Anomalous magnetic moment of the muon: theory introduction

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May 16, 2023

New Frontiers in Lepton Flavor, Pisa

funded by Swiss National Science Foundation

Magnetic moment

- relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio

- Dirac's prediction: $g_e = 2$
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- helped to establish QED and QFT as the framework for elementary particle physics
- today: probing not only QED but entire SM

Electron vs. muon magnetic moments

- influence of heavier virtual particles of mass M scales as

$$\frac{\Delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2}$$

- $(m_\mu/m_e)^2 \approx 4 \times 10^4 \Rightarrow$ muon is much more sensitive to **new physics**, but also to **EW and hadronic contributions**
- a_τ experimentally not yet known precisely enough

SM theory white paper

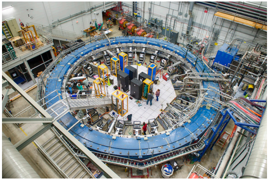
$\rightarrow$ T. Aoyama *et al.* (Muon $g-2$ Theory Initiative), Phys. Rept. **887** (2020) 1-166

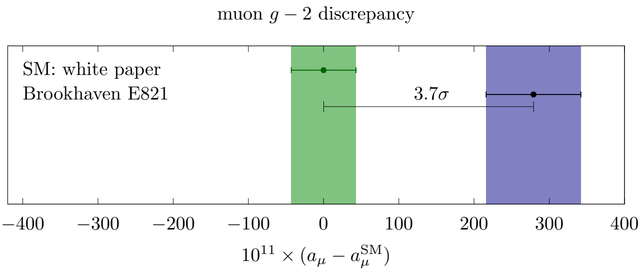
- community white paper on status of **SM calculation**
- new consensus on SM prediction, used for **comparison with FNAL 2021 result**
- many improvements on **hadronic contributions**
- since 2020: significant **new developments**

Muon anomalous magnetic moment $(g-2)_\mu$

recent and future experimental progress:

- FNAL will improve precision further: **factor of 4 wrt E821**





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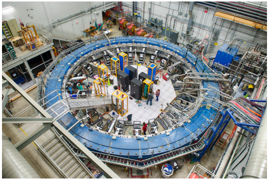
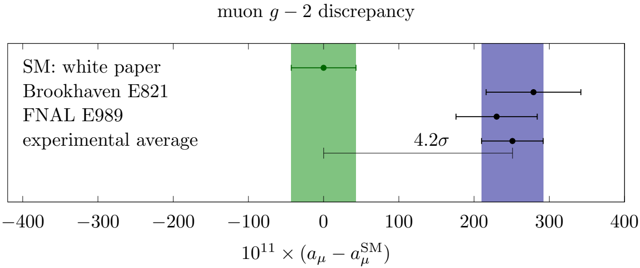


Photo: Glukicov (License: CC-BY-SA-4.0)



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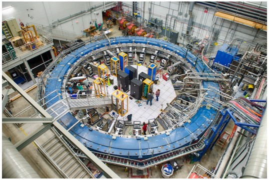
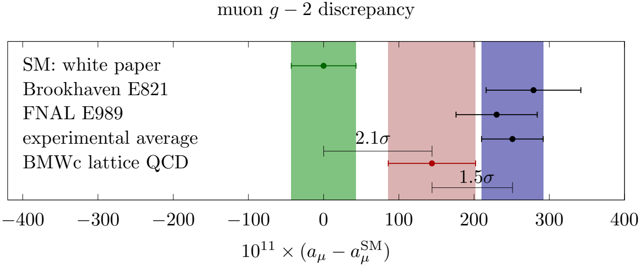




$(g-2)_\mu$: theory vs. experiment

- discrepancy between SM theory white paper and experiment $\mathbf{4.2\sigma}$
- theory error completely dominated by **hadronic effects**
- **tension** emerging between **lattice QCD** and **hadronic cross-section data**
- new $e^+e^- \to \pi^+\pi^-$ data from CMD-3 agree with lattice, incompatible with previous experiments

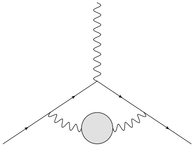
QED and electroweak contributions

- full $\mathcal{O}(\alpha^5)$ calculation by Kinoshita et al. 2012 (involves 12672 diagrams)
- EW contributions (EW gauge bosons, Higgs) calculated to two loops (three-loop terms negligible)

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
QED total	$116\,584\,718.931$	0.104
EW	153.6	1.0
theory total	$116\,591\,810$	43

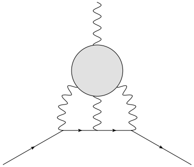
Hadronic contributions

- quantum corrections due to the strong nuclear force
- much smaller than QED, but **dominate uncertainty**



- hadronic vacuum polarization (HVP)

$$a_\mu^{\text{HVP}} = 6845(40) \times 10^{-11}$$



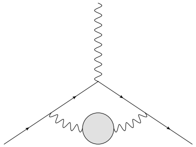
- hadronic light-by-light scattering (HLbL)

$$a_\mu^{\text{HLbL}} = 92(18) \times 10^{-11}$$

Theory vs. experiment

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
QED total	$116\,584\,718.931$	0.104
EW	153.6	1.0
HVP	$6\,845$	40
HLbL	92	18
SM total (white paper 2020)	$116\,591\,810$	43
experiment (E821+E989)	$116\,592\,061$	41
difference exp−theory	251	59

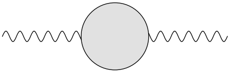
Hadronic vacuum polarization (HVP)



- at present evaluated via **dispersion relations** and cross-section input from $e^+e^- \rightarrow$ hadrons
- intriguing discrepancies between e^+e^- experiments $\Rightarrow$ treated as additional systematic uncertainty
- lattice QCD making fast progress
- $\mathbf{2.1\sigma}$ **tension** between dispersion relations and BMWc lattice results → S. Borsanyi *et al.*, Nature (2021)

Hadronic vacuum polarization (HVP)

photon HVP function:



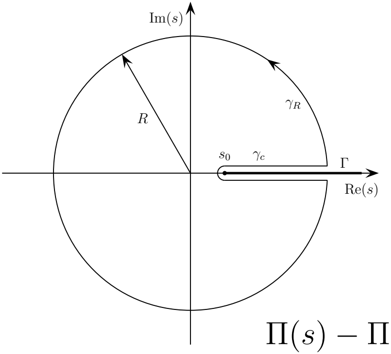
$$= i(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

unitarity of the S-matrix implies the optical theorem:

$$\mathrm{Im}\Pi(s) = \frac{s}{e(s)^2}\sigma(e^+e^- \to \mathrm{hadrons})$$

Dispersion relation

causality implies **analyticity**:



Cauchy integral formula:

$$\Pi(s) = \frac{1}{2\pi i} \oint_\gamma \frac{\Pi(s')}{s'-s} ds'$$

deform integration path:

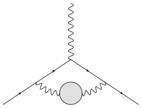
$$\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\Pi(s')}{(s'-s-i\epsilon)s'} ds'$$

HVP contribution to $(g-2)_\mu$

$$a_\mu^{\text{HVP}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\text{thr}}}^{\infty} ds\, \frac{\hat{K}(s)}{s}\,\sigma\big(e^+e^- \to \text{hadrons}(+\gamma)\big)$$

- basic principles: unitarity and analyticity
- direct **relation to data**: total hadronic cross section $\sigma\big(e^+e^- \to \text{hadrons}(+\gamma)\big)$
- dedicated e^+e^- program (BaBar, Belle, BESIII, CMD-3, KLOE, SND)

Hadronic vacuum polarization



- final white-paper number: data-driven evaluation

$$a_\mu^{\text{LO HVP, pheno}} = 6\,931(40) \times 10^{-11}$$

- white-paper 2020 average of published lattice results

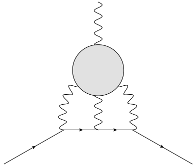
$$a_\mu^{\text{LO HVP, lattice average}} = 7\,116(184) \times 10^{-11}$$

- newest complete lattice-QCD result by BMWc

  → S. Borsanyi *et al.*, Nature (2021)

$$a_\mu^{\text{LO HVP, BMWc}} = 7\,075(55) \times 10^{-11}$$

Hadronic light-by-light (HLbL)



- previously based only on hadronic models
- our work: **dispersive framework** based on unitarity and analyticity, replacing hadronic models step by step
- **hadronic models** only for subdominant contributions
- matching to **asymptotic constraints**

Dispersive representation

→ Colangelo, Hoferichter, Procura, Stoffer, JHEP **09** (2015) 074, JHEP **04** (2017) 161

- write down a double-spectral (Mandelstam) representation for the HLbL tensor
- split the HLbL tensor according to the sum over intermediate (on-shell) states in unitarity relations

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{box}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\pi}_{\mu\nu\lambda\sigma} + \dots$$

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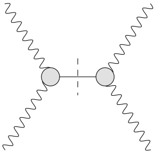
one-pion intermediate state



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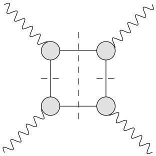
two-pion intermediate state in both channels



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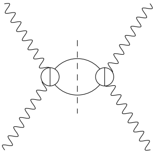
two-pion intermediate state in first channel



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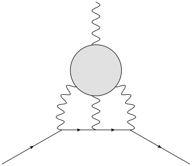
higher intermediate states

Hadronic light-by-light scattering



- dispersion relations + hadronic models (LO, without charm)

$$a_\mu^{\text{HLbL, pheno}} = 89(19) \times 10^{-11}$$

- lattice-QCD results

$a_\mu^{\text{HLbL, lattice}} = 79(35) \times 10^{-11}$ → T. Blum *et al.*, PRL **124** (2020) 132002

$a_\mu^{\text{HLbL, lattice}} = 106.8(15.9) \times 10^{-11}$ → E.-H. Chao *et al.*, EPJC **81** (2021) 651

$a_\mu^{\text{HLbL, lattice}} = 124.7(14.9) \times 10^{-11}$ → T. Blum *et al.*, 2304.04423 [hep-lat]

HLbL overview

→ T. Aoyama *et al.*, Phys. Rept. **887** (2020) 1-166

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
π^0, η, η'-poles	93.8	4.0
pion/kaon box	−16.4	0.2
S-wave $\pi\pi$ rescattering	−8	1
scalars, tensors	−1	3
axials	6	6
light quarks, short distance	15	10
c-loop	3	1
HLbL total (LO)	92	19

HLbL: recent progress

- asymptotic constraints
  $\rightarrow$ Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez, JHEP **10** (2020) 203; JHEP **04** (2021) 240; JHEP **02** (2023) 167

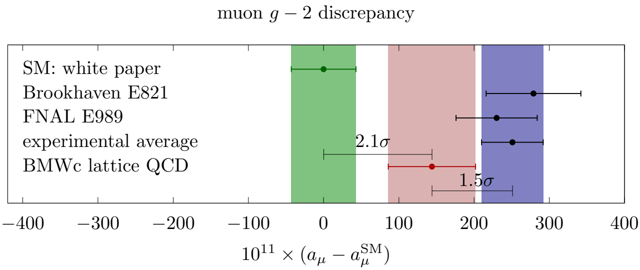
- scalar contributions: $a_\mu^{\text{HLbL}}[\text{scalars}] = -9(1) \times 10^{-11}$
  $\rightarrow$ Danilkin, Hoferichter, Stoffer, PLB **820** (2021) 136502

- first steps towards including axials in dispersive framework: $\rightarrow$ Zanke, Hoferichter, Kubis, JHEP **07** (2021) 106, Colangelo, Hagelstein, Hoferichter, Laub, Stoffer, EPJC **81** (2021) 702

- holographic-QCD models point to rather large axial contribution $\rightarrow$ Cappiello et al., PRD **102** (2020) 016009, Leutgeb, Rebhan, PRD **101** (2020) 114015; arXiv:2108.12345 [hep-ph]

- beyond spin 1: new dispersive framework in $g-2$ limit
  $\rightarrow$ Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125

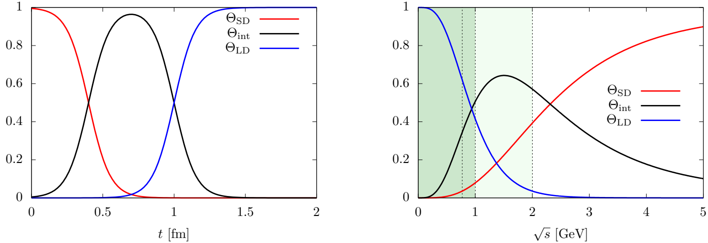
Tension between R-ratio and lattice



Tension between R-ratio and lattice

- **2.1σ tension** between R-ratio and BMWc lattice-QCD for HVP
- increases to **3.7σ for intermediate Euclidean window**
- recent results from ETMC, Mainz, RBC/UKQCD confirm BMWc intermediate window
- motivates **ongoing scrutiny** of R-ratio results

Euclidean window quantities



- smooth window weight functions in Euclidean time

  → Blum et al. [RBC/UKQCD], PRL **121** (2018) 022003

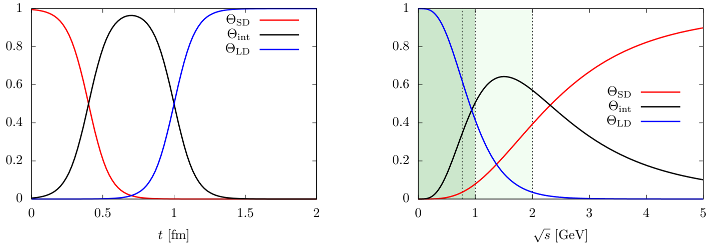
- total discrepancy:

  $$a_\mu[\text{BMWc}] - a_\mu[\text{WP20}] = 14.4(6.8) \times 10^{-10}$$

- intermediate window: → Colangelo et al., PLB **833** (2022) 137313

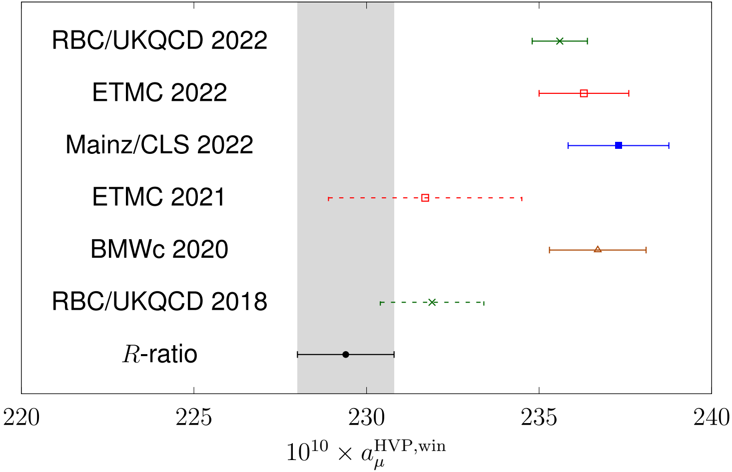
  $$a_\mu^{\text{int}}[\text{BMWc}] - a_\mu^{\text{int}}[e^+e^-] = 7.3(2.0) \times 10^{-10}$$

Euclidean window quantities



- using form of weight functions:
  at least $\sim 40\%$ from **above 1 GeV**
- assumptions:
  - rather uniform shifts in low-energy $\pi\pi$ region
  - no significant negative shifts

Results for intermediate window



R-ratio result: → Colangelo et al., PLB **833** (2022) 137313

Tension with lattice QCD

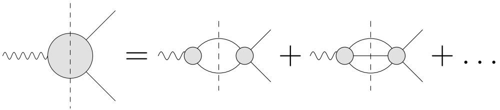
$\rightarrow$ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

- implications of changing HVP?
- modifications at high energies affect **hadronic running of $\alpha_{\text{QED}}^{\text{eff}}$** $\Rightarrow$ clash with global EW fits

  $\rightarrow$ Passera, Marciano, Sirlin (2008), Crivellin, Hoferichter, Manzari, Montull (2020), Keshavarzi, Marciano, Passera, Sirlin (2020), Malaescu, Schott (2020)
- lattice studies point at region < 2 GeV
- $\pi\pi$ **channel** dominates
- relative changes in other channels would need to be huge

Two-pion contribution to HVP

- $\pi\pi$ contribution amounts to more than 70% of HVP contribution
- responsible for a similar fraction of HVP uncertainty
- can be expressed in terms of **pion vector form factor** $\Rightarrow$ constraints from analyticity and unitarity

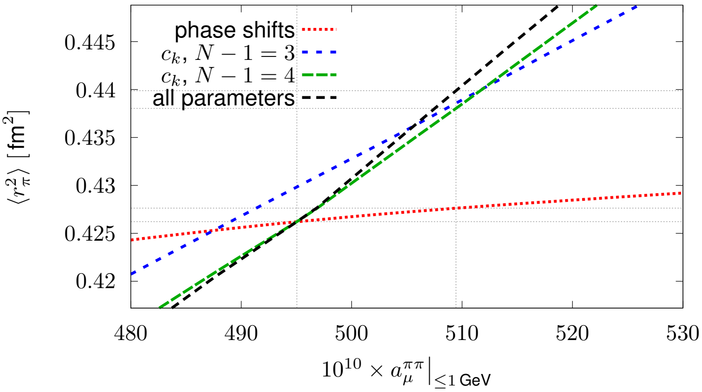


$\rightarrow$ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

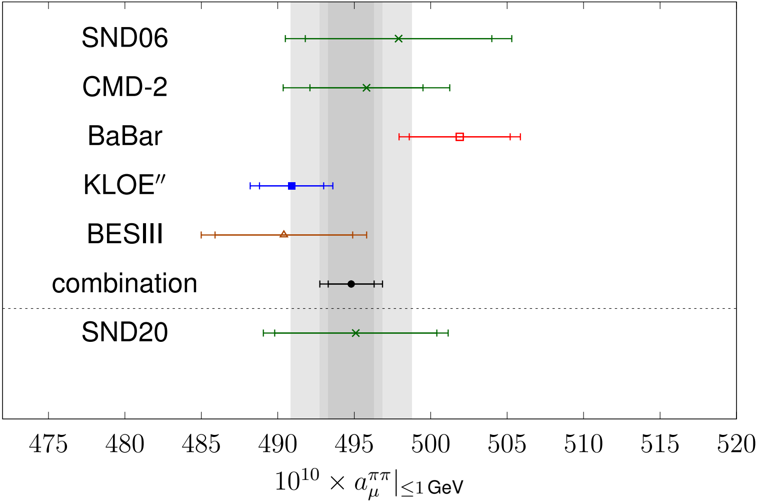
$\rightarrow$ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

correlations between $a_\mu^{\pi\pi}$ and $\langle r_\pi^2 \rangle$



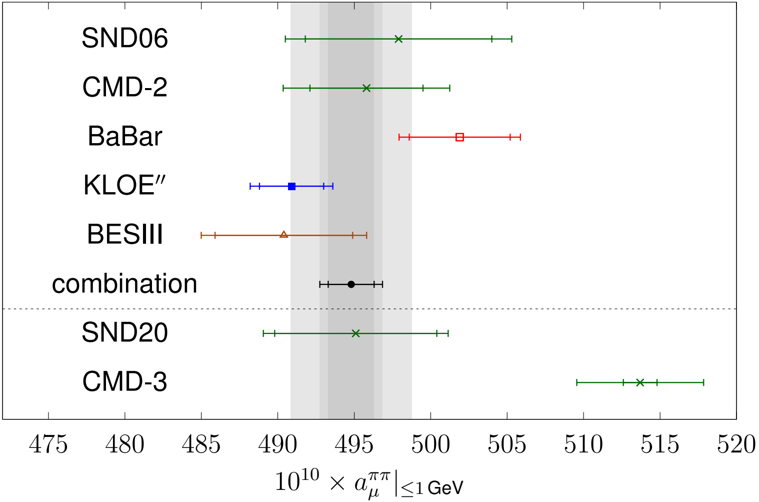
Result for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006
Colangelo, Hoferichter, Kubis, Stoffer, JHEP **10** (2022) 032



More tensions: CMD-3

→ F. Ignatov et al. (CMD-3), 2302.08834 [hep-ex]

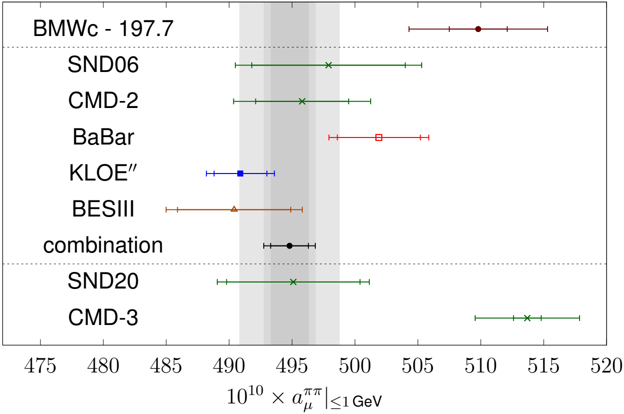


CMD-3 vs. all the rest

discrepancy	$a_\mu^{\pi\pi}\big\|_{[0.60,0.88]\,\text{GeV}}$	$a_\mu^{\pi\pi}\big\|_{\leq 1\,\text{GeV}}$	int window
SND06	1.8σ	1.7σ	1.7σ
CMD-2	2.3σ	2.0σ	2.1σ
BaBar	3.3σ	2.9σ	3.1σ
KLOE″	5.6σ	4.8σ	5.4σ
BESIII	3.0σ	2.8σ	3.1σ
SND20	2.2σ	2.1σ	2.2σ
Combination	4.2σ (6.1σ)	3.7σ (5.0σ)	3.8σ (5.7σ)

(discrepancies in brackets exclude systematic effect due to BaBar–KLOE tension)

Result for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV



Assumption: suppose all changes occur in $\pi\pi$ channel < 1 GeV

$$\Rightarrow a_\mu^{\mathrm{total}}[\mathrm{WP20}] - a_\mu^{2\pi,<1\,\mathrm{GeV}}[\mathrm{WP20}] = 197.7 \times 10^{-10}$$

Overview

Summary

- FNAL 2021 result increased tension with white-paper SM value to $\mathbf{4.2\sigma}$
- **intriguing tension** between lattice HVP and R-ratio
- long-standing discrepancy between BaBar/KLOE
- new CMD-3 results **disagree** with other e^+e^- results
- Euclidean windows useful tools for detailed scrutiny
- unitarity/analyticity enable **independent checks** via pion VFF and $\langle r_\pi^2 \rangle$
- final FNAL precision goal calls for **further improvement** also in HLbL