IFIC (CSIC-UV), València

Resonance Chiral Theory and its predictions,

case of 2pi and 3pi final states in TAUOLA MC:

status report

O. Shekhovtsova

work done together with
Z. Was *and* P. Roig

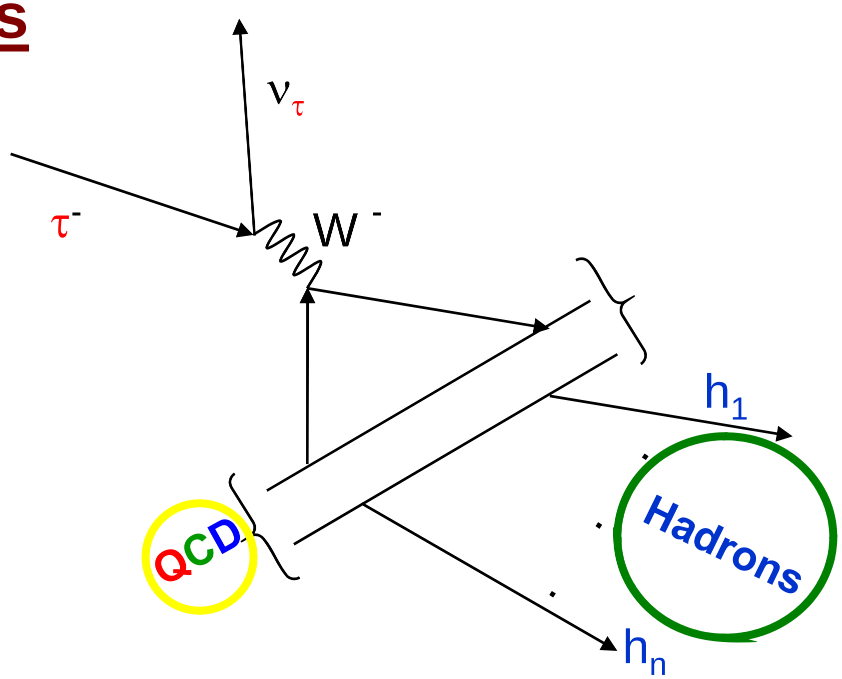
- Theoretical calculation. Resonance Chiral Theory.
- TAUOLA, implementation RCHT results for $3\pi/2\pi$ modes.

  Tests numerical stability
- Comparison with BABAR for 3π mode
- Conclusion (inspiration for future work)

Basics of hadronic tau decays

$$M = \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u}(\nu_\tau)\gamma^\mu(1-\gamma_5)u(\tau)T_\mu$$

$$d\Gamma = \frac{G_F^2}{4M_\tau^2}|V_{CKM}|^2 d\Phi^{(n+1)} L_{\mu\nu}T^\mu T^{\nu *}$$



T_μ=<Hadrons|(V-A)$_\mu$ $e^{iS_{QCD}}$|0>=Σ_i(Lorentz Structure)^i $F_i(Q^2,s_j)$

3 pions: 3 Lorentz independent structure
2 pions: 1 Lorenz independen structure (vector)

$F_i(Q^2,s_j)$ → **RχT**

Resonance Chiral Theory
(Chiral Theory with the explicit inclusion of ρ and a_1 mesons)

G.Ecker et al., Nucl. Phys B321, 311 (1989)

1. To enlarge the energy region for ChPT (≤1.5 GeV) the resonance fields ($V_{\mu\nu}$, $A_{\mu\nu}$) should be added in explicit way
2. $E \ll m_\rho$ the singularity associated with the pole of a resonance propagator is replaced by momentum expansion
3. The main contribution to the coupling constants (at least up to p^4) comes from the meson resonance exchange

The antisymmetric tensor field for the vector (axial-vector) meson

About the parameter values: experimental data or theoretical relations?

Parameters: f_π, F_V, G_V, F_A

for example →

$$\Gamma(\rho \to e^+e^-) = \frac{4\pi\alpha^2 F_V^2}{3m_\rho}$$

$$\Gamma(\rho \to \pi\pi) = \frac{G_V^2 m_\rho^3}{48\pi f_\pi^4}\left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{3/2}$$

$$\Gamma(a_1 \to \pi\gamma) = \frac{\alpha F_a^2 m_a}{24 f_\pi^2}\left(1 - \frac{m_\pi^2}{m_a^2}\right)^3$$

Theory: $F_V G_V = f_\pi^2$, $F_V^2 - F_A^2 = f_\pi^2$, $F_V^2 M_V^2 = F_A^2 M_A^2$

$2F_V G_V \neq F_V^2$ $\quad$ M_V, M_A – *mass of vector (axial-) nonet*

chiral and large N_c limit:

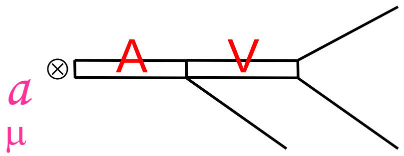
$M_\rho \sim 770$ MeV, $M_A \sim 998$ MeV

Three pion mode: $\tau^- \to (3\pi)^- \nu_\tau$

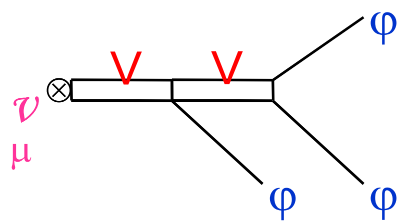
Hadronic tensor: $\tau \to h_1(p_1)h_2(p_2)h_3(p_3)\nu_\tau \qquad T^\mu = V_1^\mu F_1 + V_2^\mu F_2 + Q F_4 + i\varepsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} F_5$

$$V_1^\mu = (g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2})(p_2 - p_3)_\nu \ , \qquad V_2^\mu = (g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2})(p_1 - p_3)_\nu \ ,$$

$$Q^\mu = p_1^\mu + p_2^\mu + p_3^\mu$$



$V^\mu_{\frac{1}{2}} = (g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2})(p_{\frac{2}{1}} - p_3)_\nu$

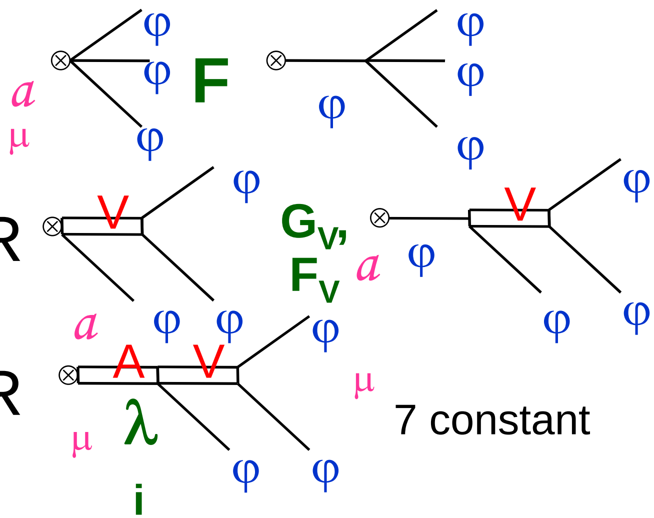


$i\varepsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} F_5$ **NOT !**

χPT

RχT, 1R

RχT, 2R



$$F_{1,2} = F_{1,2}^\chi + F_{1,2}^R + F_{1,2}^{RR} \ ,$$

$$F_4 = F_p \sim m_\pi^2 / Q^2$$

D. Gomez-Dumm, A. Pich, J. Portoles, P. Roig
(arXiv:0911.4436)

V = *ρ*, *ρ'* $\quad A = a_1$

How to include ρ‘ ?

ρ : F_V, G_V ⟶ ρ’ : F_{V1}, G_{V1}

Too many constants

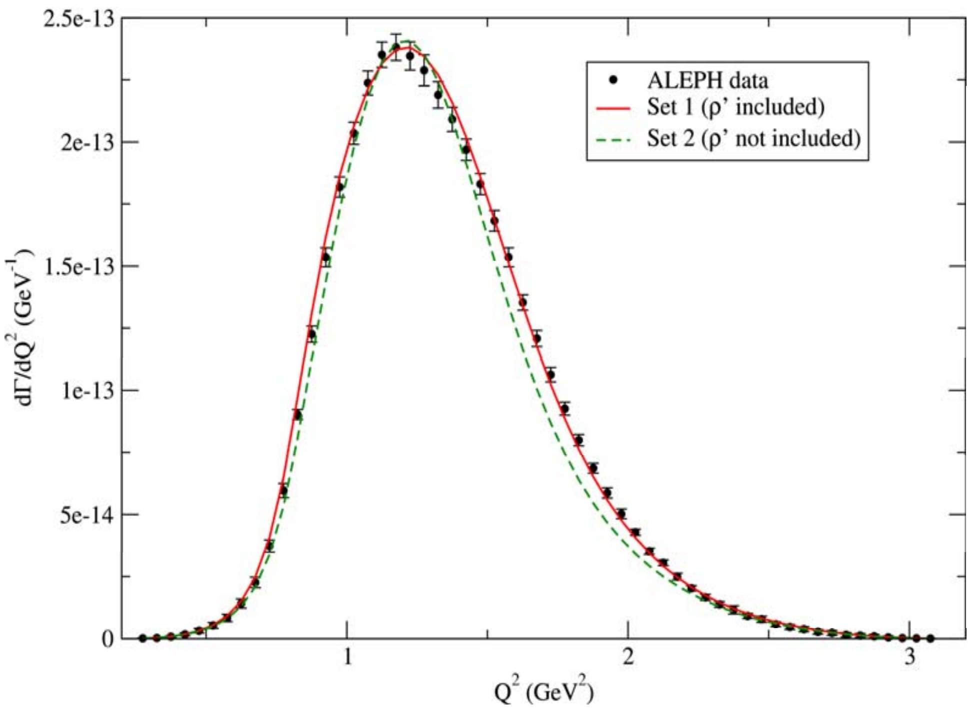
Following K&S: $$\frac{1}{M_\rho^2 - q^2 - iM_\rho\Gamma_\rho(q^2)} \longrightarrow \frac{1}{1+\beta_{\rho'}}\left[\frac{1}{M_\rho^2 - q^2 - iM_\rho\Gamma_\rho(q^2)} + \frac{\beta_{\rho'}}{M_{\rho'}^2 - q^2 - iM_{\rho'}\Gamma_{\rho'}(q^2)}\right]$$

$$\Gamma_\rho(s) = \frac{M_\rho\, s}{96\,\pi\, F^2}\left[\sigma_\pi^3\,\theta(s - 4m_\pi^2) + \frac{1}{2}\sigma_K^3\,\theta(s - 4m_K^2)\right]$$

$$\Gamma_{\rho'}(q^2) = \Gamma_{\rho'}(M_{\rho'}^2)\,\frac{M_{\rho'}}{\sqrt{q^2}}\left(\frac{p(q^2)}{p(M_{\rho'}^2)}\right)^3 \theta(q^2 - 4\,m_\pi^2)\,,$$

$$\sigma_P = \sqrt{1 - 4m_P^2/s}.$$

$$p(x) = \frac{1}{2}\sqrt{x - 4m_\pi^2}\,.$$



a1 width:

$$\Gamma_{a_1}^{\pi,K}(Q^2) = \frac{-1/n!}{192(2\pi)^3 F_A^2 M_{a_1}}\left(\frac{M_{a_1}^2}{Q^2} - 1\right)^2 \int \mathrm{d}s\,\mathrm{d}t\; T_1^{\pi,K\mu}\, T_{1\,\mu}^{\pi,K*}$$

$$\Gamma_{a_1}(Q^2) = \Gamma_{a_1}^{\pi}(Q^2)\,\theta(Q^2 - 9m_\pi^2) + \Gamma_{a_1}^{K}(Q^2)\,\theta(Q^2 - (2m_K + m_\pi)^2)$$

$$T_{1+}^{\mu} = V_1^\mu\, F_1 + V_2^\mu\, F_2\,.$$

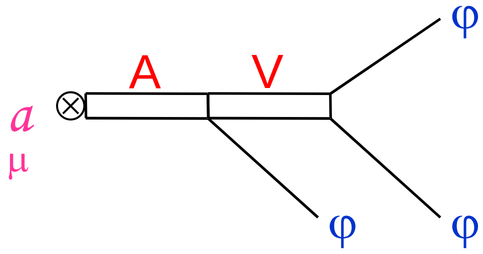
$$F_V = 0.180\,\mathrm{GeV}\;,\quad F_A = 0.149\,\mathrm{GeV}\;,\quad \beta_{\rho'} = -0.25\,,$$
$$M_V = 0.775\,\mathrm{GeV}\;,\quad M_{K^*} = 0.8953\,\mathrm{GeV}\;,\quad M_{a_1} = 1.120\,\mathrm{GeV}$$

and

$$F_V = 0.206\,\mathrm{GeV}\;,\quad F_A = 0.145\,\mathrm{GeV}\;,\quad \beta_{\rho'} = 0\,,$$
$$M_V = 0.775\,\mathrm{GeV}\;,\quad M_{K^*} = 0.8953\,\mathrm{GeV}\;,\quad M_{a_1} = 1.115\,\mathrm{GeV}$$

TAUOLA

Published version: J.H.Kuhn, A. Santamaría, Z. Phys. C48 (1990) 445



V = ρ , ρ'   A = a_1

3 pion:

- Vector Meson Dominance $$BW_R(x^2) = \frac{M_R^{\,2}}{M_R^{\,2} - x^2 - i\sqrt{x^2}\,\Gamma_R(x^2)}$$
- Asymptotic behaviour ruled by **QCD**
- χPT $\mathcal{O}(p^2)$ **YES** χPT $\mathcal{O}(p^4)$ **NO**

$$T^{\chi PT}_{\pm\mu}\Big|_{1^+} = \mp\frac{2\sqrt{2}}{3F}\left[\left(1+\frac{3\,s}{2\,M_V^2}\right)V_{1\mu} + \left(1+\frac{3\,t}{2\,M_V^2}\right)V_{2\mu}\right] + \text{chiral loops} + \mathcal{O}(p^6)$$

$$T^{(KS)}_{\pm\mu} \xrightarrow{s,t\ll M_V^2} \mp\frac{2\sqrt{2}}{3F}\left[\left(1+\frac{s}{M_V^2}\right)V_{1\mu} + \left(1+\frac{t}{M_V^2}\right)V_{2\mu}\right]$$

hep-ph/0213283, G. Gómez-Dumm, A. Pich, J.Portolés

RχT 3pion in TAUOLA *(only F_1, F_2 for the moment)*

$m_{\pi+} = m_{\pi 0}$

Check of precision integration

comparison with numerical integration of analytical formulae
(Gauss method integration)

- $F_1 = 1$, $F_2 = 0$ *no singularity phase space*

  MC: $\sigma = (2.7402 \pm 0.0004)$ E-17GeV Analyt: $\sigma = (2.7408 \pm 0.00005)$E-17GeV

- F_1 = true, $F_2 = 0$

  MC: $\sigma = (1.8720 \pm 0.0004)$ E-13GeV Analyt: $\sigma = (1.8720 \pm 0.0004)$E-13GeV

- F1 = true, F2 = true

  MC: $\sigma = (4.2013 \pm 0.0006)$ E-13GeV Analyt: $\sigma = (4.2014 \pm 0.0008)$ E-13GeV

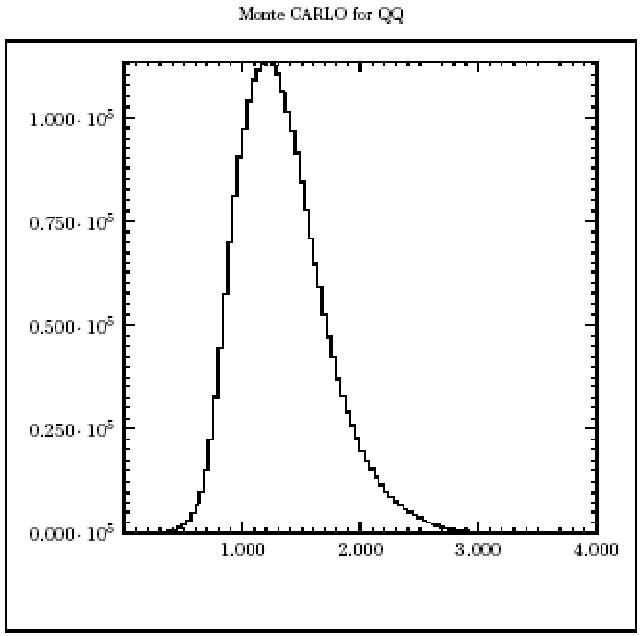
$m_{\pi+} \neq m_{\pi 0}$ for phase space F1 = true, F2 = true

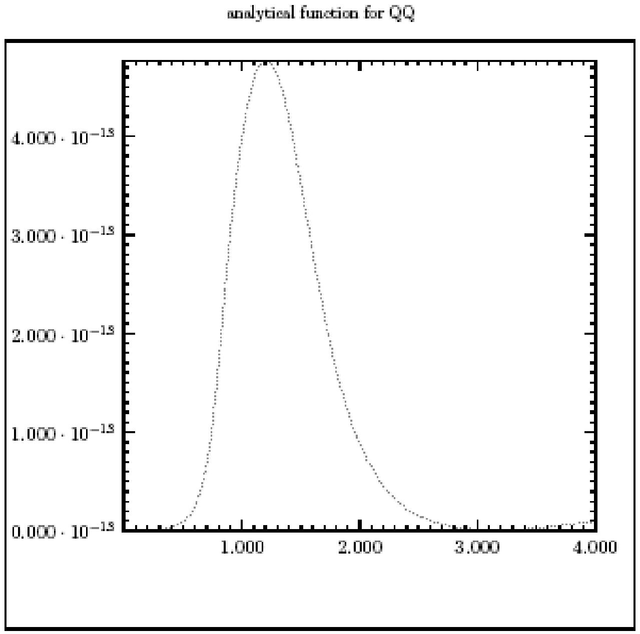
MC: $\sigma = (4.2046 \pm 0.0006)$ E-13GeV

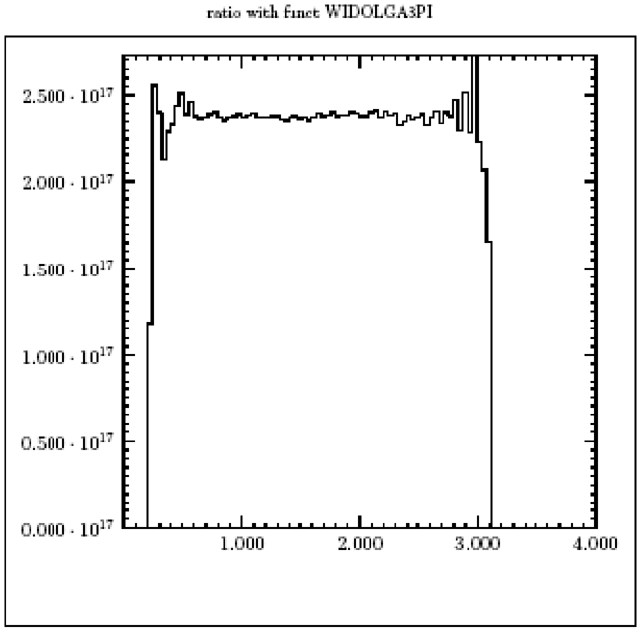
<u>Elementary technical test:</u>

Comparison with linearly interpolated spectrum *(F_1 = true, F_2 = true)*

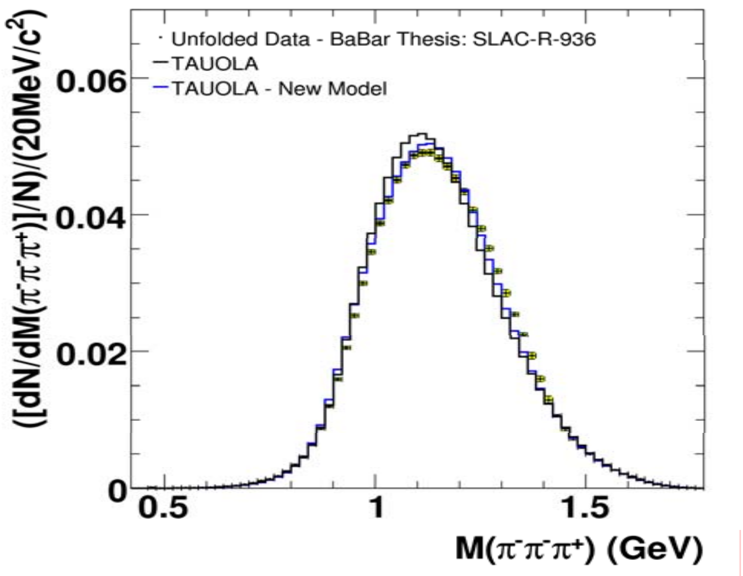
Difference is ~10% (first 5 points from 1000) at the beginning of the spectrum and less than 0.5%

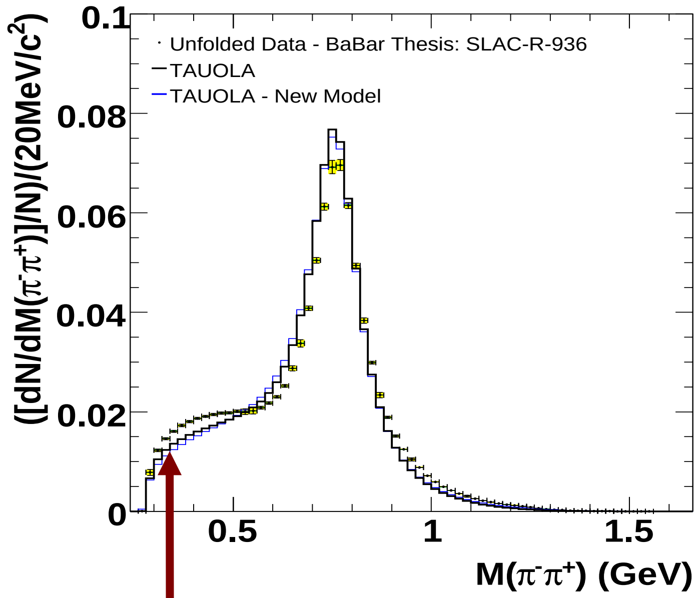






<u>Comparison with BABAR data</u>: *Ian M. Nugent, (Victoria U.) . SLAC-R-936, Dec16, 2009. Ph.D. Thesis (Advisor: Dr. J. Michael Roney).*





<u>F_4 will help (scalar contribution)?</u>

<u>Other th. or exp. problems?</u>

<u>The status</u>: without real fit, better result *for q2 spectrum should not be expected*

MORE WORK NEEDED

angular distributions should help to separate scalar from vector

Two pion case: $\tau \rightarrow \pi\pi\nu$

Hadronic tensor $\quad T^{\mu} = (p_1 - p_2)^{\mu} F_{\pi}(q^2)$

RχT model for pion form factor

$$F(q^2) = \left(\frac{m_{\upsilon}^2 + (\gamma e^{-i\phi_1} + \delta e^{-i\phi_2})s}{D_{\rho}(q^2)} - \frac{\gamma s e^{-i\phi_1}}{D_{\rho'}(q^2)} - \frac{\delta s e^{-i\phi_2}}{D_{\rho''}(q^2)} \right) e^{-((\pi\pi + KK)loops)}$$

Features:

correct up to χPT $\mathcal{O}(p^4)$ behavior

correct high energy behaviour

Parameters from BELLE spectrum

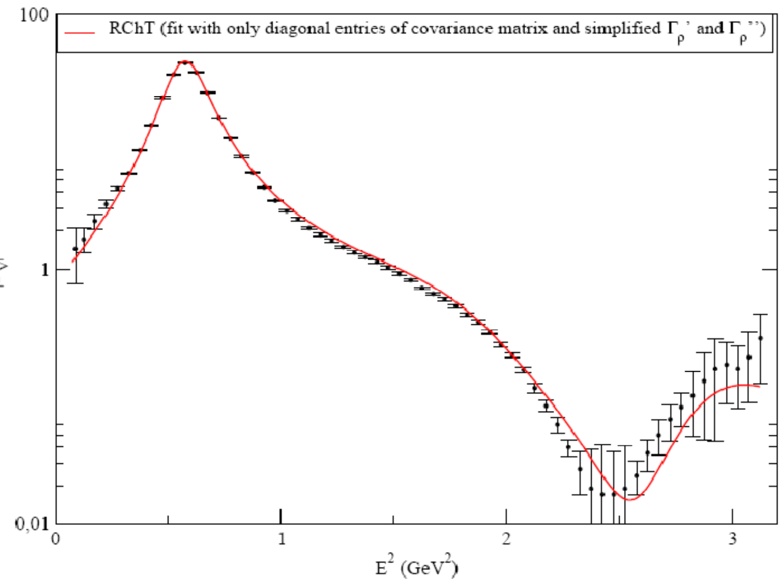
Phys. Rev. D 78 (2008) 072006

Checks:

comparison with linearly interpolated spectrum

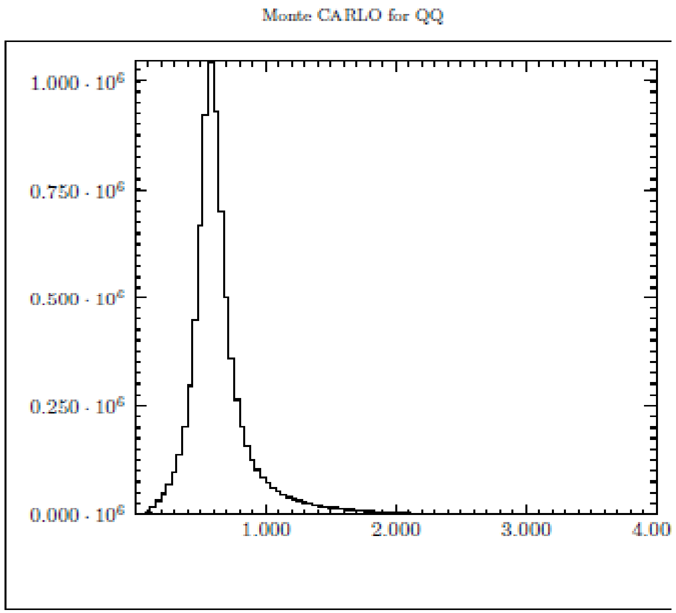
comparison MC and analytical results

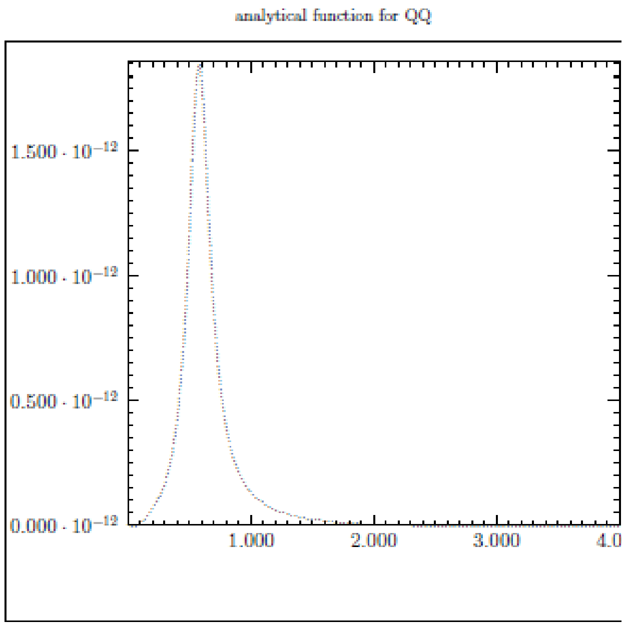
after Gauss integration

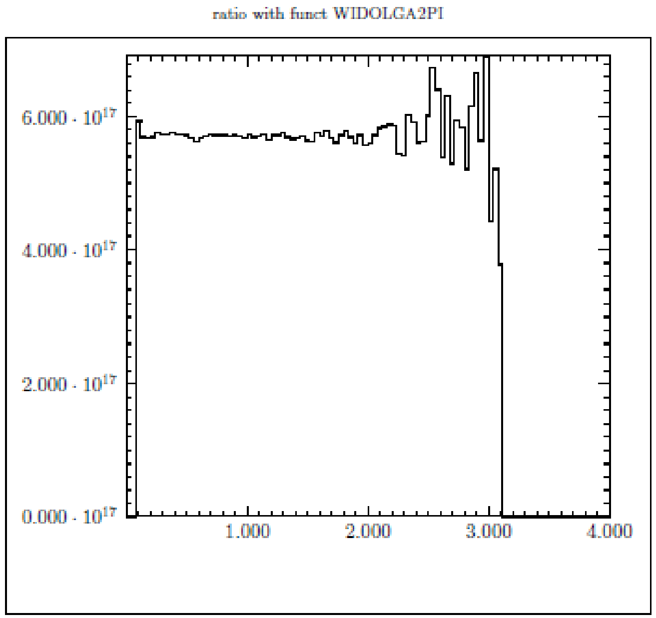


Technical test:

Monte Carlo results (8e6 events)







MC: σ = (5.617±0.0005) E-13GeV   Analyt: σ = (5.616±0.0005)E-13GeV

CONCLUSIONS

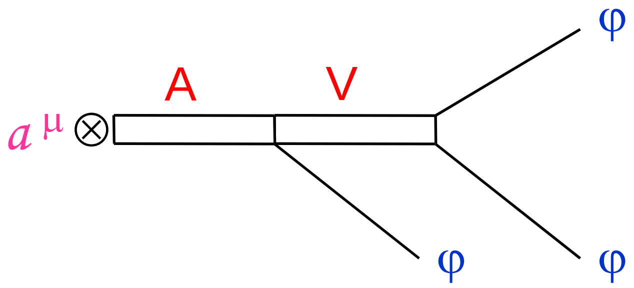
- **Theoretical results in RχT for 2 and 3 pion modes are put in TAUOLA.**
- **All work is performed with support of code manager SVN.**
- **A starting version for code development TAUOLA is Belle/BaBar version. That guarrantees that once work is completed can be installed into collaboration software.**
- **Once BaBar and/or Belle will confirm the upgrades to be valuable and fits to the data will be improved the SVN reference will make installation of such upgrade to C++ and FORTRAN applications of TAUOLA at LHC rather straightforward too.**

WORK IN PROGRESS

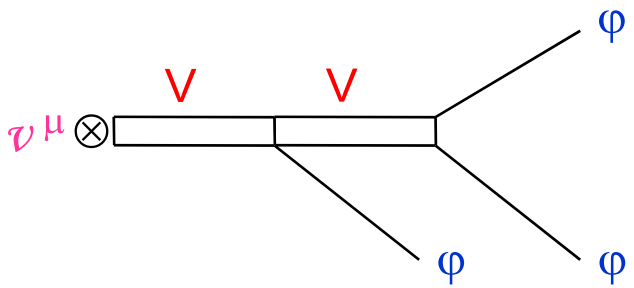
BACKUP SLIDES

K-S-like works & HADRONIZATION IN TAUOLA

(Finkemeier, Mirkes '95,'96)
(Finkemeier, Kühn, Mirkes '96)



$$\to V_{\substack{1\mu \\ 2\mu}} = \left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \right) (p_{\substack{2 \\ 3}} - p_1)^\nu$$



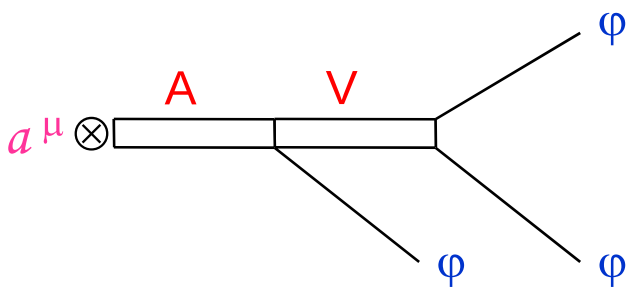
$$\to i\varepsilon_{\mu\nu\rho\sigma}\, p_1^{\nu}\, p_2^{\rho}\, p_3^{\sigma}$$

χ,1R & 2R obtained from:

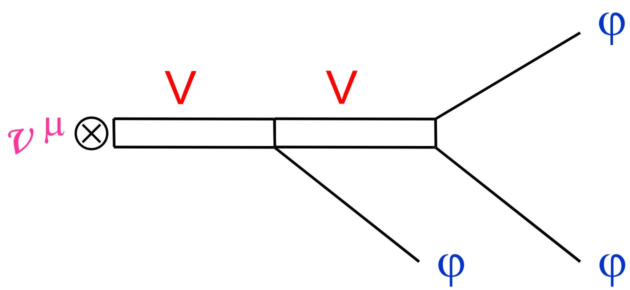
$$\frac{M_{R1}^2}{M_{R1}^2 - x^2 - i\sqrt{x^2}\,\Gamma_{R1}(x^2)} \, \frac{M_{R2}^2}{M_{R2}^2 - y^2 - i\sqrt{y^2}\,\Gamma_{R2}(y^2)}$$

K-S-like works & HADRONIZATION IN TAUOLA

(Finkemeier, Mirkes '95,'96)
(Finkemeier, Kühn, Mirkes '96)



Some allowed $\rho^{0,-}$, K^{*0} contributions are lacking in the modes: $K^+K^-\pi^-$, $K^-K^0\pi^0$, $K^-\pi^-\pi^+$, $K^0\pi^0\pi^-$



$$\left\{ \begin{array}{l} M_{\rho'}^{V_\mu} \neq M_{\rho'}^{A_\mu} \\ \Gamma_{\rho'}^{V_\mu} \neq \Gamma_{\rho'}^{A_\mu} \end{array} \right.$$

$$3\,Multiplets^{V_\mu} \neq 2\,Multiplets^{A_\mu}$$

χPT: The low-energy EFT of QCD

(Gasser & Leutwyler '84, '85)

$$\phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$$

Goldstone Bosons

$$SU(3)_L \otimes SU(3)_R \to SU(3)_V$$

$$u(x) = \exp\left(\frac{i\phi(x)}{\sqrt{2}F}\right), \quad u_\mu = i\left[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\right]$$

$$\chi = 2B_0(s+ip), \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u\chi u$$

$$f_\pm^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u$$

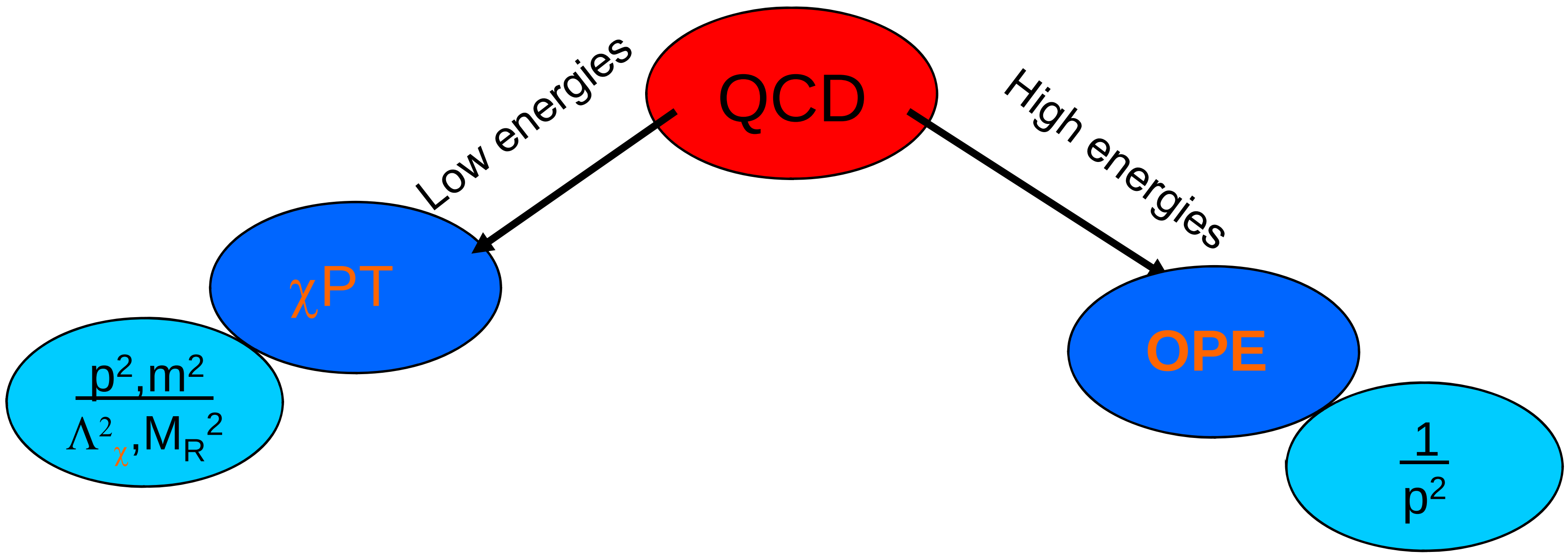
$$\mathcal{L}^{(2)}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

$$X \to h(g,\Phi)\, X\, h(g,\Phi)^\dagger$$

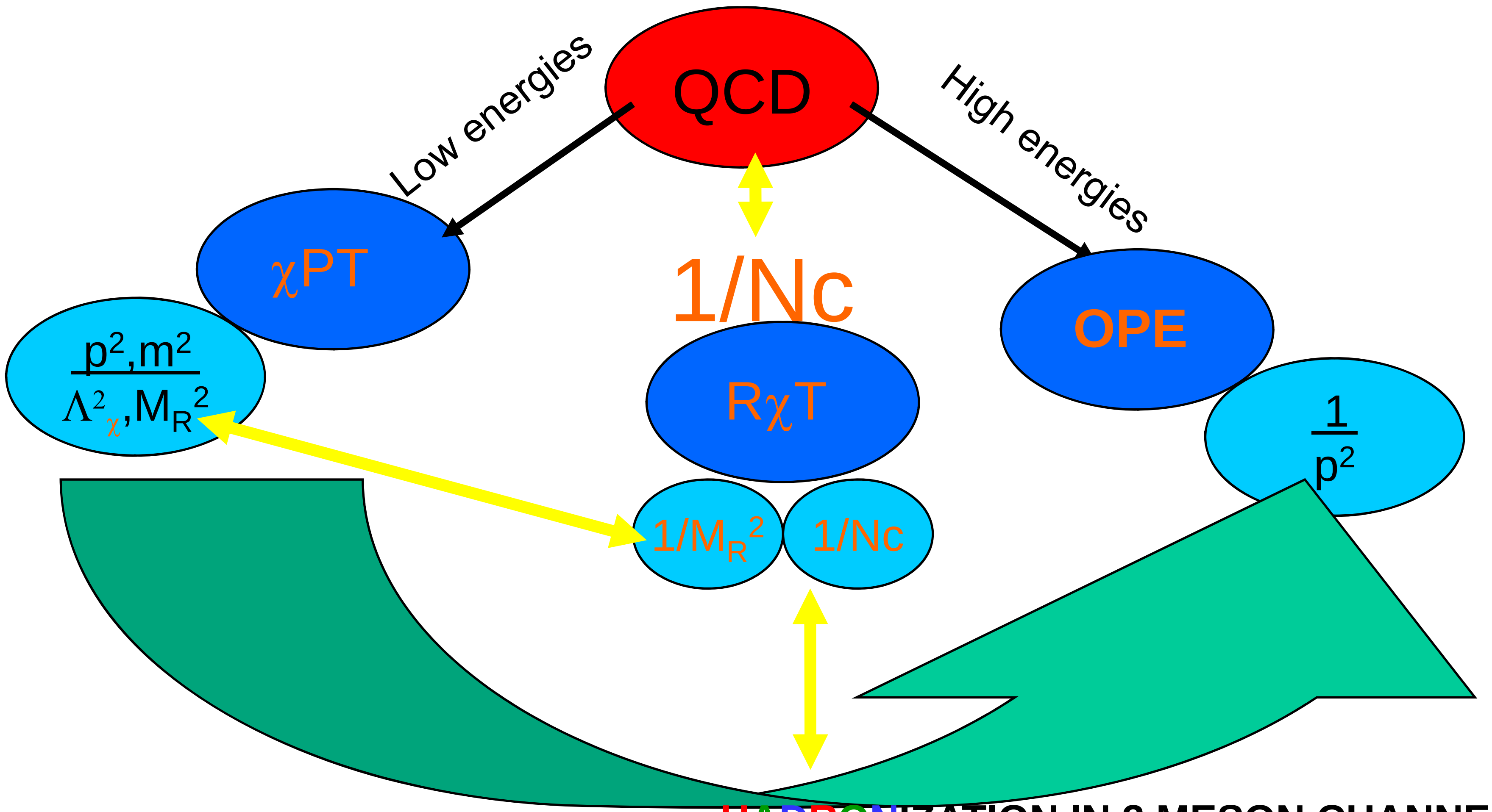
$$\mathcal{L}^{(4)}_\chi = L_1\langle u_\mu u^\mu\rangle^2 + \ldots + L_4\langle u_\mu u^\mu\rangle\,\langle\chi_+\rangle + \ldots + L_7\langle\chi_-\rangle^2 + \ldots - iL_9\langle f_+^{\mu\nu}u_\mu u_\nu\rangle + \ldots$$

$\mathcal{L}^{(4)}_{\chi,\,WZW}$ in the odd-intrinsic parity sector

RχT matching to the OPE allows it to reproduce QCD high-energy behaviour:



RχT matching to the OPE allows it to reproduce QCD high-energy behaviour:



Resonances+ Goldstone Bosons

TOOLS : RχT

(Ecker, Gasser, Pich, De Rafael '89)

(Ecker, Gasser, Leutwyler, Pich, De Rafael '89)

$$\mathcal{L}^{(P_I=+)}{}_{R\chi T} = \mathcal{L}^{(2)}{}_{\chi} + \mathcal{L}^{kin}{}_{V,A} + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{VAP};$$

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle$$

$$\mathcal{L}_A = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle$$

Antisymmetric tensor formalism

$$V_{\mu\nu}(x) = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix}_{\mu\nu}$$

$$\mathcal{L}_{VAP} = \sum_{i=1}^{5} \lambda_i O^i(V_{\mu\nu}, A^{\mu\nu}, \phi) = \lambda_1 \langle [V_{\mu\nu}, A^{\mu\nu}]\chi_-\rangle + \ldots$$

(Gómez Dumm, Pich, Portolés '04)

VMD

$$\mathcal{L}^{(P_I=-)}{}_{R\chi T} = \mathcal{L}^{(4)}{}_{\chi(WZW)} + \mathcal{L}_{VJP} + \mathcal{L}_{VVP} + \mathcal{L}_{VPPP};$$

$$\mathcal{L}_{VJP} = \sum_{i=1}^{7} \frac{c_i}{M_V} O^i(V_{\mu\nu}, j^\nu, \partial^\mu \phi) = \frac{c_5}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\left\{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha}\right\}u^\sigma\rangle + \ldots$$

$$\mathcal{L}_{VVP} = \sum_{i=1}^{5} d_i O^i(V_{\mu\nu}, V_{\rho\sigma}, \phi) = d_1 \varepsilon_{\mu\nu\rho\sigma}\langle\left\{V^{\mu\nu}, V^{\rho\alpha}\right\}\nabla_\alpha u^\sigma\rangle + \ldots$$

(Ruiz-Femenía, Pich, Portolés '03)

$$\mathcal{L}_{VPPP} = \sum_{i=1}^{5} \frac{g_i}{M_V} O^i(V_{\mu\nu}, \phi) = \frac{g_4}{M_V}\varepsilon_{\mu\nu\alpha\beta}\langle\left\{V^{\mu\nu}, u^\alpha u^\beta\right\}\chi_-\rangle + \ldots$$

(Gómez Dumm, Pich, Portolés, R. to appear)

Resonances+ Goldstone Bosons

TOOLS : RχT

(Ecker, Gasser, Pich, De Rafael '89)

(Ecker, Gasser, Leutwyler, Pich, De Rafael '89) ,…

$$\mathcal{L}^{(P_I=+)}{}_{R\chi T} = \mathcal{L}^{(2)}{}_{\chi} + \mathcal{L}^{kin}{}_{V,A} + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{VAP};$$

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle$$

$$\mathcal{L}_A = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle$$

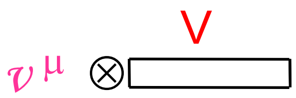
$$\mathcal{L}_{VAP} = \sum_{i=1}^{5} \lambda_i O^i (V_{\mu\nu}, A^{\mu\nu}, \phi) = \lambda_1 \langle [V_{\mu\nu}, A^{\mu\nu}] \chi_- \rangle + \ldots$$

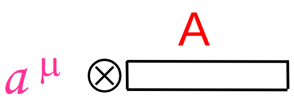
$$\mathcal{L}^{(P_I=-)}{}_{R\chi T} = \mathcal{L}^{(4)}{}_{\chi(WZW)} + \mathcal{L}_{VJP} + \mathcal{L}_{VVP} + \mathcal{L}_{VPPP};$$

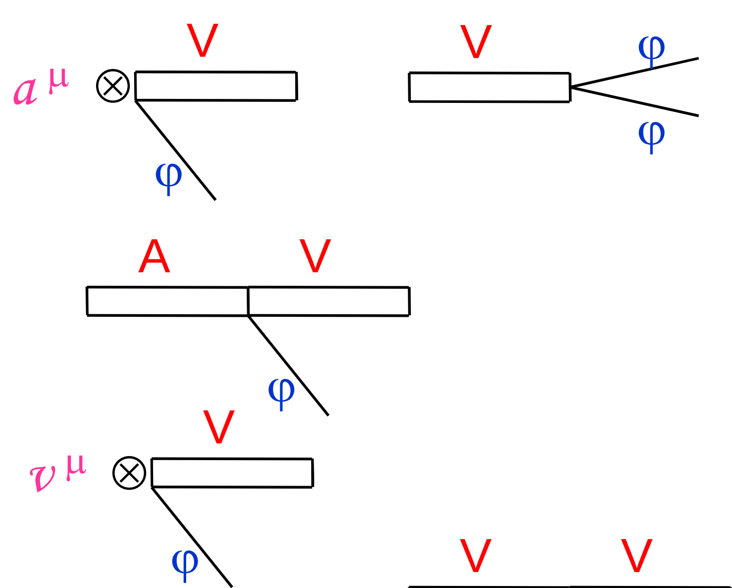
$$\mathcal{L}_{VJP} = \sum_{i=1}^{7} \frac{c_i}{M_V} O^i (V_{\mu\nu}, j^\nu, \partial^\mu \phi) = \frac{c_5}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \left\{ \nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha} \right\} u^\sigma \rangle + \ldots$$

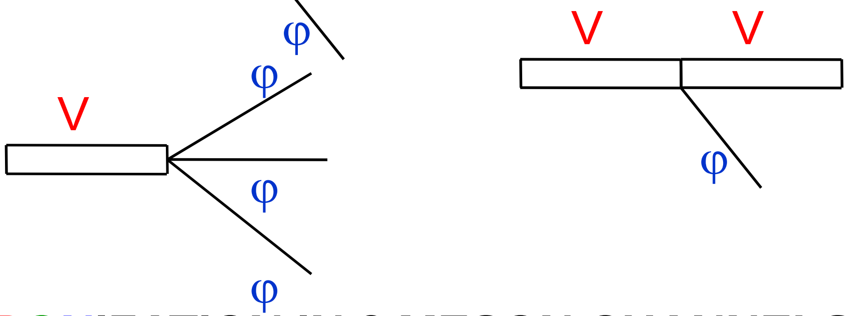
$$\mathcal{L}_{VVP} = \sum_{i=1}^{5} d_i O^i (V_{\mu\nu}, V_{\rho\sigma}, \phi) = d_1 \varepsilon_{\mu\nu\rho\sigma} \langle \left\{ V^{\mu\nu}, V^{\rho\alpha} \right\} \nabla_\alpha u^\sigma \rangle + \ldots$$

$$\mathcal{L}_{VPPP} = \sum_{i=1}^{5} \frac{g_i}{M_V} O^i (V_{\mu\nu}, \phi) = \frac{g_4}{M_V} \varepsilon_{\mu\nu\alpha\beta} \langle \left\{ V^{\mu\nu}, u^\alpha u^\beta \right\} \chi_- \rangle + \ldots$$









EFT 09
València

HADRONIZATION IN 3 MESON CHANNELS
Pablo Roig (IFIC & INFN & TUM)

CLEO analyses & KS-like models

(Liu '03) (CLEO-III '04)

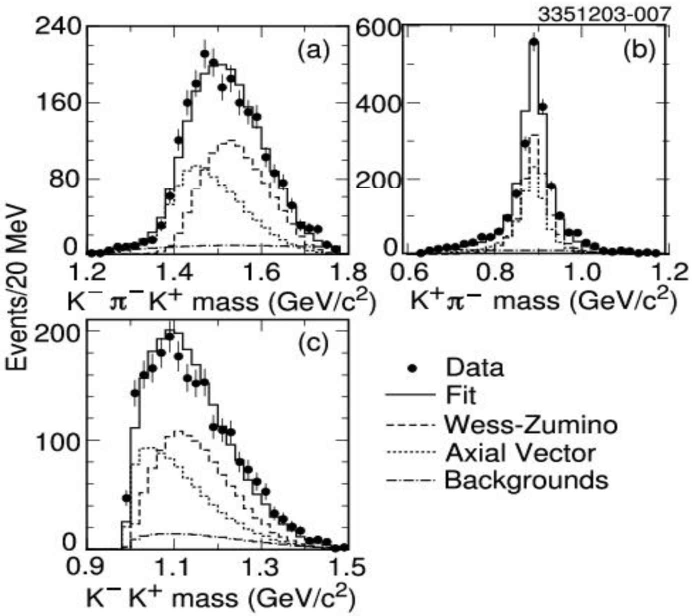
CLEO

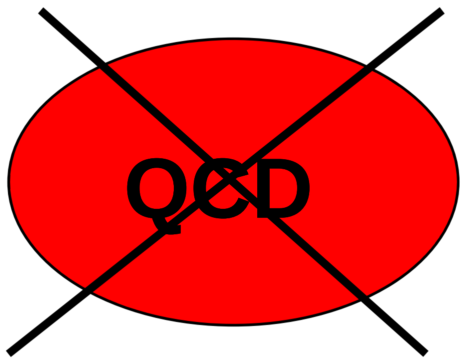
$$\tau^- \to K \bar{K} \pi^- \nu_\tau$$

$$F_V = -\underbrace{\frac{1}{2\sqrt{2}\pi^2 F^3}}_{L^{(4)}_{\chi,WZW}} \sqrt{R_B} \underbrace{\sum_i BW_i}_{\xrightarrow{Q^2 \to 0} 1}$$

$$BW_{V,A}(x = s_i, Q^2) = \frac{M_{V,A}^2}{M_{V,A}^2 - x - i\sqrt{x}\,\Gamma_{V,A}(x)}$$

R_B: **1.80±0.53**

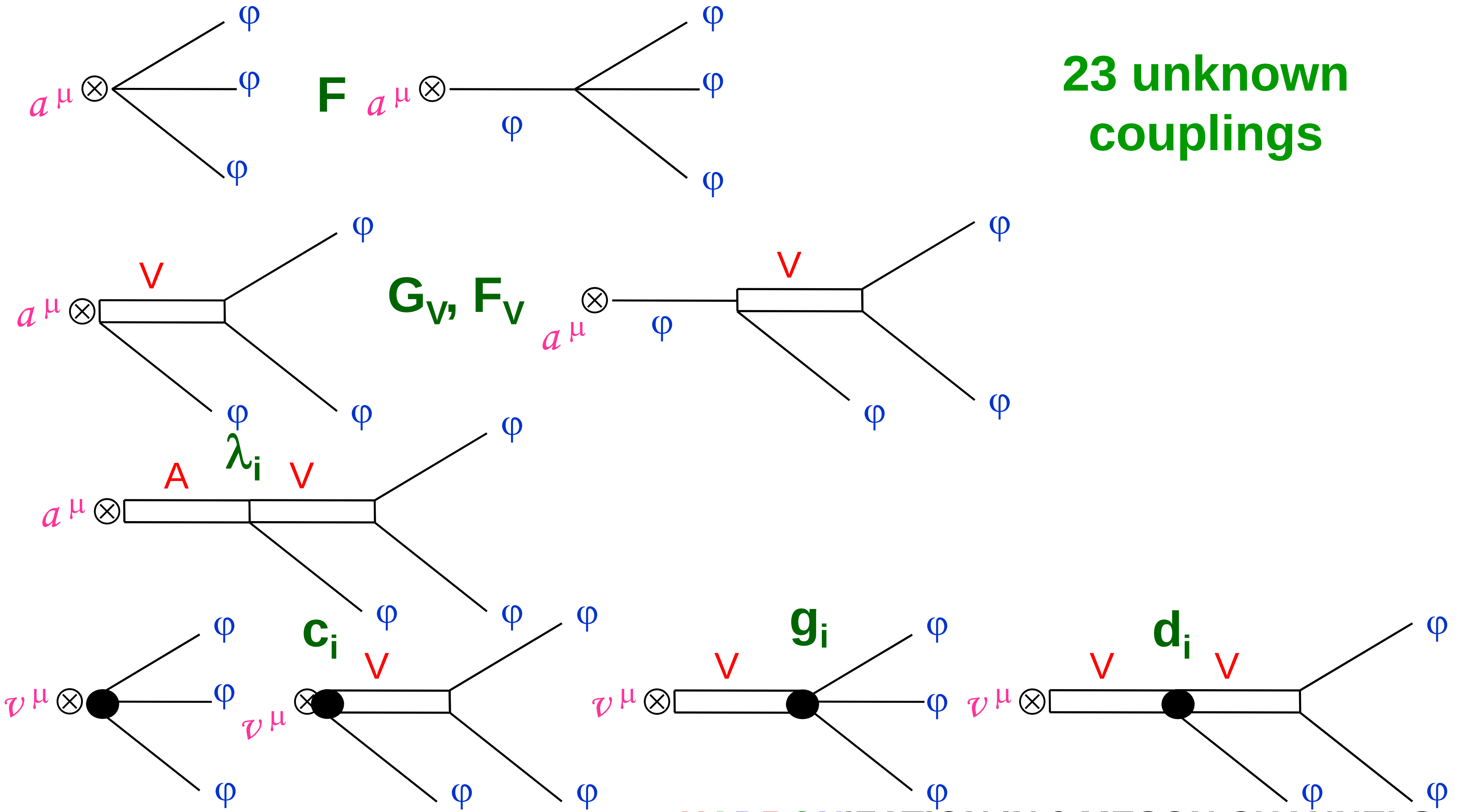




EFT 09
València

HADRONIZATION IN 3 MESON CHANNELS
Pablo Roig (IFIC & INFN & TUM)

RχT APPLIED



23 unknown couplings

EFT 09
València

HADRONIZATION IN 3 MESON CHANNELS
Pablo Roig (IFIC & INFN & TUM)

The axial-form factor and the a_1: $\tau^- \to (3\pi)^- \nu_\tau$

(Gómez-Dumm, Pich, Portolés '00)

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F^2}\left[\sigma^3_\pi \Theta(s-4m_\pi^2) + \frac{1}{2}\sigma^3_K \Theta(s-4m_K^2)\right]$$

(This work)

$$\Gamma_{a_1}(Q^2) = \Gamma_{a_1}^{3\pi}(Q^2) + \Gamma_{a_1}^{K\overline{K}\pi}(Q^2) + \Gamma_{a_1}^{(K\pi)^0 K^0}(Q^2),$$

$$\Gamma_{a_1}^{3\pi}(Q^2) = \frac{1}{48(2\pi)^3 M_{a_1}}\left(\frac{Q^2}{M_{a_1}^2}\right)\iint ds\,dt\left(F_1' V_{1\mu} + F_2' V_{2\mu}\right)\cdot$$

$$\left(F_1'^{\dagger} V_{1\mu} + F_2'^{\dagger} V_{2\mu}\right), \quad F_i' = F_i \frac{M_{a_1}^2 - Q^2}{\sqrt{2} F_A Q^2}$$