

Machine Learning exotic hadrons

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Summary. — We show that a neural network trained with synthetic differential intensities calculated with scattering length approximated amplitudes classifies the $P_c(4312)^+$ signal as a virtual state located at 4th Riemann sheet with very high certainty. This is in line with the results of other analyses but surpasses them by providing a simultaneous evaluation of probabilities of competing scenarios, like e.g. the interpretation as a bound state. Using the Shapley Additive Explanations we identified the energy bins which are key for the physical interpretation.

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1. – Introduction

Since the observation of the first tetraquark candidate $\chi_{c1}(3872)$ at Belle [1], the analysis of the structure and production mechanisms of exotic hadrons has been at the center of both experimental and theoretical effort of hadron physics community. The $P_c(4312)^+$ state observed in the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay at LHCb [2] as a peak in the $J/\psi p$ invariant mass distribution drew a lot of attention. **If confirmed as a resonance, $P_c(4312)^+$ would necessarily consist of five valence quarks.** The state's proximity to the $\Sigma_c^+ \bar{D}^0$ threshold pointed towards hadronic molecule interpretation but other interpretations followed. In [3] a bottom-up approach was taken, where a general analytical structure of the unitary amplitude was exploited. The minimally biased model pointed towards interpretation of the $P_c(4312)$ as a virtual state. Here we also exploit the bottom-up approach but rather than constraining the model parameters with experimental, data we use an artificial neural network to map the structure of the amplitude poles onto physically interpretable classes.

2. – Physics basis

We consider a two channel model with channels $J/\psi p$ and $\Sigma_c^+ \bar{D}^0$ labelled 1 and 2, respectively. The starting point of the model construction is the coupled channel form of the inverse amplitude which reads

$$(1) \quad T_{ij}^{-1} = M_{ij} - ik_i \delta_{ij},$$

where k_i are channel momenta and M_{ij} is a matrix which was shown in [4] to be singularity free in the energy region close to threshold, and thus can be Taylor expanded there. **Further we use the first term of this expansion, i.e. a constant matrix M_{ij} .** The channel momenta compatible with this approximation are defined as $k_i = \sqrt{s - s_i}$, where

$$(2) \quad s_1 = (m_p + m_{J/\psi})^2 \quad \text{and} \quad s_2 = (m_{\Sigma_c^+} + m_{\bar{D}^0})^2$$

are threshold energies in respective **channels**. The explicit form of the elastic amplitude for the pJ/ψ **channel reads**

$$(3) \quad T_{11} = \frac{M_{22} - ik_2}{(M_{11} - ik_1)(M_{22} - ik_2) - M_{12}^2}.$$

where M_{11} , M_{22} and M_{12} are model parameters. It is assumed that the state under study has a well defined spin and a contribution of other partial waves boils down to the incoherently added background to be parametrized as a first order polynomial in energy squared $B(s) = b_0 + b_1 s$. **In this study we are going to model the $J/\psi p$ invariant mass distribution observed in the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay. The Λ_b^0 decay vertex function similarly to the background can be parametrized as the first degree polynomial $P_1(s) = p_0 + p_1 s$.** So, the differential **energy-dependent** intensity reads

$$(4) \quad \frac{dN}{d\sqrt{s}} = \rho(s) [|P_1(s)T_{11}(s)|^2 + B(s)],$$

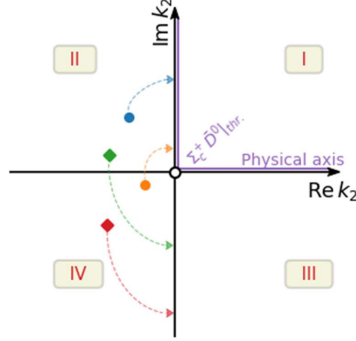


Fig. 1. – Pole structure in the complex k_2 -plane. Poles marked with circles evolve to bound states while those marked with diamonds to virtual states with decreasing inter-channel coupling (figure adopted from [6]).

with $\rho(s)$ being the phase space factor. In principle, Eq. 4 contains also the term describing a transition from $\Sigma_c^+ \bar{D}^0$ to $J/\psi p$ channel. Here, however, we are mostly concerned with the pole structure of the amplitude on the 4-sheeted Riemann surface, which for the T_{21} is the same as for T_{11} . So, the effect of T_{12} can be absorbed into $P_1(s)$ and $B(s)$ functions (see [3] for detailed discussion). To account for the finite mass resolution of the experiment, Eq. 4 was convolved with the gaussian with mass dependent width, see Supplementary Material of [5] for details. By construction, the amplitude of Eq. (3) has 4 poles but only 2 of them, being complex conjugate, lie close to the physical region on either 2-nd or 4-th Riemann sheet and are relevant for the shape of the observed signal. With decreasing M_{12} , i.e. when two channels decouple, these poles approach the imaginary k_2 -axis with positive value of $\text{Im}k_2$ for the bound state and negative one for the virtual state. In the scattering length approximation this sign is controlled by the sign of M_{22} parameter (positive for the virtual state, negative for the bound state). Thus our classification problem consists in mapping the intensity shape into 4 combinations of bound vs. virtual states on either 2-nd or 4-th sheet. In what follows we label these classes $b|2$, $b|4$, $v|2$ and $v|4$, respectively.

3. – Neural network model

The intensity function of Eq. (4) depends on 7 parameters - M_{11} , M_{22} , M_{12} , p_0 , p_1 , b_0 and b_1 . The training data set of 10^5 samples was produced by randomly sampling the model parameters within suitably chosen intervals, see [6] for details on training data generation and ANN architecture. The artificial neural network (ANN) classifier has been trained by feeding model intensities split into 65 energy bins to the input layer and matching them with respective class assignments using the categorical cross entropy as a loss function. To make the ANN model more robust against experimental uncertainties, some portion of the Gaussian noise was added to the simulated data. In Fig.2 we show how the classification accuracy depends on the level of noise (5% noise was used in actual prediction, which is our estimate of the average error percentage as related to the maximum intensity) as well as the model accuracy on the validation data set, which was held out during training. Noteworthy, the accuracy exceeds 90% for all classes.

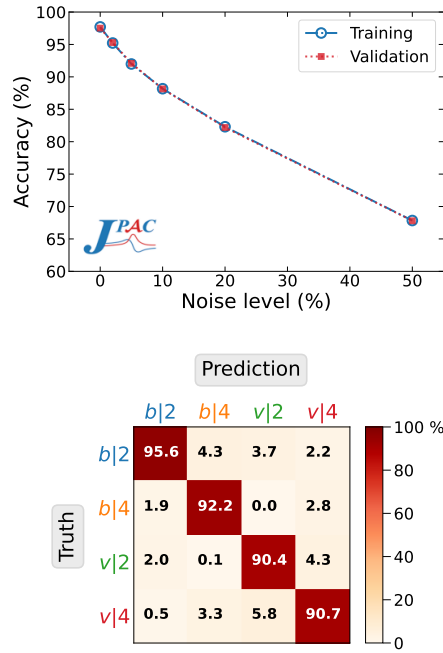


Fig. 2. – ANN classification accuracy as a function of the noise level (upper panel) and model accuracy on the validation data set (lower panel).

4. – Results

Here we discuss the results of the analysis applied to the $J/\psi p$ invariant mass distribution weighted with $\cos \theta_{P_c}$ i.e. the value of the cosine of the angle between final state kaon and the J/ψ as measured in the P_c^+ rest frame. See [6] for results obtained for other datasets. To obtain statistically significant predictions from the ANN classification model a distribution of class assignments has been produced in two ways. First, a

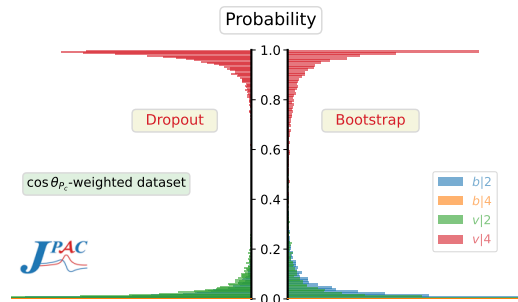


Fig. 3. – Histograms reflecting the class assignment probabilities obtained with dropout and bootstrap methods.

bootstrap method was used [7] where a set of 10^4 examples was produced by sampling

with Gaussian distribution around experimental central values of intensity with standard deviation equal to experimental error. This sample was fed to the trained model and produced the 4-class distribution. Secondly, a dropout method was used [8], where the Gaussian noise was simulated by randomly switching-off some weights in the trained model. Histograms of class assignment probabilities based on two methods are shown in Fig. 3. Both of them indicate that $v|4$, i.e. a virtual state corresponding to the pole on 4-th Riemann sheet is by far a most probable class assignment. Finally the SHapley

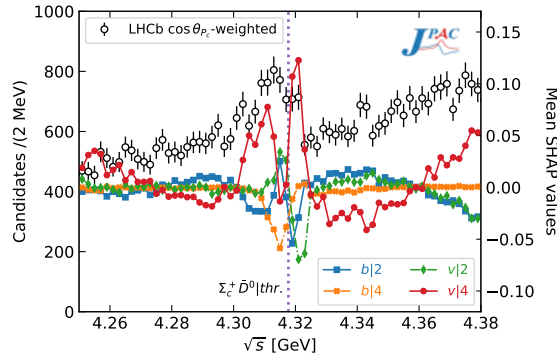


Fig. 4. – SHAP values for four possible class assignments and energy bins around $P_c(4312)$ maximum.

Additive exPlanations (SHAP) [9] were used to identify energy bins which are most important for the ANN model to choose a particular class. As shown in Fig. 4 the energy bins close to the $\Sigma_c^+ \bar{D}^0$ threshold are decisive for $v|4$ class assignment, which provides an *ex post* justification of the scattering length approximation of the amplitude. **It is worth mentioning that this analysis can be extended in several directions like taking into account the interference with the background, microscopically motivated description of the production mechanism or reducing the bias related to limited intervals of parameters used for the training dataset generation.**

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