

Microwave muonium spectroscopy

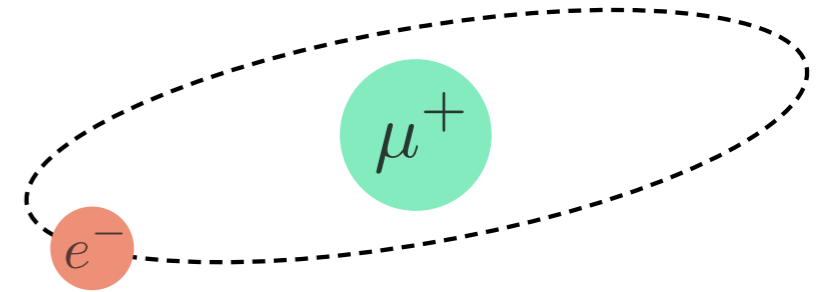


Ryoto Iwai (KEK)

Muon4Future 2023, Venice Italy

Muonium (Mu)

- Light hydrogen isotope but purely leptonic
- Good probe in precision physics
 - Theory: interaction precisely predicted by QED
 - Experiment: moderate mass ($m_\mu = 105 \text{ MeV}$, $\tau_\mu = 2.2 \text{ }\mu\text{s}$)

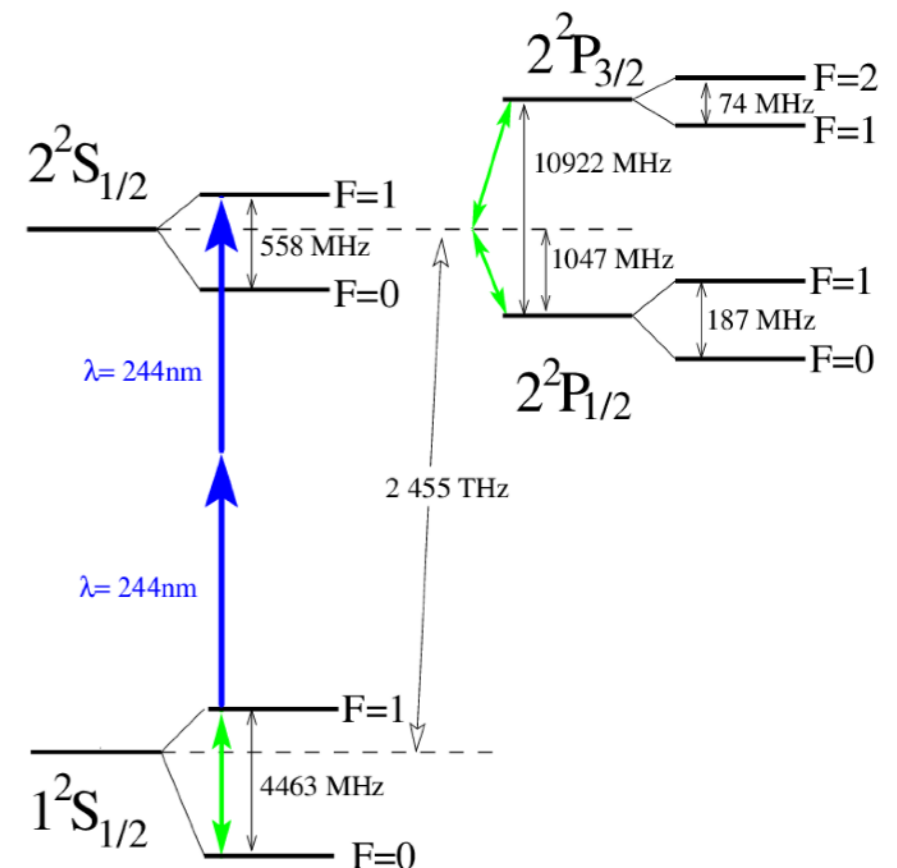


- Experimental advantages in measuring energy levels involved with ground state

- 1S-2S: laser spectroscopy



- Hyperfine splitting (HFS): microwave spectroscopy

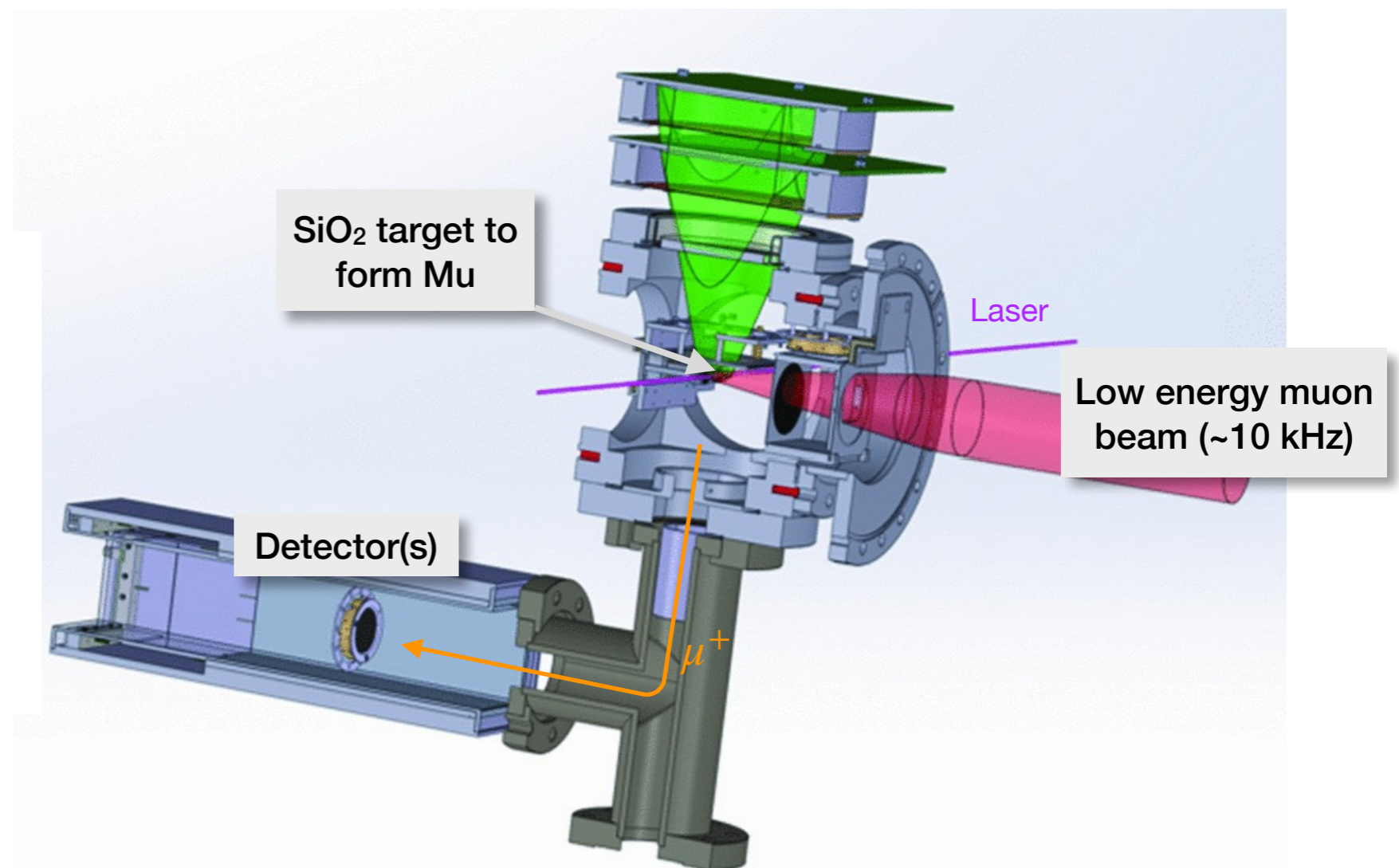
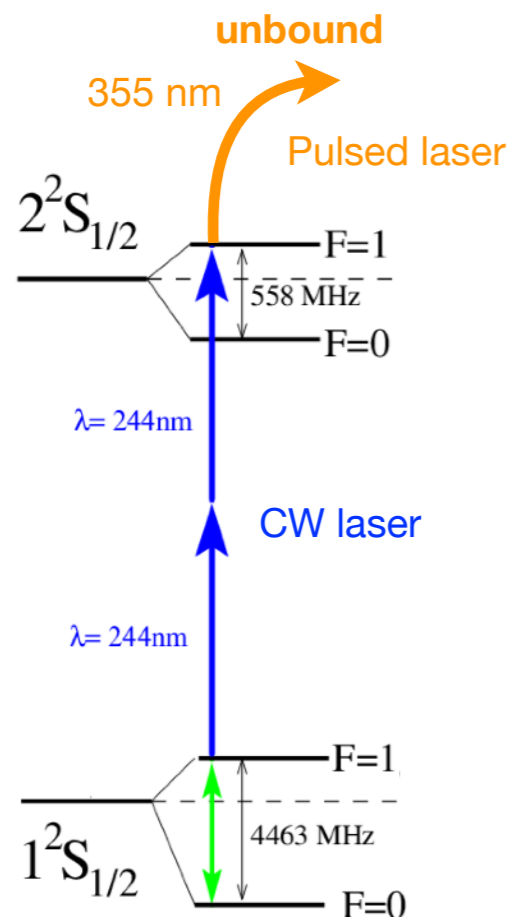


K. Jungmann, Past, Present and Future of Muonium

Mu 1S-2S laser spectroscopy

- Mu-MASS at PSI
- Doppler free two photon excitation
- Goal: ν_{1S-2S} with 4 ppt and m_e/m_μ with 1 ppb

$$\nu_{1S-2S} \approx \frac{3}{4} \frac{R_\infty c}{1 + m_e/m_\mu}$$



I. Cortinovis, et al., Eur. Phys. J. D 77, 66 (2023)

Energy of Mu HFS

- Theoretical formula

$$\Delta_{\text{HFS}} = \frac{16}{3} Z^4 \alpha^2 \frac{m_e}{m_\mu} \left(1 + \frac{m_e}{m_\mu} \right)^{-3} cR_\infty + \Delta_{\text{QED}} + \Delta_{\text{QCD}} + \Delta_{\text{EW}}$$

237 Hz -65 Hz

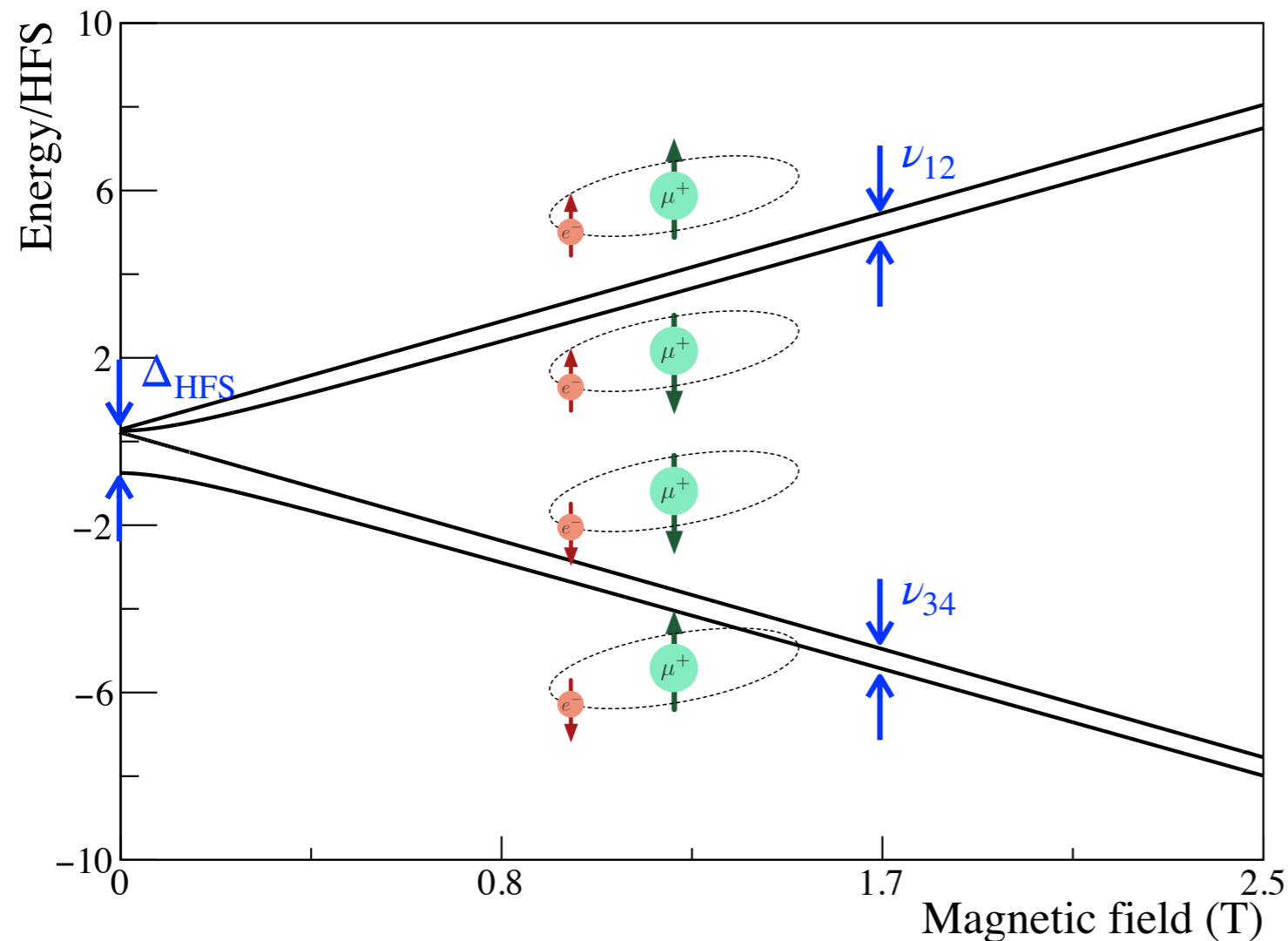
Theoretical prediction: 4.463 302 872(515) GHz [120 ppb] M. I. Eides, Phys. Lett. B 795 113 (2019)

Experimental precision: 4.463 302 776(51) GHz [11 ppb] W. Liu *et al.*, Phys. Rev. Lett. 82 711 (1999)

- Present theoretical uncertainty dominated by knowledge of m_e/m_μ (512 Hz)
 - Soon reduced by Mu 1S-2S laser spectroscopies down to <10 Hz
 - Then, precise test of Standard Model will be possible
 - Further inputs from experimental side will be needed!

Measurement principle of Mu HFS

- External magnetic field splits the Δ_{HFS} into two Zeeman sub-levels, ν_{12} and ν_{34}
- Direct measurement of Δ_{HFS} without B-field or use $\Delta_{\text{HFS}} = \nu_{12} + \nu_{34}$ with B-field
- Measurement with B-field also provide magnetic moment ratio $\mu_{\mu}/\mu_p \propto \nu_{12} - \nu_{34}$



$$\nu_{12} = -\frac{\mu_{\mu}B}{h} + \frac{\Delta_{\text{HFS}}}{2} [(1+x) - \sqrt{1+x^2}]$$

$$\nu_{34} = \frac{\mu_{\mu}B}{h} + \frac{\Delta_{\text{HFS}}}{2} [(1-x) + \sqrt{1+x^2}]$$

x : dimensionless quantity $\propto B$

Magnetic moment ratio μ_μ/μ_p

- Muon anomalous magnetic moment still puzzled

$$a_\mu = \frac{g - 2}{2}$$

- a_μ can be experimentally determined via

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$$

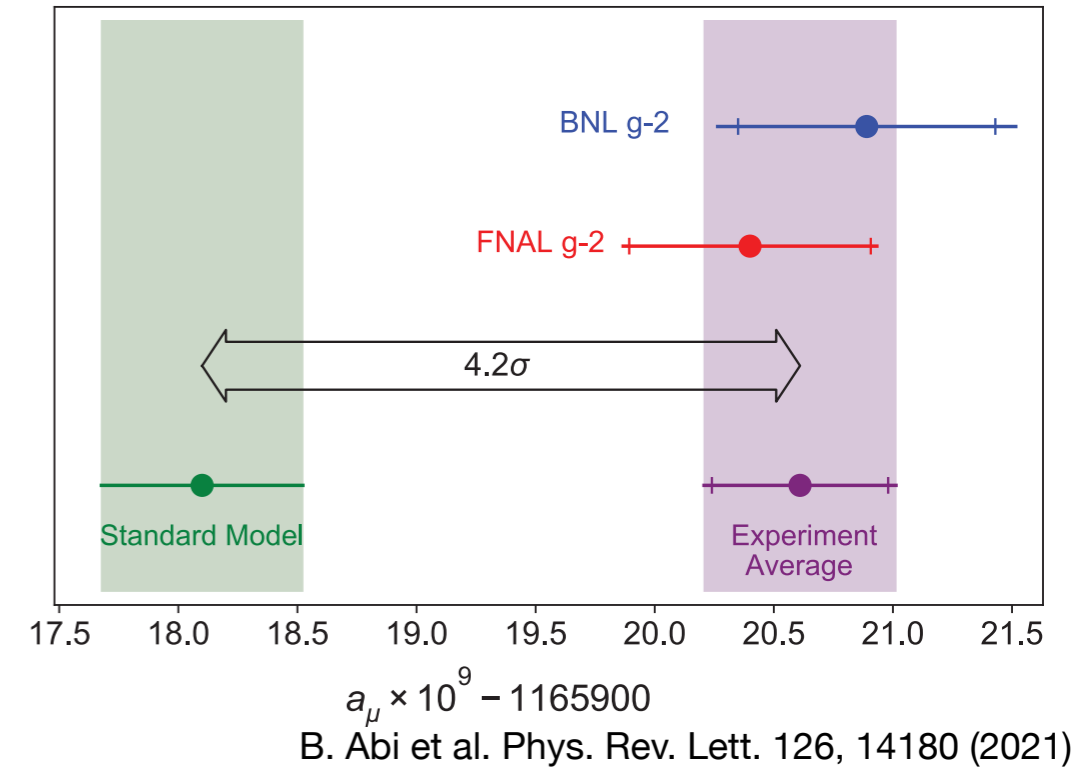
μ_μ/μ_p : from Mu HFS measurement (120 ppb)

W. Liu et al., Phys. Rev. Lett. 82 711 (1999)

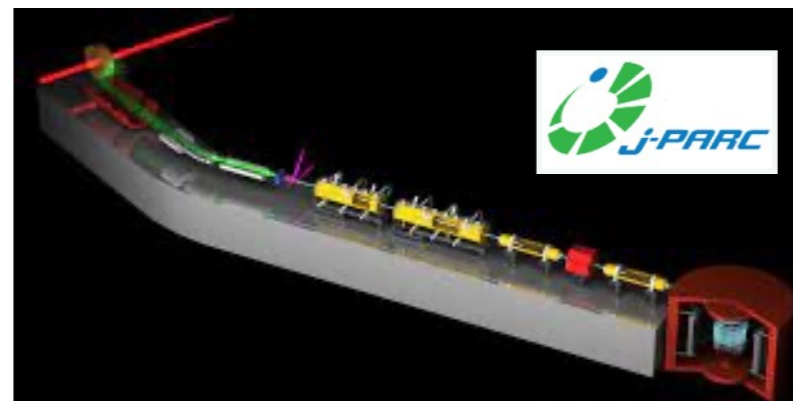
$\omega_a = \omega_s - \omega_c$: from stage ring experiments (350 ppb)

Abi et al. Phys. Rev. Lett. 126, 14180 (2021)

ω_p : NMR precession frequency of proton



- Reduction of uncertainty in μ_μ/μ_p contributes future (ongoing) measurements of a_μ down to ~ 100 ppb

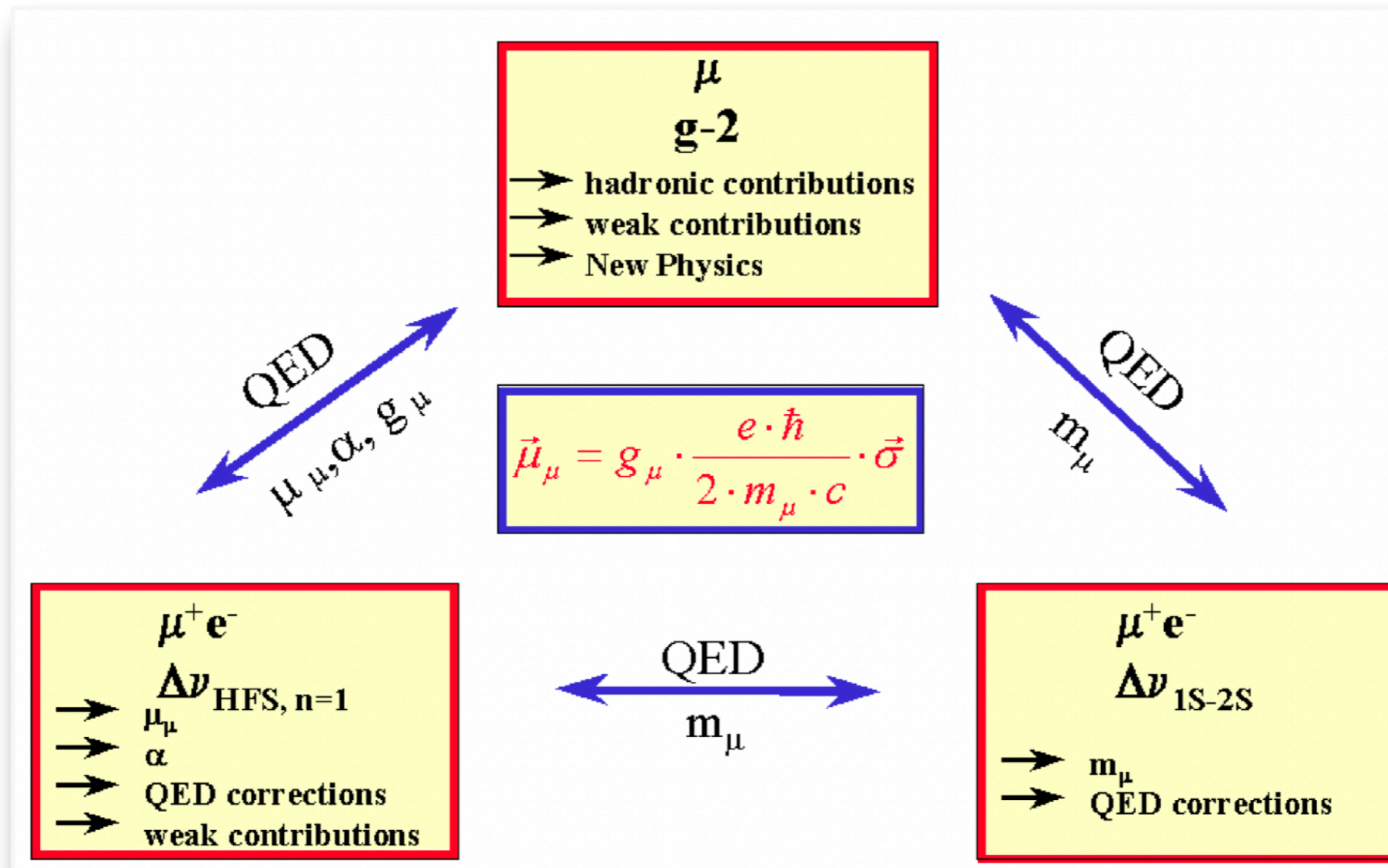


M. Abe, et al. Prog. Theor. Exp. Phys., 053C02 (2019)



All in all...

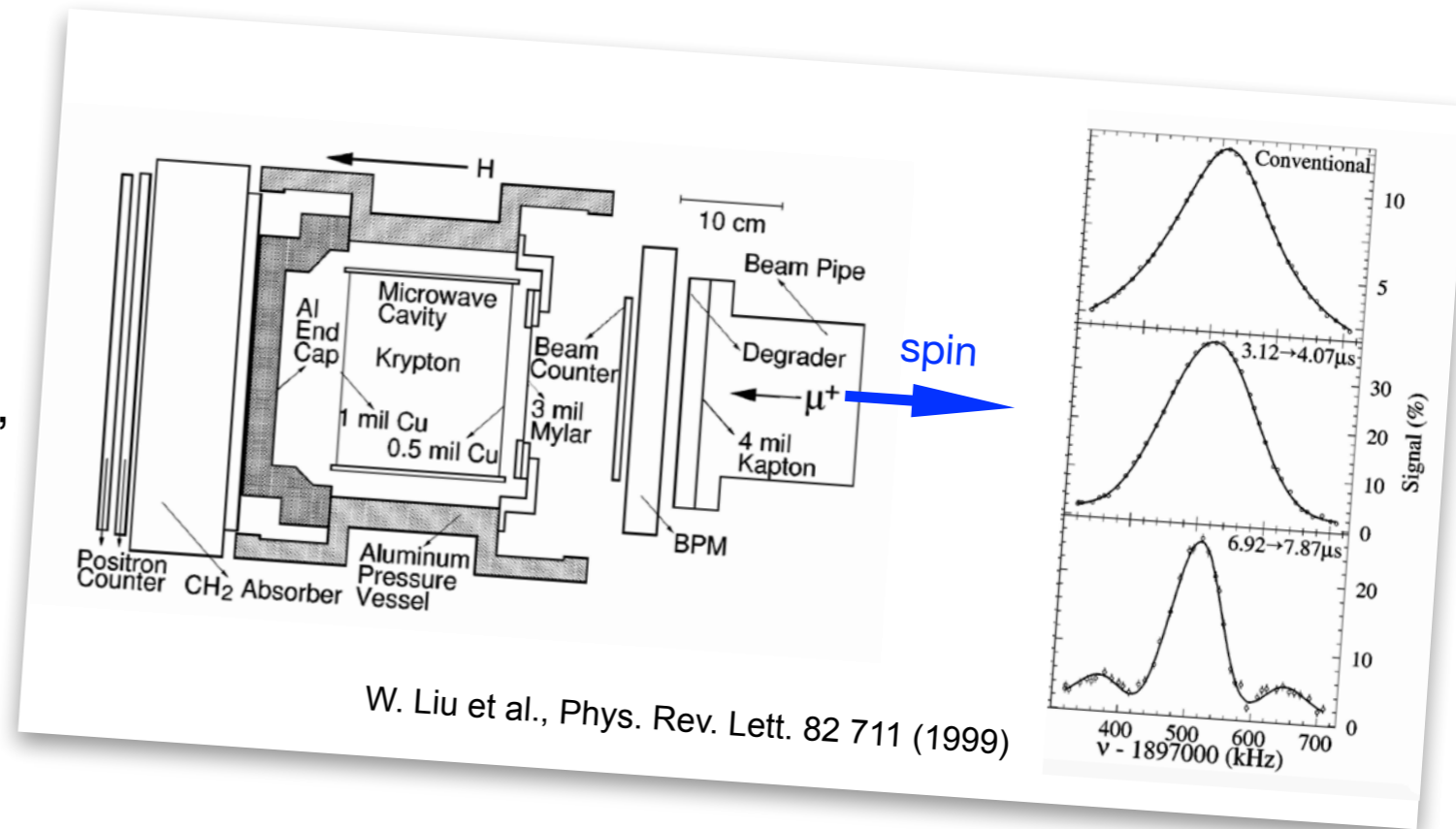
- Mu HFS, Mu 1S-2S and muon g-2 are closely related



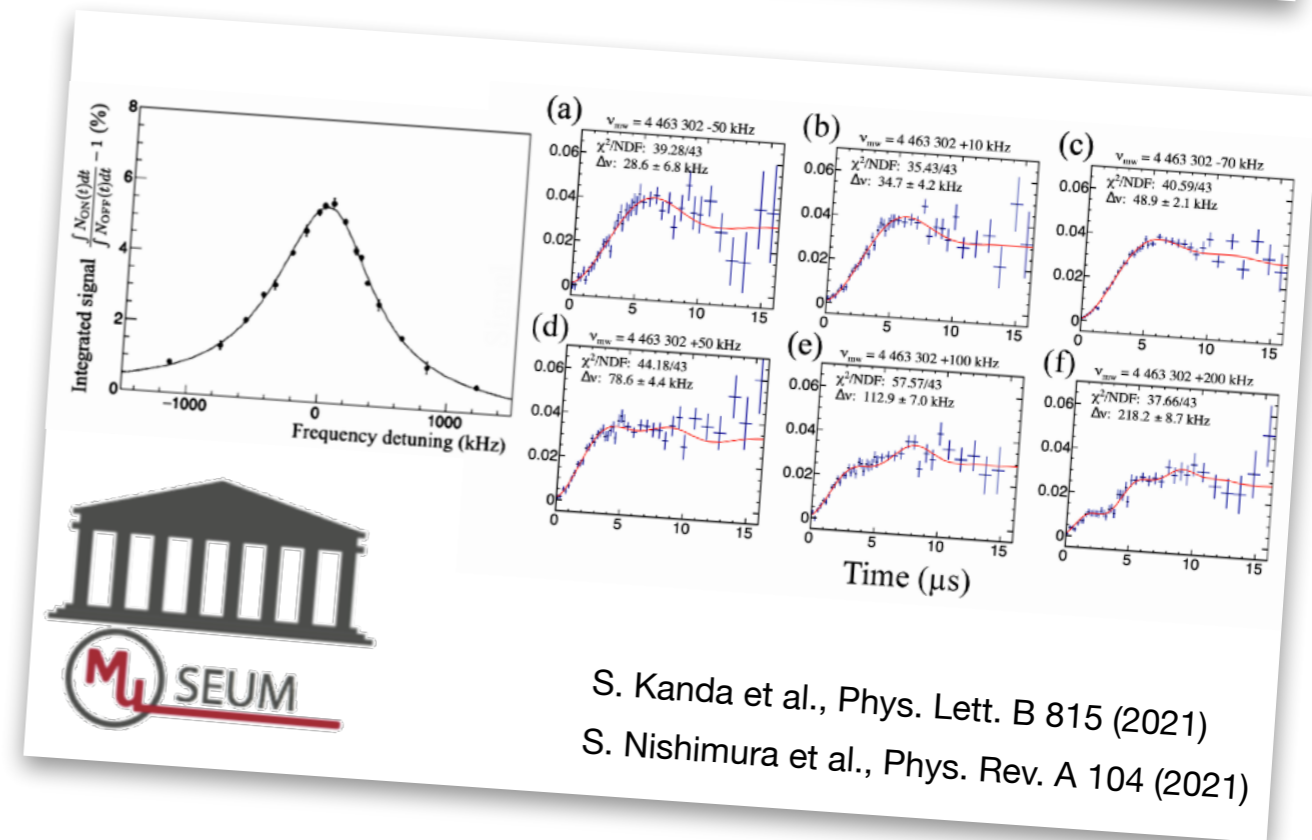
K. Jungmann, Past, Present and Future of Muonium

Measurements of Mu HFS

- Previous work at LAMPF
 - Mu formed in Kr gas
 - Spin flipped by microwave field
 - Chopped CW beam to use “old muonium”
 - Statistically limited

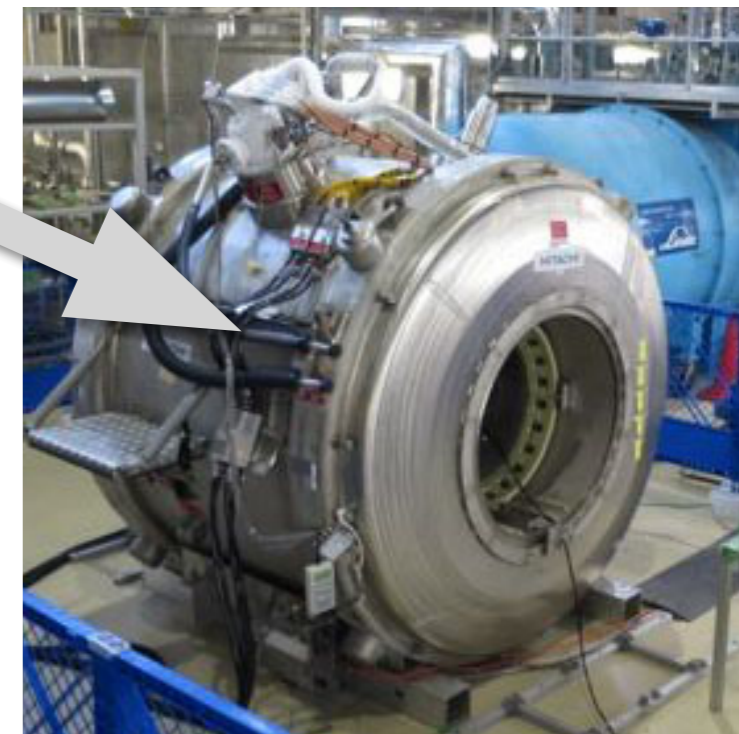
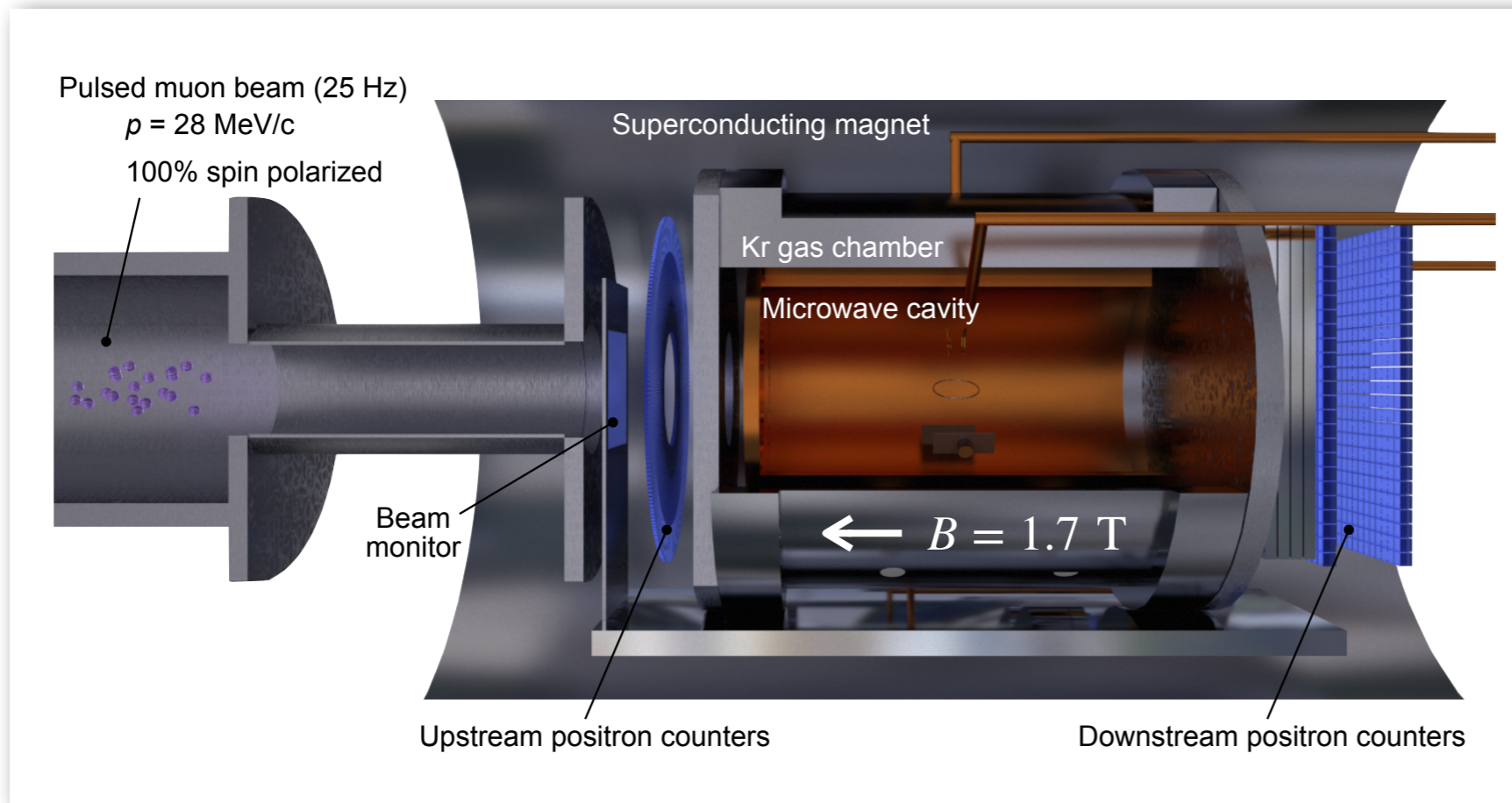


- New measurement at J-PARC by MuSEUM
 - Utilize pulsed muon beam
 - Completed measurements without B-field (Δ_{HFS} with 160 ppb)
 - Goal: Δ_{HFS} with 1 ppb and μ_{μ}/μ_p with 10 ppb with B-field



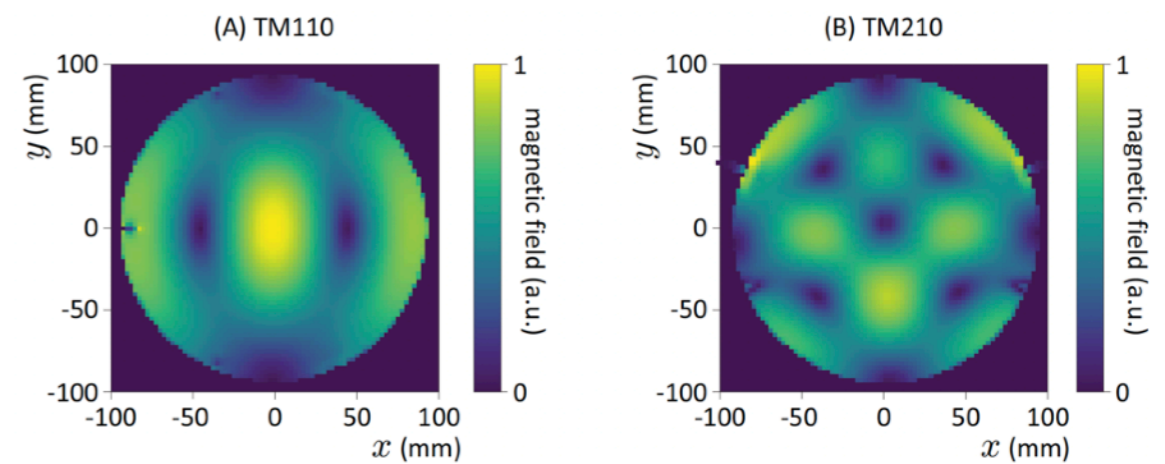
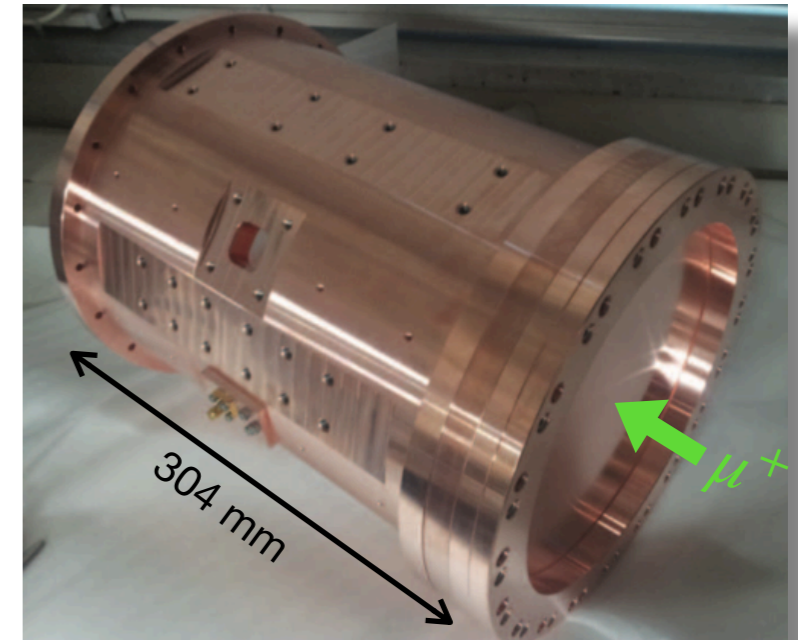
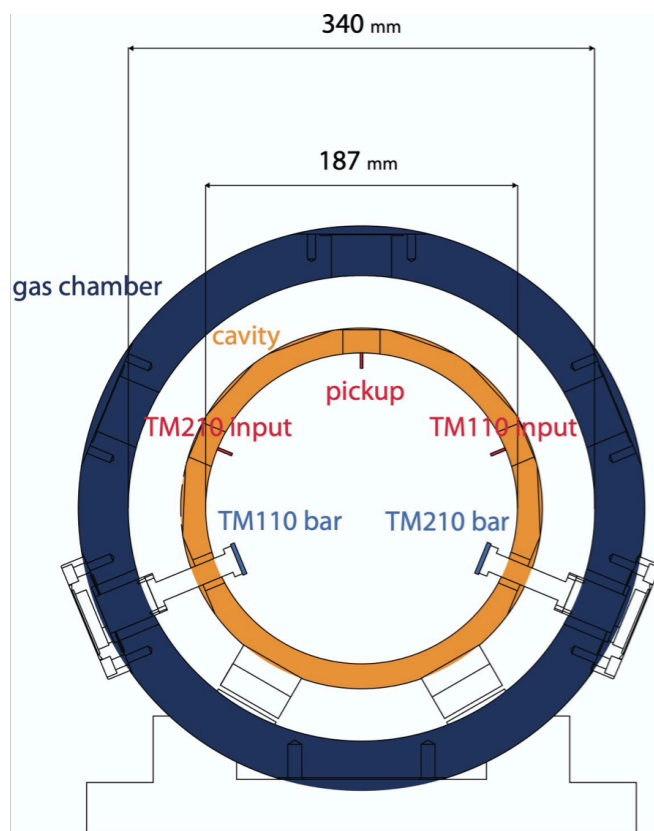
2023: first measurement with magnetic field and new H-line

- Statistics boosted
 - H-line: highest muon rate ($10^8 \mu^+/s$)
 - 1.7 T field: keep initial polarization and increase acceptance of decay positrons
- Plus
 - $\Delta_{\text{HFS}} = \nu_{12} + \nu_{34}$ independent from accuracy of measuring B-field
 - $\mu_{\mu}/\mu_p \propto \nu_{12} - \nu_{34}$ can be extracted



Apparatus with 1.7 T field: cylindrical microwave cavity

- TM110 and TM210 modes for $\nu_{12} = 1.90$ GHz and $\nu_{34} = 2.57$ GHz at 1.7 T
 - 1st and 2nd fundamental modes
 - Uniform field distribution in the axial direction
- Two modes independently tunable
- Q-factors > 5000

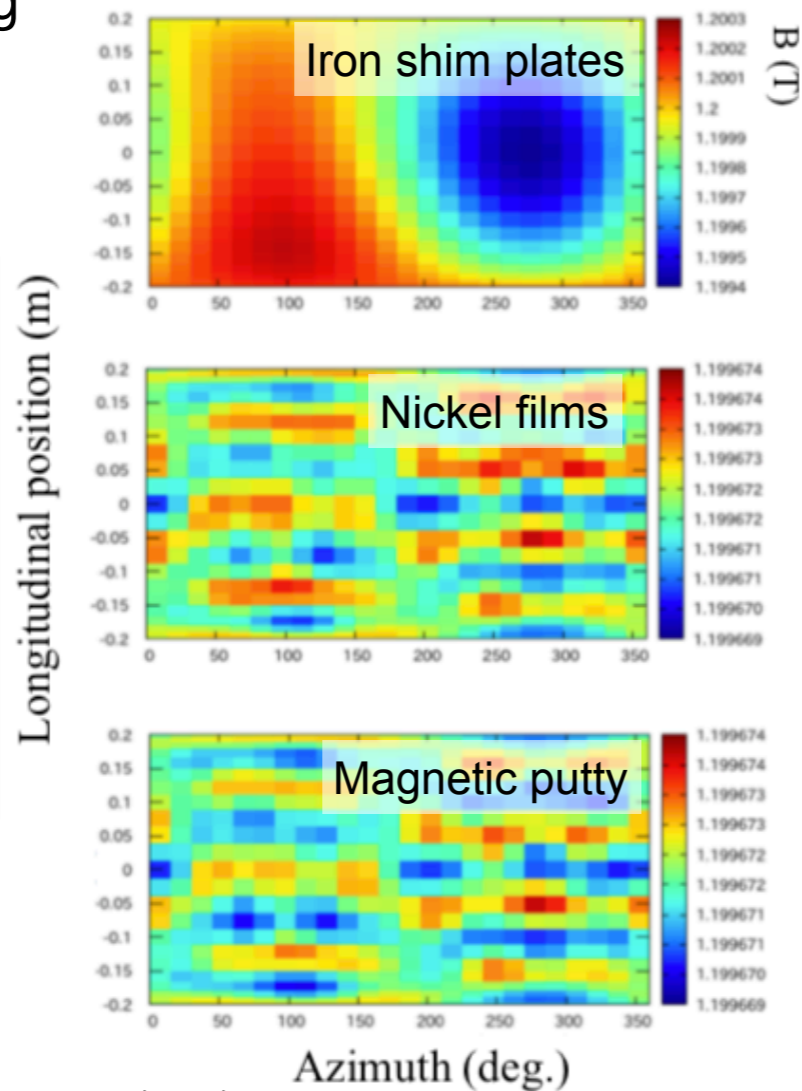
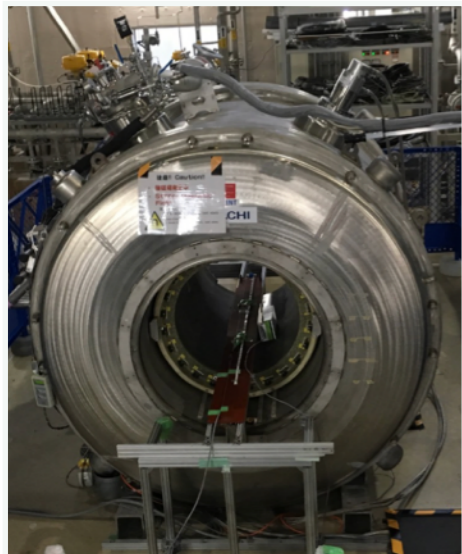


K.S. Tanaka et al. PTEP 2021, 053C01 (2021)

Apparatus with 1.7 T field: field controlling/mapping systems

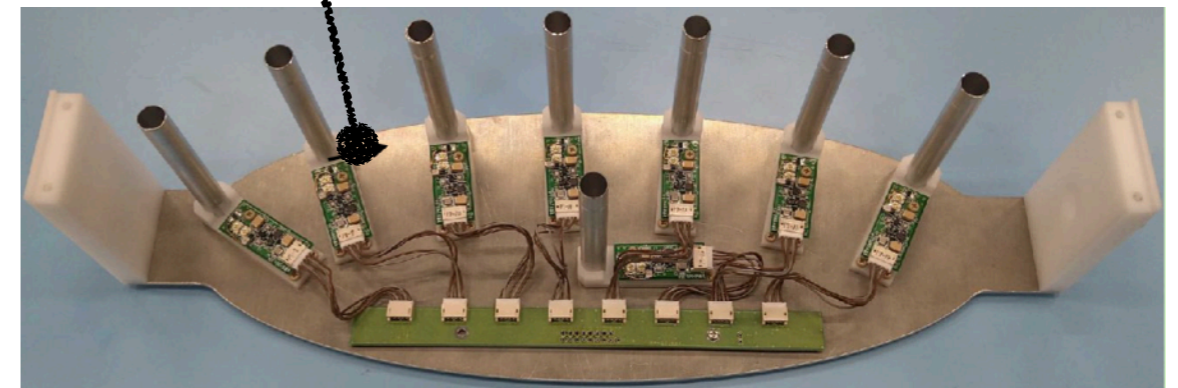
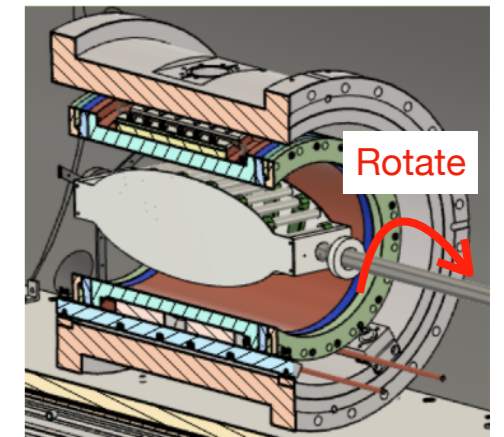
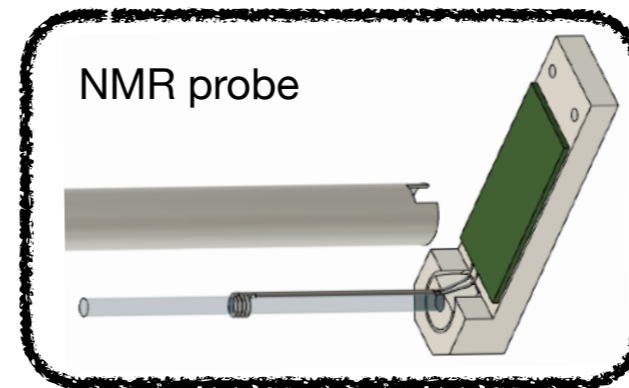
- Uniformity and stability of the field are keys for measuring μ_μ/μ_p
 - 0.2 ppm (peak-to-peak) in spectroscopy volume
 - ± 0.1 ppm stability during the measurement

Passive shimming



M. Abe, magn. reson. med. sci., 16, 4 (2017)
K. Sasaki, et al., IEEE Trans. Appl. Supercond., 32, 6 (2022)

Mapping system for spectroscopy region



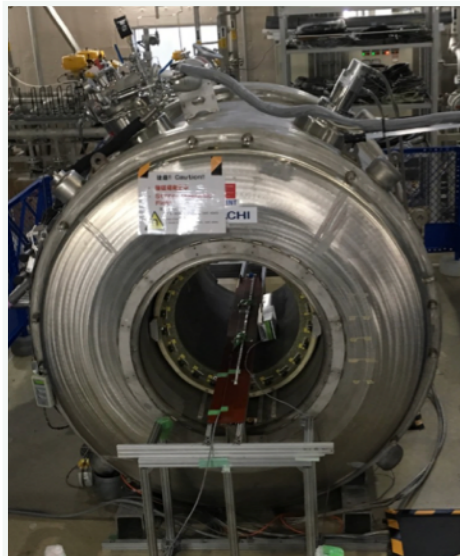
+ larger mapping system (magnet bore), stability monitoring probes, calibrating standard NMR probe (15 ppb), ...

H. Tada et al., IEEE Trans. Appl. Supercond., 10, 1109 (2022)
H. Tada, poster at FKK 2023

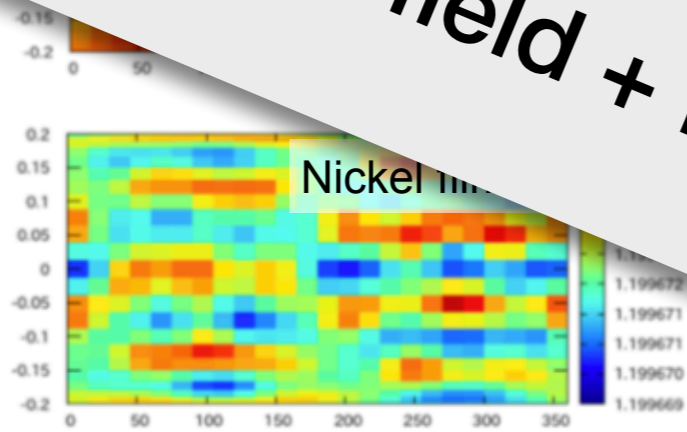
Apparatus with 1.7 T field: field controlling/mapping systems

- Uniformity and stability of the field are keys for measuring μ_μ/μ_p
 - 0.2 ppm (peak-to-peak) in spectroscopy volume
 - ± 0.1 ppm stability during the measurement

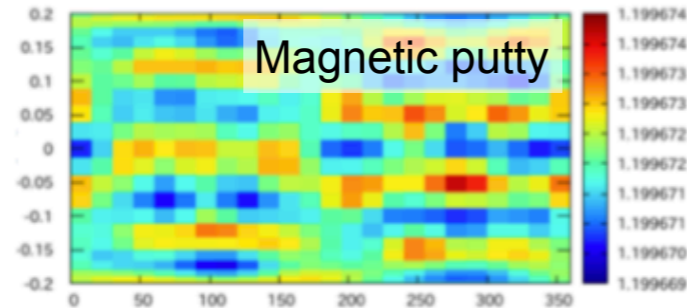
Passive shim



Longitudinal position (m)



Nickel III

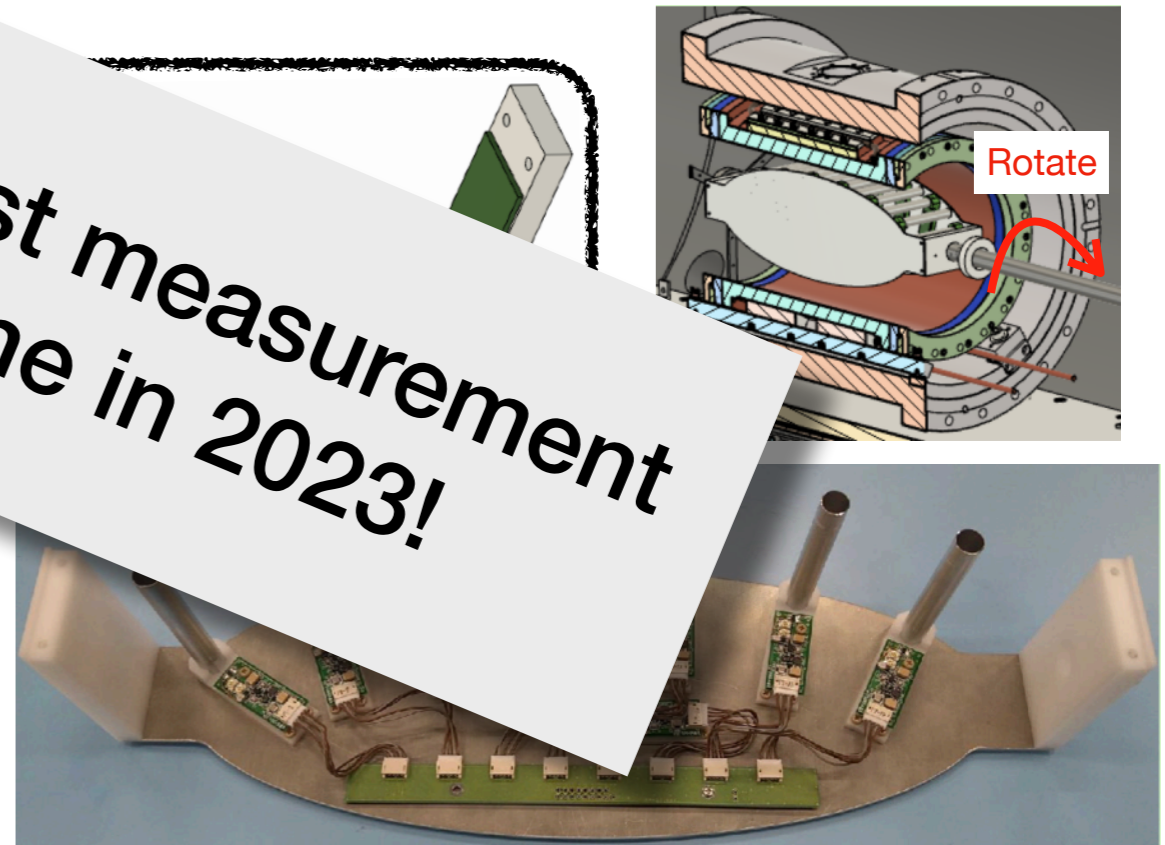


Magnetic putty

Azimuth (deg.)

M. Abe, magn. reson. med. sci., 16, 4 (2017)
K. Sasaki, et al., IEEE Trans. Appl. Supercond., 32, 6 (2022)

Mapping system for spectroscopy region



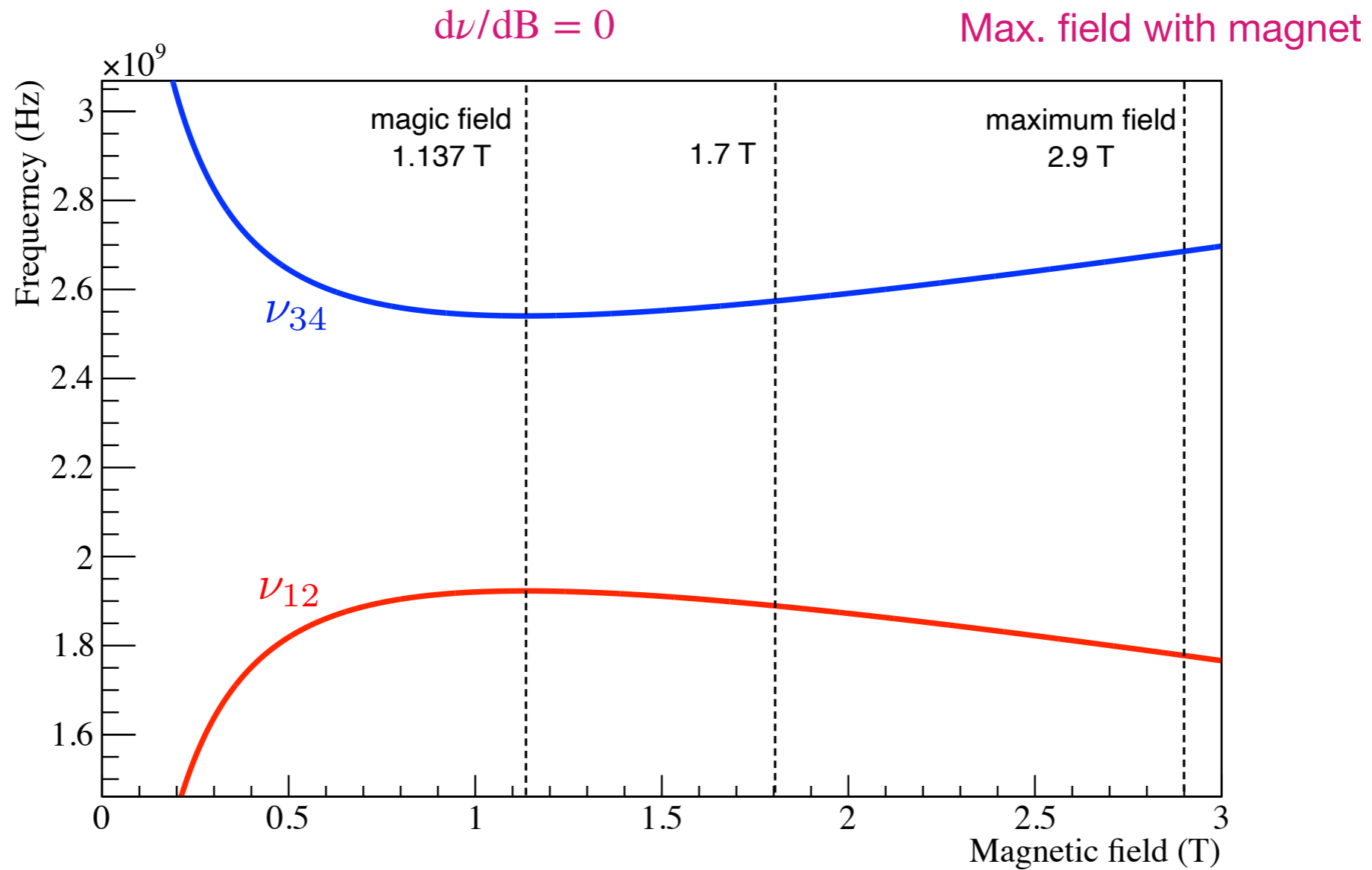
+ larger mapping system (magnet bore), stability monitoring probes, calibrating standard NMR probe (15 ppb), ...

H. Tada et al., IEEE Trans. Appl. Supercond., 10, 1109 (2022)
H. Tada, poster at FKK 2023

Stay tuned for our first measurement with B-field + H-line in 2023!

From near future to future

- Now, let's talk about the future after measuring at 1.7 T field
- Measurements at different B-fields for more precise determination of μ_μ/μ_p ?



Problem with cylindrical cavity

- Cylindrical microwave: only generate modes for 1.7 T due to $F_{\text{TM}110}/F_{\text{TM}210} \sim \text{constant}$

$$F_{\text{TM}mn0} = \frac{cx_{mn}}{n\pi D}$$

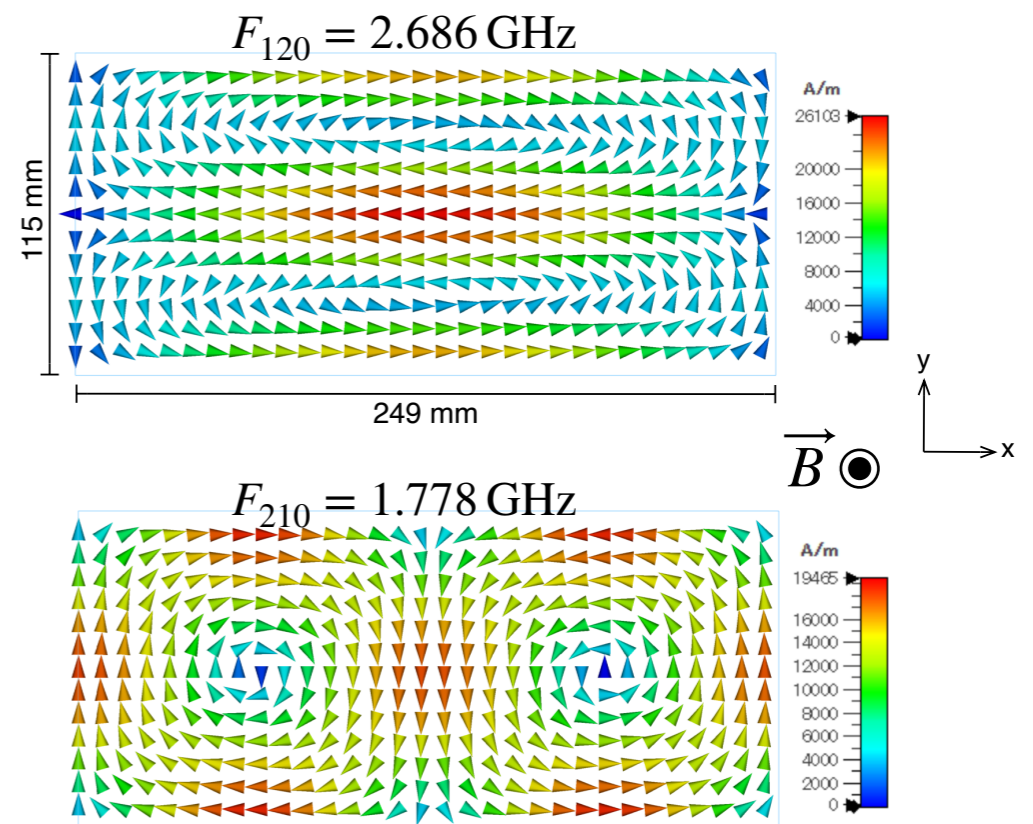
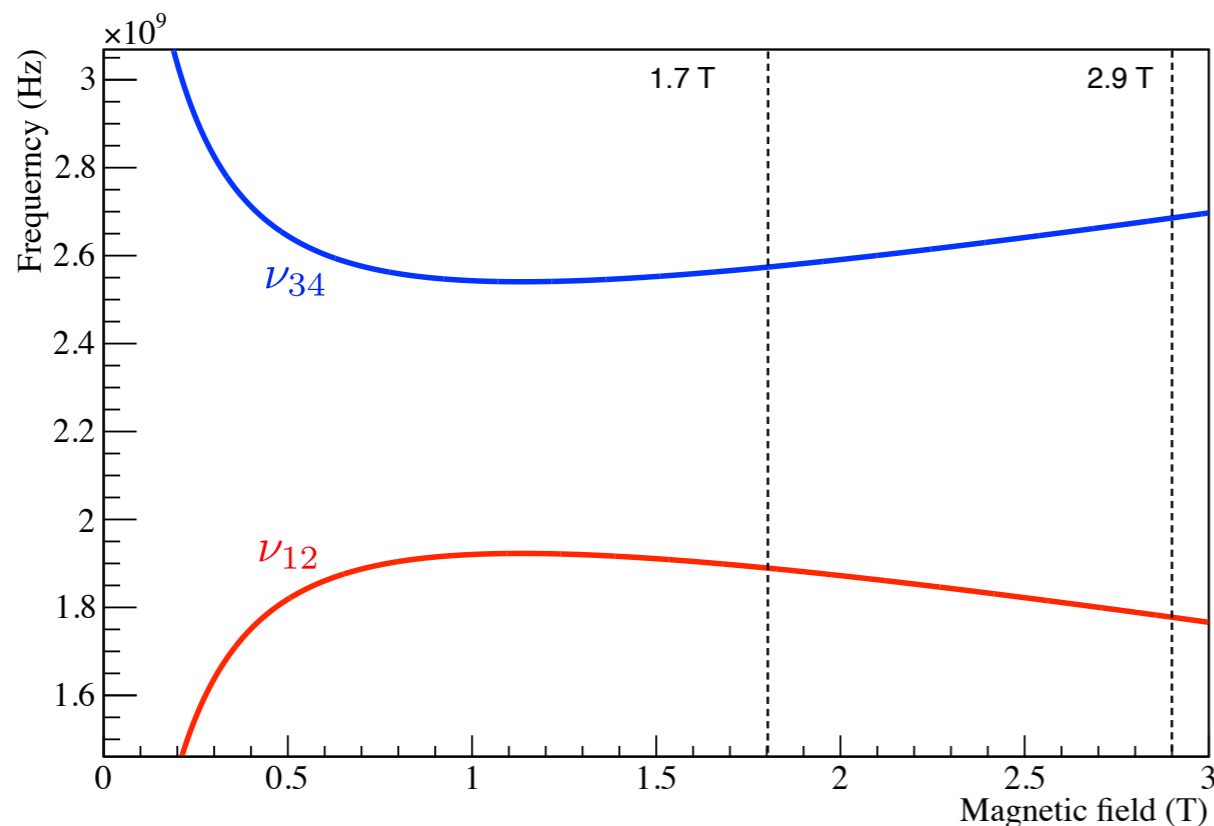
c : Speed of light
 x_{mn} : n th root of Bessel function $J_m(x)$
 D : Cavity diameter

- Rectangular cavity: more degrees of freedom

$$F_{mnl} = \frac{c}{2\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2}$$

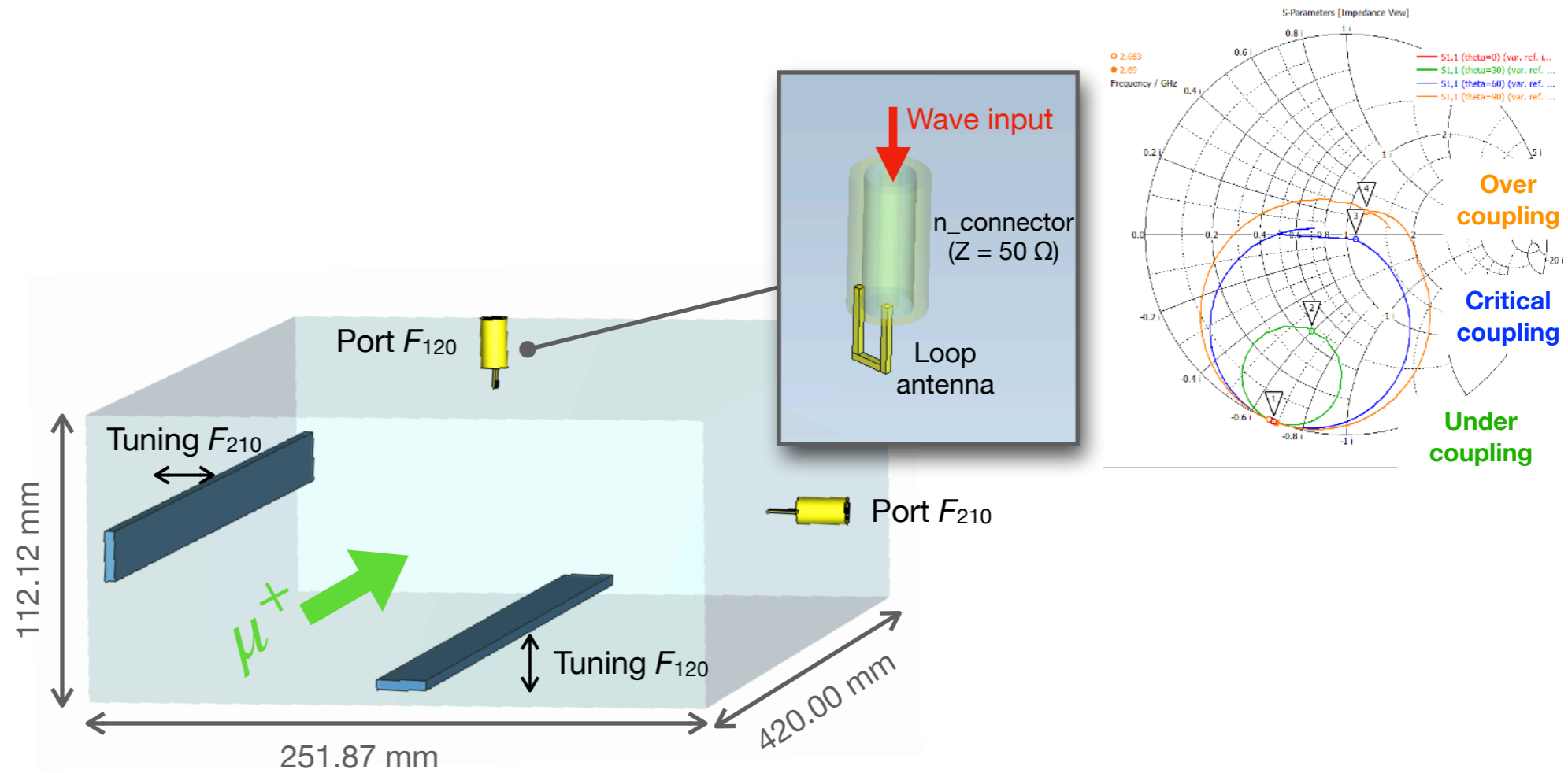
μ_r and ϵ_r : Relative permeability and permittivity
 m, n, l : Mode numbers
 a, b, d : Cavity dimensions

- e.g. cavity dimensions for 2.9 T field

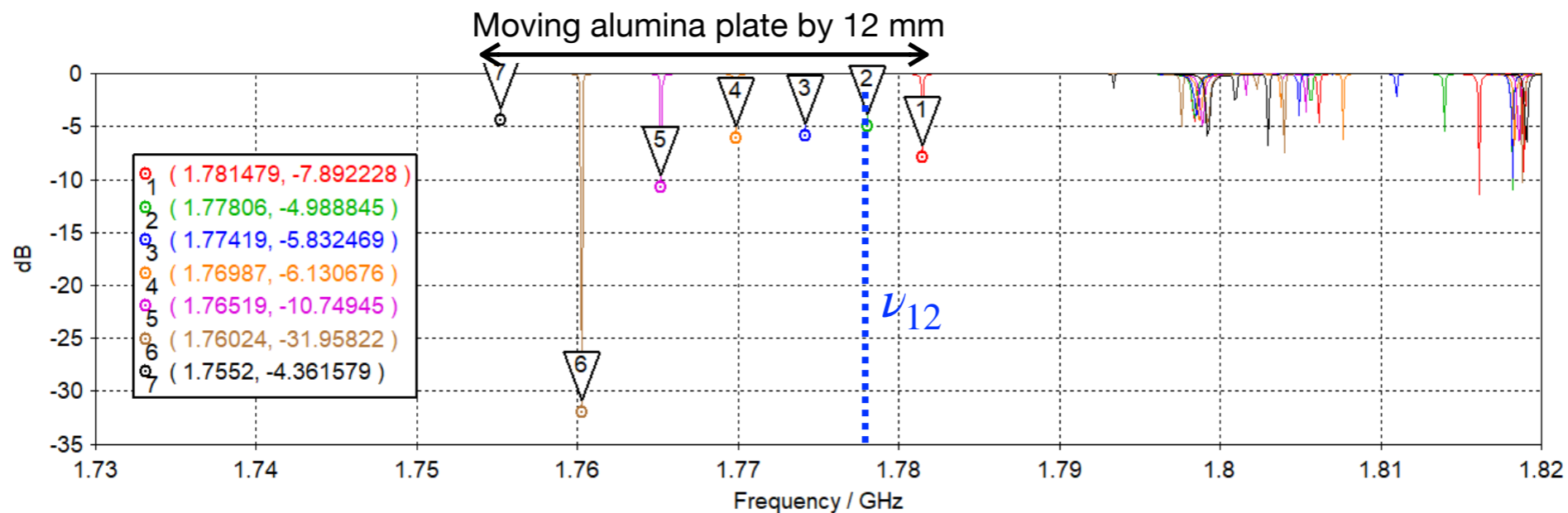


Rectangular shaped cavity: design

- Cavity dimensions
- Wave ports
- Tuning scheme

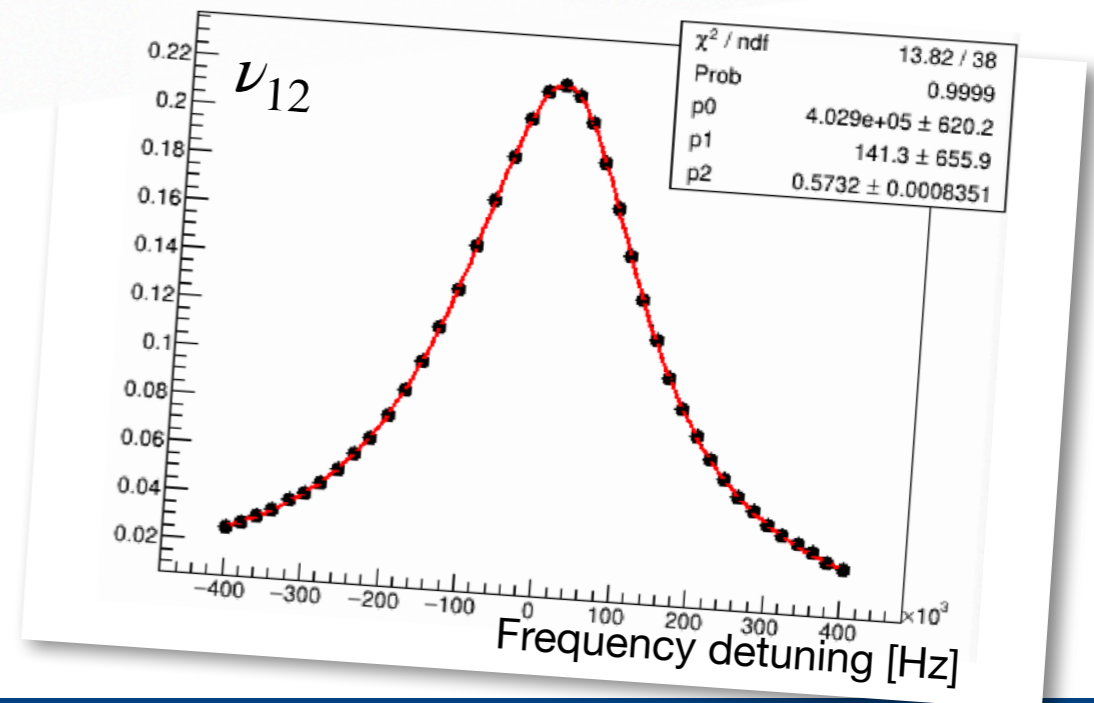
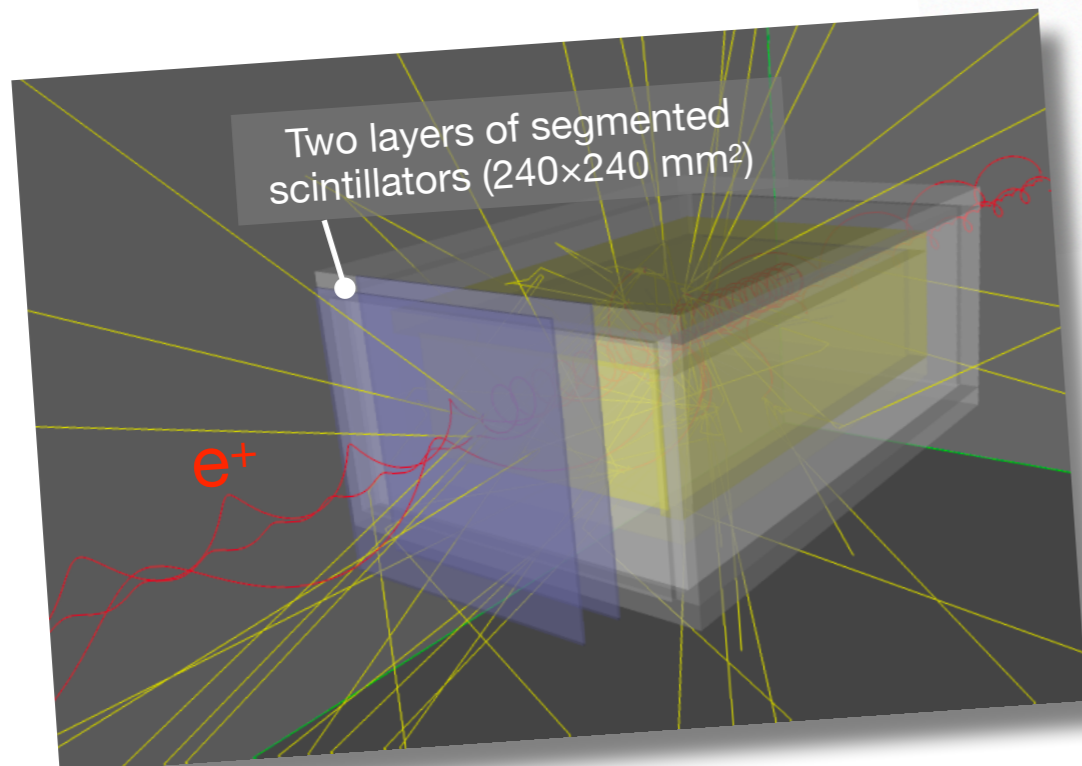
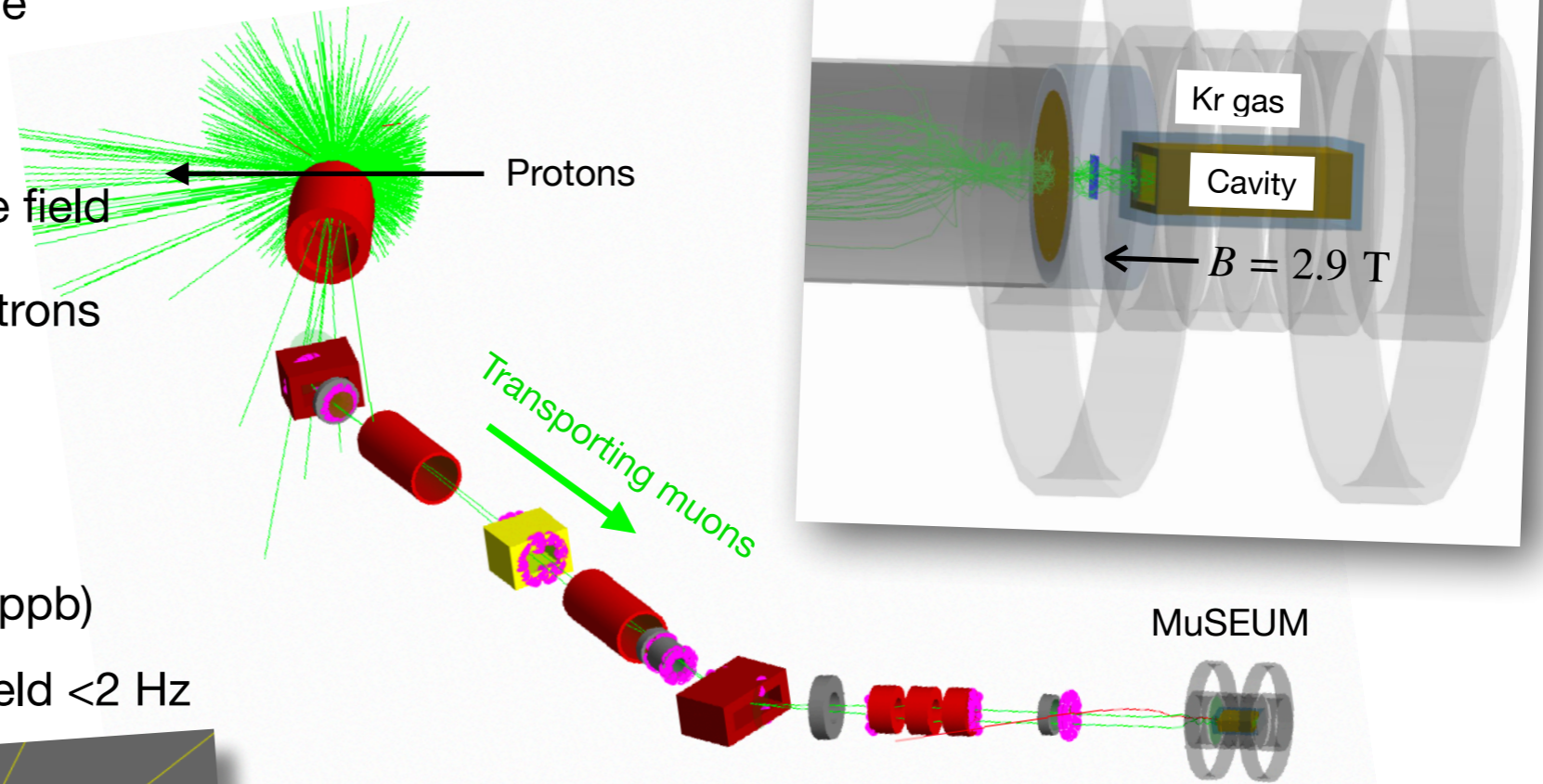


- Simulated frequency sweep (F_{210})



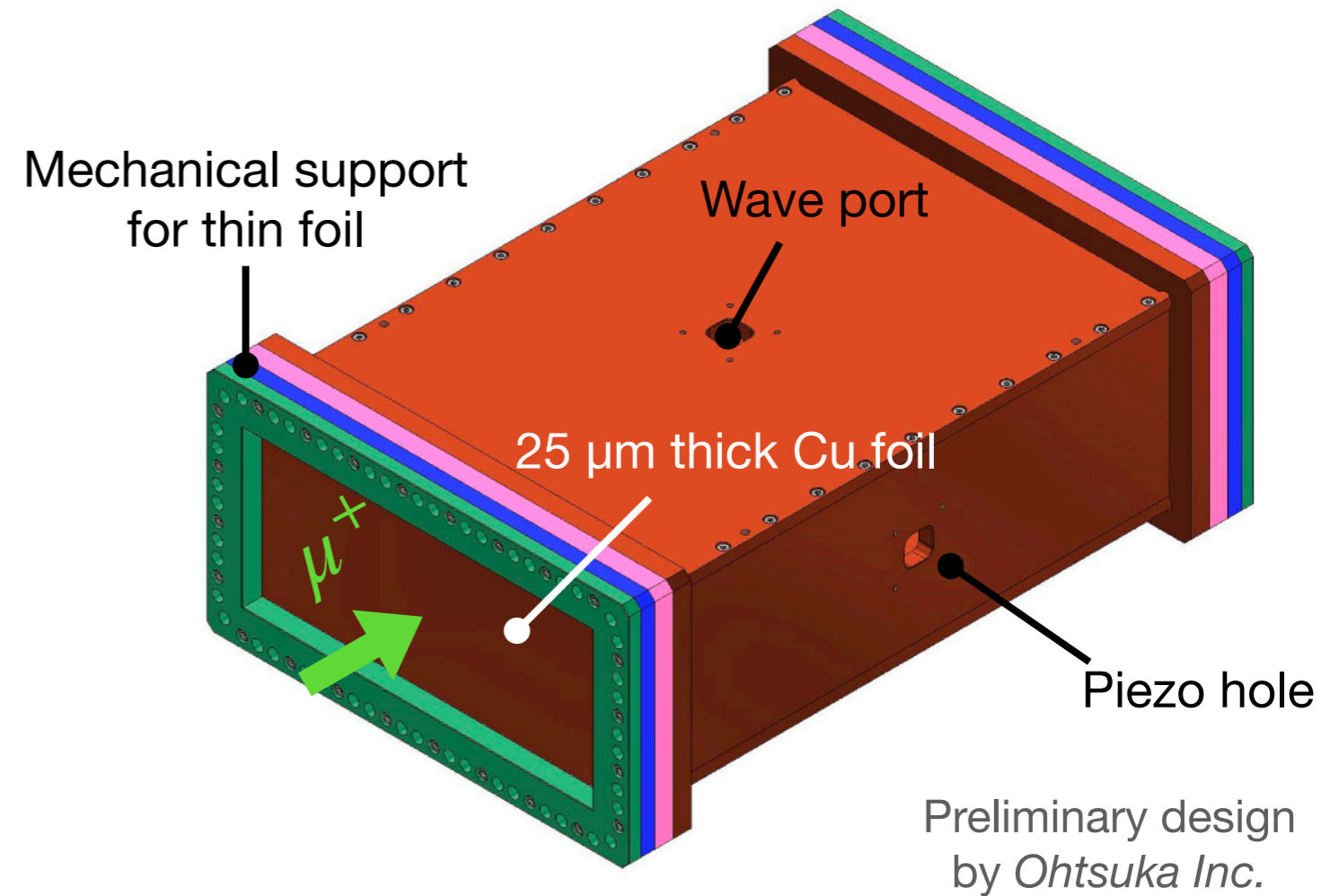
Rectangular shaped cavity: simulations

- Check performance with rectangular shaped cavity + 2.9 T
 - Injection of muon beam from H-line
 - Stopping distribution
 - Spin-state evolution by microwave field
 - Detection efficiency of decay positrons
- Statistics seems to be promising
 - $\Delta(\nu_{12} - \nu_{34}) < 5 \text{ Hz}$ (μ_μ/μ_p with $<10 \text{ ppb}$)
 - Sys. uncertainty of measuring B-field $<2 \text{ Hz}$



Rectangular shaped cavity: production

- Mechanical design nearly completed
- Production in 2023



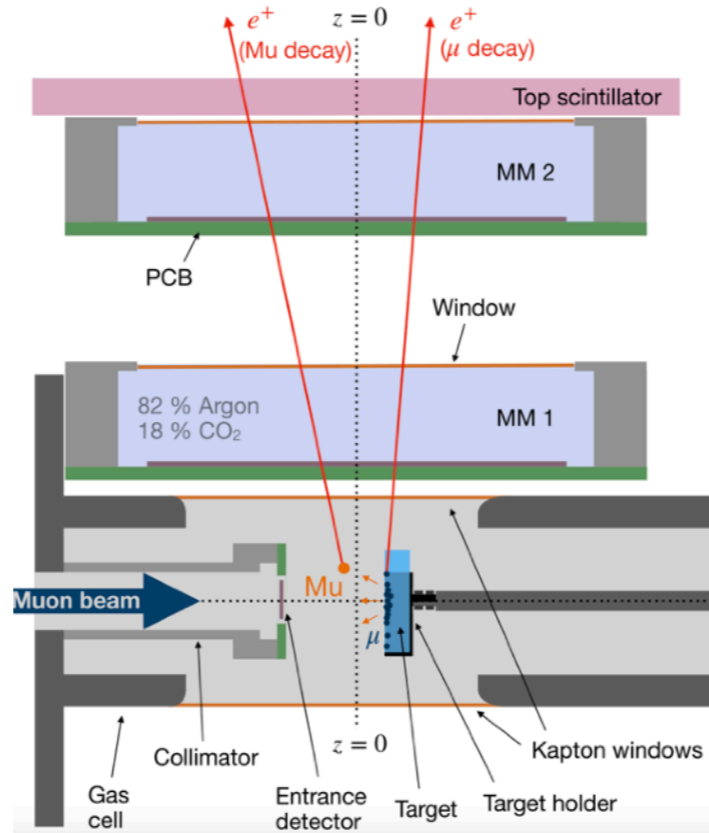
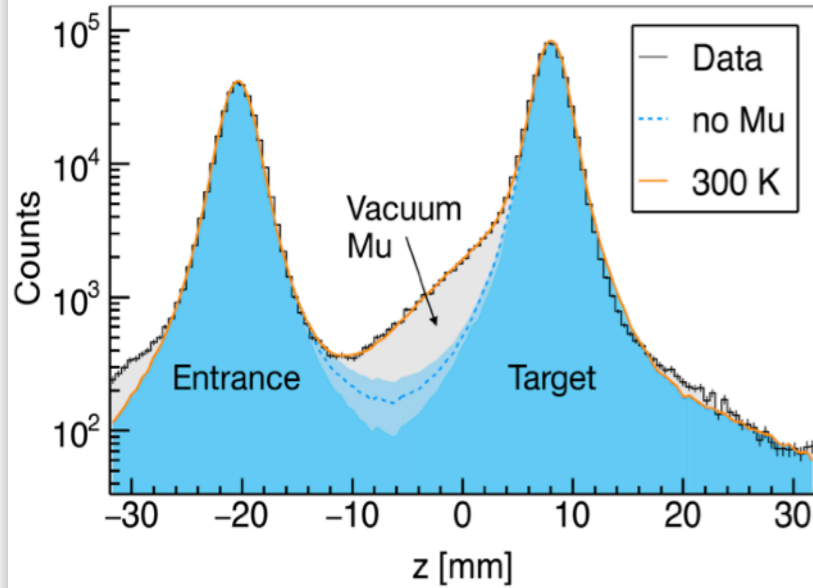
Conclusions

- Precision Mu spectroscopies are powerful tool to precisely test the Standard Model
- Mu microwave spectroscopy for precisely determining Δ_{HFS} and μ_{μ}/μ_p
 - MuSEUM integrating the setup towards the first measurement with 1.7 T field and H-line in 2023
 - Further future plans (2023~) for measurements at different B-fields with new rectangular microwave cavities

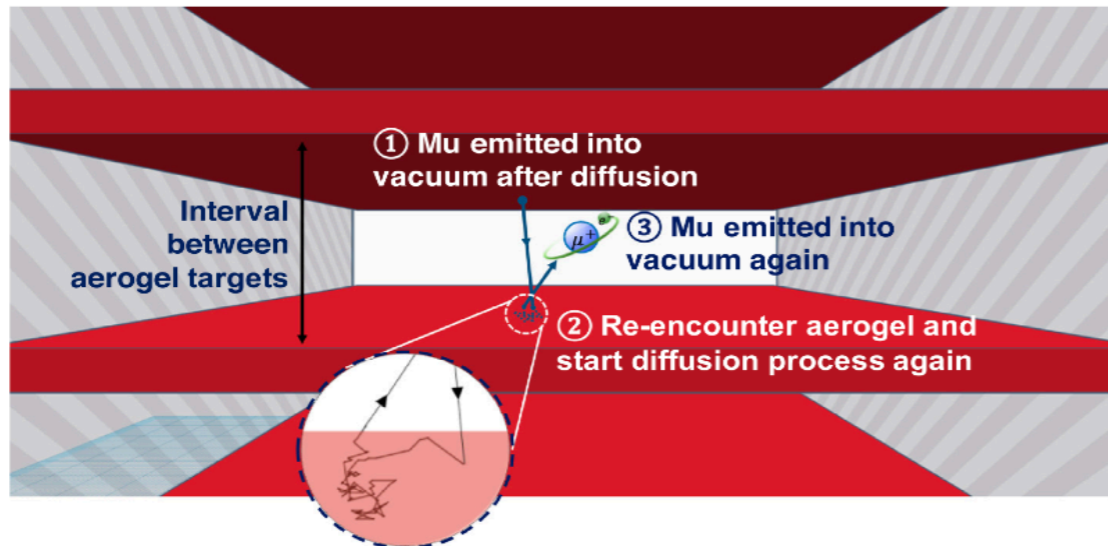
Recent activities on vacuum Mu sources

Aerogel for laser experiments

A. Antognini, et al., Phys. Rev. A 106, 052809 (2022)

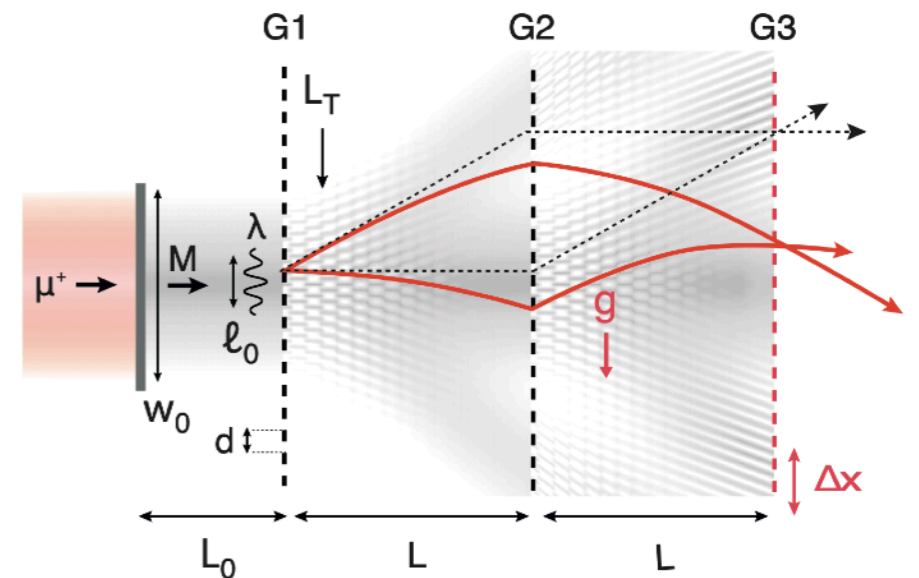
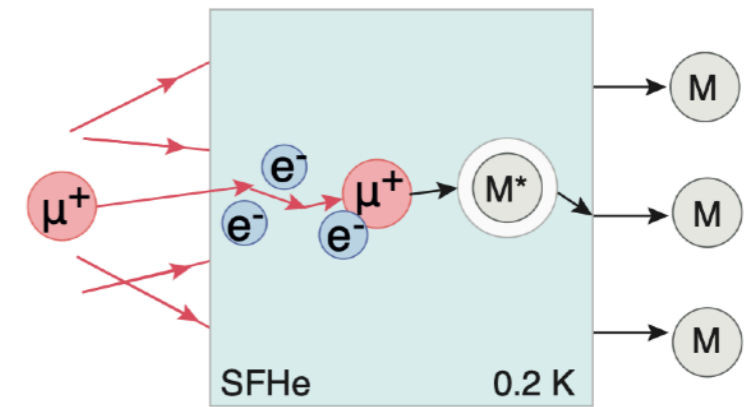


C. Zhang, et al., Nucl. Instrum. Methods Phys. Res., A 1042 (2022)



Superfluid He for gravity experiment

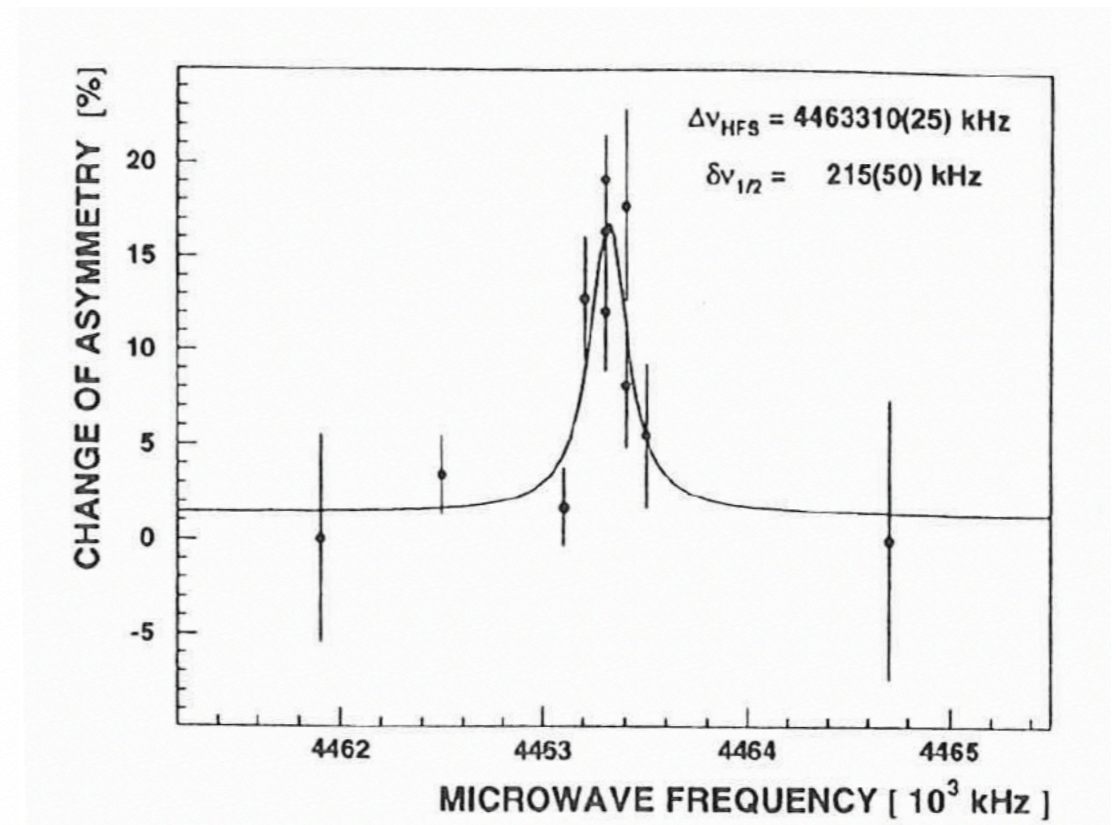
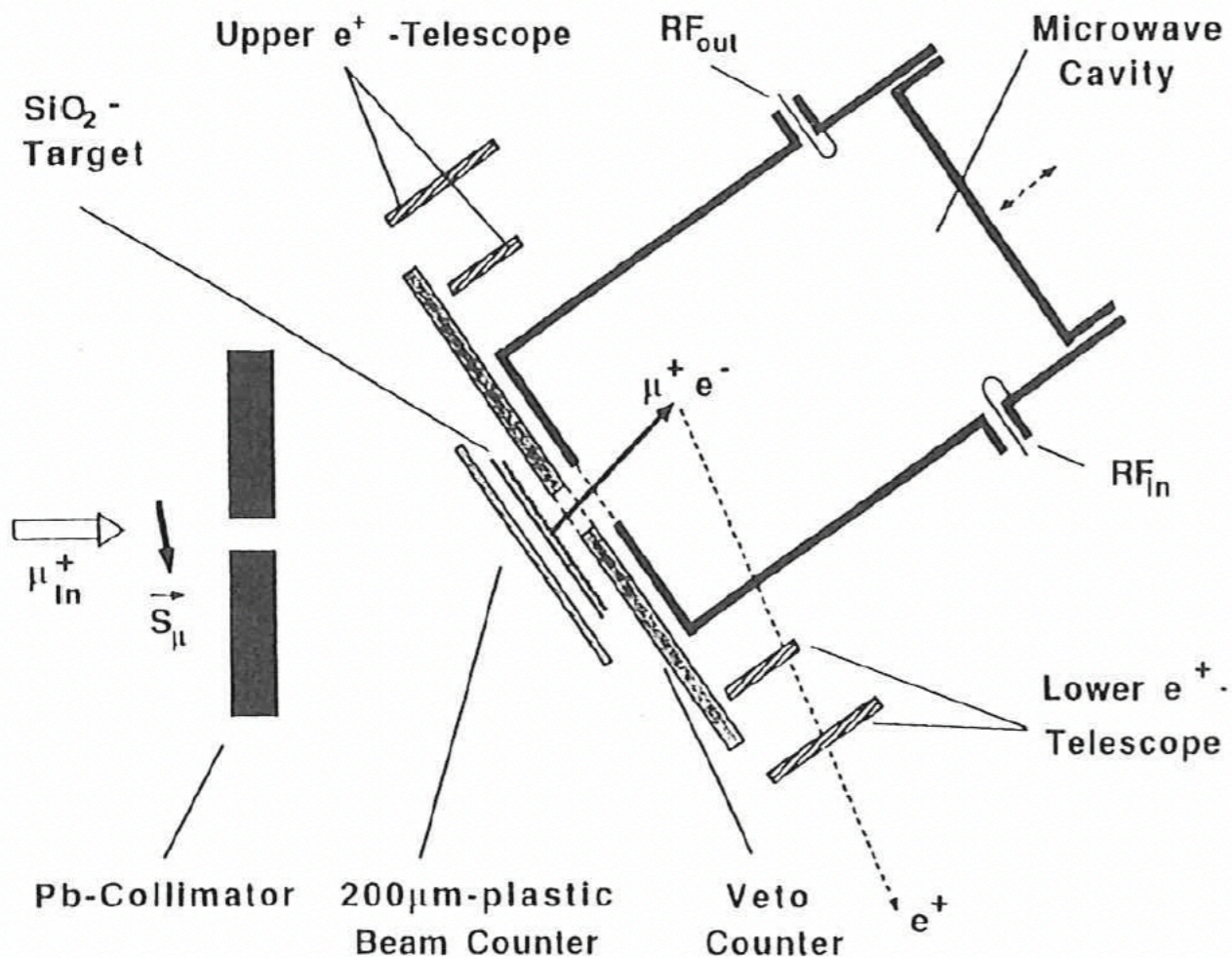
A. Soter and A. Knecht, SciPost Phys. Proc. 5, 031 (2021)



Backup

Rectangular cavity: previous experiment

- HFS measurement of vacuum Mu
- Utilized sub-surface muon beam of $\pi M3$ beamline at PSI
- Muons spin flipped to reduce beam related background
 - Used different frequency mode (F301)



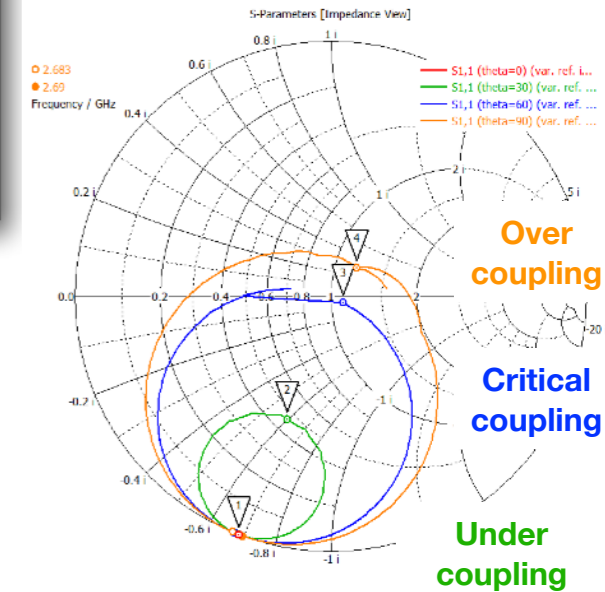
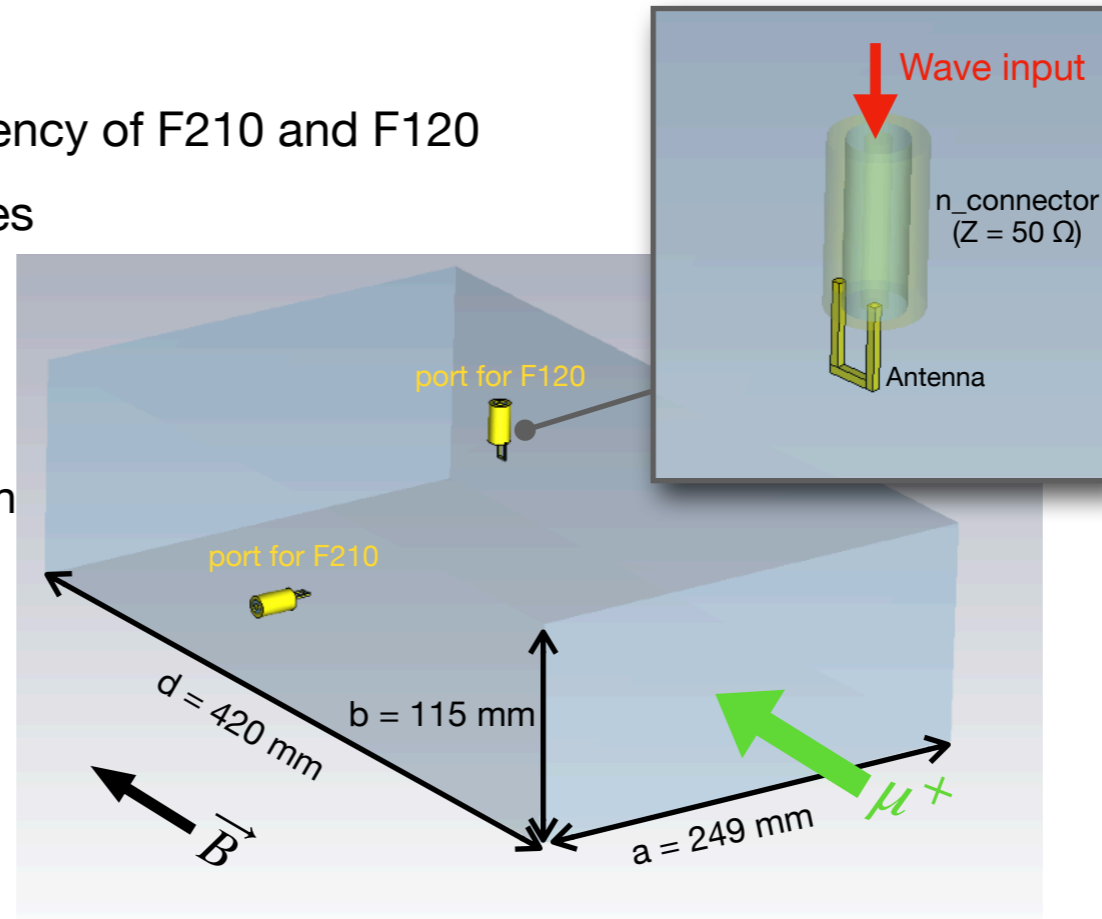
K Jungmann, et. al., Appl. Phys. B 60 (1995)

Rectangular cavity: cavity dimensions and ports

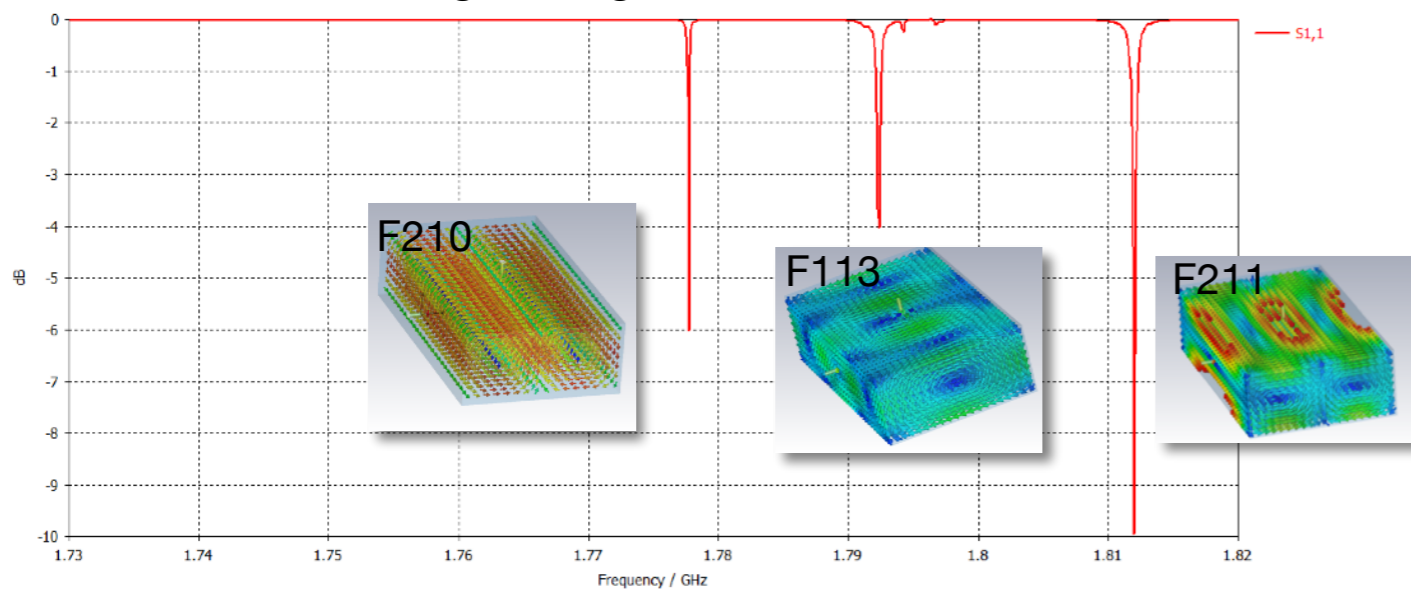
- Cavity dimensions
 - Width and height: To match frequency of F210 and F120
 - Depth: To avoid overlapping modes

- Wave ports

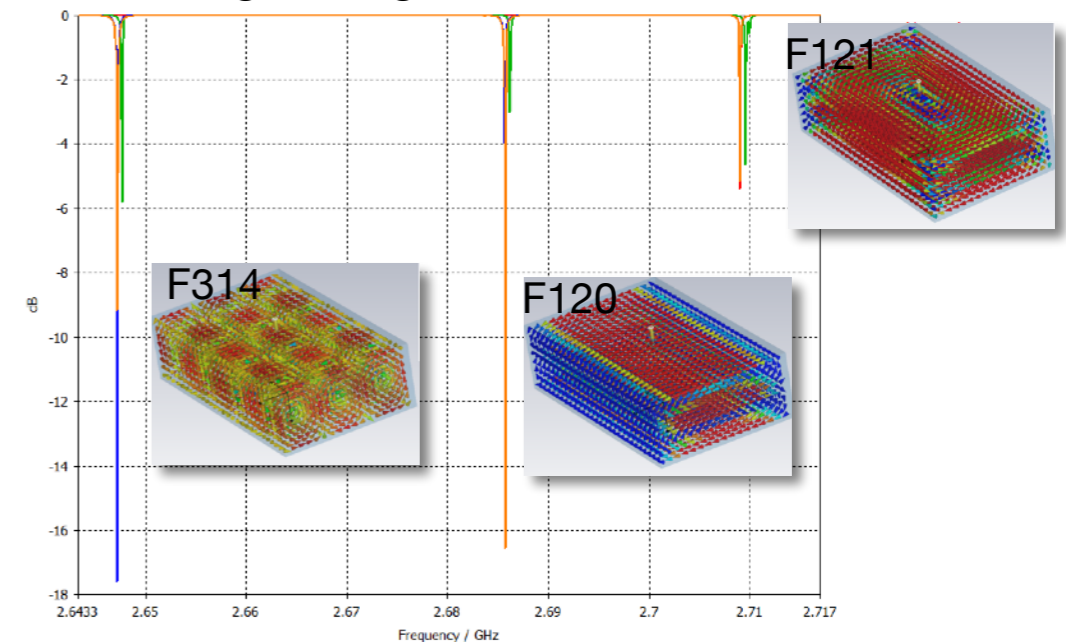
- Loop antennas: Critical coupling with magnetic field of F210 and F120



Neighboring modes of F210



Neighboring modes of F120



Rectangular cavity: tuning scheme 1

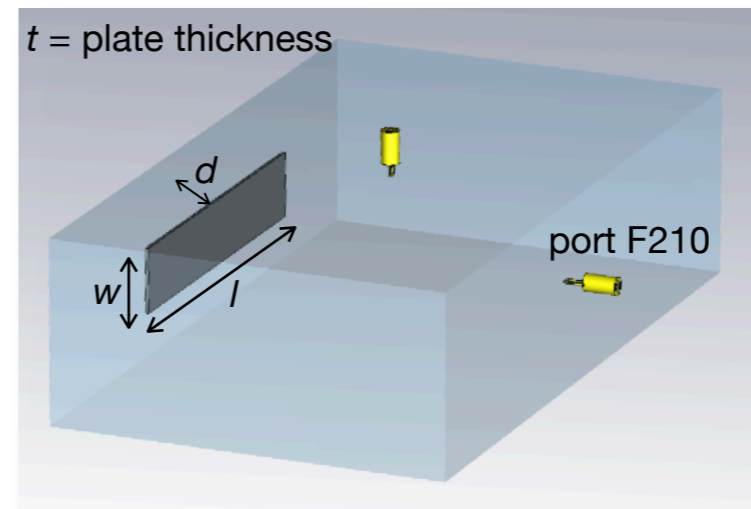
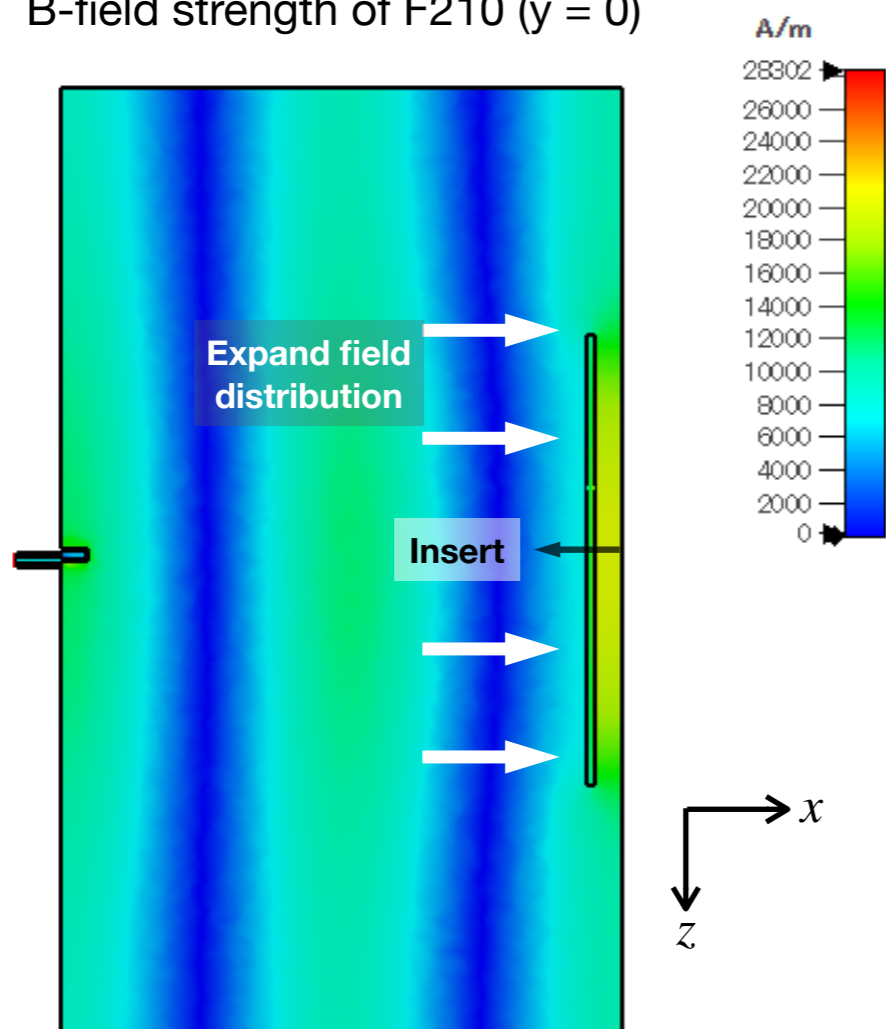
- Insert alumina bar ($\epsilon_r = 9.5$) to shift frequencies
- Naively from the cavity perturbation theory

$$\frac{\omega - \omega_0}{\omega_0} \simeq \frac{-\int_{V_0} (\Delta\epsilon |\bar{E}_0|^2 + \Delta\mu |\bar{H}_0|^2) dv}{\int_{V_0} (\epsilon |\bar{E}_0|^2 + \mu |\bar{H}_0|^2) dv}$$

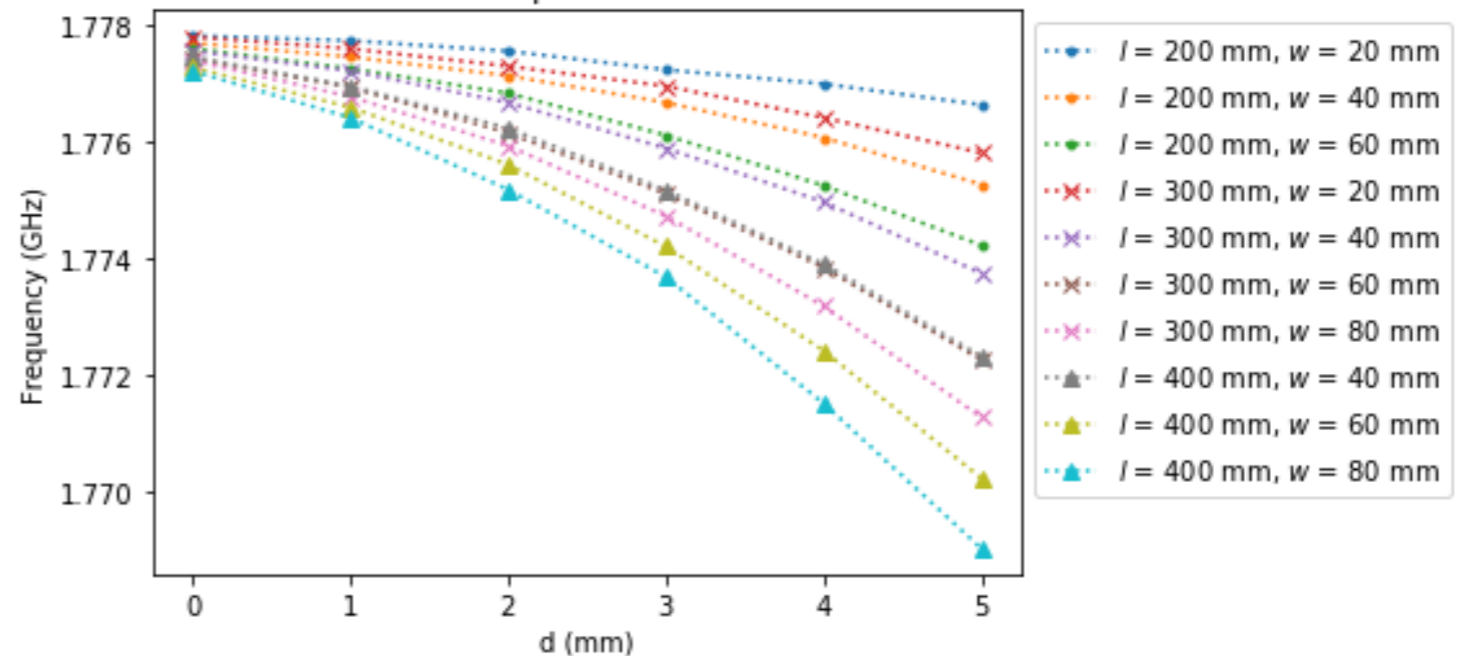
$$\Delta\epsilon = (\epsilon_r - 1)\epsilon_0, \Delta\mu = (\mu_r - 1)\mu_0$$

D. Pozar, Microwave Engineering, Wiley (2012)

B-field strength of F210 ($y = 0$)

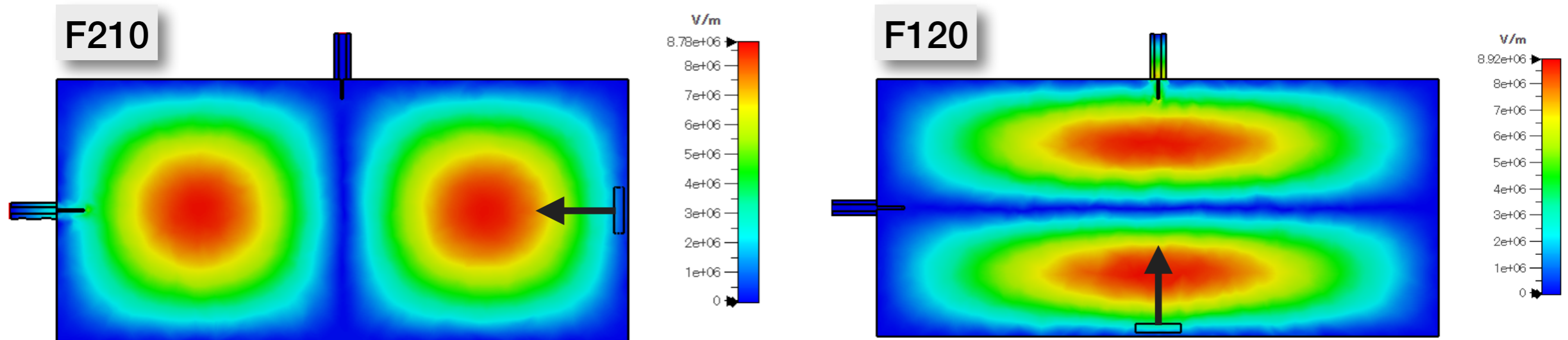


Alumina plate, $t = 4$ mm

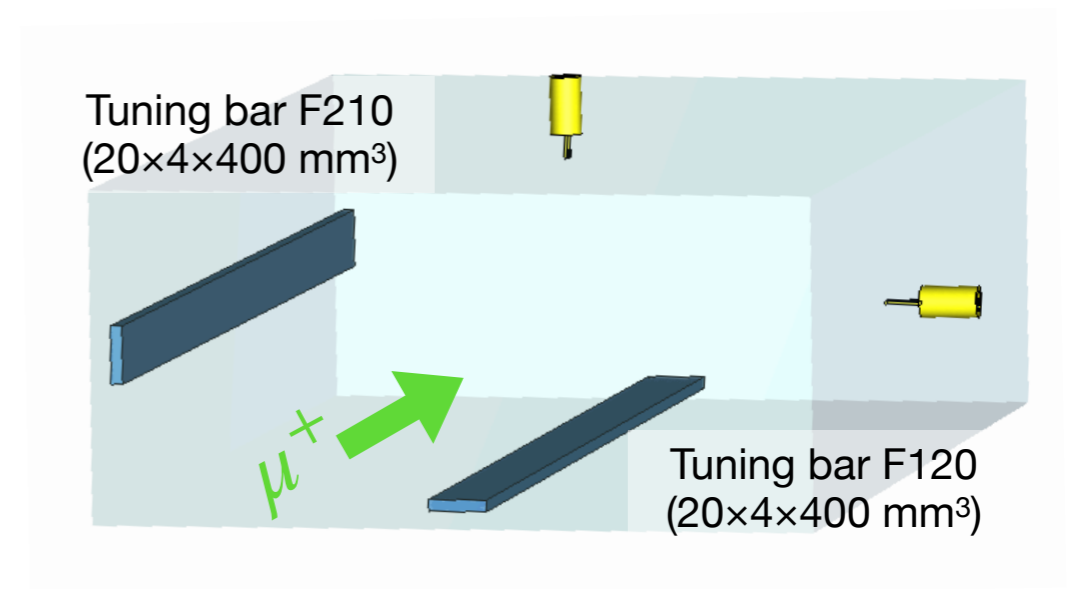
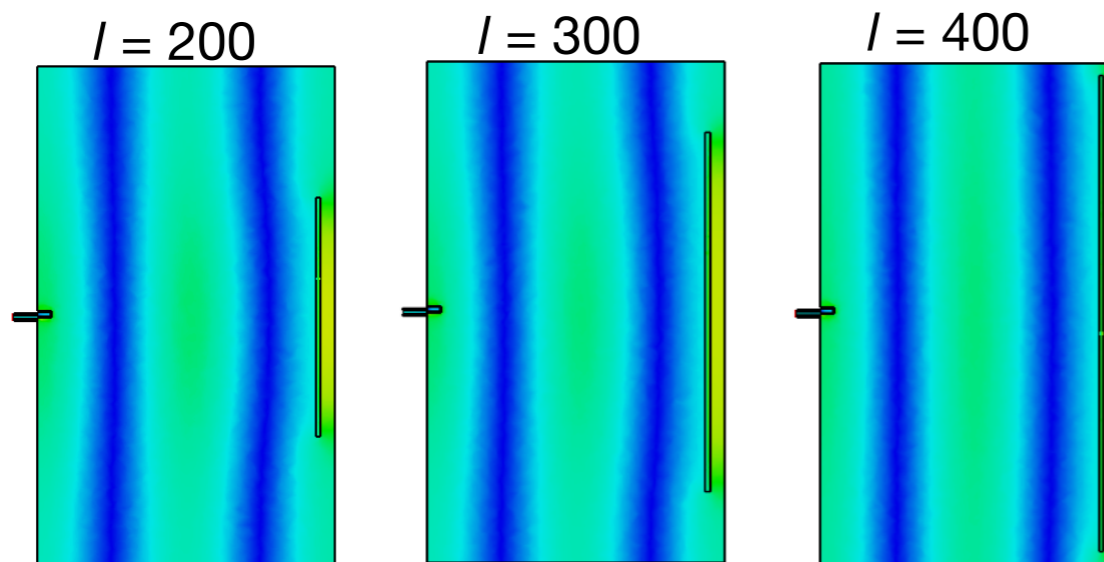


Rectangular cavity: tuning scheme 2

- Locations to insert two bars
 - At maximum E-field strength regions

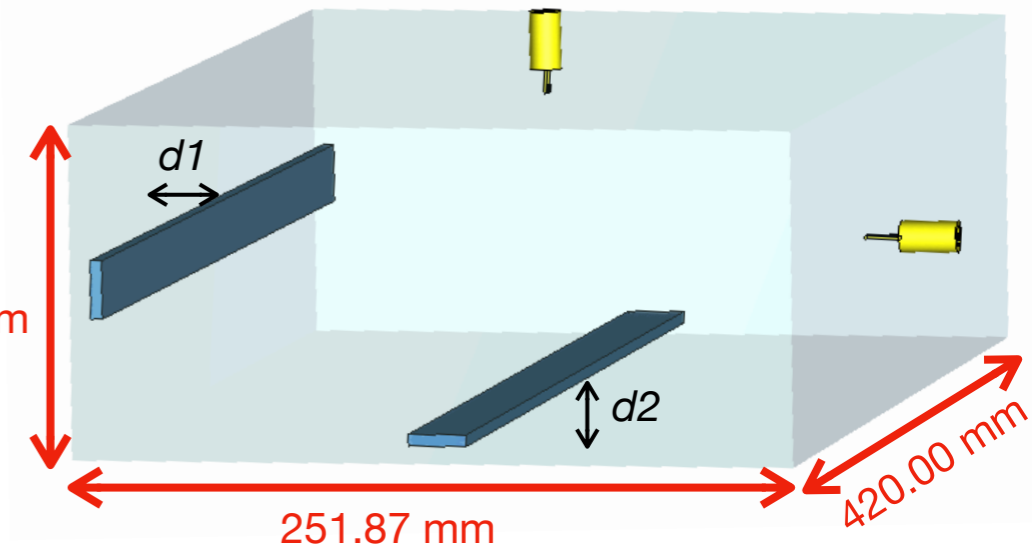


- Dimensions of the bar
 - Need longer length for uniformity in the beam axis direction

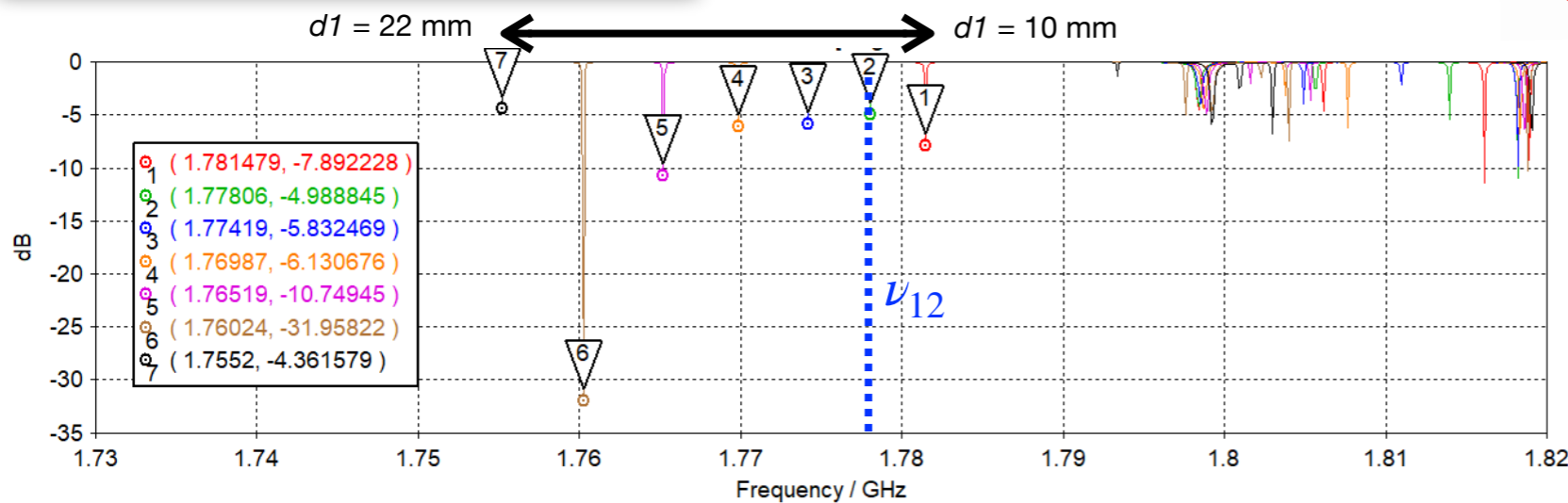


Rectangular cavity: final dimensions

- Final cavity dimensions is optimized by taking into account frequency shifts with tuning bars

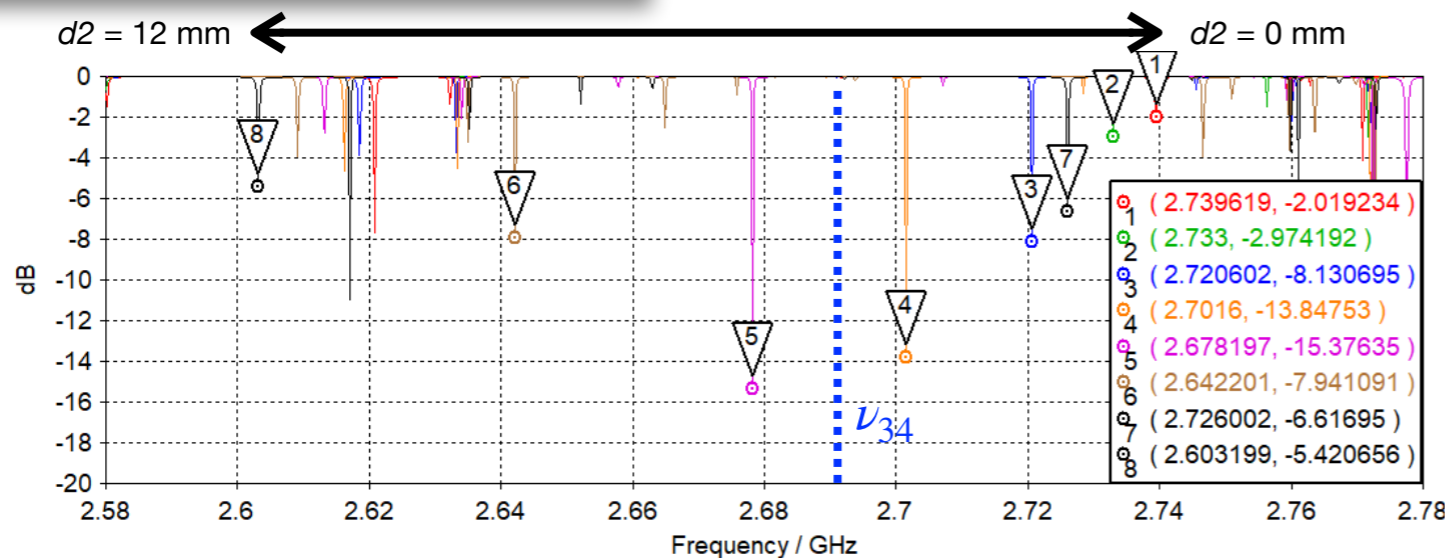


F210 (starting from $d1 = 10$ mm)



- Sweep range ~25 MHz
- Very clean!

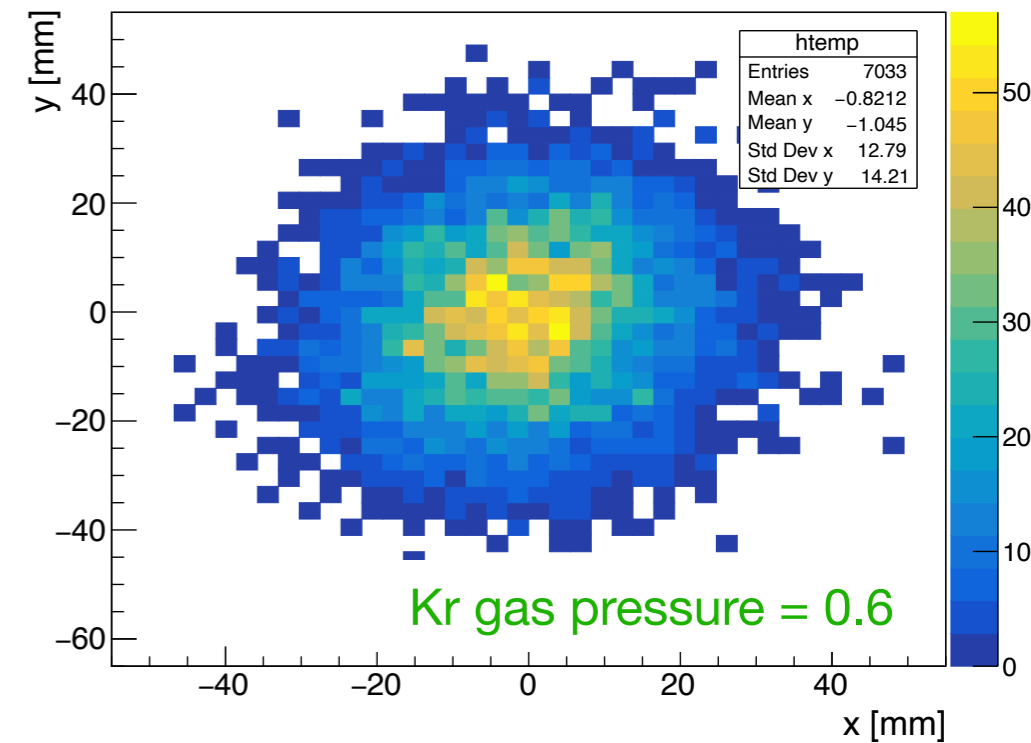
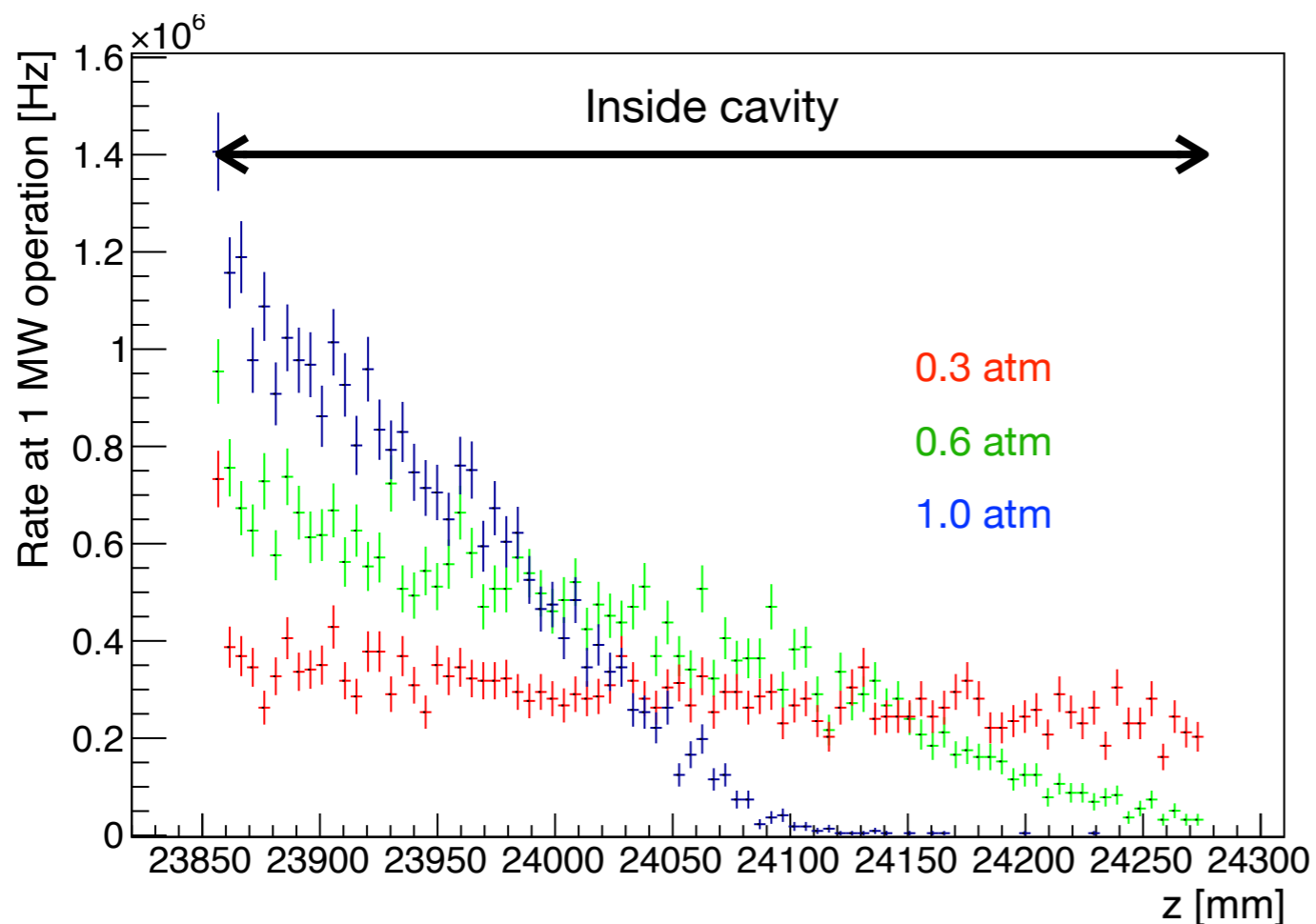
F120 (starting from $d2 = 0$ mm)



- Sweep range ~136 MHz
- Overlapping with F314 at ~2.63 GHz

Muon stopping distribution in the cavity

- Transverse beam spot is smaller than cavity height (112 mm)
- While, more muons stopped in upstream region. Strong focusing of the muon beam with 2.9 T field results in larger energy losses at upstream components

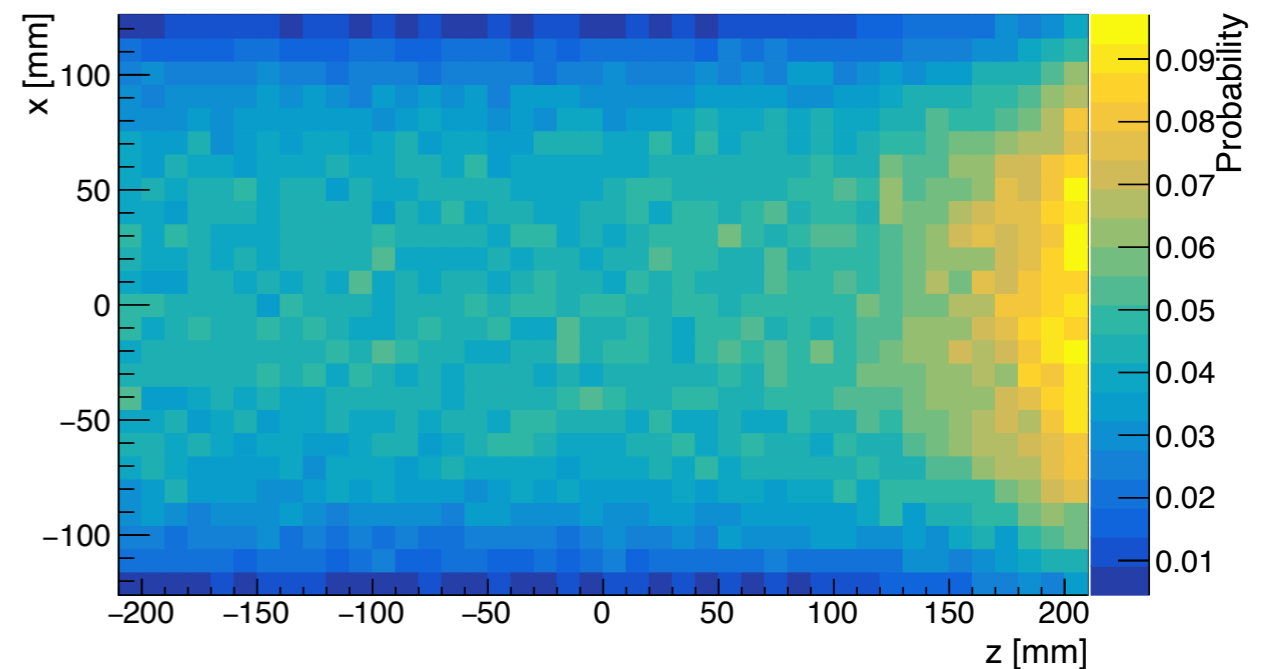
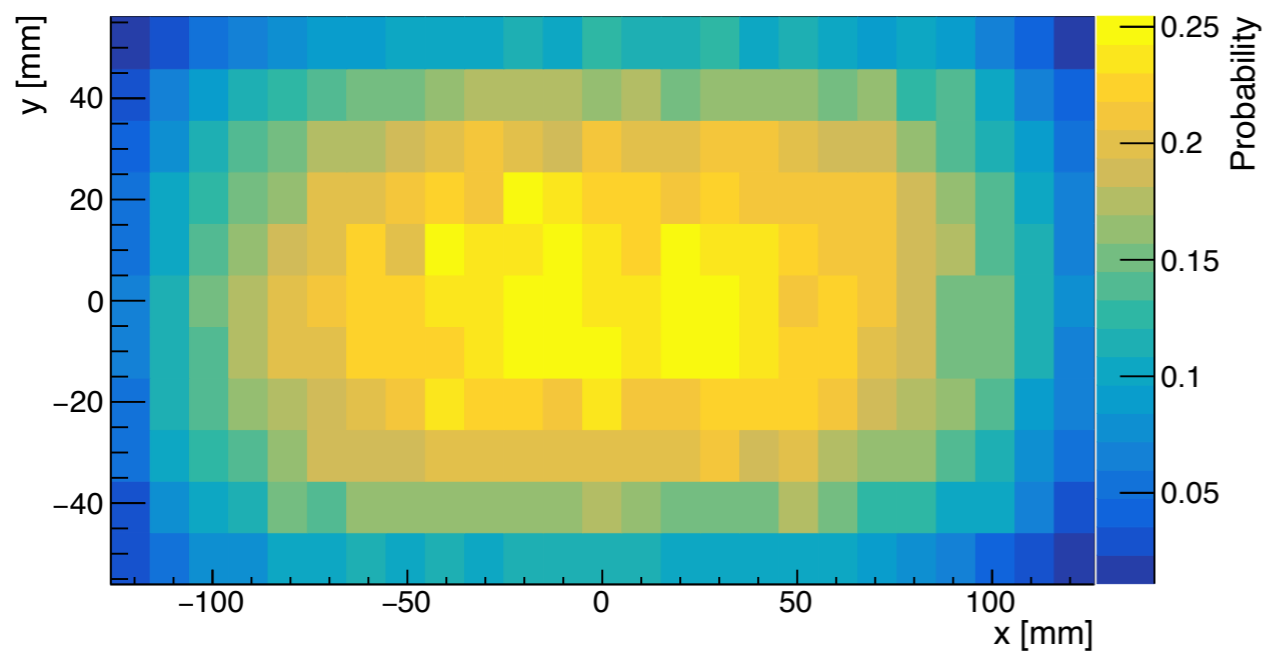
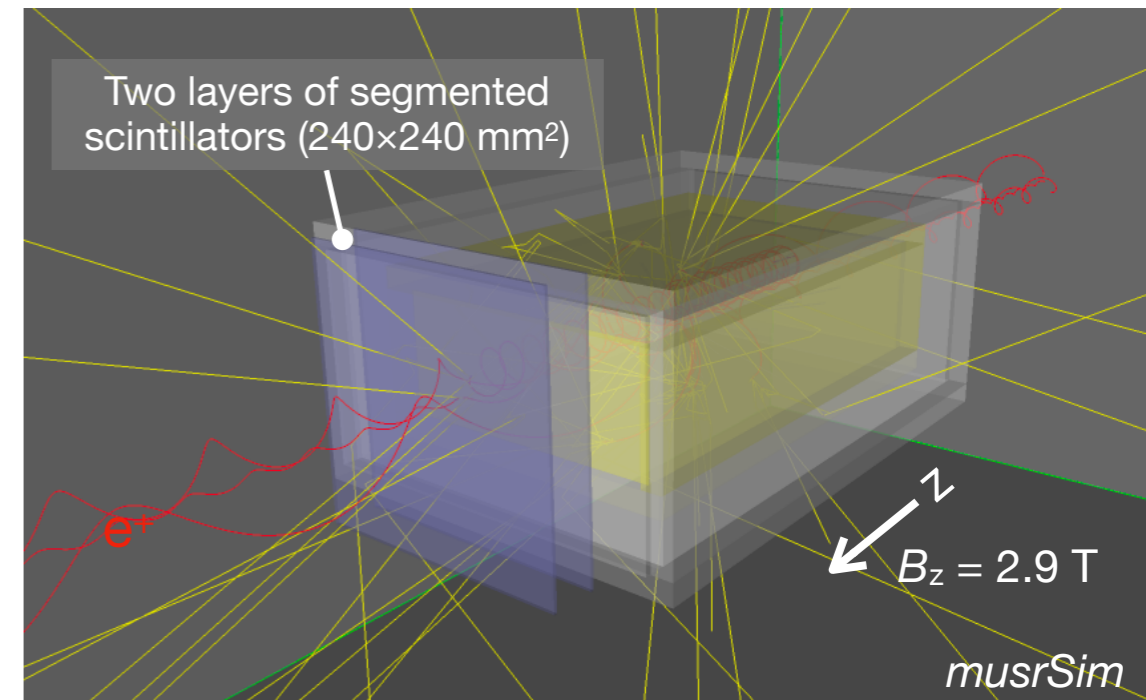


Muon stopping rate in cavity

Kr gas pressure	Stopping rate at 1 MW
0.3 atm	25.1 MHz
0.6 atm	32.4 MHz
1.0 atm	29.4 MHz

Detection efficiency of decay positrons

- Positron detectors placed downstream side of the cavity
- Simulated 3D-map of the detection efficiency
- 2.9 T field helping to swept away positrons from inside the cavity
- Effect of pile-up to be investigated

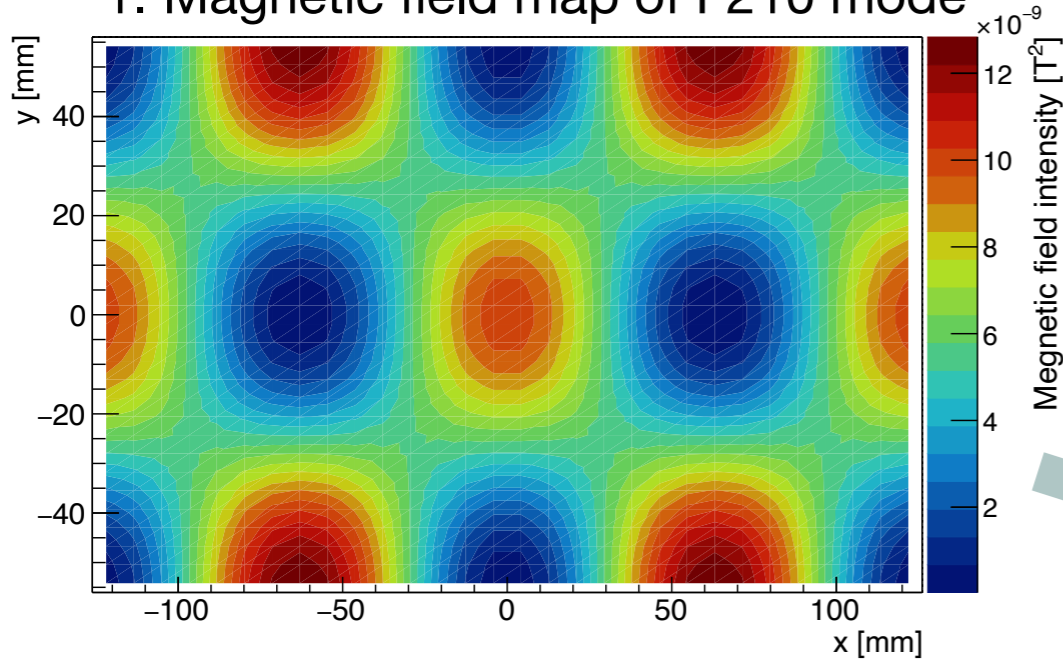


N.B. For coincidence hits of two layers, all muons spins directed in +z-direction

ToyMC simulation of resonance line shape

- Example for simulating ν_{12} transition
- Statistical and systematic uncertainties can be calculated with this simulation package

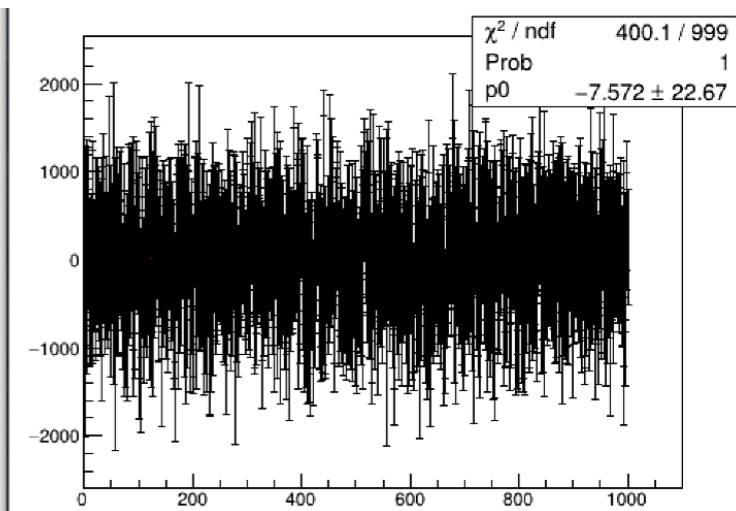
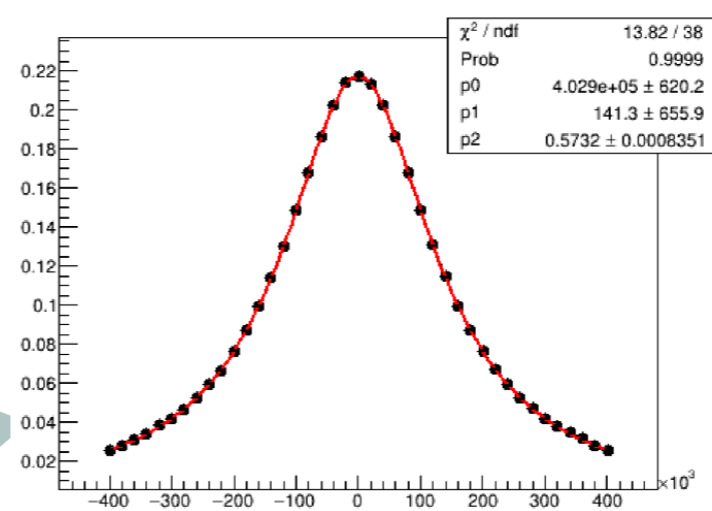
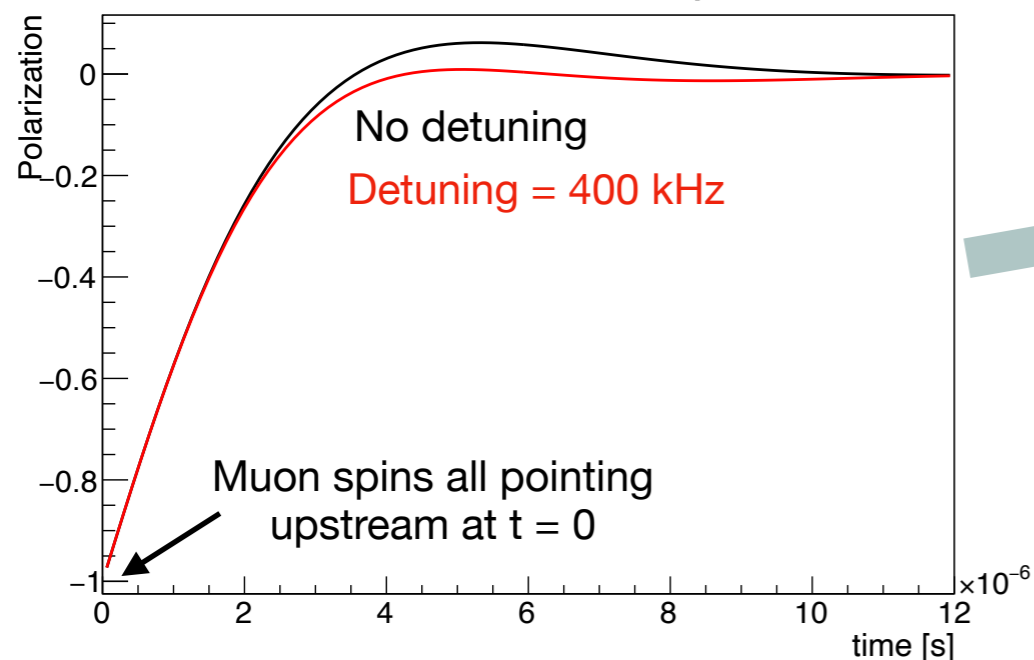
1. Magnetic field map of F210 mode



3. Muon stopping distribution in the cavity (previous page)

4. Detection efficiency map for decay positrons (previous page)

2. Time evolution of muon's polarization



1S-2S laser spectroscopies

- Doppler-free two-photons excitation
- Determine muon mass with