The compositeness and the role of the interaction range

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Summary. — A new approach is discussed on how to obtain information on the compositeness of molecular states from combined information of the scattering length of the hadronic components, the effective range, and the binding energy, in which the range of the interaction is considered explicitly. This approach can improve the original formula of Weinberg, which was just obtained only in the limit of very small binding and zero range interaction. An application to three different cases, deuteron, $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$, has been done. The results show that we can determine simultaneously the value of the compositeness within a certain range, as well as get qualitative information on the range of the interaction.

1. – Introduction

We start from the pioneer work of Weinberg formalism for deuteron [1], which was obtained in the limit of very small binding and zero range interaction in coordinate space. The scattering length a, effective range r_0, and scattering matrix are given

$$
\begin{aligned}
a &= R\left[\tfrac{2X_W}{1+X_W} + O(\tfrac{R_{\rm typ}}{R})\right], \\
r_0 &= R\left[-\tfrac{1-X_W}{X_W} + O(\tfrac{R_{\rm typ}}{R})\right], \\
f &= \tfrac{1}{k\cot\delta - ik} \approx \tfrac{1}{-\frac{1}{a}+\frac{1}{2}r_0 k^2 - ik}\,,
\end{aligned}
\tag{1}
$$

respectively.

The formulas of Weinberg are derived assuming a zero range interaction and small binding energies. We aim at removing these restrictions in the present work.

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Hence, our aim here is to propose a new method in order to improve Weinberg's formalism. We hope to determine simultaneously the compositeness and range of the interaction from the combined properties of scattering length, effective range and binding energy. We successfully applied our new formalism to three different cases with both small and large binding energy [2].

2. – Formalism

2˙1. *meson-meson system*. – We start from a potential written in momentum space as

$$\langle \boldsymbol{p}'|V|\boldsymbol{p}\rangle = V(\boldsymbol{p}',\boldsymbol{p}) = V\,\theta(q_{\max}-p')\theta(q_{\max}-p)\,, \tag{2}$$

with $q_{\max}$ the range of the potential in momentum space. Its inverse would provide the range of the interaction in coordinate space. Next we solve the Bethe-Salpeter equation with this potential to obtain the T-matrix (four momentum).

By using Cauchy's residues, the q^0 integration is readily done

$$\begin{aligned} T(\boldsymbol{p}',\boldsymbol{p}) &= V(\boldsymbol{p}',\boldsymbol{p}) \\ &+ \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} V(\boldsymbol{p}',\boldsymbol{q})T(\boldsymbol{q},\boldsymbol{p})\frac{w_1(\boldsymbol{q})+w_2(\boldsymbol{q})}{2w_1(\boldsymbol{q})w_2(\boldsymbol{q})}\,\frac{1}{s-(w_1(\boldsymbol{q})+w_2(\boldsymbol{q}))^2+i\epsilon}\,, \end{aligned} \tag{3}$$

with $w_i(\boldsymbol{q}) = \sqrt{\boldsymbol{q}^2+m_i^2}$. By expanding in a power series, we obtain the algebraic equation

$$T = [1-VG]^{-1}V\,, \tag{4}$$

with G the loop function of meson-meson system. In Ref. [3] we ever discussed how to construct an effective potential $V_{\rm eff}$ and evaluate the compositeness of finding molecular state in one channel, in which the energy dependence potential was taken the linear function for meson-meson system,

$$V = V_{\rm eff} = V_0 + \beta\,(s-s_0)\,. \tag{5}$$

The scattering matrix is written

$$T(s) = \frac{1}{[V_0+\beta(s-s_0)]^{-1}-G(s)}\,, \tag{6}$$

it has a pole at s_0, thus we can obtain

$$V_0 = \frac{1}{G(s_0)}\,. \tag{7}$$

Next we establish the connection of amplitude with quantum mechanics

$$\frac{1}{C\Big\{[\frac{1}{G(s_0)}+\beta(s-s_0)]^{-1}-G(s)\Big\}} \approx \frac{1}{-\frac{1}{a}+\frac{1}{2}r_0k^2-ik}\,, \tag{8}$$

with C normalization constant.

Now we obtain at threshold

$$8\pi\sqrt{s_{\rm th}}\Big\{\big[\frac{1}{G(s_0)}+\beta(s_{\rm th}-s_0)\big]^{-1}-{\rm Re}G(s_{\rm th})\Big\}=\frac{1}{a}\,, \tag{9}$$

and the derivative to k^2 at threshold

$$\begin{aligned}-\frac{1}{2}r_0 &= \frac{8\pi}{2\sqrt{s_{\rm th}}}\Big[\big[\frac{1}{G(s_0)}+\beta(s_{\rm th}-s_0)\big]^{-1}-{\rm Re}G(s)_{\rm th}\Big]\frac{s}{w_1(k)w_2(k)}|_{s_{\rm th}}\\ &+8\pi\sqrt{s_{\rm th}}\Big[-\beta\big[\frac{1}{G(s_0)}+\beta(s_{\rm th}-s_0)\big]^{-2}-\frac{\partial{\rm Re}[G(s)]}{\partial s}|_{s^+_{\rm th}}\Big]\frac{s}{w_1(k)w_2(k)}|_{s_{\rm th}}\,.\end{aligned} \tag{10}$$

Finally we evaluate Z of the nonmolecular compositeness

$$\begin{aligned}Z &= -g^2\,G(s_0)^2\,\beta\,,\\ g^2 &= \lim_{s\to s_0}(s-s_0)T(s)=\frac{1}{-G(s_0)^2\beta-\frac{\partial G}{\partial s}|_{s_0}}\,.\end{aligned} \tag{11}$$

2$\cdot$2. *nucleon-nucleon system.* – An extension to nucleon-nucleon case, similar formalism can be derived for deuteron. The potential is now

$$V=V_0+\beta(E-E_0)\,. \tag{12}$$

We also obtain at threshold

$$\tfrac{2\pi E_{\rm th}}{m_1m_2}\Big\{\big[\tfrac{1}{G(E_0)}+\beta(E-E_0)\big]^{-1}-{\rm Re}G(E_{\rm th})\Big\}=\tfrac{1}{a}\,, \tag{13}$$

and the derivative to k^2 at threshold

$$\begin{aligned}-\frac{1}{2}r_0 &= \frac{2\pi}{m_1m_2}\Big[[\frac{1}{G(E_0)}+\beta(E_{\rm th}-E_0)]^{-1}-{\rm Re}G(E)_{\rm th}\Big]\frac{E}{2E_1(k)E_2(k)}|_{E_{\rm th}}\\ &+\frac{2\pi E_{\rm th}}{m_1m_2}\Big[-\beta[\frac{1}{G(E_0)}+\beta(E_{\rm th}-E_0)]^{-2}-\frac{\partial{\rm Re}[G(E)]}{\partial E}\Big]\frac{E}{2E_1(k)E_2(k)}|_{E_{\rm th}}\,.\end{aligned} \tag{14}$$

Finally we evaluate Z of the nonmolecular compositeness

$$\begin{aligned}Z &= -g^2\,G(E_0)^2\,\beta\,,\\ g^2 &= \lim_{E\to E_0}(E-E_0)T(E)=\frac{1}{-G(E_0)^2\beta-\frac{\partial G}{\partial E}|_{E_0}}\,,\\ \beta &= \frac{1}{E_{\rm th}-E_0}\Big\{\big[\frac{1}{a}\frac{1}{2\pi}\frac{m_1m_2}{m_1+m_2}+{\rm Re}\,G(E_{\rm th})\big]^{-1}-\frac{1}{G(E_0)}\Big\}\,.\end{aligned} \tag{15}$$

3. – Results

In Fig. 1 we show the calculated results of nonmolecular compositeness Z and effective range r_0 for deuteron with the $\mathrm{q_{max}}$. It is seen that $q_{\max} \geq 140$ MeV, r_0^{theory} is close to r_0^{exp}, and below this value there is a noticeable disagreement. The small $q_{\max}$ indicates that the range of the nucleon-nucleon interaction in coordinate space is rather large. This provides a picture far closer to the actual molecular nature than Weinberg's equations. We also find that when $q_{\max} > 140$ MeV, $Z < 0$, thus this situation should be discarded. Starting from $q_{\max} = 100$ MeV, $Z < 0.25$, indicating a strong molecular component; at $q_{\max} = 140$ MeV, $Z = 0$, deuteron is a molecular state.

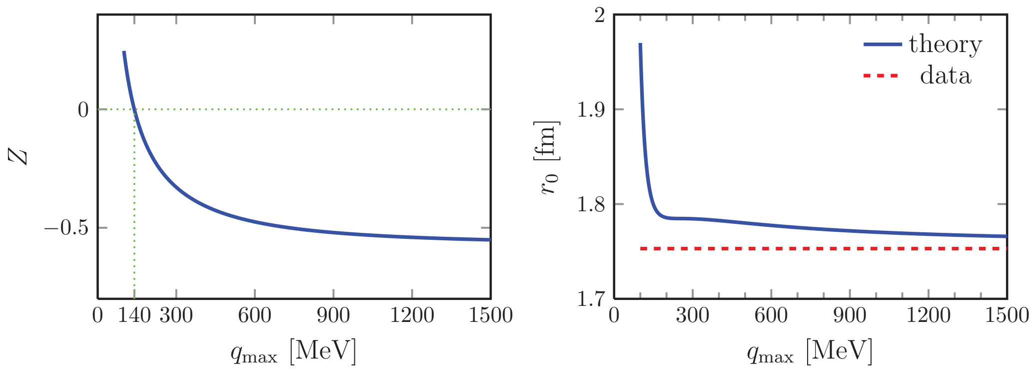

Fig. 1. – The calculated results of Z (left) and r_0 (right) for deuteron state.

In Fig. 2 we show the first case of meson-meson system, the calculated results for $D^*_{s_0}(2317)$. We can see the good agreement with those in Ref. [4] obtained from QCD lattice analysis of the finite volume levels. When $q_{\max} > 400$MeV, we can see the agreement between r_0^{theory} and r_0^{exp}. When $q_{\max} \geq 725$ MeV, $Z \leq 0$, indicating the range of the interaction with light vector exchange; at $q_{\max} = 400$ MeV, $Z < 0.4$, indicating a DK molecular component with probability larger than 60%.

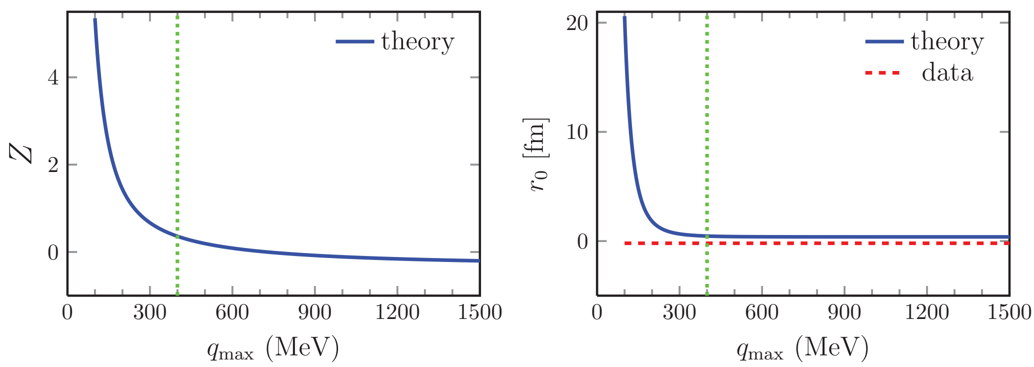

Fig. 2. – The calculated results of Z (left) and r_0 (right) for $D^*_{s_0}(2317)$ state.

In Fig. 3 we show another case of meson-meson system for $D^*_{s_1}(2460)$, the calculated results are also in good agreement with the findings of [4]. However, independent of $q_{\max}$, the Z never becomes zero and reaches a value of 0.2 for large $q_{\max}$. We can see that $0.3 < Z < 0.6$, indicating a KD^* molecular component with probability $\geq 40\%$, which shows that the ηD_s^* channel is mostly responsible for the remaining probability.

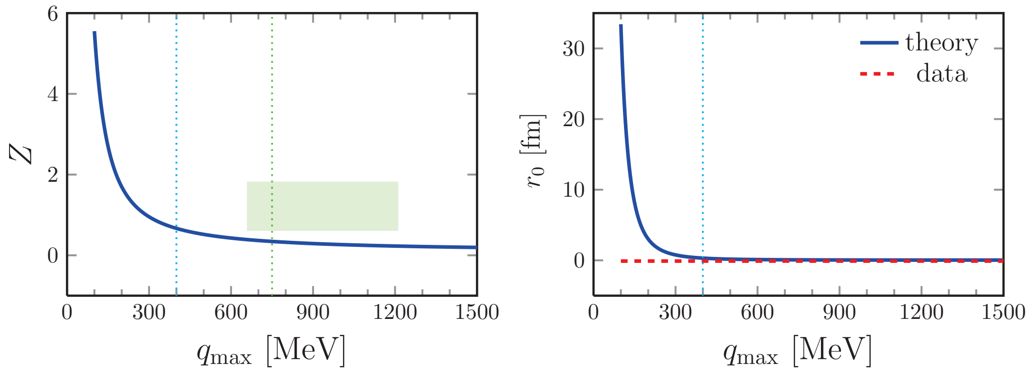

Fig. 3. – The calculated results of Z (left) and r_0 (right) for $D_{s1}^*(2460)$ state.

4. – Summary

We propose an new approach to evaluate simultaneously the effective range and non-molecular compositeness in order to improve the Weinbergs formalism which was obtained in the limit of very small binding and zero range interaction in coordinate space. We find that the range of the interaction is very important in describing meson-meson and nucleon-nucleon systems. The combined information of scattering length, the effective range, and the binding energy could provide a fair information on the $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$, both cases are bound by about $40-45$ MeV. The deuteron has a longer range in coordinate space than the cases of $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$. The molecular compositeness can be determined simultaneously within a certain range, as well as get qualitative information on the range of the interaction for the three different cases with small or larger binding.

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