# Status of the Muon $g_\mu - 2$

## Experiment versus Theory

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$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\vec{s} \qquad \text{and} \qquad \underbrace{g_\mu = 2}_{\text{Dirac}}(1 + a_\mu)$$

$$a_\mu = \frac{1}{2}(g_\mu - 2) \quad : \quad \text{anomalous magnetic moment}$$

World average experimental value:
(dominated by the BNL–experiment)

$$a_\mu(\text{exp.}) = 11\ 659\ 202.3(15.1) \times 10^{-10}\ [1.3\text{ppm}]\ (\text{Sep.'01})$$

$$\boxed{a_\mu(\text{exp.}) = 11\ 659\ 203 \qquad (8) \times 10^{-10}\ [0.7\text{ppm}]\ (\text{Sep.'02})}$$

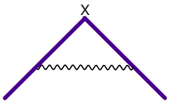
Standard Model "prediction" (September 2001) was:

$$a_\mu^{\text{SM}} = (11\ 659\ 159.7 \pm 6.7) \times 10^{-10}\,.$$

- Last year's $2.6\sigma$ "discrepancy" $\Rightarrow$ *avalanche* of theoretical papers.

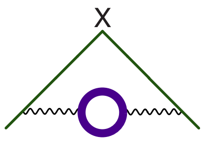
- Today I shall review the present status on the Standard Model prediction.

# Some Theoretical Comments



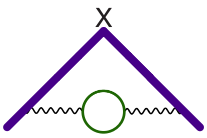
$\Rightarrow \quad a_\mu = a_e = \frac{1}{2}\frac{\alpha}{\pi}$ Schwinger '48

- Loops with *different masses* $\Rightarrow a_\mu \neq a_e$
  - Internal *LARGE* masses decouple:



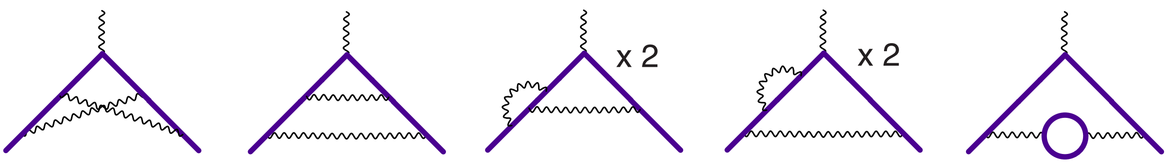

$$\Rightarrow \left[\left(\frac{1}{3}\right)\left(\frac{1}{15}\right)\left(\frac{m_\mu}{m_\tau}\right)^2 + \mathcal{O}\left(\frac{m_\mu^4}{m_\tau^4}\log\frac{m_\tau}{m_\mu}\right)\right]\left(\frac{\alpha}{\pi}\right)^2$$

  - Internal *SMALL* masses give rise to log's of mass ratios:




$$\Rightarrow \Big[\underbrace{\left(\frac{2}{3}\right)}_{\beta_1}\left(\frac{1}{2}\right)\log\frac{m_\mu}{m_e} - \frac{25}{36} + \mathcal{O}\left(\frac{m_e}{m_\mu}\right)\Big]\left(\frac{\alpha}{\pi}\right)^2$$

- Two loops: *7 Feynman diagrams (with common fermion lines)*




$$a_l^{(4)} = \left\{\frac{197}{144} + \frac{1}{12}\pi^2 - \frac{1}{2}\pi^2\ln 2 + \frac{3}{4}\zeta(3)\right\}\left(\frac{\alpha}{\pi}\right)^2$$

Peterman '57
Sommerfield '57

- Three loops: *72 Feynman diagrams (which I can show you)* Laporta– Remiddi '96

- Four loops: *891 Feynman diagrams (which I wan't show you)* Kinoshita (in progress)

# The Muon Anomaly

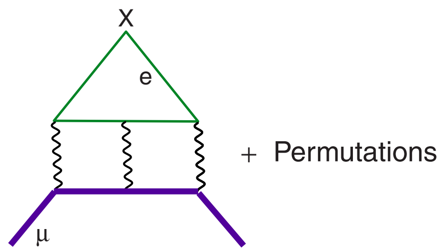
$$a_\mu = [a_e]_{\rm QED} + \underbrace{a_\mu(e,\tau) + a_\mu(\text{hadrons})}_{SU(3)\times SU(2)\times U(1)}$$

- Vacuum Polarization from electron loops
  - Enhanced by QED short–distance logarithms
  - Obey Renormalization Group Equation $[\alpha \Rightarrow \alpha(m_\mu)]$

$$\left(m_e\frac{\partial}{\partial m_e} + \beta(\alpha)\alpha\frac{\partial}{\partial\alpha}\right)a_\mu^{(\infty)}\left(\frac{m_\mu}{m_e},\alpha\right) = 0$$

Lautrup–de Rafael '74

- Light–by–Light Scattering from electron loops




  - Enhanced by QED infrared logarithms

Kinoshita *et al* '69
Laporta–Remiddi '93

$$a_\mu^{(3)}|_{\rm l.byl.} = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \cdots\right]\left(\frac{\alpha}{\pi}\right)^3 = 20.947\ldots\left(\frac{\alpha}{\pi}\right)^3$$

- Vacuum Polarization and Light–by–Light Scattering
  from tau loops, suppressed by Mass Decoupling (but explicitly known)

$$a_\mu(\text{QED}) = (11\ 658\ 470.57 \pm 0.29) \times 10^{-10}$$

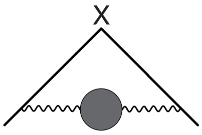
Recall that at present:

$$a_\mu(\text{Exp.}) = (11\ 659\ 203 \quad \pm \quad 8) \times 10^{-10}$$

♣ Is this discrepancy due to the SM Hadronic–EW Interactions ?

# Hadronic Vacuum Polarization and $g_\mu - 2$

- All estimates are based on the spectral representation



$$\sigma(t)_{e^+e^-\to \text{hadrons}} = \frac{4\pi^2\alpha}{t}\underline{\frac{1}{\pi}\text{Im}\Pi(t)}$$

- Adler Function (Relevant QCD Green's Function)

$$\underline{\mathcal{A}(Q^2)} = -Q^2\frac{\partial\Pi(Q^2)}{\partial Q^2} = \int_0^\infty dt \frac{Q^2}{(t+Q^2)^2}\underline{\frac{1}{\pi}\text{Im}\Pi(t)}$$

$$a_\mu^{(\text{h. v.p.})} = \frac{\alpha}{\pi}\int_0^1 dx \frac{1}{x}(1-x)(1-x/2)\underline{\mathcal{A}\left(\frac{x^2}{1-x}m_\mu^2\right)}$$

- Large $N_c$ QCD *Minimal Hadronic Ansatz* Approximation

$$\mathcal{A}(Q^2) = \left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) e^2 \left\{ 2f_V^2 M_V^2 \frac{Q^2}{(Q^2+M_V^2)^2} + \frac{N_C}{16\pi^2}\frac{4}{3}\frac{Q^2}{Q^2+s_0}(1+\cdots)\right\}$$

  – No $1/Q^2$ term in the OPE $\Rightarrow$

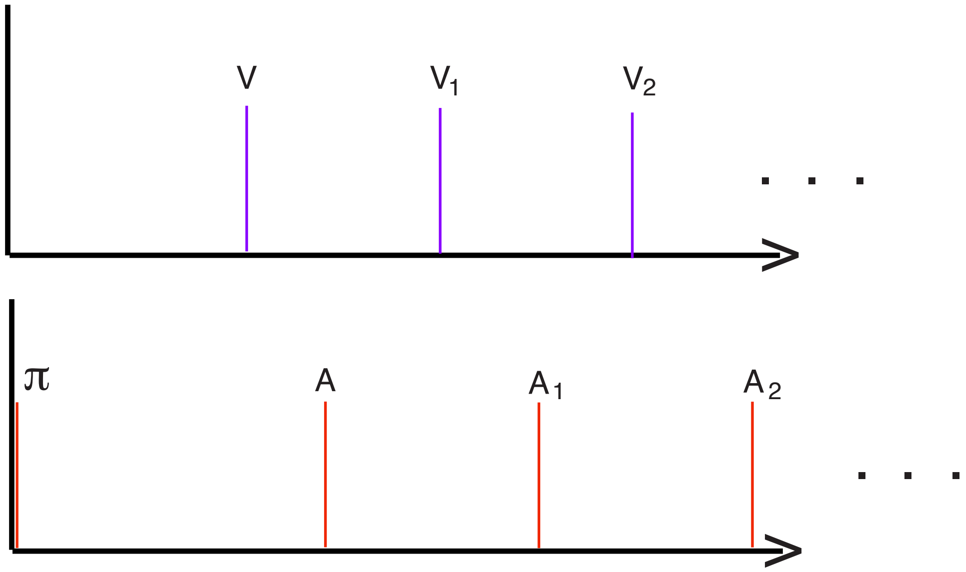
$$2f_V^2 M_V^2 = \frac{N_C}{16\pi^2}\frac{4}{3}s_0\left(1+\frac{3}{8}\frac{\alpha_s(s_0)}{\pi}+\cdots\right)$$

  – Chiral loops (two–pion states) subleading in $1/N_C$

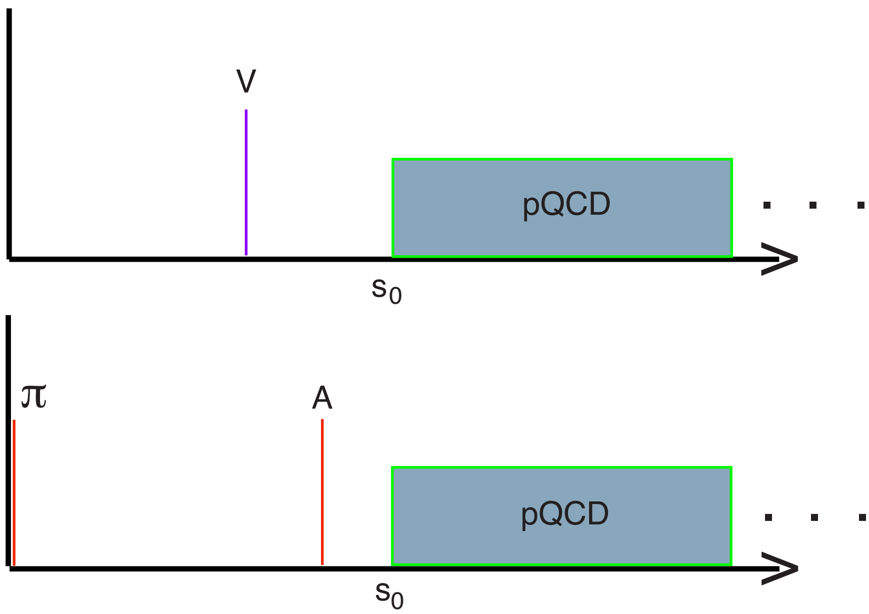
$$a_\mu^{(\text{h. v.p.})} \simeq (5.7 \pm \underbrace{1.7}_{30\%\ \text{sys. error}}) \times 10^{-8}$$

$$\left[ a_\mu^{(\text{h. v.p.})} = (6.924 \pm 0.062)\times 10^{-8} \qquad \textit{Davier–Höcker '98} \right]$$

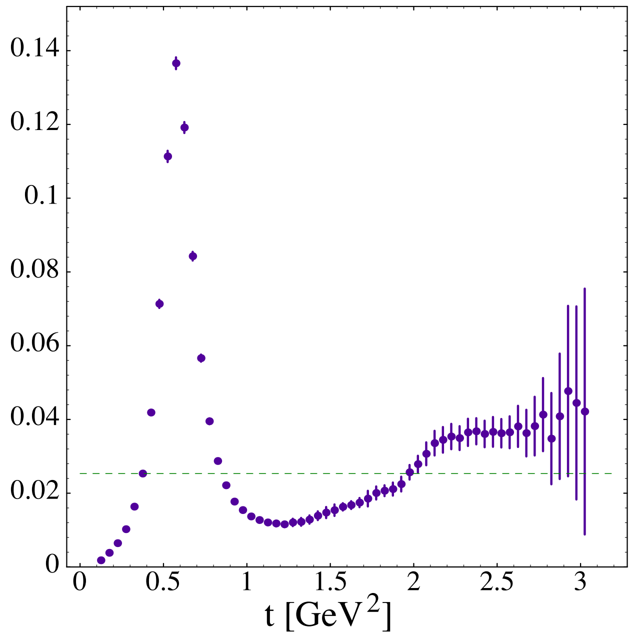
# The Large–$N_c$ World

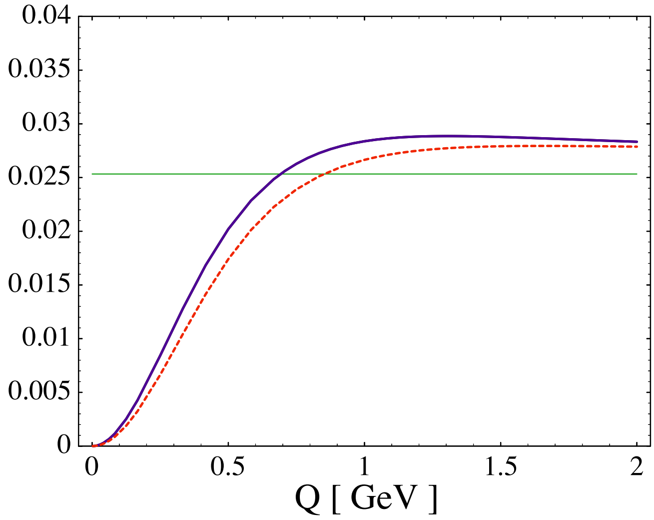



*Minimal Hadronic Approximation*



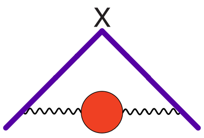
# The Vector Spectral Function (ALEPH–Data)
## versus
## The Adler Function



# Hadronic Vacuum Polarization Results

- All calculations are based on the spectral representation




$$a_\mu^{(\text{h. v.p.})} = \frac{\alpha}{\pi}\int_0^\infty \frac{dt}{t}\,\frac{1}{\pi}\mathrm{Im}\Pi(t)\ \int_0^1 \frac{x^2(1-x)}{x^2+\frac{t}{m_\mu{}^2}(1-x)}dx$$

$$\sigma(t)_{e^+e^-\to\text{hadrons}} = \frac{4\pi^2\alpha}{t}\frac{1}{\pi}\mathrm{Im}\Pi(t)$$

Compilation from Recent Estimates (with comments):

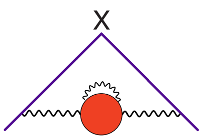
| **Authors** | **Contribution to** $a_\mu \times 10^{10}$ |
|---|---|
| Davier–Höcker | $692.4 \pm 6.2$ |
| Jegerlehner | $697.40 \pm 10.45$ |
| Narison (last v.) | $703.6 \pm 7.6$ |
| de Trocóniz–Ynduráin (v5) | $695.2 \pm 6.4$ |
| Davier *et al* CMD-2 | $684.7 \pm 6.0_{\text{exp}} \pm 3.6_{\text{rad}}$ |
| Davier *et al* ALEPH-CLEO | $701.9 \pm 4.7_{\text{exp}} \pm 1.2_{\text{rad}} \pm 3.8_{SU(2)}$ |
| Hagiwara *et al* | $683.1 \pm 5.9_{\text{exp}} \pm 2.0_{\text{rad}}$ |

- Problems with *possible* double counting
- Higher Order Hadronic Vacuum Polarization

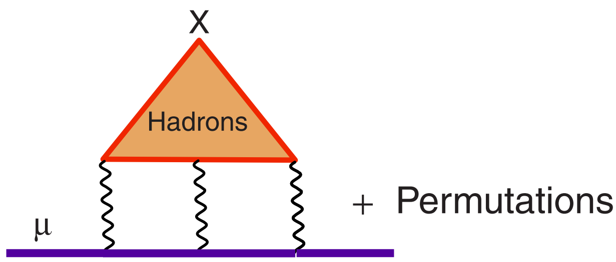
  Calmet, Narison, Perrottet, de Rafael '77; Krause '97

$$a_\mu^{(\text{h.o.-h. v.p.})} = -10.0\ (0.6) \times 10^{-10}$$

- Disagreement, at present, on Hadronic EM self–energy estimates:



# Hadronic L–by–L Scattering



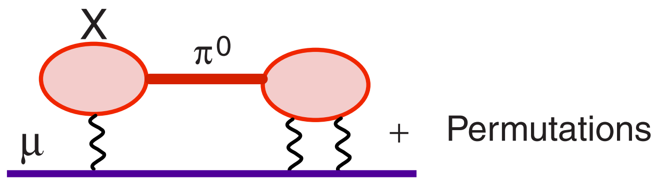

- All Estimates (so far) are model dependent
- Progress in identifying dominant regions of virtual momenta
- Most of the recent estimates use models *compatible* with

  low–energy $\chi$PT behaviour and Large–$N_c$ counting rules (de R '94)

  - ENJL–model Bijnens, Pallante, Prades '96
  - Vector Gauge Model Hayakawa – Kinoshita '98
  - MHA to Large–$N_c$ QCD Knecht–Nyffeler '01

- In these calculations, the dominant contribution comes from

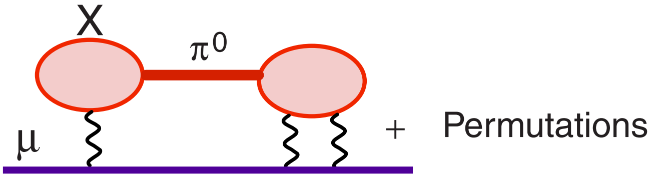
  twice the anomalous VVP vertex




♣ $a_\mu^{(\pi^0 \text{ l by l})} = +(5.8 \pm 1.0) \times 10^{-10}$ Knecht–Nyffeler '01

$$a_\mu^{(\text{H l by l})} = (+8 \pm 4.0) \times 10^{-10}$$

# The Knecht–Nyffeler Calculation




$$a_\mu^{(\pi^0 \text{ l. by l.})} = \int_0^\infty dQ_1^2 \int_0^\infty dQ_2^2 \;\; \mathcal{W}(Q_1^2, Q_2^2)\mathcal{H}(Q_1^2, Q_2^2)$$

$\mathcal{H}(Q_1^2, Q_2^2)$ is a convolution of two $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$ form factors.

In Large–$N_c$ QCD:

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)\Big|_{N_C\to\infty} = \sum_{ij} \frac{c_{ij}(q_1^2, q_2^2)}{(q_1^2 - M_i^2)(q_2^2 - M_j^2)} \,,$$

with constraints on $c_{ij}(q_1^2, q_2^2)$ from Long–Distance and Short–Distance QCD

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0) = -\frac{N_C}{12\pi^2 F_0} \,,$$

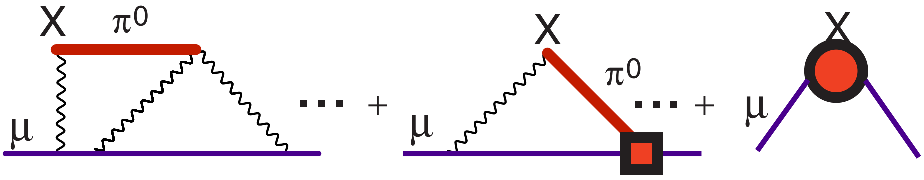
$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}[\lambda^2 q^2, (p-\lambda q)^2] = \frac{2}{3}\frac{F_0}{q^2}\left\{\frac{1}{\lambda^2} + \frac{1}{\lambda^3}\frac{q\cdot p}{q^2}\right\}$$

plus other *phenomenological* constraints

♣ $$a_\mu^{(\pi^0 \text{ l. by l.})} = +(5.8 \pm 1.0) \times 10^{-10}$$

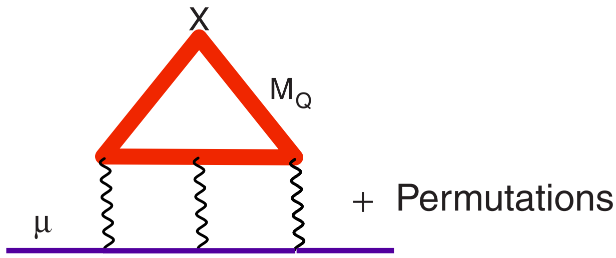
# Effective Field Theory Approach

Knecht–Nyffeler–Perrottet–de Rafael '01




$$a_\mu^{(\pi^0)} = \left(\frac{\alpha}{\pi}\right)^3 \left\{ \underbrace{\frac{N_c^2}{48\pi^2}\frac{m_\mu^2}{F^2}} \log^2\left(\frac{\mu}{m}\right) + \mathcal{O}\left[\log\left(\frac{\mu}{m}\right)\right] + \kappa(\mu) \right\}$$

- Whatever UV– $\mu$, the coefficient of $\log^2 \mu$ is an exact QCD result
- The Knecht–Nyffeler Calculation reproduces this leading behaviour when $M_\rho^2 \longrightarrow \infty$
- The CQM, is NOT an effective theory of QCD !!!



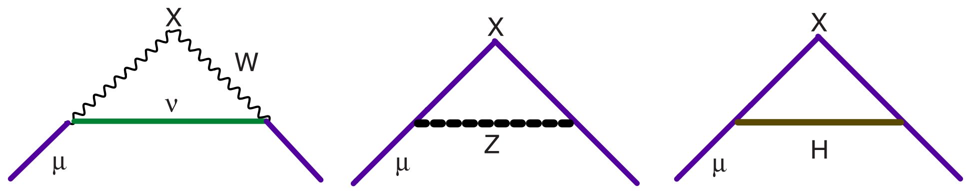

$$a_\mu^{(\mathrm{CQM})} = \left(\frac{\alpha}{\pi}\right)^3 N_C \frac{2}{9} \left\{ \underbrace{\left[\frac{3}{2}\zeta(3) - \frac{19}{16}\right]}_{0.616} \left(\frac{m_\mu}{M_Q}\right)^2 + \mathcal{O}\left[\left(\frac{m_\mu}{M_Q}\right)^4 \log^2\left(\frac{M_Q}{m_\mu}\right)\right] \right\}$$

Laporta–Remiddi '93

# Weak Interactions

- One Loop

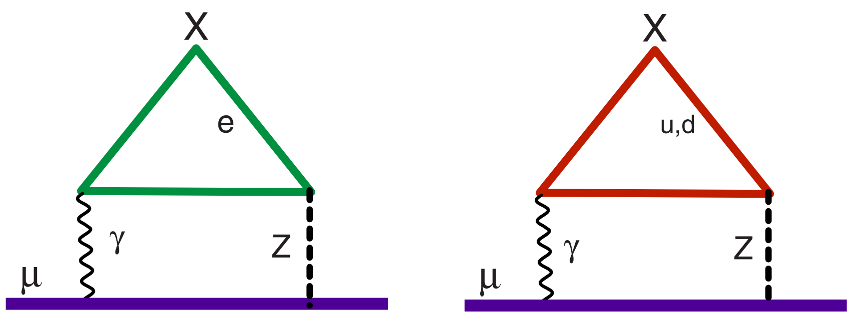
  Bardeen–Gastmans–Lautrup 72, Altarelli–Cabbibo–Maiani 72, Jackiw–Weinberg 72, Bars–Yoshimura 72, Fujikawa–Lee–Sanda 72.




$+38.9 \times 10^{-10}$ $-19.4 \times 10^{-10}$ $\leq 3.3 \times 10^{-14}$

- Two Loops
  - Possible large terms of $\mathcal{O}\left[\left(\frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\right) \times \frac{\alpha}{\pi} \log \frac{M^2}{m_l^2}\right]$
  - Separation of LEPTONS and QUARKS no longer possible

    Peris–Perrottet–de Rafael '95, Czarnecki–Krause–Marciano '95




$$a_\mu^{EW} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2 - \underbrace{\left(\frac{\alpha}{\pi}\right)(159\pm4)}_{22\%}\right]$$

$$=(15.2 \pm 0.1) \times 10^{-10}$$ Knecht–Peris–Perrottet–de Rafael 02

# Electroweak Results from $\gamma\gamma Z$

*Knecht–Peris–Perrottet–de Rafael 02*

- Third Generation:

$$\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\left[-3\log\frac{M_Z^2}{m_\tau^2}-\log\frac{M_Z^2}{m_b^2}-\frac{8}{3}\log\frac{m_t^2}{M_Z^2}+\frac{8}{3}+\mathcal{O}\left(\frac{M_Z^2}{m_t^2}\log\frac{m_t^2}{M_Z^2}\right)\right]$$

$$=\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times(-30.6)\,.$$

- Second and First Generation:

$$\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times\left\{-3\log\frac{M_Z^2}{m_\mu^2}-\frac{5}{2}\right.$$

$$-3\log\frac{M_Z^2}{m_\mu^2}+4\log\frac{M_Z^2}{m_c^2}-\frac{59}{6}+\frac{8}{9}\pi^2$$

$$\left.+\left[\frac{4}{3}\log\frac{M_Z^2}{m_\mu^2}+\frac{2}{3}+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)\right]+4.57\pm1.80+0.04\pm0.02\right\}$$

$$=\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\times(-28.5\pm1.8)\,,$$

# Full Electroweak Contribution to Two Loops

♣ $$a_\mu^{EW}=\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2-\left(\frac{\alpha}{\pi}\right)(159\,\pm\,4)\right]$$

$$=(15.2\,\pm\,0.1)\times10^{-10}\,,$$

- Includes error from Higgs contribution Czarnecki–Krause–Marciano '95
- The two–loop contribution reduces the one–loop result by 22%

# Summary of SM Contributions

- Leptonic QED contributions

$$a_{\rm QED}(\mu) = \; 11\;658\;470.57 \pm 0.29 \times 10^{-10} \qquad * \;\; * \;\; *$$

- Hadronic Contributions
  - Vacuum Polarization ( Hagiwara *et al* Sept. 02)

$$a_{\rm hadronic}^{\rm (VP)} = [\underbrace{683.1 \pm 7.9}_{\rm LO} - \underbrace{10.0 \pm 0.6}_{\rm HO}]) \times 10^{-10}$$

(Questions of Radiative Corrections and Double Counting...)

  - Light–by–Light (After Knecht–Nyffeler's hadronic light-by-light calculation)

$$a_{\rm hadronic}^{\rm (light\ by\ light)} = \underbrace{(8 \pm 4)}_{\rm could\ be\ improved} \times 10^{-10} \quad *$$

- Electroweak Contributions (after the calculation reported here)

$$a_{\rm EW} = (15.2 \pm 0.1) \times 10^{-10} \qquad * \;\; *$$

- Total Standard Model Contribution

$$a_\mu^{\rm SM} = (11\;659\;167 \quad \pm 9) \times 10^{-10}$$
$$a_\mu^{\rm exp} = (11\;659\;203 \quad \pm 8) \times 10^{-10}$$

♣ $$\boxed{a_\mu^{\rm exp} - a_\mu^{\rm SM} = (36 \pm 12) \times 10^{-10} \qquad 3\sigma}$$

**Be prepared for another avalanche of theorrtical papers !**