"$\rho - \gamma$ mixing and e+e- vs. τ spectral functions"

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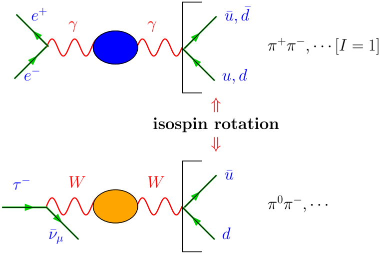
Outline

- The τ vs. e^+e^- problem
- A minimal model: VMD + sQED
- $F_\pi(s)$ with $\rho - \gamma$ mixing at one-loop
- Applications: a_μ and $B^{\mathrm{CVC}}_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$
- Summary and Outlook

The τ vs. e^+e^- problem

A need for a calculation of hadronic vacuum polarization from hadron production data.

A good idea: enhance e^+e^-–data by isospin rotated/corrected τ–data + CVC



ALEPH–Coll., (OPAL, CLEO), Alemany, Davier, Höcker 1996, Belle–Coll. Fujikawa, Hayashii, Eidelman 2008

We relate the τ and e^+e^- data:

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+e^- \to X^0$$

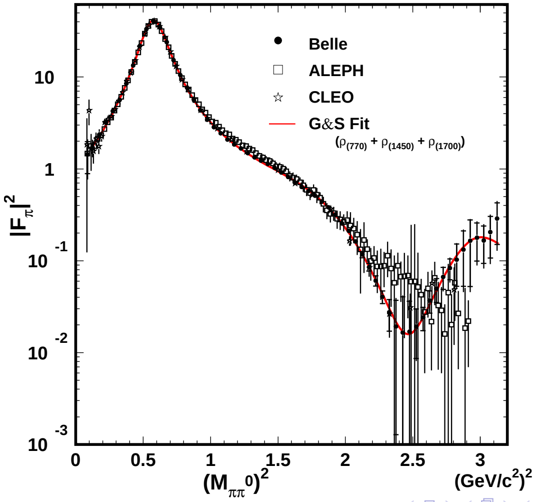
where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^- \to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \leq M_\tau$$

in terms of the τ spectral function v_1.
Mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution) which is dominating in a_μ^{had} (72%)

$I = 1 \sim 75\%$; $I = 0 \sim 25\%$ τ–data cannot replace e^+e^-–data

Data: ALEPH 97, ALEPH 05, OPAL, CLEO and
most recent measurement from *Belle* (2008):



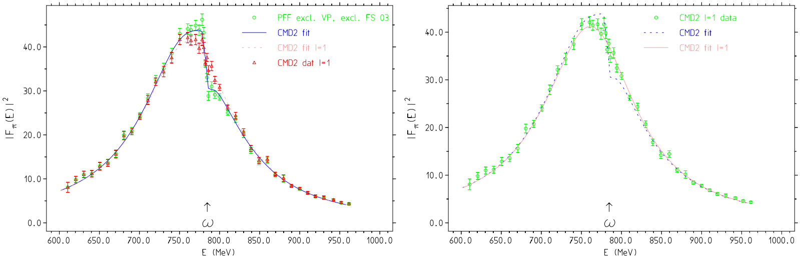
e^+e^-**–data*= data corrected for isospin violations:**

In e^+e^- **channel** $\rho - \omega$ **mixing due isospin violation be quark mass difference** $m_u \neq m_d \;\Rightarrow$

I=0 component; to be subtracted for comparison with τ **data**

$|F(s)|^2 =$ **(**$|F(s)|^2$**–data) /** $\left|\left(1 + \frac{\epsilon s}{(s_\omega - s)}\right)\right|^2$ **with** $s_\omega = (M_\omega - \frac{i}{2}\Gamma_\omega)^2$

ϵ **determined by fit to the data:** $\epsilon = 0.00172$

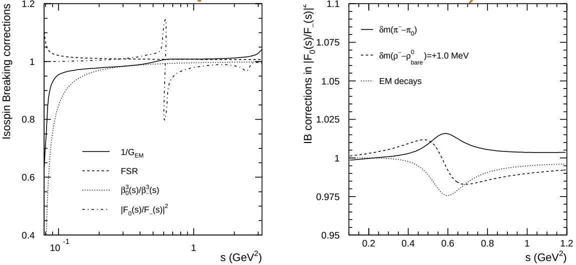


CMD-2 data for $|F_\pi|^2$ **in** $\rho - \omega$ **region together with Gounaris-Sakurai fit. Left before subtraction right after subtraction of the** ω**.**

I=0 component to be added to τ **data for calculating** a_μ^{had} **!**

Other isospin-breaking corrections

(Cirigliano et al. 2002, López Castro el al. 2007)

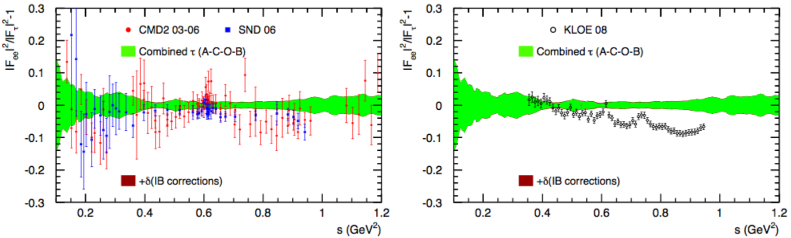


Left: Isospin-breaking corrections $G_{\rm EM}$, FSR, $\beta_0^3(s)/\beta_-^3(s)$ and $|F_0(s)/F_-(s)|^2$.

Right: Isospin-breaking corrections in $I=1$ part of ratio $|F_0(s)/F_-(s)|^2$:

- π mass splitting $\delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$,
- ρ mass splitting $\delta m_\rho = m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$, and
- ρ width splitting $\delta\Gamma_\rho = \Gamma_{\rho^\pm} - \Gamma_{\rho^0}$.

New isospin corrections applied shift in mass and width [as advocated by S. Ghozzi and F. Jegerlehner in 2003!!!] plus changes [López Castro, Toledo Sánchez et al 2007] below the ρ which Davier et al say are not understood!
New BaBar radiative return $\pi\pi$ spectrum in much better agreement, in particular with *Belle* τ spectrum!



e^+e^- vs τ spectral functions: $|F_{ee}|^2/|F_\tau|^2 - 1$ as a function of s. Isospin-breaking (IB) corrections are applied to τ data with its uncertainties included in the error band.

Possible origin of problems:

- **Radiative corrections involving hadrons fully under control?**
- **IB in parameter shifts:** $m_{\rho^+} - m_{\rho^0}$, $\Gamma_{\rho^+} - \Gamma_{\rho^0}$ **fully known?**

Key problem: on basis of commonly used Gounaris-Sakurai type parametrizations

e^+e^- **vs.** τ **fit with same formula** $\Rightarrow$ **differ in parameters only: NC vs. CC process** δM_ρ, $\delta\Gamma_\rho$, **mixing coefficients etc.**

Other possible source: do we really understand quantum interference?

- e^+e^-: $|F_\pi^{(e)}(s)|^2 = |F_\pi^{(e)}(s)[I=1] + F_\pi^{(e)}(s)[I=0]|^2$ **what we need and measure**
- τ: $|F_\pi^{(\tau)}(s)[I=1]|^2$ **measured in** τ**-decay**
- $ee+\tau$: $|F_\pi^{(e)}(s)|^2 \simeq |F_\pi^{(e,\tau)}(s)[I=1]|^2 + |F_\pi^{(e)}(s)[I=0]|^2$ **??? usual approximation**

Need theory $\rightarrow$ **specific model for the complex amplitudes**

A minimal model: VMD + sQED

Effective Lagrangian $\mathcal{L} = \mathcal{L}_{\gamma\rho} + \mathcal{L}_{\pi}$

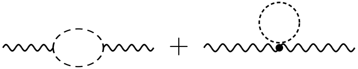
$$\mathcal{L}_\pi = D_\mu \pi^+ D^{+\mu} \pi^- - m_\pi^2 \pi^+ \pi^- ; \quad D_\mu = \partial_\mu - i\, e\, A_\mu - i\, g_{\rho\pi\pi}\, \rho_\mu$$

$$\mathcal{L}_{\gamma\rho} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{M_\rho^2}{2} \rho_\mu \rho^\mu + \frac{e}{2\, g_\rho} \rho_{\mu\nu} F^{\mu\nu}$$

The Feynman rules in momentum space are:

$$\begin{array}{lclclcl}
A^\mu \pi\pi & \hat{=} & -i\, e\, (p+p')^\mu & , & \rho^\mu \pi\pi & \hat{=} & -i\, g_{\rho\pi\pi}\, (p+p')^\mu \\
A^\mu A^\nu \pi\pi & \hat{=} & 2\, i\, e^2\, g^{\mu\nu} & , & \rho^\mu \rho^\nu \pi\pi & \hat{=} & 2\, i\, g_{\rho\pi\pi}^2\, g^{\mu\nu} \\
A^\mu \rho^\nu \pi\pi & \hat{=} & 2\, i\, e\, g_{\rho\pi\pi}\, g^{\mu\nu} & , & A^\mu \rho^\nu & \hat{=} & -i\, e/g_\rho\, (p^2\, g^{\mu\nu} - p^\mu p^\nu).
\end{array}$$

Self-energies: pion loops to $\gamma - \rho$ vacuum polarizations

$$-\mathrm{i}\, \Pi^{\mu\nu\;(\pi)}_{\gamma\gamma}(q) = $$  .

bare $\gamma - \rho$ transverse self-energy functions

$$\Pi_{\gamma\gamma} \;=\; \frac{e^2}{48\pi^2}\, f(q^2),\;\; \Pi_{\gamma\rho} = \frac{e g_{\rho\pi\pi}}{48\pi^2}\, f(q^2) \quad \text{and} \quad \Pi_{\rho\rho} = \frac{g^2_{\rho\pi\pi}}{48\pi^2}\, f(q^2),$$

where

$$f(q^2) \equiv q^2\, h(q^2) = \left(B_0(m_\pi, m_\pi; q^2)\,(q^2 - 4\, m_\pi^2) - 4\, A_0(m_\pi) - 4\, m_\pi^2 + \frac{2}{3} q^2 \right).$$

Explicitly, in the $\overline{\rm MS}$ scheme (μ the $\overline{\rm MS}$ renormalization scale)

$$h(q^2) \equiv f(q^2)/q^2 \quad = \quad 2/3 + 2\,(1-y) - 2\,(1-y)^2\,G(y) + \ln\frac{\mu^2}{m_\pi^2}\,,$$

where $y = 4m_\pi^2/s$ **and** $G(y) = \frac{1}{2\beta_\pi}\left(\ln\frac{1+\beta_\pi}{1-\beta_\pi} - i\,\pi\right)$**, for** $q^2 > 4\,m_\pi^2$**.**
Mass eigenstates, diagonalization: renormalization conditions are such that the matrix is diagonal and of residue unity at the photon pole $q^2 = 0$ **and at the** ρ **resonance** $s = M_\rho^2$**,**
[$\Pi_{\cdot\cdot}(0) = 0\,, \Pi'_{\gamma\gamma}(q^2) = \Pi_{\gamma\gamma}(q^2)/q^2$**]**

$$\begin{aligned}
\Pi^{\rm ren}_{\gamma\gamma}(q^2) &= \Pi_{\gamma\gamma}(q^2) - q^2\,\Pi'_{\gamma\gamma}(0) \doteq q^2\,\Pi'^{\rm ren}_{\gamma\gamma}(q^2) \\
\Pi^{\rm ren}_{\gamma\rho}(q^2) &= \Pi_{\gamma\rho}(q^2) - \frac{q^2}{M_\rho^2}\,\mathbf{Re}\,\Pi_{\gamma\rho}(M_\rho^2) \\
\Pi^{\rm ren}_{\rho\rho}(q^2) &= \Pi_{\rho\rho}(q^2) - \mathbf{Re}\,\Pi_{\rho\rho}(M_\rho^2) - (q^2 - M_\rho^2)\,\mathbf{Re}\,\frac{d\Pi_{\rho\rho}}{ds}(M_\rho^2)
\end{aligned}$$

Propagators = inverse of symmetric 2×2 self-energy matrix

$$\hat{D}^{-1} = \begin{pmatrix} q^2 + \Pi_{\gamma\gamma}(q^2) & \Pi_{\gamma\rho}(q^2) \\ \Pi_{\gamma\rho}(q^2) & q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) \end{pmatrix}$$

inverted $\Rightarrow$

$$\begin{aligned} D_{\gamma\gamma} &= \frac{1}{q^2 + \Pi_{\gamma\gamma}(q^2) - \frac{\Pi^2_{\gamma\rho}(q^2)}{q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)}} \\ D_{\gamma\rho} &= \frac{-\Pi_{\gamma\rho}(q^2)}{(q^2 + \Pi_{\gamma\gamma}(q^2))(q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)) - \Pi^2_{\gamma\rho}(q^2)} \\ D_{\rho\rho} &= \frac{1}{q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) - \frac{\Pi^2_{\gamma\rho}(q^2)}{q^2 + \Pi_{\gamma\gamma}(q^2)}} \, . \end{aligned}$$

To diagnolize the mixed propagator we perform

i) **Infinitesimal (perturbative) rotation**

$$\begin{pmatrix} A_b \\ \rho_b \end{pmatrix} = \begin{pmatrix} 1 & -\Delta_0 \\ \Delta_0 & 1 \end{pmatrix} \begin{pmatrix} A' \\ \rho' \end{pmatrix} \quad (1)$$

diagonalizing the mass matrix at one-loop.

ii) **Upper diagonal matrix wave function renormalization**

$$\begin{pmatrix} A' \\ \rho' \end{pmatrix} = \begin{pmatrix} \sqrt{Z_\gamma} & -\Delta_\rho \\ 0 & \sqrt{Z_\rho} \end{pmatrix} \begin{pmatrix} A_r \\ \rho_r \end{pmatrix}$$

which allows to normalize the residues to one for the γ- and ρ-propagator.

Such that:

$$\begin{aligned} A_b &= \sqrt{Z_\gamma} A_r - (\Delta_\rho + \Delta_0)\, \rho_r \\ \rho_b &= \sqrt{Z_\rho} \rho_r + \Delta_0\, A_r\,. \end{aligned}$$

Diagonalization $\Rightarrow$ physical ρ acquires a direct coupling to the electron

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i\, e_b\, A_{b\mu})\, \psi_e$$
$$\Downarrow$$
$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i\, e\, A_\mu + i\, g_{\rho ee} \rho_\mu)\, \psi_e$$

with $g_{\rho ee} = e\,(\Delta_\rho + \Delta_0)$, where in our case $\Delta_0 = 0$.

Resonance parameters $\Leftrightarrow$ location s_P of the pole of the propagator

$$s_P - m^2_{\rho^0} + \Pi_{\rho^0 \rho^0}(s_P) - \frac{\Pi^2_{\gamma\rho^0}(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0,$$

with $s_P = \tilde{M}^2_{\rho^0}$ complex.

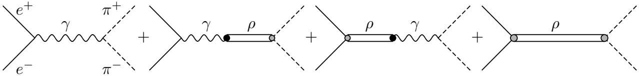
$$\tilde{M}^2_\rho \equiv \left(q^2\right)_{\mathrm{pole}} = M^2_\rho - i\; M_\rho\; \Gamma_\rho$$

$F_\pi(s)$ with $\rho - \gamma$ mixing at one-loop

The $e^+e^- \to \pi^+\pi^-$ matrix element in sQED is given by

$$\mathcal{M} = -i\, e^2\, \bar{v}\gamma^\mu u\, (p_1 - p_2)_\mu\, F_\pi(q^2)$$

with $F_\pi(q^2) = 1$. In our extended VMD model we have the four terms



Diagrams contributing to the process $e^+e^- \to \pi^+\pi^-$.

$$F_\pi(s) \propto e^2\, D_{\gamma\gamma} + e g_{\rho\pi\pi}\, D_{\gamma\rho} - g_{\rho ee}\, e D_{\rho\gamma} - g_{\rho ee} g_{\rho\pi\pi}\, D_{\rho\rho}\,,$$

Properly normalized (VP subtraction: $e^2(s) \to e^2$):

$$F_\pi(s) = \left[e^2\, D_{\gamma\gamma} + e\,(g_{\rho\pi\pi} - g_{\rho ee})\, D_{\gamma\rho} - g_{\rho ee} g_{\rho\pi\pi}\, D_{\rho\rho}\right] / \left[e^2\, D_{\gamma\gamma}\right]$$

Typical couplings

$g_{\rho\pi\pi\,\mathrm{bare}} = 5.8935$, $g_{\rho\pi\pi\,\mathrm{ren}} = 6.1559$, $g_{\rho ee} = 0.018149$, $x = g_{\rho\pi\pi}/g_\rho = 1.15128$.

We note that the precise s-dependence of the effective ρ-width is obtained by evaluating the imaginary part of the ρ self-energy:

$$\mathbf{Im}\, \Pi_{\rho\rho} = \frac{g^2_{\rho\pi\pi}}{48\,\pi}\, \beta_\pi^3\, s \equiv M_\rho\, \Gamma_\rho(s),$$

which yields

$$\Gamma_\rho(s)/M_\rho = \frac{g^2_{\rho\pi\pi}}{48\,\pi}\, \beta_\pi^3\, \frac{s}{M_\rho^2}; \quad \Gamma_\rho/M_\rho = \frac{g^2_{\rho\pi\pi}}{48\,\pi}\, \beta_\rho^3; \quad g_{\rho\pi\pi} = \sqrt{48\,\pi\, \Gamma_\rho/(\beta_\rho^3\, M_\rho)}.$$

In our model, in the given approximation, the on ρ-mass-shell form factor reads

$$F_\pi(M_\rho^2) = 1 - i\,\frac{g_{\rho ee}\,g_{\rho\pi\pi}}{e^2}\,\frac{M_\rho}{\Gamma_\rho}, \quad |F_\pi(M_\rho^2)|^2 = 1 + \frac{36}{\alpha^2}\,\frac{\Gamma_{ee}}{\beta_\rho^3\,\Gamma_\rho},$$

$$\Gamma_{\rho ee} = \frac{1}{3}\,\frac{g_{\rho ee}^2}{4\pi}\,M_\rho, \quad g_{\rho ee} = \sqrt{12\pi\,\Gamma_{\rho ee}/M_\rho}.$$

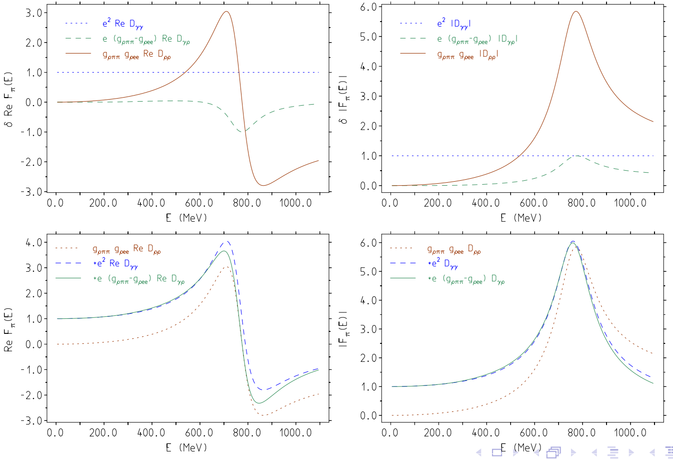
While in Gounaris-Sakurai (GS) formula

$$F_\pi^{\mathrm{GS}}(s) = \frac{-M_\rho^2 + \Pi_{\rho\rho}^{\mathrm{ren}}(0)}{s - M_\rho^2 + \Pi_{\rho\rho}^{\mathrm{ren}}(s)}, \quad \Gamma_{\rho ee}^{\mathrm{GS}} = \frac{2\alpha^2\,\beta_\rho^3 M_\rho^2}{9\,\Gamma_\rho}\,(1 + d\,\Gamma_\rho/M_\rho)^2\,.$$

GS does not involve $g_{\rho ee}$ resp. $\Gamma_{\rho ee}$ in a direct way, as normalization is fixed by applying an overall factor $1 + d\,\Gamma_\rho/M_\rho \equiv 1 - \Pi_{\rho\rho}^{\mathrm{ren}}(0)/M_\rho^2 \simeq 1.089$ to enforce $F_\pi(0) = 1$ (in our approach "automatic" by gauge invariance).

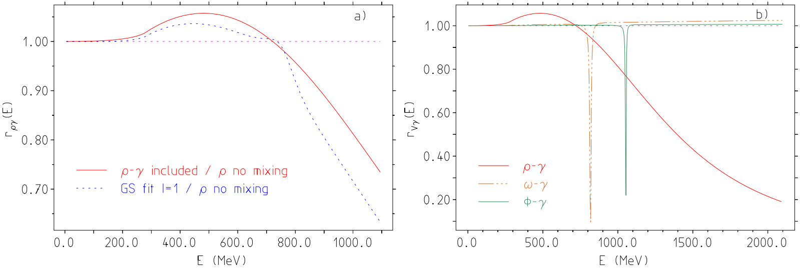
The interferemce of terms in $F_\pi^{(e)}$

Real parts and moduli of the 3 individual and added terms normalized to the sQED term are displayed:



Detailed comparison, in terms of the ratio

$$r_{\rho\gamma}(s) \equiv \frac{|F_\pi(s)|^2}{|F_\pi(s)|^2_{D_{\gamma\rho}=0}}$$



a) Ratio of $|F_\pi(E)|^2$ with mixing vs. no mixing. Same ratio for GS fit with PDG parameters. b) The same mechanism scaled up by the branching fraction $\Gamma_V/\Gamma(V \to \pi\pi)$ for $V = \omega$ and ϕ. In the $\pi\pi$ channel the effects for resonances $V \neq \rho$ are tiny if not very close to resonance.

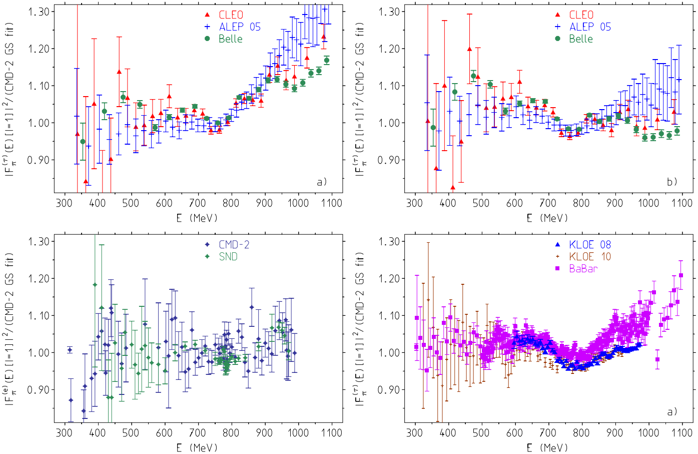
If mixing not included in $F_0(s)$ $\Rightarrow$ total correction formula on spectral functions

$$v_0(s) = r_{\rho\gamma}(s)\, R_{\rm IB}(s)\, v_-(s)$$

$$R_{\rm IB}(s) = \frac{1}{G_{\rm EM}(s)}\,\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

- $G_{\rm EM}(s)$ **electromagnetic radiative corrections**
- $\beta_0^3(s)/\beta_-^3(s)$ **phase space modification by** $m_{\pi^0} \neq m_{\pi^\pm}$
- $|F_0(s)/F_-(s)|^2$ **incl. shifts in masses, widths etc**

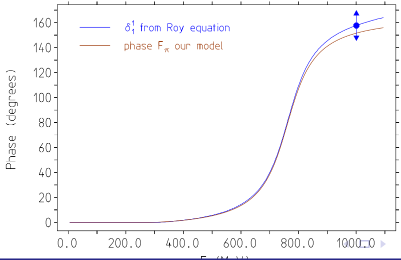
Final state radiation correction FSR(s) and vacuum polarization effects $(\alpha/\alpha(s))^2$ and I=0 component ($\rho - \omega$) we have been subtracted from all e^+e^--data.



$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit): a) τ data uncorrected for $\rho - \gamma$ mixing, and b) after correcting for mixing. Lower panel: e^+e^- energy scan data [left] and e^+e^- radiative return data [right]

Comparison of $\pi\pi$ rescattring with Colangelo-Leutwyler's from first principles approach

One of the key ingredients in this approach is the strong interaction phase shift $\delta_1^1(s)$ of $\pi\pi$ (re)scattering in the final state. We compare the phase of $F_\pi(s)$ in our model with the one obtained by solving the Roy equation with $\pi\pi$-scattering data as input. We notice that the agreement is surprisingly good up to about 1 GeV. It is not difficult to replace our phase by the more precise exact one.



Applications: a_μ and $B^{\rm CVC}_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$

How does the new correction affect the evaluation of the hadronic contribution to a_μ? To the lowest order in terms of e^+e^--data, represented by $R(s)$, we have

$$a_\mu^{\rm had,LO}(\pi\pi) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds\, R^{(0)}_{\pi\pi}(s) \frac{K(s)}{s},$$

with the well-known kernel $K(s)$ and

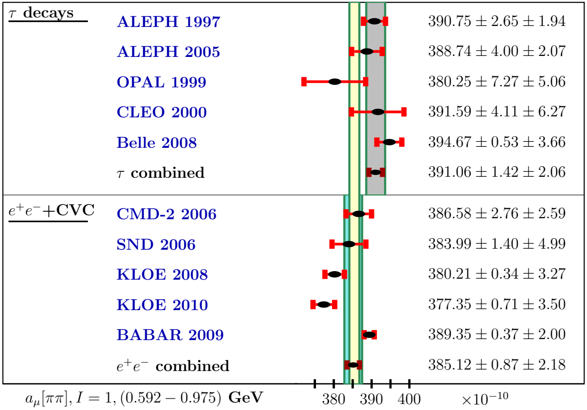
$$R^{(0)}_{\pi\pi}(s) = (3s\sigma_{\pi\pi})/4\pi\alpha^2(s)) = 3v_0(s).$$

Note that the $\rho - \gamma$ interference is included in the measured e^+e^--data, and so is its contribution to $a_\mu^{\rm had}$. In fact $a_\mu^{\rm had}$ is intrinsic an e^+e^--based “observable” (neutral current channel).

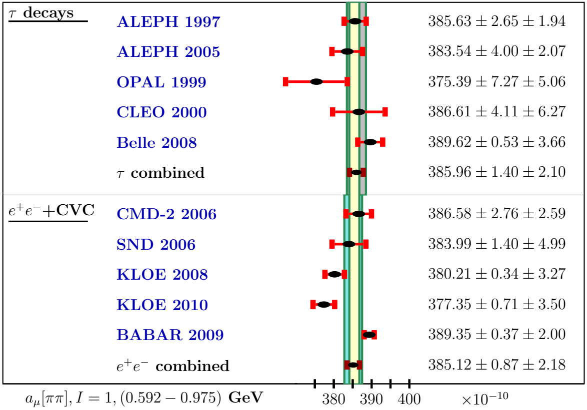
How to utilize τ data: subtract CVC violating corrections

- **traditionally** $v_-(s) \to v_0(s) = R_{\rm IB}(s)\, v_-(s)$
- **our correction** $v_-(s) \to v_0(s) = r_{\rho\gamma}(s)\, R_{\rm IB}(s)\, v_-(s)$

Result for the I=1 part of $a_\mu^{\rm had}[\pi\pi]$: $\delta a_\mu^{\rm had}[\rho\gamma] \simeq (-5.1 \pm 0.5) \times 10^{-10}$



I=1 part of $a_\mu^{\mathrm{had}}[\pi\pi]$

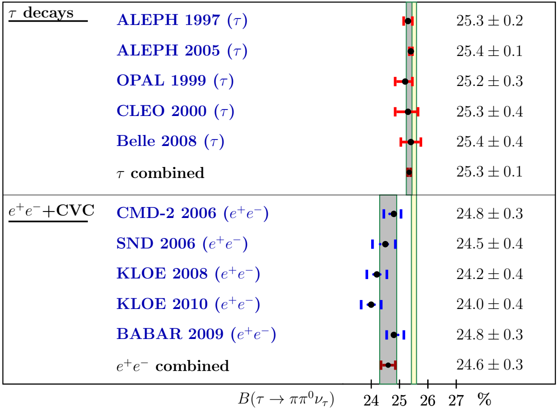


I=1 part of $a_\mu^{\rm had}[\pi\pi]$

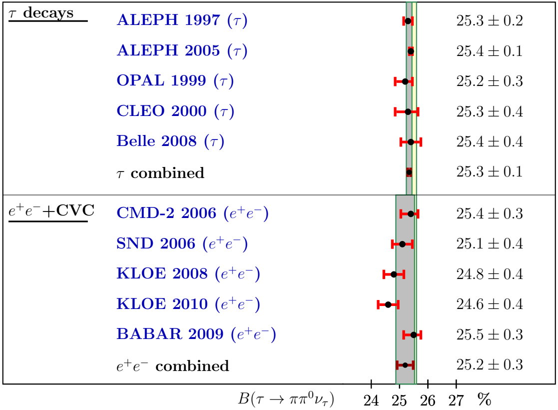
The $\tau \to \pi^0 \pi \nu_\tau$ branching fraction $B_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi \pi^0)/\Gamma_\tau$ is another important quantity which can be directly measured. This "τ-observable" can be evaluated in terms of the I=1 part of the $e^+e^- \to \pi^+\pi^-$ cross section, after taking into account the IB correction $v_0(s) \to v_-(s) = v_0(s)/R_{\rm IB}(s)/r_{\rho\gamma}(s)$,

$$B^{\rm CVC}_{\pi\pi^0} = \frac{2 S_{\rm EW} B_e |V_{ud}|^2}{m_\tau^2} \int_{4m_\pi^2}^{m_\tau^2} ds\, R^{(0)}_{\pi^+\pi^-}(s) \left(1 - \frac{2}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \frac{1}{r_{\rho\gamma}(s)\, R_{\rm IB}(s)} .$$

where here we also have to "undo" the $\rho - \gamma$ mixing which is absent in the charged isovector channel. The shift is $\delta B^{\rm CVC}_{\pi\pi^0}[\rho\gamma] = +0.62 \pm 0.06$ **%**



Branching fractions $B(\tau \to \pi\pi^0\nu_\tau)$



Branching fractions $B(\tau \to \pi\pi^0\nu_\tau)$

Summary and Conclusions

VMD+sQED EFT:

- **proper ρ propagator self-energy effects for S form factor ($\rho \to \pi\pi$)**
- **pion-loop effects in $\rho - \gamma$ mixing contributes sizable interferences**

Note: so far PDG parameters masses, withs, branching fractions etc. of resonances like ρ^0 all extracted from data assuming GS like form factors (model dependent!)
Pattern:

- **moderate positive interference (up to +5%) below ρ, substantial negative interference (-10% and more) above the ρ (must vanish at $s = 0$ and $s = M_\rho^2$)**

- remarkable agreement with pattern of e^+e^- vs τ discrepancy
- shift of the τ data to lie perfectly within the ballpark of the e^+e^- data

Efective field theory is the basic tool!

- $\rho - \gamma$ correction function $r_{\rho\gamma}(s)$ entirely fixed from neutral channel
- τ data provide independent information

What does it mean for the muon $g-2$?

- it looks we have fairly reliable model to include τ data to improve a_μ^{had}
- there is no τ vs. e^+e^- alternative of a_μ^{had}

For the lowest order hadronic vacuum polarization (VP) contribution to a_μ we find

$$a_\mu^{\mathrm{had,LO}}[e,\tau] = 690.96(1.06)(4.63) \times 10^{-10} \qquad (e+\tau)$$

$$a_\mu^{\rm the} = 116591797(60) \times 10^{-11} \;\; a_\mu^{\rm exp} = 116592080(54)(33) \times 10^{-11}$$
$$a_\mu^{\rm exp} - a_\mu^{\rm the} = (283 \pm 87) \times 10^{-11}$$

$3.3\,\sigma$

(Höcker 2010) (theory-driven analysis)

$a_\mu^{\rm had,LO}[e] \quad = (692.3 \pm 1.4 \pm 3.1 \pm 2.4 \pm 0.2 \pm 0.3) \times 10^{-10}$ **(e^+e^- based),**

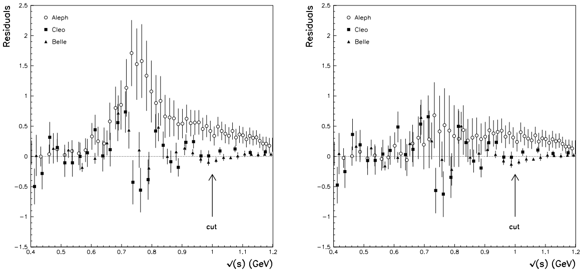
$a_\mu^{\rm had,LO}[e,\tau] = (701.5 \pm 3.5 \pm 1.9 \pm 2.4 \pm 0.2 \pm 0.3) \times 10^{-10}$ **(e^+e^-+τ based),**

- **Note: ratio $F_0(s)/F_-(s)$ could be measured within lattice QCD, without reference to sQED or other hadronic models.**
- **Including $\omega, \phi, \rho', \rho'', \cdots$ requires to go to appropriate Resonance Lagrangian extension (e.g HLS model (Benayoun et al.))**

Backup slides

The HLS model calculation of $F_\pi(e)$ and $F_\pi(\tau)$

Benayoun et al 09
includes $\rho - \gamma$ mixing as well. The Figure shows τ data vs. residual distribution in the fit of τ data: Left: BELLE+CLEO, Right: ALEPH+BELLE+CLEO (from Benayoun et al 09))



The model yields good simultaneous fits e^+e^-- and τ-data.

Our results in Tables

Isovector (I=1) contribution to $a_\mu^{\mathrm{had}} \times 10^{10}$ from the range [0.592 - 0.975] GeV from selected experiments. First entry: results from τ-data after standard isospin breaking (IB) corrections. Second entry: results from τ-data after applying in addition the $\rho-\gamma$ mixing corrections $r_{\rho\gamma}(s)$, with fitted values for M_ρ, Γ_ρ and $\Gamma_{\rho ee}$ [$M_\rho = 775.65\ MeV, \Gamma_\rho = 149.99\ MeV, \mathcal{B}[(\rho \to ee)/(\rho \to \pi\pi)] = 4.10 \times 10^{-5}$]. For the $\rho-\omega$ mixing we subtracted 2.67×10^{-10}. Errors are statistical, systematic, isospin breaking and $\rho-\gamma$ mixing, assuming a 10% uncertainty for the latter. Final state radiation is not included.

Data	standard IB corrections	incl. $\rho - \gamma$ mixing
ALEPH 1997	390.75(2.69)(1.97)(1.45)	385.63(2.65)(1.94)(1.43)(0.50)
ALEPH 2005	388.74(4.05)(2.10)(1.45)	383.54(4.00)(2.07)(1.43)(0.50)
OPAL 1999	380.25(7.36)(5.13)(1.45)	375.39(7.27)(5.06)(1.43)(0.50)
CLEO 2000	391.59(4.16)(6.81)(1.45)	386.61(4.11)(6.72)(1.43)(0.50)
BELLE 2008	394.67(0.53)(3.66)(1.45)	389.62(0.53)(3.66)(1.43)(0.50)
average	391.06(1.42)(1.47)(1.45)	385.96(1.40)(1.45)(1.43)(0.50)
CMD-2 2006		386.34(2.26)(2.65)
SND 2006		383.99(1.40)(4.99)
KLOE 2008		380.24(0.34)(3.27)
KLOE 2010		377.35(0.71)(3.50)
BABAR 2009		389.35(0.37)(2.00)
average		385.12(0.87)(2.18)
all e^+e^- data		385.21(0.18)(1.54)
$e^+e^- + \tau$		385.42 (0.53)(1.21)

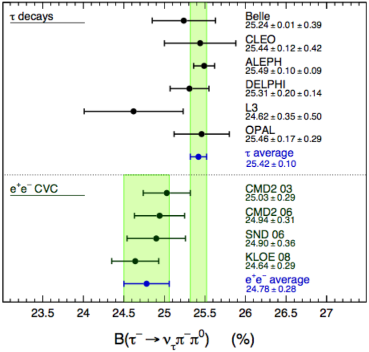
Calculated branching fractions in % from selected experiments. Experimental data completed down to threshold and up to m_τ by corresponding world averages where necessary. The experimental world average of direct branching fractions is $B^{\rm CVC}_{\pi\pi^0} = 25.51 \pm 0.09$ % .

τ data	$B_{\pi\pi^0}$[%]	e^+e^- data	$B^{\rm CVC}_{\pi\pi^0}$[%]
ALEPH 97	$25.27 \pm 0.17 \pm 0.13$	CMD-2 06	$25.40 \pm 0.21 \pm 0.28$
ALEPH 05	$25.40 \pm 0.10 \pm 0.09$	SND 06	$25.09 \pm 0.30 \pm 0.28$
OPAL 99	$25.17 \pm 0.17 \pm 0.29$	KLOE 08	$24.82 \pm 0.29 \pm 0.28$
CLEO 00	$25.28 \pm 0.12 \pm 0.42$	KLOE 10	$24.65 \pm 0.29 \pm 0.28$
Belle 08	$25.40 \pm 0.01 \pm 0.39$	BaBar 09	$25.45 \pm 0.18 \pm 0.28$
combined	$25.34 \pm 0.06 \pm 0.08$	combined	$25.20 \pm 0.17 \pm 0.28$

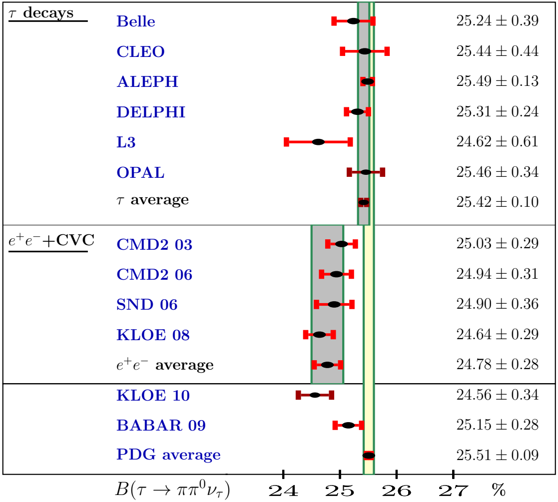
For the direct τ branching fractions the first error is statistical the second systematic. For e^+e^-+CVC the first error is experimental the second error includes uncertainties of the IB correction +0.06 from the new mixing effect. Remaining problems seem to be experimental.

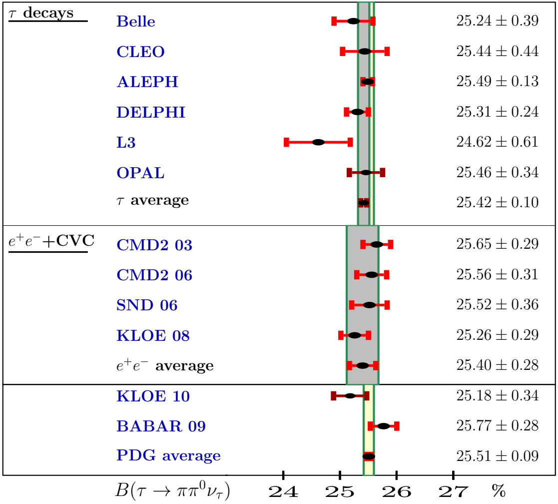
$B^{\mathrm{CVC}}_{\pi\pi^0}$: Most recent results of Davier et al

Pre BaBar:	**25.42 $\pm$ 0.10 %**			for τ
	24.78 $\pm$ 0.28 %	$\overset{+\rho\gamma}{\Rightarrow}$	**25.40 $\pm$ 0.28 $\pm$ 0.06 %**	for e^+e^- + **CVC**
New BaBar:	**25.15 $\pm$ 0.28 %**	$\overset{+\rho\gamma}{\Rightarrow}$	**25.77 $\pm$ 0.28 $\pm$ 0.06 %**	for e^+e^- + **CVC**



shift $\delta B^{\mathrm{CVC}}_{\pi\pi^0}[\rho\gamma] = +0.62 \pm 0.06$ **%**

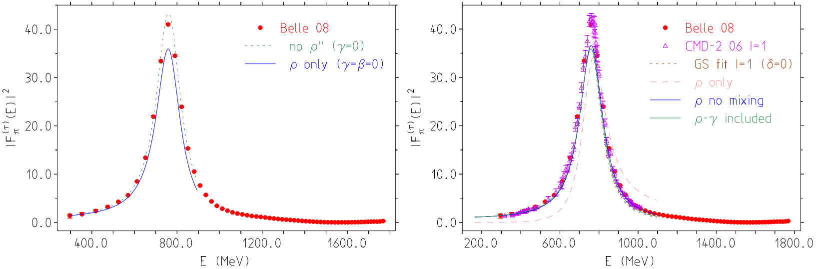

τ decays
Belle 25.24 ± 0.39
CLEO 25.44 ± 0.44
ALEPH 25.49 ± 0.13
DELPHI 25.31 ± 0.24
L3 24.62 ± 0.61
OPAL 25.46 ± 0.34
τ average 25.42 ± 0.10
e+e− +CVC
CMD2 03 25.03 ± 0.29
CMD2 06 24.94 ± 0.31
SND 06 24.90 ± 0.36
KLOE 08 24.64 ± 0.29
e+e− average 24.78 ± 0.28
KLOE 10 24.56 ± 0.34
BABAR 09 25.15 ± 0.28
PDG average 25.51 ± 0.09
B(τ → ππ0ντ)
24
25
26
27
%


τ decays
Belle
CLEO
ALEPH
DELPHI
L3
OPAL
τ average
e^+e^-+CVC
CMD2 03
CMD2 06
SND 06
KLOE 08
e^+e^- average
KLOE 10
BABAR 09
PDG average
25.24 ± 0.39
25.44 ± 0.44
25.49 ± 0.13
25.31 ± 0.24
24.62 ± 0.61
25.46 ± 0.34
25.42 ± 0.10
25.65 ± 0.29
25.56 ± 0.31
25.52 ± 0.36
25.26 ± 0.29
25.40 ± 0.28
25.18 ± 0.34
25.77 ± 0.28
25.51 ± 0.09
$B(\tau \to \pi\pi^0\nu_\tau)$
24
25
26
27
%

$\rho - \omega$ mixing

see our paper e-Print: arXiv:1101.2872 for an first attempt in the field theory approach. A complete treatment requires an extension of the model to include the $\omega \to \pi\pi\pi^0$ and $\omega \to \pi^0\gamma$ channel. (see from Benayoun et al 09)

Relation to data



Left: GS fits of the Belle data and the effects of including higher states ρ' and ρ'' at fixed M_ρ and Γ_ρ. Right: Effect of $\gamma - \rho$ mixing in our simple EFT model

Parameters: $M_\rho = 775.5\ MeV$, $\Gamma_\rho = 143.85\ MeV$, $\mathcal{B}[(\rho \to ee)/(\rho \to \pi\pi)] = 4.67 \times 10^{-5}$, $e = 0.302822$, $g_{\rho\pi\pi} = 5.92$, $g_{\rho ee} = 0.01826$.