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Chavdar Dutsov :: Paul Scherrer Institute

Systematic effects in the search for the muon EDM using the frozen-spin method

On behalf of the muonEDM collaboration

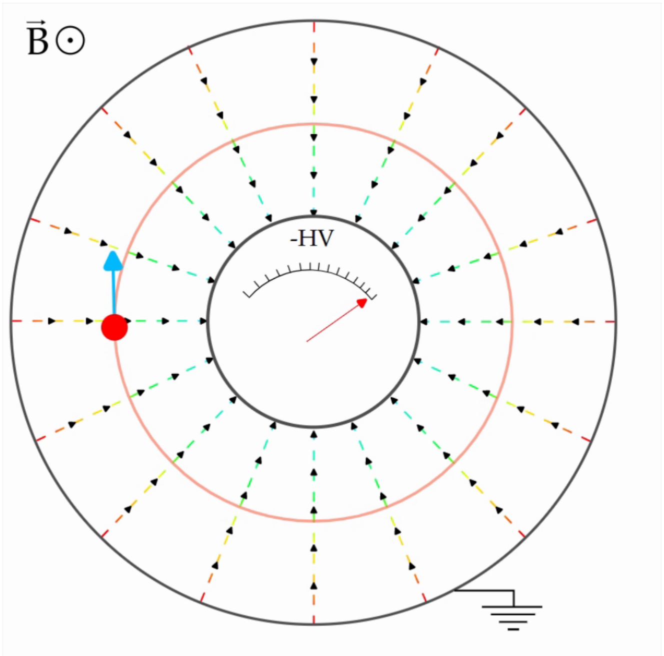
16 May 2023 – Pisa

Frozen spin technique in a nutshell

- Relativistic spin precession of a charged particle (Thomas-BMT equation) →

$$\vec{\Omega} = -\frac{e}{m_0}\left[\underbrace{\cancel{a\vec{B} + \left(\frac{1}{\gamma^2 - 1} - a\right)\frac{\vec{\beta}\times\vec{E}}{c}}}_{\text{g-2 term}} + \underbrace{\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)}_{\text{EDM term}}\right]$$

- By applying an appropriate radial E-field to the muon we negate the *g-2* term.
- Ideally any observed spin precession would be due to a non-zero EDM.
- Asymmetry between upstream and downstream emitted decay positrons.
- **Some asymmetry could still be observed due to systematic effects**



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Systematic effects

- Effects that lead to a *real* or *apparent* precession of the spin around the radial axis that are not related to the EDM.

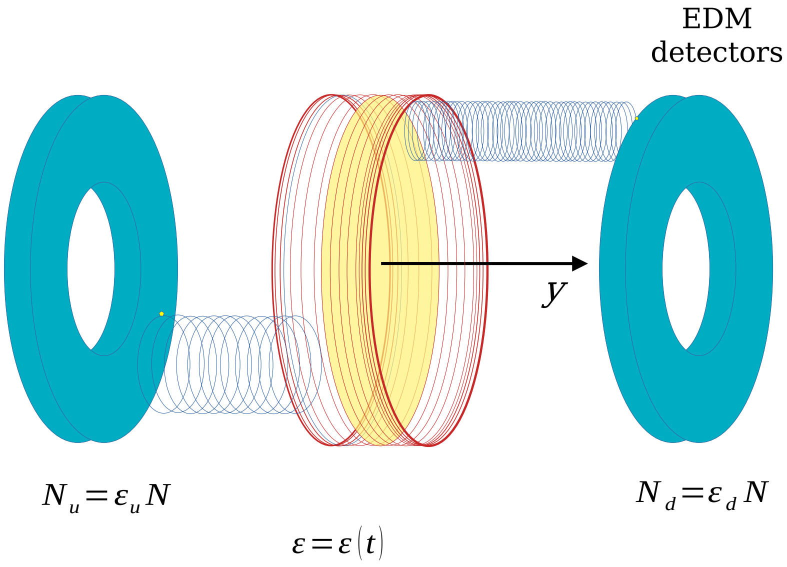
- Types of systematic effects:
  - Early to late variation of detection efficiency of the EDM detectors ***(apparent)***
  - Coupling of the anomalous magnetic moment with the EM fields of the experimental setup ***(real)***
    - Dynamical phase $$\vec{\Omega}_{\mathrm{MDM}} = -\frac{e}{m_0}\left[a\vec{B} - a\frac{\gamma-1}{\gamma}\frac{(\vec{\beta}\cdot\vec{B})\vec{\beta}}{\beta^2} + \left(\frac{1}{\gamma^2-1} - a\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$
    - Geometric phase $$\gamma_n[C] = i\oint_C \langle n,t|\left(\nabla_R|n,t\rangle\right) dR$$

Early-to-late detection efficiency changes

- Strong pulsed magnetic field → eddy currents, noise, heat in detectors and associated electronics.

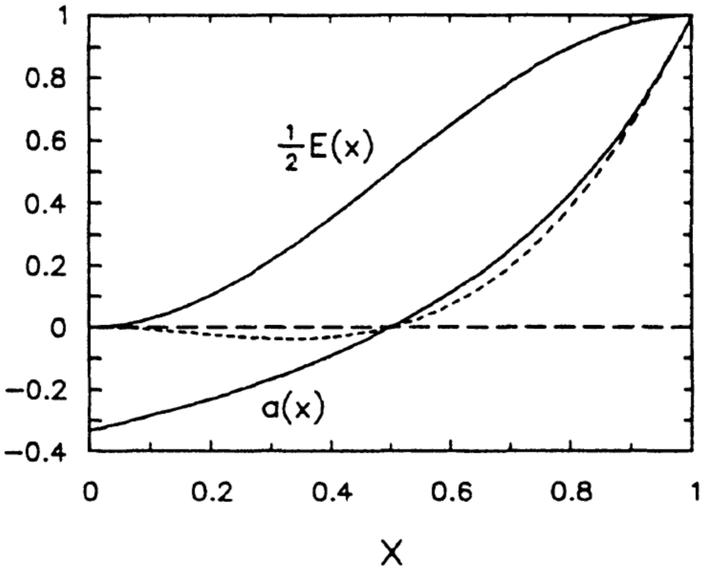
- **Time-dependent changes in the detection efficiency of a set of detectors will be seen as a false EDM signal.**

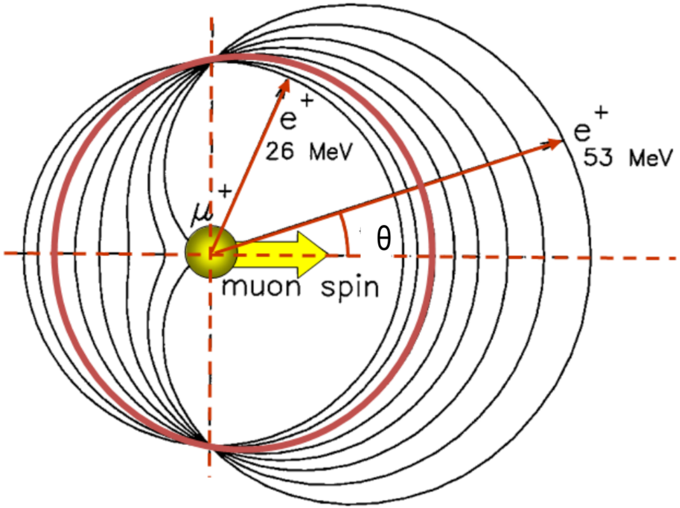
- Significant only for low-energy positrons that would produce a weak signal and could be missed by the detectors.



Kinematics of Michel decay positrons

- For high positron energies – preferentially emitted in the direction of the muon spin.
- Energy spectrum and **directional** asymmetry as a function of the fractional energy $x = E/E_{max}$:

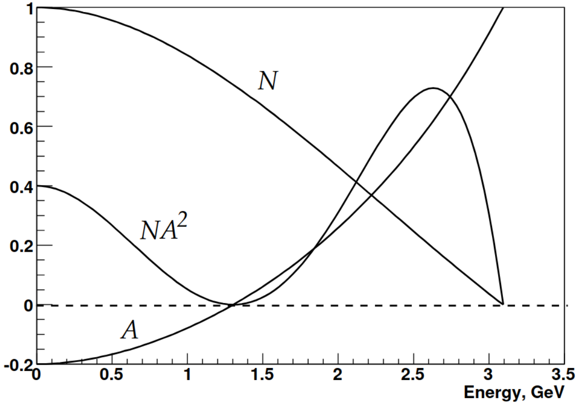


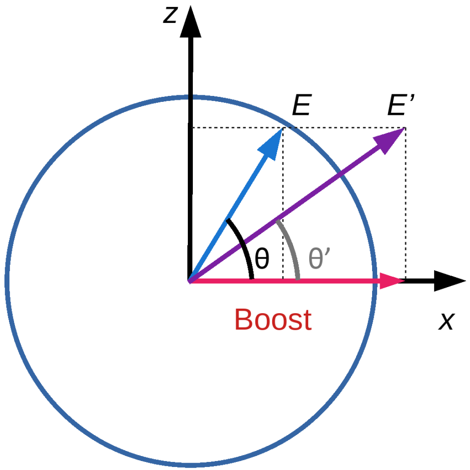


Kinematics – g-2 experiments

- For high momentum muons the angular distribution is Lorentz boosted along the momentum.
- For large boosts practically all decay positrons are emitted in the forward direction – **no directional asymmetry**.
- **Intensity asymmetry:** Dependence of the number of decay positrons at a given enrgy on the spin.

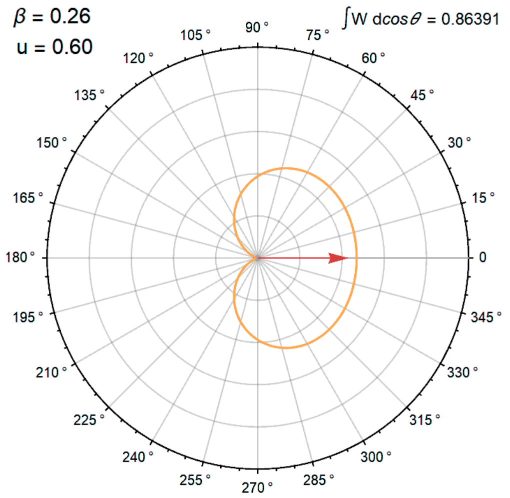
arXiv:1501.06858

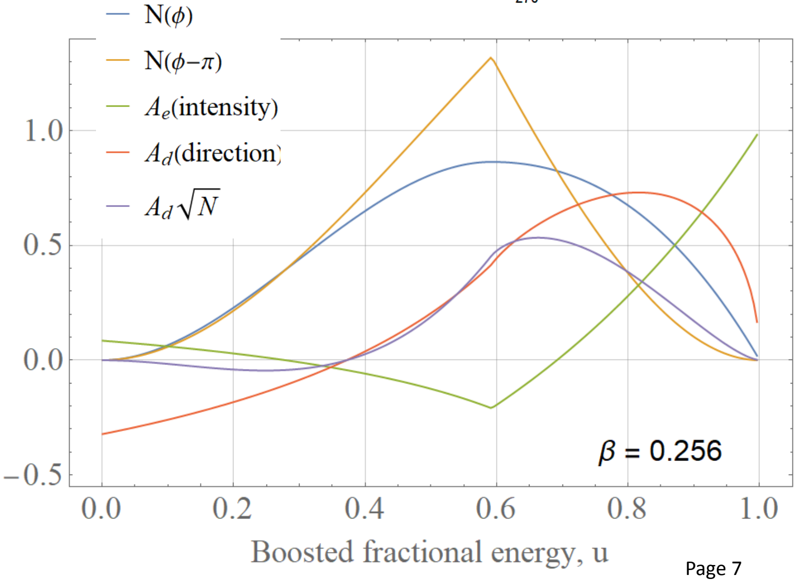




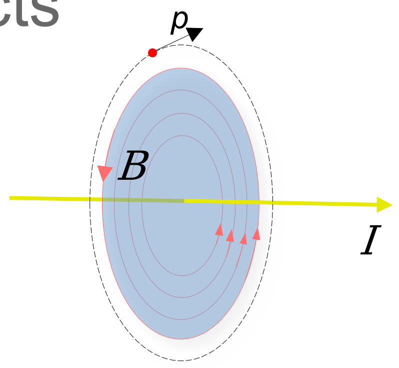
muEDM kinematics

- The first stage – 28 MeV/c surface muons.
- Both **directional** and **intensity** dependence on the spin direction.
- Precession due to the g-2 can be measured using intensity asymmetry.
- Precession due to EDM can be measured from the direction of emitted positrons:
  - up-down asymmetry.
- Only positrons above 25 MeV contribute significantly to the asymmetry.





Real spin precession systematic effects



- Only four sources of spin precession that could lead to EDM-like signal:

- Net azimuthal B-field
  (zero if no current flows through the area enclosed by the orbit)

$$\left\langle \left(\vec{\Omega}_{\mathrm{MDM}}\right)_{\theta} \right\rangle = -\frac{e}{m} \langle B_{\theta}(t) \rangle$$

- Time variable radial B-field
  (constraints on the pulse width and decay time of the magnetic kick)

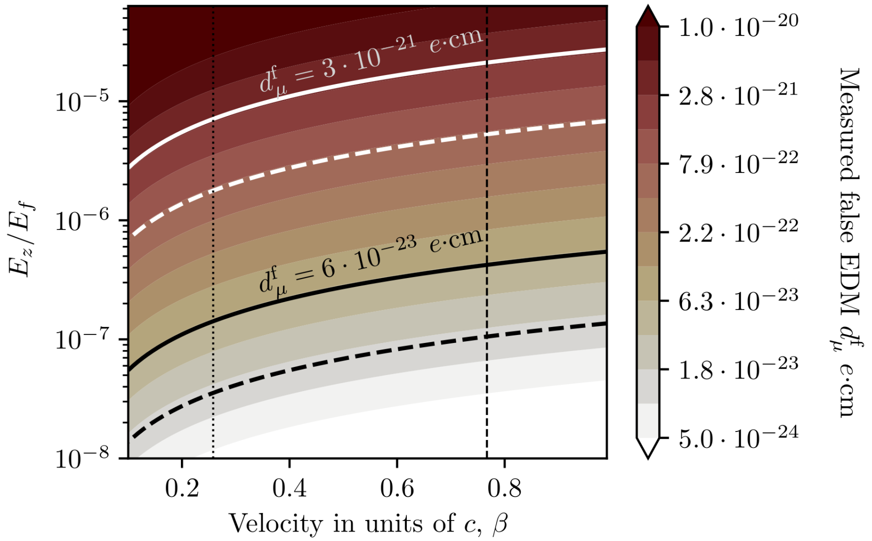
- Net longitudinal E-field

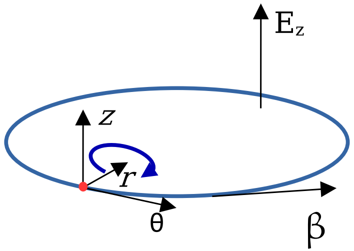
$$\left\langle \left(\vec{\Omega}_{\mathrm{MDM}}\right)_{\rho} \right\rangle = -\frac{ea}{mc} \left\langle \left(1 - \frac{1}{a(\gamma^2 - 1)} - \frac{1}{\beta_{\theta}^2}\right) \beta_{\theta} \right\rangle \langle E_z \rangle$$

- Geometric phases

Constraints on the average longitudinal E-field

- Limit on the average E_z field as a function of the muon velocity shown as a fraction of the radial component:
- **Effect cancels if particles are injected alternatively CW and CCW and subtracting counts in the detectors.**
- CW and CCW orbit directions are done by switching the B-field direction.





$$\left\langle \left(\vec{\Omega}_{\mathrm{MDM}}\right)_\rho \right\rangle = -\frac{ea}{mc} \left\langle \left(1 - \frac{1}{a(\gamma^2 - 1)} - \frac{1}{\beta_\theta^2}\right) \beta_\theta \right\rangle \langle E_z \rangle$$

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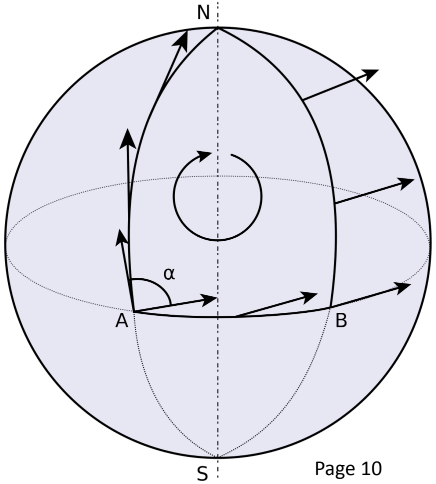
Geometric (Berry's) phase

- The geometric phase is a phase difference acquired over the course of a cycle in parameter space.
- Parallel transport of a vector around a closed loop.
- The angle by which it twists is proportional to the area inside the loop:
  - In classical parallel transport it's equal.
  - In quantum mechanics it's -½ (fermions).
- If oscillations around two axes are combined we can observe a phase shift (false EDM) **even if the the time average of each oscillation is zero.**



Calculation of geometric phases

- Spin precesses around axis **x** with amplitude C_1 and frequency Ω_x, and around **y** with amplitude C_2 and frequency Ω_y. Phase difference between the two β_0.

$$x = C_1 \sin(\Omega_x t), \quad y = C_2 \sin(\Omega_y t + \beta_0)$$

- The movement of the spin encloses an area A on some abstract surface. The area can be calculated from Green's theorem:

$$A = \frac{1}{2}\int_{t_0}^{t_1} (xy' - yx')dt$$

- The geometric phase as a function time is then:

$$\alpha(t;\omega_x,\omega_y,\beta_0) = \frac{1}{2}\frac{\Omega_x\Omega_y}{\omega_x\omega_y}\int \left(\omega_y\cos(\omega_y t+\beta_0)\sin(\omega_x t) - \omega_x\cos(\omega_x t)\sin(\omega_y t+\beta_0)\right)dt =$$

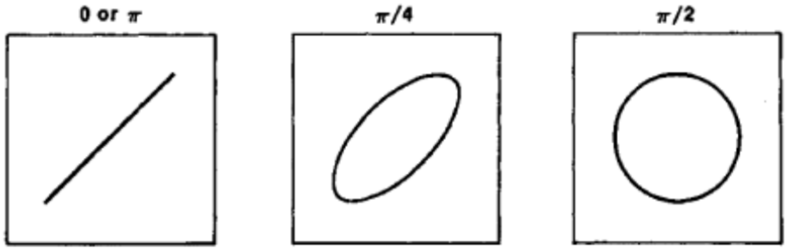
$$= \frac{1}{4}\frac{\Omega_x\Omega_y}{\omega_x\omega_y}\left[\frac{\omega_x-\omega_y}{\omega_x+\omega_y}\cos((\omega_x+\omega_y)t+\beta_0) - \frac{\omega_x+\omega_y}{\omega_x-\omega_y}\cos((\omega_y-\omega_x)t+\beta_0)\right]$$

Calculation of geometric phases

- In the case where the two oscillations have the same frequency the geometric phase is:

$$\alpha(t;\omega,\beta_0) = \frac{1}{2}\frac{\Omega_x\Omega_y}{\omega^2}\int \left(\omega\cos(\omega t+\beta_0)\sin(\omega t) - \omega\cos(\omega t)\sin(\omega t+\beta_0)\right) dt = \boxed{-\frac{1}{2\omega}\Omega_x\Omega_y t\sin(\beta_0)}$$

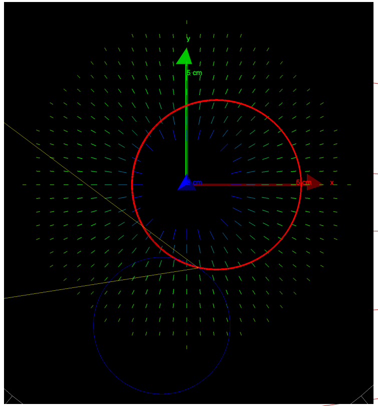
- The motion of the spin in this case is an ellipse with eccentricity defined by the phase difference between oscillations
  - no phase difference: ellipse looks like a line – no geometric phase
  - π/2 phase difference: ellipse is a circle and maximum area

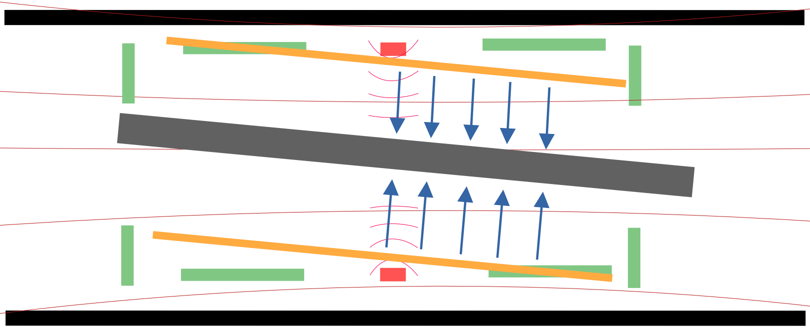


Example of a geometric phase accumulation

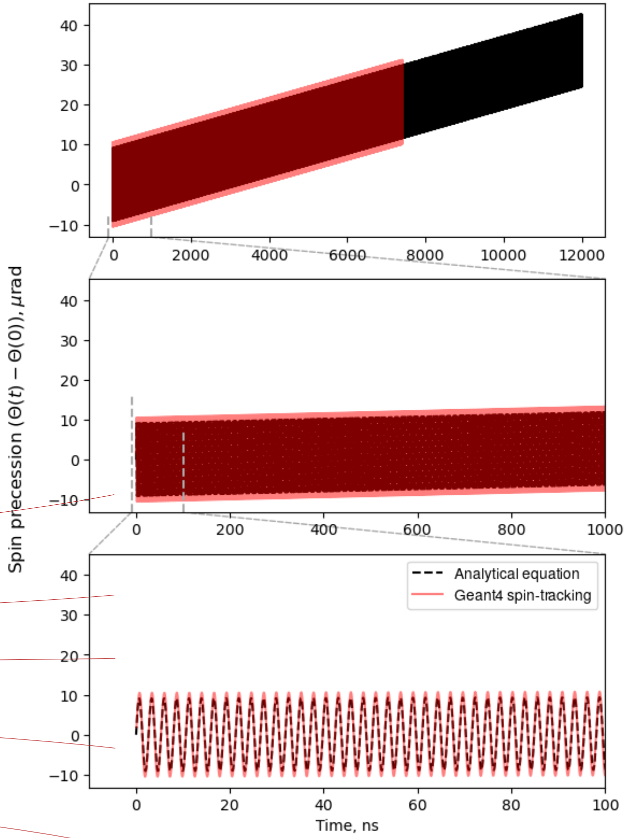
- Spin precession due to misalignment of E-field:
  - longitudinal oscillations due to stronger and weaker freeze field *(cyclotron frequency)*
  - radial oscillations due to longitudinal E-field oscillating between upstream and downstream directions *(cyclotron frequency)*

$$\alpha(t;\omega,\beta_0) = -\frac{1}{2\omega}\Omega_x\Omega_y t\sin(\beta_0)$$





Tilted and shifted E-field with respect to the center of the muon orbit



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Conclusions

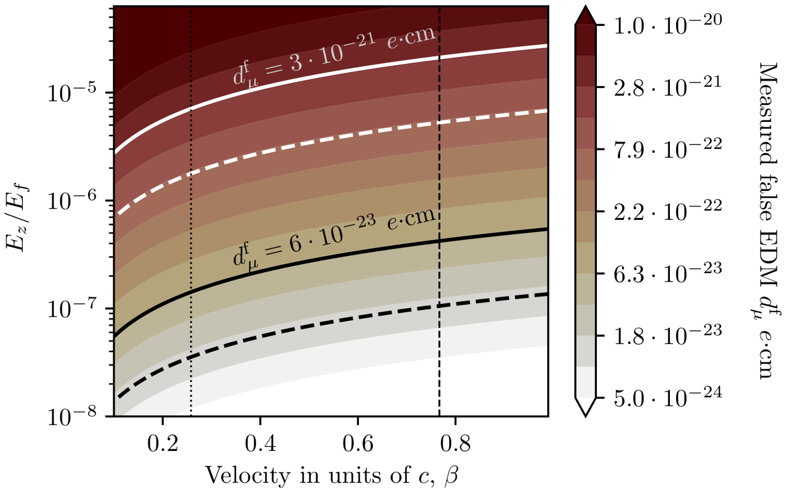
- Major sources of possible systematic effects in the experiemnt are:
- Early-to-late variations in the detection efficiency of the EDM detectors:
  - Only higher energy positrons above 25 MeV contribute to the measured asymmetry.
- Non-zero longitudinal E-field component:
  - Effect cancels if we alternate between CW and CCW injections, determined by the polarity of the main solenoid.
- An analytical description of the geometric phases was developed and tested with G4 simulations:
  - Places constraints on the E-field uniformity and alignment of the injection.

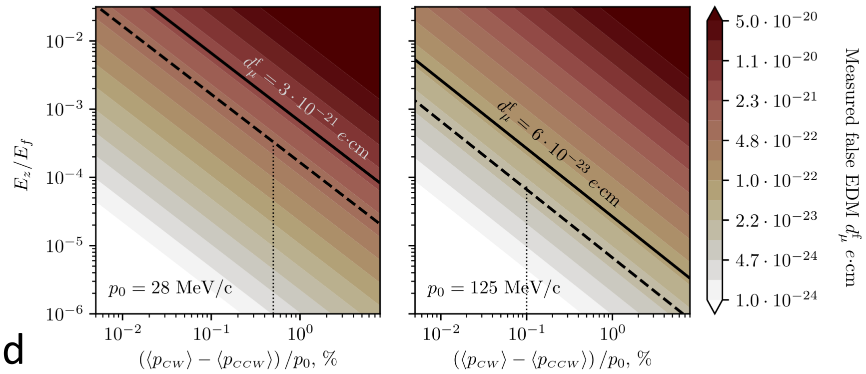
Thank you for the attention!

muonEDM collaboration kick-off meeting May 2022 (Pisa, Italy) →

Net longitudinal E-field

- Very stringent limit on the net longitudinal E-field.
- The effect changes sign with β.
  (change in the longitudinal B-field direction)
- This places limits on the average muon momentum for CW and CCW injections.
- The cancellation works only if:
  - the muon orbits are in the same place for CW/CCW
  - E-field non-uniformity does not lead to too different average E_z fields for CW/CCW injections.

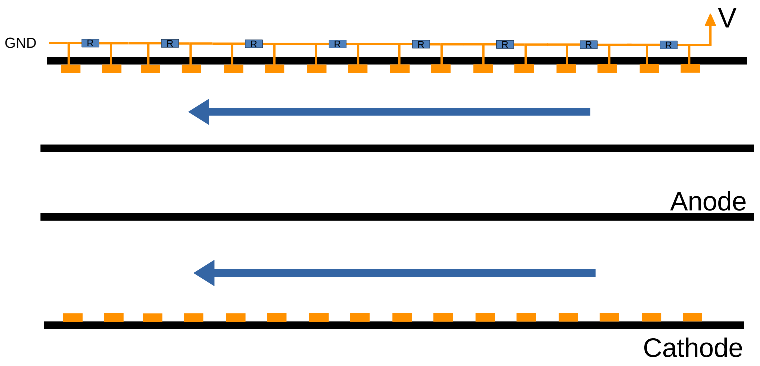


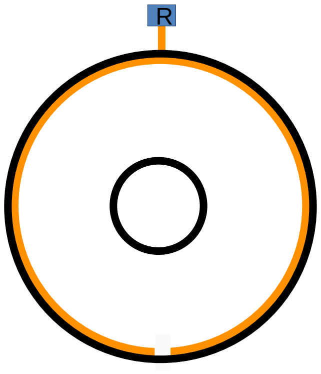


$$\left\langle \left(\vec{\Omega}_{\mathrm{MDM}}\right)_\rho \right\rangle = -\frac{ea}{mc} \left\langle \left(1 - \frac{1}{a(\gamma^2 - 1)} - \frac{1}{\beta_\theta^2}\right) \beta_\theta \right\rangle \langle E_z \rangle$$

Artificial longitudinal E-field

- One can generate a longitudinal E-field with controllable intensity.
- Could be used to verify that we can observe EDM-like signal.
- Could be tuned to cancel the intrinsic average E-field of an imperfect electrode, e.g.:
  - tune field until no signal is seen for CW, then measure CCW.
  - would increase real signal to systematic effect ratio.



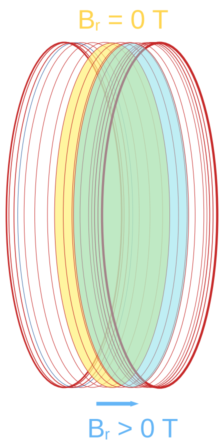


Limits on real spin precession effects

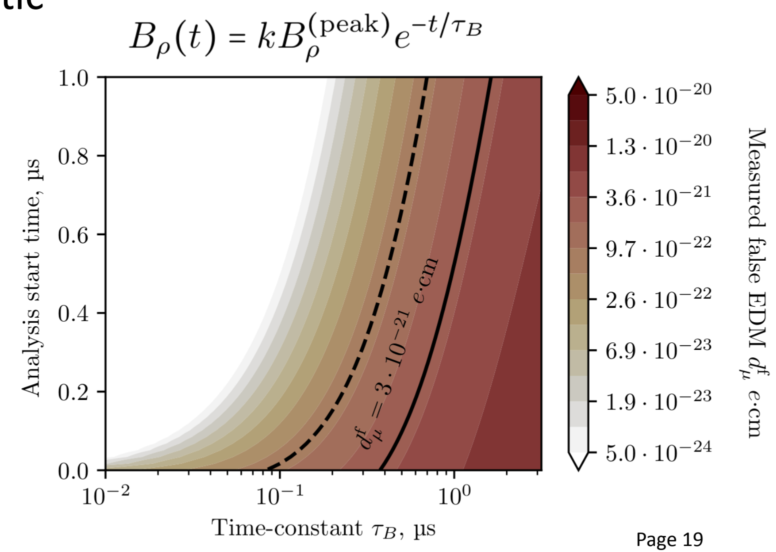
Systematic effect	Constraints	Phase I		Phase II	
		Expected value	**Syst.** **($\times 10^{-21}\,e\cdot$cm)**	**Expected value**	**Syst.** **($\times 10^{-23}\,e\cdot$cm)**
Cone shaped electrodes (longitudinal E-field)	Up-down asymmetry in the electrode shape	$\Delta_R < 30$ µm	0.75	$\Delta_R < 7$ µm	1.5
Electrode local smoothness (longitudinal E-field)	Local longitudinal electrode smoothness	$\delta_R < 3$ µm	0.75	$\delta_R < 0.7$ µm	1.5
Residual B-field from kick	Decay time of kicker field	< 50 ns	$< 10^{-2}$	< 50 ns	0.5
Net current flowing muon orbit area	Wiring of electronics inside the orbit	< 10 mA	$< 10^{-2}$	< 10 mA	0.3
Early-to-late detection efficiency change	Shielding and cooling of detectors	–		–	
Resonant geometrical phase accumulation	Misalignment of central axes	Pitch < 1 mrad Offset < 2 mm	2×10^{-2}	Pitch < 1 mrad Offset < 2 mm	0.15
TOTAL			1.1		2.2

*assuming electrode shape does not depend on magnetic field orientation

Time-variable radial B-field

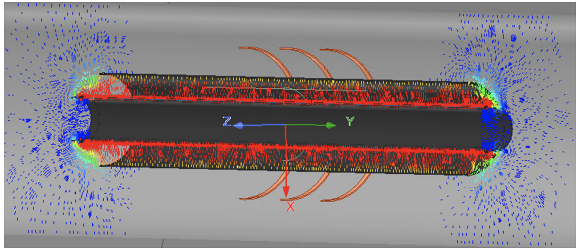


- A static radial B-field would create an offset in the position of the equilibrium orbit.
  - Not a problem since we are interested in the change in the phase with time.*
- A time-variable B_r can come from the magnetic kick or eddy currents caused by it.
- Assuming an exponentially decaying residual B-field after the kicker pulse one can determine limits on the decay constant as a fucntion of the analysis start time.
- Radial B-fields of less than 5 µT would not induce a significant spin precession.

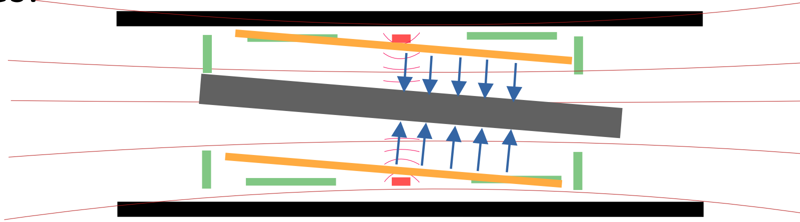


* may or may not induce Berry's phases

Sources of E_y field: electrode alignment



Misalignment of the electric field



- The E field of an infinitely long coaxial cylinders is:

$$\vec{E}(\vec{r}) = \frac{V}{\log\frac{b}{a}}\begin{pmatrix} x/r^2 \\ y/r^2 \\ 0 \end{pmatrix}$$

- Shifting the field by r_0 and rotating by α gives:

$$\vec{E}' = R_y(\alpha)\vec{E}(R_y^{-1}(\alpha)\vec{r} + \vec{r}_0) = V_0\begin{pmatrix} \frac{\xi}{\rho^2}\cos\alpha \\ \frac{\upsilon}{\rho^2} \\ -\frac{\xi}{\rho^2}\sin\alpha \end{pmatrix}$$

$$V_0 = \frac{V}{\log\frac{b}{a}},\ \upsilon = y + y_0,\ \xi = x_0 + x\cos\alpha - z\sin\alpha \text{ and } \rho^2 = \xi^2 + \upsilon^2$$

- Then average the new field out over a circular orbit:

$$\langle E'(\rho,\zeta)\rangle = \frac{1}{2\pi}\int_0^{2\pi} E'\,d\phi$$

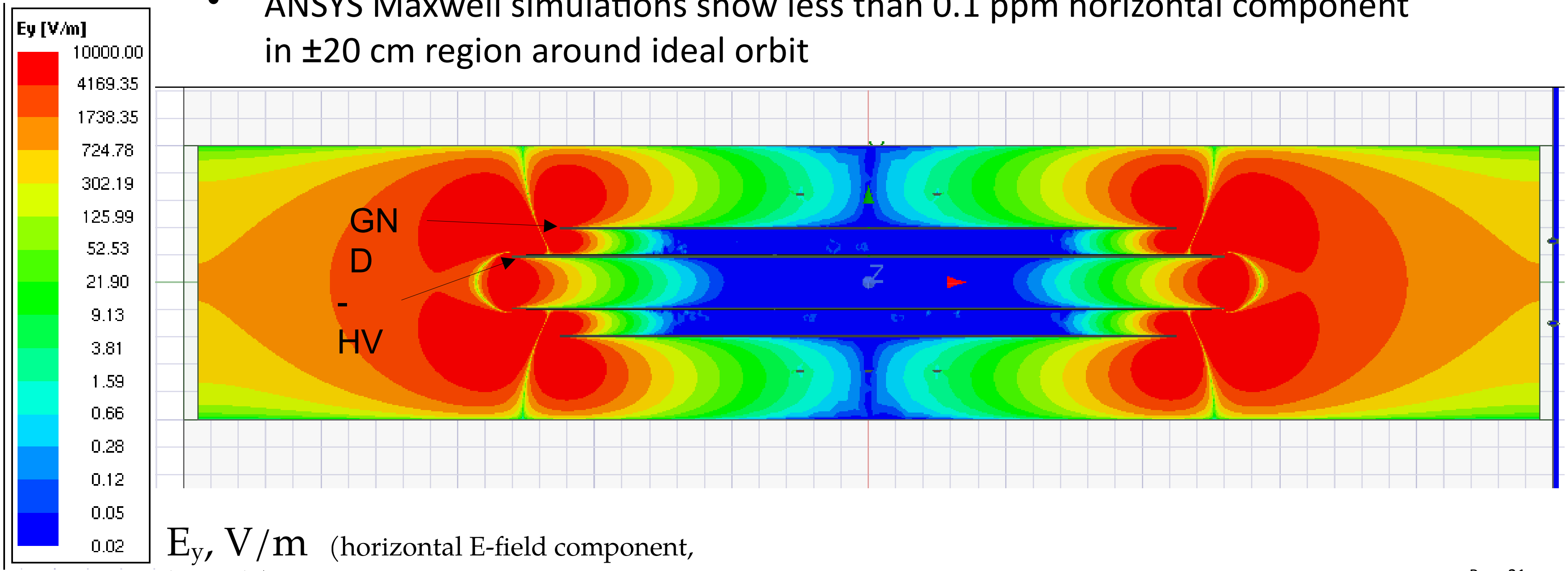
- It can be shown (numerically for now) that:

$$\boxed{\langle E'(\rho,\zeta)\rangle = \langle E(\rho,\zeta)\rangle}$$

- For a circular orbit the misalignment of the anode or cathode cannot introduce a net horizontal E-field (that was not there before)
- It also does not affect the ‘frozen spin’ condition

Sources of E_y field: fringe fields

- The assumption for infinite coaxial cylinders holds if there are negligible fringe field in the region of interest
- ANSYS Maxwell simulations show less than 0.1 ppm horizontal component in ±20 cm region around ideal orbit

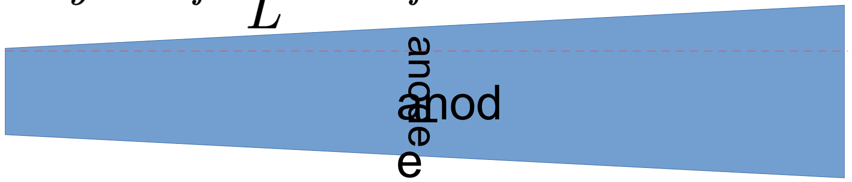


E_y, V/m (horizontal E-field component, logscale)

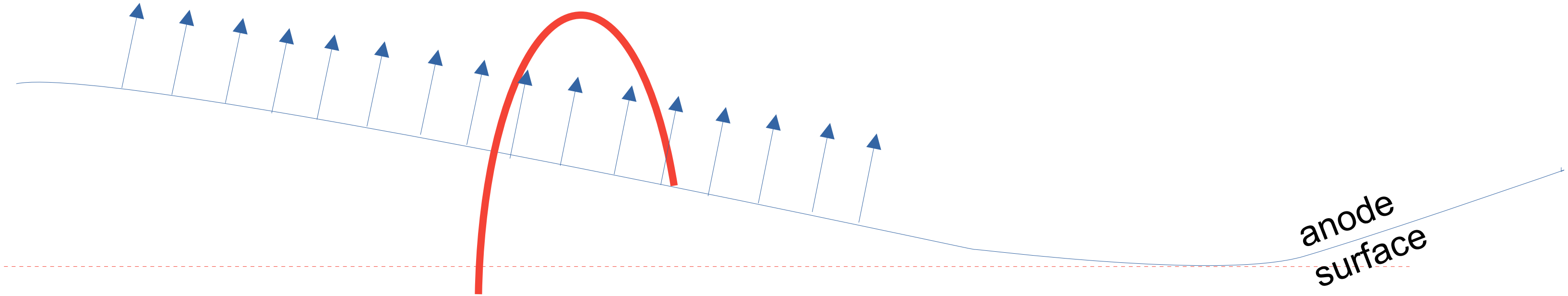
Sources of E_y field: electrode non-uniformity

- Tapered cone shaped electrodes

$$E_y \approx E_f \frac{\Delta_R}{L} \approx E_f \alpha$$



- Smoothness of the electrodes close to the muon orbit (few centimeters)



- Generally sub-micrometer surface smoothness is possible with common machining and polishing techniques

- Cylindricity in the order of 50 nm is measurable even on large samples

Requirements for electrode uniformity

- The condition to assume ideal uniformity is such that E_y is kept below the 0.5 ppm from E_f target
- If the radius of the anode or cathode is larger on one side than on the other then there will be a field component in the vertical direction
  - Using a small angle approximation:

    $$E_y \approx E_f \frac{\Delta_R}{L} \approx E_f \alpha$$

  - The radius of the anode or cathode should not change with more than Δ_R=150 nm (precursor) and Δ_R=300 nm (final) along its length (L ~ 50 cm)

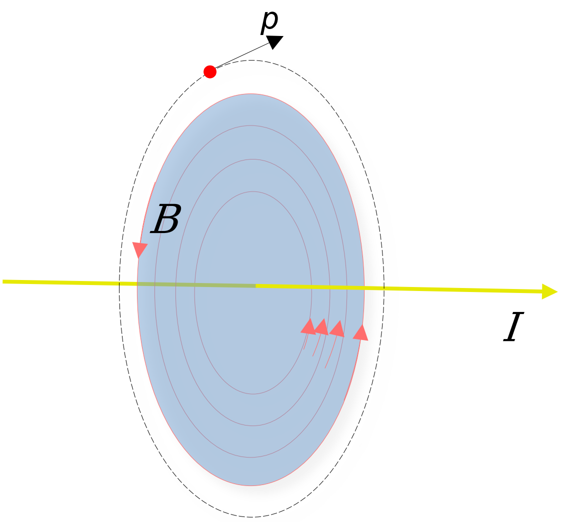
Requirements for electrode uniformity

- Another source of a net vertical field would be a small protrusion on the surface of the electrode
- The E-field of a cylinder with radius R and surface charge density σ at a distance r is: $$E_f(r) = \frac{\sigma R}{\varepsilon_0 r}$$
- The E-field from a small protrusion with area A at a distance Δ_z from the storage ring can be calculated from Coulomb's law: $$E_y(\Delta_z) = \frac{1}{4\pi\varepsilon_0}\frac{\sigma A}{\Delta_z^2}$$
- Thus the maximum allowed area for a protrusion is: $$A = 4\pi\frac{\Delta_z^2}{r}\frac{E_y}{E_f}$$
- Assuming a spherical cap like protrusion at 2 cm from the storage ring – maximum allowed height $h = 40\ \mu m$

Limit on the B-field parallel to the momentum

$$\langle \Theta_{\hat{z}} \rangle = -\frac{ea}{m_0} \langle B_z \rangle t.$$

- Non-zero average B_z field if there is electric current flowing through the area enclosed by the muon orbit
- Write net current!
- From Biot-Savart's law we can give a limit on the systematics due to such current
- Assuming non-insulated wire at the center of the orbit:
  - Precursor: I < 250 mA
  - Final experiment: I < 40 mA



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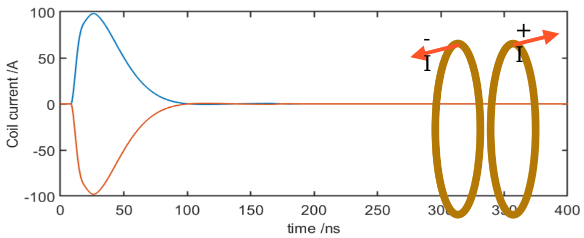
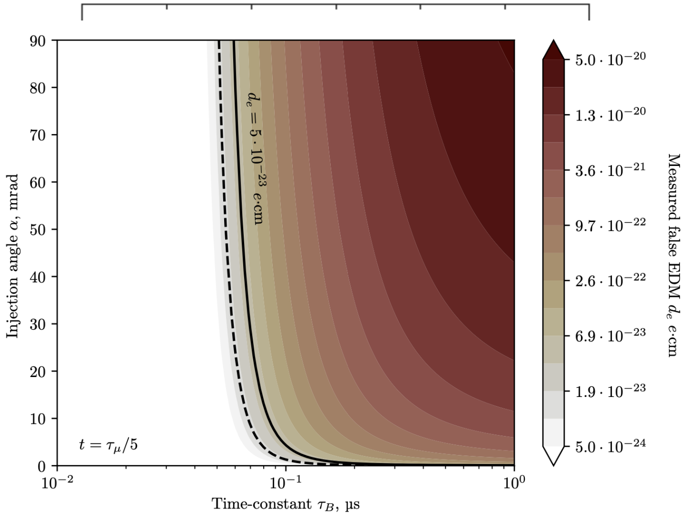
Limit on the radial *B*-field



- Limit on the kicker field decay time with relation to the injection angle

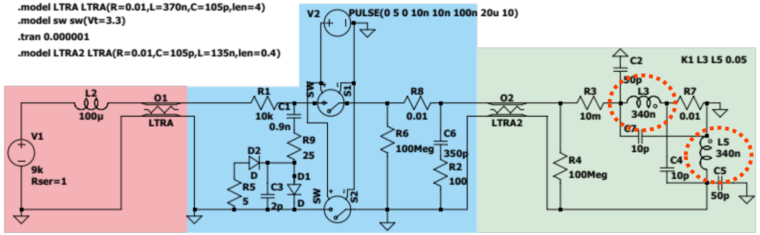
- Assumptions:
  - half-sine kicker field intensity
  - end of the kick is considered to be at the 10% from maximum livel
  - exponential decay of the ringing signal with time constant τ_B
  - the limit is such that the influence of the residual field is less than a given d_e at ~400 ns time

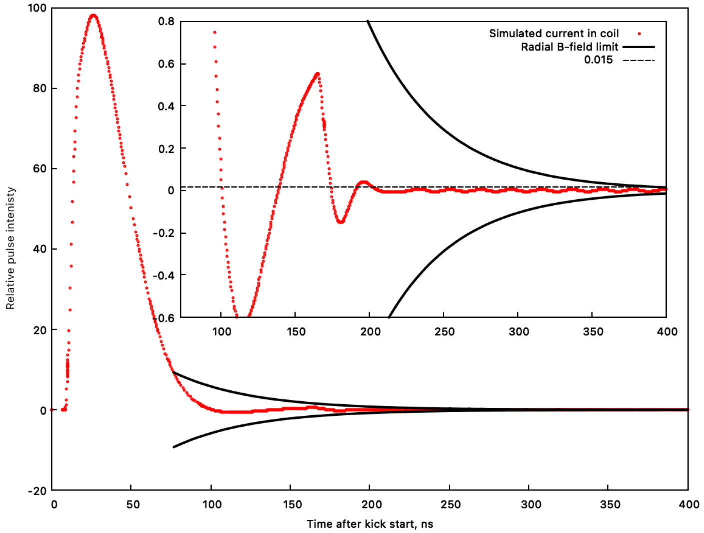


- *Note: the constraint is lower for later times and stronger for earlier times*

Limit on the radial *B*-field

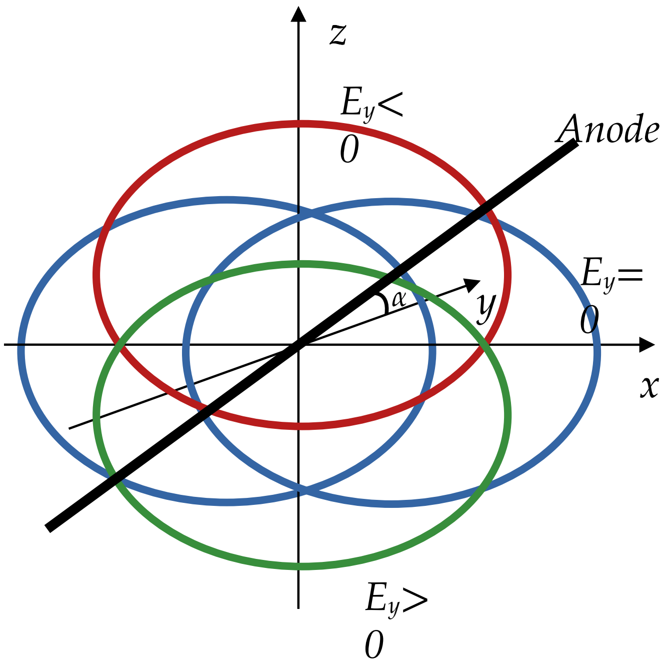
- Simulated short current pulse for the two anti-Helmholtz coils
- The solid black line shows the limiting decay time for an exponentially decaying pulse that goes below the limit at 400 ns (overshoot or undershoot)
- The influence of the simulated kicker field to the observed spin precession is negligible after 200 ns





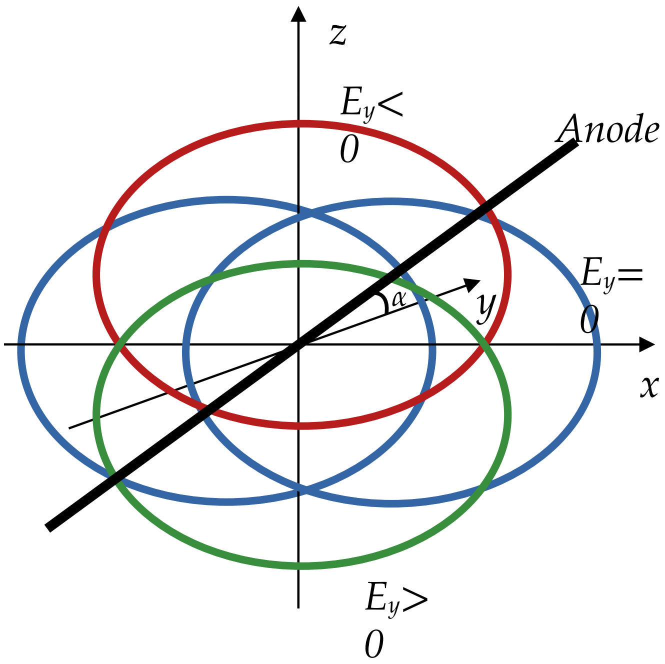
Non-circular muon orbit

- A non-zero average E_y field can be generated if the orbit of the muons is eliptical and at the same time it is not perpendicular to the axis of the anode.
  - the average field will be zero if the center of the orbit lies on the x axis
  - it is positive if it lies on the z axis above zero and negative if below zero
- In the general case the orbit will be eccentric due to the inward radial Lorentz force from the freeze field



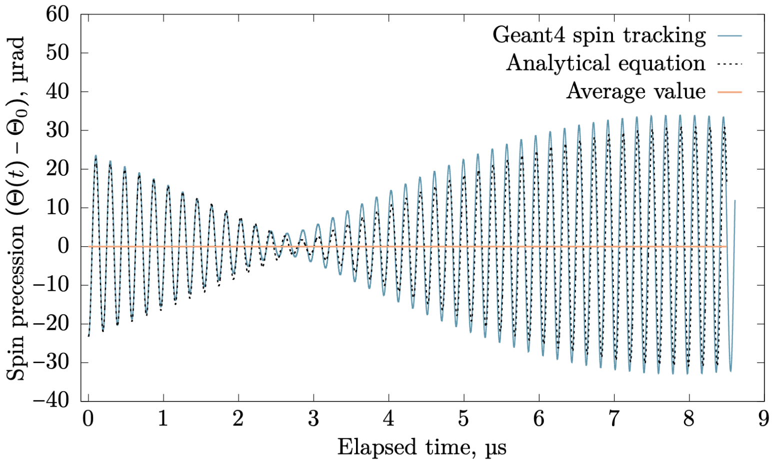
Non-circular muon orbit

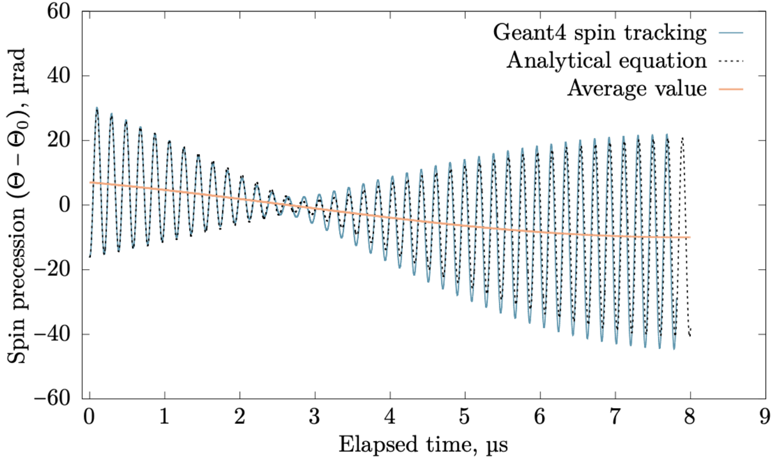
- The effect was observed also in the Geant4 simulations and is consisten with the analytical estimate
- Calculations with the analytical equations show that for $\alpha = 0.1°$ and orbit displacements up to 5 mm the eccentricity of the orbit should be kept below 0.1
- The eccentricity caused by the freeze field is significantly lower and does not pose a problem
- This effect could constrain the magnetic field uniformity (analysis pending)



Comparison with G4

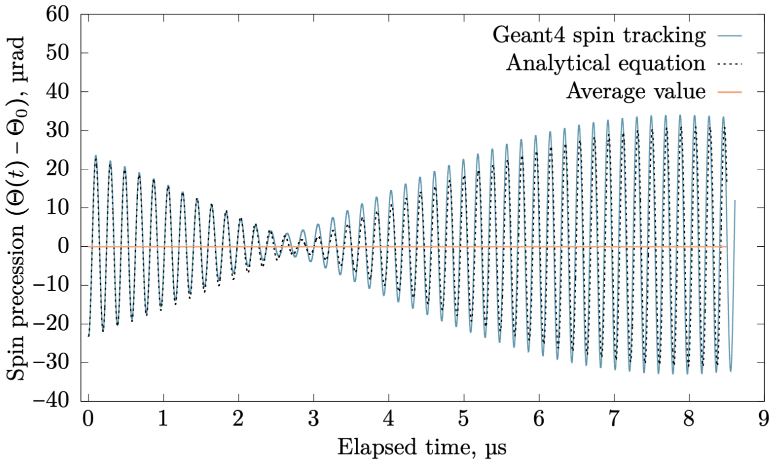
- Compared the analytical equations with a Geant4 simulation with the same parameters (weakly focusing coil current, radius; inital spin vector; etc…)
- In both attempts the frozen spin condition is not perfectly met (for illustration)
- Top: $E_y = 0$; Bottom: $E_y = E_{freeze}/10^6$
  - Note: bottom trend is similar to EDM of 10^{-21} *e*.cm

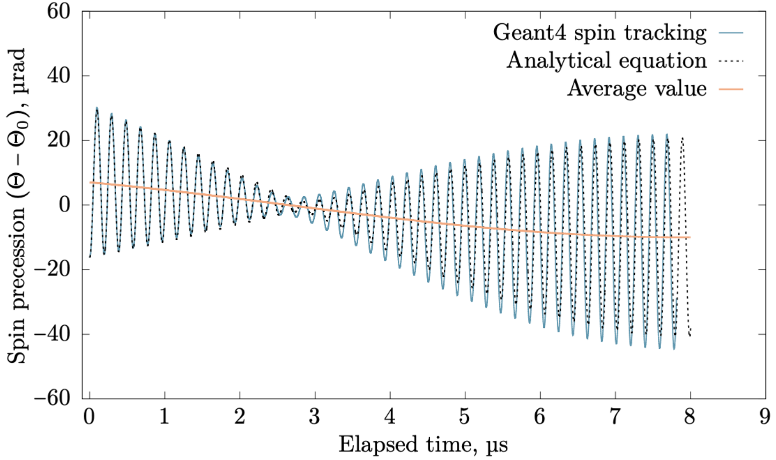




Comparison with G4

- Note that the oscillation frequency is not perfect as the fields are described by first order approximation
- Nevertheless, the equations describe the spin precession well in a very general scenario





Systematic effects

- The observed change in the asymmetry between the upstream and downstream detectors due to a non-zero EDM is to a first approximation equal to:

$$\dot{A} = \frac{\eta}{2}\frac{e}{m_0}\beta_z B_y \alpha P.$$

- Where $\eta \sim 10^{-9}$ for $d_e = 10^{-23}\, e.\mathrm{cm}$
- It has a similar meaning as the anomalous magnetic moment $a \sim 10^{-3}$
- One can expect that effects due to the magnetic dipole moment should be suppressed to a level better than 10^{-6}

$$\vec{\Omega} = -\frac{e}{m_0}\left[\underbrace{a\vec{B} + \left(\frac{1}{\gamma^2 - 1} - a\right)\frac{\vec{\beta}\times\vec{E}}{c}}_{\text{MDM}} + \underbrace{\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)}_{\text{EDM}}\right]$$

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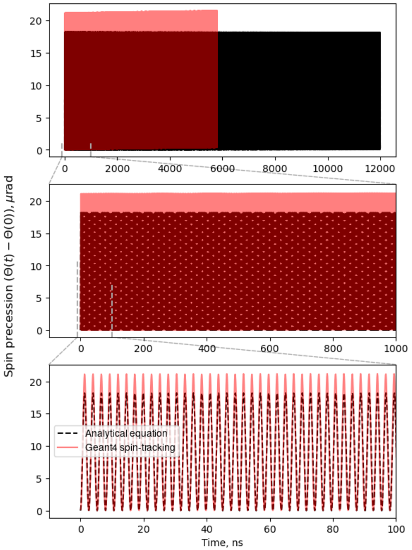

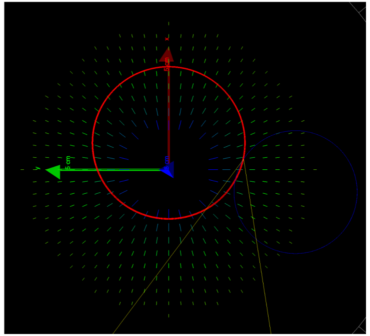

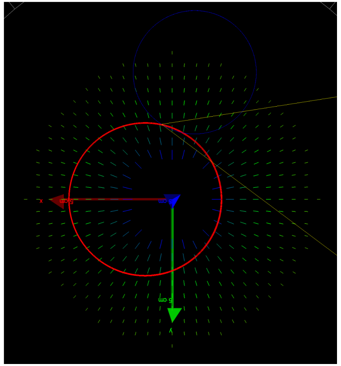

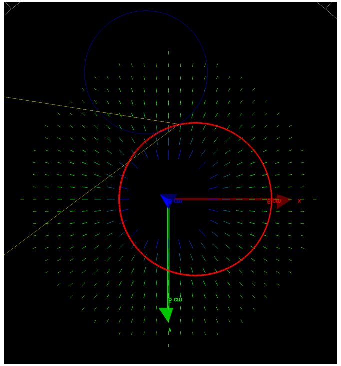

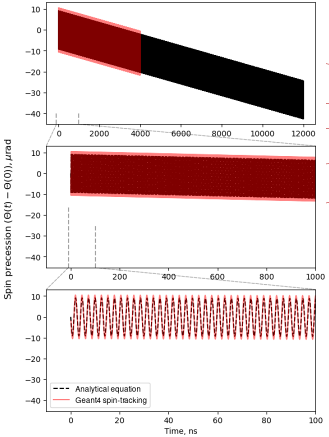

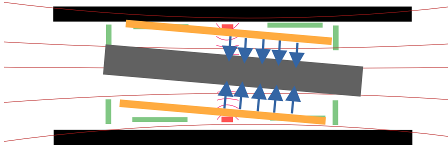

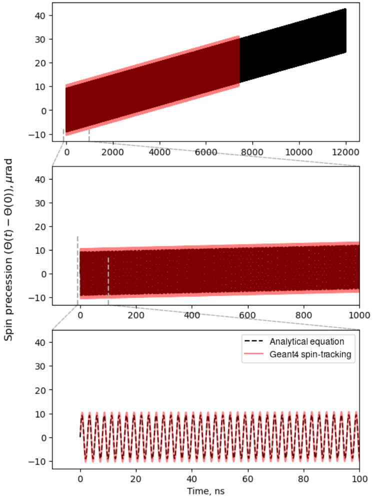

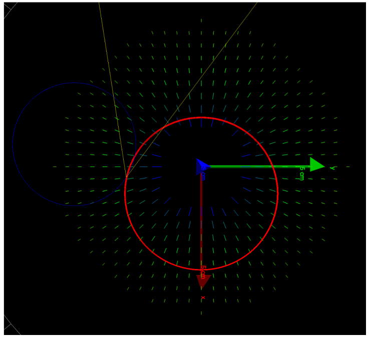

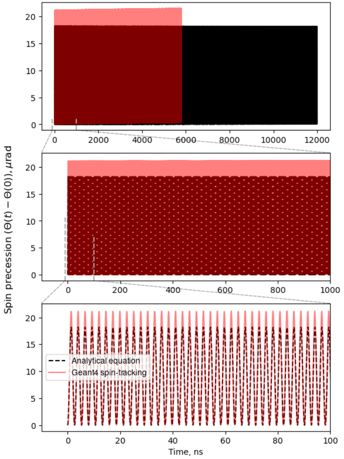

$$\alpha(t;\omega,\beta_0) = -\frac{1}{2\omega}\Omega_x\Omega_y t\sin(\beta_0)$$