



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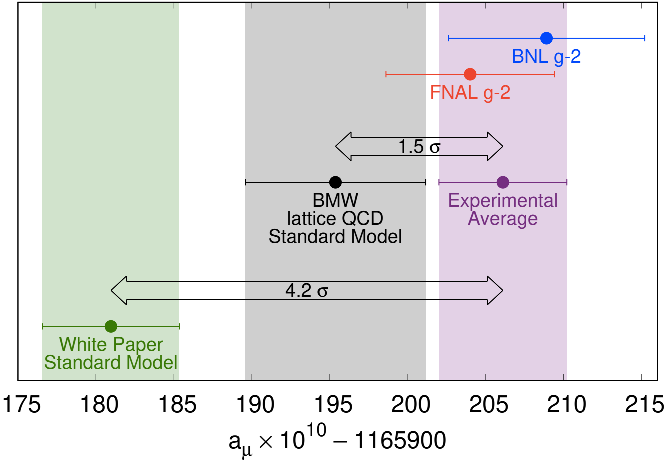
Tension: take-home message #1 full g-2

Systematic/statistical error ratios: lattice ≈2; R-ratio ≈4



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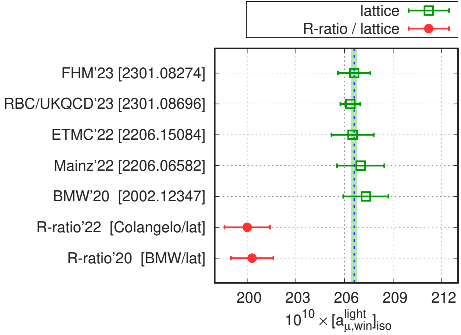

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