Experimental input to the hadronic corrections of the muon $g-2$

C. F. Redmer

Johannes Gutenberg-University Mainz, Mainz, Germany

Summary. — The hadronic contributions to the anomalous magnetic moment of the muon completely limit its Standard Model prediction. The recent advances in lattice QCD have shown tensions with the long established data-driven methods. A thorough assessment of the experimental inputs is necessary to identify potential improvements, which will allow to settle the situation in future.

1. – Introduction

The anomalous magnetic moment of the muon is one of the most precisely determined observables in the Standard Model (SM). It is referred to in terms of the muon anomaly $a_\mu = (g-2)_\mu/2$, which describes the relative deviation of the muon g_μ factor from Dirac's prediction $g_\mu = 2$. The most recent experimental determination at Fermilab yields a new world average value of $a_\mu^{\mathrm{Exp}} = 116592059(22) \times 10^{-11}$ [1]. Considering the SM prediction published by the "Muon $g-2$ Theory Initiative" [2], the long-standing tension between the direct measurement and the prediction of a_μ is increased to more than $5\,\sigma$. In principle, this discrepancy might be seen as a hint of New Physics. However, new tensions within the SM prediction do not allow to draw this conclusion, yet.

The SM prediction [2] of the muon anomaly contains contributions from all interactions of the SM. The largest contribution comes from QED and has been determined with high accuracy in perturbation theory up to the 10th order [3]. Also the contribution due to the weak interaction is determined using perturbative methods. It is found to be small and its uncertainty is well under control [4]. The contributions of the strong interaction to the absolute value of a_μ are small. However, since at the relevant energy scale the strong interaction cannot be treated perturbatively, the corresponding contributions so far completely dominate the total uncertainty of the SM prediction of a_μ.

The contribution of the strong interaction is separated into two parts: The hadronic vacuum polarization a_μ^{HVP}, which is the larger part making up for more than 80% of the hadronic uncertainties, and the hadronic Light-by-Light scattering a_μ^{HLbL}. Only recently it became possible to perform non-perturbative calculations using lattice-QCD at competitive accuracy. The long established approaches to determine the hadronic contributions to a_μ are data-driven. Using dispersion relations, a_μ^{HVP} can be directly related to hadronic cross sections measured in e^+e^- collisions [5, 6]. The determination of a_μ^{HLbL} uses transition form factors of mesons and meson systems as input [7]. Data-driven approaches allow to systematically improve the theory prediction by providing more precise measurements of the data input.

2. – Hadronic vacuum polarization

Using the optical theorem, the contribution of the hadronic vacuum polarization to a_μ can be directly related to hadronic cross sections σ_{had} in e^+e^- annihilations. At leading order the contribution can be calculated using the dispersion integral

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{2m_\pi}^{\infty} \mathrm{d}s \frac{R(s)K(s)}{s^2} \;,$$

where $K(s)$ is a known kernel function [8] and $R(s)$ is the ratio of the Born cross sections of hadron and muon pair production in e^+e^- collisions. Both, the kernel function and the hadronic cross sections are proportional to $1/s$, making the contribution from processes at center-of-mass energies $\sqrt{s} < 1\,\mathrm{GeV}$ most important. This mass region is dominated by the $\rho(770)$ and $\omega(782)$ resonances. Thus, the production cross sections of the corresponding decay products $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ are the most important inputs to the dispersion integral. The cross section of $e^+e^- \to \pi^+\pi^-$, which is proportional to the pion form factor, makes up for more than two thirds of $a_\mu^{\mathrm{HVP,LO}}$ and about one third of the uncertainty. A correct and reliable determination of these cross sections, and especially of the pion form factor, is essential for the prediction of a_μ.

Two complementary approaches are established in experiments to measure hadronic cross sections. In an energy scan measurement the center-of-mass energy of the accelerator is varied to measure the cross section at different energies. The result is a set of individual measurements, typically with very good energy resolution. The other approach exploits events, where a hard photon is emitted from the initial state. This initial state radiation (ISR) is a continuous process, which lowers the nominal $\sqrt{s}$ of the e^+e^- collision to an effective center-of-mass energy $\sqrt{s'} = \sqrt{s - 2\sqrt{s}E_{\gamma_{\mathrm{ISR}}}}$ for the hadron production, where $E_{\gamma_{\mathrm{ISR}}}$ is the energy of the radiated photon. In this way, a scan measurement can be performed from a single accelerator setting with homogeneous data taking conditions and a consistent normalization over the full energy range, down to the threshold region. The energy resolution is however limited by the detector and an additional uncertainty arises from the so-called radiator function, which is used to convert the radiative cross section of the ISR process to the non-radiative cross section needed to calculate $a_\mu^{\mathrm{HVP,LO}}$. Since the ISR process is suppressed by a factor α, the cross section measurements are performed at high intensity accelerators, like ϕ, τ-charm and B-factories.

The most precise measurements of the dominating $e^+e^- \to \pi^+\pi^-$ process, which enter the 2020 SM prediction of the Theory Initiative, are obtained exploiting the ISR technique. The BaBar and the KLOE collaborations have determined the contribution $a_\mu^{\mathrm{HVP,LO}}$ with 0.7% and with 0.6% accuracy, respectively [9, 10]. The result of the KLOE collaboration is a combination of three individual measurements using different analysis and normalization strategies, which all lead to consistent results. Nevertheless, there is a discrepancy between the BaBar and KLOE results of about three standard deviations, which currently lacks explanation. The discrepancy also drives the uncertainty of the combined data sets used in the prediction of the Muon $g-2$ Theory Initiative. Other measurements entering this combination are performed using ISR [11, 12] and energy

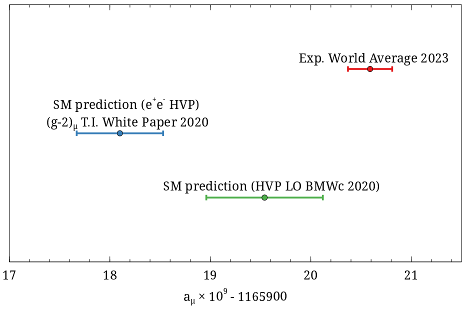

Fig. 1. – Comparison of world average of $a_\mu^{\rm exp}$ (red) [1] with 2020 SM prediction (blue) [2] and the SM prediction where the HVP(LO) contribution is replaced by the BMWc result (green) [18].

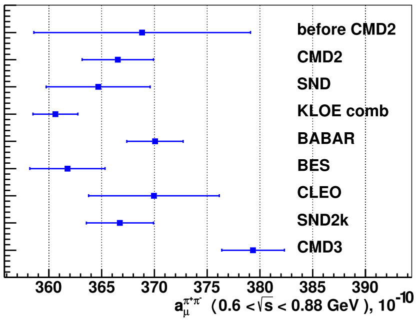

Fig. 2. – Taken from Ref. [22]. Contributions to $a_\mu^{\rm HVP,LO}$ from the $\pi^+\pi^-(\gamma)$ cross sections measured by [9, 10, 11, 12, 13, 14, 15, 16, 17] and [22] in the range $0.6 \le \sqrt{s}[{\rm GeV}] \le 0.88$.

scans [13, 14, 15, 16, 17], however, the spread of the results and their precision does not allow to discern the two most precise measurements. A conservative merging procedure is used to combine all experimental results, taking into account the tensions of the pion form factor measurements.

The resulting value for $a_\mu^{\rm HVP,LO}$ creates a tension of about five standard deviations with the latest world average value of $a_\mu^{\rm Exp}$ from experiment. In the mean time the first lattice QCD determination of $a_\mu^{\rm HVP,LO}$ with competitive, i.e. sub-percent precision is published by the BMWc collaboration [18]. Its result is in clear tension with the dispersive determination, at a level of two standard deviations, being in much better agreement with $a_\mu^{\rm Exp}$, as shown in Fig.1. Other lattice QCD calculations of a related quantity by other collaborations [19, 20, 21] seem to confirm the findings of the BMWc collaboration. Thus, new measurements of hadronic cross sections are called for, which can help to scrutinize the current situation and point out short comings of the current procedures.

Very recently, a new, high accuracy measurement of the pion form factor has been presented by the CMD-3 collaboration [22]. The energy scan measurement performed at the VEPP2000 collider in Novosibirsk provides competitive statistical accuracy, clearly improving on previous measurements of the CMD, CMD-2, and SND collaborations. The resulting value of $a_\mu^{\rm HVP,LO}$ is larger than the corresponding findings of all previous measurements, as illustrated in Fig. 2, and almost in good agreement with the BMWc lattice result. A series of systematic checks is performed establishing the final uncertainty of 0.8%. The disagreement with the previous measurements, especially with the previous CMD-2 and SND results [13, 14, 15, 16, 17] is not yet understood, giving rise to intense discussions.

3. – Hadronic Light-by-Light scattering

The contribution due to hadronic Light-by-Light scattering $a_\mu^{\rm HLbL}$ cannot be related to only a single measurable quantity. In a counting scheme attributed to de Rafael [23], the processes contributing to $a_\mu^{\rm HLbL}$ can be separated into pseudoscalar transitions, pion and kaon s-wave rescattering as well as loops and and boxes, contributions due to tensor, scalar, and axial states, and a short distance contribution. In addition to the first competitive lattice QCD calculations of a_μ^{HLbL}, the Muon $g-2$ Theory Initiative made use of data-driven estimates for the individual contributions to avoid model dependent predictions as much as possible. The most important contributions of pseudoscalar transitions and pion/ kaon loops are well under control [2]. The remaining contributions need further input for improvements.

It was demonstrated that experimental information is most relevant at momentum transfers up to 1 GeV [24]. The corresponding experimental input for the pseudoscalar transitions are transition form factors of the light pseudoscalar mesons. Most of the available data is currently provided by the B-factories [25, 26], albeit, due to kinematic restrictions, at large momentum transfers. Preliminary data from the BESIII collaboration shows significant improvements in the relevant energy region [27]. Moreover, the data confirm the dispersive construction [28] and the lattice QCD calculation [29] of the π^0 transition form factor, which are used in the current prediction of a_μ^{HLbL}. The available data has been determined as a function of a single virtuality, while transition form factors are in general depending on two virtualities. Due to the strong dependence of the cross section on the involved virtualities, double-virtual measurements are experimentally challenging. So far only a single measurement is reported for the η' meson [30].

Experimental information for the pion and kaon loops can be derived from the partial waves of the pion pair production in two-photon collisions. Currently, only the Belle collaboration reports information on neutral pion and kaon pair production as a function of a single virtuality [31, 32]. Complementary information is expected from the BESIII collaboration on the production of charged and neutral pion pairs [27].

Most important for further improvements of a_μ^{HLbL} are additional experimental information on axial vector mesons, e.g. the $f_1(1285)$. Currently, only a single measurement is reported by the L3 collaboration [33]. While data is provided in the most relevant energy region, the dependence of an effective transition form factor on the virtuality is not directly measured, but deduced from correlations found in simulations. More precise measurements are needed and can be provided from BaBar, BESIII, and Belle-II data.

4. – Perspectives

The hadronic contributions to a_μ continue to be the limiting factor in the SM prediction. The contribution of a_μ^{HLbL} shows a consistent picture among data-driven and lattice QCD evaluations. Additional data input and further advances in the calculations will bring improvements of the SM prediction.

A more critical situation is currently found in the evaluations of $a_\mu^{\mathrm{HVP,LO}}$. The traditional data-driven evaluation drives the deviation of the SM prediction of a_μ from the direct measurement. The lattice result on HVP by the BMWc collaboration, however, would mend the deviation. Similarly, the recent measurement of CMD-3 would reduce the tension. Thus, it is of utmost importance to understand on the one hand the differences between the new and previous CMD results, and on the other hand provide new and independent measurements at other facilities in order to identify potential shortcomings of previous procedures, which might have caused the discrepancy. Several activities have been initiated as the BaBar and KLOE collaborations did not evaluate the complete data sets, yet. BESIII and Belle-II are taking new data to provide new results with improved strategies. Finally, the SND collaboration has access to the same amount of data as the CMD-3 collaboration, therefore new results are to be expected. A coordinated effort within the Muon $g-2$ Theory Initiative has been started to eventually resolve the current tensions.

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