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The role of positive charge conjugation states in the Glueball Resonance Gas

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Summary. — In this note, we present our recent analyses of the thermodynamic properties of the glueball resonance gas. We observe that the dominant contribution to the thermodynamic quantities, such as pressure, trace anomaly, and entropy, is coming from the free glueball gas with the states having positive charge conjugation ($C = +$). A comparison of these states obtained from LQCD and functional methods within the glueball resonance gas model is also presented.

1. – Introduction

Quantum chromodynamics (QCD) can be described in the high-temperature regime by a perturbative quark-gluon plasma, while in the low-temperature regime, by a gas of weakly interacting hadrons. The two regimes are separated by the pseudo-critical temperature T_c, which represents the confinement-deconfinement cross-over transition of QCD whose value is still under debate [1, 2, 3, 4, 5]. In the case of Yang-Mills (YM) (QCD without quarks), the T_c represents the transition (of the first order) between the bound states of gluons (glueballs) and gas of gluons at low and high temperatures, respectively.

At finite temperature, several approaches have been developed to study QCD [6, 7, 8, 9]. One typically compares the outcomes of the lattice QCD (LQCD) to the hadron resonance gas model (HRG) below T_c. This procedure can be repeated in the YM case. Instead of HRG, one has a subset of it, that we call a Glueball Resonance Gas (GRG) [10].

The study on the pomeron ($J^{PC} = (\text{even})^{++}$) and odderon ($J^{PC} = (\text{odd})^{--}$) trajectories suggests the existence of glueballs [11, 12]. We observe that, within the GRG, the contribution provided by the $C = +$ states is predominant, since the contribution to the thermodynamic (TD) quantities, e.g., pressure and trace anomaly, is dominated by the lightest resonances (0^{++} and 2^{++}).

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We have studied the GRG in [10] by considering the mass spectra of glueballs from Refs. [13, 14, 15]. In this work, we compare the results for the TD quantities coming from the positive charge conjugation states obtained from the LQCD [15] and the functional method [16].

2. – Results

The TD of the GRG can be described by using the total, dimensionless energy density $\hat{\epsilon}$

$$\hat{\epsilon} = \sum_{i=1}^{N} \frac{(2J_i+1)}{T^4} \int_0^\infty \frac{k^2}{2\pi^2} \frac{\sqrt{k^2+m_i^2}}{\exp\left[\frac{\sqrt{k^2+m_i^2}}{T}\right]-1} dk\,, \tag{1}$$

and the pressure $\hat{p}$:

$$\hat{p} = -\sum_{i=1}^{N} \frac{(2J_i+1)}{T^3} \int_0^\infty \frac{k^2}{2\pi^2} \ln\left(1-e^{-\frac{\sqrt{k^2+m_i^2}}{T}}\right) dk\,, \tag{2}$$

where J_i is the total spin of the i-th state. In the description of the GRG, other two quantities are also relevant, i.e., the dimensionless trace anomaly $\hat{I}$ and the entropy density $\hat{s}$:

$$\hat{I} = \hat{\epsilon} - 3\hat{p}\,, \quad \hat{s} = \hat{p} + \hat{\epsilon}\,. \tag{3}$$

These quantities play a central role in Ref. [10], where the comparison between the LQCD TD data from Ref. [3] and GRG model constructed the lattice spectra from [13, 14, 15] was performed. The results have shown that the GRG with the most recent lattice work on the glueball masses [15] better describes LQCD TD data. In Fig. 1, this comparison is shown for the normalized pressure, with the addition of the statistical errors (not present in Ref. [10]).

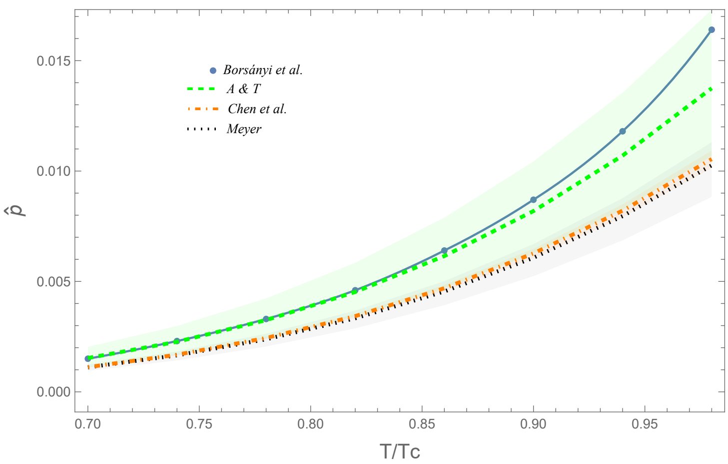

Fig. 1. – Pressure of the GRG as a function of the temperature for three different sets of the LQCD mass spectra [13, 14, 15], compared to the same quantity evaluated in Ref. [3].

Even when considering the errors in the pressure, we confirm the masses from Ref. [15] are favored. We remind that the TD results provided in Ref. [3] are given in functions of T/T_c; thus, the same quantities calculated from the GRG model must be presented for this ratio. However, lattice parameters used to calculate the mass spectra in [13, 14, 15] imply different T_c values for each line in Fig 1. For more details, see Ref. [10].

In Ref [10], we included the excited states by using Regge trajectory fitted from the glueballs with the quantum numbers $J^{PC} = 0^{++}, 2^{++}, 0^{-+}, 2^{-+}, 1^{+-}$ using the spectrum of Ref. [15]. The results do not significantly change, even with the addition of states up to radial quantum number $n = 10$. As we see from Fig. 2, among these states, the ones with positive charge conjugation provide the main contribution to the pressure, while those with $C = -$ have a negligible effect up to the vicinity of T_c.

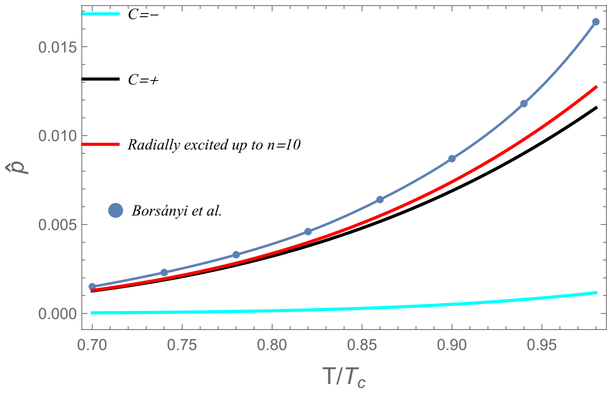

Fig. 2. – Pressure obtained in Ref. [3] compared to the GRG by considering the spectrum of Ref. [15] and all excited stated up to $n = 10$ (red), the $C = +$ states (black) and the $C = -$ states (cyan).

As an additional task, we compare the (favored) masses of [15] with the mass predictions obtained with functional methods [16]. The comparison is possible by neglecting the $C = -$ contribution from [15]. In Table I, the masses that will be used in the GRG model are reported.

$n\,J^{PC}$	M[MeV]		$n\,J^{PC}$	M[MeV]	
	Huber et al. [16]	A & T [15]		Huber et al. [16]	A & T [15]
1 0^{++}	1850(130)	1653(26)	**1 0**$^{-+}$	2580(180)	2561(40)
2 0^{++}	2570(210)	2842(40)	**2 0**$^{-+}$	3870(120)	3540(80)
1 2^{++}	2610(180)	2376(32)	**1 2**$^{-+}$	2740(140)	3070(60)
2 2^{++}	3640(240)	3300(50)	**2 2**$^{-+}$	4300(190)	3970(70)
1 3^{++}	3370(50)	3740(70)	**1 4**$^{++}$	4140(30)	3690(80)

Table I. – *The spectra of glueballs with $C = +$ reported in Refs. [15, 16].*

In Refs. [15, 16], the values are reported using the same value of the lattice parameter $r_0^{-1} = 418(5)$ MeV. By considering the relation between T_c and r_0 [17]:

$$T_c = 1.26(7) \cdot 0.614(2) \cdot r_0^{-1} \,, \tag{4}$$

we obtain the common value T_c = 323 ± 18 MeV. Three TD values (pressure, trace anomaly, and entropy) are shown in Fig. 3. One can see that the values obtained using the GRG model with the masses from [16] are lower than those from [15]. This is due mainly to the effect of the lightest states since a slight increase in the mass of the *i*-th glueball is reflected in a sizeable decrease in the TD quantities. However, in both cases, a slight discrepancy is still present with the lattice TD results [3] (which includes all glueballs contribution).

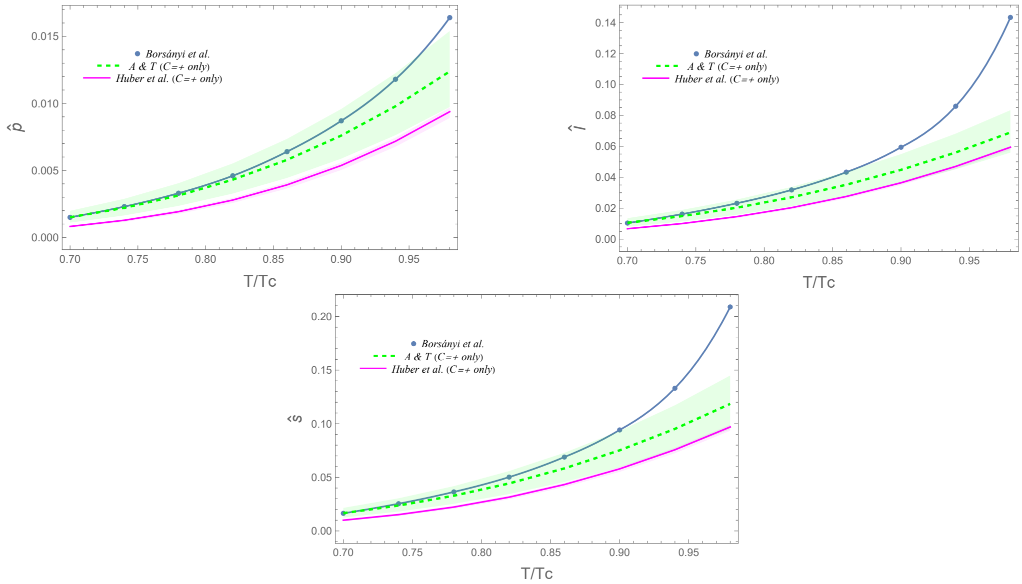

Fig. 3. – Pressure (top, left), trace anomaly (top, right), and entropy (bottom) as a function of T/T_c. The green dashed, and magenta continuous, plots (from the GRG model with the spectra (C = + only) from [15] and [16] respectively) are reported with errors due to the mass uncertainties. The comparison is done with the lattice data from [3].

3. – Conclusions

We revisited the results reported in Ref. [10] by considering the statistical errors in the mass spectrum, confirming that GRG with the glueball spectrum from [15] better describes the LQCD date from [3]. Among the states, the ones with positive charge conjugation provide the dominant contribution. Along this line, we present the comparison between the LQCD spectra of [15] with the one observed by functional methods [16], and we evaluate TD quantities for both C = + spectra within GRG.

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