# Hadronic Light-by-Light Scattering in Muon *g* – 2: from the INT Seattle HLbL Workshop

Fred Jegerlehner\* HU Berlin/DESY Zeuthen, fjeger@physik.hu-berlin.de

Working Group on Radiative Corrections and Generators for Low Energy Hadronic Cross Section and Luminosity Frascati, March 28-29, 2011

F. Jegerlehner

Radio MonteCarLow WG meeting, Frascati, 2011

## Abstract

A summary of the "INT Workshop on The Hadronic Light-by-Light Contribution to the Muon Anomaly" held at INT Seattle, February 28 - March 4, 2011. Goal of the WS was working out a "White Paper" in support of the new g - 2experiment planned (and approved) at Fermilab. Working groups:

Models for HLbL: de Rafael, Bijnens, Nyffeler, Vainshtein, and others

HLbL from lattice QCD: Blum, Jansen, Hashimoto, Kronfeld

Data for HLbL: J., Denig, Morriciani, Eidelman, Czyż

Original idea for the WS: previous attempts to get funding for an upgraded BNL muon g - 2 experiment failed also because experts raised doubts whether theory is able to predict HLbL for Muon g - 2. Lee Roberst spokesperson of E989 Collaboration got upset about HLbL theoreticians. David Herzog proposes INT WS to work out convincing evidence that theory is able to provide reliable estimates of  $a_{\mu}$ (HLbL). Decision for the shutdown of TEVATRON, changed situation for particle physics in US: muon g - 2 at Fermilab now first priority, second is support for super KEK B,...

Topics of Talk:

The hadronic LbL: setup and problems
Models and Controversies
Data constrain Models
A role for lattice QCD
Present & Future

The Good News first:

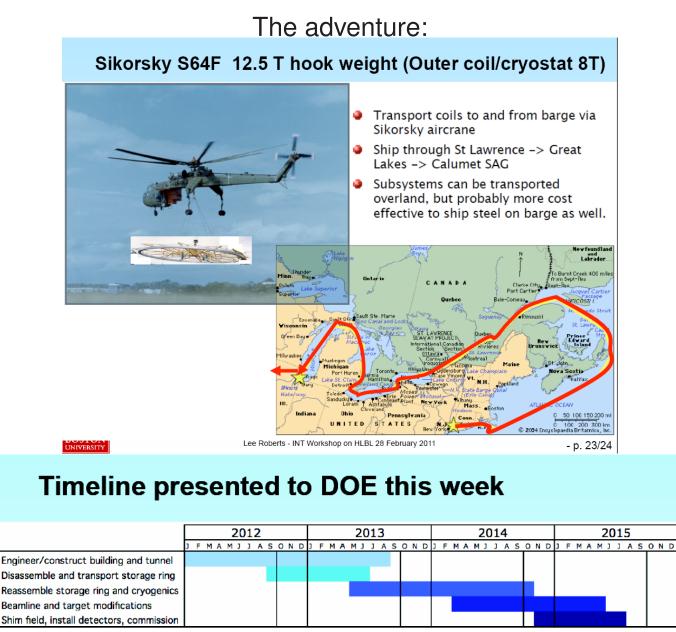
### Fermilab E989: Approved January 2011

- Re-locate the (g 2) storage ring to Fermilab
- Use the many proton storage rings to form the ideal proton beam
- Use one of the antiproton rings as a 900 m decay line to produce a pure muon beam
- Accumulate 21 times the statistics
- Improve the systematic errors
- Final goal: At least a factor of 4 more precise over E821



Lee Roberts - INT Workshop on HLBL 28 February 2011

- p. 22/24



F. Jegerlehner

Radio MonteCarLow WG meeting, Frascati, 2011

## On this timescale it's essential that the theory improve

- Lowest-order hadronic
  - BaBar and Belled have additional unanalyzed data
    - especially important for multihadron channels
  - VEPP2000 at Novosibirsk
    - CMD3
    - SND
- HLBL
  - Agreement among theorists and additional work
  - KLOE 2 photon physics
  - BES, Mainz



Lee Roberts - INT Workshop on HLBL 28 February 2011

- p. 25/25

## The new muon g - 2: Fermilab E989

 $\delta a_{\mu} = 16 \times 10^{-11}$  by 2015

•Magnetic field:  $\frac{\delta \langle B \rangle_{\mu}}{\langle B \rangle_{\mu}} \le 2 \times 10^{-8}$ 

Requires 10% error on HLbL

HLbL white paper in progress

Present:

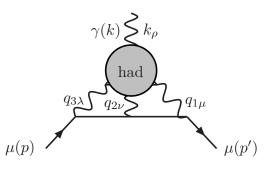
 $\Box a_{\mu}^{\exp} = 116\,592\,089(63) \times 10^{-11} ; \ a_{\mu}^{SM} = 116\,591\,793 \pm 51 \times 10^{-11}$ 

E989: statistics 21×; total error factor 4 more precise  $\sigma_{stat} = 0.1 \text{ ppm} \\ \sigma_{syst} = 0.1 \text{ ppm}$   $\sigma_{tot} = 0.14 \text{ ppm}$ 

 $\Box a_{\mu}^{\exp} = 11659x xxx(16) \times 10^{-11}$ 

## The hadronic LbL: setup and problems

Hadrons in  $\langle 0|T\{A^{\mu}(x_1)A^{\nu}(x_2)A^{\rho}(x_3)A^{\sigma}(x_4)\}|0\rangle$ 



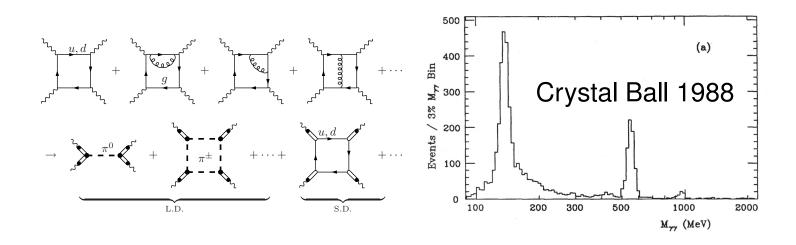
Key object full rank-four hadronic vacuum polarization tensor

$$\Pi_{\mu\nu\lambda\rho}(q_1, q_2, q_3) = \int d^4x_1 d^4x_2 d^4x_3 e^{i(q_1x_1+q_2x_2+q_3x_3)} \\ \times \langle 0 | T\{j_{\mu}(x_1)j_{\nu}(x_2)j_{\lambda}(x_3)j_{\rho}(0)\} | 0 \rangle$$

- non-perturbative physics
- general covariant decomposition involves 138 Lorentz structures of which
- ♦ 32 can contribute to g 2

- ✤ fortunately, dominated by the pseudoscalar exchanges  $\pi^0$ ,  $\eta$ ,  $\eta'$ , ... described by the effective Wess-Zumino Lagrangian
- generally, pQCD useful to evaluate the short distance (S.D.) tail
- off-shell form factors needed not directly accessible to experiment!
- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar Goldstone bosons as well as the vector mesons which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large N<sub>c</sub> inspired ansätze, and others

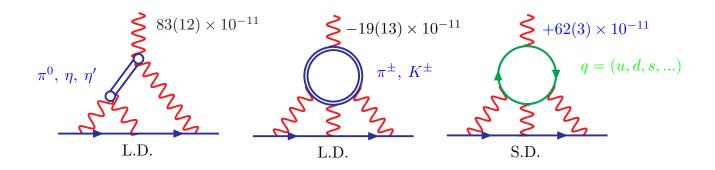
Need appropriate low energy effective theory  $\Rightarrow$  amount to calculate the following type diagrams



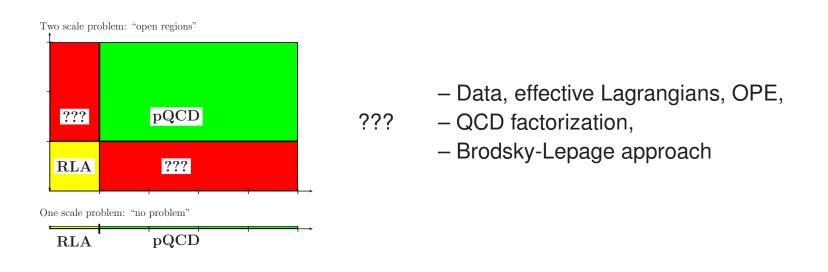
Data show almost background free spikes of the PS mesons! Substantial background form quark loop is absent (seems to contradict large quark-loop contribution as obtained in SDA). Clear message from data: fully non-perturbative, evidence for PS dominance. However, no information about axial mesons (Landau-Yang theorem). Illustrates how data can tell us where we are.

Low energy expansion in terms of hadronic components: theoretical models vs experimental data

➡ KLOE, KEDR, BES, BaBar, Belle, ?



LD requires low energy effective hadronic models: simplest case  $\pi^0 \gamma \gamma$  vertex Basic problem:  $(s, s_1, s_2)$ -domain of  $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$ ; here  $(0, s_1, s_2)$ -plane



## **Models and Controversies**

Low energy effective field theory Traditional approach: low energy effective Lagrangians: HLS, ENJL (resonance chiral theory) Kinoshita et al., Bijnens et al, matching and double counting problems

□ Large  $N_c$  QCD inspired approach Novel approach: refer to quark—hadron duality of large- $N_c$  QCD, hadron spectrum known, infinite series of narrow spin 1 resonances 't Hooft 79 ⇒ no matching problem (resonance representation has to match quark level representation) De Rafael 94, Knecht, Nyffeler 02

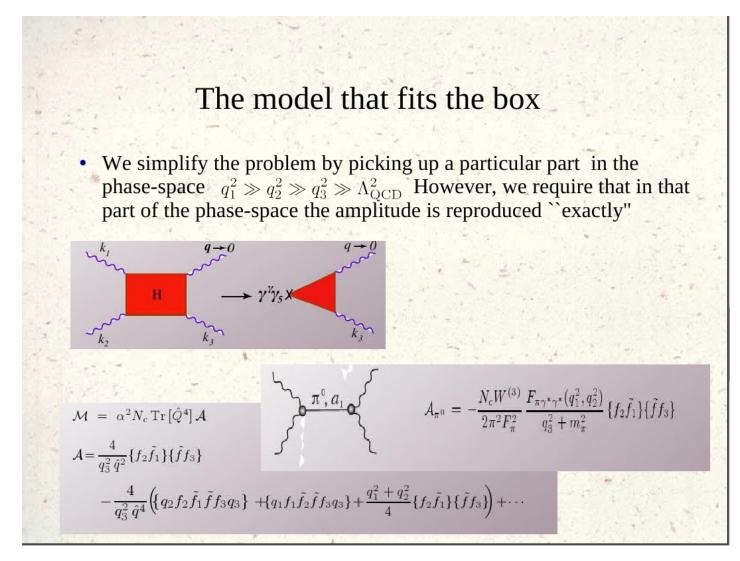
other new approaches:

HLbL from string theory

Cappiello, Catá, D' Ambrosio

 QCD based numeric Schwinger-Dyson/Bethe-Salpeter equations approach Goecke, Fischer, Williams

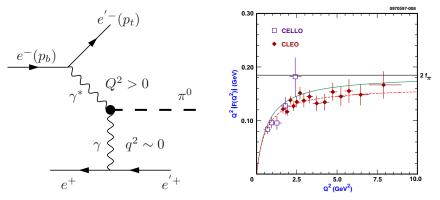
### The Melnikov-Vainshtein constraint and model



F. Jegerlehner

Constraints for on-shell pions (pion pole approximation)

- ★ The constant  $e^2 \mathcal{F}_{\pi^0 \gamma \gamma}(m_{\pi}^2, 0, 0) = \frac{e^2 N_c}{12\pi^2 f_{\pi}} = \frac{\alpha}{\pi f_{\pi}} \approx 0.025 \text{ GeV}^{-1}$  well determined by  $\pi^0 \rightarrow \gamma \gamma$  decay rate (from Wess-Zumino Lagrangian); experimental improvement needed!
- ♦ Information on  $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_{\pi}^2, -Q^2, 0)$  from  $e^+e^- \rightarrow e^+e^-\pi^0$  experiments



CELLO and CLEO measurement of the  $\pi^0$  form factor  $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_{\pi}^2, -Q^2, 0)$  at high space–like  $Q^2$ . outdated now by BABAR?

Brodsky–Lepage interpolating formula gives an acceptable fit.

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_{\pi}^2, -Q^2, 0) \simeq \frac{1}{4\pi^2 f_{\pi}} \frac{1}{1 + (Q^2/8\pi^2 f_{\pi}^2)} \sim \frac{2f_{\pi}}{Q^2}$$

Inspired by pion pole dominance idea this FF has been used mostly (HKS,BPP,KN) in the past, but has been criticized recently (MV and FJ07).

■ Melnikov, Vainshtein: in chiral limit vertex with external photon must be non-dressed! i.e. use  $\mathcal{F}_{\pi^0\gamma^*\gamma}(0,0,0)$ , which avoids eventual kinematic inconsistency, thus no VMD damping ⇒result increases by 30% !

□ In *g* – 2 external photon at zero momentum ⇒ only  $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$  not  $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_{\pi}^2, -Q^2, 0)$  is consistent with kinematics. Unfortunately, this off–shell form factor is not known and in fact not measurable and CELLO/CLEO constraint does not apply!. Obsolete far off-shell pion (in space-like region). Can we check such questions experimentally or in lattice QCD?

#### Present status:

#### **Pseudoscalar exchanges**

Model for $\mathcal{F}_{P^{(*)}\gamma^*\gamma^*}$	$a_\mu(\pi^0) imes 10^{11}$	$a_\mu(\pi^0,\eta,\eta') imes 10^{11}$
modified ENJL (off-shell) [BPP]	59(9)	85(13)
VMD / HLS (off-shell) [HKS,HK]	57(4)	83(6)
LMD+V (on-shell, $h_2=0$ ) [KN]	58(10)	83(12)
LMD+V (on-shell, $h_2=-10~{ m GeV}^2$ ) [KN]	63(10)	88(12)
LMD+V (on-shell, constant FF at ext. vertex) [MV]	77(7)	114(10)
nonlocal $\chi$ QM (off-shell) [DB]	65(2)	—
LMD+V (off-shell) [N]	72(12)	99(16)
AdS/QCD (off-shell ?) [HoK]	69	107
AdS/QCD/DIP (off-shell) [CCD]	65.4(2.5)	—
DSE (off-shell) [FGW]	58(7)	84(13)
[PdRV]	—	114(13)
[JN]	72(12)	99(16)

BPP = Bijnens, Pallante, Prades '95, '96, '02 (ENJL = Extended Nambu-Jona-Lasinio model); HK(S) = Hayakawa, Kinoshita, Sanda '95, '96; Hayakawa, Kinoshita '98, '02 (HLS = Hidden Local Symmetry model); KN = Knecht, Nyffeler '02; MV = Melnikov, Vainshtein '04; DB = Dorokhov, Broniowski '08 ( $\chi$ QM = Chiral Quark Model); N = Nyffeler '09; HoK = Hong, Kim '09; CCD = Cappiello, Catà, D'Ambrosio '10 (used AdS/QCD to fix parameters in DIP (D'Ambrosio, Isidori, Portolés) ansatz); FGW = Fischer, Goecke, Williams '10, '11 (Dyson-Schwinger equation) Reviews on LbyL: PdRV = Prades, de Rafael, Vainshtein '09; JN = Jegerlehner, Nyffeler '09

### A. Nyffeler, Seattle

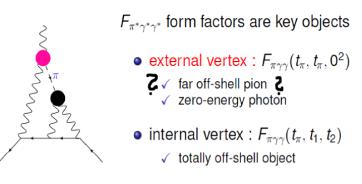
- p. 29

## **Data constrain Models**

Details in Seattle talks:<br/>Dario MorricianiKLCAchim DenigBallSimon EidelmanBellHenryk CzyżEKH

KLOE small angle tagger (low energy  $\pi^0 \gamma \gamma$ ) BaBar and BES results and plans Belle and KEDR results and plans [work in progress] EKHARA a Monte Carlo for  $\gamma^* \gamma^*$  physics

### Pion exchange in hadronic LbL

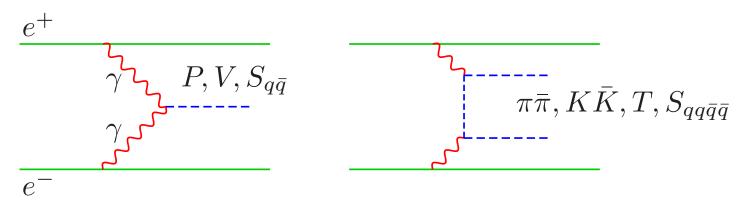


3

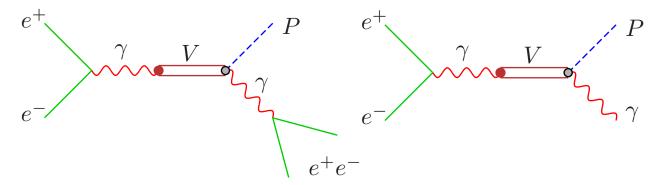
These form factors were never measured

H. Czyż, IF, UŚ, Katowice, EKHARA 2.0+ ...

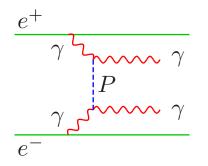
Overview (Eidelman incl progress report for Belle, KEDR):



mostly single-tag events: KLOE, KEDR (taggers), BaBar, Belle, BES III (high luminosity)



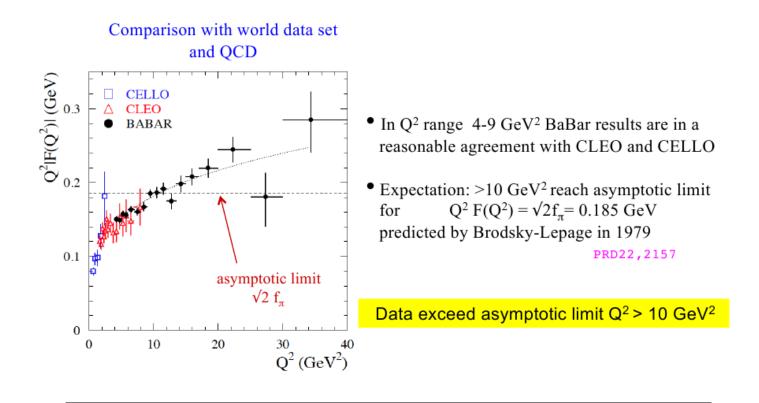
Dalitz-decays:  $\rho, \omega, \phi \to \pi^0(\eta) e^+ e^-$  Novosibirsk, NA60, JLab, Mainz, Bonn, Jülich, BES



would be interesting, but is buried in the background

### BaBar

## The $\pi^0$ Transition Form Factor

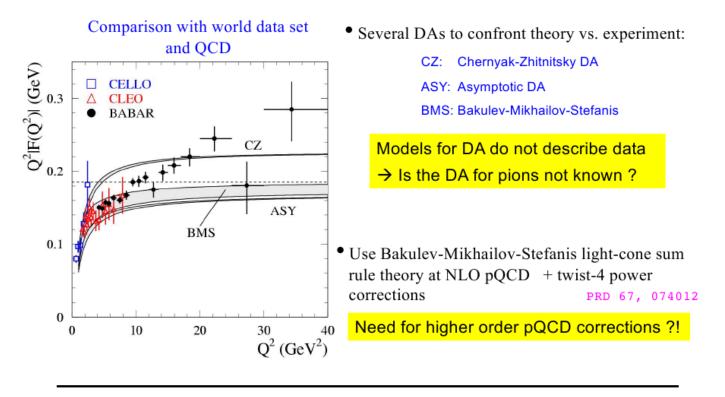


Achim Denig

Meson Transition FFs at BaBar

10

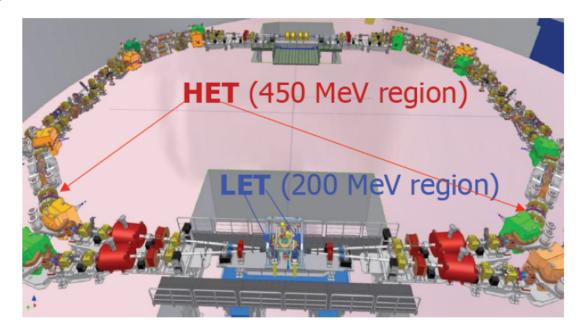
## The $\pi^0$ Transition Form Factor



Achim Denig

Meson Transition FFs at BaBar

## Start of new KLOE-2 experiment under way:KLOE-2 experiment $\gamma\gamma \rightarrow \pi^0, \eta, \cdots$

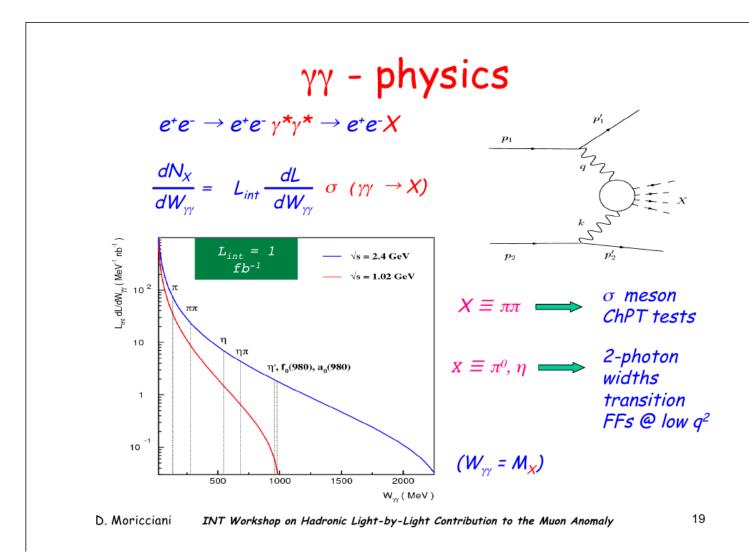


The  $\phi(1020)$  meson factory DA $\Phi$ NE(Frascati)+ KLOE detector + small angle taggers

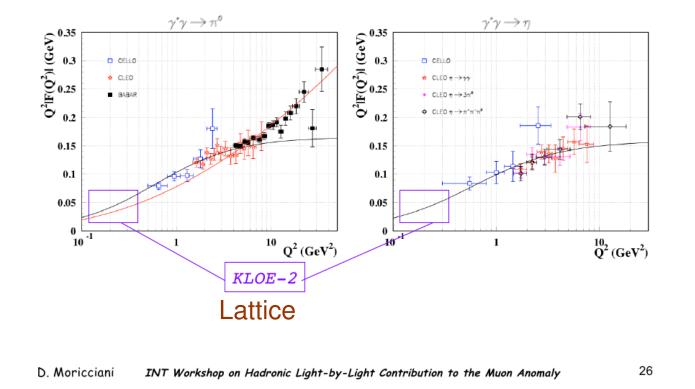
Sergiy IVASHYN (Katowice, Kharkov)

 $r^0\gamma\gamma$ 

21 / VI / 2010 @ Mainz 28 / 66



## KLOE-2 contribution ??



### Lattice (Shoij Hashimoto)

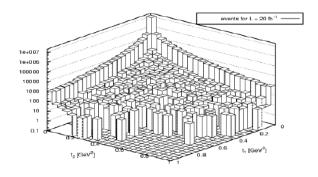
MC Simulation with EKHARA Generator

H. Czyż, S. Ivashyn et al. [http://prac.us.edu.pl/~ekhara]

Tagging:

- □ single tagging LET: tagged invariant  $t_1$  close to zero, promising range  $0.05 \text{ GeV}^2 < t_2 < 0.4 \text{ GeV}^2$
- LET-LET and LET-HET double tagging is not possible
- $\Box$  LET + central: promising range 0.18 GeV<sup>2</sup> <  $t_2$  < 0.4 GeV<sup>2</sup>
- $\Box$  single tagging HET: tagged invariant  $t_1$  close to zero  $\Rightarrow$   $t_2$  also close to zero
- HET-HET double tagging is possible but both photons quasi-real ⇒ good for measurement of  $\pi^0 \rightarrow \gamma \gamma$  width, pion practically at rest



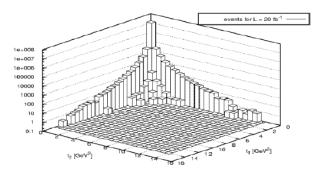


•  $\sqrt{s} = 3 \text{ GeV}$ ,  $\int \mathscr{L} dt = 20 \text{ fb}^{-1}$ (~ 9 months at  $\mathscr{L} = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ )

### Single-tag measurement at BES-III

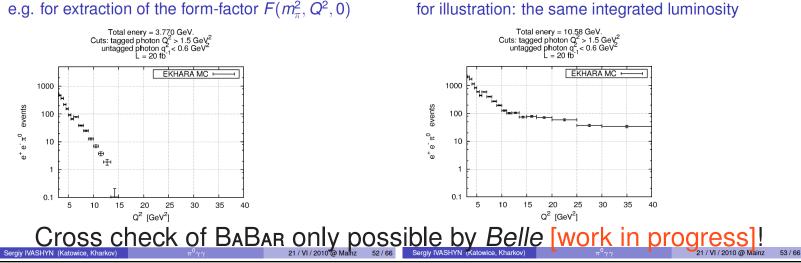
e.g. for extraction of the form-factor  $F(m_{\pi}^2, Q^2, 0)$ 





•  $\sqrt{s} = 3.770 \text{ GeV}, \quad \int \mathscr{L} dt = 20 \text{ fb}^{-|1|}$ (~ 9 months at  $\mathscr{L} = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ )

### cf. Single-tagging at BaBar energy



F. Jegerlehner

Radio MonteCarLow WG meeting, Frascati, 2011

## Axial exchanges: $a_1, f'_1, f_1$

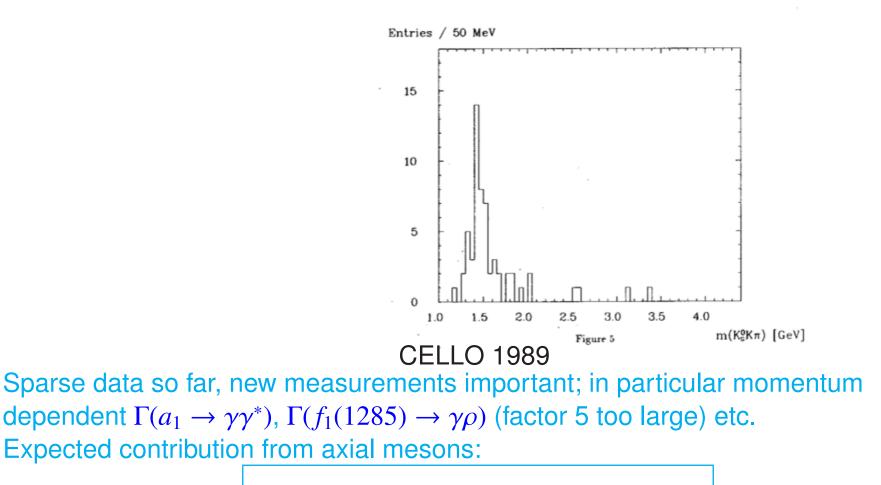
Axial exchanges Landau-Yang Theorem:  $\mathcal{A}$  (axial meson  $\rightarrow \gamma\gamma$ )=0

```
e.g. Z^0 \not\approx \gamma \gamma, while Z^0 \rightarrow \gamma e^+ e^- \checkmark
```

Why  $a_{\mu}[a_1, f'_1, f_1] \sim 25 \times 10^{-11}$  so large?

untagged \(\gamma\) y \(\rightarrow A\) no signal!
single-tag \(\gamma^\*\) y \(\rightarrow A\) strong peak is \(Q^2 >> m\_f^2\)

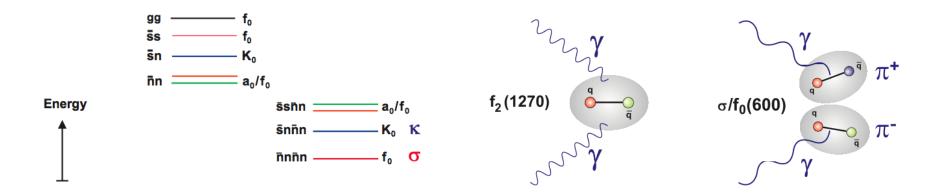
 $\sigma(\gamma^*\gamma \to f_1 \to K^0_{\rm s}K\pi)$ 

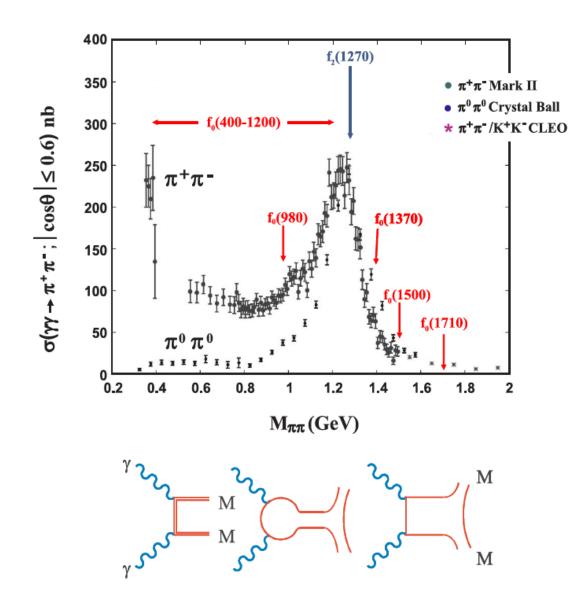


$$a_{\mu}[a_1, f'_1, f_1] \sim (28.13 \pm 5.63) \times 10^{-11}$$

## Scalar exchanges: $a_0, f'_0, f_0, \cdots$

Mesons:  $M(q\bar{q})$ ,  $M(qq\bar{q}\bar{q}\bar{q})$ , glueballs mixing Experimental: Crystal Ball, Mark II, Belle! Theory: Mennessier, Pennington et al., Mousallam et al., Achasov et al., ...





Strong tensor meson resonance in  $\pi\pi$  channel  $f_2(1270)$ 

So: expect usual pion-loop in HLbL plays role like pion-loop in VP. i.e. like missing the  $\rho$ .

Need to explicitly include tensor mesons

Scalars everywhere. Many scalars many small contributions may sum up to substantial effect!

Expected contribution from  $q\bar{q}$  scalars:

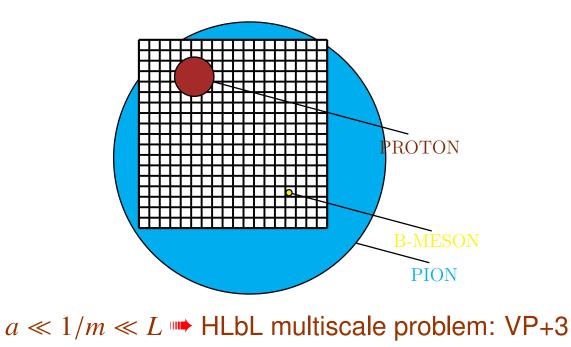
$$a_{\mu}[a_0, f'_0, f_0] \sim (-5.98 \pm 1.20) \times 10^{-11}$$

So far nobody has evaluated  $qq\bar{q}\bar{q}$  in SU(3) sector [u, d, s] many possible states, which individually are expected rather small

## A role for lattice QCD

• Hadronic contributions to g - 2 = integrals over physical cross sections or hadronic amplitudes

• in fact all can be represented as integrals over space-like "form-factors" • directly accessible to lattice QCD :  $\langle j_{em}^{\mu}(x_1) j_{em}^{\nu}(x_2) j_{em}^{\rho}(x_3) j_{em}^{\sigma}(x_4) \rangle$  or integral of it.



Hadronic LbL difficult, challenging long term project:

### HLbL Blum et al.

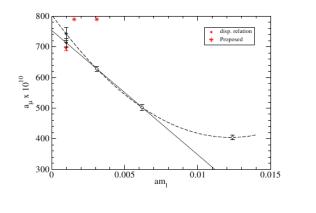
## Summary/Outlook: light-by-light contributions ( $O(\alpha^3)$ )

- Pure QED calculation on the lattice roughly reproduces the perturbative result. Encouraging.
- Full hadronic contribution is O(10<sup>2</sup>) times smaller, still swamped by the statistical noise
- Small volumes, poor statistics. Try
  - Volume (low-mode) averaging for the loop
  - Larger volumes
  - More statistics, i.e. more QED configurations per QCD configuration
  - conventional calculation using "all-to-all" propagator
- multi-quark loops not yet attempted

### The test case hadronic VP:

VP – Aubin & Blum:

The order  $\alpha^2$  hadronic contribution to g-2



Extrapolate  $m_l \rightarrow m_{u,d}$ 

Simple linear and quadratic chiral extrapolations consistent with  $e^+e^- \rightarrow {\rm hadrons}~{\rm result}$ 

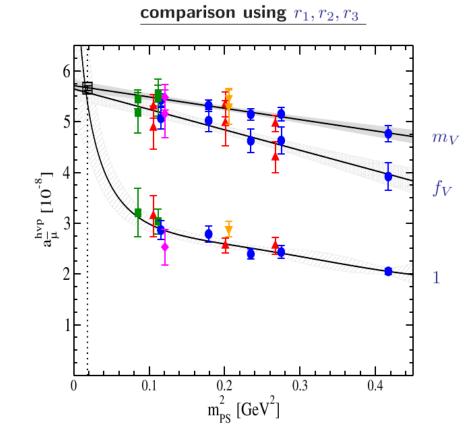
$$\begin{split} a_{\mu}^{HLO} &= (713 \pm 15) \times 10^{-10} \text{ (linear)} \\ a_{\mu}^{HLO} &= (742 \pm 21) \times 10^{-10} \text{ (quad)} \\ \text{(statistical errors only).} \end{split}$$

[Aubin, Blum, Phys. Rev. D, 2006]

Fit	quenched	$am_l = 0.0124$	$am_l = 0.0062$	$am_l = 0.0031$
Poly 3	381 (63)	370(49)	445(43)	542(24)
Poly 4	588 (142)	410(91)	639(123)	729(59)
А	366.6 (7.0)	412.3 (7.8)	516.0 (9.5)	646.9 (8.1)
В		403.9 (7.8)	502.1 (9.5)	628.0 (8.1)
С		403.9 (7.8)	502.1 (9.5)	628.0 (8.1)

23

The problem of extrapolation: VP Jansen et al



 $a_{\mu}$  direct, as a function of  $M_{\rho}$  and of  $f_{\rho}$ 

### Some preliminary numbers

- experimental value:  $a_{\mu,N_f=2}^{\mathrm{hvp,exp}} = 5.66(05)10^{-8}$
- from our old analysis:  $a_{\mu,N_f=2}^{\rm hvp,old}=2.95(45)10^{-8}$
- $\rightarrow$  misses the experimental value
- $\rightarrow~$  order of magnitude larger error
- from our new analysis:  $a_{\mu,N_f=2}^{\text{hvp,new}} = 5.66(11)10^{-8}$
- $\rightarrow~$  error (including systematics) almost matching experiment

### looks like very promising progress!

## **Present & Future**

**D** Role of Melnikov-Vainshtein constraint still under debate (is virtual photon dressed or undressed at external  $\pi^0 \gamma \gamma$  vertex?)

Role of quark loop: is it an independent contribution? solving Schwinger-Dyson equation approach yields very large value.

□ Large  $Q^2$  behavior of  $\mathcal{F}_{\pi^{0*}\gamma\gamma^*}(m_{\pi}^2, -Q^2, 0)$  from BaBar shows much weaker fall-off than expected by theory

□ New muon g - 2 experiment is on its way !!!

Need to improve accuracy for the hadronic light-by-light contribution.

• New input form  $\gamma\gamma$  physics to constrain theoretical models for HLbL (KLOE-2,BES,MAINZ)

• Challenge for theory: radiative corrections needed

• Question of asymptotic behavior seen by BaBar, will likely be settled by Belle

Can we check controversial dressed/undressed (i.e damping or not?) at external vertex? Can Primakoff-effect plus DR help?

Lattice QCD makes big progress: we may expect relevant results for constraining models

□ Not to forget: urgent improvement of VP mandatory [lattice QCD may become competitive] (Novosibirsk: CMD3, SND, unanalised data from BaBar & Belle?)

Viva g = 2! Let's go to work!