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Charm meson and charm-meson molecule in an expanding hadron gas

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Summary. — We present our studies on the evolution of charm mesons after the kinetic freeze-out of the expanding hadron gas produced in a heavy-ion collision and the thermal correction to a loosely bound charm-meson molecule in a pion gas. The $\pi D^* \to \pi D^*$ reaction rates have t-channel singularities that give contributions inversely proportional to the thermal width of the D. The ratio of the D^0 and D^+ production rate can differ significantly from those predicted using the measured D^* branching fractions. The thermal correction to a loosely bound charm-meson molecule in a pion gas comes primarily from the complex thermal energy shift of the charm-meson constituents.

1. – Introduction

The charm mesons that are most easily observed in high-energy experiments are the pseudoscalar mesons D^+ and D^0 and the vector mesons D^{*+} and D^{*0}. A remarkable aspect of D and D^* is that the D^*-D mass splittings are all very close to the pion mass m_π. A previously unrecognized consequence is that there are charm-meson reactions with t-channel singularities. The simplest reactions with a t-channel singularity are $\pi D^* \to \pi D^*$, which can proceed through the decay $D^* \to D\pi$ followed by the inverse decay $\pi D \to D^*$. The t-channel singularity arises because the exchanged D can be on shell. A general discussion of t-channel singularities was presented in Ref. [1]. One situation in which the effects of t-channel singularities may be observable is in the production of charm mesons from ultrarelativistic heavy-ion collisions. In a hadron gas, the most divergent term from a t-channel singularity in the reaction rate of $\pi D^* \to \pi D^*$ is replaced by a term inversely proportional to the thermal width of the exchanged D. After short

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introductions to the expanding hadron gas from a heavy-ion collision and charm-meson mass shifts and thermal widths, in Sec. **4**, we briefly review the effects of t-channel singularities on the evolution of charm mesons in the expansion of the hadron gas after kinetic freeze-out (see Refs. [2, 3] for more details).

Nature has provided us with at least two exquisite examples of loosely bound hadronic molecules: $X(3872)$ which was discovered by the Belle collaboration in 2003 [4], and $T_{cc}^{+}(3875)$ which was discovered by the LHCb collaboration in 2021 [5]. Their tiny binding energies can be exploited to develop quantitative treatments of their properties, including their interactions with other hadrons. The simplest effective field theory (EFT) that can be applied to a loosely bound charm-meson molecule is a zero-range effective field theory (ZREFT) for nonrelativistic charm mesons, in which they interact only through contact interactions [6]. The behavior of a loosely bound charm-meson molecule in the hadron resonance gas near the hadronization temperature is a very challenging problem. A much simpler problem is the behavior in the hadron gas near the kinetic freeze-out temperature. In Sec. **5**, we show our results of the thermal correction to a loosely bound charm-meson molecule in a pion gas near the kinetic freeze-out temperature (see Ref. [7] for more details).

2. – Expanding hadron gas

The central collision of relativistic heavy ions is believed to produce a quark-gluon plasma consisting of deconfined quarks and gluons which then evolves into a hadron resonance gas (HRG) consisting of hadrons. At the kinetic freeze-out temperature T_{kf} around 115 MeV, the HRG goes out of thermal equilibrium. Afterwards, the momentum distributions of the hadrons are those fixed at temperature T_{kf}. A simple model for the volume of the system after the kinetic freeze-out is longitudinal expansion at the speed of light and transverse expansion at the same speed v_{kf} as at kinetic freeze-out.

Near kinetic freeze-out, the most abundant hadrons in the hadron gas are pions. The abundance of kaons is smaller by about a factor of 5 and other hadrons are even less abundant. The hadron gas can be approximated by a pion gas. The number density for pions in chemical and thermal equilibrium at temperature T is $\mathfrak{n}_\pi^{(\mathrm{eq})} = \int d^3q(2\pi)^{-3}(e^{\omega_q/T}-1)^{-1}$ where $\omega_q = \sqrt{m_\pi^2 + q^2}$ and m_π is the pion mass. The momentum distribution $\mathfrak{f}_\pi$ of the pions is the Bose-Einstein distribution: $\mathfrak{f}_\pi(\omega_q) = (\mathfrak{n}_\pi/\mathfrak{n}_\pi^{(\mathrm{eq})})(e^{\omega_q/T}-1)^{-1}$.

3. – Charm-meson mass shifts and thermal widths

In a pion gas, charm-meson properties are modified by the interactions with the pion gas. The modifications can be described by the self-energy $\Pi(p)$. Its contribution includes a zero-temperature part, a thermal pion part and a thermal charm-meson part. We ignore the zero-temperature part, because it is taken into account in the parameters of the EFT. We ignore the thermal charm-meson part, because the thermal distribution of the charm meson has an exponential suppression factor $\exp(-M/T)$, where M is the charm-meson mass. The thermal pion part can alternatively be represented by the coherent pion forward-scattering. The thermal mass shift and thermal width of a charm meson can be obtained from its self-energy.

At leading order in the pion interactions and leading order in m_π/M, the D and D^* mass shifts δM and δM_* are insensitive to isospin splittings:

$$\delta M \approx (3g_\pi^2/2f_\pi^2)\,\mathfrak{n}_\pi\, m_\pi\, \langle 1/\omega_q\rangle\,, \qquad \delta M_* = -\delta M/3, \tag{1}$$

where $g_\pi = 0.520$, $f_\pi = 131.7$ MeV is the pion decay constant, and $\langle 1/\omega_q \rangle$ is the thermal average of $1/\omega_q$ over the pion momentum distribution. For a pion gas at $T_{\rm kf} = 115\,{\rm MeV}$, $\delta M = 1.257$ MeV. The thermal widths of D and D^* are sensitive to isospin splittings only through the rates $\Gamma_{*b,d}$ for the decays $D^{*b} \to D^d \pi$. The thermal widths of D^a and D^{*a} in the hadron gas are

$$\delta\Gamma_a \approx 3\,\mathfrak{f}_\pi(m_\pi) \sum_c \Gamma_{*c,a}, \qquad \delta\Gamma_{*a} \approx \mathfrak{f}_\pi(m_\pi) \sum_c \Gamma_{*a,c}. \tag{2}$$

For a pion gas at $T_{\rm kf} = 115\,{\rm MeV}$, $\delta\Gamma_+ = 32.6$ keV, $\delta\Gamma_0 = 118.9$ keV, $\delta\Gamma_{*+} = 35.2$ keV and $\delta\Gamma_{*0} = 15.3$ keV. Thermal mass shifts and thermal widths have significant effects on some reaction rates for pions and charm mesons in the expanding hadron gas created by a heavy-ion collision.

4. – Evolution of charm-meson ratios in an expanding hadron gas

We denote the numbers of D^0, D^+, D^{*0}, and D^{*+} by N_0, N_+, N_{*0}, and N_{*+}. The observed numbers of D^0 and D^+ can be predicted in terms of the numbers $(N_a)_0$ and $(N_{*a})_0$ before D^* decays and the measured branching fraction B_{+0} for $D^{*+} \to D^0\pi^+$:

$$N_0 = (N_0)_0 + (N_{*0})_0 + B_{+0}\,(N_{*+})_0\,, \tag{3a}$$
$$N_+ = (N_+)_0 + 0 + (1 - B_{+0})\,(N_{*+})_0\,, \tag{3b}$$

where the last two terms come from D^{*0} and D^{*+} decays, respectively. These simple relations have been assumed in all previous analyses of charm-meson production. We will show that Eqs. (3) can be modified by t-channel singularities. The evolution of the number density $\mathfrak{n}_{D^{(*)}}(\tau)$ of a charm meson in the expanding hadron gas with the proper time τ can be described by a first-order differential equation. The number density decreases because of the increasing volume $V(\tau)$, but it can also be changed by reactions. The time derivative of $\mathfrak{n}_{D^{(*)}}$ has positive contributions from reactions with $D^{(*)}$ in the final state and negative contributions from reactions with $D^{(*)}$ in the initial state. The most relevant reactions include the decays $D^* \to D\pi$ and $D^* \to D\gamma$, the scattering reactions $\pi D \to \pi D$, $\pi D \to D\gamma$, and $\pi D^* \to \pi D^*$ that change the charm-meson flavor, and the scattering reactions $\pi D \to \pi D^*$ and $\pi D^* \to \pi D$ that change the charm-meson spin. We define the fraction of a charm meson with flavor a by $f_{D^{(*)a}} = \mathfrak{n}_{D^{(*)a}}/(\mathfrak{n}_{D^+} + \mathfrak{n}_{D^0} + \mathfrak{n}_{D^{*+}} + \mathfrak{n}_{D^{*0}})$. The proper-time evolution of the fractions $f_{D^{(*)a}}$ with all those most relevant reactions is presented as solid lines in Fig. 1. The fraction f_{D^*} decreases exponentially to 0 on time scales comparable to the D^* lifetimes. The ratio of the multiplicities of D^0 and D^+ is predicted to increase from 1.044 at kinetic freezeout to 2.100 at the detector.

The decays of D^* into $D\pi$ or $D\gamma$ are 1-body reactions that give contributions to the rate equations for charm-meson number densities that are not suppressed by any powers of the pion number density. The evolution of the charm-meson fractions with only decays of D^* is presented as dashed lines in Fig. 1. The t-channel singularities in the reactions $\pi D^* \to \pi D^*$ produce additional 1-body terms. If we only keep the 1-body terms with the vacuum values of D^* decay widths, the evolution equations can be solved analytically. The difference between this ratio and the naive prediction with only D^* decays is -0.079 ± 0.006, which differs from 0 by about 13 standard deviations.

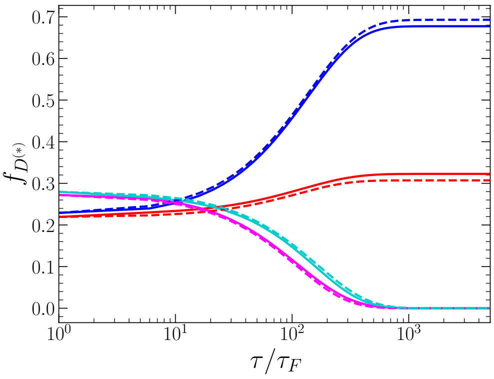

Fig. 1. – Proper-time evolution of the charm-meson fractions f_{D^0} (blue), f_{D^+} (red), $f_{D^{*0}}$ (cyan), and $f_{D^{*+}}$ (magenta). The four fractions add up to 1.

5. – Charm-meson molecule in an pion gas

A loosely bound charm-meson molecule X can be produced by a geometric series of contact interactions between D^* and D in the vacuum, as shown in Fig. 2. The sum of the geometric series of diagrams is

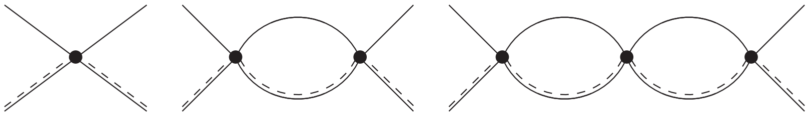

Fig. 2. – The first few diagrams in the geometric series of D^*D contact interaction.

$$\frac{i(2\pi/\mu)}{-\gamma_X+\sqrt{-2\mu\,(E_{\rm cm}+i\Gamma_*/2)}}. \tag{4}$$

where μ is the reduced mass of D and D^*, Γ_* is the decay width of D^*, and $E_{\rm cm}$ is the center-of-mass energy relative to the D^*D threshold. This amplitude has a square-root branch point in E at $E_{\rm cm}=0$. If the binding momentum $\gamma_X>0$, it also has a nearby pole on the real axis at $E_{\rm cm}=-\gamma_X^2/(2\mu)$.

In a pion gas, the properties of a charm-meson molecule would be corrected by the coherent forward scattering of an on-shell pion. For simplicity, we calculate the D^*D self-energy only for charm-meson pairs with kinetic energies of order m_π^2/M. The amplitude with thermal corrections can be repressed as

$$\frac{i(2\pi/\mu)\,Z_X}{-(\gamma_X+\delta\gamma_X)+\sqrt{-2\mu\big[E_{\rm cm}-(\delta\varepsilon_*+\delta\varepsilon)-\zeta_X P^2/(2M_X)+i\Gamma_*/2\big]}}, \tag{5}$$

where $\delta\gamma_X$ is the thermal correction to the binding momentum, Z_X is the residue factor for the molecule in a pion gas, $\delta\varepsilon_{(*)}$ is the thermal rest energy of $D^{(*)}$, ζ_X is a constant determined by charm-meson kinetic mass, and P is the total 3-momentum.

The zero of the denominator in Eq. (5) determines the energy-momentum relation for X. The pole energy of X with constituents $D^{*a}D^b$ and with zero 3-momentum is

$$E_X = (\delta\varepsilon_{*a} + \delta\varepsilon_b) - i\,\Gamma_{*a}/2 - (\gamma_X + \delta\gamma_X)^2/(2\mu). \tag{6}$$

The change of pole energy is the sum of the thermal energy shifts of the two charm-meson constituents and a contribution from the thermal binding momentum $\delta\gamma_X$. The pole energies of $X(3872)$ and $T_{cc}^+(3875)$ in a pion gas at $T = 0$ and $T = 115$ MeV are presented in Table. **5**. For both charm-meson molecules, the real and imaginary parts of the contribution $\delta\gamma_X$ are smaller than those from the energy shifts of the constituents by orders of magnitude.

TABLE I. – *Pole energy and the contribution of pole energy from charm-meson constituents for* $X(3872)$ *and* $T_{cc}^+(3875)$ *at* $T = 0$ *and* $T = 115$ *MeV. The energies are in MeV.*

	E_X ($T = 0$)	E_X ($T = 115$ MeV)	$\delta\varepsilon + \delta\varepsilon_*$ ($T = 115$ MeV)
$X(3872)$	$0.025 - 0.140\,i$	$+1.64 - 0.21\,i$	$+1.61 - 0.07\,i$
$T_{cc}^+(3875)$	$-0.36 - 0.024\,i$	$+1.20 - 0.10\,i$	$+1.56 - 0.08\,i$

6. – Conclusion

We studied the evolution of charm mesons after the kinetic freeze-out of an expanding hadron gas produced by a central heavy-ion collision. We have identified a simple aspect of charm-meson reactions, namely t-channel singularities, that leads to a prediction for the D^0/D^+ ratio that differs significantly from conventional expectations. The identification of an observable effect from t-channel singularities will stimulate the search for other such effects in hadronic, nuclear, and particle physics.

We calculated the thermal energy of a loosely bound charm-meson molecule in a pion gas. Our results suggest that loosely bound charm-meson molecules, such as $X(3872)$ and $T_{cc}^+(3875)$, can remain loosely bound and narrow in the thermal environment of a hadron gas at sufficiently low temperature. Further studies of $X(3872)$ along with studies of $T_{cc}^+(3875)$ in heavy-ion collisions will provide essential insights into the behavior of loosely bound hadronic molecules.

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