Theory overview of muon g-2 and EDM

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Plan of the talk

1. **The high-intensity frontier**
   - Experimental stutus/prospects
2. **“Old muon g-2 puzzle” (pre BMW 2021)**
   - Possible new physics interpretations
3. **Leptonic g-2, EDMs & LFV interrelationship**
   - Model independent considerations
4. **“New muon g-2 puzzle” (post BMW 2021)**
   - Possible new physics interpretations
5. **Conclusions and future prospects**

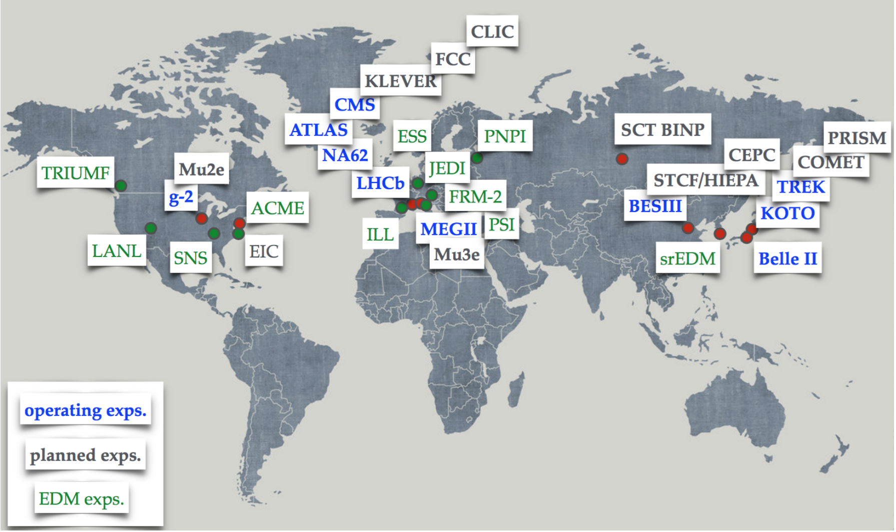
Where to look for New Physics at low-energy?

- **Processes very suppressed or even forbidden in the SM**
  - LFV processes ($\mu \to e\gamma$, $\mu \to e$ in N, $\tau \to \mu\gamma$, $\tau \to 3\mu$,...)
  - CPV effects in the leptonic (e, μ) and neutron EDMs
  - FCNC & CPV in $B_{s,d}$ & D decay/mixing amplitudes

- **Processes predicted with high precision in the SM**
  - EWPO as $(g-2)_\mu$: $\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (2.51 \pm 0.59) \times 10^{-9}$ (4.2σ discrepancy!)
  - LFUV in $M \to \ell\nu$ (with $M = \pi, K, B$), $B \to D^{(*)}\ell\nu$, $B \to K\ell\ell'$, τ and Z decays

- **High-intensity frontier: A collective effort to determine the NP symmetries**

Experimental status



Experimental status

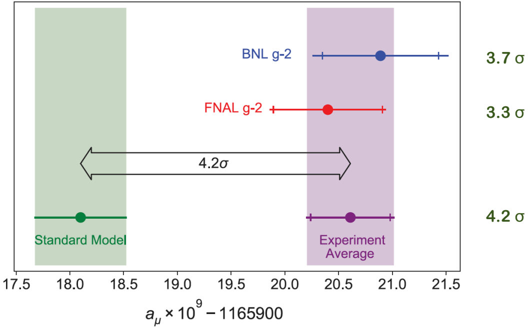
Process	Present	Experiment	Future	Experiment
$\mu \to e\gamma$	4.2×10^{-13}	MEG	$\approx 6 \times 10^{-14}$	MEG II
$\mu \to 3e$	1.0×10^{-12}	SINDRUM	$\approx 10^{-16}$	Mu3e
μ^- Au $\to e^-$ Au	7.0×10^{-13}	SINDRUM II	?	
μ^- Ti $\to e^-$ Ti	4.3×10^{-12}	SINDRUM II	?	
μ^- Al $\to e^-$ Al	$-$		$\approx 10^{-16}$	COMET, MU2e
$\tau \to e\gamma$	3.3×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to \mu\gamma$	4.4×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to 3e$	2.7×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$\tau \to 3\mu$	2.1×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$d_e(\mathrm{e\ cm})$	1.1×10^{-29}	ACME	$\sim 3 \times 10^{-31}$	ACME III
$d_\mu(\mathrm{e\ cm})$	1.8×10^{-19}	Muon (g-2)	$\sim 10^{-22}$	PSI

Table: Present and future experimental sensitivities for relevant low-energy observables.

- So far, only upper bounds. Still excellent prospects for exp. improvements.
- We can expect a NP signal in all above observables below the current bounds.

Experimental status of the muon $g-2$

- **April 7$^{\text{th}}$ 2021: Muon $g-2$ experiment at FNAL confirms BNL!**



$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

- **FNAL aims at 16×10^{-11}. A new FNAL release expected by this summer!** [see Foster's talk]
- **Muon $g-2$ proposal at J-PARC: Phase-1 with similar BNL precision.**

New Physics for the muon $g-2$: at which scale?

- **Δa_μ discrepancy at $\sim 4.2\,\sigma$ level:**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

  - NP is at the weak scale ($\Lambda \approx v$) and weakly coupled to SM particles.*
  - NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.
  - NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.

*Favoured by the *hierarchy problem* and by a WIMP DM candidate but disfavoured by the LEP and LHC bounds (supersymmetry being the most prominent example).

[For a through compilation of models, see Athron, Balazs, Jacob, Kotlarski, Stockinger, Stockinger-Kim, '21.]

$\Lambda \approx v$: SUSY and the muon $(g-2)$

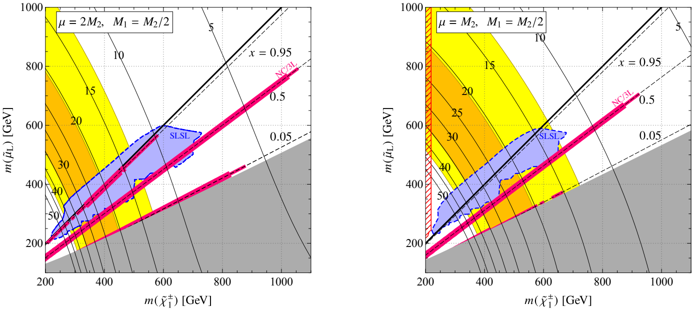
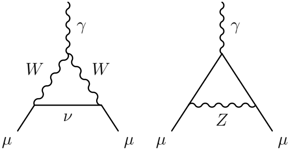
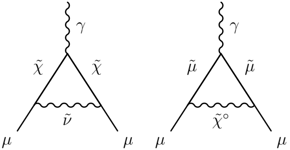


Figure: LHC Run 2 bounds on SUSY scenario for the muon $g-2$ anomaly for $\tan\beta = 40$. Orange (yellow) regions satisfy the muon $g-2$ anomaly at the 1σ (2σ) level [Endo et al., '20].





$$(a_\mu^{\rm SM})_{weak} \approx \frac{g^2 m_\mu^2}{32\pi^2 M_W^2} \approx 2 \times 10^{-9}$$

$$a_\mu^{\rm SUSY} \approx \frac{g^2 m_\mu^2 \tan\beta}{32\pi^2 \tilde{m}^2} \quad \underbrace{\approx 2 \times 10^{-9}}_{\tilde{m} = 500\,\mathrm{GeV}\ \&\ \tan\beta = 40}$$

$\Lambda \lesssim 1$ GeV: Axion-like Particles and the muon $(g-2)$

Axion-like Particle effective Lagrangian

$$\mathcal{L} = e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{c_{\mu\mu}}{2} \frac{\partial^\nu a}{\Lambda} \bar{\mu} \gamma_\nu \gamma_5 \mu$$

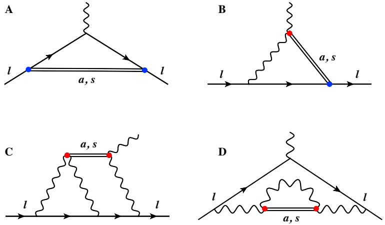


Figure: Contributions of a scalar 's' and a pseudoscalar 'a' ALP to the $(g-2)_\ell$.

[Marciano, Masiero, PP, Passera '16]

[Cornella, P.P., Sumensari '19]

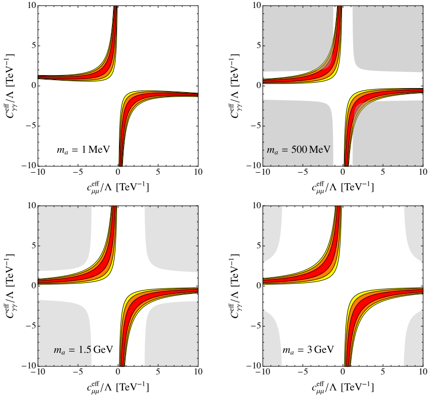


Figure: Δa_μ regions favoured at 68% (red), 95% (orange) and 99% (yellow) CL. Gray regions are excluded by the BaBar search $e^+e^- \to \mu^+\mu^- + \mu^+\mu^-$ [Bauer, Neubert, Thamm, '17]

$$\Delta a_\mu = \frac{m_\mu^2}{\Lambda^2} \left[\frac{12\alpha^3}{\pi} C_{\gamma\gamma}^2 \ln^2 \frac{\Lambda^2}{m_\mu^2} - \frac{(c_{\mu\mu})^2}{16\pi^2} h_1\left(\frac{m_a^2}{m_\mu^2}\right) - \frac{2\alpha}{\pi} c_{\mu\mu} C_{\gamma\gamma} \ln \frac{\Lambda^2}{m_\mu^2} \right]$$

- **SMEFT Lagrangian relevant for Δa_ℓ** [Buttazzo and P.P., '20]

$$\mathcal{L} = \sum_{V=B,W} \frac{C^{\ell}_{eV}}{\Lambda^2} \left(\bar{\ell}_L \sigma^{\mu\nu} e_R\right) H V_{\mu\nu} + \sum_{q=c,t} \frac{C^{\ell q}_{T}}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$

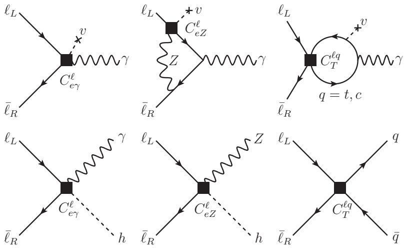


Figure: SMEFT Feynman diagrams for the g-2 (upper row) and scattering processes (lower row): $H = v + h/\sqrt{2}$

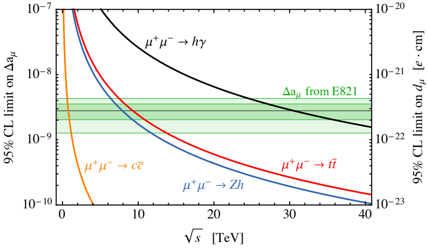


Figure: 95% C.L. reach on Δa_μ vs $\sqrt{s}$ from various processes assuming $\mathcal{L} = \left(\sqrt{s}/10\,\mathrm{TeV}\right)^2 \times 10\,\mathrm{ab}^{-1}$.

$$\Delta a_\mu \sim \frac{m_\mu v}{\Lambda^2} C_{eV,T} \quad \Longleftrightarrow \quad \sigma_{\mu\mu\to f} \sim \frac{s}{\Lambda^4} |C_{eV,T}|^2 \qquad (f = e\gamma, eZ, q\bar{q})$$

- **At high energy $\sigma_{\mu\mu\to f}$ can compete with Δa_μ to test the very same NP!**

On leptonic dipoles: $\ell \to \ell' \gamma$

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R \sigma_{\mu\nu} A_{\ell\ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu\nu} A^\star_{\ell\ell'} \ell_R\right) F^{\mu\nu} \qquad \ell, \ell' = e, \mu, \tau\,,$$

  - **Branching ratios of $\ell \to \ell'\gamma$** [see Teixeira's and Renner's talks]

$$\frac{\mathrm{BR}(\ell \to \ell'\gamma)}{\mathrm{BR}(\ell \to \ell' \nu_\ell \bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right).$$

  - **Δa_ℓ and leptonic EDMs**

$$\Delta a_\ell = 2m_\ell^2\, \mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\, \mathrm{Im}(A_{\ell\ell})\,.$$

  - **"Naive scaling":** a broad class of NP theories contributes to Δa_ℓ and d_ℓ as

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \qquad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}}\,.$$

Model-independent predictions

- $\mathrm{BR}(\ell_i \to \ell_j \gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3 \times 10^{-13} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4 \times 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{\mu\tau}}{10^{-2}}\right)^2$$

- **EDMs vs.** $(g-2)_\mu$

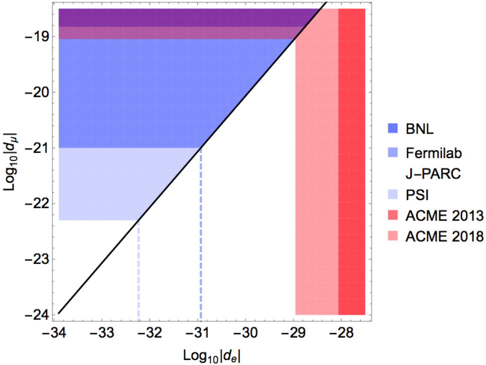
$$d_e \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-29} \left(\frac{\phi_e^{CPV}}{10^{-5}}\right) \; e\,\mathrm{cm}\,,$$

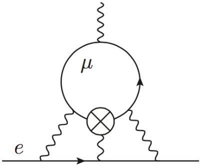
$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 2 \times 10^{-22} \, \phi_\mu^{CPV} \; e\,\mathrm{cm}\,,$$

- **Main messages:**
  - $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$ **requires a nearly flavor and CP conserving NP**
  - **Large effects in the muon EDM** $d_\mu \sim 10^{-22}\, e\,\mathrm{cm}$ **are still allowed!**

[Giudice, P.P., & Passera, '12]

Experimental status of the muon EDM





$$d_\mu \leq 10^{-21}\, e\,\mathrm{cm} \left(\frac{d_e}{10^{-31}\, e\,\mathrm{cm}} \right)$$

[Crivellin, Hoferichter & Schmidt-Wellenburg, ’18, see Schmidt-Wellenburg’s talk]

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22}\, \phi_\mu^{CPV}\; e\,\mathrm{cm}\,,$$

[Giudice, PP & Passera, ’12]

Testing new physics with the electron $g-2$

- **Longstanding muon $g-2$ anomaly**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

- **Testing the muon $g-2$ anomaly through the electron $g-2$**

$$\frac{\Delta a_e}{\Delta a_\mu} = \frac{m_e^2}{m_\mu^2} \qquad \Longleftrightarrow \qquad \Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}$$

  - a_e has never played a role in testing NP effects. From $a_e^{\rm SM}(\alpha) = a_e^{\rm EXP}$, we extract α which was is the most precise value of α up to 2018!
  - The situation has now changed thanks to th. and exp. progresses.
  - α can be extracted from atomic physics and a_e used to perform NP tests!

- **$\Delta a_e \lesssim 10^{-13}$ will bring a_e to play a pivotal role in probing new physics!**

[Giudice, P.P, & Passera, '12]

The Standard Model prediction of the electron $g-2$

- **Status of Δa_e as of 2012**

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM} = -9.2\,(8.1) \times 10^{-13},$$

$$\delta a_e \times 10^{13}: \quad (0.6)_{\rm QED4}, \;\; (0.4)_{\rm QED5}, \;\; (0.2)_{\rm HAD}, \;\; (7.6)_{\delta\alpha}, \;\; (2.8)_{\delta a_e^{\rm EXP}}.$$

  - The errors from QED4 and QED5 will be reduced soon to 0.1×10^{-13} [Kinoshita]
  - We expect a reduction of $\delta a_e^{\rm EXP}$ to a part in 10^{-13} (or better). [Gabrielse]
  - Work is also in progress for a significant reduction of $\delta\alpha$. [Nez]

- **Status of Δa_e as of 2018: 2.4σ discrepancy** [Parker et al., Science, '18]

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM}(\alpha_{\rm Berkeley}) = -8.8\,(3.6) \times 10^{-13}$$

$$\delta a_e \times 10^{13}: \quad (0.1)_{\rm QED5}, \;\; (0.1)_{\rm HAD}, \;\; (2.3)_{\delta\alpha}, \;\; (2.8)_{\delta a_e^{\rm EXP}}.$$

- **Status of Δa_e as of 2020: 1.6σ discrepancy** [Morel et al., Nature, '20]

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM}(\alpha_{\rm LKB2020}) = 4.8\,(3.0) \times 10^{-13}$$

$$\delta a_e \times 10^{13}: \quad (0.1)_{\rm QED5}, \;\; (0.1)_{\rm HAD}, \;\; (0.9)_{\delta\alpha}, \;\; (2.8)_{\delta a_e^{\rm EXP}}.$$

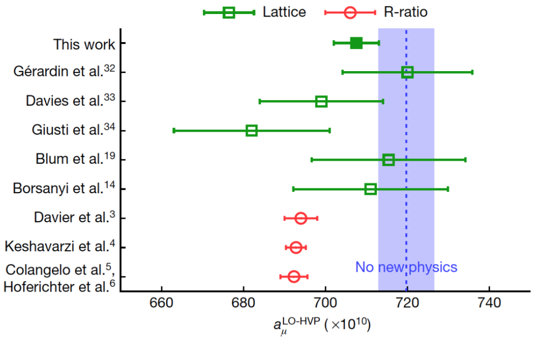
- **Status of Δa_e as of 2022:** $(2.8)_{\delta a_e^{\rm EXP}}$ reduced to $(1.3)_{\delta a_e^{\rm EXP}}$ [Gabrielse et al., '22]

HLO contribution from lattice QCD

- **Great progress also in lattice QCD, where spacetime is modeled as a discrete grid of points. The BMW collaboration reached a 0.8% precision!**

$$a_\mu^{HLO} = 7075(23)_{stat}(50)_{syst}\,[55]_{tot} \times 10^{-11}$$

- **2–2.5σ tension with the "data-driven" evaluations.**



Borsanyi et al (BMWc), Nature 2021

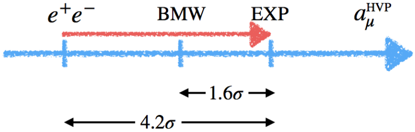
[see Colangelo's and Fodor's talks]

“New muon g-2 puzzle”

$$(a_\mu^{\rm HVP})_{\rm EXP} = a_\mu^{\rm EXP} - a_\mu^{\rm SM,\,rest}$$

$$(a_\mu^{\rm HVP})_{e^+e^-}^{\rm WP20} = 6931(40) \times 10^{-11}$$

$$(a_\mu^{\rm HVP})_{\rm BMW} = 7075(55) \times 10^{-11}$$



“new puzzle”: if BMW is correct, the “old” g-2 discrepancy (4.2σ) would be basically gone

however, this brings in a new tension with e^+e^- data (2.2σ)

Here, NP in $\sigma_{\rm had}(e^+e^- \to \text{hadrons})$ such that [LDL, Masiero, Paradisi, Passera 2112.08312]

1. $(a_\mu^{\rm HVP})_{e^+e^-}^{\rm WP20} \approx (a_\mu^{\rm HVP})_{\rm EXP}$
2. the approximate agreement between BMW and EXP is not spoiled
3. w/o a direct contribution $a_\mu^{\rm NP}$ (i.e. NP not in muons)

Muon g-2 $\rightleftarrows$ $\Delta\alpha$ connection

- Can Δa_μ be due to a missing contribution in $\sigma_{\rm had}$?

[Marciano, Passera, Sirlin 2008 & 2010;
Keshavarzi, Marciano, Passera, Sirlin 2020.
See also Crivellin, Hoferichter, Manzari, Montull 2020;
Malaescu, Schott 2020;
Colangelo, Hoferichter, Stoffer 2020]

→ *a upward shift of $\sigma_{\rm had}$ induces an increase of $\Delta\alpha^{(5)}_{\rm had}(M_Z)$*

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha_{\rm lep}(M_Z) - \Delta\alpha^{(5)}_{\rm had}(M_Z) - \Delta\alpha_{\rm top}(M_Z)}$$

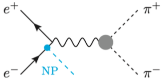
- disfavoured by the EW fit (at about 2σ), if the shift happens at $\sqrt{s} \gtrsim 1\,\text{GeV}$

[Keshavarzi, Marciano, Passera, Sirlin 2020]

→ *selects light NP inducing a sub-GeV modification of $\sigma_{\rm had}$*

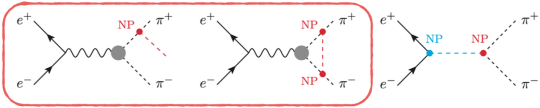
Light New Physics in $\sigma_{\rm had}$

- Light new physics inducing a sub-GeV modification of $\sigma_{\rm had}$ is the only possibility



1. NP coupled only to electrons → severe bounds

[See however
Darmé, Grilli di Cortona, Nardi 2112.09139
NP in Bhabha scattering ? → backup slides]



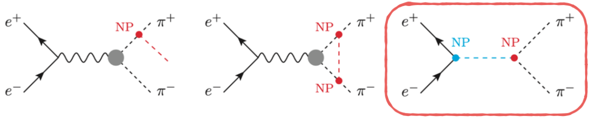
2. NP coupled only to hadrons

FSR effects due to NP should be included into $\sigma_{\rm had}(s)$, not easy to be accounted for…
(depend on exp. cuts and mass of NP)

→ *however, we know that in the QED case*

$$(a_\mu^{\rm HVP})^{\rm FSR}_{e^+e^-} \approx 50 \times 10^{-11} \quad \longleftrightarrow \quad |(a_\mu^{\rm HVP})_{\rm BMW} - (a_\mu^{\rm HVP})^{\rm WP20}_{e^+e^-}| \approx 150 \times 10^{-11}$$

Light New Physics in $\sigma_{\rm had}$

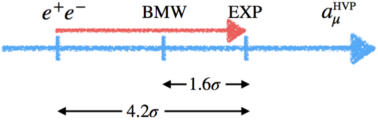


3. NP coupled both to hadrons and electrons

$$(a_\mu^{\rm HVP})_{e^+e^-} = \frac{\alpha}{\pi^2}\int_{m_{\pi^0}^2}^{\infty}\frac{{\rm d}s}{s}K(s)\,{\rm Im}\,\Pi_{\rm had}(s) = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty}{\rm d}s\,K(s)\sigma_{\rm had}(s)$$

$$\sigma_{\rm had} = \sigma_{\rm had}^{\rm SM} + \Delta\sigma_{\rm had}^{\rm NP}$$

$$\boxed{\sigma_{\rm had} - \Delta\sigma_{\rm had}^{\rm NP}}$$



should be "subtracted" by NP,
since NP does not contribute to HVP at the LO
(but it contributes at the LO to the x-section)

→ *a positive sift on* $(a_\mu^{\rm HVP})_{e^+e^-}$ *requires* $\Delta\sigma_{\rm had}^{\rm NP} < 0$ *(negative interference)*

A new light Z' vector boson

- Requirements:

  1. $\sigma_{\rm had}$ modified at $\sqrt{s} \lesssim 1\,\mathrm{GeV}$ (mostly $\pi^+\pi^-$ channel)
  2. A sizeable negative interference with the SM

sub-GeV mediator

couples to u, d quarks (and electrons)

tree-level mediator

couples via a vector current

a light spin-1 mediator with vector couplings to first generation SM fermions

$$\mathcal{L}_{Z'} \supset (g_V^e\, \bar{e}\gamma^\mu e + g_V^q\, \bar{q}\gamma^\mu q) Z'_\mu \qquad q = u, d \qquad m_{Z'} \lesssim 1\ \mathrm{GeV}$$

- It can be shown that (neglecting iso-spin breaking corrections due to NP)

$$\frac{\sigma_{\pi\pi}^{\rm SM+NP}}{\sigma_{\pi\pi}^{\rm SM}} = \left| 1 + \frac{g_V^e (g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}} \right|^2$$

A new light Z' vector boson

1. Semi-leptonic processes

$e^+e^- \to q\bar{q}$ *has been measured with per-cent accuracy at LEP-II*

$$\frac{\sigma_{qq}^{\mathrm{SM+NP}}}{\sigma_{qq}^{\mathrm{SM}}} \approx 1 + 2\frac{g_V^e g_V^q}{e^2 Q_q} \longrightarrow |g_V^e g_V^q| \lesssim 4.6 \cdot 10^{-4} |Q_q| \qquad (\epsilon \lesssim 3.3 \cdot 10^{-3})$$

2. Leptonic processes

- for $m_{Z'} \lesssim 0.3$ GeV ($Z' \to e^+e^-$ is the main decay mode)

$e^+e^- \to \gamma Z'$ *@ BaBar* $\longrightarrow$ $g_V^e \lesssim 2 \cdot 10^{-4}$

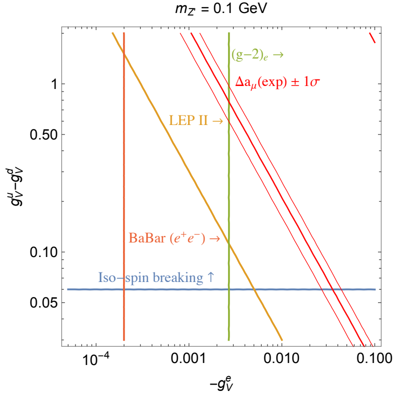
3. Iso-spin breaking observables

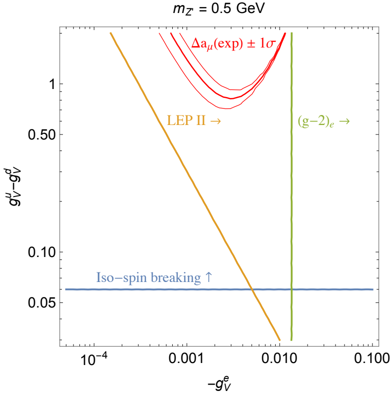
charged vs. neutral pion mass^2 difference $\Delta m^2 = m_{\pi^+}^2 - m_{\pi^0}^2$

$$(\Delta m^2)_{Z'} \sim \frac{(g_V^u - g_V^d)^2}{(4\pi)^2} \Lambda_\chi^2 \qquad (\Lambda_\chi \approx 1\ \mathrm{GeV})$$

$\longrightarrow$ $|g_V^u - g_V^d| \lesssim 0.06$ [Rescaling lattice QCD calculation of Frezzotti et al 2112.01066]

A new light Z' vector boson



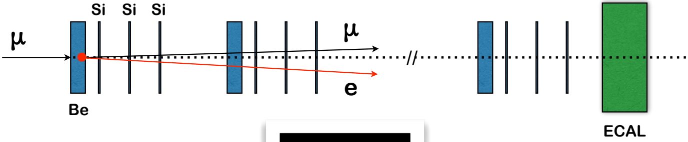


Independent exp. bounds prevent to solve the “new muon g-2 puzzle”!

MUonE: a new determination of $\Delta\alpha_{\mathrm{had}}$

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.

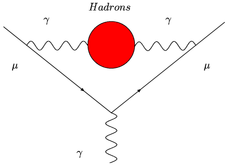


Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

[Courtesy by M. Passera]

- Letter of Intent submitted to CERN SPSC in 2019 [see Pocanik's talk]

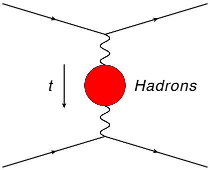
- **The leading hadronic contribution $a_\mu^{\rm HLO}$ computed via the timelike formula:**



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\rm had}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\,\left(s/m_\mu^2\right)}$$

- **Alternatively, simply exchanging the x and s integrations:**



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{\rm had}(t)$ is the hadronic contribution to the running of α in the spacelike region: $a_\mu^{\rm HLO}$ can be extracted from scattering data!

- **The extraction of $\Delta\alpha_{\rm had}$ is not contaminated by NP!** [Masiero, PP, Passera, 2020]

Outlook

- The muon $g-2$ represents the most longstanding hint of New Physics now, thanks to the E989 experiment at FNAL, growing to 4.2σ.
- LQCD results by the BMWc weaken the muon $g-2$ discrepancy to 1.6σ but they are in tension with the EW-fit and $e^+e^- \to hadrons$ experimental data:
  - The MUonE experiment can provide an independent measure of $\Delta\alpha_{\rm had}$ which is not contaminated by new physics effects.
- Both heavy New Physics ($v \lesssim \Lambda \lesssim 100\,\mathrm{TeV}$) and ligh New Physics ($\Lambda \lesssim 1\mathrm{GeV}$) scenarios have the potential to account for the muon $g-2$ anomaly.
  - Different scenarios can be disentangled by dedicated searches at running or future experiments such as Belle II and a high-energy Muon Collider .
- If the muon $g-2$ anomaly will survive, we expect relevant enhancements in leptonic EDMs (especially in the muon EDM) and LFV physics.
- Testing New Physics effects in the electron $g-2$ at the 10^{-13} is not too far! This will bring a_e to play a pivotal role in probing New Physics in the leptonic sector.

Message: an exciting Physics program is in progress at the Intensity Frontier!