The Dispersive Matrix perspective on Lepton Flavour Universality observables

Work in collaboration with G. Martinelli, M. Naviglio and S. Simula

[PRD ’21 (2105.02497), PRD ’21 (2105.07851), PRD ’22 (2105.08674), EPJC ‘22 (2109.15248), PRD ‘22 (2204.05925)]

Ludovico Vittorio (LAPTh & CNRS, Annecy, France)

New Frontiers in Lepton Flavor

Pisa, 15th May 2023

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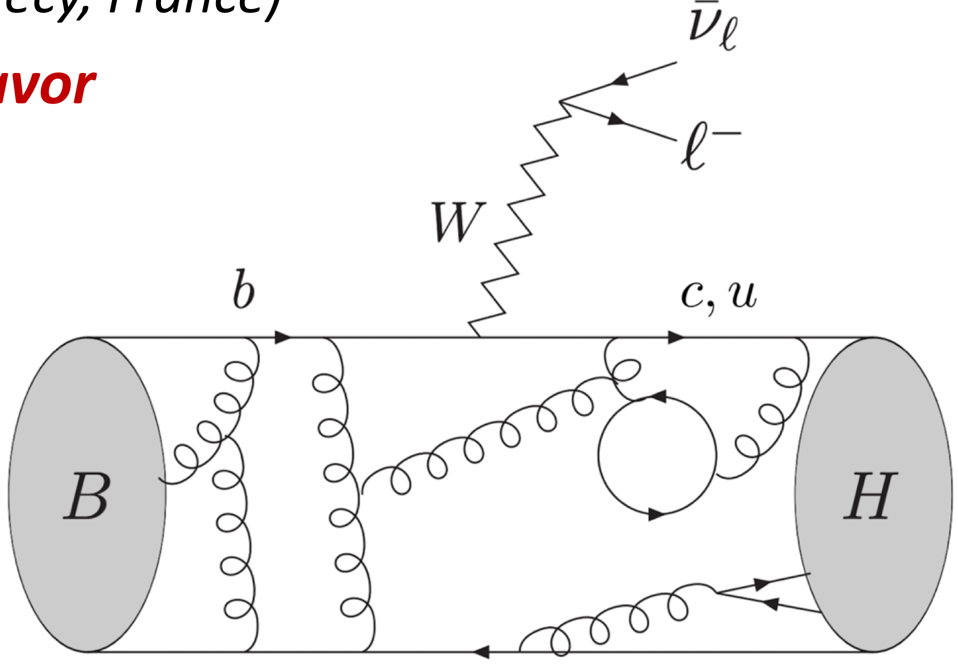
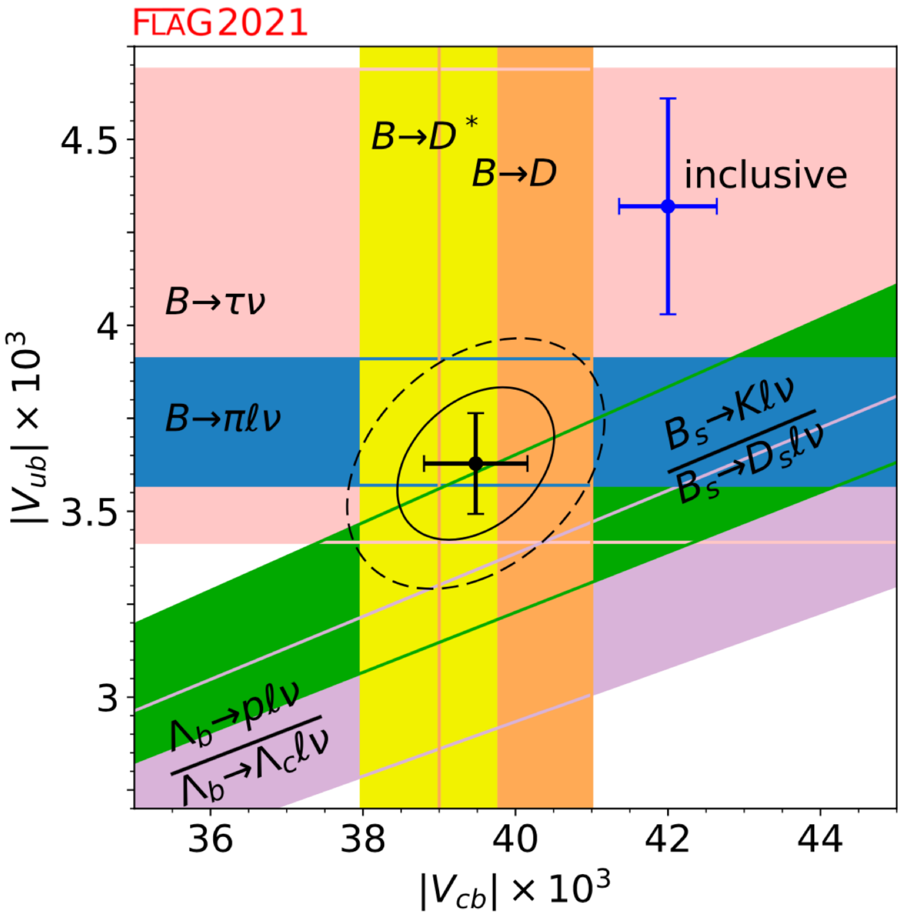


(from J.Phys.G 46 (2019) 2, 023001)

Many challenges in $b \to c$ decays at present

Although there is *no direct evidence for New Physics from experiments*, many phenomenological puzzles need a solution:

1. CKM matrix elements puzzles



FLAG Review 2021 [EPJC '22 (2111.09849)]

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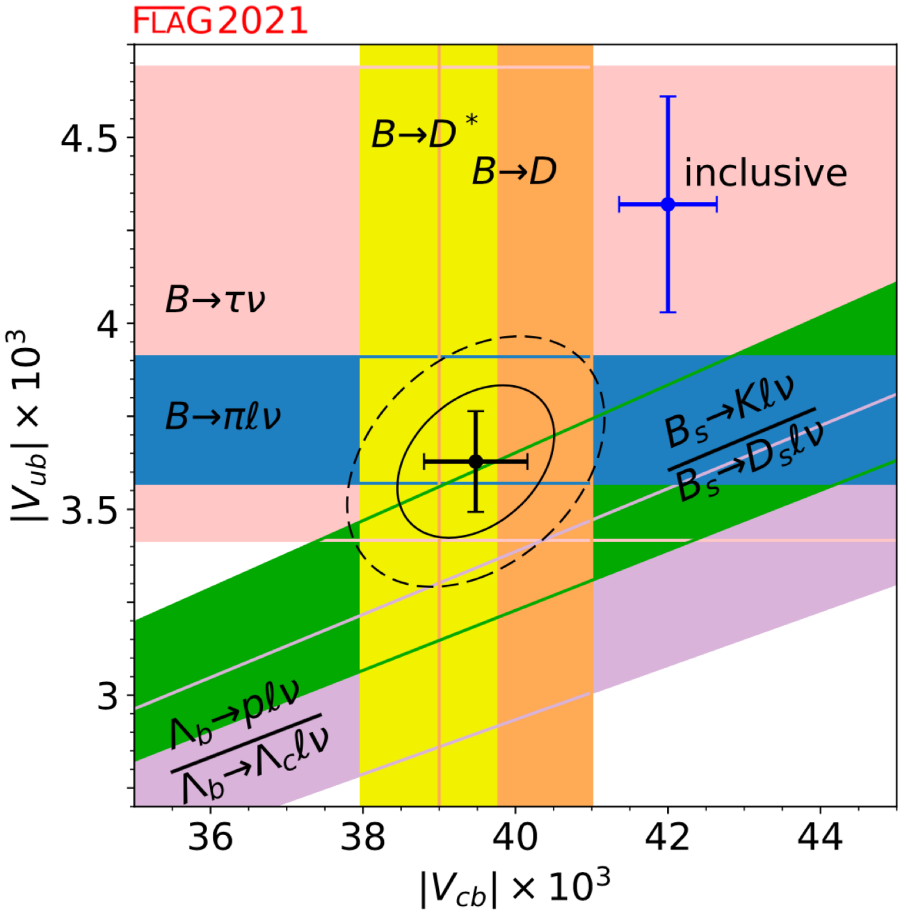

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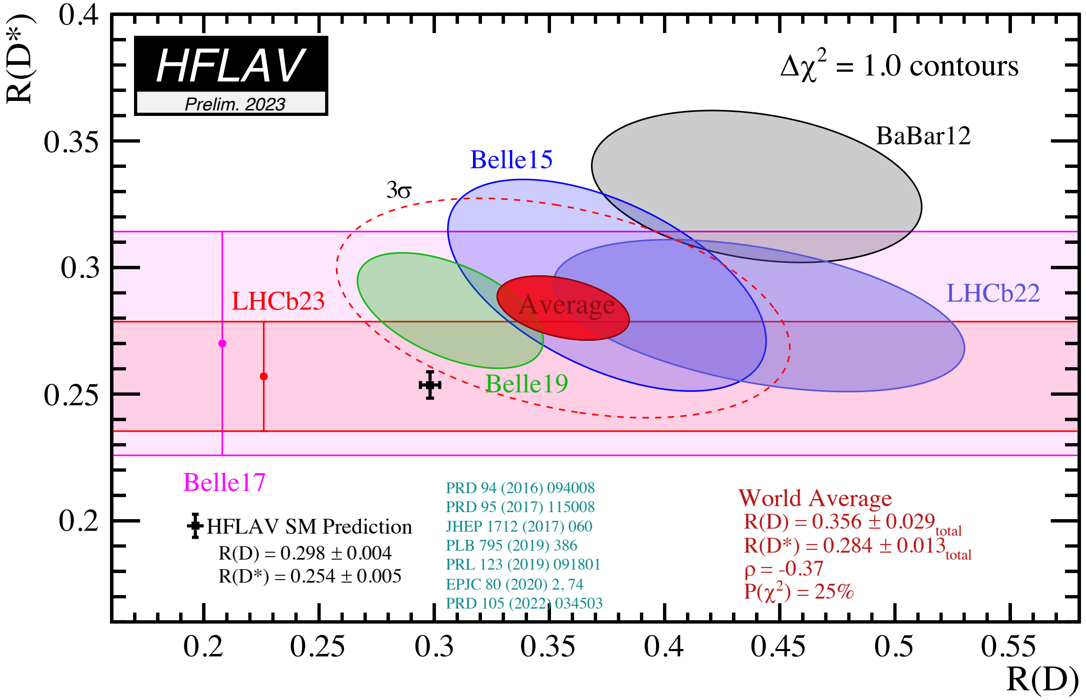

HFLAV Collaboration, PRD '23 [arXiv:2206.07501] (updated plot)

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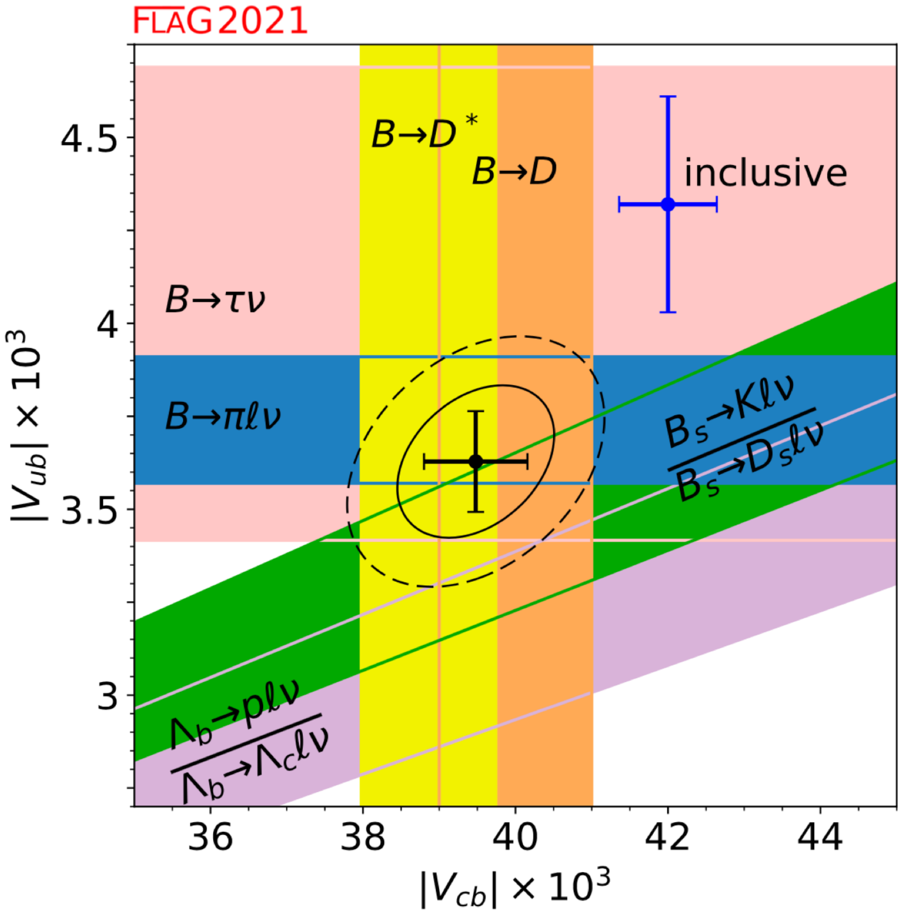

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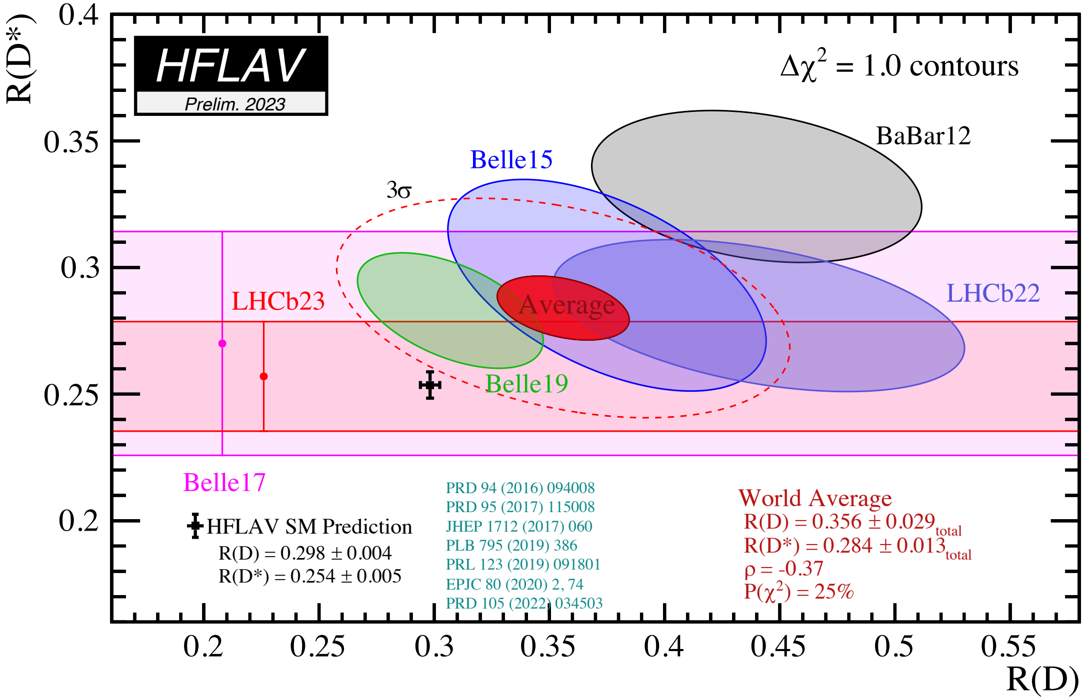

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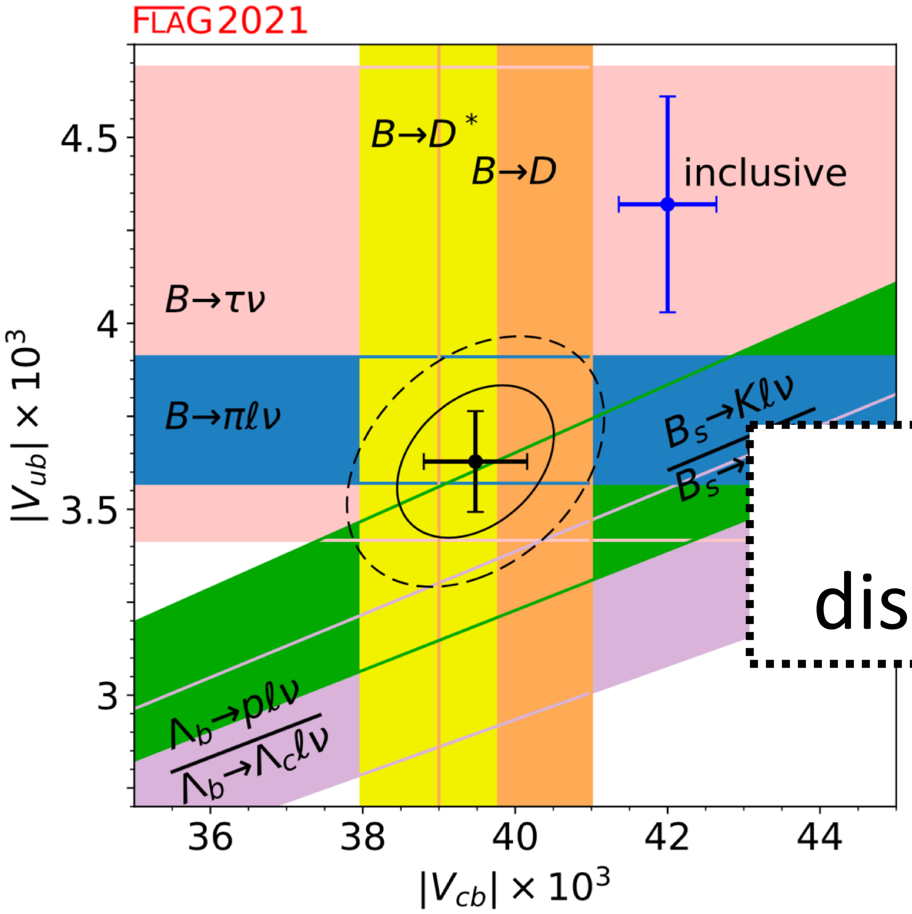

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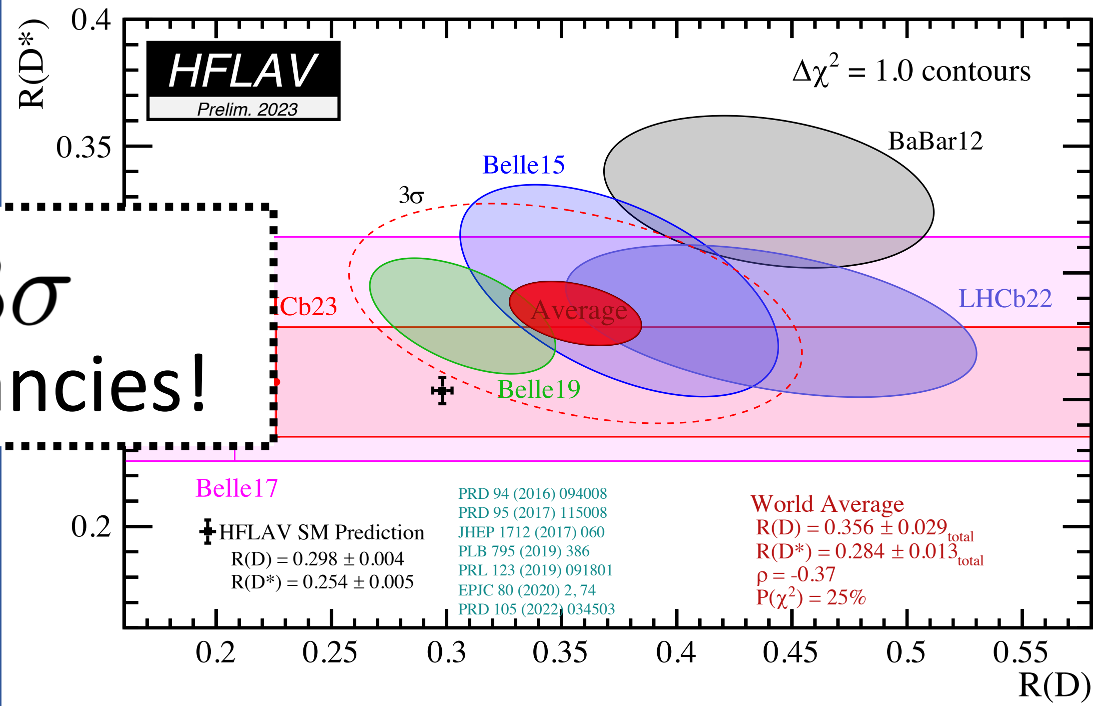


HFLAV Collaboration, PRD '23 [arXiv:2206.07501] (updated plot)

$\sim 3\sigma$ discrepancies!

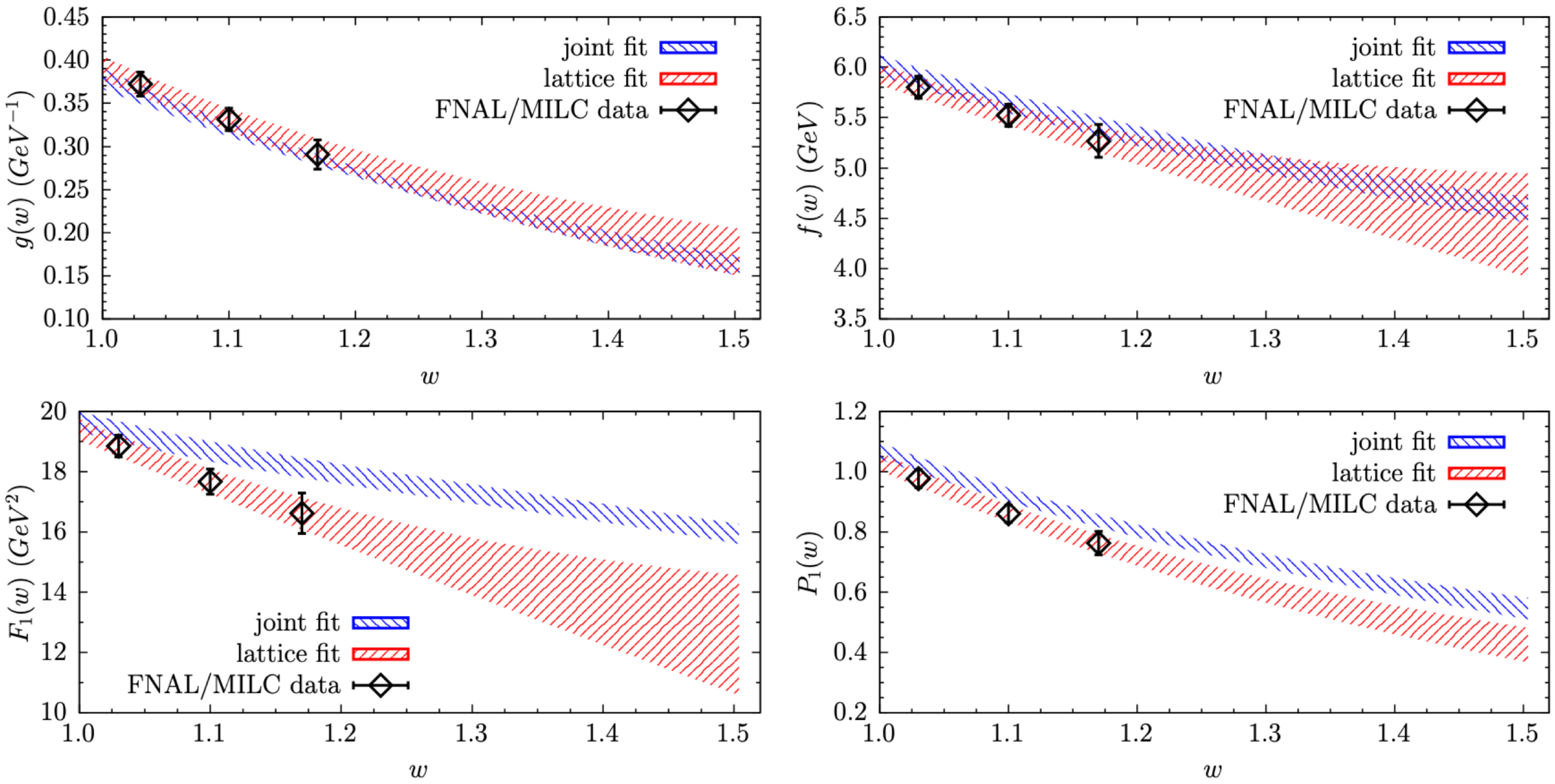
Why not doing a global fit of lattice and exp. data?

We have at our disposal both experimental data from Belle Collaboration (new data in arXiv:2301.07529 [hep-ex])

$$d\Gamma/dx, \;\; x = w, \cos\theta_l, \cos\theta_v, \chi$$

Belle Coll., PRD '19 [arXiv:1809.03290]

and lattice data computed by FNAL/MILC Collaborations in EPJC '22 [arXiv:2105.14019], which have performed **two fits**:



joint fit:
BGL fit of LQCD points + Belle + BaBar exp. data

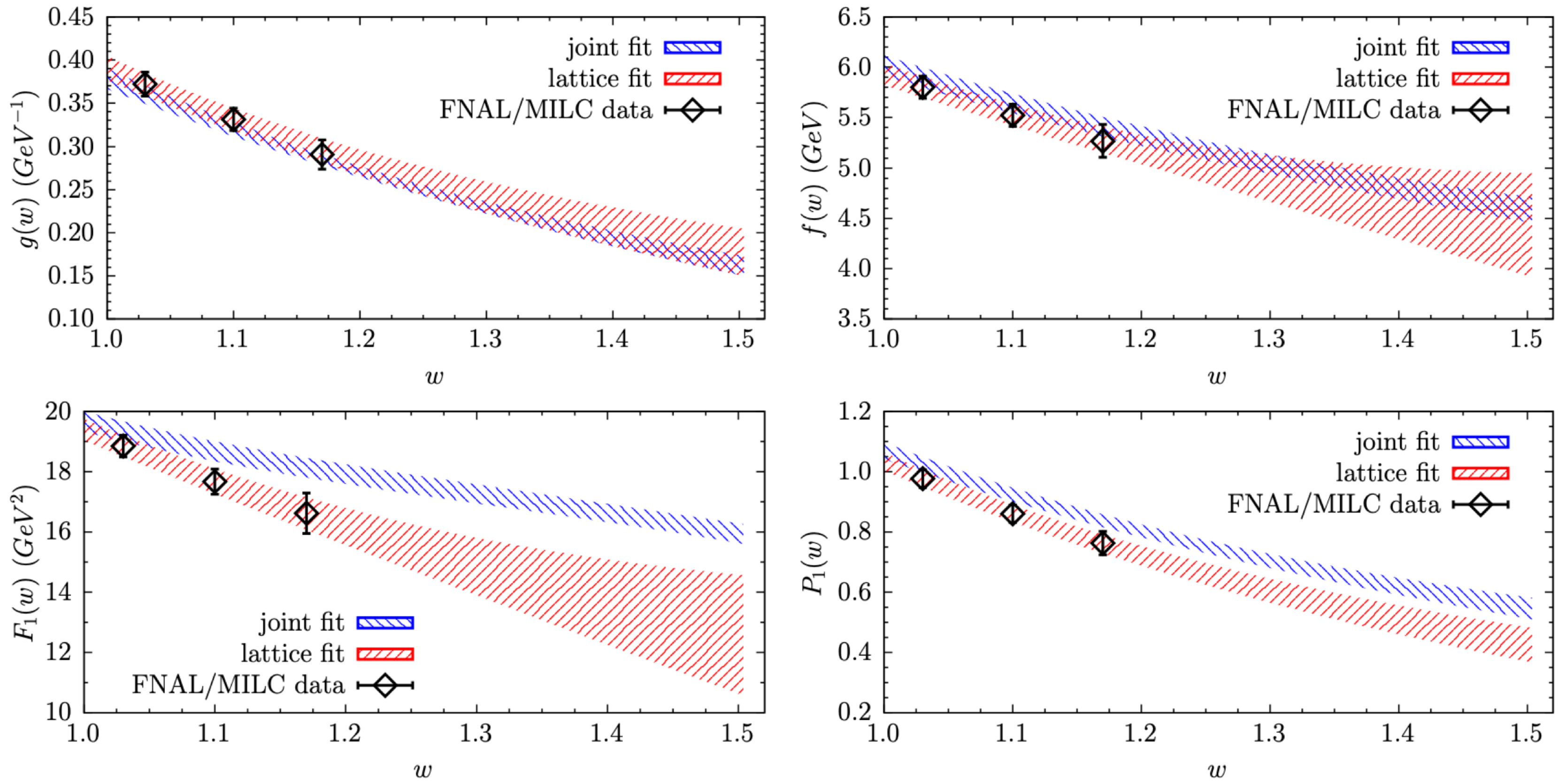
$R(D^*) = 0.2483 \pm 0.0013$

lattice fit:
quadratic BGL fit of LQCD points only

$R(D^*) = 0.265 \pm 0.013$

Why not doing a global fit of lattice and exp. data?

Which value of R(D*) should we trust?



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The central role of the Form Factors (FFs) in excl. semil. *B* decays

- Production of a pseudoscalar meson (*i.e. D,π*):

$$\frac{d\Gamma}{dw} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{48\pi^3} \frac{4r\, m_D^3\, (m_B + m_D)^2\, (w^2-1)^{3/2}}{(1+r)^2} \boxed{|f_+(w)|^2}$$

- Production of a vector meson (*i.e. D**):

$$\frac{d\Gamma(B \to D^*(\to D\pi)\ell\nu)}{dw\, d\cos\theta_\ell\, d\cos\theta_v\, d\chi} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{4(4\pi)^4} 3 m_B m_{D^*}^2 \sqrt{w^2-1}$$

$$\times\, B(D^* \to D\pi)\{(1-\cos\theta_\ell)^2 \sin^2\theta_v |H_+|^2$$
$$+ (1+\cos\theta_\ell)^2 \sin^2\theta_v |H_-|^2 + 4\sin^2\theta_\ell \cos^2\theta_v |H_0|^2$$
$$- 2\sin^2\theta_\ell \sin^2\theta_v \cos 2\chi H_+ H_-$$
$$- 4\sin\theta_\ell (1-\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_+ H_0$$
$$+ 4\sin\theta_\ell (1+\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_- H_0\},$$

$$H_\pm(w) = \boxed{f(w)} \mp m_B m_{D^*} \sqrt{w^2-1}\, \boxed{g(w)}$$

$$H_0(w) = \frac{\boxed{\mathcal{F}_1(w)}}{\sqrt{m_B^2 + m_D^2 - 2 m_B m_D w}}$$

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relation between the momentum transfer and the recoil

$$\boxed{q^2 = m_B^2+m_P^2-2m_Bm_P\,w}$$

If the lepton is **NOT** massless? Two other FFs!

$$\boxed{f_0(w)}\ \text{(pseudoscalar)},\ \boxed{P_1(w)}\ \text{(vector)}$$

The Dispersive Matrix (DM) method

Our goal is to describe the FFs using a novel, non-perturbative and model independent approach: starting from the available LQCD computations of the FFs in the high-q^2 (or low-w) regime, we extract the FFs behaviour in the low-q^2 (or high-w) region!

- Pioneering works from S. Okubo [PRD, 3 (1971); PRD, 4 (1971)], C. Bourrely et al [NPB, 189 (1981)] and L. Lellouch [NPB, 479 (1996)]
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The resulting description of the FFs

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- **is independent of any assumption on the functional dependence of the FFs on the momentum transfer**
- **can be applied to theoretical calculations of the FFs, but also to experimental data**
- **keep theoretical calculations and experimental data separated**
- **is universal: it can be applied to any exclusive semileptonic decays of mesons and baryons**

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How does it work?

The DM method

Let us focus on a generic FF *f*: **we will determine f(t) with f(t_i) known at positions t_i (i=1, …, N)**

$$z(t) = \frac{\sqrt{\frac{t_+ - t}{t_+ - t_-}} - 1}{\sqrt{\frac{t_+ - t}{t_+ - t_-}} + 1}$$

$$t_\pm \equiv (m_B \pm m_D)^2$$

t: momentum transfer

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How? We define

- inner product $$\langle h_1 | h_2 \rangle = \int_{|z|=1} \frac{dz}{2\pi i z} \bar{h}_1(z) h_2(z)$$
- auxialiary function $$g_t(z) \equiv \frac{1}{1 - \bar{z}(t) z}$$

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We build up the matrix M of the scalar products of ϕf, g_t, g_{t1}, …, g_{tN} :

$$\mathbf{M} = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

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The DM method

CENTRAL ISSUE: since **M** contains only inner products, by construction its determinant is semipositive definite

$$\det \mathbf{M} \geq 0 \quad \Rightarrow \quad f_{\rm lo}(z) \leq f(z) \leq f_{\rm up}(z)$$

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t: momentum transfer

DISPERSION RELATIONS:

$$0 \leq \langle \phi f | \phi f \rangle \leq \chi(q^2)$$

$$\left(\frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z} |\phi(z, q^2) f(z)|^2 \leq \chi(q^2) \right)$$

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How to implement the DM method in practice

In a schematic way**, the steps to be implemented are**:

- Generation of input data for the DM method through the mean values and the covariances associated to the LQCD data

- «Filtering» of input data: we obtain **the subset** of events passing the unitarity filters and the kinematical constraint(s);

- Evaluation of the FFs at several values of the momentum transfer;

- Computation of the integral of the theoretical differential decay width (d.d.w.) for each of the experimental q2-bins

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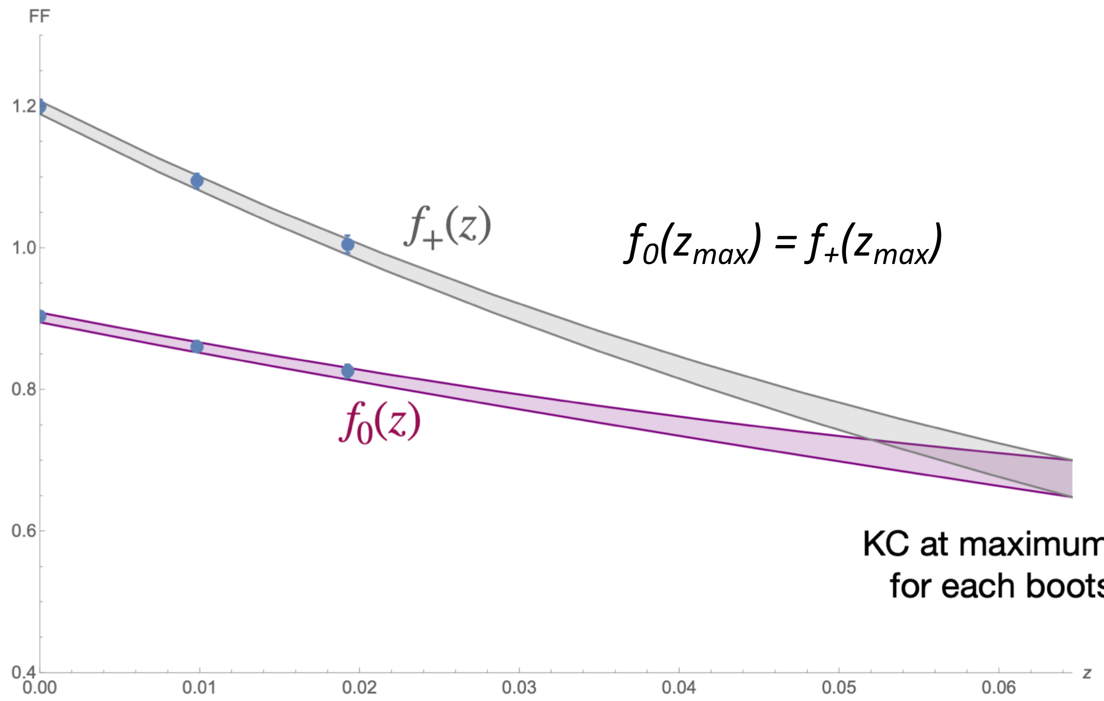
→ ***Simple implementation!***

The simplest example: semileptonic **B → D** decays

In **PRD ’21 (arXiv:2105.08674)**, our DM method has been applied to B → D decays:

- **3 FNAL/MILC data for each FF**: final results contained in **PRD ‘15 (arXiv:1503.07237)**



$f_0(z_{max}) = f_+(z_{max})$

KC at maximum recoil
for each bootstrap

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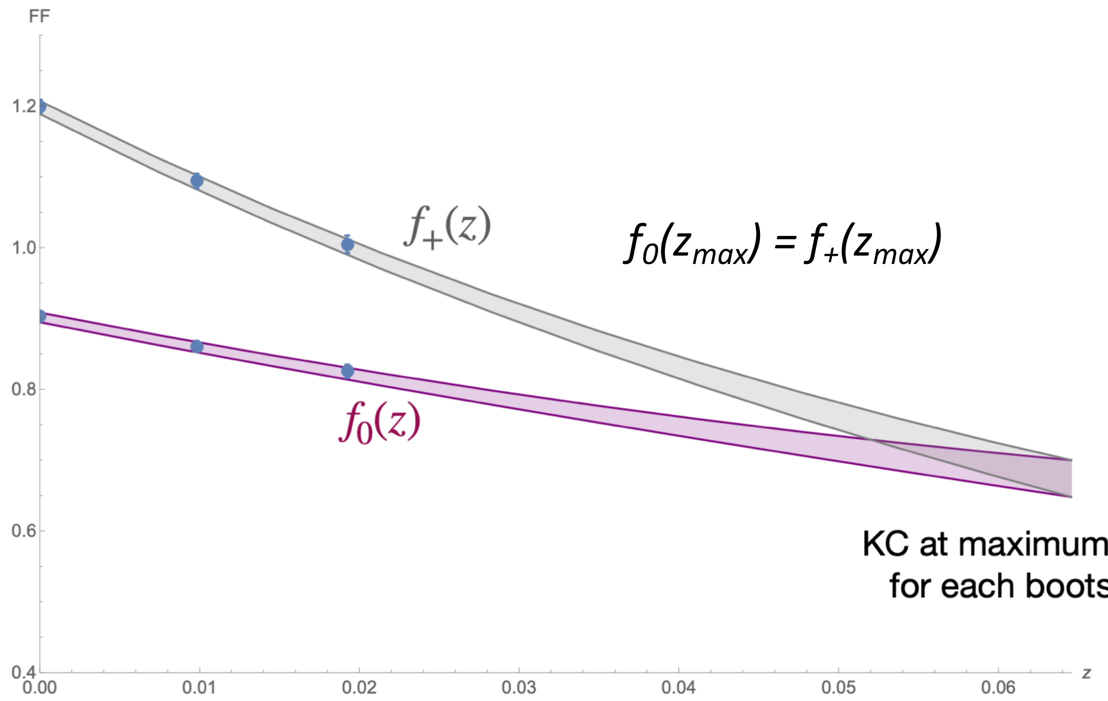

Recalling that for production of a pseudoscalar meson (i.e. D) in case of ***massive*** *lepton:*

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{24\pi^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \left[|\vec{p}_D|^3 \left(1 + \frac{m_\ell^2}{2q^2}\right) |f^+(q^2)|^2 + m_B^2 |\vec{p}_D| \left(1 - \frac{m_D^2}{m_B^2}\right)^2 \frac{3m_\ell^2}{8q^2} |f^0(q^2)|^2 \right]$$

$$R(D) = 0.296 \pm 0.008$$

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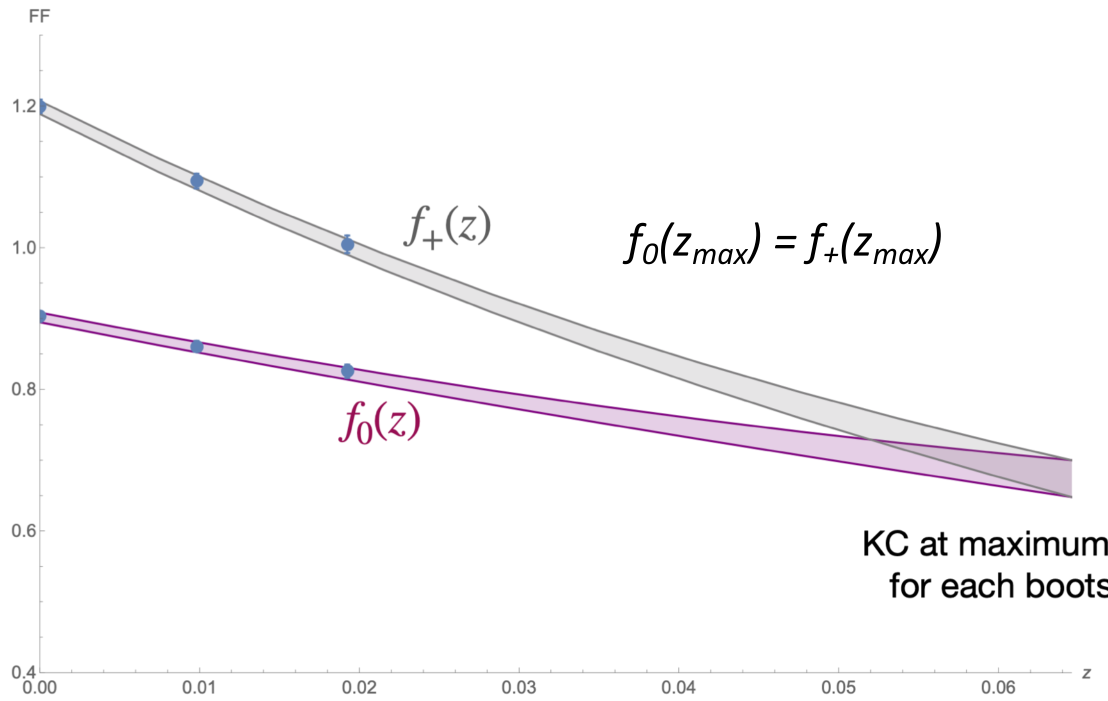

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FULLY-THEORETICAL ESTIMATE!

The "problematic" semileptonic **B → D*** channel

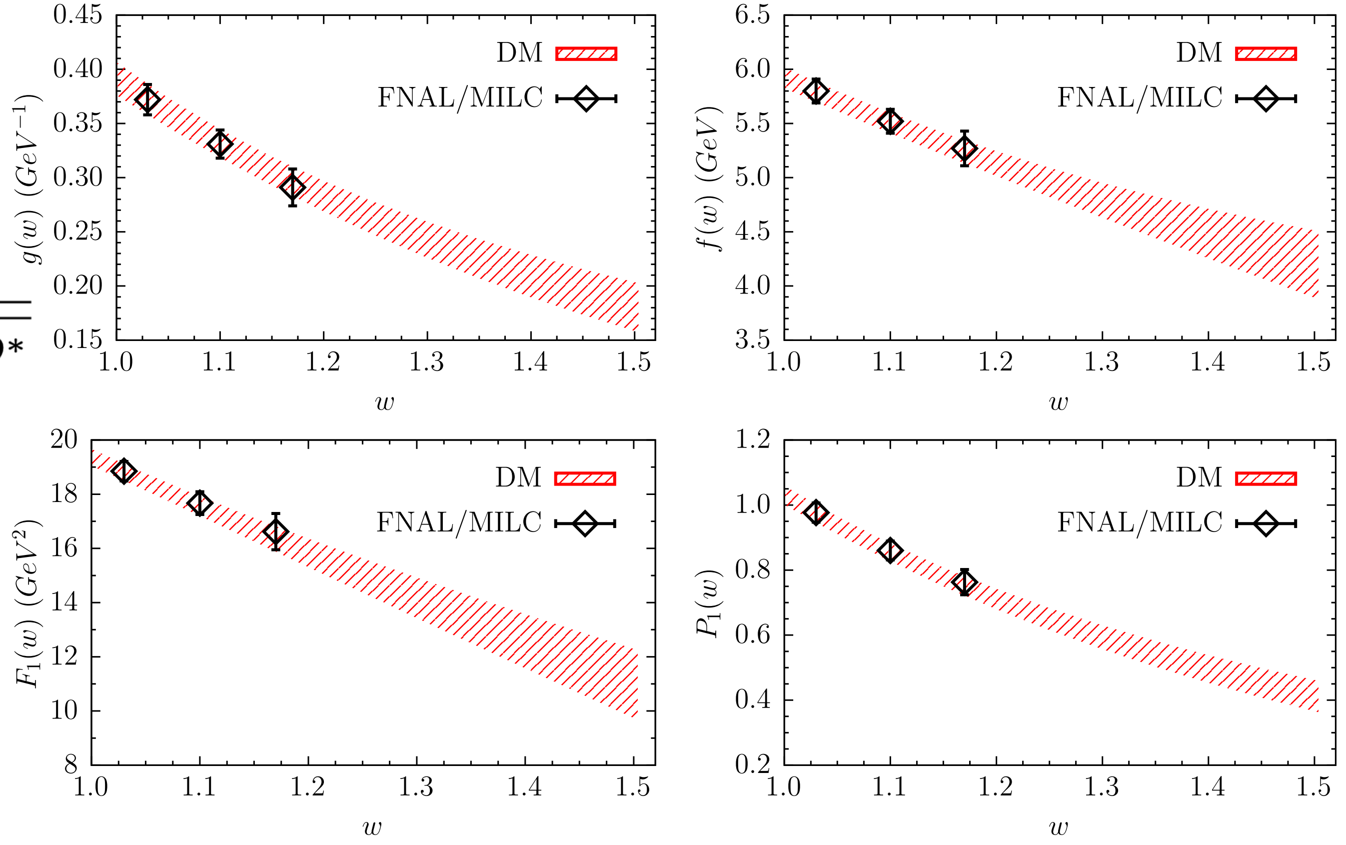
In **EPJC '22 (arXiv:2109.15248)**, we have studied the final results of the FNAL/MILC computations of the FFs

- **3 FNAL/MILC data (diamonds) for each FF**: final results contained in **EPJC '22 arXiv:2105.14019 [hep-lat]**

Two kinematical constraints (KCs):

$$\mathcal{F}_1(1) = (m_B - m_{D^*}) f(1)$$

$$P_1(w_{max}) = \frac{\mathcal{F}_1(w_{max})}{(1+w_{max})(m_B - m_{D^*})\sqrt{m_B m_{D^*}}}$$



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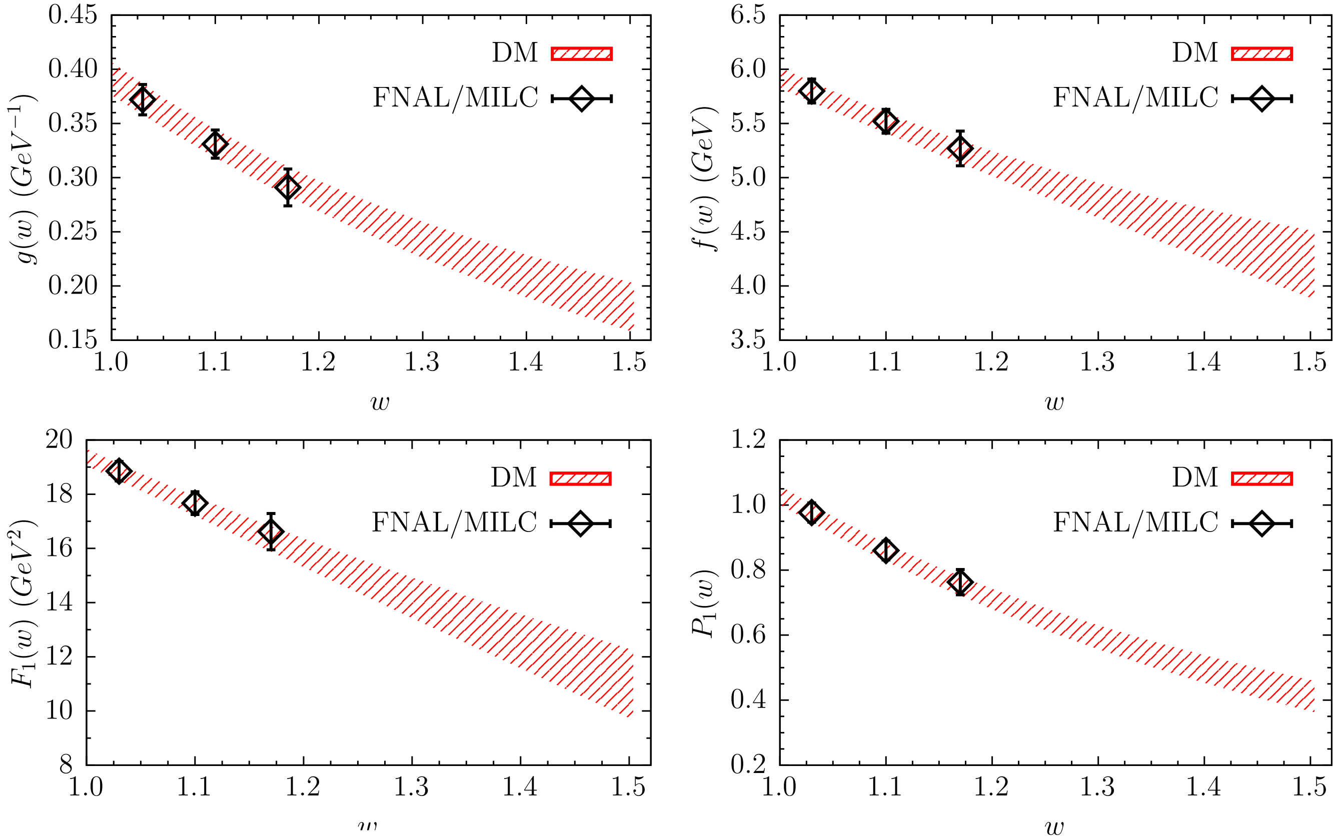
*Using the unitarity bands of the FFs, we can compute a new expectation value of the anomaly **R(D*)**:*

DM prediction:

$$R(D^*) = 0.275 \pm 0.008$$

FULLY-THEORETICAL

Less than 1σ compatibility!



$$R(D^*)|_{\rm exp} = 0.284 \pm 0.013$$

HFLAV Coll. (https://hflav-eos.web.cern.ch/hflav-eos/semi/winter23_prel/html/RDsDsstar/RDRDs.html)

Why not doing a global fit of lattice and exp. data

Let us come back to one of the first slide:

Note that one can use **<u>also</u>** experimental data (in addition to the LQCD ones) to constrain the shape of the FFs…

$$d\Gamma/dx, \quad x = w, \cos\theta_l, \cos\theta_v, \chi$$

Belle Coll.: arXiv:1702.01521, PRD '19 [arXiv:1809.03290]

Let us see this in detail: *let us consider the BGL fits performed by FNAL/MILC Collaborations in EPJC '22 arXiv:2105.14019*

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Basics of BGL: the hadronic FFs corresponding to definite spin-parity can be represented as an expansion, originating from unitarity, analyticity and crossing symmetry, in terms of the conformal variable *z*, for instance

$$g(z) = \frac{1}{\sqrt{\chi_{1^-}(q_0^2)}} \frac{1}{\phi_g(z, q_0^2)\, P_{1^-}(z)} \sum_{n=0}^{\infty} a_n\, z^n$$

Unitarity:

$$\sum_{n=0}^{\infty} a_n^2 \leq 1$$

See M. Bordone's talk

Boyd, Grinstein and Lebed, Phys. Lett. B353, 306 (1995)
Boyd, Grinstein and Lebed, Nucl. Phys. B461, 493 (1996)
Boyd, Grinstein and Lebed, Phys. Rev. D 56, 6895 (1997)

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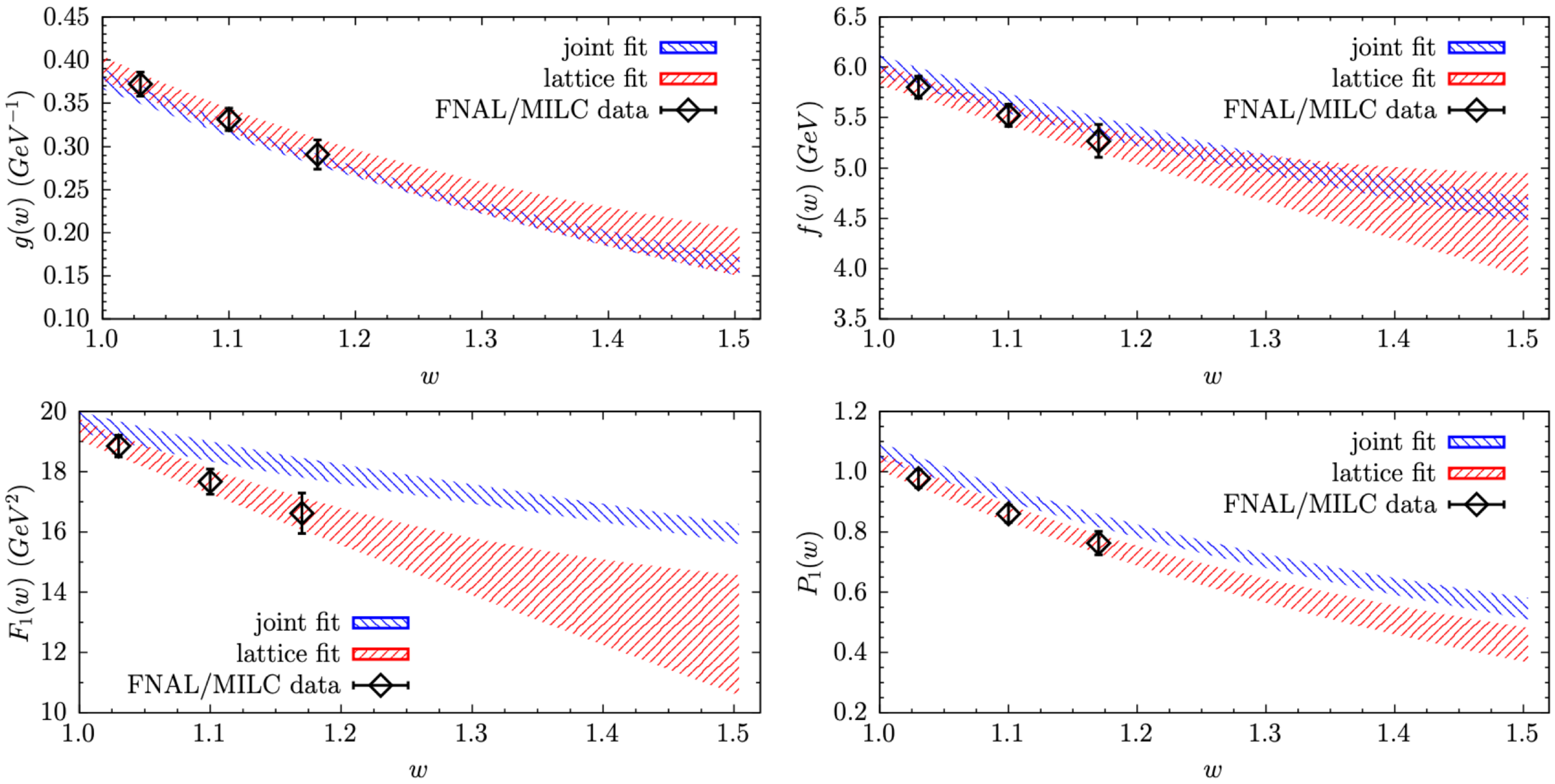

joint fit:
BGL fit of LQCD points + Belle + BaBar exp. data

$R(D^*) = 0.2483 \pm 0.0013$

lattice fit:
quadratic BGL fit of LQCD points only

$R(D^*) = 0.265 \pm 0.013$

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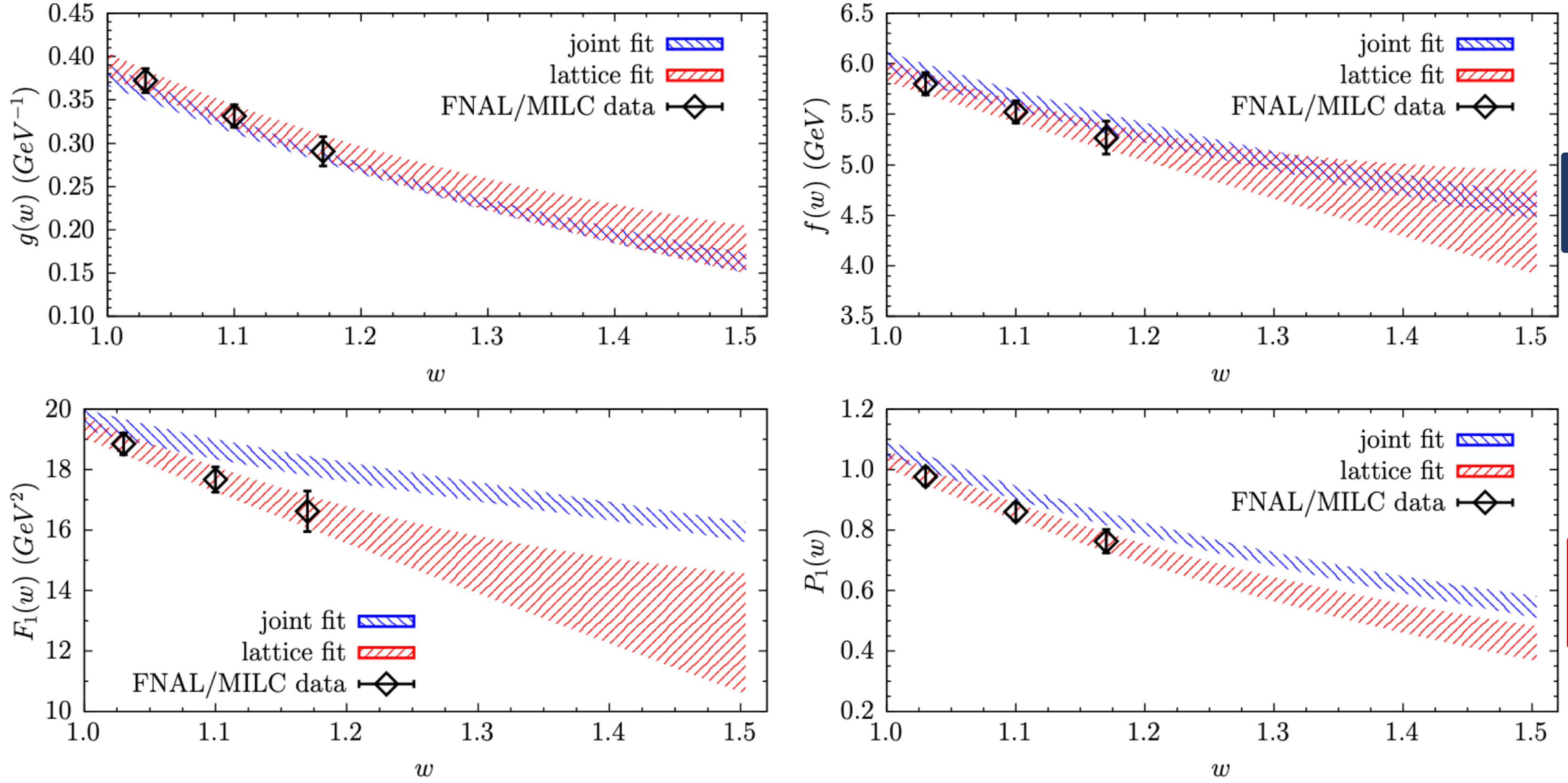

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$$\boxed{\chi^2/d.o.f. = 126/84}$$

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$$\boxed{\chi^2/d.o.f. = 0.64/3}$$

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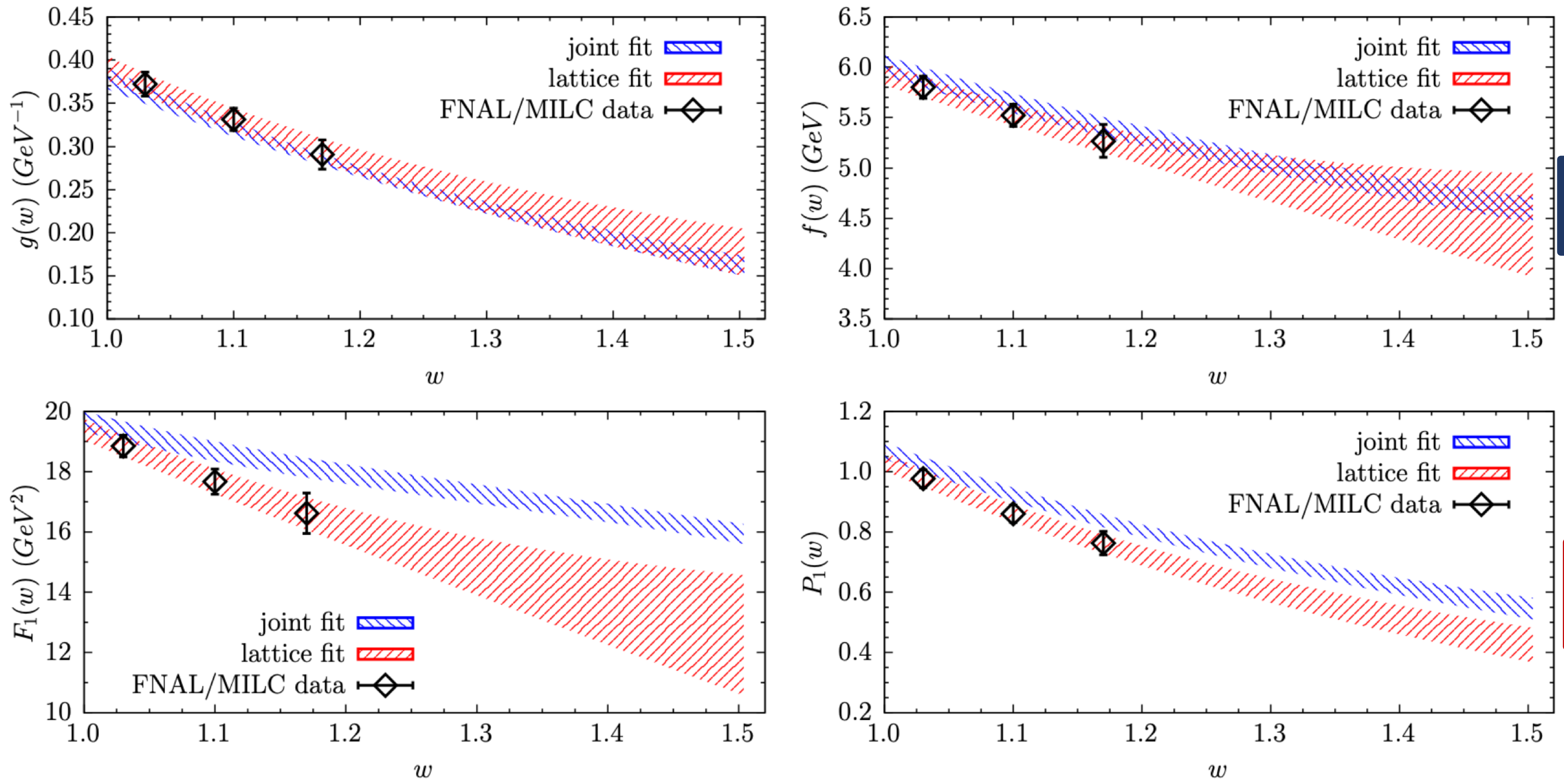
Why not doing a global fit of lattice and exp. data

OUR UNDERSTANDING: to avoid any bias in the description of the final shape of the FFs, we want to first analyse the lattice data and *then* compare the results with experiments!



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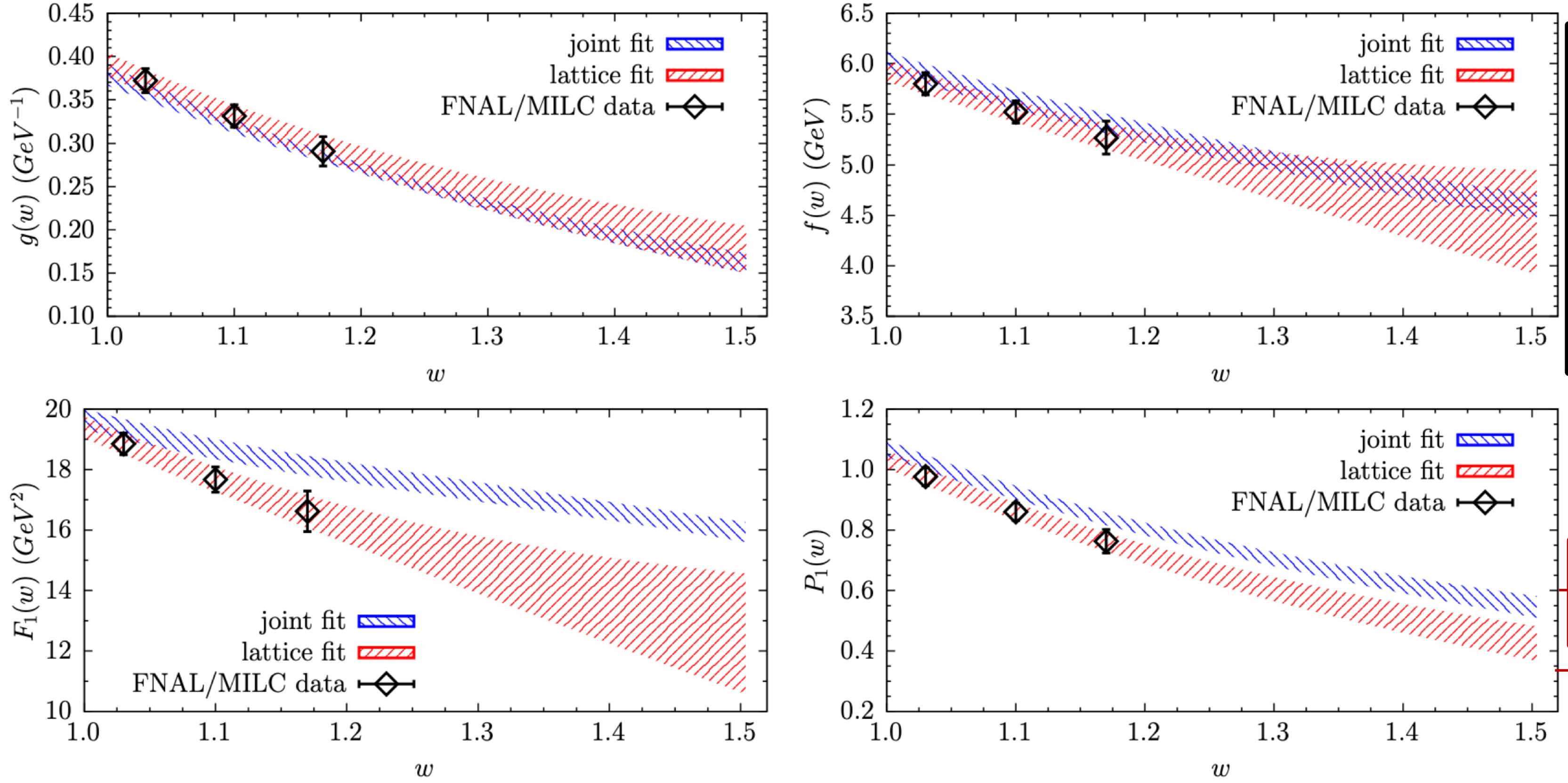
Why not doing a global fit of lattice and exp. data

EQUAL TO THE DM result!

$$R(D^*) = 0.275 \pm 0.008$$

FULLY-THEORETICAL



Moreover: if all the KCs are exaclty implemented:

$\chi^2/d.o.f. = 1.39/4$

$R(D^*) = 0.274(10)$

lattice fit:
quadratic BGL fit of LQCD points only

$\boxed{\cancel{\chi^2/d.o.f. = 0.64/3}}$

$\cancel{R(D^*) = 0.265 \pm 0.013}$

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$$R(D^*)^{\mathrm{JLQCD}} = 0.252(22)$$

Vaquero's talk @ Flavour@TH Workshop, CERN, May 2023

Conclusions

The Dispersion Matrix method allows for a **first-principle, non-perturbative and completely model-independent extrapolation of the behaviour of the hadronic FFs in the whole kinematic region**, starting from existing LQCD data.

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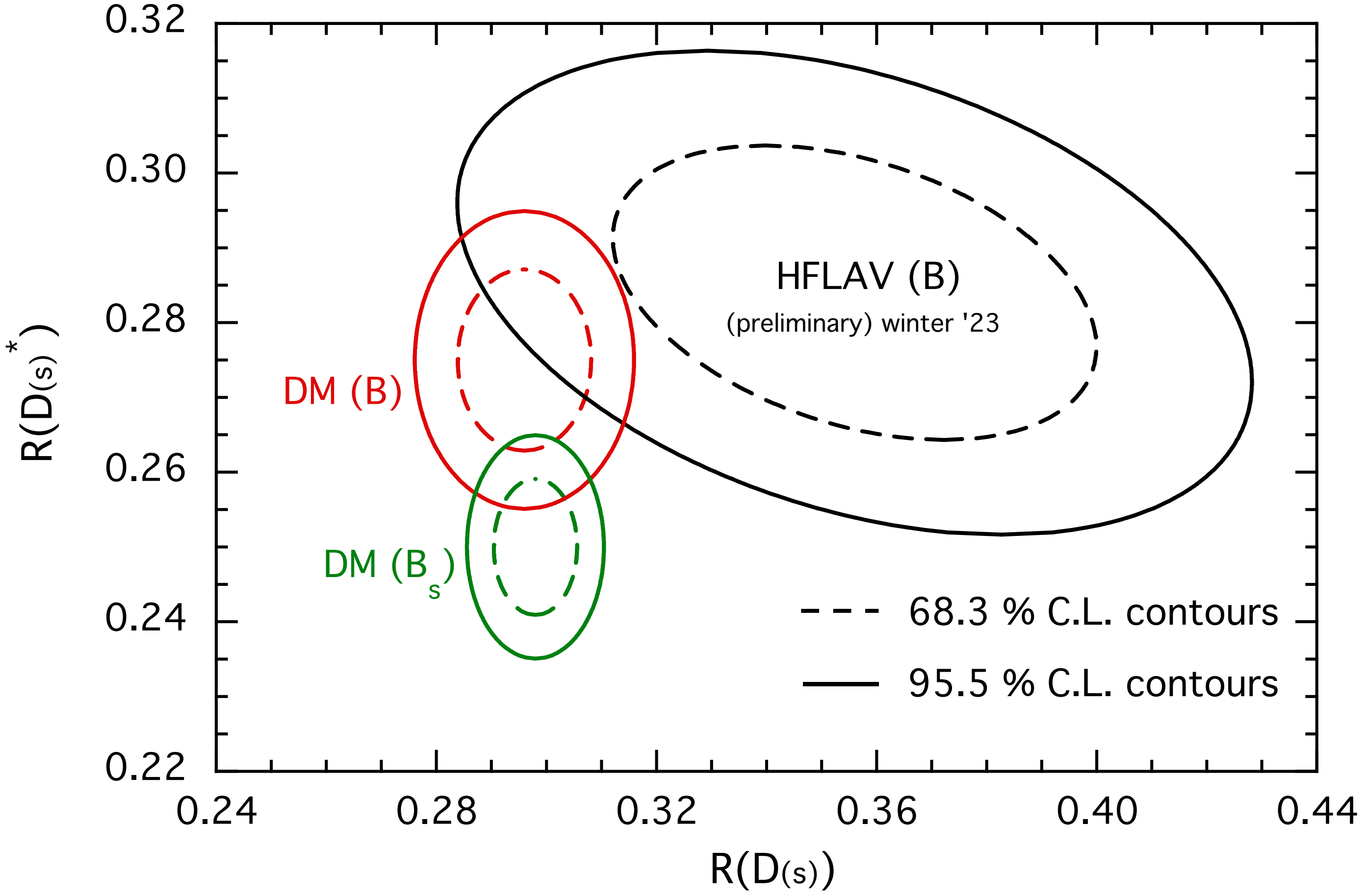
iv) Combine all the available lattice data to have a final answer for the shape of the FFs

Conclusions

LFU observables



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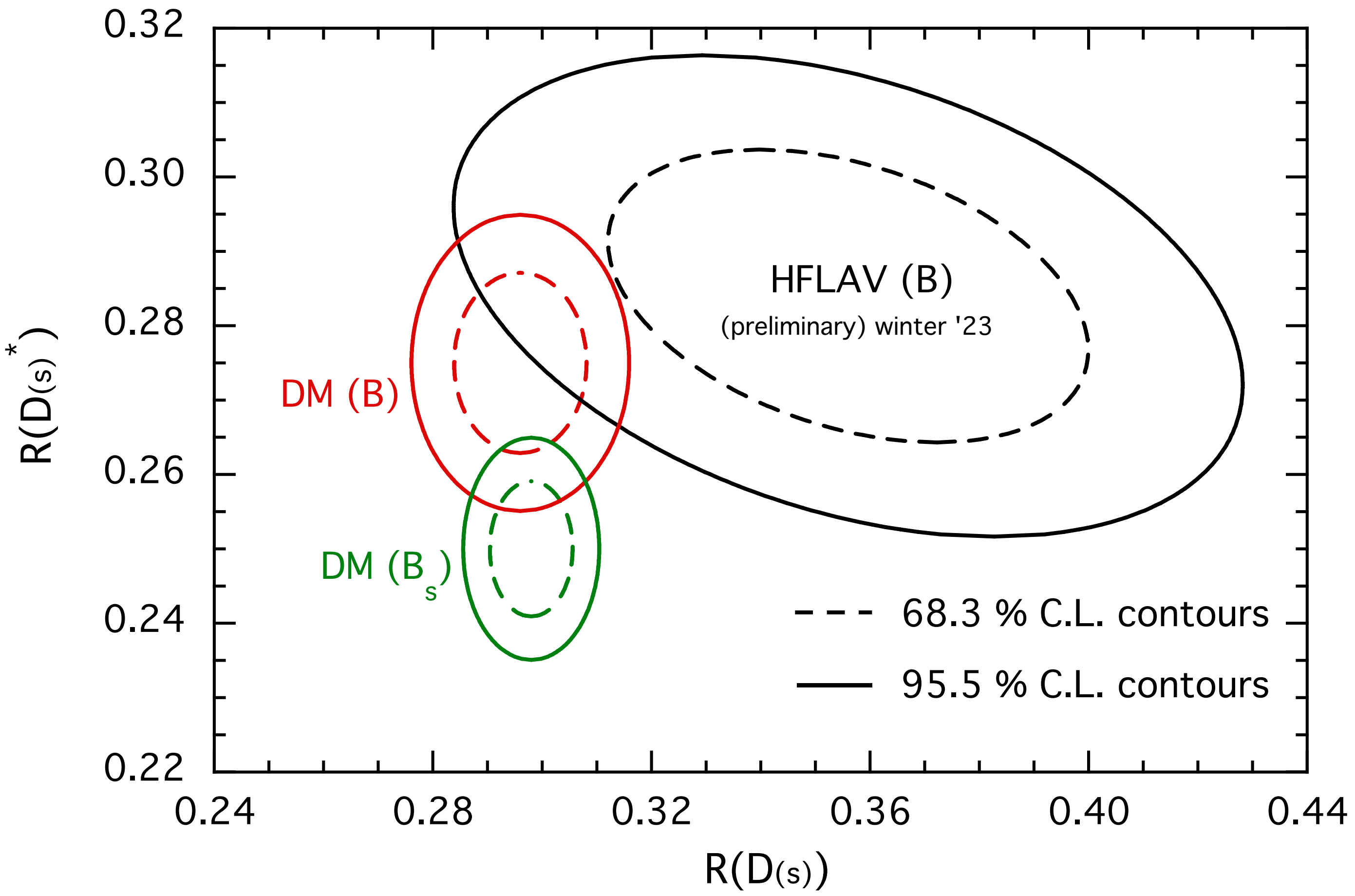

By using (and trusting) FNAL/MILC lattice data, the R(D(*)) anomalies are practically gone…

If you are interested in semileptonic Bs → Ds decays, we can discuss about the DM application to these decays (back-up slides)

<u>THANKS FOR</u>

<u>YOUR ATTENTION!</u>

BACK-UP SLIDES

The DM method

The positivity of the original inner products guarantee that $\det \mathbf{M} \geq 0$: the solution of this inequality can be computed analitically, bringing to

LOWER bound $\boxed{\beta - \sqrt{\gamma}} \leq f(z) \leq \boxed{\beta + \sqrt{\gamma}}$ *UPPER bound*

$$\beta = \frac{1}{d(z)\phi(z)} \sum_{j=1}^{N} f_j \phi_j d_j \frac{1 - z_j^2}{z - z_f} \qquad \gamma = \frac{1}{d^2(z)\phi^2(z)} \frac{1}{1 - z^2} \left[\chi - \sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} \right]$$

UNITARITY FILTER: unitarity is satisfied if γ is semipositive definite, namely if

$$\chi \geq \sum_{i,j=1} N f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}$$

Unitarity is built-in!

Statistical and systematic uncertainties

How can we finally combine all the N_U lower and upper bounds of both the FFs??

One bootstrap event case:

after a single extraction, we have one value of the lower bound f_L and one value of the upper one f_U for each FF. Assuming that the true value of each FF can be **everywhere inside the range (f_U - f_L) with equal probability**, we associate to the FFs a *flat* distribution

$$P(f_{0(+)}) = \frac{1}{f_{U,0(+)} - f_{L,0(+)}} \Theta(f_{0(+)} - f_{L,0(+)}) \Theta(f_{U,0(+)} - f_{0(+)})$$

Many bootstrap events case:

how to mediate over the whole set of bootstrap events? Since the lower and the upper bounds of a generic FF are deeply correlated, we will assume a multivariate Gaussian distribution:

$$P(f_L, f_U) = \frac{\sqrt{\det \rho}}{2\pi} \exp\left[-\frac{\rho_{up,up}(f_U - \langle f_U \rangle)^2 + \rho_{lo,lo}(f_L - \langle f_L \rangle)^2 + 2\rho_{lo,up}(f_U - \langle f_U \rangle)(f_L - \langle f_L \rangle)}{2} \right]$$

In conclusion, we can combine the bounds of each FF in a final mean value and a final standard deviation, defined as

$$\langle f \rangle = \frac{\langle f_L \rangle + \langle f_U \rangle}{2},$$

NO PARAMETRIZATION ADOPTED!!!

$$\sigma_f = \frac{1}{12} (\langle f_U \rangle - \langle f_L \rangle)^2 + \frac{1}{3} (\sigma^2_{f_{lo}} + \sigma^2_{f_{up}} + \rho_{lo,up} \sigma_{f_{lo}} \sigma_{f_{up}})$$

Kinematical Constraints (KCs)

REMINDER: after the unitarity filter we were left with *$N_U < N$ survived events!!!*

Let us focus on the pseudoscalar case. Since by construction the following *kinematical constraint* holds

$$\boxed{f_0(0) = f_+(0)}$$

we will filter only the $N_{KC} < N_U$ events for which the two bands of the FFs intersect each other @ *$t = 0$*.
Namely, for each of these events we also define

$$\phi_{lo} = \max[F_{+,lo}(t=0), F_{0,lo}(t=0)]$$

$$\phi_{up} = \min[F_{+,up}(t=0), F_{0,up}(t=0)]$$

From WE theorem

$$\langle D(p_D)|\, V^\mu \,|B(p_B)\rangle \;=\; f_+(p_B + p_D)^\mu + f_-(p_B - p_D)^\mu$$

One then defines

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2} f_-(q^2)$$

$$\langle D(p_D)\,|\,V^\mu\,|\,B(p_B)\rangle = f^+(q^2)\left(p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu\right) + f^0(q^2)\frac{m_B^2 - m_D^2}{q^2} q^\mu$$

Kinematical Constraints (KCs)

We then consider a **modified matrix**

$$\mathbf{M_C} = \begin{pmatrix} \langle\phi f|\phi f\rangle & \langle\phi f|g_t\rangle & \langle\phi f|g_{t_1}\rangle & \cdots & \langle\phi f|g_{t_n}\rangle & \langle\phi f|g_{t_{n+1}}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle & \langle g_t|g_{t_{n+1}}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle & \langle g_{t_1}|g_{t_{n+1}}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle & \langle g_{t_n}|g_{t_{n+1}}\rangle \\ \langle g_{t_{n+1}}|\phi f\rangle & \langle g_{t_{n+1}}|g_t\rangle & \langle g_{t_{n+1}}|g_{t_1}\rangle & \cdots & \langle g_{t_{n+1}}|g_{t_n}\rangle & \langle g_{t_{n+1}}|g_{t_{n+1}}\rangle \end{pmatrix}$$

with $t_{n+1} = 0$. Hence, we compute the new lower and upper bounds of the FFs in this way. For each of the N_{KC} events, we extract $N_{KC,2}$ values of $f_0(0) = f_+(0) \equiv f(0)$ with uniform distribution defined in the range $[\phi_{lo}, \phi_{up}]$. Thus, for both the FFs and for each of the N_{KC} events we define

$$\begin{aligned} F_{lo}(t) &= \min[F_{lo}^1(t), F_{lo}^2(t), \cdots, F_{lo}^{N_{KC,2}}(t)], \\ F_{up}(t) &= \max[F_{up}^1(t), F_{up}^2(t), \cdots, F_{up}^{N_{KC,2}}(t)] \end{aligned}$$

Non-perturbative computation of the susceptibilities

In **PRD '21 [arXiv:2105.07851]**, we have presented the results of the first computation on the lattice of the susceptibilities for the $b \to c$ quark transition, using the N_f=2+1+1 gauge ensembles generated by ETM Collaboration.

How are they defined? The starting point is the HVP tensor:

$$\Pi^V_{\mu\nu}(Q) = \int d^4x \; e^{-iQ\cdot x} \langle 0| T\left[\bar{b}(x)\gamma^E_\mu c(x)\; \bar{c}(0)\gamma^E_\nu b(0)\right] |0\rangle$$
$$= -Q_\mu Q_\nu \Pi_{0^+}(Q^2) + (\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu)\Pi_{1^-}(Q^2)$$

To compute the susceptibilities on the lattice, we start from the Euclidean correlators:

$$\chi_{0^+}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^+}(Q^2)\right] = \int_0^\infty dt\; t^2 j_0(Qt)\; C_{0^+}(t)\;, \xrightarrow{W.\,I.} \frac{1}{4}\int_0^\infty dt'\, t'^4\, \frac{j_1(Qt')}{Qt'}\left[(m_b - m_c)^2 C_S(t') + Q^2 C_{0^+}(t')\right]$$

$$\chi_{1^-}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^-}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\; t^4 \frac{j_1(Qt)}{Qt}\; C_{1^-}(t)$$

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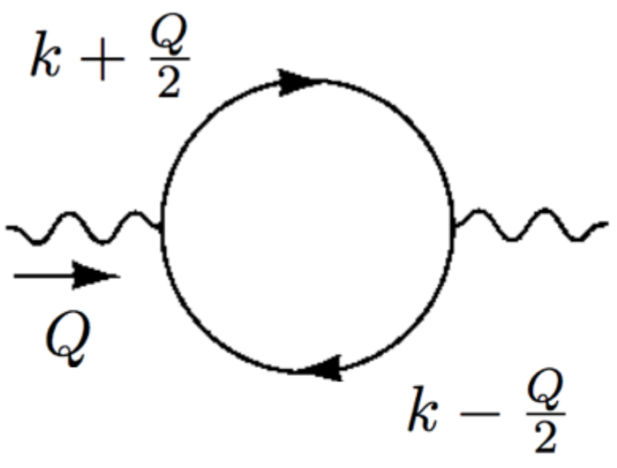
$$\chi_{1^+}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^+}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\; t^4 \frac{j_1(Qt)}{Qt}\; C_{1^+}(t)$$

Contact terms & perturbative subtraction

In **twisted mass LQCD**:

$$\boxed{\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\right],}$$



Thus, by separating the *longitudinal* and the *transverse* contributions, we can compute the susceptibilities for all the spin-parity quantum numbers **in the free theory on the lattice**, *i.e.* at order $\mathcal{O}(\alpha_s^0)$ using twisted-mass fermions!

$$\chi_j^{free} = \boxed{\chi_j^{LO}} + \boxed{\chi_j^{discr}}$$

LO term of PT @ $\mathcal{O}(\alpha_s^0)$ contact terms and discretization effects @ $\mathcal{O}(\alpha_s^0 a^m)$ with $m \geq 0$

Perturbative subtraction:

$$\chi_j \to \chi_j - \left[\chi_j^{free} - \chi_j^{LO}\right]$$

ETMC ratio method & final results

For the extrapolation to the physical *b*-quark point we have used the ETMC ratio method:

$$R_j(n; a^2, m_{ud}) \equiv \frac{\chi_j[m_h(n); a^2, m_{ud}]}{\chi_j[m_h(n-1); a^2, m_{ud}]} \boxed{\frac{\rho_j[m_h(n)]}{\rho_j[m_h(n-1)]}} \xrightarrow[\lim_{n\to\infty} R_j(n) = 1]{\text{to ensure that}} \boxed{\begin{array}{l}\rho_{0^+}(m_h) = \rho_{0^-}(m_h) = 1\ , \\ \rho_{1^-}(m_h) = \rho_{1^+}(m_h) = (m_h^{pole})^2\end{array}}$$

All the details are deeply discussed in **PRD '21 [2105.07851]**. In this way, we have obtained the first lattice QCD determination of susceptibilities of <u>heavy-to-heavy</u> (<u>and heavy-to-light</u>, see **JHEP '22 [2202.10285]**) transition current densities:

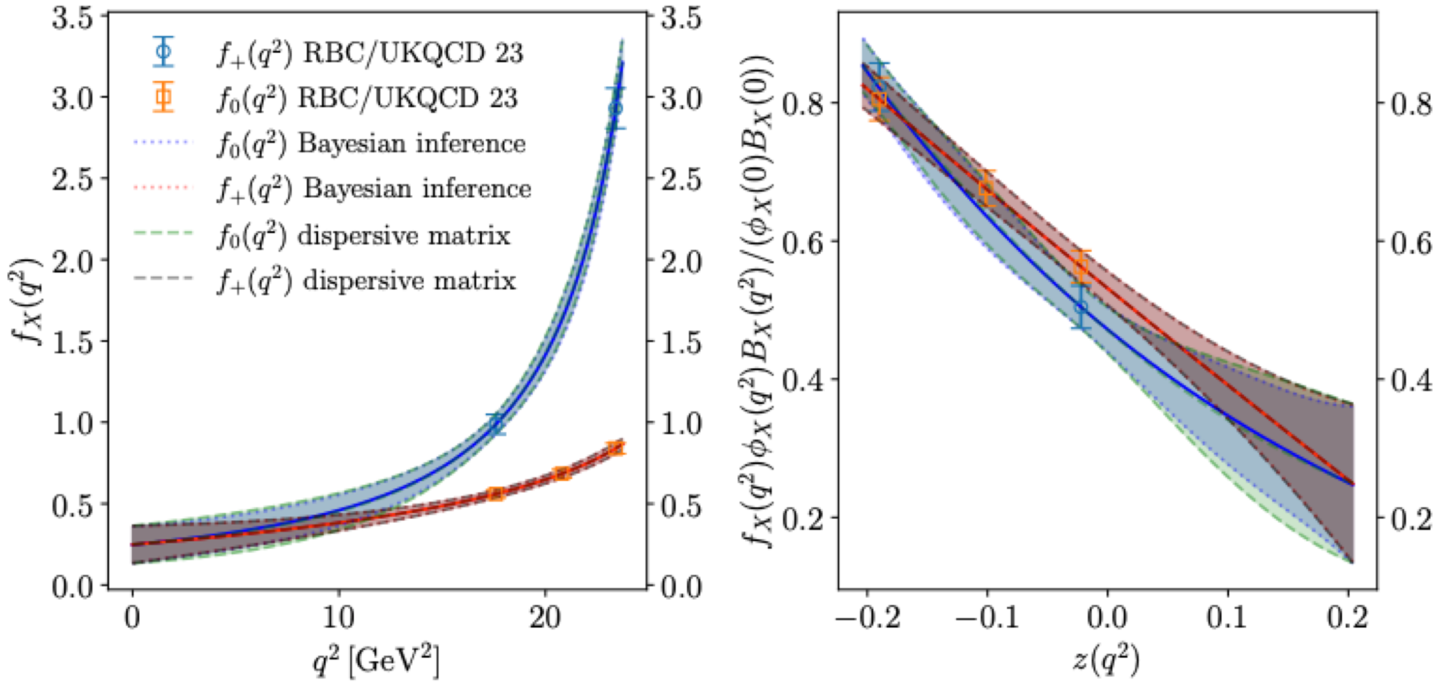
b → c

	Perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L}\,[10^{-3}]$	6.204(81)	—	7.58(59)	—
$\chi_{A_L}\,[10^{-3}]$	24.1	19.4	25.8(1.7)	21.9(1.9)
$\chi_{V_T}\,[10^{-4}\ \text{GeV}^{-2}]$	6.486(48)	5.131(48)	6.72(41)	5.88(44)
$\chi_{A_T}\,[10^{-4}\ \text{GeV}^{-2}]$	3.894	—	4.69(30)	—

Differences with PT? **~4% for 1^-, ~7% for 0^-, ~20 % for 0^+ and 1^+**

A recent counter-check of the DM method

Results III: Bayesian Inference vs Dispersive Matrix Method



- BI and DM produce equivalent results
- BI simpler to implement + easily to combine with multiple data sets
- BI provides numerical value of the coefficients

Tsang's talk @ Moriond EW 2023

Application to $B_s \to K$: **identical results!**

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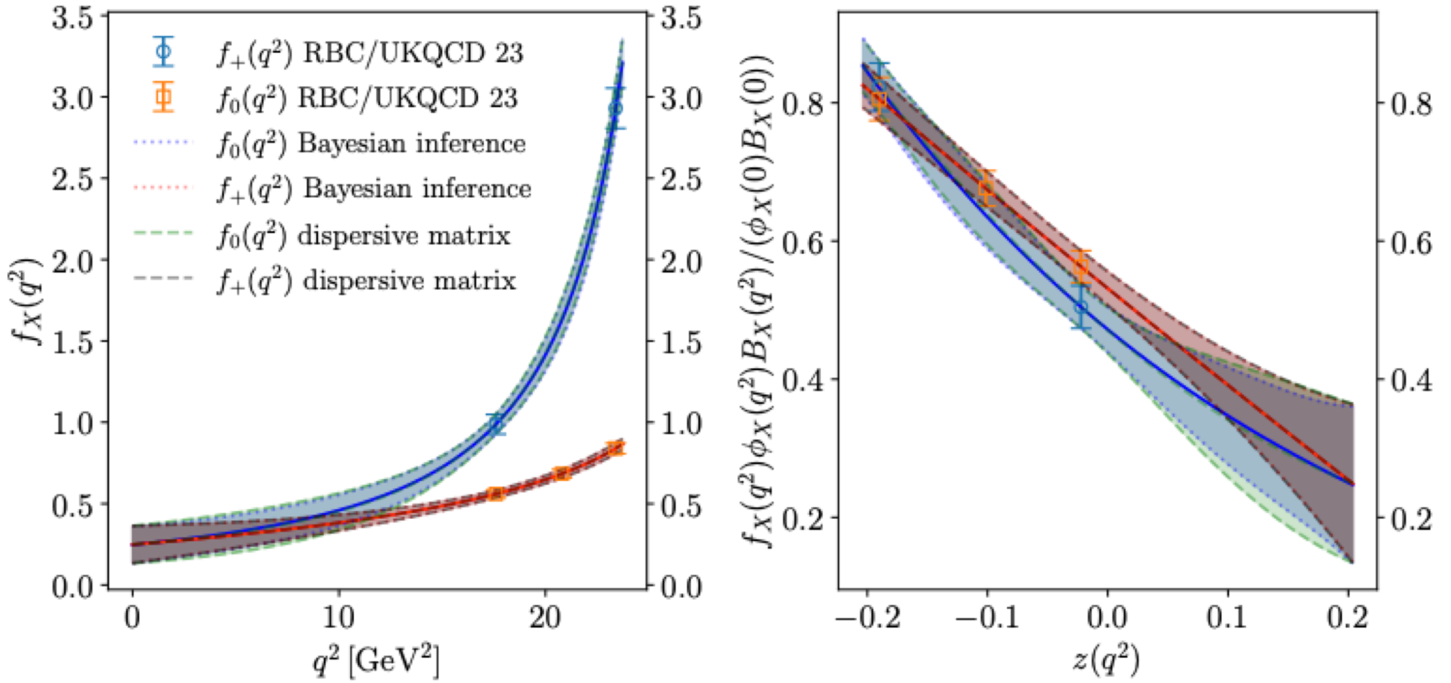

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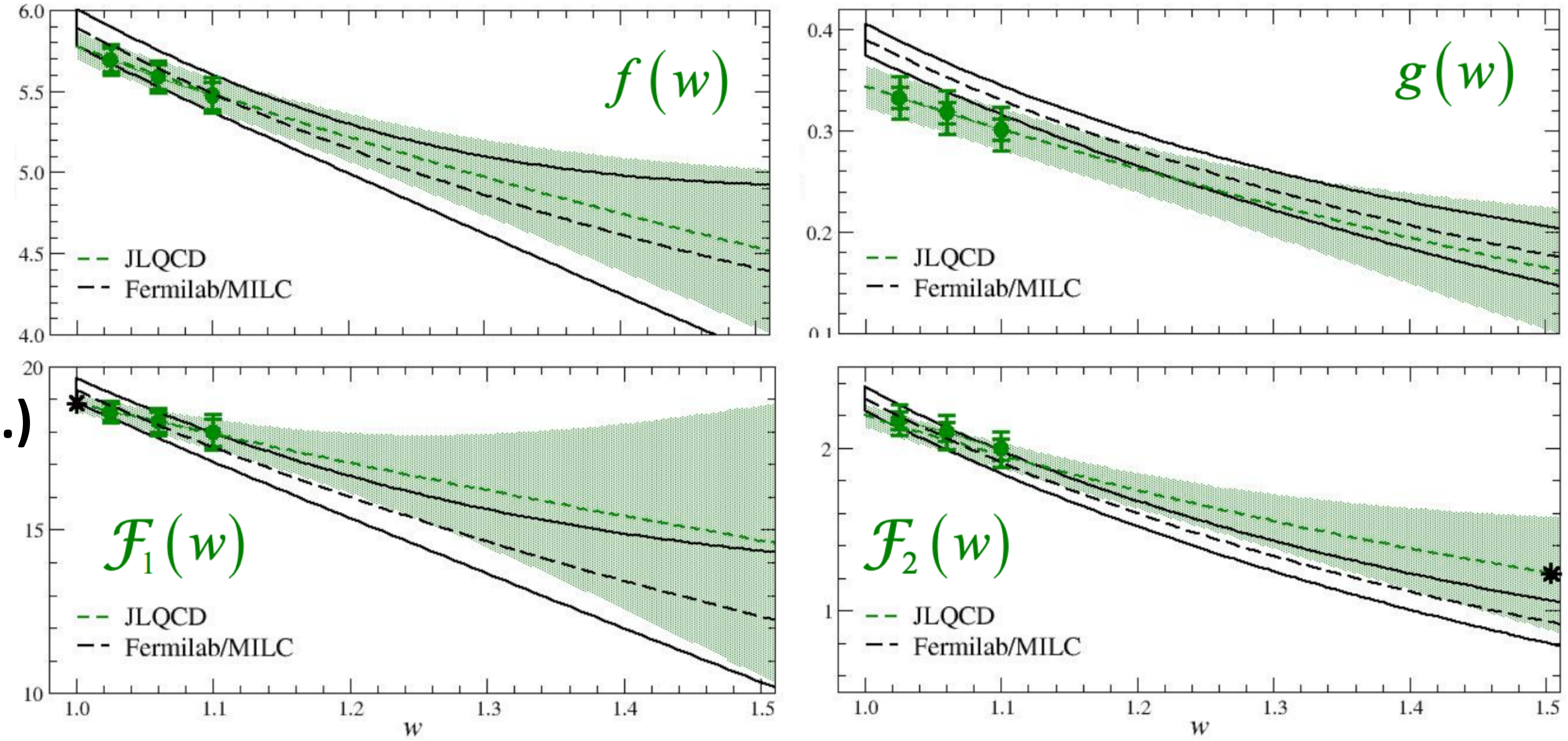
Application to $B_s \to K$: ***identical results!***

All the details of the new ***Bayesian Inference (B.I.)*** *method can be found in:*
i) arXiv:2303.11285
ii) arXiv:2303.11280

Future perspectives for LQCD data

JLQCD (prelim.)

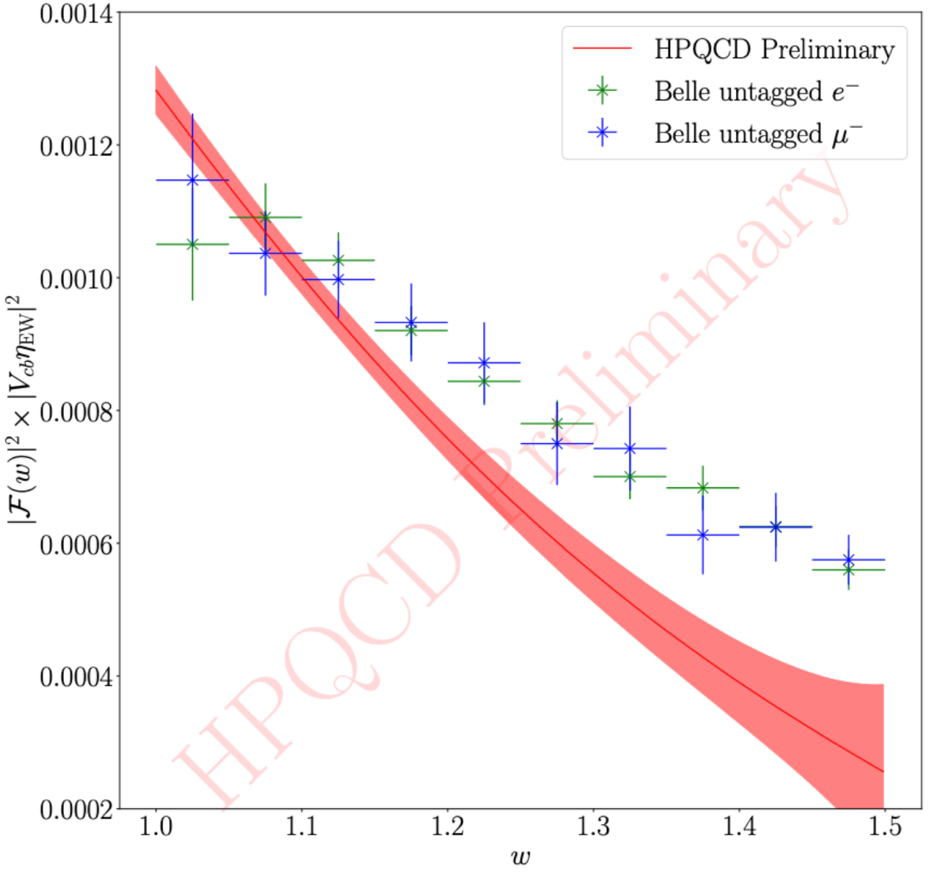


Kaneko's talk @ "Challenges in Semileptonic B decays 2022" Workshop

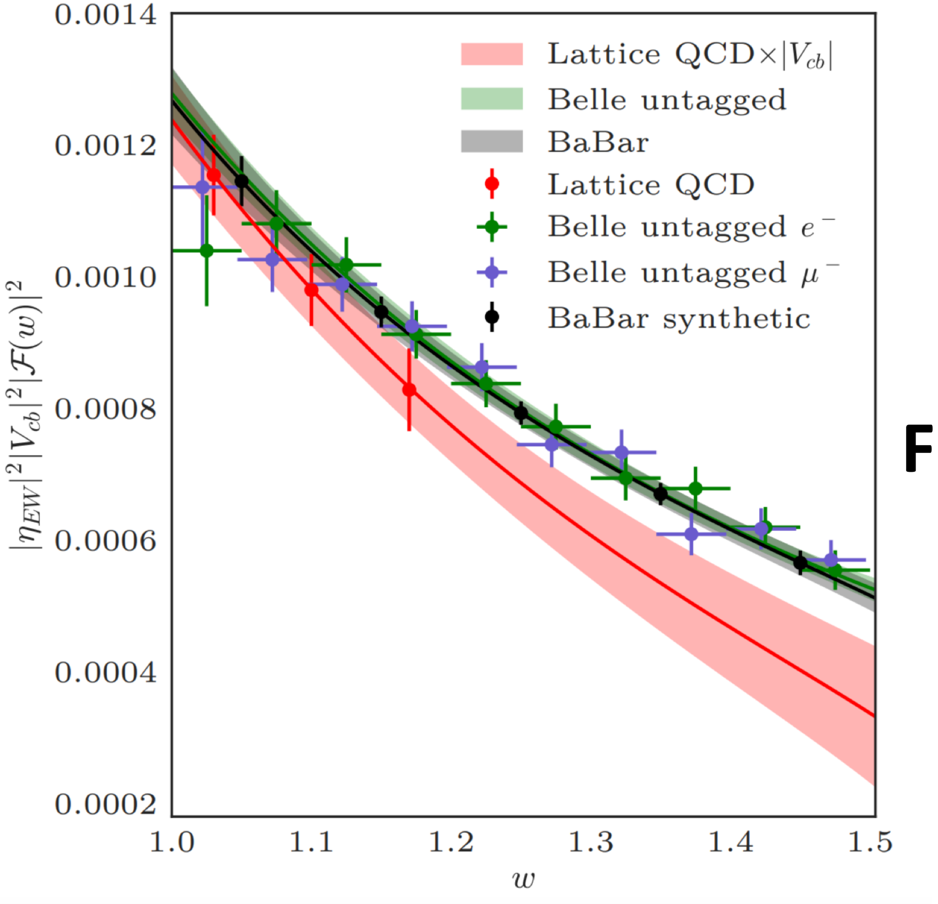
Future perspectives for LQCD data

Harrison's talk @ "Challenges in Semileptonic B decays 2022" Workshop

HPQCD (prelim.)



FNAL/MILC, arXiv:arXiv:2105.14019 [hep-lat]



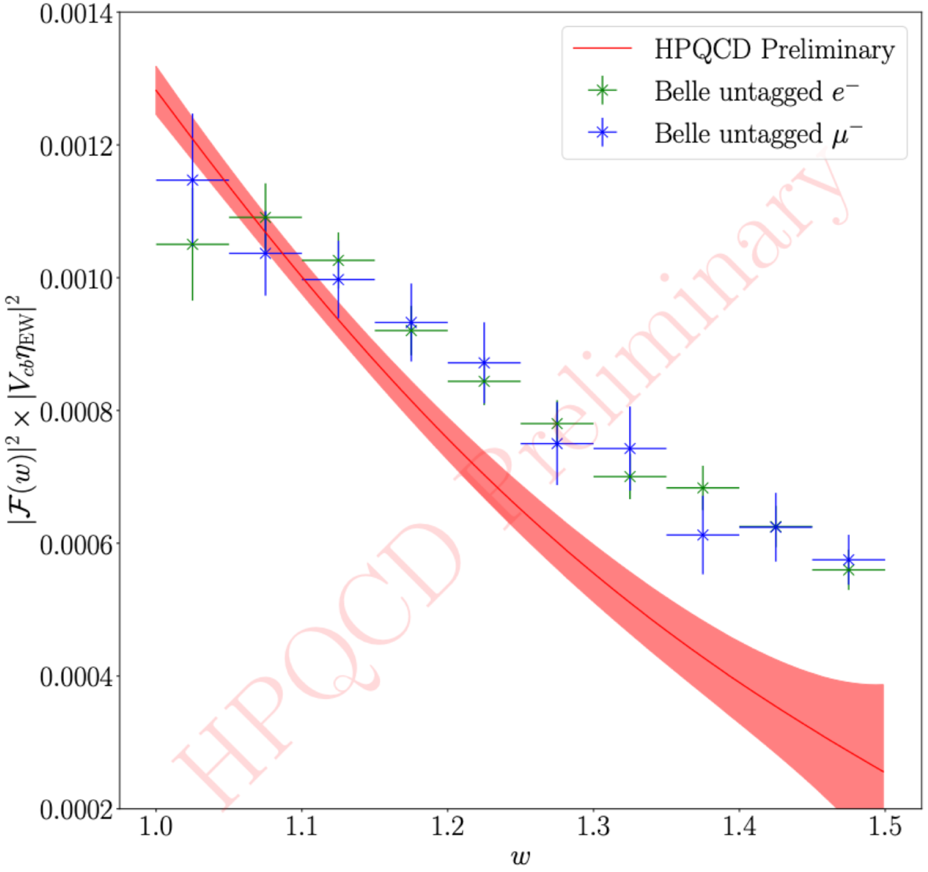
FNAL/MILC

$$\frac{\mathrm{d}\Gamma(\bar{B} \to D^* \ell \bar{\nu})}{\mathrm{d}w} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B - m_{D^*})^2 m_{D^*}^3 \sqrt{w^2 - 1}\,(w+1)^2 \times \left[1 + \frac{4w}{w+1} \frac{m_B^2 - 2w\, m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right] |\mathcal{F}(w)|^2$$

Future perspectives for LQCD data

Harrison's talk @ "Challenges in Semileptonic B decays 2022" Workshop

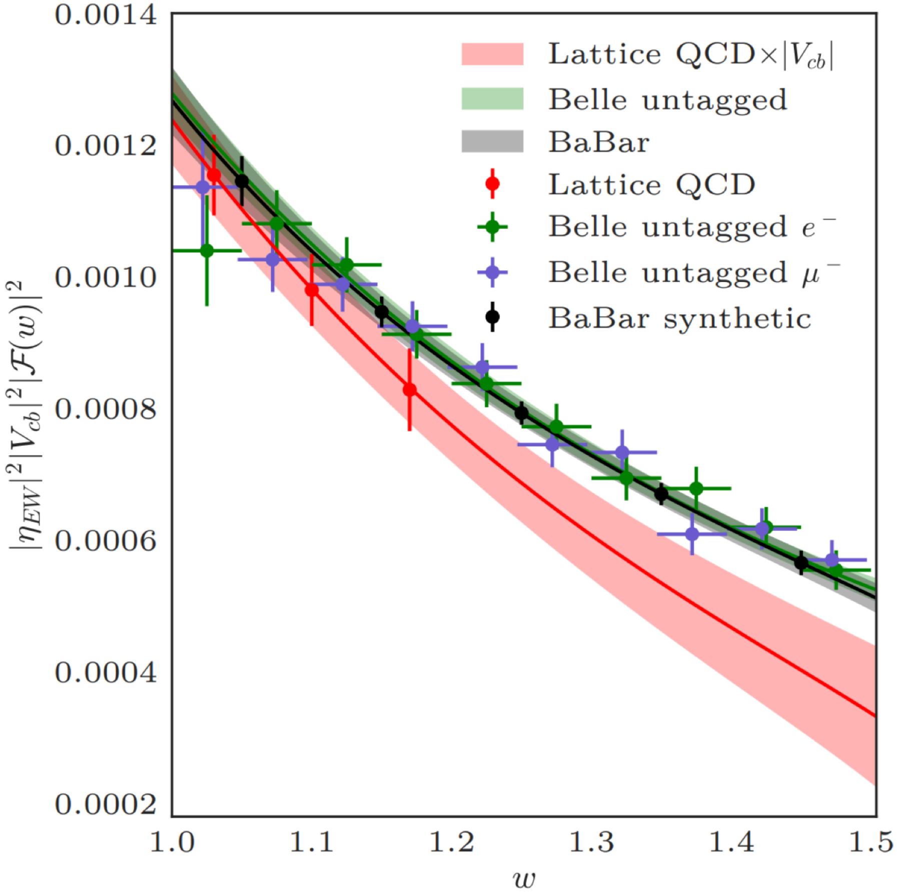
HPQCD (prelim.)



$$\frac{d\Gamma(\bar{B}\to D^*\ell\,\bar{\nu})}{dw} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}(m_B - m_{D^*})^2\, m_{D^*}^3 \sqrt{w^2-1}\,(w+1)^2 \times\left[1+\frac{4w}{w+1}\,\frac{m_B^2 - 2w\,m_B m_{D^*} + m_{D^*}^2}{(m_B-m_{D^*})^2}\right]|\mathcal{F}(w)|^2$$

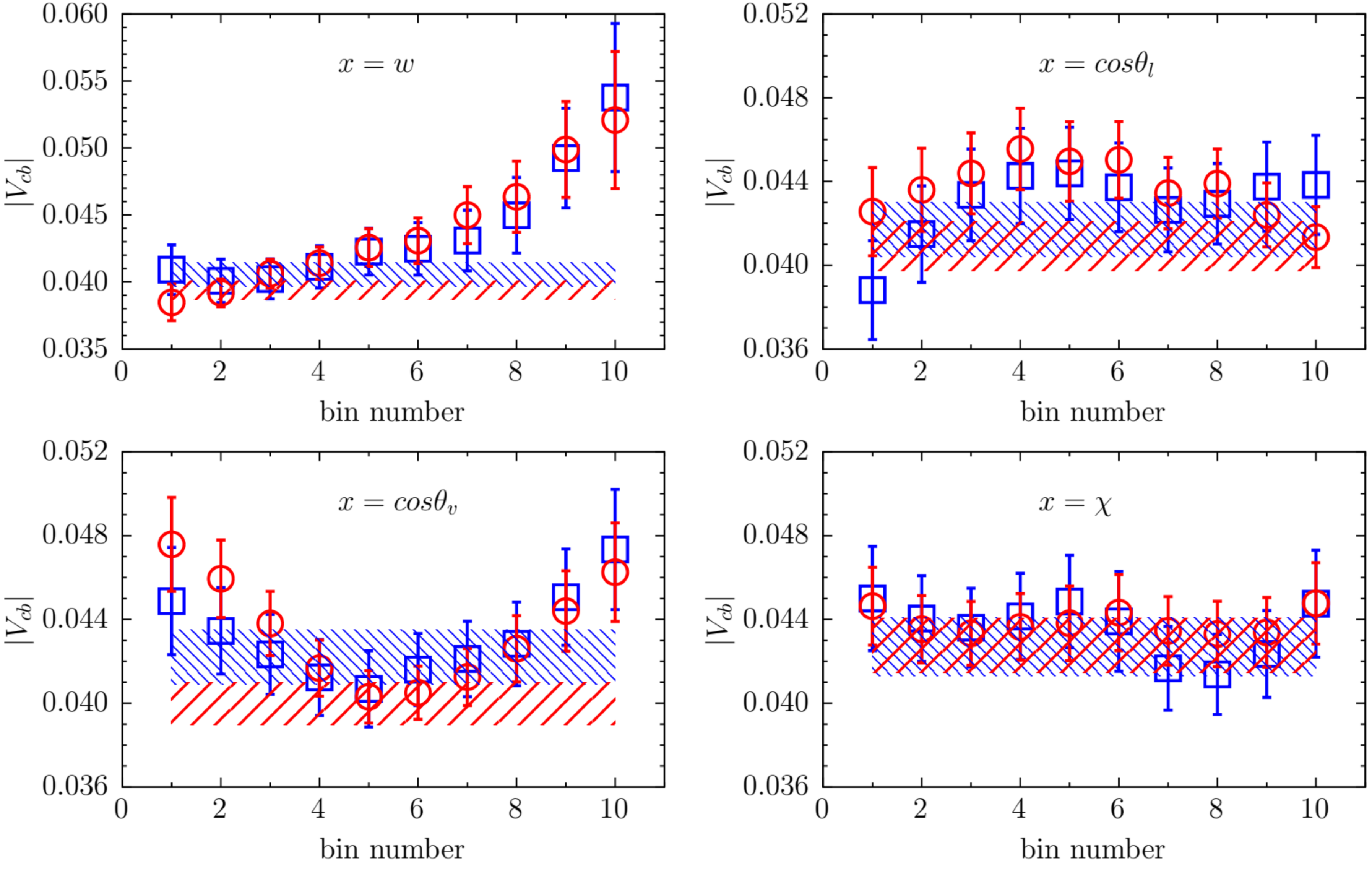
FNAL/MILC, arXiv:arXiv:2105.14019 [hep-lat]

FNAL/MILC



<u>CONCLUSION</u>: FNAL/MILC and HPQCD have similar shape, which is different from Belle & different from JLQCD (which is affected by higher uncertainties…)

Our proposal: *bin-per-bin exclusive Vcb* determination through unitarity



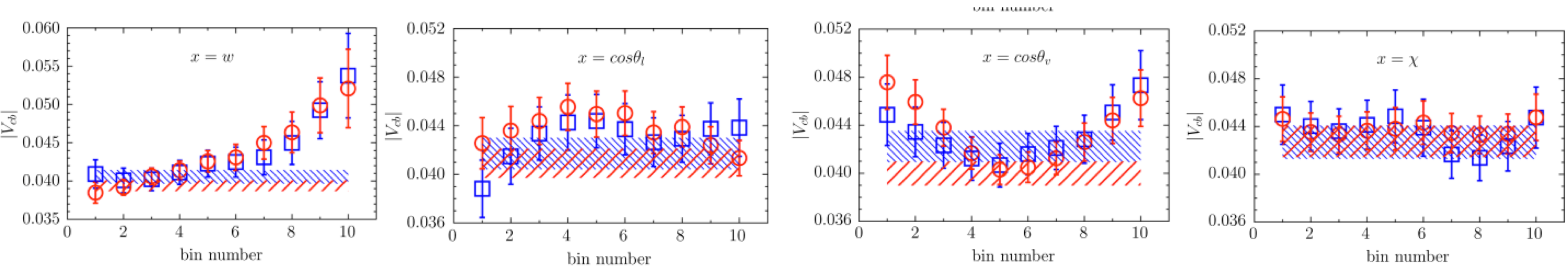
Blue squares: arXiv:1702.01521

Red points: arXiv:1809.03290

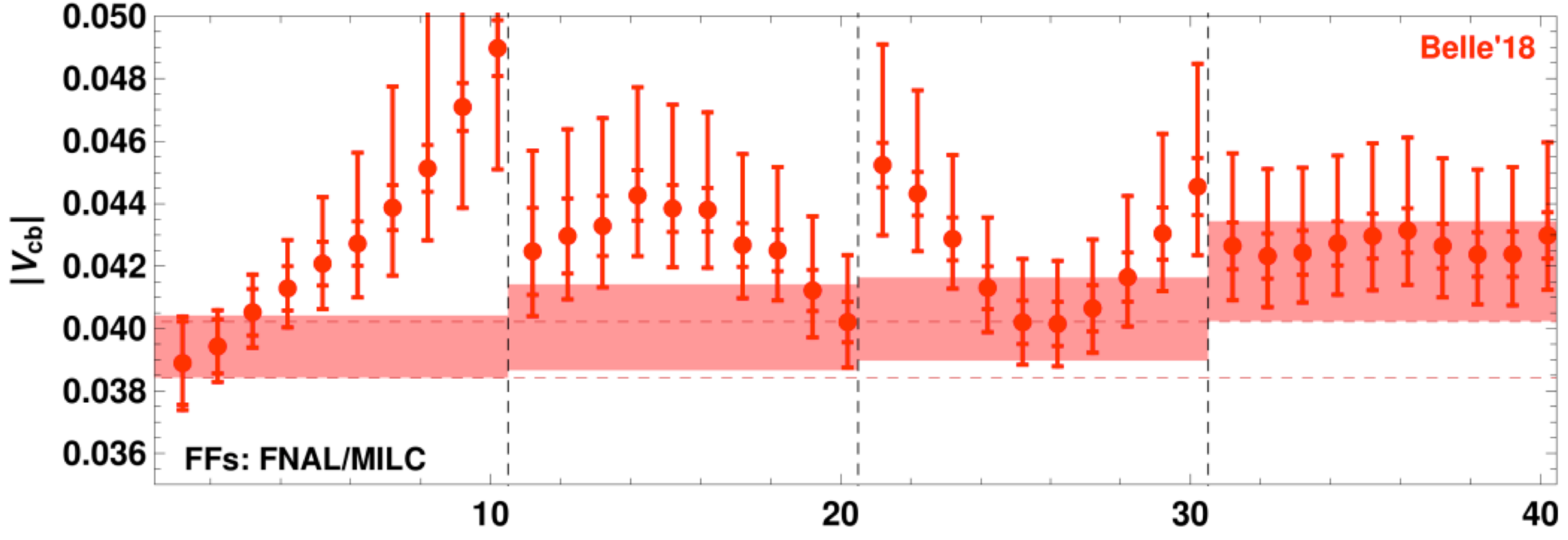
$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp.}}{(d\Gamma/dx)_i^{th.}}}$$

Critical understanding of the results obtained so far

Does the DM method modify the mean values/the correlations of the FFs?



i) Same bin-per-bin values of Vcb
ii) Same mean values for each kinematical variable!

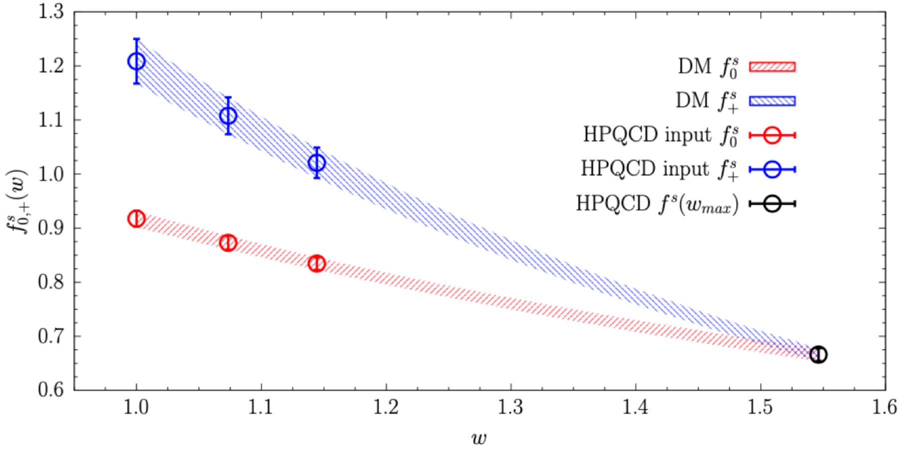


Jung's talk @ LHCb Implications Workshop 2022 (CERN)

Quick parenthesis: semileptonic $B_s \rightarrow D_s^{(*)}$ decays

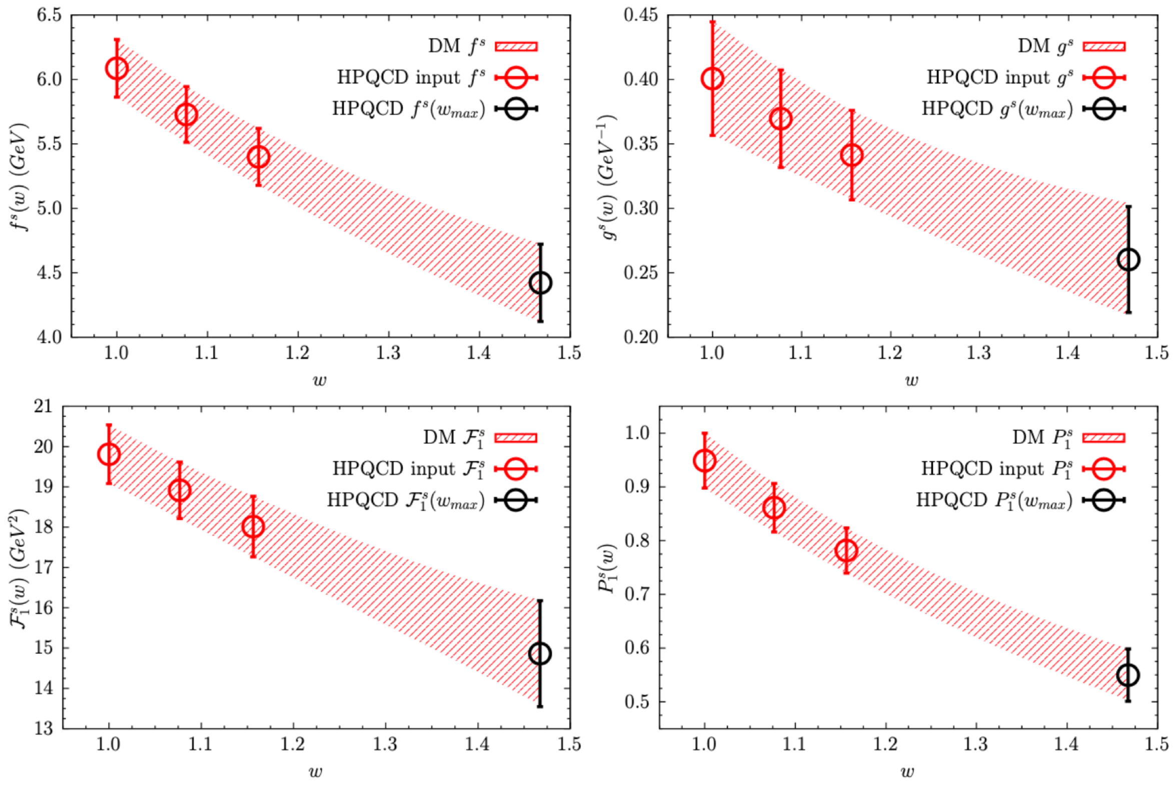
In **PRD '22 [arXiv:2204.05925]**, our DM method has been applied to **semileptonic $B_s \rightarrow D_s^{(*)}$ decays**. LQCD form factors taken from the results of the fits preformed by the HPQCD Collaboration in **PRD '20 [arXiv:1906.00701]** ($B_s \rightarrow D_s$) and **PRD '22 [arXiv:2105.11433]** ($B_s \rightarrow D_s^*$): we extract 3 data points for the FFs at small values of the recoil and apply the DM approach.

$B_s \rightarrow D_s \ell \nu_\ell$



* nice agreement in the whole kinematical range

$B_s \rightarrow D_s^* \ell \nu_\ell$



Quick parenthesis: semileptonic $B_s \to D_s^{(*)}$ decays

Without entering in the details of this analysis, phenomenological applications give the results

$$|V_{cb}|^{\rm DM} \cdot 10^3 = 41.7 \pm 1.9 \qquad \text{from } B_s \to D_s \ell \nu_\ell \text{ decays}$$

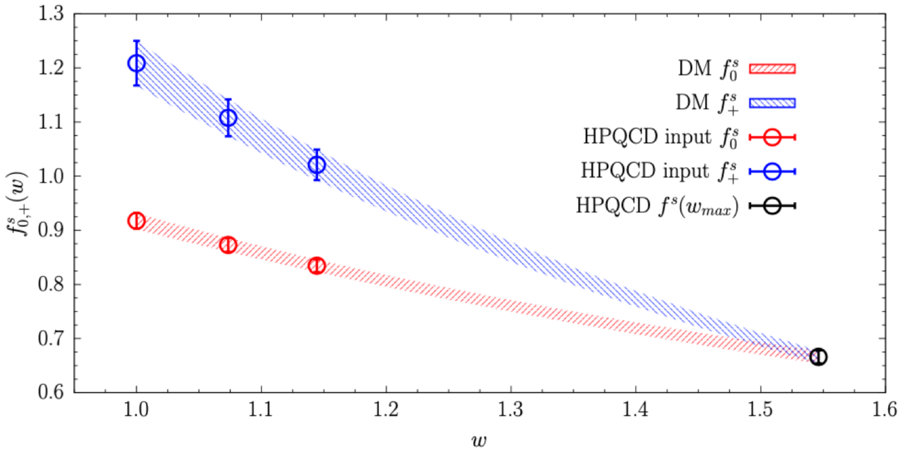
$$= 40.7 \pm 2.4 \qquad \text{from } B_s \to D_s^* \ell \nu_\ell \text{ decays}$$

through available exps. data by LHCb Collaboration (PRD '20 [2001.03225], JHEP '20 [2003.08453])

$$R(D_s) = 0.298\ (5)$$
$$R(D_s^*) = 0.250\ (6)$$

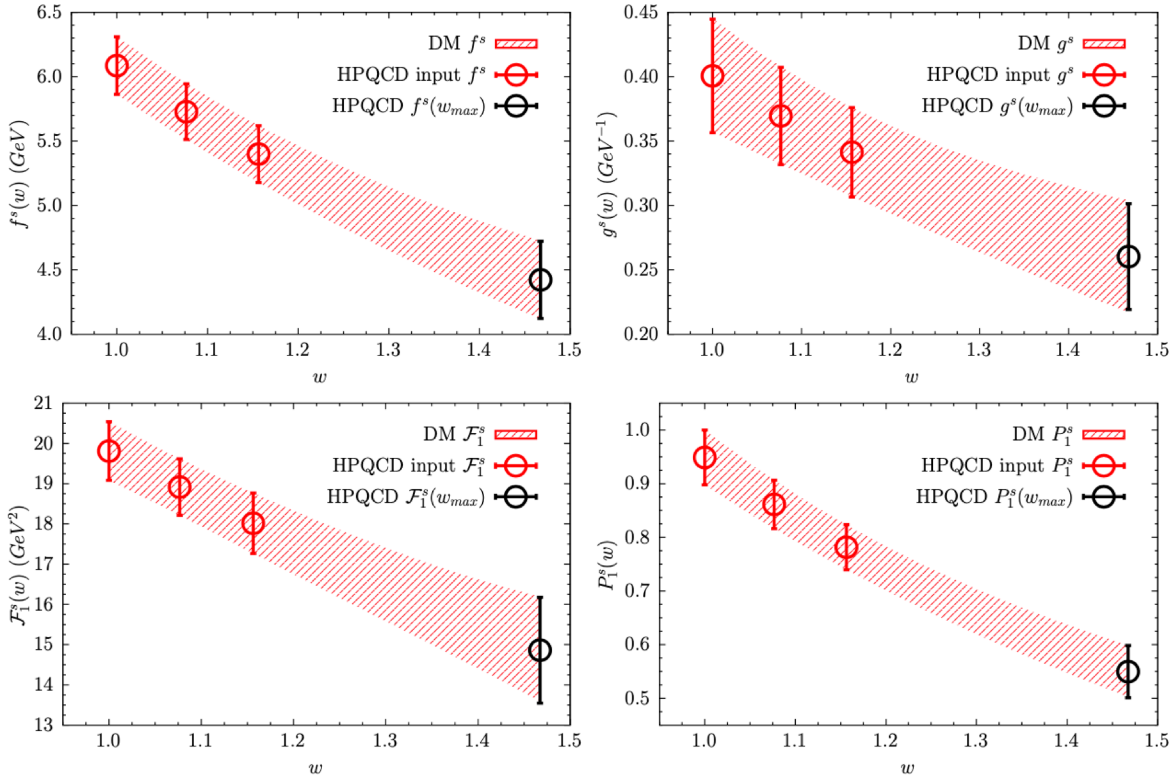
fully-theoretical

$B_s \to D_s \ell \nu_\ell$



* nice agreement in the whole kinematical range

$B_s \to D_s^* \ell \nu_\ell$



Quick parenthesis: semileptonic $B_s \to D_s^{(*)}$ decays

However, HPQCD Collaboration has recently updated their determination of lattice data for semileptonic Bs → Ds* decays!