Theory of charged current decays

Marzia Bordone

New Frontiers in Lepton Flavor

15.05.2023

Introduction

Interaction basis

$$-\mathcal{L}_{\text{Yukawa}} = Y_d^{ij}\bar{Q}_L^i H d_R^j + Y_u^{ij}\bar{Q}_L^i \tilde{H} u_R^j + \text{h.c.}$$

$Y_q \sim$ 

- Strong hierarchy between families
- Many free parameters

Mass basis

$$\mathcal{L}_{cc} \propto \bar{u}_L^i \gamma^\mu b_L^j W_\mu^+ V_{ij}$$

- Remnant of the change of basis is the CKM matrix
- The CKM is a unitary matrix

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3x3 matrices in flavour space

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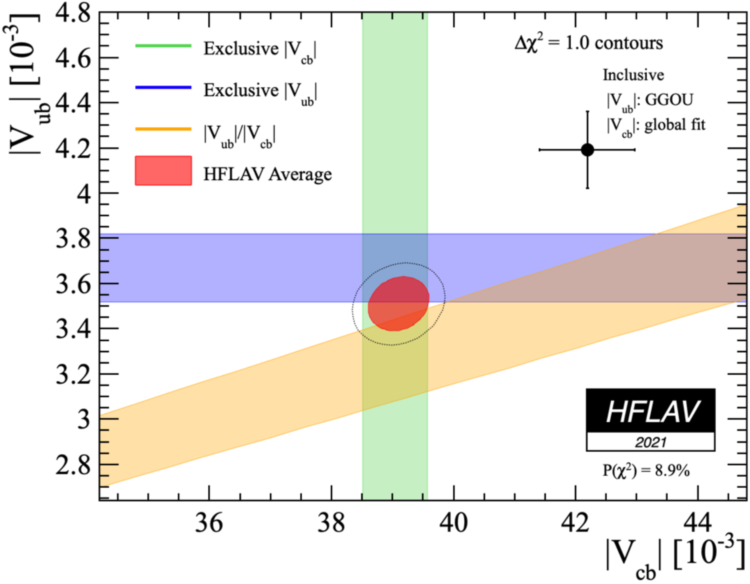
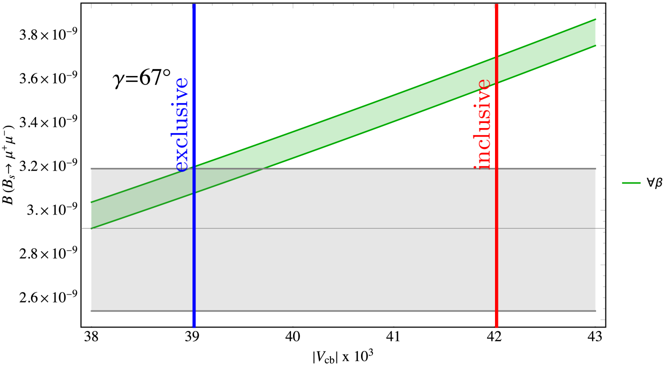


Exclusive |V_cb|
Exclusive |V_ub|
|V_ub|/|V_cb|
HFLAV Average
Δχ² = 1.0 contours
Inclusive
|V_ub|: GGOU
|V_cb|: global fit
HFLAV
2021
P(χ²) = 8.9%
|V_ub| [10^-3]
4.8
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
2.8
36
38
40
42
44
|V_cb| [10^-3]

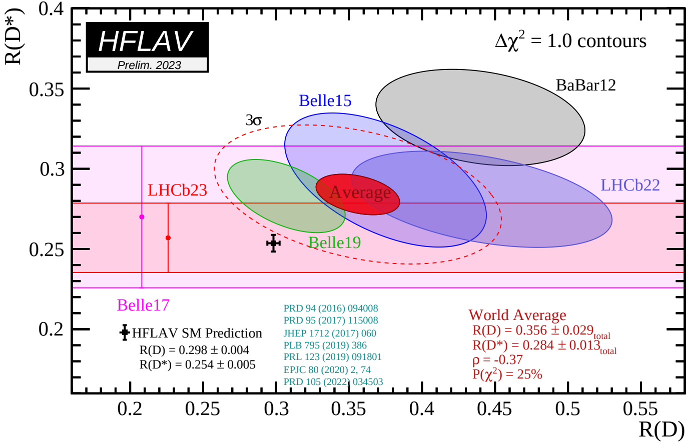
Why is V_{cb} important?

[Buras, Venturini, '21]



$$\mathcal{B}(\bar{B}_s \to \mu^+\mu^-) \sim |V_{tb}V_{ts}^*|^2 \sim |V_{cb}|^2[1+\mathcal{O}(\lambda^2)]$$

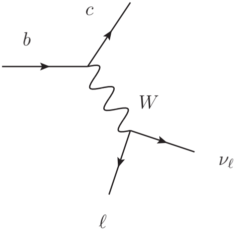
Lepton Flavour Universality tests



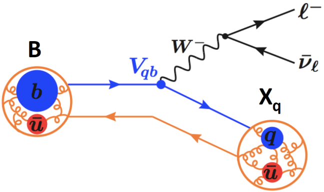
$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu})} \sim 3.2\sigma$$

Other modes ($R_{J/\psi}$, R_{Λ_c}) are less significant

Partonic vs Hadronic



$\mu_{\text{partonic}} = m_b$



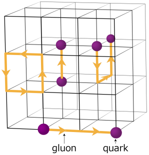
$\mu_{\text{hadronic}} = \Lambda_{\text{QCD}}$

Fundamental challenge to match partonic and hadronic descriptions

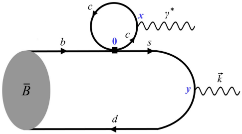
How can we tame the non-perturbative monsters

Exclusive decays

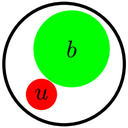
$$\langle H_c | J_\mu | H_b \rangle = \sum_i S_\mu^i \mathcal{F}_i$$



- Lattice QCD



- QCD SR, LCSR



- HQET (exploit $m_{b,c} \to \infty$ limit) + Data driven fits

- Dispersive analysis $\Rightarrow$ see Ludovico's talk!

How can we tame the non-perturbative monsters

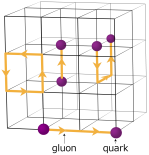
Exclusive decays

$$\langle H_c|J_\mu|H_b\rangle = \sum_i S^i_\mu \mathcal{F}_i$$

form factor

scale Λ_{QCD}

independent Lorentz structures



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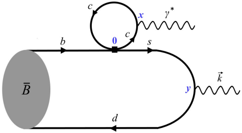

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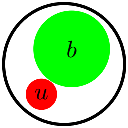

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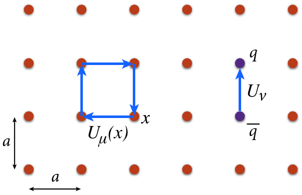
Lattice QCD

- Lattice QCD does not rely on perturbative expansion to perform calculations $\Rightarrow$ perfect environment to calculate non-perturbative quantities

- Lattice QCD uses a discretised space-time, with lattice spacing denoted as a



- fermions occupy sites on the lattice

- gauge fields are links between sites

- the lattice spacing a acts as a regulator $\Rightarrow$ QFT built on lattice is finite

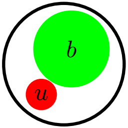
- physical results are obtained taking the continuum limit $a \to 0$

- in practice, lattice QCD calculations are limited only by computational resources and efficiency of the implementation $\Rightarrow$ leads to statistical and systematic uncertainties

Lattice QCD: uncertainties

- Continuum Limit: controlling the discretisation errors
- Infinite Volume limit: finite space-time might induce shifts of physical quantities from the measured ones
- Chiral extrapolation: extrapolation of m_u and m_d (or equivalently m_π)
- Heavy quark mass extrapolation to the physical limit
- Operator matching: matching of operators on the lattice with lattice regularisation scheme onto the continuum

Heavy Quark Effective Theory

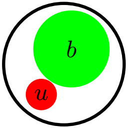


- The H_b momentum is mostly carried by the b quark

$$p^\mu = m_b v^\mu + k^\mu$$

- The residual momentum: $k^\mu \sim \Lambda_{\rm QCD}$

Heavy Quark Effective Theory



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$$\mathcal{L}_{\sf eff} = \mathcal{L}_\infty + \mathcal{O}(1/m_q)$$

mass independent

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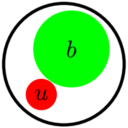

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$$\mathcal{L}_{\mathsf{eff}} = \mathcal{L}_\infty + \mathcal{O}(1/m_q)$$

mass independent

1. At leading power, all heavy quarks are the same
2. Intrinsic spin-flavour symmetry relates the various form factors

$b \to c$ case

- For $b \to c$ transitions, we have $m_b, m_c \to \infty$ but m_c/m_b finite
- Spin-flavour symmetry relates all $B^{(*)} \to D^{(*)}$ form factors
- The HQET provides a reduction of the free parameters
- At zero recoil ($q^2 = q^2_{\mathrm{max}}$), the form factors are normalised

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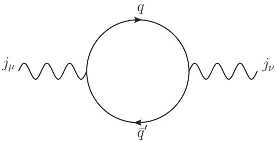
Apart from the zero-recoil point, parameters in the Heavy Quark Expansion are unknown a priori, and have to be determined from other dynamical sources

Sum rules



$$\Pi_{\mu\nu}(q^2) = i \int d^4x \, e^{iqx} \langle 0 | T \left\{ J^{\mu}(x), J^{\nu,\dagger}(x) \right\} | 0 \rangle$$

- In the region $\mathrm{Re}(q^2) < 0$: the correlation function $\Pi_{\mu\nu}(q^2)$ is analytic
- For $-q^2 \ll \Lambda^2_{\mathrm{QCD}}$: quarks propagate at short distances

If both conditions are fulfilled, $\Pi_{\mu\nu}(q^2)$ can be expanded in a local OPE

$$\Pi(q^2) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds \frac{\mathrm{Im}\Pi(s)}{s - q^2}$$

Quark-Hadron Duality

Amplitudes computed in perturbative QCD can be approximated by amplitudes computed treating hadrons as fundamental particles

$$\underbrace{\int_{s_{th}}^{\infty} ds \frac{\mathrm{Im}\Pi(s)^{\mathrm{OPE}}}{s-q^2}}_{\text{calculable}} \approx \underbrace{\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi(s)^{\mathrm{had}}}{s-q^2}}_{\text{unknown}}$$

We can extract information on the hadronic parameters

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calculable | unknown

We can extract information on the hadronic parameters

Spectral representation:

$$2\ \mathrm{Im}\,(\Pi)_{\mu\nu} = \sum_n \langle 0|j_\mu|n\rangle\langle n|j_\nu|0\rangle d\tau_n (2\pi)^4 \delta^{(4)}(q-p_n)$$

contain hadronic parameters

The exclusive form factors

- Non perturbative methods evaluate the form factors in precise kinematic points
- The kinematic dependence must be inferred
- The most used ones are:

  $\Rightarrow$ BGL parametrisation **[Boyd, Grinstein, Lebed, '95]**

  $\Rightarrow$ CLN parametrisation + updates **[Caprini, Lellouch, Neubert, '95]**

  $\Rightarrow$ Dispersive Matrix **[Martinelli, Simula, Vittorio, '21]**

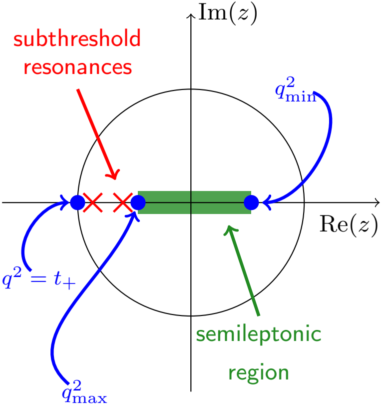
BGL

- Model independent parametrisation
- Uses analytical properties of the form factors
- Conformal mapping

$$q^2 \mapsto z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

with t_+ pair production threshold and $t_0 < t_+$

The z-expansion



- in the complex plane form factors are real analytic functions
- q^2 is mapped onto the conformal complex variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

  $t_+ = (m_{H_{\rm in}} + m_{H_{\rm fin}})^2$ and t_0 can be chosen to minimise $z_{\rm max}$
- q^2 is mapped onto a disk in the complex z plane, where $|z(q^2, t_0)| < 1$
- being z small, we can expand any form factor in z and truncate the series at relatively low orders

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  with t_+ pair production threshold and $t_0 < t_+$
- $|z| \ll 1$, for $B \to D^{(*)}$: $|z_{\max}| = 6\%$
- We can expand as

$$F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k \quad \text{and} \quad \sum_{k=0}^{n_i} |a_k^i|^2 < 1$$

  P_i: Blaschke factors, ϕ_i: outer functions $\Rightarrow$ known quantities
- Bounds + $|z_{\max}|$ $\Rightarrow$ expect rapid convergence
- a_k^i need to be determined (from data, lattice, sum rules, etc.)

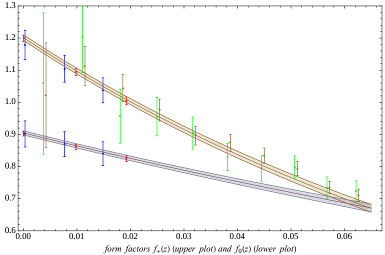
$B \to D\ell\bar{\nu}$

[Bigi, Gambino, '16]

- The "easy" case:
  - $\Rightarrow$ only two form factors
  - $\Rightarrow$ the D is "stable" on the lattice
- Two datasets available, in excellent agreement
- Two lattice determinations available, in excellent agreement

BGL Fit lattice + data

$$|V_{cb}^D| = (40.5 \pm 1.0) \times 10^{-3} \qquad R_D = 0.299 \pm 0.003$$



Inputs:

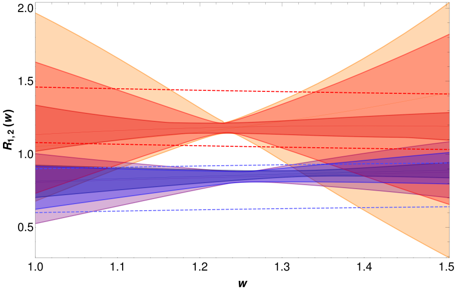
- FNAL/MILC'15
- HPQCD'16
- Babar'09
- Belle'16

$B \to D^* \ell \bar{\nu}$

[Gambino, Jung, Schacht, '19]

Inputs:

- Belle '18 differential distribution in the 4 kinematical variables
- LCSR at $q^2 = 0$
- Unitarity constraints on the form factors parameters
- Lattice points at $q^2 = q^2_{\max}$
- Form factors expanded up to z^2



$$|V_{cb}^{D^*}| = (39.2^{+1.4}_{-1.2}) \times 10^{-3}$$
$$R_{D^*} = 0.253^{+0.007}_{-0.006}$$

CLN

- CLN uses Heavy Quark Effective Theory at $1/m_b$
- Use ansatz at $\mathcal{O}(1/m_b)$ and $\mathcal{O}(\alpha_s)$

$$F_i = F_i(q^2 = q^2_{\max}) \times \left[\left(a_i + b_i \frac{\alpha_s}{\pi} \right) \xi + \frac{\Lambda_{\mathrm{QCD}}}{2m_b} \sum_j c_{ij} \xi^j_{\mathrm{SL}} + \frac{\Lambda_{\mathrm{QCD}}}{2m_c} \sum_j d_{ij} \xi^j_{\mathrm{SL}} \right]$$

- Only 1 leading and 3 sub-leading Isgur Wise function contribute but they are not known a priori
- The form factors in $B^{(*)} \to D^{(*)}$ are correlated

CLN ansatz is inconsistent:

- Use $F_i(q^2 = q^2_{\max})$ from other source (e.g. Lattice) to properly normalize form factors
- Use QCDSR for sub-leading IW functions w/o error estimates
- No proper inclusion of errors from higher orders

Note: when CLN was introduced these assumptions were justified as experimental sensitivity was low and allowed fits with a small set of parameters

HQET with $1/m_c^2$

- With the current precision can go beyond CLN and include higher order corrections

At order $1/m$, α_s, $1/m_c^2$:

$$F_i = \left(a_i + b_i \frac{\alpha_s}{\pi}\right) \xi + \frac{\Lambda_{\rm QCD}}{2m_b} \sum_j c_{ij} \xi^j_{\rm SL} + \frac{\Lambda_{\rm QCD}}{2m_c} \sum_j d_{ij} \xi^j_{\rm SL} + \left(\frac{\Lambda_{\rm QCD}}{2m_c}\right)^2 \sum_j g_{ij} \xi^j_{\rm SSL}$$

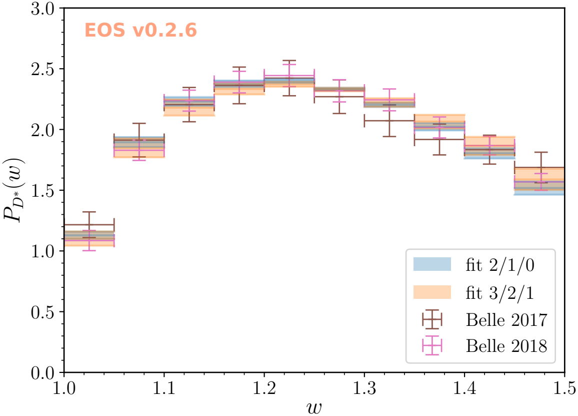
- More conservative use of QCDSR (including uncertainties)
- Can leverage new theory inputs like LCSR and Lattice beyond zero recoil.
- Inclusion of $1/m_c^2$ corrections highly motivated because they are naively of the same size of $1/m_b$, α_s, and α_s/m_c corrections
- data independent determination of the IW functions are possible

[1703.05330,1801.01112,1908.09398,1912.09335,2206.11281]

$B \to D^{(*)}$ at $1/m_c^2$

[MB, Jung, van Dyk '19]
[MB, Gubernari, Jung, van Dyk '19]

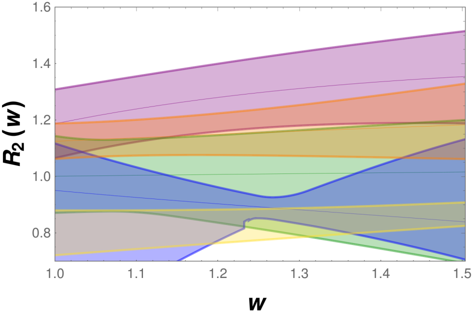
- QCD Sum Rules, LCSR, Lattice at $q^2 = q^2_{\max}$ for $B \to D^*$, Lattice for $B \to D$



$$|V_{cb}^{D}| = (40.7 \pm 1.1) \times 10^{-3}$$

$$|V_{cb}^{D^*}| = (38.8 \pm 1.4) \times 10^{-3}$$

Lattice calculations at $q^2 \neq q^2_{\rm max}$

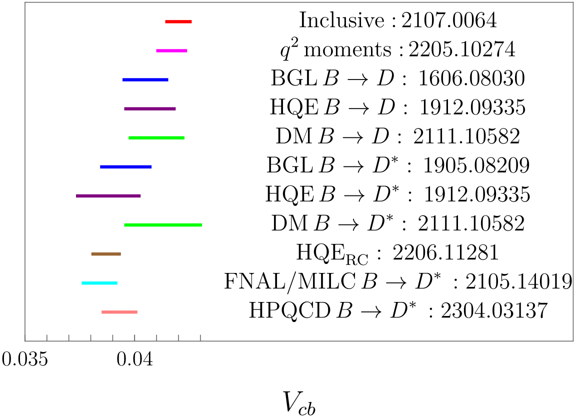


- FNAL/MILC '21
- HQE@$1/m_c^2$
- Exp data (BGL)
- JLQCD
- HPQCD '23

- Tensions between different lattice determinations, experimental data and non-lattice theory determination
- No consensus yet, ongoing checks
- Lattice calculations drift R_{D^*} to higher values
- New Belle analysis available

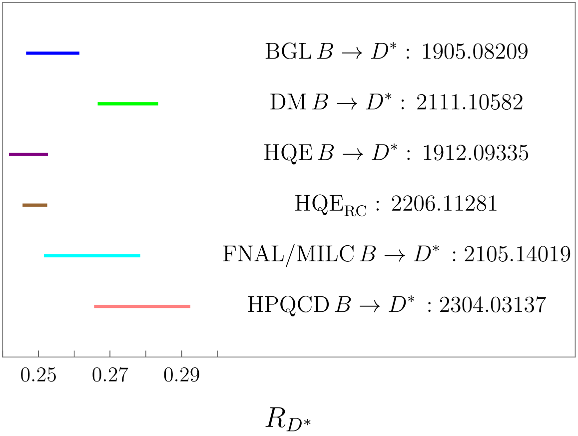
Summary

The V_{cb} puzzle



- There is a **spread** between inclusive and exclusive determinations of V_{cb}
- Discussion going on different inputs both from experimental and theoretical point of view
- New Belle analysis data are just out, stay tuned for the results!

R_{D^*}



- Spread between lattice-based and non-lattice based calculations
- Lattice-based determinations are not yet included in HFLAV

Summary

- Charged current decays provide the means to probe the Standard Model at high accuracy
- This requires a high control of hadronic matrix elements
- A lot of work has been done in recent years both from theoretical and experimental points of view
  - $\Rightarrow$ The V_{cb} puzzle is far from being resolved!
  - $\Rightarrow$ Personal opinion: this is one of the biggest problem in flavour physics nowadays
  - $\Rightarrow$ New Lattice results are impressive, but they need further investigation
  - $\Rightarrow$ New experimental analyses are out, results are yet to come, but all data are welcome!

Appendix

Theory framework

$$\Gamma = \frac{1}{m_B}\mathrm{Im}\int d^4x\langle B(p)|T\left\{\mathcal{H}^\dagger_{\mathrm{eff}}(x)\mathcal{H}_{\mathrm{eff}}(0)\right\}|B(p)\rangle$$

Theory framework

$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p) | T \left\{ \mathcal{H}^\dagger_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0) \right\} | B(p) \rangle$$

$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

Theory framework

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$$\uparrow$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p)|\mathcal{O}_{n+3,i}|B(p)\rangle$ are non perturbative

  $\Rightarrow$ They need to be determined with non-perturbative methods, e.g. Lattice QCD

  $\Rightarrow$ They can be extracted from data

  $\Rightarrow$ With large n, large number of operators

Theory framework

$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p) | T \left\{ \mathcal{H}^\dagger_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0) \right\} | B(p) \rangle$$

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$\uparrow$

loss of predictivity

Theory framework

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[1 + a_1\left(\frac{\alpha_s}{\pi}\right) + a_2\left(\frac{\alpha_s}{\pi}\right)^2 + a_3\left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_\pi^2}{m_b^2}$$

$$+ \left(g_0 + g_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_G^2(m_b)}{m_b^2} + d_0\frac{\rho_D^3}{m_b^3} - g_0\frac{\rho_{LS}^3}{m_b^3} + \ldots\Big]$$

$$\mu_\pi^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v(i\vec{D})^2 b_v|B\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b_v|B\rangle_\mu$$

- Coefficients of the expansions are known
- Ellipses stands for higher orders

Theory framework

NEW

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$$+ \left(g_0 + g_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_G^2(m_b)}{m_b^2} + d_0\frac{\rho_D^3}{m_b^3} - g_0\frac{\rho_{LS}^3}{m_b^3} + \ldots\Big]$$

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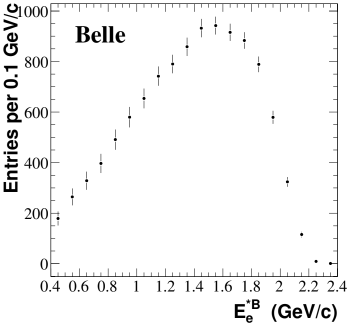
- Ellipses stands for higher orders

How do we constrain the OPE parameters?



- Lepton energy and hadronic invariant mass distributions can be used to extract non perturbative information
- Moments of the kinematic distributions

$$\langle E_\ell^n \rangle = \frac{\int_{E_\ell > E_{\ell,\mathrm{cut}}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell,\mathrm{cut}}}}$$

$$R^* = \frac{\int_{E_\ell > E_{\ell,\mathrm{cut}}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}{\int dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

- Similar definition for hadronic mass moments

- The moments give access to the distribution, but not to the normalisation
- They admit an HQE as the rate

$\Rightarrow$ No $\mathcal{O}(\alpha_s^3)$ terms are known yet

Scheme conventions

The semileptonic width has a strong dependence on m_b: $\Gamma_0 \sim m_b^5$

Suitable choice for the mass scheme is needed:

- Pole mass scheme
  - $\Rightarrow$ Renormalon ambiguity
  - $\Rightarrow$ Perturbative series is factorially divergent

$$\Gamma_{sl} \sim \sum_k k! \left(\frac{\beta_0}{2} \frac{\alpha_s}{\pi} \right)^k$$

- We choose to use to b-quark mass and the non perturbative parameters in the kinetic scheme

[Bigi, Shifman,Uraltsev,Vainshtein]

$$m_b^{kin}\mu = m_b^{OS} - [\bar{\Lambda}(\mu)]_{\rm pert} - \frac{[\mu_\pi^2(\mu)]_{\rm pert}}{2m_b^{kin}(\mu)}$$

$$\mu_\pi^2(0) = \mu_\pi^2(\mu) - [\mu_\pi^2(\mu)]_{\rm pert}$$

$$\rho_D^3(0) = \rho_D^3(\mu) - [\rho_D^3(\mu)]_{\rm pert}$$

  - $\Rightarrow$ Wilsonian cutoff $\mu = 1\,\mathrm{GeV}$
  - $\Rightarrow$ Kinetic scheme tailored on the HQE

- We express the charm mass in the $\overline{\mathrm{MS}}$ scheme

	experiment	values of E_{cut}(GeV)	Ref.
R^*	BaBar	0.6, 1.2, 1.5	[26, 27]
ℓ_1	BaBar	0.6, 0.8, 1, 1.2, 1.5	[26, 27]
ℓ_2	BaBar	0.6, 1, 1.5	[26, 27]
ℓ_3	BaBar	0.8, 1.2	[26, 27]
h_1	BaBar	0.9, 1.1, 1.3, 1.5	[26]
h_2	BaBar	0.8, 1, 1.2, 1.4	[26]
h_3	BaBar	0.9, 1.3	[26]
R^*	Belle	0.6, 1.4	[28]
ℓ_1	Belle	1, 1.4	[28]
ℓ_2	Belle	0.6, 1.4	[28]
ℓ_3	Belle	0.8, 1.2	[28]
h_1	Belle	0.7, 1.1, 1.3, 1.5	[29]
h_2	Belle	0.7, 0.9, 1.3	[29]
$h_{1,2}$	CDF	0.7	[31]
$h_{1,2}$	CLEO	1, 1.5	[32]
$\ell_{1,2,3}$	DELPHI	0	[33]
$h_{1,2,3}$	DELPHI	0	[33]

- Theoretical uncertainties are necessary for the fit stability [Gambino, Schwanda, '13]
- Different treatments yield to slightly different results, but all compatible
- The value of $|V_{cb}|$ is simply extracted as [Alberti, Gambino, Healey, Nandi, '14]

$$|V_{cb}| = \sqrt{\frac{\mathcal{B}_{c\ell\bar{\nu}}}{\tau_B \Gamma_{sl}}} = (42.21 \pm 0.78) \times 10^{-3}$$

Inclusion of $\mathcal{O}(\alpha_s^3)$ results

[Fael, Schönwald, Steinhauser, '20]

b-quark mass:

$$m_b^{kin}(1\,\mathrm{GeV}) = [4169 + 259_{\alpha_s} + 78_{\alpha_s^2} + 26_{\alpha_s^3}]\,\mathrm{MeV} = (4526 \pm 15)\,\mathrm{MeV}$$

50% **reduction!**

Semileptonic width

$\Rightarrow$ $\mu = 1\,\mathrm{GeV}$, $\mu_b = m_b^{kin}$, $\mu_c = 3\,\mathrm{GeV}$

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[0.9257 - 0.1163_{\alpha_s} - 0.0349_{\alpha_s^2} - 0.0097_{\alpha_s^3}\Big]$$

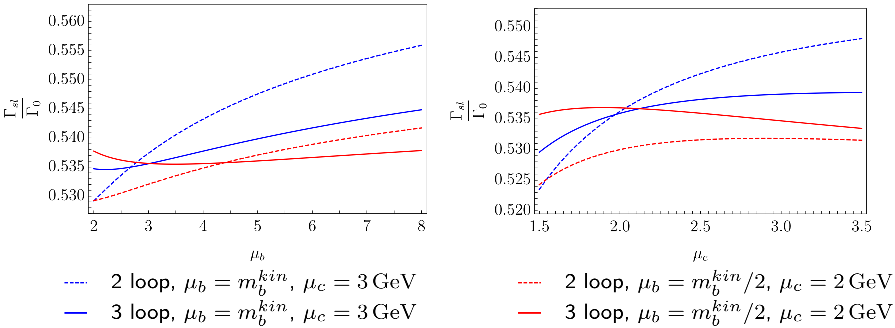
$\Rightarrow$ $\mu = 1\,\mathrm{GeV}$, $\mu_b = m_b^{kin}/2$, $\mu_c = 2\,\mathrm{GeV}$

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[0.9257 - 0.1138_{\alpha_s} - 0.0011_{\alpha_s^2} + 0.0104_{\alpha_s^3}\Big]$$

$\boxed{\textbf{residual uncertainty} \sim 0.5\%}$

Residual uncertainty

[MB, Capdevila, Gambino, '21]



- Residual scale dependence

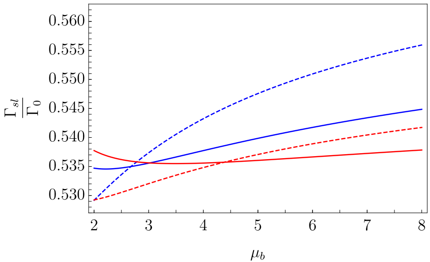
  $\Rightarrow$ Milder including $\mathcal{O}(\alpha_s^3)$

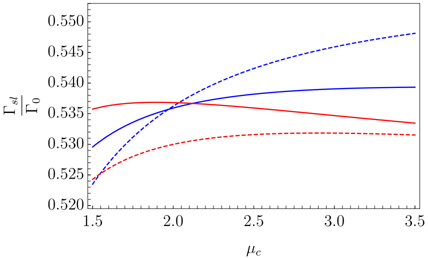
  $\Rightarrow$ We choose $\mu_c = 2\,\mathrm{GeV}$, $\mu_b = m_b^{kin}/2$ and $\mu = 1\,\mathrm{GeV}$ to minimize scale dependence

- Other sources of uncertainties e.g. higher corrections to the HQE parameters yield to smaller residual uncertainties

Residual uncertainty

[MB, Capdevila, Gambino, '21]





2 loop, $\mu_b = m_b^{kin}$, $\mu_c = 3\,\text{GeV}$
3 loop, $\mu_b = m_b^{kin}$, $\mu_c = 3\,\text{GeV}$
2 loop, $\mu_b = m_b^{kin}/2$, $\mu_c = 2\,\text{GeV}$
3 loop, $\mu_b = m_b^{kin}/2$, $\mu_c = 2\,\text{GeV}$

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  $\Rightarrow$ Milder including $\mathcal{O}(\alpha_s^3)$

  $\Rightarrow$ We choose $\mu_c = 2\,\text{GeV}$, $\mu_b = m_b^{kin}/2$ and $\mu = 1\,\text{GeV}$ to minimize scale dependence

- Other sources of uncertainties e.g. higher corrections to the HQE parameters yield to smaller residual uncertainties

1.2% **residual uncertainty**

The semileptonic fit

[MB, Capdevila, Gambino, '21]

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_g(m_b)$	ρ_{LS}	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.092	0.477	0.185	0.306	−0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

- Constraints from FLAG $N_f = 2+1+1$: $\overline{m}_b(\overline{m}_b) = 4.198(12)\,\text{GeV}$ and $\overline{m}_c(\overline{m}_c) = 0.988(7)\,\text{GeV}$
- No new experimental input wrt to the one in 1411.6560
- The central value of V_{cb} is stable
- Without constraints on m_b, we extract $\overline{m}_b(\overline{m}_b) = 4.210(22)\,\text{GeV}$

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$$V_{cb} = 42.16(32)_{exp}(30)_{th}(25)_\Gamma \cdot 10^{-3}$$

Higher power corrections

- At $\mathcal{O}(1/m^4)$ the number of operators become large
  - $\Rightarrow$ 9 at dim 7
  - $\Rightarrow$ 18 at dim 8

Lowest Lying State Saturation Approximation: [Mannel, Turczyk, Uraltsev, '10]

$$\langle B|\mathcal{O}_1\mathcal{O}_2|B\rangle = \sum_n \langle B|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|B\rangle$$

At dimension 6 the LLSA works well:

$$\rho_D^3 = \epsilon\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\mu_G^2 \qquad \epsilon \sim 0.4\,\mathrm{GeV}$$

- Large corrections to the LLSA are possible [Gambino, Mannel, Uraltsev, '12]
- 60% gaussian uncertainty on higher order parameters

$$V_{cb} = 42.00(53) \times 10^{-3}$$

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$$\langle B|\underbrace{\mathcal{O}_1\mathcal{O}_2}_{iD_\alpha \dots iD_\rho}|B\rangle = \sum_n \langle B|\mathcal{O}_1\underbrace{|n\rangle\langle n|}_{\text{complete set of states}}\mathcal{O}_2|B\rangle$$

At dimension 6 the LLSA works well:

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$$V_{cb} = 42.00(53) \times 10^{-3}$$

What about New Physics?

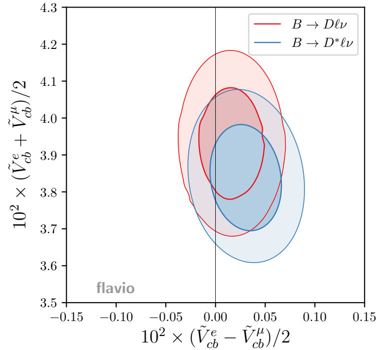
[Jung, Straub 2018]

- If we allow LFUV between μ and electrons

$$\tilde{V}_{cb}^{\ell} = V_{cb}(1 + C_{V_L}^{\ell})$$

- Fitting data from Babar and Belle

$$\frac{\tilde{V}_{cb}^{e}}{\tilde{V}_{cb}^{\mu}} = 1.011 \pm 0.012$$



$$\frac{1}{2}(\tilde{V}_{cb}^{e} + \tilde{V}_{cb}^{\mu}) = (3.87 \pm 0.09)\%$$
$$\frac{1}{2}(\tilde{V}_{cb}^{e} - \tilde{V}_{cb}^{\mu}) = (0.022 \pm 0.023)\%$$

Inclusive V_{cb} from q^2 moments [Bernlochner et al., '22]

An alternative for the inclusive determination

- q^2 moments

$$R^* = \frac{\int_{q^2 > q^2_{\text{cut}}} dq^2 \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \frac{d\Gamma}{dq^2}} \qquad \langle (q^2)^n \rangle = \frac{\int_{q^2 > q^2_{\text{cut}}} dq^2 (q^2)^n \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \frac{d\Gamma}{dq^2}}$$

- Exploits HQE to reduce numbers of higher dimensional operators [Fael, Mannel, Vos, '18]

- Preliminary result:

$$|V_{cb}| = (41.69 \pm 0.63) \times 10^{-3}$$

What's the issue with the previous determination?

- The q^2 moments require a measurement of the branching ratio with a cut in q^2 which is not available yet
- By extrapolating from the current available measurements, the branching ratio is lower then what used
- If the same branching ratios is used, the two methods give the **same** result

The results for inclusive V_{cb} are stable

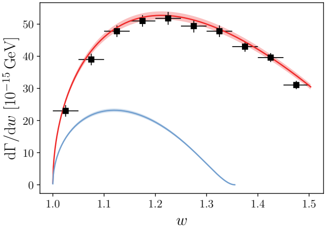
Comparison with Bernlochner et al '22

Differences:

- Different power counting in the HQET expansion
  - $\Rightarrow$ Less freedom in higher-order corrections
- Avoid the use of LCSR results
- Partial α_s^2 corrections
- Partial inclusion of the latest FNAL/MILC results

Observations:

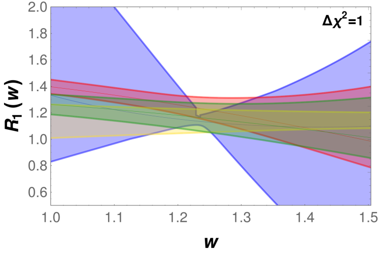
- $1/m_c^2$ corrections are necessary
- Uncertainties are overall small



$$R_D = 0.288(4)$$
$$R_{D^*} = 0.249(3)$$
$$|V_{cb}| = 38.7(6) \times 10^{-3}$$

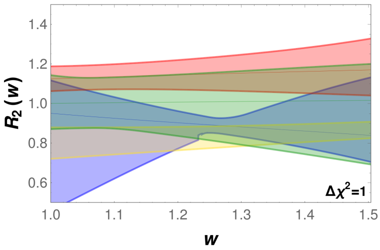
FNAL/MILC at $q^2 \neq q^2_{\max}$ [2105.14019]

Major breakthrough: FNAL/MILC released Lattice QCD data for the whole q^2 region



- FNAL/MILC 21
- HQET@$1/m_c^2$
- BGL w/ exp data
- JLQCD

Good compatibility

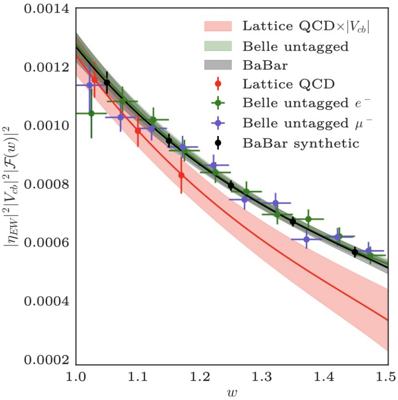


- Differences with other theory determinations
- Differences with experimental data
  - ⇒ Soften with new Belle 23 analysis, but still there
- JLQCD has better agreement with all determinations

Further investigation is needed

FNAL/MILC at $q^2 \neq q^2_{\max}$ [2105.14019]

- FNAL/MILC released Lattice QCD data for the whole q^2 region
- First data set for lattice-only driven determination of $B \to D^*$ form factors



- Combined fit with data has $\chi^2/\mathrm{dof} > 1$
- Poor compatibility with current experimental dataset
- $|V_{cb}|$ is rather low

$$|V_{cb}| = (38.40 \pm 0.74) \times 10^{-3}$$

- Soon results from HPQCD and JLQCD