

## Employing Approximate Symmetries for Hidden Pole Extraction

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**Summary.** — Recent lattice analyses of the  $D\pi$  scattering by Hadron Spectrum Collaboration(HadSpec) report only one pole in the  $D_0^*$  channel. This is in odds with the unitarised chiral perturbation theory analyses, which predict the  $D_0^*(2300)$  as the interplay of two poles. We provide an explanation for this contradiction — the existence of a hidden pole. We further show that the hidden pole can be better extracted from the lattice data by imposing SU(3) flavour constraints on the fitting amplitudes.

### 1. – Introduction

From unitarized chiral perturbation theory analyses, the structure of  $D_0^*(2300)$  and  $D_1(2430)$  can be understood as the interplay of two poles, corresponding to two scalar/axial-vector isospin doublet states with different SU(3) flavor content [1–7]. These states emerge from non-perturbative dynamics of  $D$  mesons scattering off the Goldstone boson octet. This two pole picture solves various problems that the experimental observation posed [6]. However, in the recent lattice analysis from HadSpec of  $D\pi - D\eta - D_s\bar{K}$  coupled channel system at higher pion masses, only one pole was reported in the  $D_0^*$  channel, while it was not possible to extract reliable parameters of a second pole from the lattice data [8]. We here show that a higher pole does indeed exist in the lattice data albeit on hidden sheets and propose the use of SU(3) flavour constraints to better capture its location

### 2. – Analytic Properties of the T-matrix

Except for branch cuts and poles the T-matrix is analytic over the whole complex plane. Right-hand cuts appear at every channel threshold, doubling the number of Riemann sheets. The sign of the imaginary part of the centre of mass momentum( $\text{Im}(p_{cm})$ ) is used to label the sheets; see tab.2 of ref. [9] for details. The physical sheet is the

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Riemann sheet in which  $\text{Im}(p_{cm}) > 0$  for all channels. Depending on location, the poles correspond to bound states, virtual states or resonances. Hidden poles are resonance poles located on a sheet beyond the point at which it is connected directly to the physical sheet. Hence their effect on the amplitude can only be felt at the threshold, where the corresponding sheet is connected to the physical sheet. For example, the blue and orange points in fig. 1 are the poles which would produce a 'conventional' resonance signal. While the black point would correspond to a hidden pole. And its effect on the amplitude would be felt at the second channel threshold. Poles located on the sheets closest to the physical sheet have the largest influence on the amplitude.

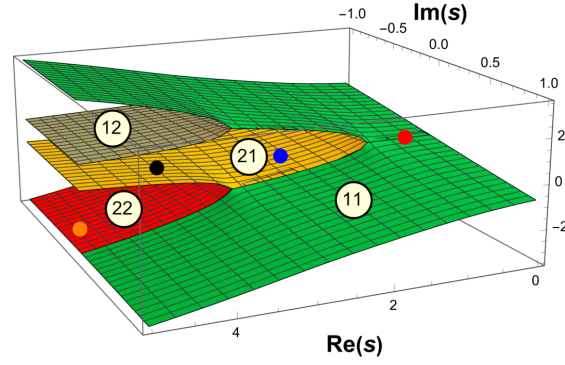


Fig. 1. – Illustration shows the sheet structure in case of a two channel case. The pole shown in red correspond to a bound state while blue, black and orange represent resonances.

### 3. – Analysis of the Lattice Study Amplitude

The analysis in ref. [8] was done by using a sizeable set of K-matrix parametrisations of the kind

$$(1) \quad K_{ij} = \frac{(g_i^{(0)} + g_i^{(1)}s)(g_j^{(0)} + g_j^{(1)}s)}{m^2 - s} + \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)}s,$$

where  $i$  and  $j$  label the various channels,  $g_i^{(n)}, \gamma_{ij}^{(n)}$  and  $m$  are real parameters determined from the fit to the lattice data. For the explicit form of the T-matrix see ref. [8, 9]. A pion mass of  $m_\pi = 391\text{MeV}$  was used in the lattice study.

**3'1. Pole Search.** – In our labelling the closest sheets to the physical sheet above the respective thresholds are the sheets labelled RS211, RS221 and RS222. The lower pole in the studied channels appear as a bound state at the pion mass used in the lattice study and is accordingly located on sheet RS111 [8]; the same was seen in UChPT [5].

In our analysis we find in addition to the reported bound state pole, additional higher poles in sheets RS211, RS221 and RS222. These additional poles were found in almost all parametrisations employed by HadSpec we analysed. Figure 3 shows the locations of these higher poles in sheet RS221 from the HadSpec amplitude parametrisations and of the most prominent higher pole from the UChPT amplitude at the unphysical masses employed in the lattice study [5].

From fig. 3 we see that their locations do not scatter wildly between the parametrisations but show a significant correlation. All higher poles are seen to lie at higher energies than the point where their respective Riemann sheet connects to the physical sheet, *i.e* they are located on hidden sheets. For example, the poles on RS221 lie beyond the  $D_s\bar{K}$  threshold. They are shielded by RS222 and their effect can hardly be seen above the  $D_s\bar{K}$  threshold. From the plot we see an increase in their widths as their distance from the threshold increases. We can see that the UChPT pole also lies in line with this correlation. To have an estimate of the effects of the poles at the respective threshold we plot the effective couplings from their residues to the different channels, with respect to the distance of the poles from the threshold. We quantified the distance as

$$(2) \quad \text{Dist} = M - M_{\text{thr}},$$

where  $M_{\text{thr}}$  is the relevant threshold and  $M$  is the real part of the pole. Since the residues also drive the widths of the poles, eq. (2) was chosen to quantify the distance. Figure 2 shows the effective couplings *vs* the distance plots. A straight line was fitted to extract the y - intercept to use as an estimate of the effect of the poles at threshold. From the plots it is seen that this value is well constrained. We interpret this as the lattice data requires, in addition to the bound state pole, a second pole. The ill constrained second pole locations are being balanced by their enhanced residue. This mechanism was also observed in the case of  $f_0(980)$  and  $a_0(980)$  [10]. This calls for a stronger constraint in the amplitude parametrisation to better confine the location of the higher hidden pole.

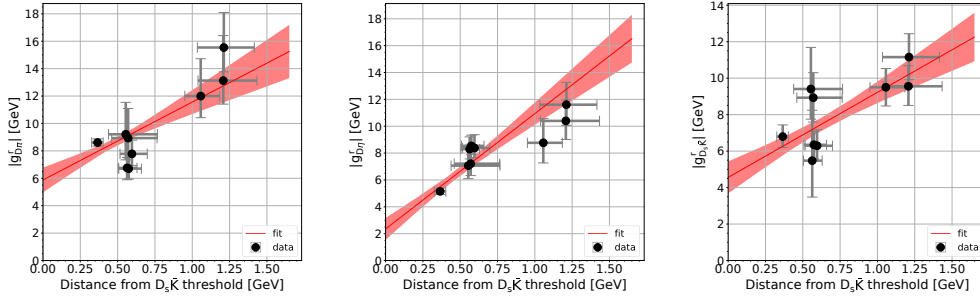


Fig. 2. – The distance of the real part of the pole from threshold versus the effective coupling of the pole to the  $D\pi$  channel (left),  $D\eta$  channel (middle) and  $D_s\bar{K}$  channel (right). The red line shows the straight line fit. The red band encloses the  $1\sigma$  uncertainty of the fit.

#### 4. – SU(3) Flavour Symmetry

We put forward a SU(3) flavour constrained K-matrix to better capture the second pole location. Employing SU(3) flavour symmetry is justified as we work with higher pion masses which leads to lower pion-kaon mass difference, moreover we are interested in the higher mass range of the second pole. In these circumstances the leading SU(3) breaking effects come from the loop functions.

The relation between the SU(3) flavour basis and the isopin-symmetric particle basis is given by

$$(3) \quad \begin{pmatrix} |[3]\rangle \\ |[6]\rangle \\ |[1\bar{5}]\rangle \end{pmatrix} = U \begin{pmatrix} |D\pi\rangle \\ |D\eta\rangle \\ |D_s\bar{K}\rangle \end{pmatrix}; U = \begin{pmatrix} -3/4 & -1/4 & -\sqrt{3/8} \\ \sqrt{3/8} & -\sqrt{3/8} & -1/2 \\ 1/4 & 3/4 & -\sqrt{3/8} \end{pmatrix}.$$

The flavour constrained K-matrix is of the form

$$(4) \quad K = \left( \frac{g_3^2}{m_3^2 - s} + c_3 \right) C_3 + \left( \frac{g_6^2}{m_6^2 - s} + c_6 \right) C_6 + c_{15} C_{15}$$

where matrices  $C_3$ ,  $C_6$  and  $C_{15}$  contain the SU(3) symmetric coupling strengths and can be read off from eq. (3). For explicit forms see eqs.(20, 21, 22) in ref. [9]. The free parameters are  $g_\alpha$ ,  $c_\alpha$  and  $m_\alpha$ . Several fits with varying number of parameters were carried out to the energy levels in the lattice rest frame. The scheme introduced in ref. [11] was followed to perform the fits.

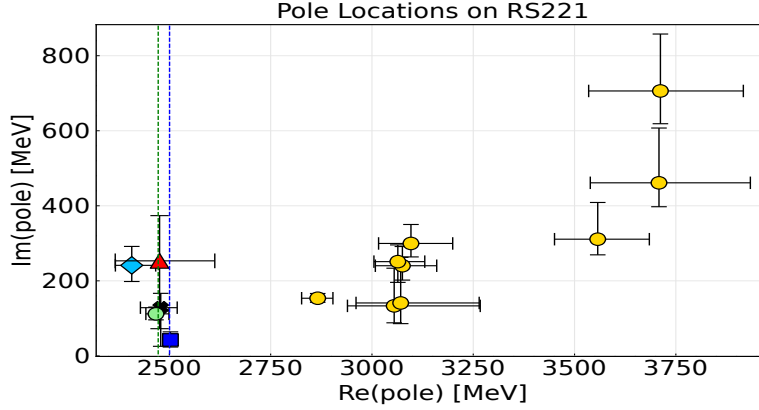


Fig. 3. – The location of poles on RS221 sheet. The  $x$ -axis and  $y$ -axis show the real and imaginary part of energy, respectively. The poles from the amplitude parametrizations employed in ref. [8] are shown in yellow. The pole from the UChPT amplitude [12] is shown in green [5]. The higher pole locations extracted from the various fits are shown in red, cyan, black and blue. The vertical green and blue lines show the  $D\eta$  and  $D_s\bar{K}$  thresholds respectively.

The higher pole locations extracted from fitting to the first four energy levels in the different lattice volumes at rest are shown in fig. 3. We see that the flavour constrained amplitude is successful in better capturing the hidden higher pole. The location of the lower bound state pole is consistent amongst all extractions. For a more in depth discussion on the quality of the fits and inclusion of higher lattice energy levels we refer the reader to ref. [9].

## 5. – Conclusion

A re-analysis of the K-matrix parametrisations provided in ref. [8] found, in addition to the reported bound state pole, additional poles on unphysical Riemann sheets in every

amplitude. Though their locations vary greatly between parametrisations, their effects on the amplitudes were comparable in all parametrisations. We identify the origin of this as their location on hidden sheets. In such scenarios their effect on the amplitude becomes visible at threshold, and their distance from threshold can be balanced by an enhanced residue. To extract the higher pole from lattice data we propose an  $SU(3)$  flavour constrain on the K-matrix. The flavour constrained amplitude is seen to be able to reproduce energy levels well and produce a pole at  $D\eta$  and  $D_s\bar{K}$  thresholds. It is interesting to observe that this pole is in fact consistent with that of the UChPT amplitudes. Such a symmetry constrained amplitude maybe used in analysing other lattice data and also experimental data involving multiple channels.

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