Existence of Ground-state solutions of 1D Relativistic Schrödinger Equation
A Numerical Method using directly minimization of the energy functional

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We study the existence of solitary wave solutions of the 1D non linear pseudo-relativistic Schrödinger cubic equation

$$i\psi_t = \left(\sqrt{m^2 - \partial_x^2} - m\right)\psi - |\psi|^2\psi$$

which describes the dynamics of a particle accelerated to a significant fraction $|v| < 1$ of the speed light $c = 1$. The classic non linear cubic term represents the contribution of de charge density to the solution.

For mass $m > 0$, we prove existence of travelling solitary waves $\psi(t,x) = \varphi_v(x - vt)\, e^{it\mu}$, where $\mu \in \mathbb{R}$ is a phase parameter.

We introduce a variational problem that yield the functions $\varphi_v \in H^{1/2}(\mathbb{R})$ as minimizers of the functional, which we call boosted ground states.

Our existence proof uses the concentration-compactness principle of J.L. Lions, trough a suitable adaptation of this problem (as in a J. Frölich, et al, work published in 2007).

The second part of this work is devoted to develop an efficient numerical method to compute profile φ_v of the above boosted ground state solution.

We make this method by directly minimizing the energy functional. This minimization process is carried out in the frequency space. We start this minimization process in an solution of an approximated differential equation of the ground state.