Status of $g-2$ and α_{em}

University of Liverpool

Thomas Teubner

Liverpool Theoretical Physics

I. $(g-2)_\mu$: Introduction

II. Recent developments in $(g-2)_\mu$; Hadronic Vacuum Polarisation contributions

- 2π: KLOE 2008 and 2010, BaBar 2009 analyses
- Radiative Return data below 2 GeV
- New HLMNT compilation; comparison with other groups; SM vs. BNL

III. $\Delta\alpha(q^2)$: Running QED coupling; $\alpha(M_Z^2)$

IV. Outlook

Thanks to my collaborators Kaoru Hagiwara, Ruofan Liao, Alan Martin and Daisuke Nomura

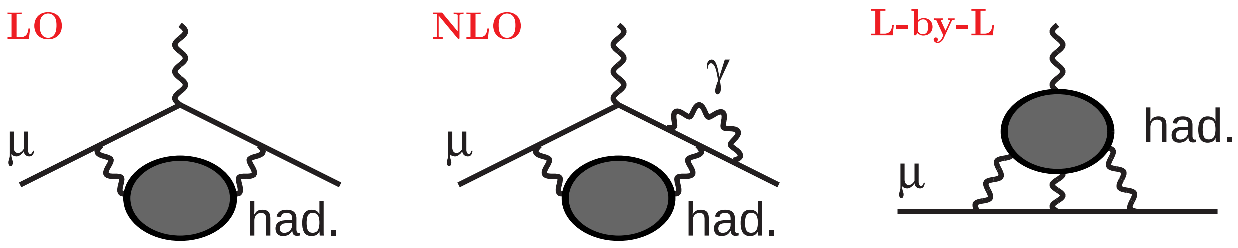
I. $(g-2)_\mu$: Introduction

- $a_\mu \;=\; (g-2)_\mu/2 \;=\; a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had} \;+ a_\mu^{\rm New\ Physics?}$
- QED: Predictions consolidated, further work (numerical five-loop) ongoing, big surprises very unprobable, error formidably small: $a_\mu^{\rm QED} = 116584718.08(15) \cdot 10^{-11}$ ✓

  Kinoshita et al.
- EW: reliable two-loop predictions, accuracy fully sufficient: $a_\mu^{\rm EW} = (154 \pm 2) \cdot 10^{-11}$ ✓

  Czarnecki et al., Knecht et al.
- Hadronic contributions: uncertainties completely dominate $\Delta a_\mu^{\rm SM}$!

$$a_\mu^{\rm had} \;=\; a_\mu^{\rm had,VP\ LO} \;+\; a_\mu^{\rm had,VP\ NLO} \;+\; a_\mu^{\rm had,Light-by-Light}$$



▶ Hadronic contributions from low γ virtualities not calculable with perturbative QCD

— Lattice simulations difficult; promising first steps, but accuracy not (yet?) sufficient

- **Light-by-Light:**
  - No dispersion relation for L-by-L. *First Principles* calculations from lattice QCD are underway by two groups: QCDSF and T Blum et al. Both approaches promising but at an early stage and no results yet.
    Also first results based on Dyson-Schwinger eqs. by C Fischer et al.
  - 'Convergence' of different recent model calculations. HMNLT numbers below use compilation from J Prades, E de Rafael, A Vainshtein: $a_\mu^{\rm L-by-L} = (10.5 \pm 2.6) \cdot 10^{-10}$
  - Compatible recent result from F Jegerlehner, A Nyffeler: $a_\mu^{\rm L-by-L} = (11.6 \pm 4.0) \cdot 10^{-10}$

→ For more details and latest news see talks by Fred Jegerlehner and Simon Eidelman.

- **Vacuum Polarisation** contributions from exp. $\boldsymbol{\sigma(e^+e^- \to \gamma^* \to hadrons)}$ data
  or from $\tau \to \nu_\tau + hadrons$ spectral functions; isospin breaking?! → talks by Robert Szafron and FJ
  via *dispersion integral* (based on analyticity and unitarity):

$$a_\mu^{\rm had,VP\ LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma_{\rm had}^0(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

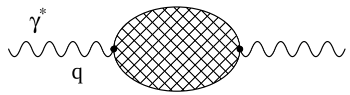
→ Kernel K ⇝ weighting towards smallest energies. $\sigma^0_{\rm had}$ the undressed cross section

→ Similar approach with different kernel functions for NLO VP contributions $a_\mu^{\rm had,VP\ NLO}$

II. Recent developments in $(g-2)_\mu$; Hadronic VP contributions

▶ Compilation of $\sigma^0_{\rm had}(s)$

- For low energies, need to sum ~ 25 exclusive channels. $[2\pi,\ 3\pi,\ KK,\ 4\pi,\ \ldots]$
- $1.43 - 2$ GeV: sum exclusive channels and/or use old inclusive data
- above ~ 2 GeV: inclusive data *or* use of perturbative QCD.
- In each channel: Data combination from many experiments, non-trivial w.r.t. error analysis/correlations/different energy ranges.
  [Different methods/machinery used by different groups.]
- Note: $\sigma^0(s)$ must be the *undressed* hadronic cross section (i.e. photon VP *subtracted* $[\sigma^0(s) = \sigma(s)\cdot(\alpha/\alpha(s))^2]$, otherwise double-counting with $a_\mu^{\rm had,VP\ NLO}$)



- but must *include final state photon radiation*.

⇝ Uncertainty in treatment of radiative corrections, especially for older data sets!
Assign additional error.   HLMNT: $\delta a_\mu^{\rm had,VP+FSR} \simeq 2\times 10^{-10}$ $[\sim 10\cdot \Delta a_\mu^{\rm EW}]$

▶ Most important channels with changes in input data since ∼2006

The main exps. for 'low' energy hadronic cross sections in e^+e^-; channels

– CMD-2, [VEPP-2M], Novosibirsk (K^+K^-, $2\pi^+2\pi^-\pi^0$, $2\pi^+2\pi^-2\pi^0$)

– SND, [VEPP-2M], Novosibirsk (K^+K^-, $K_S^0K_L^0$)

– KLOE, [DAΦNE], Frascati ($\pi^+\pi^-(\gamma)$, $\omega\pi^0$)

– BaBar, [PEP-II], SLAC, Stanford ($\pi^+\pi^-(\gamma)$, $K^+K^-\pi^0$, $K_S^0\pi K$, $2\pi^+2\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-\eta$, $2\pi^+2\pi^-2\pi^0$, $KK\pi\pi$, $K^+K^-K^+K^-$)

– BELLE, [KEKB], KEK, Tsukuba

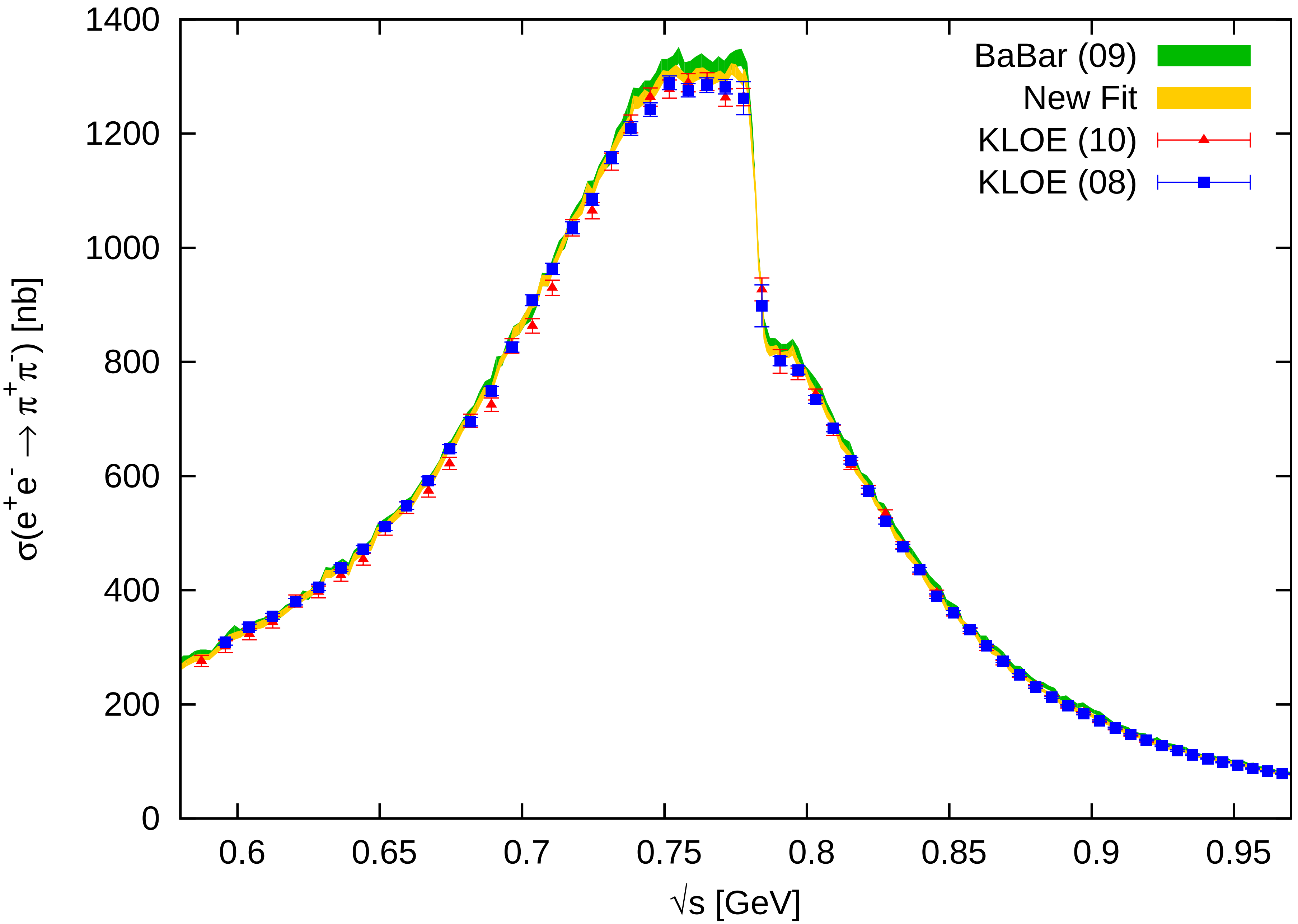
– BES, [BEPC], Beijing (inclusive $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ data)

– CLEO, [CESR], Cornell (inclusive R)

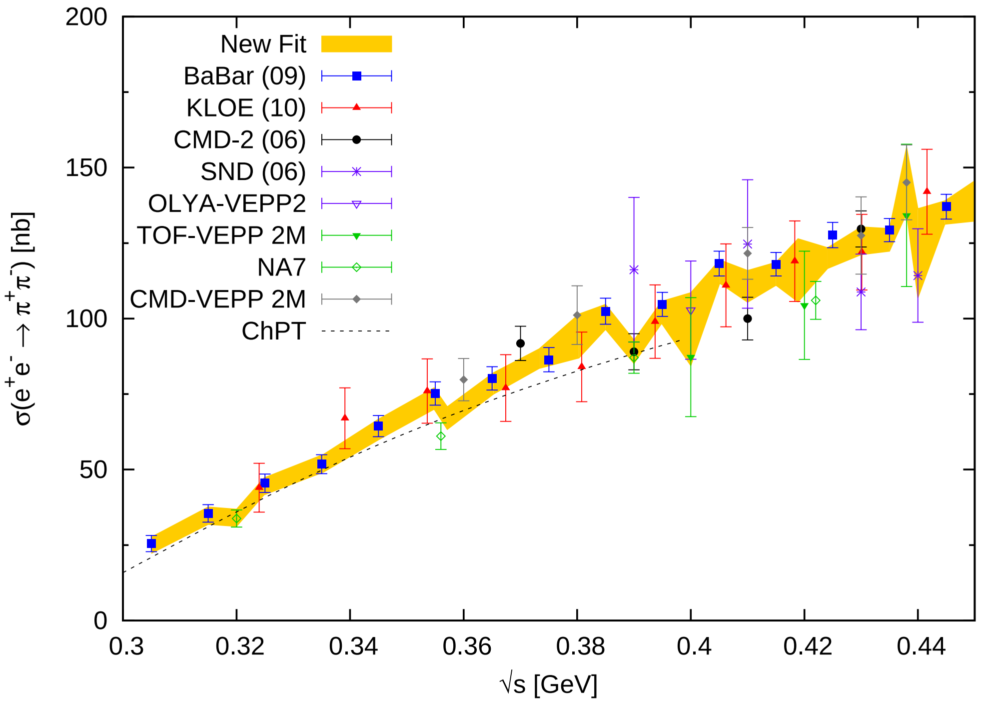
- In principle inclusion of new data in updated analyses straightforward...

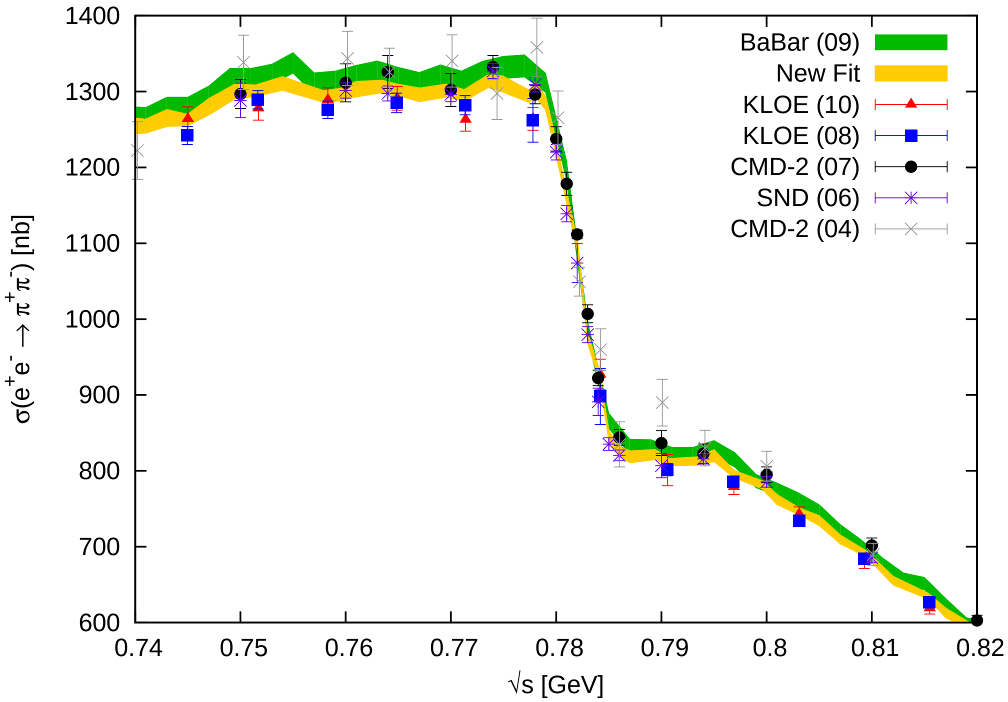
  Concentrate on two cases where not: most important $\mathbf{2\pi}$ and the $\mathbf{1.43 - 2}$ GeV region.

► The most important 2π channel ($> 70\%$) 879 data points, overall picture fine



Zoom in low energy (2π threshold) and ρ-peak / ρ-ω interference region



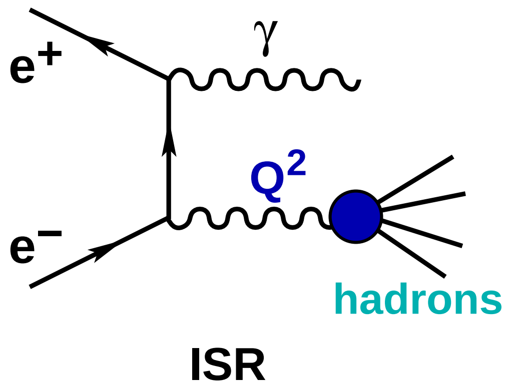


- 'Direct Scan': Very good agreement between data from CMD-2 and SND, fully consistent with earlier data.
- Low energy points crucial for recent improvements of $a_\mu^{\pi\pi}$.
- 'Radiative Return': *KLOE* and *BaBar* show slight tension with the Direct Scan data, and with each other;

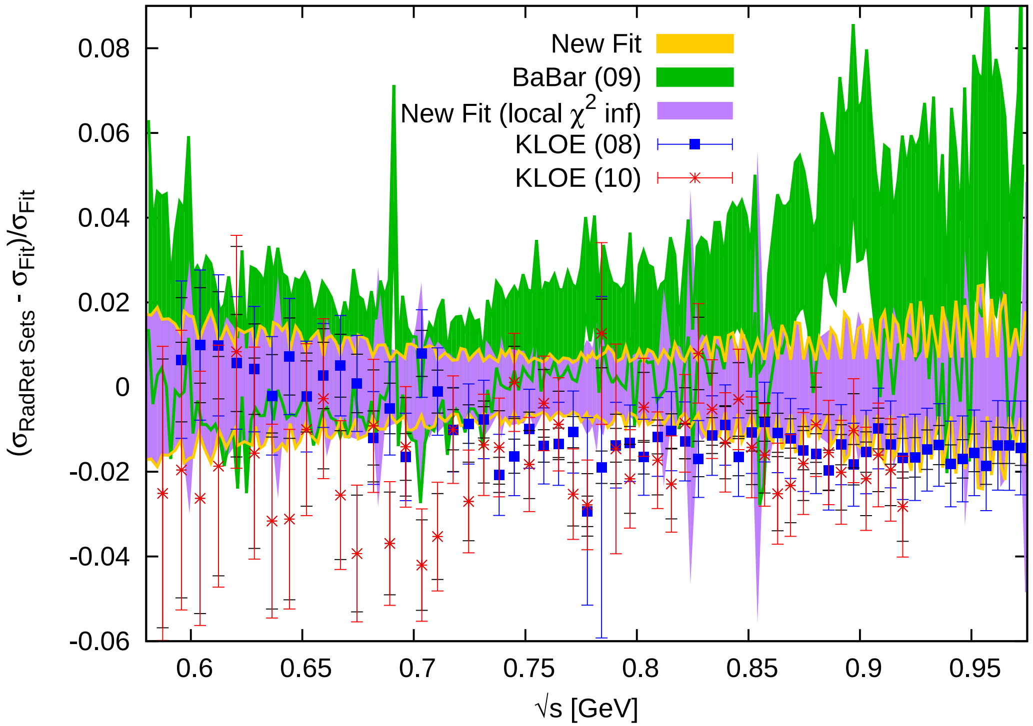
$\longrightarrow$ Differences in shape and BaBar high at medium and higher energies:

KLOE 08/10 and BaBar 09 $\pi\pi(\gamma)$ Radiative Return data compared to combination of all

Radiative Return (at fixed e^+e^- energy) has recently developed (TH + EXP) into a powerful method with great potential, *complementary to direct energy scan*



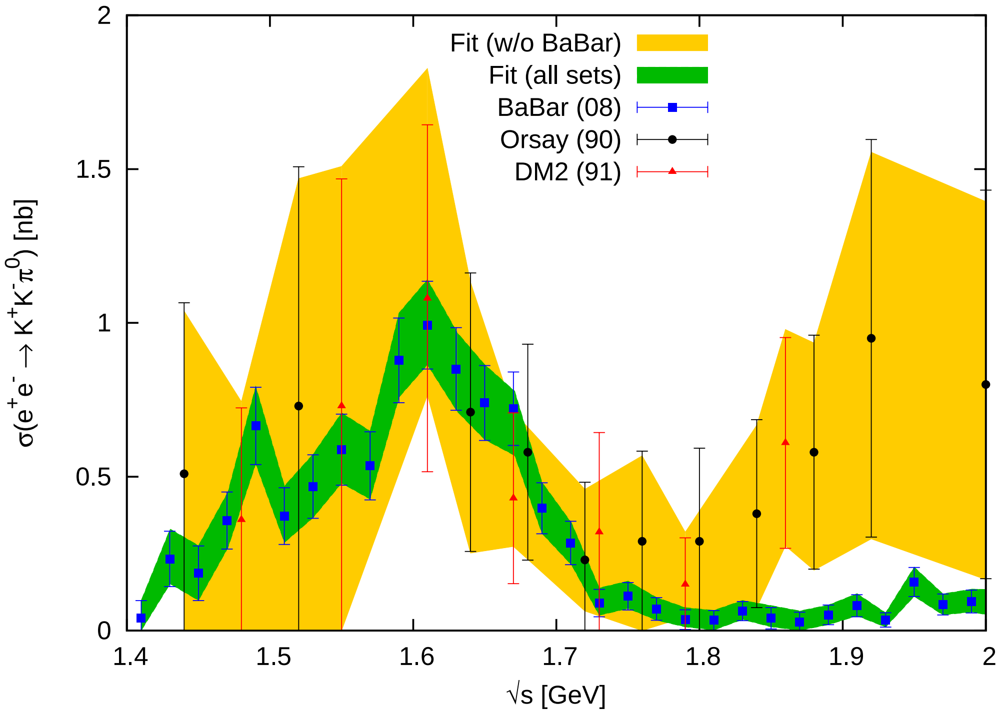
Normalised diff. of cross sections [HLMNT]



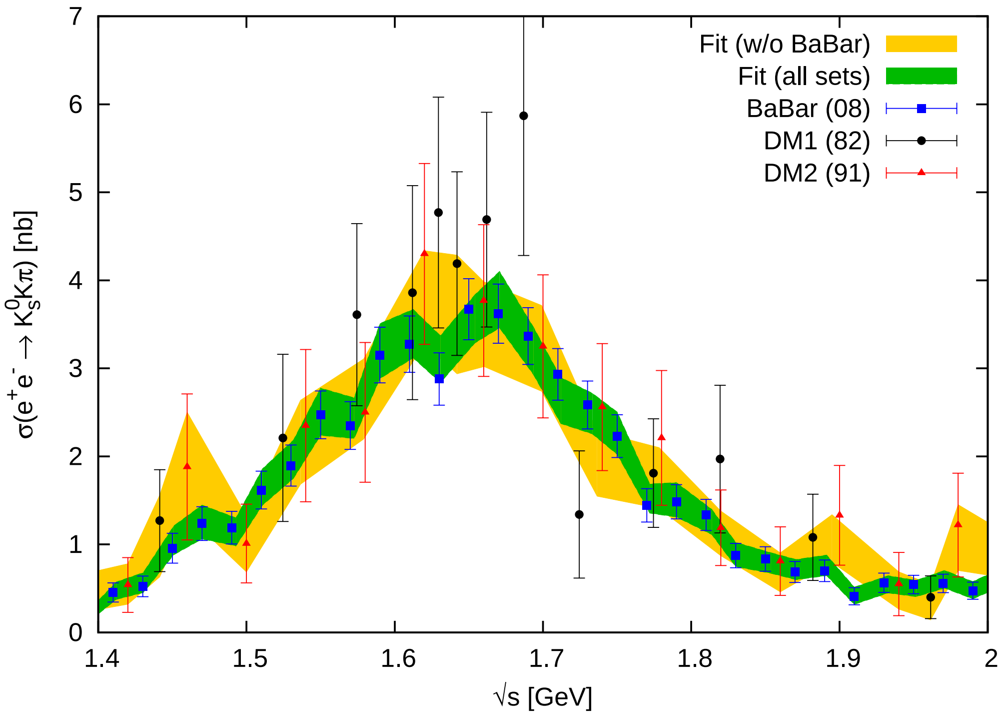
- Method used first by 'meson factories', where high statistics compensates α/π suppression of γ radiation.
- Results for 2π channel slightly different in shape, but completely different method, Monte Carlos etc.
- Comb. of all data on same footing, before integration (purple band): still good $\chi^2_{\rm min}/{\rm d.o.f.} \sim 1.5$ of fit]
  $\rightsquigarrow$ limited gain in accuracy due to 'tension'; pull-up (mainly from BaBar):
  HLMNT 10: $a_\mu^{2\pi}(0.32 - 2\,\text{GeV}) = (504.23 \pm 2.97) \cdot 10^{-10}$ [pull a_μ up by ~ 5.5 units]

▶ Region below 2 GeV: influence of recent BaBar Radiative Return analyses

$K^+K^-\pi^0$ channel



$K^0_S K\pi$ channel

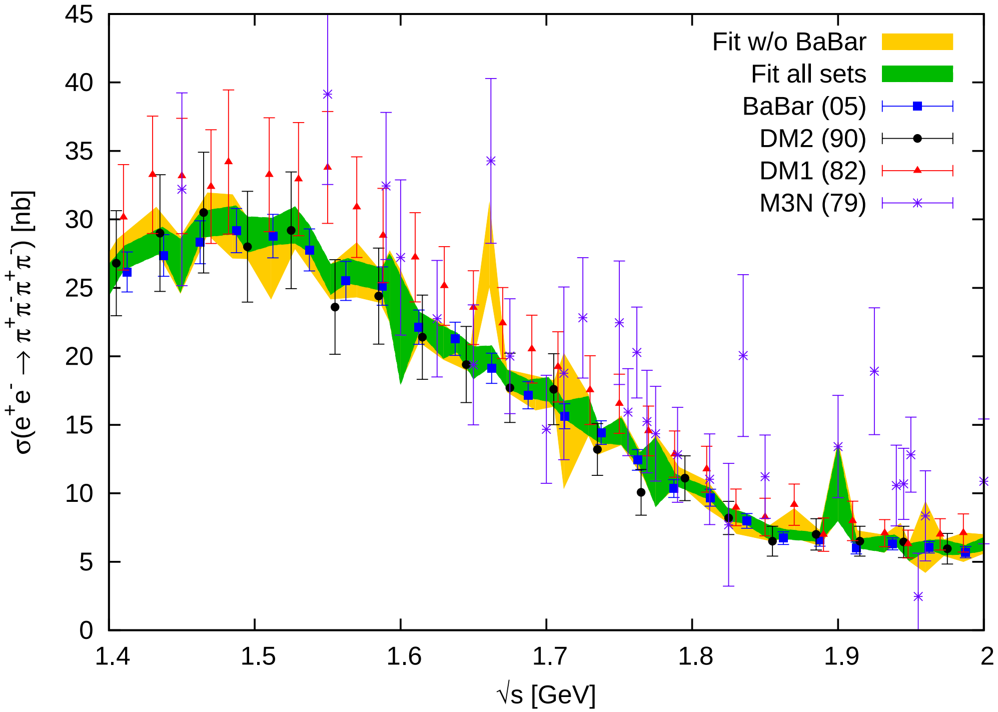


⟶ Big improvements over earlier data compilations in many channels.

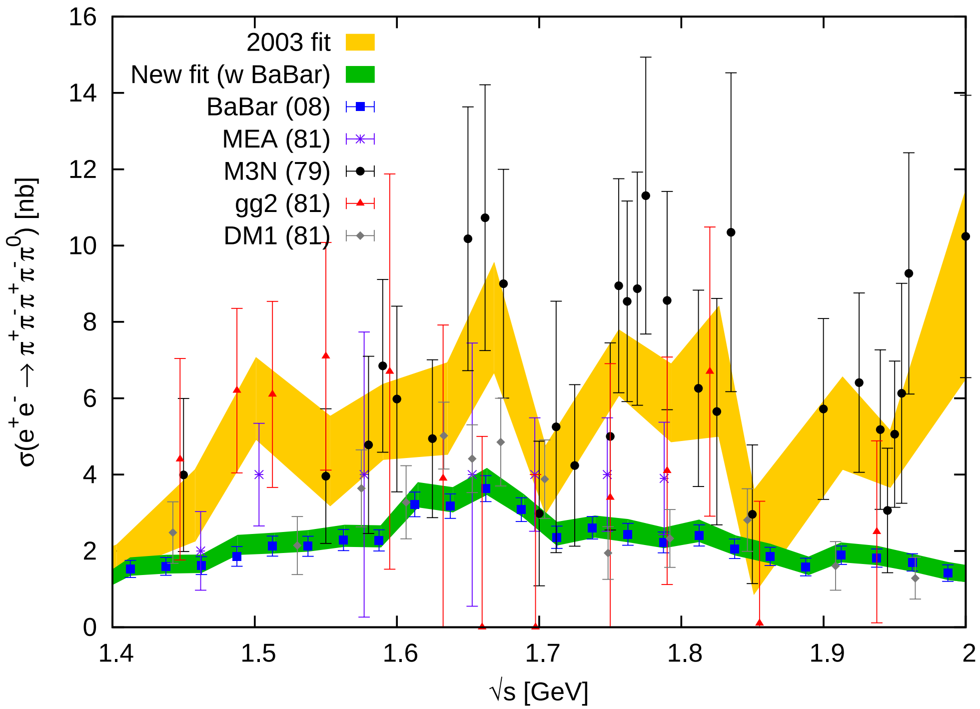
BaBar Radiative Return data lower than less precise older data in most channels.

▶ Region below 2 GeV: influence of recent BaBar Radiative Return analyses (contd)

$2\pi^+2\pi^-$ channel



$2\pi^+2\pi^-\pi^0$ channel

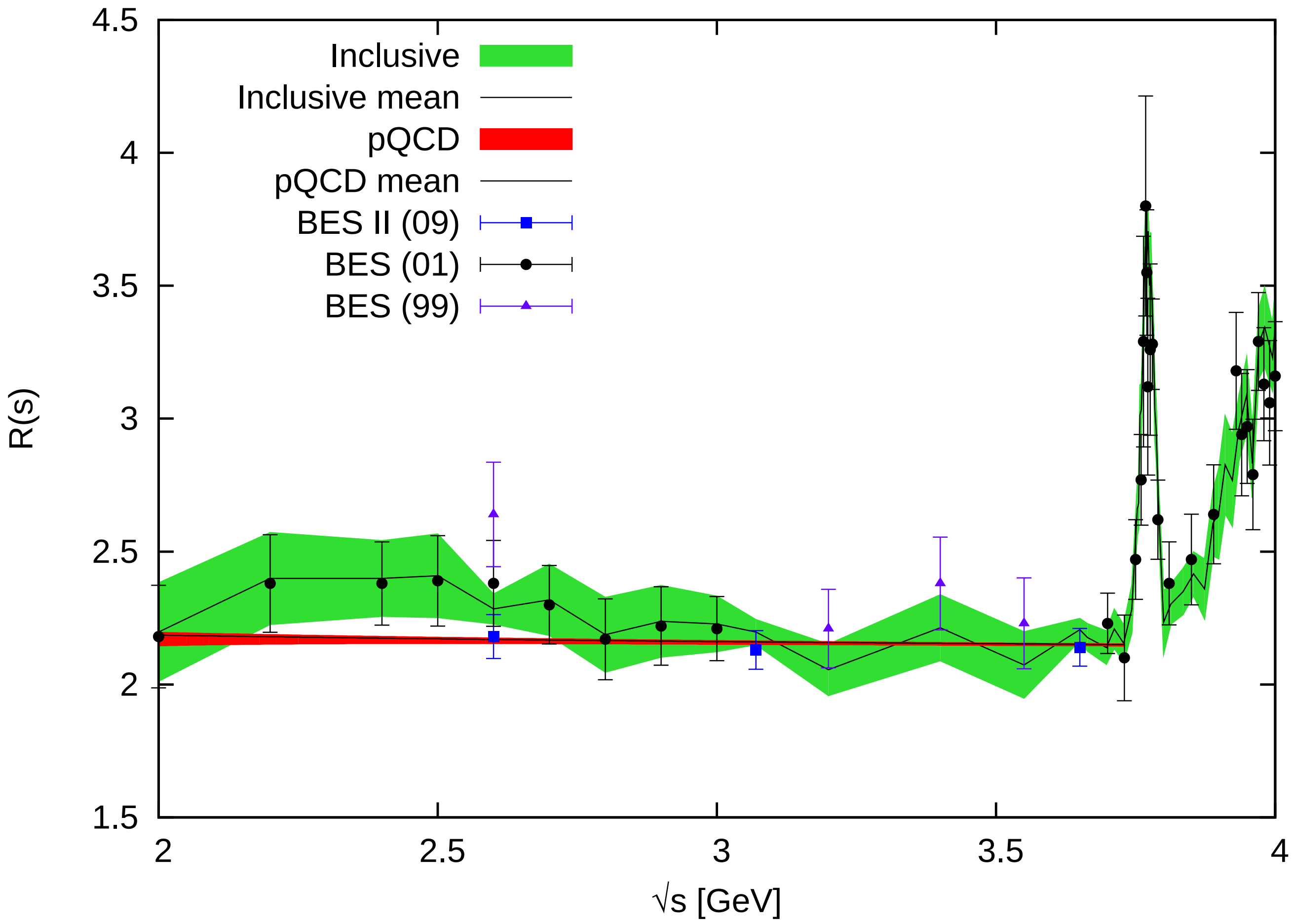


$\longrightarrow$ Error 'inflation' needed when data inconsistent,

e.g. BaBar lower than previous measurements in $2\pi^+2\pi^-\pi^0$ channel

$\leadsto$ HLMNT: Errors for $g-2$ inflated by local $\sqrt{\chi^2_{\rm min}/{\rm d.o.f.}}$ [global $\chi^2_{\rm min}/{\rm d.o.f.} = 1.4$]

Perturbative QCD vs. inclusive data above 2 GeV (below charm threshold)



- Latest BES data agree very well with pQCD [Davier et al. use pQCD from 1.8 GeV]
- R_{uds} from pQCD mostly below data fit in region above 2 GeV
- HLMNT use pQCD only for $2.6 < \sqrt{s} < 3.7$ GeV and with (larger) BES errors
  [would have small shift downwards ($\sim -1.4 \cdot 10^{-10}$ for a_μ) if used from 2 GeV]

The different SM contributions numerically

HLMNT 11 (prel.)

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	$116\,584\,718.08 \pm 0.15$	up to 5-loop (Kinoshita+Nio, Passera)
	(was $116\,584\,719.35 \pm 1.43$)	▶ incl. recent updates of α
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		(agrees very well with Knecht+Peris+Perrottet+deRafael)
LO hadr.	$\mathbf{6923 \pm 42}$	Davier *et al.* '10 (e^+e^-)
	$\mathbf{6908 \pm 47}$	F Jegerlehner + R Szafron '11 (e^+e^-)
	$\mathbf{6894 \pm 42 \pm 18}$	Hagiwara+Martin+Nomura+T '06
new:	$\mathbf{6954 \pm 37 \pm 21}$	HLMNT 11 (prel.), this analysis, comb. error 43
NLO hadr.	$\mathbf{-98.5 \pm 0.6 \pm 0.4}$	HLMNT, in agreem. with Krause '97, Alemany+D+H '98
L-by-L	105 ± 26	▶ Prades+deRafael+Vainshtein
agrees with	< 159 (95% CL)	upper bound from Erler+Toledo Sánchez from PHD
< Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \to 1.6\,\sigma$
$\sum$	$\mathbf{116591830 \pm 49}$	HLMNT 11 (prel.)

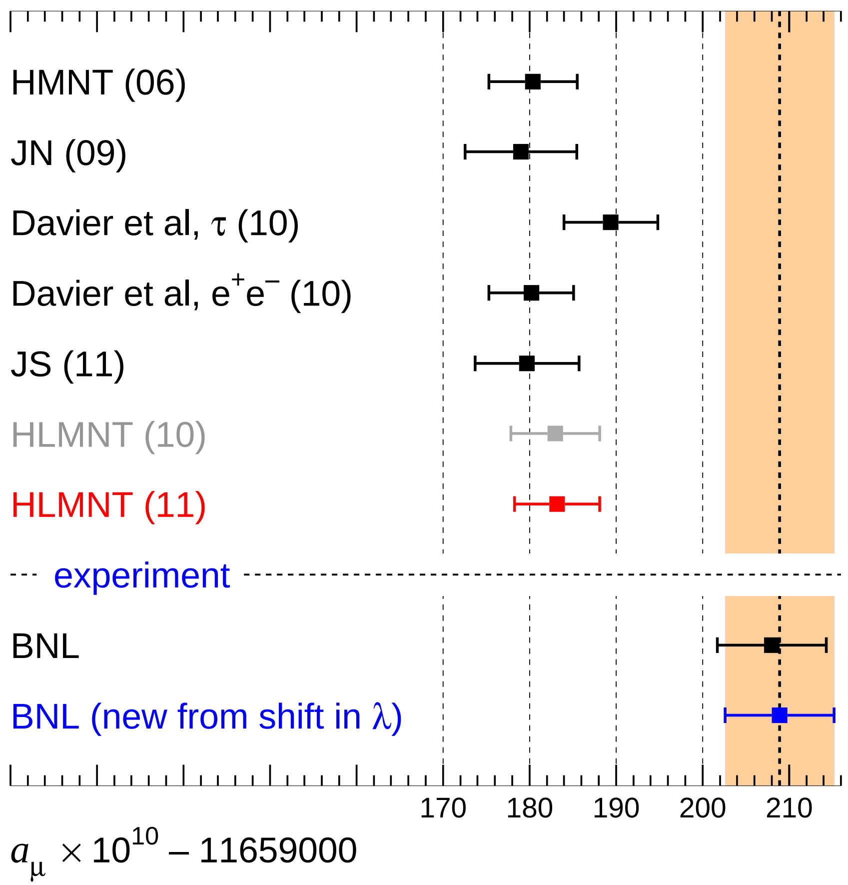
The theory prediction of $g-2$ is now slightly more precise than the BNL measurement

SM vs BNL: A sign for New Physics?

Covered storage ring (Pic. from the g–2 Collab.)

Various choices w.r.t. data, way to compile, τ (?!), L-by-L: ALWAYS $a_\mu^{\rm SM} < a_\mu^{\rm EXP}$

$a_\mu^{\rm SM}$ compared to BNL world av.



Davier et al.: $1.9/3.9/3.2\,\sigma$, '10: $3.6\,\sigma$

JN 09: $3.2\,\sigma$ [179.0 ± 6.5], JS '11: $3.3\,\sigma$

HLMNT 09: was $4.0\,\sigma$ [w/out BaBar 09 2π]

Recent changes

TH: Updated/improved LO hadronic (from e^+e^-)

[Many new data from CMD-2, SND, KLOE, BaBar, CLEO, BES. Excl. data below 2 GeV (BaBar RadRet)]

$(6894 \pm 46) \cdot 10^{-11} \longrightarrow (6954 \pm 43) \cdot 10^{-11}$

TH: Use of recent L-by-L compilation [PdeRV] $a_\mu^{\rm L-by-L} = (10.5 \pm 2.6) \cdot 10^{-10}$

EXP: Small shift of BNL's value due to CODATA's shift of muon to proton magn. moment ratio:

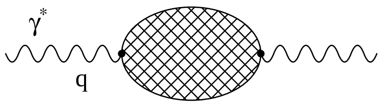
Was $a_\mu = 116\,592\,080(63) \times 10^{-11}$

$\rightarrow a_\mu = 116\,592\,089(63) \times 10^{-11}$ $(0.5ppm)$

► With this input HLMNT (prel. '11 ~ '10) get

$a_\mu^{\rm EXP} - a_\mu^{\rm TH} = (25.7 \pm 8.0) \cdot 10^{-10}$, $\sim \mathbf{3.2\sigma}$

III. The running QED coupling $\alpha(q^2)$ ... and the Higgs mass



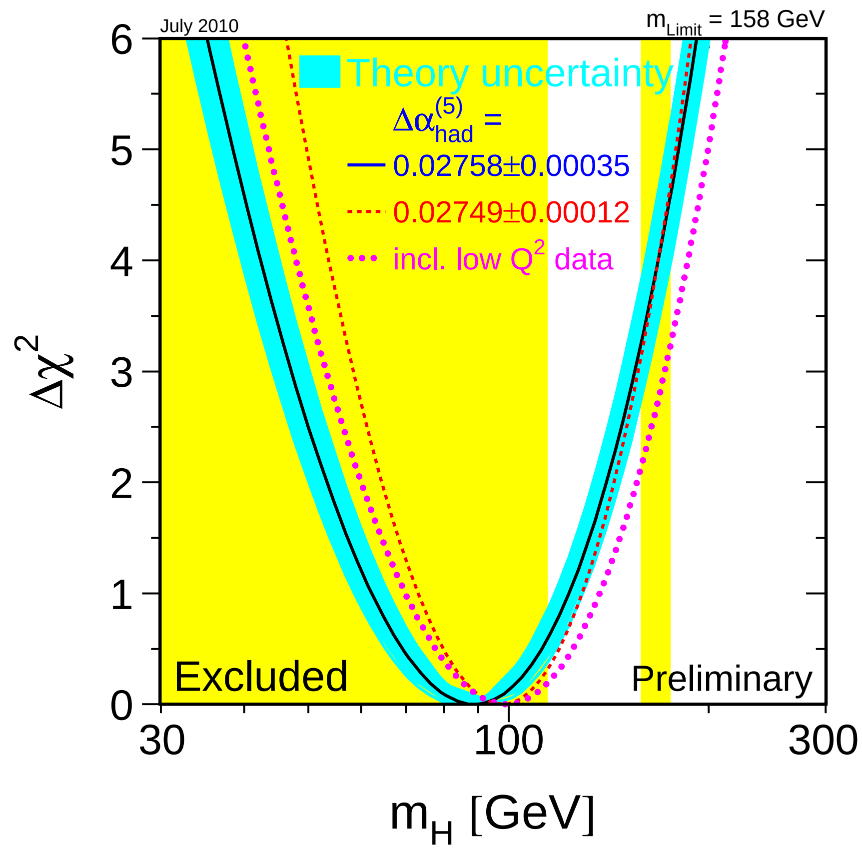
- Vacuum polarisation leads to the 'running' of α from $\alpha(q^2=0) = 1/137.035999084(51)$ to $\alpha(q^2 = M_Z^2) \sim 1/129$
- $\alpha(q^2) = \alpha / \left(1 - \Delta\alpha_{\rm lep}(q^2) - \Delta\alpha_{\rm had}(q^2)\right)$
- Again use of a dispersion relation: $\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{\alpha q^2}{3\pi} P \int_{s_{\rm th}}^{\infty} \frac{R_{\rm had}(s)\, ds}{s(s-q^2)}$
- Hadronic uncertainties $\rightsquigarrow$ α the least well known EW param. of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$!
- We find: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02759 \pm 0.00015$

  [HLMNT 11 prel.: 0.02764 ± 0.00010]

  i.e. $\alpha(M_Z^2)^{-1} = 128.953 \pm 0.020$ (HLMNT 10)

▶ HLMNT-routine for $\alpha(q^2)$ & $R_{\rm had}^{\rm data}$ available

Fit of the SM Higgs mass: LEP EWWG



- $M_H = 89\,^{+35}_{-26}$ GeV ($m_t = (173.3 \pm 1.1)$ GeV)

  ($M_H < 158$ GeV (95% CL), < 185 GeV incl. direct limit $M_H < 114$ GeV.)
- M_H moves further down with new $\Delta\alpha$.

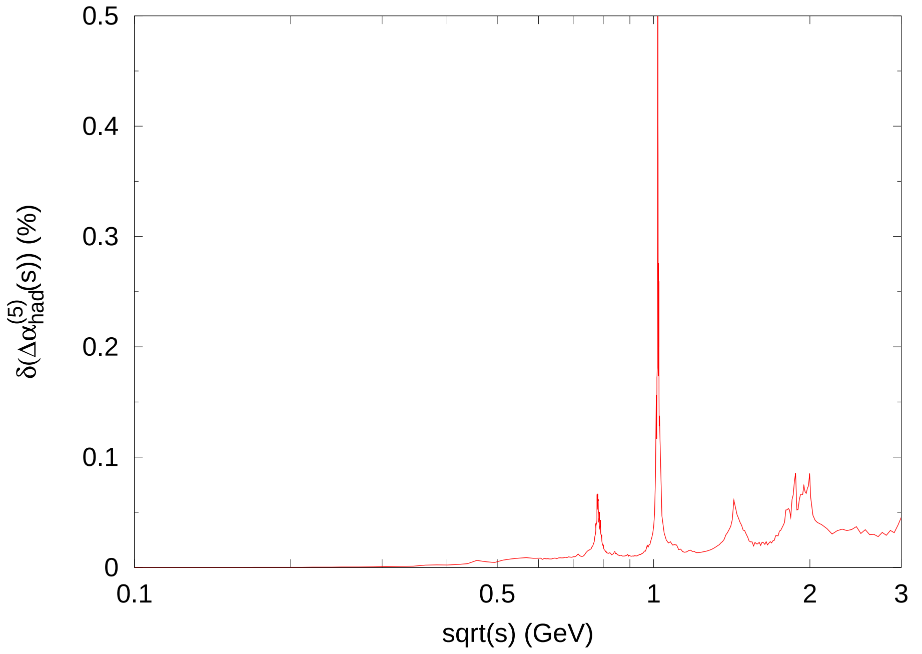
Features of the HLMNT VP code

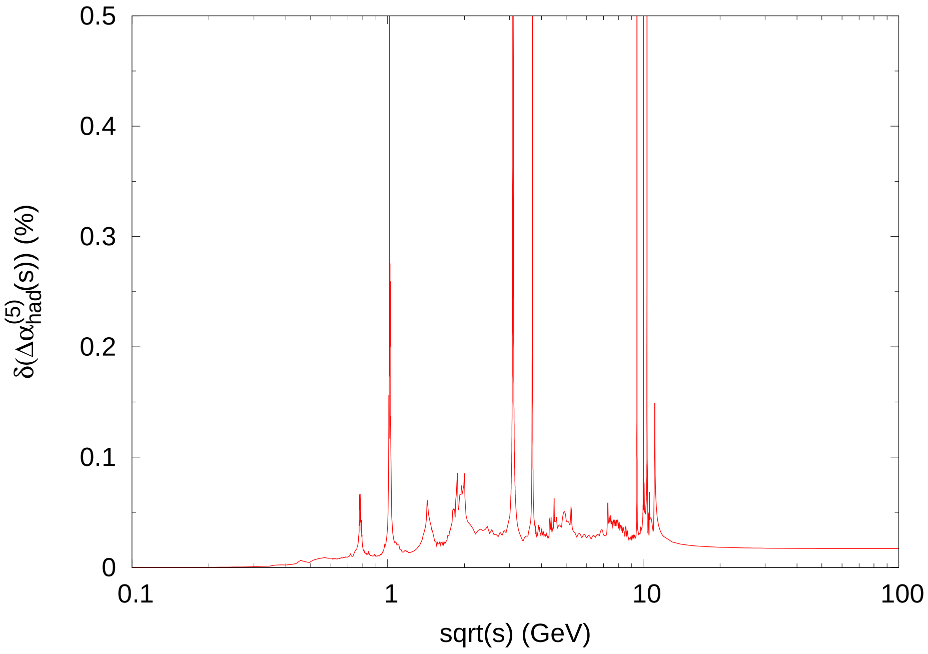
- Latest version is `VP_HLMNT_v2_0`, version 2.0, 15 July 2010
- Simple set of (standard) Fortran routines; completely standalone, no libs needed; all explanations in comment-headers
- Gives separately real and imaginary part ($\Delta\alpha(s)$ and $R(s)$)
- Tabulation/interpolation of hadronic part, for both space- and time-like region, including errors; no input data files or `rhad` installation needed
- Leptonic part coded analytically; all special function included (partly with custom made expansions)
- top contribution in the same way

→ Flag to include or exclude very narrow resonances $\boldsymbol{J/\psi}$, $\boldsymbol{\psi'}$, $\boldsymbol{\Upsilon(1-6\,S)}$
[ϕ always included via integral over final state data (3π, KK)]

- Typical accuracy $\delta\left(\Delta\alpha^{(5)}_{\rm had}(s)\right)$

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):

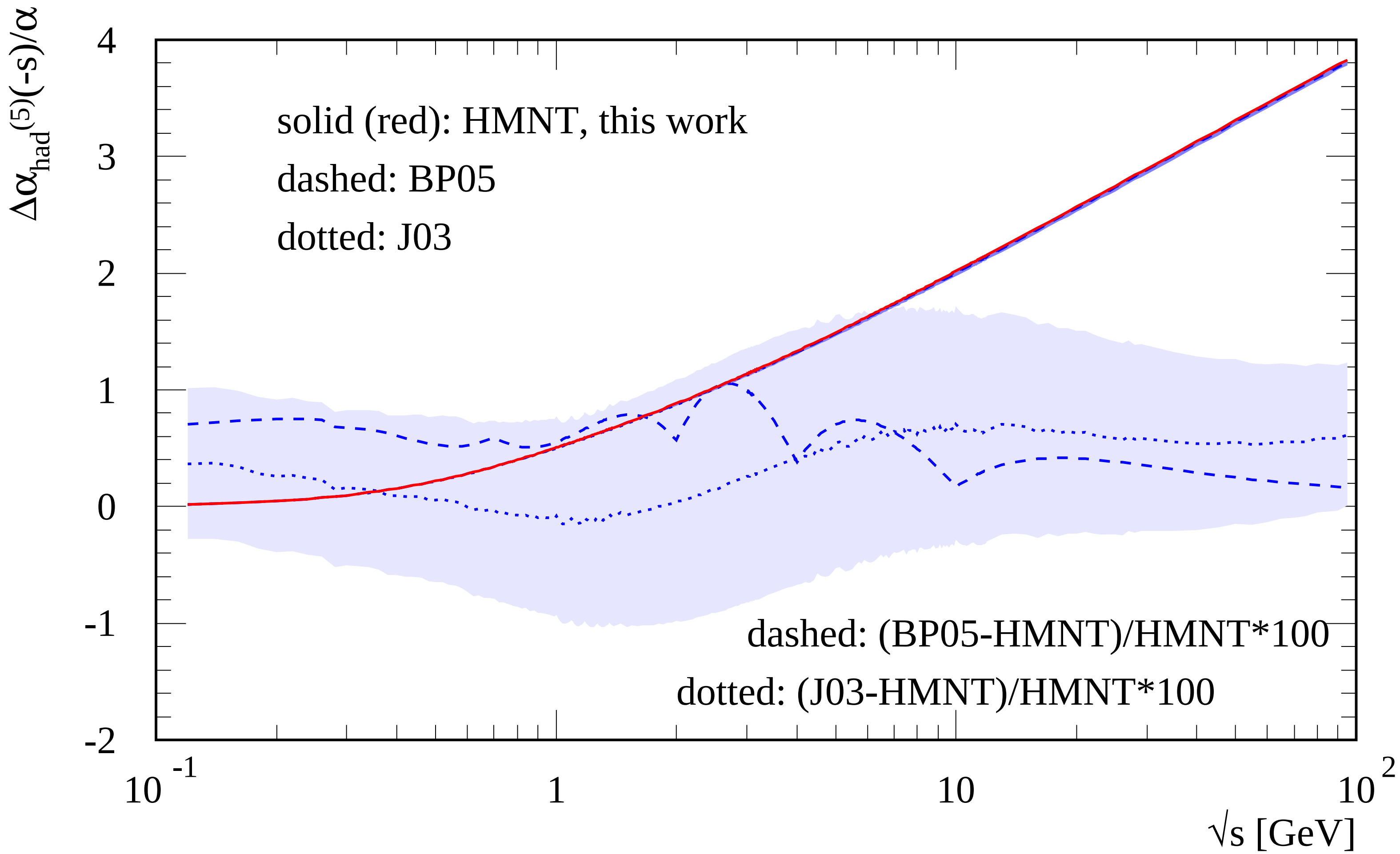




$\longrightarrow$ Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.

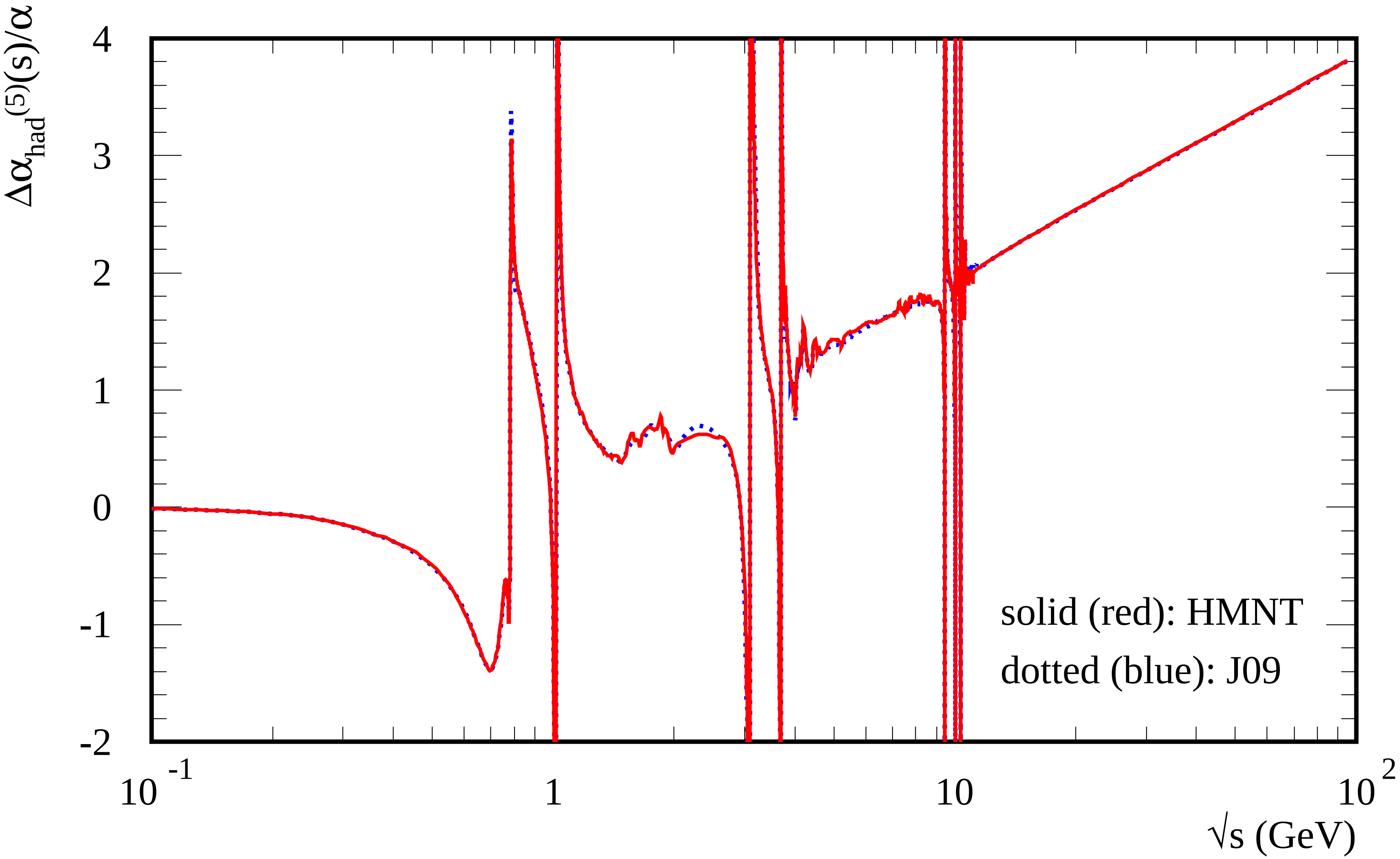
Enough in the long term? Need for more work in resonance regions.

- Comparison of Spacelike $\Delta\alpha^{(5)}_{\rm had}(-s)/\alpha$ (smooth $\alpha(q^2<0)$)



- Differences between parametrisations clearly visible but within error band (of HLMNT)
- Few-parameter formula from Burkhardt+Pietrzyk slightly 'bumpy' but still o.k.
- Encourage use of more accurate recent tabulations; $\boldsymbol{\Delta\alpha(M_Z^2)}$

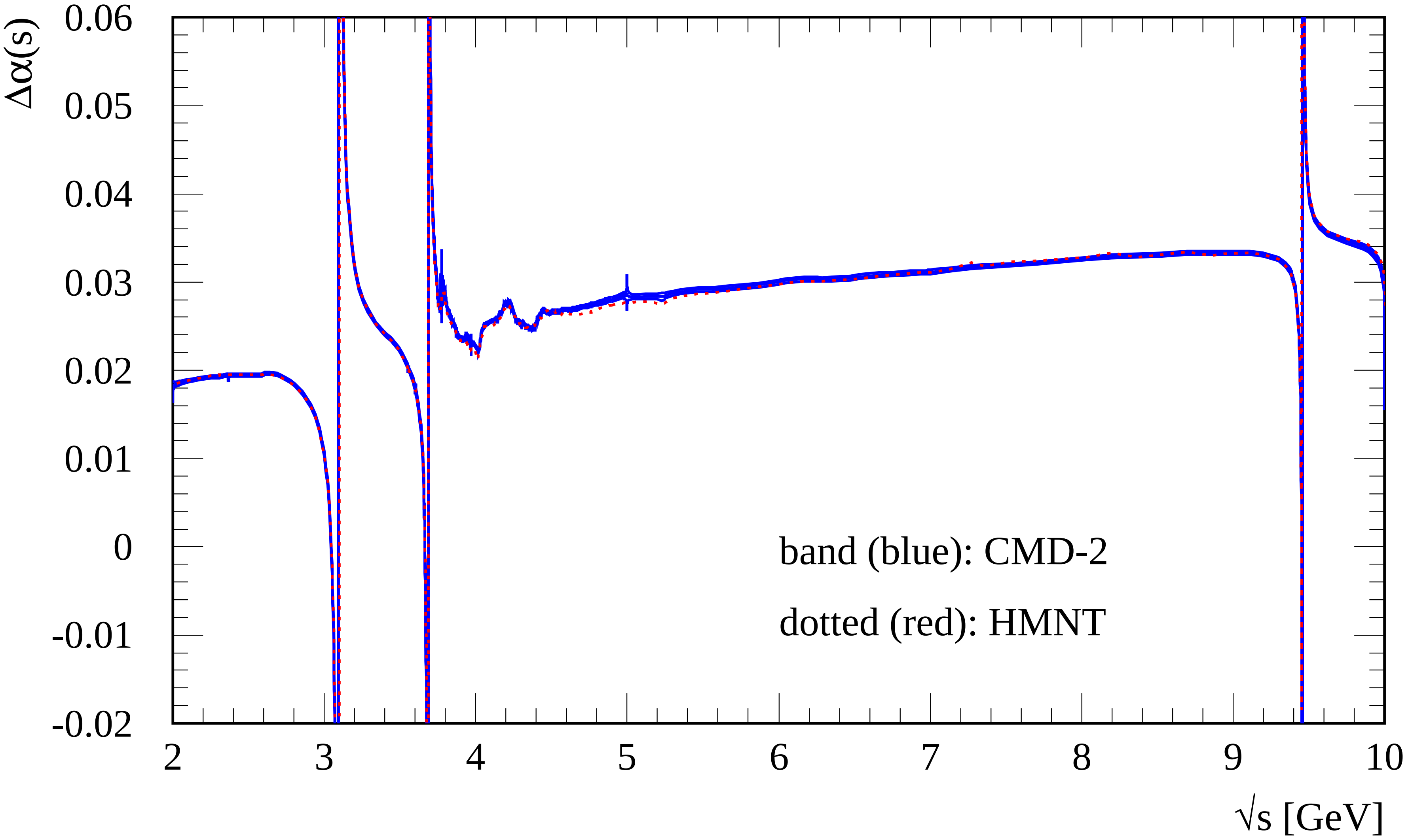
- $\Delta\alpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner's new routines



⟶ with new version big differences (with 2003 version) gone

– smaller differences remain and reflect different choices, smoothing etc.

- HLMNT compared to Novosibirsk   Timelike, $\Delta\alpha(q^2)$



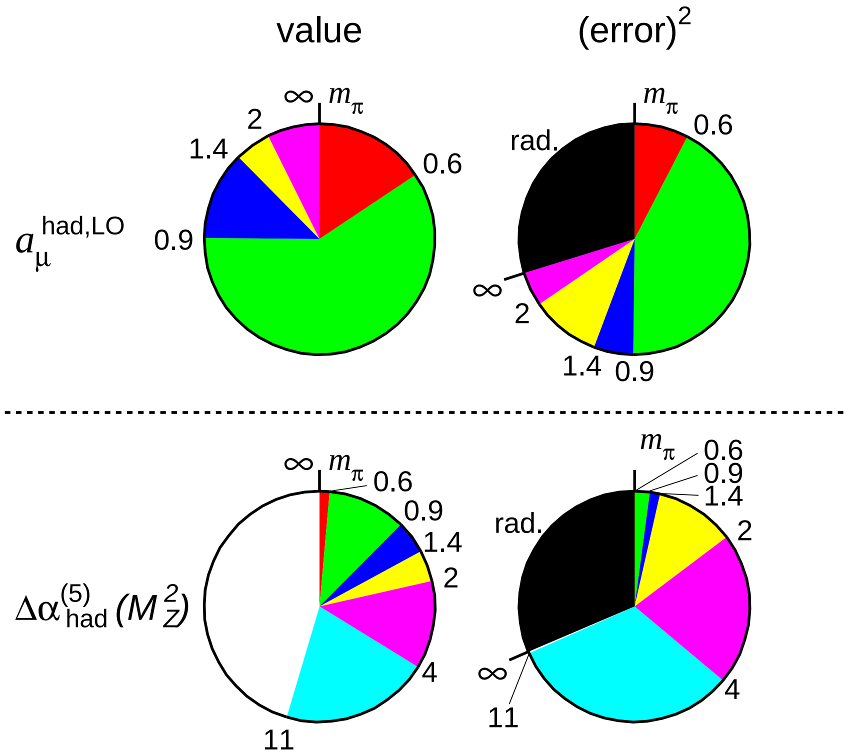
IV. Outlook

- **Further improvements** Hadronic VP still (just) the biggest error in $a_\mu^{\rm SM}$, soon l-by-l...

Prospects for further squeezing errors:

- More 'Rad. Ret.' in progress at KLOE
- Great opportunity for DAΦNE-2, very strong case for DAFNE-HE, in a few years SUPER-B
- Big improvement envisaged with CMD-3 and SND at VEPP2000
- Higher energies: BES-III at BEPCII in Beijing is on; opportunities for BELLE

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors[2]:



- New $g-2$ experiments planned at Fermilab and J-PARC. Start 2015 ?!
- Will $a_\mu^{\rm SM}$ match the planned accuracy? $\rightsquigarrow$ Light-by-Light may become limiting factor!

Conclusions

- $(g-2)_\mu$ strongly tests *all sectors* of the SM and constrains possible physics beyond.
- SM prediction consolidated in all sectors: Loops for QED + EW, many exp. data for $R_{\rm had}$ plus TH (incl. *Rad. Ret.*) for hadronic VP, low energy modelling for l-by-l.
- With the same data compilations as for $g-2$, also the hadronic contributions to $\Delta\alpha(q^2)$ have been determined; in turn $\boldsymbol{\alpha(M_Z^2)}$ has been improved considerably. M_H !?
- Interaction of TH + MC + EXP most important to achieve even higher precision.
  $\longrightarrow$ WG Radio Montecar Low
- low energy $R_{\rm had}$ is also a place to measure $\boldsymbol{\alpha_s}$ at a low scale.

▶ Discrepancy betw. SM pred. of $\boldsymbol{g-2}$ and BNL measurement persists at $> \boldsymbol{3\,\sigma}$.

▶ More to come from all sides. Clear and strong case for continued *and* new experiments!

`The coming years will be exciting, and not only for the LHC`

Extras:

$\Delta\alpha(q^2)$: Vacuum Polarisation in the space- and time-like

- Why Vacuum Polarisation / running α corrections ?

Precise knowledge of VP / $\alpha(q^2)$ needed for:

– Corrections for data used as input for $g-2$: 'undressed' $\sigma^0_{\rm had}$

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

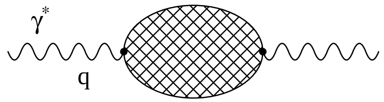
– Determination of α_s and quark masses from total hadronic cross section $R_{\rm had}$ at low energies and of resonance parameters.

– Part of higher order corrections in Bhabha scattering important for precise Luminosity determination.

– $\alpha(M_Z^2)$ a fundamental parameter at the Z scale (the least well known of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$), needed to test the SM via precision fits/constrain new physics.

$\longrightarrow$ Ingredient in MC generators for many processes.

- Dyson summation of Real part of one-particle irreducible blobs $\mathbf{\Pi}$ into the effective, real running coupling $\boldsymbol{\alpha}_{\mathrm{QED}}$:

$\Pi\ =$ 

Full photon propagator $\sim\ 1\ +\ \Pi\ +\ \Pi\cdot\Pi\ +\ \Pi\cdot\Pi\cdot\Pi\ +\ \ldots$

$$\rightsquigarrow \qquad \alpha(q^2)\ =\ \frac{\alpha}{1-\mathrm{Re}\Pi(q^2)}\ =\ \alpha\,/\,\big(1-\Delta\alpha_{\mathrm{lep}}(q^2)-\Delta\alpha_{\mathrm{had}}(q^2)\big)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2)\ =\ -\frac{q^2}{4\pi^2\alpha}\,\mathrm{P}\int_{m_\pi^2}^{\infty}\frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s-q^2}\ , \quad \sigma_{\mathrm{had}}(s)=\frac{\sigma^0_{\mathrm{had}}(s)}{|1-\Pi|^2}$$

[$\to$ σ^0 requires 'undressing', e.g. via $\cdot(\alpha/\alpha(s))^2$ $\rightsquigarrow$ iteration needed]

- Observable cross sections σ_{had} contain the $|$full photon propagator$|^2$, i.e. $|$infinite sum$|^2$.
$\to$ To include the subleading Imaginary part, use dressing factor $\frac{1}{|1-\Pi|^2}$.

Comparison of different compilations

- Timelike $\alpha(s)$ from Fred Jegerlehner's (2003 routine as available from his web-page)

$$\alpha(s = E^2) = \alpha \,/\, \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\right)$$

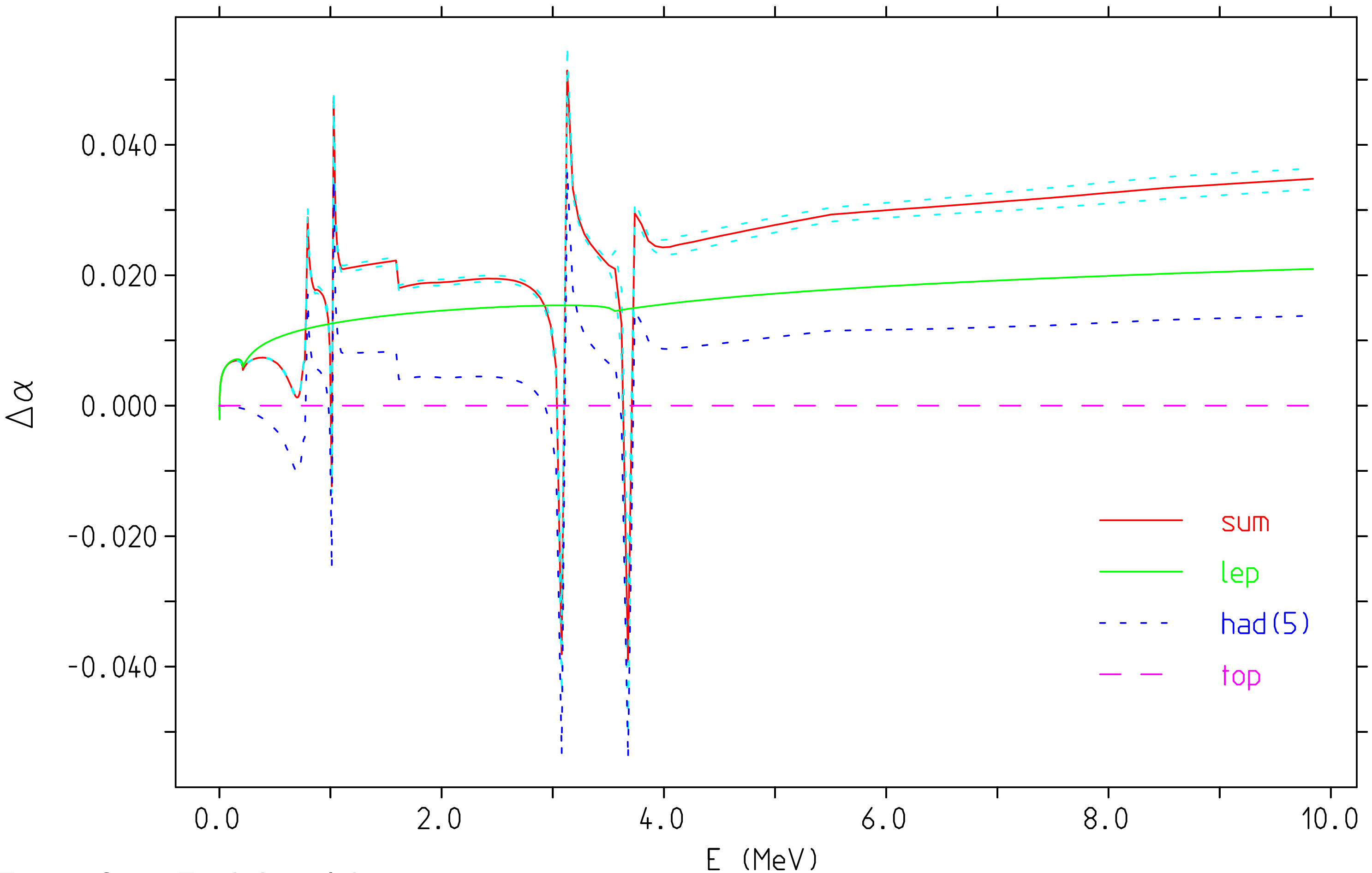
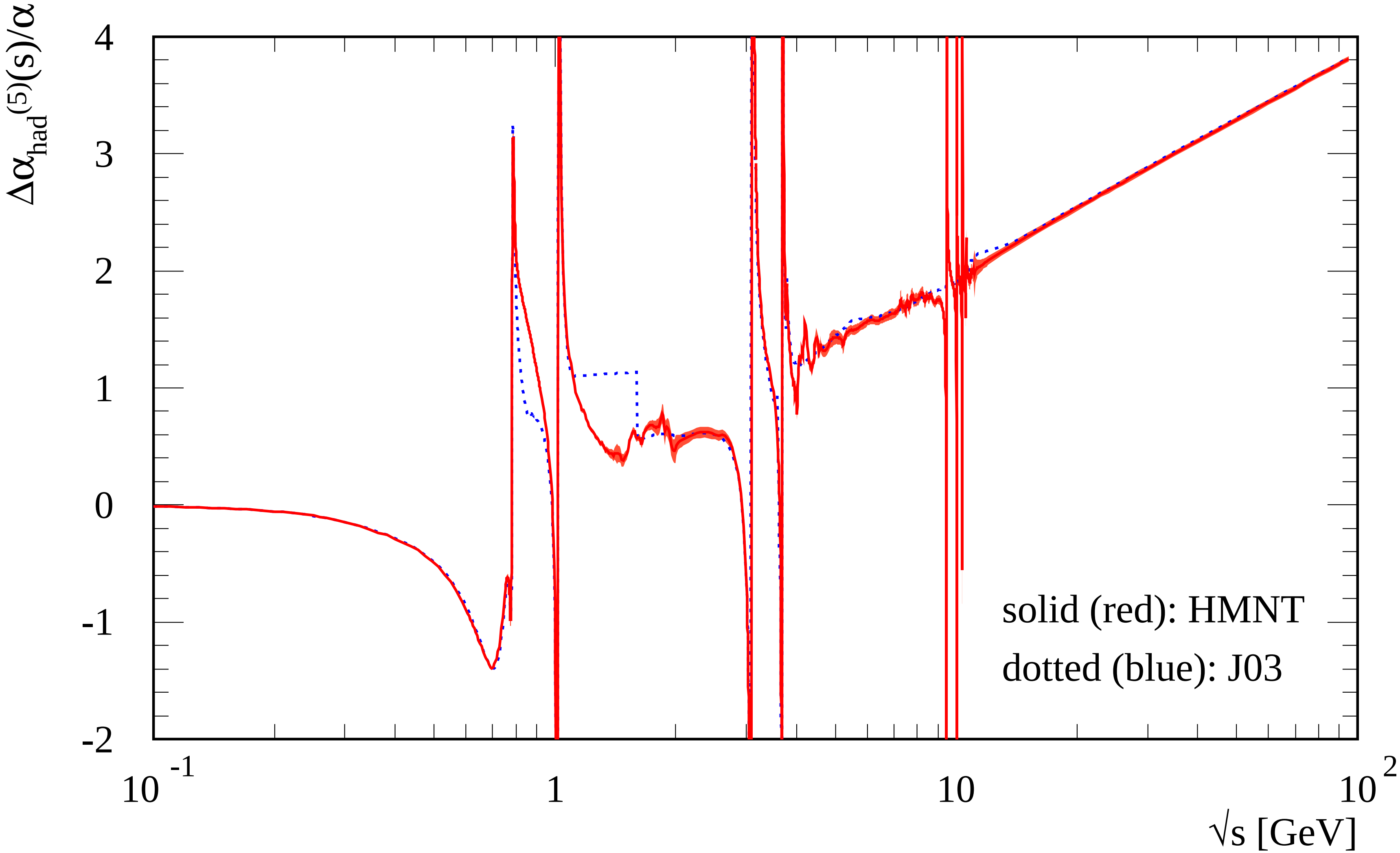


Figure from Fred Jegerlehner

Timelike $\alpha(s = q^2 > 0)$ follows resonance structure:



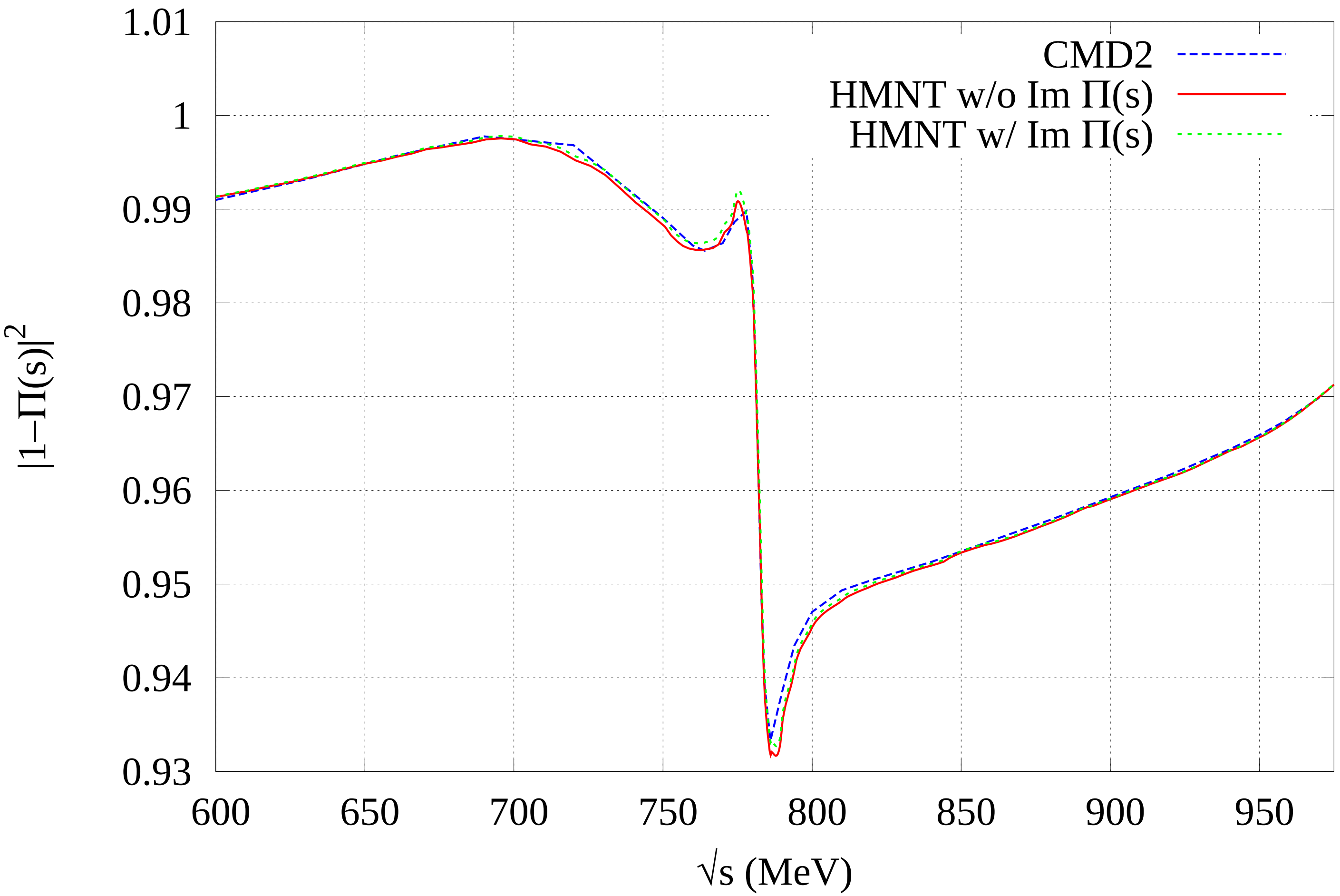
— Step below just a feature of unfortunate grid.

— Difference below 1 GeV not expected from data.

[Comparisons with other parametrisations confirm HMNT.]

- HMNT compared to Novosibirsk's parametrisation

Timelike $|1-\Pi(s)|^2 \sim (\alpha(s)/\alpha)^2$ in ρ central energy region: A relevant correction!



⟶ Small but visible differences, as expected from independent compilations.

- **What about $\Delta\alpha(M_Z^2)$?**

$\longrightarrow$ With the same data compilation of $\sigma^0_{\rm had}$ as for $g-2$ HLMNT find:

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \;=\; 0.02760 \pm 0.00015 \quad \text{(HLMNT 09 prelim.)}$$

i.e. $\alpha(M_Z^2)^{-1} \;=\; 128.947 \pm 0.020$ [HMNT '06: $\alpha(M_Z^2)^{-1} \;=\; 128.937 \pm 0.030$]

Earlier compilations:

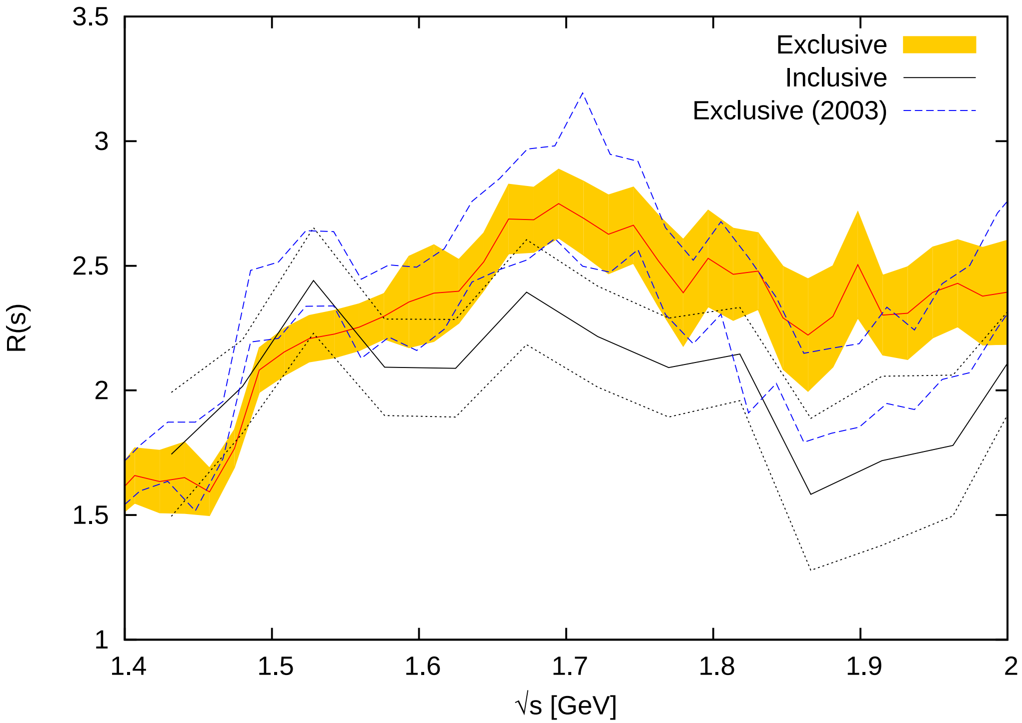
Group	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	remarks
Burkhardt+Pietrzyk '05	0.02758 ± 0.00035	data driven
Troconiz+Yndurain '05	0.02749 ± 0.00012	pQCD
Kühn+Steinhauser '98	0.02775 ± 0.00017	pQCD
Jegerlehner '08	0.027594 ± 0.000219	data driven/pQCD
($M_0 = 2.5$ GeV)	0.027515 ± 0.000149	Adler fct, pQCD
HMNT '06	0.02768 ± 0.00022	data driven

Adler function: $$D(-s) \;=\; \frac{3\pi}{\alpha} s \frac{\rm d}{{\rm d}s} \Delta\alpha(s) \;=\; -(12\pi^2) s \frac{{\rm d}\Pi(s)}{{\rm d}s}$$

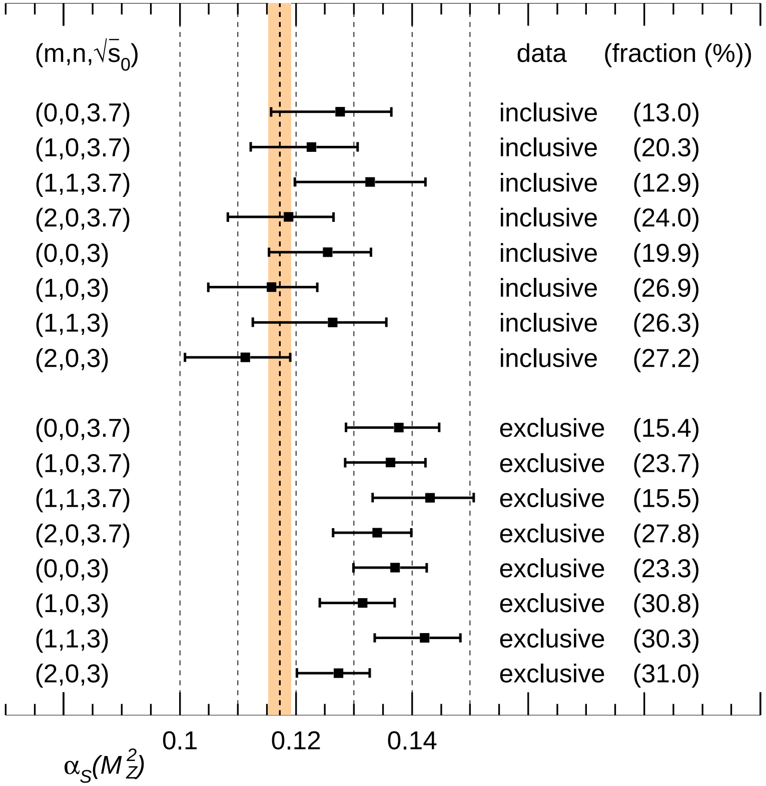
allows use of pQCD and minimizes dependence on data.

Region below 2 GeV: how reliable are the data? *inclusive* vs. *sum over exclusive*

Data blue: old excl. analysis, red/orange: new (2011)

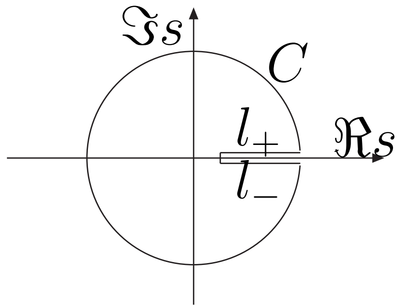


Sum-rules 'determining' α_S (2003):



- Shape similar, but normalisation different
- Question of completeness/quality of sum of exclusive data vs. reliability/systematics of old inclusive data ($\gamma\gamma2$, MEA, M3N, BBbar)
- HMNT previously (2003/06) have used incl. data, in line with sum-rule analysis

Check against perturbative QCD: QCD $\sum$-rule analysis



- Evaluate QCD $\sum$-rules of the form:

$$\int_{s_{\rm th}}^{s_0} {\rm d}s\, R(s) f(s) = \int_C {\rm d}s\, D(s) g(s)\,, \qquad \text{with} \quad D(s) \equiv -12\pi^2 s \frac{\rm d}{{\rm d}s}\left(\frac{\Pi(s)}{s}\right)$$

- The Adler D function is calculable in pQCD: $D(s) = D_0(s) + D_{\rm m}(s) + D_{\rm np}(s)\,.$

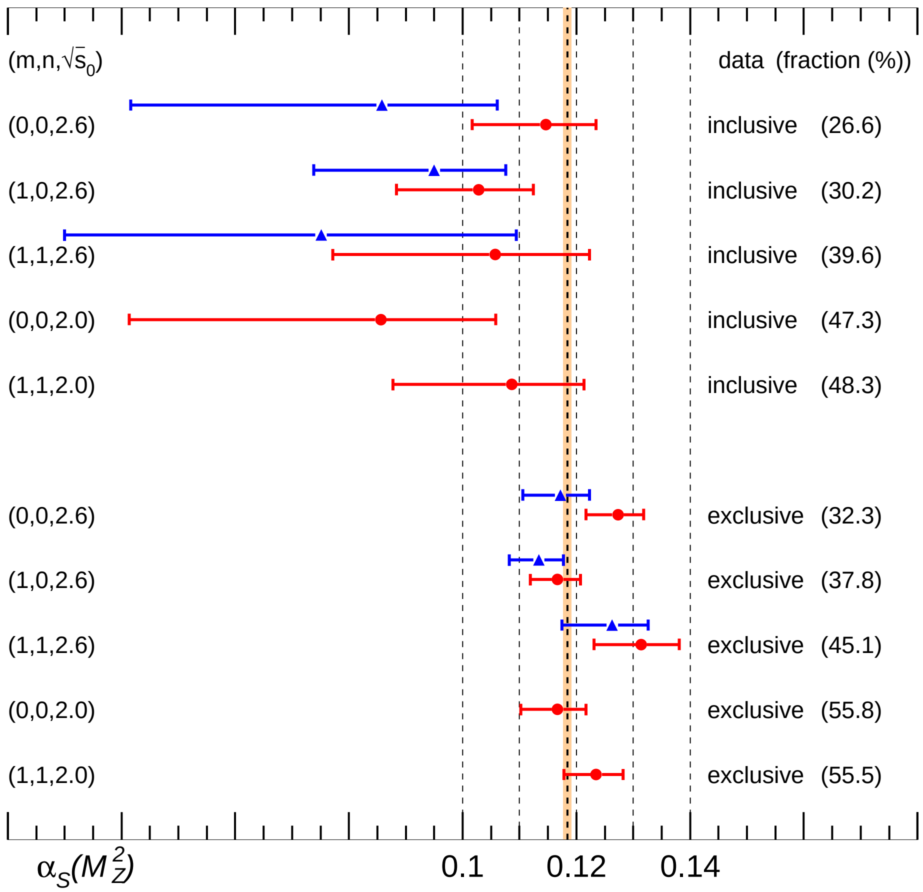
- Take $f(s) = (1 - s/s_0)^m (s/s_0)^n$ to maximise sensitivity to the required region, $g(s)$ follows.

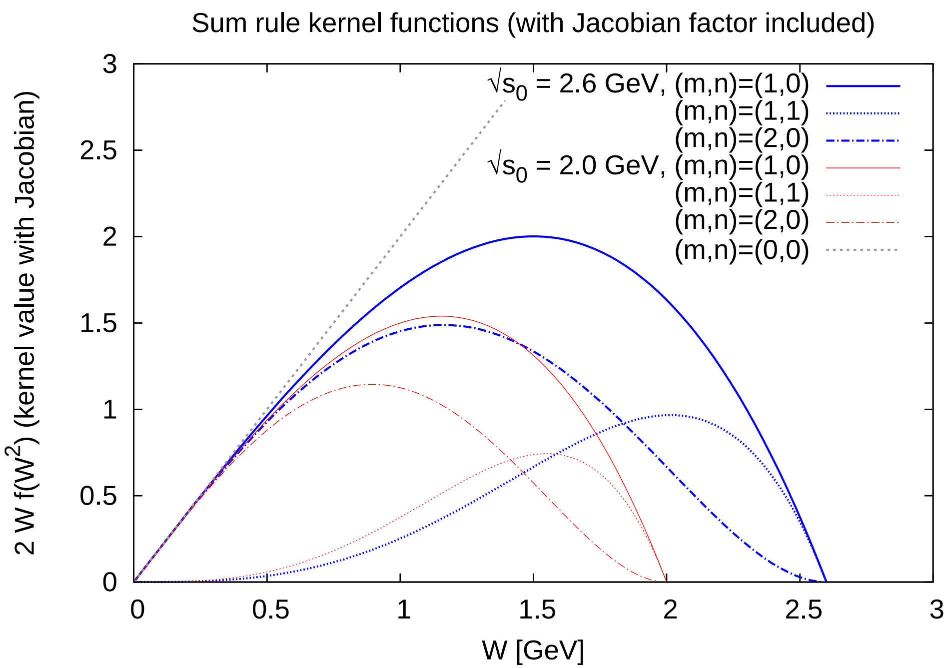
- Choose s_0 below the open charm threshold ($n_f = 3$ for pQCD).

- For $m = 1$, $n = 0$ one gets e.g.

$$\int_{s_{\rm th}}^{s_0} {\rm d}s\, R(s) \left(1 - \frac{s}{s_0}\right) = \frac{i}{2\pi} \int_C {\rm d}s\, \left(-\frac{s}{2s_0} + 1 - \frac{s_0}{2s}\right) D(s)\,.$$

▶ HLMNT's new sum-rule analysis:





- New data have changed the picture $\longrightarrow$ *sum over exclusive* agrees better with QCD
- Still rely on isospin relations for missing channels [sizeable error from $K\bar{K}\pi\pi$]
- From HLMNT 10: Use of more precise *sum over exclusive* ($\hookrightarrow$ shift up by $\sim +3\cdot 10^{-10}$)