

Quark mass dependence of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ states.

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Summary. — We study the light and heavy quark mass dependence of the low-lying charmed mesons in the framework of one-loop $\text{HH}\chi$ PT. The low energy constants are determined by analyzing the available lattice data from different LQCD simulations. Model selection tools are implemented to determine the relevant parameters as required by data with a higher precision. We analyze the HSC energy levels for DK scattering in $I = 0$ for different boosts and two pion masses and perform a global fit with these levels in addition to DK and D^*K scattering energy levels from RQCD and Prelovsek et al. Finally we extract the pion mass dependence of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ resonances.

1. – Introduction

The discovery of exotic hadrons in the heavy quark sector, which cannot be accommodated in terms of $q\bar{q}$ mesons or qqq baryons, like those with a tetraquark or pentaquark structure, has manifested the relevance of hadronic loops in order to explain the masses and other properties of many states in the hadron spectrum [1].

This results in a couple of bound states with binding energy of ~ 40 MeV with respect to the DK and D^*K thresholds, with a probability of 70% for the DK and D^*K components in the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ states, respectively. Given the large amount of parameters in one-loop $\text{HM}\chi$ PT it has not been possible yet to pin down precisely the parameters of the theory. Because of the recent progress in lattice QCD it is worth coming back to this issue. In the first section we study the quark mass dependence of the ground state charmed mesons, D , D^* , D_s and D_s^* within one-loop $\text{HM}\chi$ PT [2] by analyzing the available lattice data.

In the second section, the data of [3] are analyzed considering explicitly the coupling of DK components to $c\bar{s}$ states, including the mass of the latter as fitting parameter. The meson-meson interaction is given by the hidden gauge formalism. At leading order, this interaction is of the same type than the one of $\text{HM}\chi$ PT. The quark mass dependence

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of the D_{s0}^* (2317) properties in both, the light and charm quarks is extracted [4]. In order to perform the extrapolation, we use the analysis of charmed meson masses [5] from the previous section. Moreover, we study the sensitivity in the compositeness of Weinberg to the quark masses within the energy range considered. We compare with previous lattice QCD analyses, and, finally, we perform a global LQCD data fit of [6, 7, 8, 9, 3], extracting accurately the quark mass dependence of the D_{s0}^* (2317) and D_{s1} (2460).

2. – Quark mass dependence of the low-lying charmed mesons at one loop in HH χ PT

The formulas for the low-lying charmed meson masses (D , D^* , D_s and D_s^*) are calculated at one loop in HH χ PT in the Appendices (A.1) and (A.10) of [2]. These are,

$$(1) \quad \frac{1}{4}(M_{D_a} + 3M_{D_a^*}) = m_H + \alpha_a - \sum_{X=\pi,K,\eta} \beta_a^{(X)} \frac{M_X^3}{16\pi f^2} + \sum_{X=\pi,K,\eta} \left(\gamma_a^{(X)} - \lambda_a^{(X)} \alpha_a \right) \frac{M_X^2}{16\pi^2 f^2} \log(M_X^2/\mu^2) + c_a$$

and

$$(2) \quad (M_{D_a^*} - M_{D_a}) = \Delta + \sum_{X=\pi,K,\eta} \left(\gamma_a^{(X)} - \lambda_a^{(X)} \Delta \right) \frac{M_X^2}{16\pi^2 f^2} \log(M_X^2/\mu^2) + \delta c_a.$$

The meson masses are fitted to the lattice data collected. Given the large number of parameters, we apply the Least Absolute Shrinkage and Selection Operator (LASSO) [10] method to eliminate the less relevant parameters of the model. In order to do that we will add a penalty term to the χ^2 of the form: $P = \frac{\lambda}{10} \sum_i^n |p_i|$, where p_i are the parameters of the model. As the penalty coefficient λ increases, the LASSO method will set the less important parameters to zero in order to minimize the full penalized $\chi_F^2 = \chi^2 + P$.

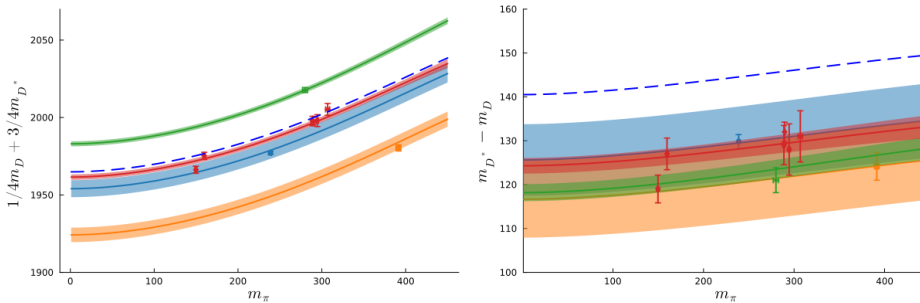
We use automatic differentiation [11] to evaluate the errors propagation with the Julia ADerrors package. We can see in Fig. 1 the final result of D and D^* meson masses fit to the lattice data.

3. – Quark mass dependence of D_{s0}^* (2317) and D_{s1} (2460)

In order to extrapolate the D_{s0} (2317) pole we need first to fit the indeterminate parameters of the $DK \rightarrow DK$ amplitude. The scattering amplitude in the infinite volume is given by the Bethe-Salpeter equation, that is,

$$(3) \quad T = \frac{V}{1 - VG},$$

where V is the scattering potential matrix and G the loop matrix. To compute the V potential for $DK \rightarrow DK$ we use the Hidden Gauge Lagrangian which corresponds to the hidden gauge formalism [12, 13], and also the lowest order of HM χ PT. For the $DK \rightarrow DK$


 Fig. 1. – Results of the D and D^* meson masses for the global analysis.

scattering we only need the lagrangian term of pseudoscalar-pseudoscalar-vector that is, $\mathcal{L}^{PPV} = ig' \text{Tr} [[\partial_i \Phi, \Phi] \mathcal{V}^i]$ where $g' = m_\rho / (2f_\pi)$, Φ is the $SU(4)$ pseudoscalar meson matrix given in [13] and \mathcal{V}^i the vector meson matrix. With this, we have the following amplitude just before projecting to partial waves

$$(4) \quad V_{DK} = -\frac{s-u}{2f^2}.$$

Note that one arrives also to this interaction through the LO HM χ PT [14, 15]. A term associated with the interaction of DK with a bare $c\bar{s}$ states of the $J_l^P = \frac{1}{2}^+$ HQSS doublet can be added. At LO in the heavy quark expansion this gives [15],

$$(5) \quad V_{\text{ex}} = \frac{V_{c\bar{s}}^2}{s - m_{c\bar{s}}^2}, \quad \text{with} \quad V_{c\bar{s}}(s) = -\frac{c}{f} \sqrt{M_D m_{c\bar{s}}} \frac{s + m_K^2 - M_D^2}{\sqrt{s}},$$

where $m_{c\bar{s}}$ is the mass of the bare $c\bar{s}$ component, and c is a dimensionless constant that provides the strength of the coupling of this component to the DK channel. In this work we will consider this coupling as a free parameter, together with $m_{c\bar{s}}$. Note that $m_{c\bar{s}}$ can vary for every LQCD simulation with a different setup. The potential $V(s)$ consistent with HQSS is then given by $V(s) = V_{DK}(s) + V_{\text{ex}}(s)$. To project to partial waves we use

$$(6) \quad V_l(s) = \frac{1}{2} \int_{-1}^1 M(s, \theta) \mathcal{P}_l(\cos(\theta)) d\cos(\theta),$$

where $\mathcal{P}_l(x)$ are the Legendre polynomials. However, as we are going to fit to lattice data we need to discretize the momenta $q = (2\pi/L)\vec{n}$ with $\vec{n} \in Z^3$. In the finite volume the scattering amplitude is given by $\tilde{T} = V/(1 - V\tilde{G})$. The final \tilde{G} that we insert to the Bethe-Salpeter equation is constructed with the elements

$$(7) \quad \tilde{G}_{fin}^{(i)co} = G_{inf}^{(i)DR} + \lim_{q_{co} \rightarrow \infty} (G_{fin}^{(i)} - G_{inf}^{(i)co}).$$

where superscripts DR and co means that we have used the dimensional and the cutoff regularization method and the inf , fin subscripts means infinite or finite volume respectively. With this, the value of the cutoff is cancelled between the last two terms of Eq. (7). Fixing the dimensional regularization scale $\mu = 1000$ MeV we only have the

subtraction constant a as a free parameter. In order to extrapolate to the physical point we assume that the subtraction constant can be written as a first order Taylor expansion of the squared pion mass as $a = a_1 + a_2 m_\pi^2$. We determine the two fitting parameters a_1 , a_2 . We perform a global fit analyzing the energy levels of [3] for DK scattering in $I = 0$ and the DK and D^*K scattering levels from [9, 7, 6]. As fitting parameters, we have, c , $m_{c\bar{s}}$, a_1 and a_2 for the subtraction constants. We consider two situations, one where the bare $c\bar{s}$ component is not included, i.e., $V_{\text{ex}} = 0$, and other with $V_{\text{ex}} \neq 0$.

In Fig. 2 we plot the binding energies and mass splitting of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ states as a function of the pion mass in both fits. The binding energies of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are compatible with the experimental data for Fit II, while for the Fit I we predict the $D_{s1}(2460)$ state with less than 20 MeV from the physical value. They also increase with the pion mass for pion masses above 200 MeV, showing a quadratic behaviour. To plot Fig. 2 we have used an extrapolation of the bare mass to the physical point.

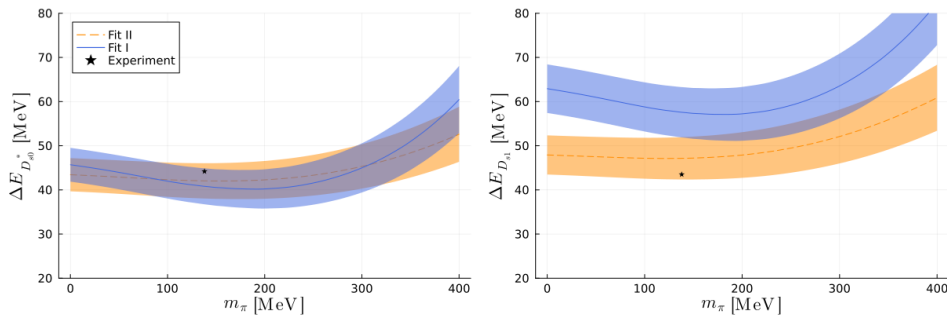


Fig. 2. – Binding energies of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ for physical charm and strange quark masses.

Finally, in Fig. 3 we also show the pion mass dependence of the compositeness, the coupling constant and the scattering length. The compositeness is lower for the Prelovsek data set. The scattering length is in reasonably good agreement with previous works [8, 14]. However, the errors extracted here are smaller. The scattering length obtained here is lower than in [14].

REFERENCES

- [1] Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner, Qian Wang, Qiang Zhao, and Bing-Song Zou. Hadronic molecules. *Rev. Mod. Phys.*, 90(1):015004, 2018. [Erratum: *Rev.Mod.Phys.* 94, 029901 (2022)].
- [2] Elizabeth Ellen Jenkins. Heavy meson masses in chiral perturbation theory with heavy quark symmetry. *Nucl. Phys. B*, 412:181–200, 1994.
- [3] Gavin K. C. Cheung, Christopher E. Thomas, David J. Wilson, Graham Moir, Michael Peardon, and Sinéad M. Ryan. DK $I = 0, D\bar{K}1 = 0, 1$ scattering and the $D_{s0}^*(2317)$ from lattice QCD. *JHEP*, 02:100, 2021.
- [4] F. Gil-Domínguez and R. Molina. Quark mass dependence of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ resonances. 6 2023.
- [5] Fernando Gil-Domínguez and Raquel Molina. Quark mass dependence of the low-lying charmed mesons at one loop in $\text{HH}\chi\text{PT}$. *Phys. Lett. B*, 843:137997, 2023.

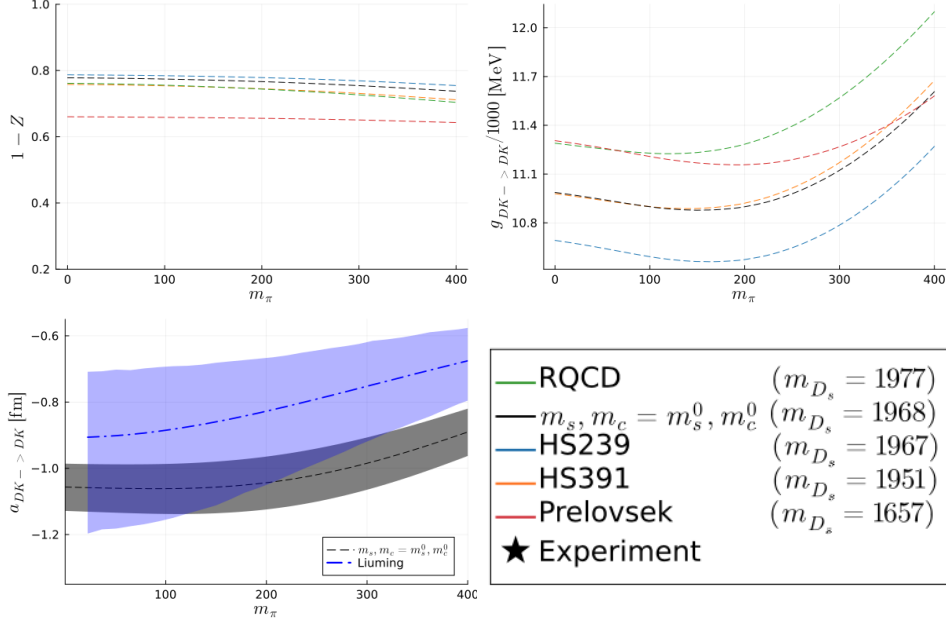


Fig. 3. – Fit II: Result of the global fit for the scattering length as a function of the pion mass in comparison with the result obtained in [14] (dashed line with grey band).

- [6] C. B. Lang, Luka Leskovec, Daniel Mohler, Sasa Prelovsek, and R. M. Woloshyn. Ds mesons with DK and D*K scattering near threshold. *Phys. Rev. D*, 90(3):034510, 2014.
- [7] Daniel Mohler, C. B. Lang, Luka Leskovec, Sasa Prelovsek, and R. M. Woloshyn. D_{s0}^* (2317) Meson and D-Meson-Kaon Scattering from Lattice QCD. *Phys. Rev. Lett.*, 111(22):222001, 2013.
- [8] A. Martínez Torres, E. Oset, S. Prelovsek, and A. Ramos. Reanalysis of lattice QCD spectra leading to the D_{s0}^* (2317) and D_{s1} (2460). *JHEP*, 05:153, 2015.
- [9] Gunnar S. Bali, Sara Collins, Antonio Cox, and Andreas Schäfer. Masses and decay constants of the D_{s0}^* (2317) and D_{s1} (2460) from $N_f = 2$ lattice QCD close to the physical point. *Phys. Rev. D*, 96(7):074501, 2017.
- [10] Baptiste Guegan, John Hardin, Justin Stevens, and Mike Williams. Model selection for amplitude analysis. *JINST*, 10(09):P09002, 2015.
- [11] Alberto Ramos. Automatic differentiation for error analysis of Monte Carlo data. *Comput. Phys. Commun.*, 238:19–35, 2019.
- [12] R. Molina, T. Branz, and E. Oset. A new interpretation for the D_{s2}^* (2573) and the prediction of novel exotic charmed mesons. *Phys. Rev. D*, 82:014010, 2010.
- [13] D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas. Dynamically generated open and hidden charm meson systems. *Phys. Rev. D*, 76:074016, 2007.
- [14] Liuming Liu, Kostas Orginos, Feng-Kun Guo, Christoph Hanhart, and Ulf-G. Meissner. Interactions of charmed mesons with light pseudoscalar mesons from lattice QCD and implications on the nature of the D_{s0}^* (2317). *Phys. Rev. D*, 87(1):014508, 2013.
- [15] Miguel Albaladejo, Pedro Fernandez-Soler, Juan Nieves, and Pablo G. Ortega. Contribution of constituent quark model $c\bar{s}$ states to the dynamics of the D_{s0}^* (2317) and D_{s1} (2460) resonances. *Eur. Phys. J. C*, 78(9):722, 2018.