Understanding the nature of the controversial $\rho(1250)$ meson through the covariant representation of hadrons

T. Maeda(1)(*), T. Komada(1), K. Yamada(2) and M. Oda(2)

(1) *Junior College Funabashi Campus, Nihon University, Chiba, Japan*
(2) *Research Institute of Science and Technology, College of Science and Technology, Nihon University, Tokyo, Japan*

Summary. — Recent unitary multichannel reanalysis of elastic $\pi\pi$-scattering data by Hammoud *et al.* has offered a clear evidence for the existence of five isovector vector mesons with masses below 1.8 GeV, including the controversial $\rho(1250)$. More recently, following to their results, we have found further evidence for the existence of some of these states by analyzing the cross section data for the $e^+e^- \to \omega\pi$ process. However, some of these states do not fit to results of typical constituent quark models and the PDG suggested assignments. In this work, in order to clarify the "$q\bar{q}$" assignments for these ρ states and predict the missing states, we examine their squared mass spectra in a framework of the covariant oscillator quark model, where we propose covariant wave functions of those states. We also investigate their couplings to a pion to check the validity of their assignments.

1. – Introduction

According to the Particle Data Group (PDG) summary table [1], the excited states of the $\rho(770)$ meson are the $\rho(1450)$ and $\rho(1700)$ in the mass region below 1.8 GeV. Their masses are in good agreement with the predictions for the 2^3S_1 and 1^3D_1 states in the typical constituent quark potential model by Godfrey and Isgur (GI) [2]. However, in a recent reanalysis of elastic P-wave $\pi\pi$ phase shifts and inelasticities [3] it was revealed that there existed five states below 2.0 GeV; $\rho(770)$, $\rho(1250)$, $\rho(1450)$, $\rho(1600)$, and $\rho(1800)$. Following to their results, we analyzed the cross section data for the $e^+e^- \to \omega\pi$ process and found further evidence for the existence of some of these states (see, these proceedings [4]). Obviously some of these states do not fit to the results of the GI model and the suggested $q\bar{q}$ quark-model assignments by PDG. In particular, the existence of the extremely low mass $\rho(1250)$ has a long history and is still the subject of long-standing experimental and theoretical controversy. At present it has not yet been accepted for

(*) maeda.tomohito@nihon-u.ac.jp

inclusion in the PDG list, but some relevant observations are listed under the entry for the $\rho(1450)$ [1] and the nature of $\rho(1250)$ has not yet been clarified.

On the other hand, we have been developing the covariant oscillator quark model (COQM) [5, 6, 7, 8] as a covariant extension of the non-relativistic quark model (NRQM) for many years, and have successfully applied it not only to static problems but also to various reactions of hadrons, where the relativistic treatment is required. The COQM treats the squared mass operator of relevant hadron systems, rather than the Hamiltonian as in the NRQM. As a result, the squared mass spectra of hadron systems are given by the linear rising Regge trajectories for both the orbital quantum number l and the radial quantum number n_r, consistent with the well-known phenomenological behaviors of the so-called orbital and radial trajectories.

In this work, based on the analysis of the Regge trajectories for the ρ mesons, we examine their squared mass spectra in the framework of the COQM, where we propose covariant wave functions (WFs) of these states. Then we clarify the "$q\bar{q}$" assignments for the five ρ states mentioned above and predict the missing states, and investigate their one-pion couplings to check the validity of their assignments.

2. – Phenomenological analysis of squared mass spectra and a possible classification of ρ mesons

In the COQM the squared mass spectra of relevant hadrons are given as

$$M^2_{l,n_r} = l\Omega_l + n_r\Omega_r + \text{const.} \quad (l, n_r = 0, 1, 2, \cdots), \tag{1}$$

where l and n_r are the orbital and radial quantum numbers respectively, leading to the linear rising Regge trajectories. In this analysis we treat the respective slope Ω_N $(N = l, n_r)$ as phenomenological parameters. Note that the slope parameters are related to the size parameters of the space-time WFs in our model. Here we assume the existence of the five ρ states below 1.8 GeV reported in the Ref. [3], and consider the existence of some states around 2 GeV from the Ref. [1]. Then, by using the mass relation (1), we discuss a possible classification of these states in terms of the "$q\bar{q}$" systems.

The leading orbital Regge trajectory is shown in Fig. (1a), from which we obtain $\Omega_l = 1.14$ GeV^2 by reading its slope. As pointed out in Ref. [3], in the naive assignments based on the NRQM, the five ρ mesons can be assigned, in order of increasing mass, as $\rho(1250) = 2^3S_1$, $\rho(1450) = 1^3D_1$, $\rho(1600) = 2^3S_1$ and $\rho(1800) = 2^3D_1$. Based on these assignments, the squared mass spectra are plotted for the radial quantum number in Fig. (1b). The masses of the respective states are well represented by linear trajectories with the slope close to the leading orbital trajectory, though slightly different for the S-wave trajectory. However, there is a problem with this assignments. The main reason is that, in the conventional constituent quark potential models, it is difficult to make the predicted mass of the 2^3S_1 state smaller than that of the GI model. A possible solution is discussed in Ref. [9] where the bare 2^3S_1 quark-model state at 1.45 GeV can become the physical $\rho(1250)$ due to the dynamical effect of the meson-meson coupling channel. However, it is also pointed out that it is difficult to shift the calculated mass of the 3^3S_1 state in the GI model from 2.0 GeV to 1.6 GeV.

In the following, we propose a novel classification from the COQM point of view. Here we assume the universality of the slope parameters ($\Omega_l = \Omega_{n_r} = 1.14$GeV^2). In the constituent quark potential model, including the Ref. [2], the assignments $\rho(770) = 1^3S_1$ and $\rho(1450) = 2^3S_1$ as the usual $q\bar{q}$ bound states are plausible. As shown in Fig. (1c), we can assign $\rho(1800) = 3^3S_1$ since it lies on the trajectory together with these states. Furthermore, by drawing the trajectory of the corresponding D-wave excited states using the relation (1), we found that $\rho(2000)$ and $\rho(2270)$ (taken from "Further states" of the particle listings in Ref. [1]) are naturally assigned for the 2^3D_1 and 3^3D_1 states, respectively.

On the other hand, Fig. (1d) shows that the controversial $\rho(1250)$ is well described by the another radial S-wave trajectory together with $\rho(1600)$ and $\rho(1900)$. In addition, we found that $\rho(2150)$ is a possible candidate as the another 2^3D_1 state by drawing the corresponding D-wave trajectory. Although the states in Fig. (1d) are different from the conventional states in Fig. (1c), it is suggested that these states have a similar level structure. In the next section, we discuss that the relativistic framework of the COQM allows us to construct wave functions corresponding to these states.

In Tab.(I) we summarizes the results of our new classification. Including the experimentally missing states, the masses of the corresponding mesons are given as theoretical predictions, with the underlined values used as input.

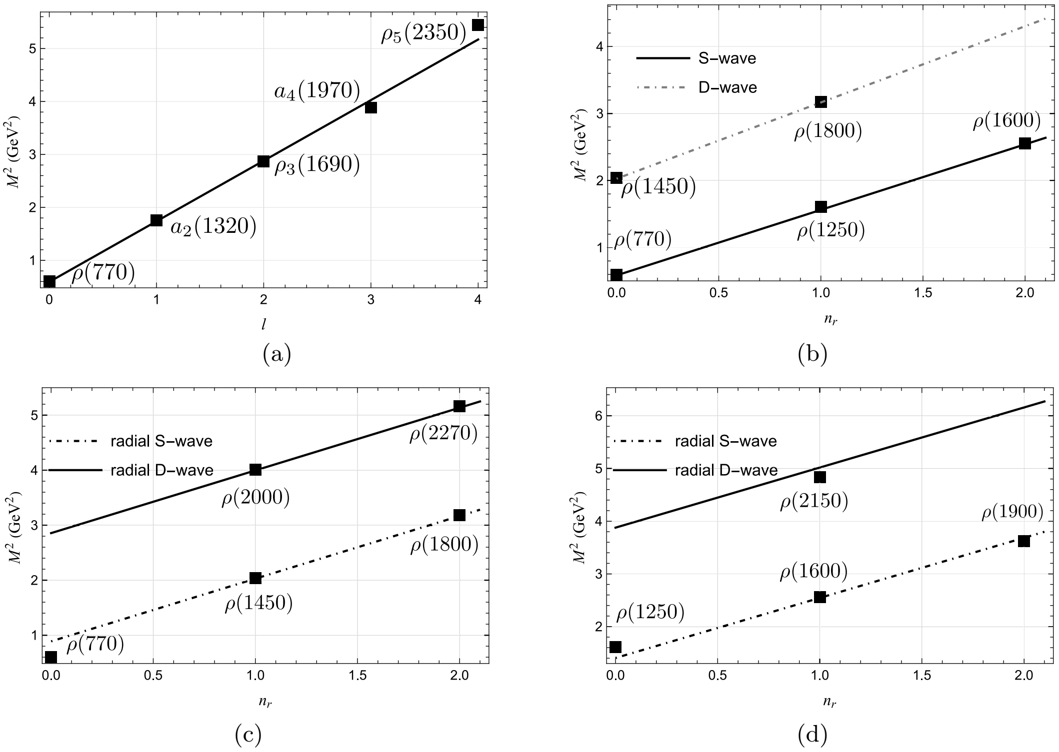

Fig. 1.: Squared mass trajectories versus number of quanta N for excited ρ mesons: (a) Leading orbital trajectory ($N = l$), (b) Radial S- and D-wave trajectories ($N = n_r$) with naive assignments, (c) , (d) Radial S- and D-wave trajectories ($N = n_r$) with our new assignments. The masses of the corresponding mesons (filled squares) are taken from Ref. [3] for states below 1.8 GeV and Ref. [1] for the other states.

TABLE I.: A possible assignment of observed states and predicted mass of missing states.

$(n_r+1)^{2S+1}L_J$	1^3S_1	2^3S_1	1^3D_1	3^3S_1	2^3D_1	3^3D_1
States in Fig.(1c)	$\rho(770)$	$\rho(1450)$	-	$\rho(1800)$	$\rho(2000)$	$\rho(2270)$
Mass (GeV)	0.77	$\underline{1.42}$	1.69	1.78	2.00	2.27
States in Fig.(1d)	$\rho(1250)$	$\rho(1600)$	-	$\rho(1900)$	$\rho(2150)$	-
Mass (GeV)	1.18	$\underline{1.60}$	1.92	1.92	2.20	2.44

3. – Covariant representation for the WFs of the excited ρ-mesons and study of one-pion couplings

In the COQM [6], the WF of composite mesons are described as

$$\Psi(X,x)_{\alpha}^{(\pm)\beta} = \frac{1}{\sqrt{2P_0}} e^{\mp iPX} f^{(l,n_r)}(x,P)^{\nu\rho\cdots} \left(W(P)_{\alpha}{}^{\beta}\right)_{\nu\rho\cdots} \tag{2}$$

where the P^μ, X^μ, x^μ represent the center of mass (CM) momentum, the CM coordinate, the relative coordinate of $q\bar{q}$ system, respectively. The α and β denote the Dirac spinor indices for respective constituents. The WF consists of a space-time part $f(x,P)$ and a spin part $W(P)$, where each part is covariantly extended. More concretely, the space-time WF is given by the oscillator eigen-functions of the squared mass operator and the spin WF by the direct product of Dirac spinors boosted with of the velocity of the hadrons. Since the spin WF has 16 independent components, we can adopt the components not previously included in the NRQM to construct the WF of excited vector mesons.

In the following, we limit our discussion to the description of the WF for $\rho(1250)$. Note that, in this scheme, the physical state of $\rho(1250)$ is described by the possible superposition of relevant components included in the COQM, not just those included in the NRQM. As a matter of fact, the analysis of the Regge trajectory suggests that the WF of $\rho(1250)$ contains a large amount of the another types of 1^3S_1 component in this framework. To check the validity of this assignments, we study the strong coupling with one-pion by using our covariant WF. In this study, we treat the emitted pion as a point-like Nambu-Goldstone particle. The effective coupling [10] with one-pion consistent with low-energy theorem is given by

$$i\int d^4X\mathcal{L} = \int d^4X \int d^4x \ \mathrm{tr}\left(\bar{\Psi}(x,X)\frac{g_A}{\sqrt{2}f_\pi}2\gamma_5\sigma_{\mu\nu}\overleftrightarrow{\partial_1^{\nu}}\Psi(x,X)\right)\partial_1^\mu\phi_\pi + (1\leftrightarrow 2), \tag{3}$$

where $\partial_{1,2} = \frac{1}{2}\partial_X \pm \partial_x$. In actual calculations we take the parameters as $f_\pi = 93$ MeV and $g_A \simeq 1$. Note that the Gaussian parameter of the oscillator WF is uniquely determined to be $\beta = 0.38$ GeV by the study of the Regge trajectory. Using the above action, the decay amplitudes of "$\rho(1250)$"$\to \rho\pi, \omega\pi$ can be evaluated in a conventional manner. In Tab.(II), we show the calculated results of the coupling to $\pi\pi$ and $\omega\pi$ channel for each possible component that can couple to physical $\rho(1250)$. Here respective effective couplings are defined as

$$\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi}\varepsilon_{abc}\rho_\mu^a\pi^b\partial^\mu\pi^c, \quad \mathcal{L}_{\rho\omega\pi} = \frac{g_{\rho\omega\pi}}{M}\delta_{ab}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\rho_\nu^a\partial_\alpha\omega_\beta\pi^b. \tag{4}$$

TABLE II.: Effective couplings calculated with the corresponding components of $\rho(1260)$. The $g_{\rho\pi\pi}$ and $g_{\rho\omega\pi}$ are defined as Eq. (4).

Component	Internal WF $\Phi(x, P)$	$g_{\rho\pi\pi}$	$g_{\omega\rho\pi}$
$2^3S_1{}^{(++)}$	$f_0(x,P)\frac{-\gamma^\mu\epsilon_\mu(P)}{2\sqrt{2}}(1-\not{v})$	1.08	2.57
$1^3S_1{}^{(--)}$	$f_0(x,P)\frac{\gamma^\mu\epsilon_\mu(P)}{2\sqrt{2}}(1+\not{v})$	6.98	0.949
$1^1P_1{}^{(+-)}$	$\sqrt{2\beta^2}x^\mu f_0(x,P)\epsilon_\mu(P)\frac{1}{2}$	0	15.9
$1^3P_1{}^{(+-)}$	$\sqrt{2\beta^2}x^\mu f_0(x,P)\epsilon_{\mu\nu}(P)\frac{-\gamma^\nu\gamma_5}{2}$	0	6.9

Concerning the conventional $2^3S_1{}^{(++)}$ component, the couplings to both mode are quite small due to the WF nodes and boosting effects. This fact makes it difficult to explain $\rho(1250)$ as conventional $2^3S_1{}^{(++)}$. Instead, the $1^3S_1^{(--)}$ state is found to couple strongly to $\pi\pi$ and to be broad, consistent with the result in Ref. [3]. On the other hand, as confirmed by our analysis, there seems to be a certain coupling $\rho(1250) \to \omega\pi$ [4]. Interestingly, the P-wave components with negative norm couple only to the $\omega\pi$ mode. Thus, the inclusion of such contributions would allow us to fully reproduce the results of analysis.

4. – Concluding Remarks

In order to clarify the "$q\bar{q}$" assignments for excited ρ mesons, including the controversial $\rho(1250)$, we have studied their squared mass spectra in a framework of the COQM. As a result, we identify two different S-wave radial trajectories, the conventional states ($S^{(++)}$) to which $\rho(770)$, $\rho(1450)$, $\rho(1800)$ belong, and the another states ($S^{(--)}$) to which $\rho(1250)$, $\rho(1600)$, $\rho(1900)$ can be assigned. In addition, we point out the possible existence of $\rho(1.69)$ and $\rho(1.92)$ as the $1D^{(++)}$ and $1D^{(--)}$ states. A closer look at the PDG data for $\rho(1700)$ suggests a clear difference in width between data with masses around 1.6 GeV and those with masses around 1.8 GeV. This fact may give some clues about the existence of the two missing 1^3D_1 states in our classification.

We have also examined their couplings to a pion to check the validity of their assignments. The results indicate that the WF of $\rho(1250)$ can be described primarily as a superposition of $1^3S_1{}^{(--)}$ and $1^{1,3}P_1{}^{(+-)}$ states, rather than the conventional $2^3S_1{}^{(++)}$ state, in the COQM.

REFERENCES

[1] WORKMAN R. L. *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.*, **2022** (2022) 083C01.
[2] GODFREY S.,ISGUR N., *Phys. Rev. D*, **32** (1985) 189.
[3] HAMMOUD N., KAMINSKI R., NAZARI V., RUPP G., *Phys. Rev. D*, **102** (2020) 054029.
[4] KOMADA T., MAEDA T., TSURU T., YAMADA K., these proceedings.
[5] ISHIDA S., ODA M., *Prog. Theor. Phys.*, **89** (1993) 1033.
[6] ISHIDA S., ISHIDA M. Y., ODA M.,*Prog. Theor. Phys.*, **93** (1995) 939.
[7] ISHIDA S., YAMADA K., *Phys. Rev. D*, **35** (265) 1987.
[8] ISHIDA S., YAMADA K., ODA M.,*Phys. Rev. D*, **40** (1989) 1497.
[9] BEVEREN E.V., RUPP G., RIJKEN T. A., DULLEMOND C.,*Phys. Rev. D*, **27** (1983) 1527.
[10] FEYNMAN R. P., KISLINGER M., RAVNDAL. F,*Phys. Rev. D*, **3** (1970) 2706.